Post-TOV modelling of relativistic stars and astrophysical applications

$$G_{\mu\nu} = 8\pi \, T_{\mu\nu} \; ?$$

(based on PRD 92, 2015 & 94, 2016)

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Outline

- Neutron stars as gravity probes.
- Gravity-matter degeneracy in neutron stars.
- The post-TOV formalism (interior and exterior).
- Astrophysical applications: redshift, X-ray bursts and QPOs.
- Further/future extensions of the formalism.

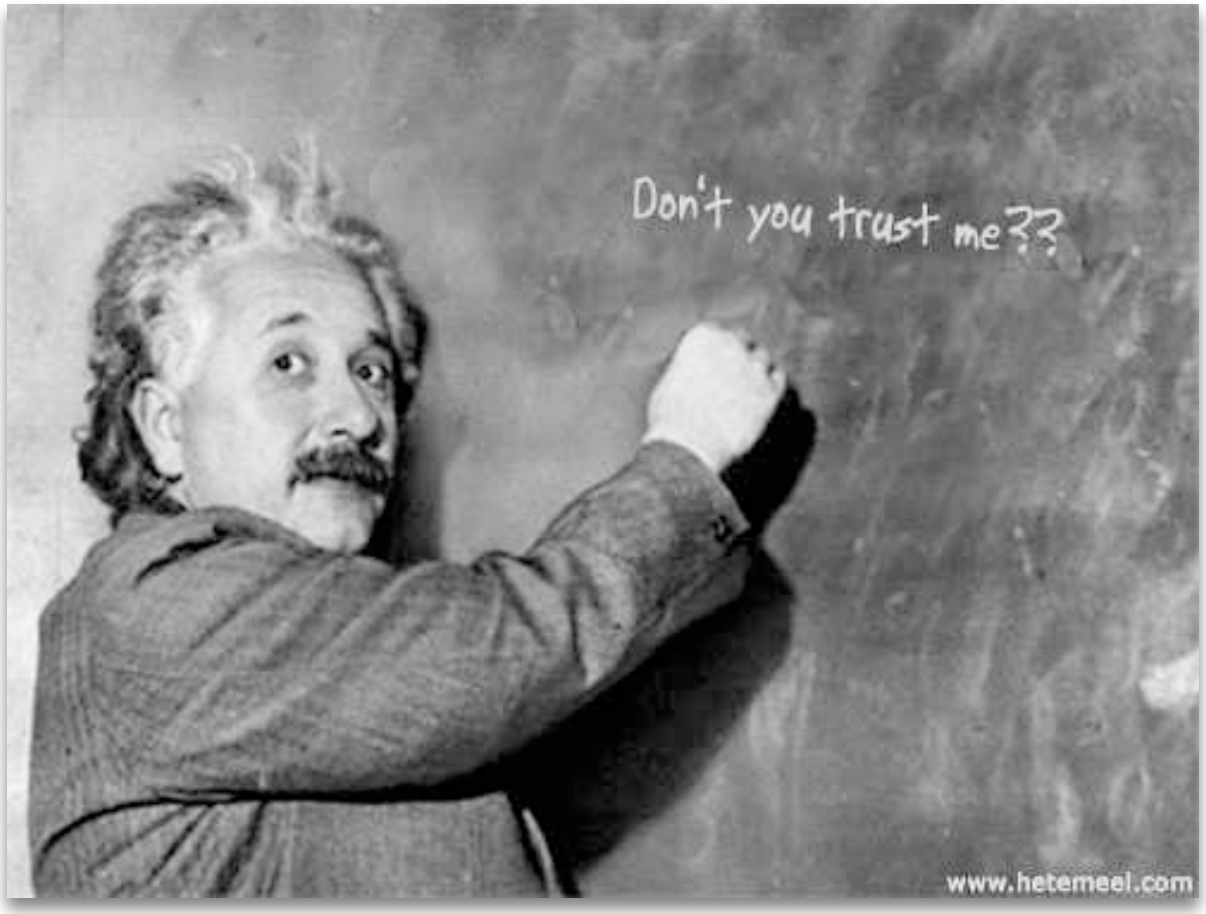
GR-exit?

- General Relativity is arguably the most elegant theory invented so far and is, most likely, *The* theory of (classical) gravity.

- Good science: test all theories, no matter how elegant!

- Build phenomenology
—> how "special" is GR?

- A host of modified theories of gravity on the market.



The zoo of gravity theories

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [30]	[31–33]
Multiscalar	S	✗	✓	✓	✓	✓	✓?	[34]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [35, 36]	[37]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[38]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [39]	[40]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[41–44]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[43–46]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([47])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[16]
Galileon	S	✗	✓	✓	✓	✓	✓?	[16, 48]
Nondynamical fields								
Palatini $f(R)$	–	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	–	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVeS	SVT	✗	✓	✓	✓	✓	?	[33]
$f(R)\mathcal{L}_m$	?	?	✓	✓	✓	✗	?	
$f(T)$	?	✗	✓	✗	✓	✓	?	[49]

[Berti et al. 2015]

Testing strong gravity

Testing GR

Black Holes

Probably the "cleanest" probes of gravity, although the Kerr metric (and its geodesics) is not unique to GR.

Neutron stars

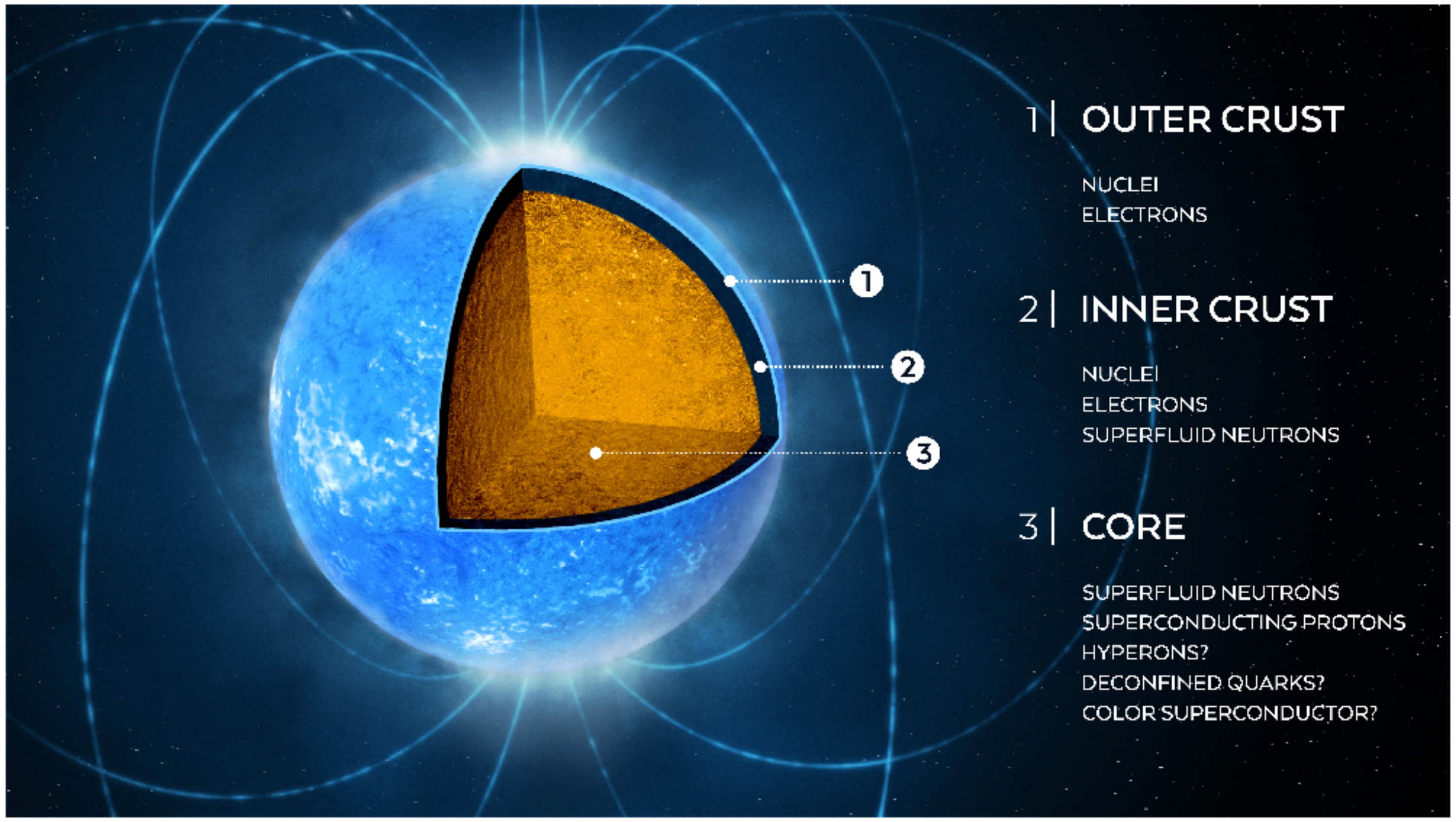
Many observational "handles" but intrinsically plagued by a matter-gravity degeneracy.

Cosmology

Need to go beyond the scale factor $\alpha(t)$, typically this can be "reverse-engineered" by the extra degrees of freedom. The vacuum energy puzzle persists in other theories.

Neutron stars on a napkin

- Relativistic objects $R/M \sim 5$.
- Supra-nuclear density matter.
- Lots of exotic physics (superfluidity, superconductivity, deconfined quarks ...).
- Fast rotation, strong B-fields.
- Future experiments (SKA, NICER ...) should provide accurate NS information (masses, radii).

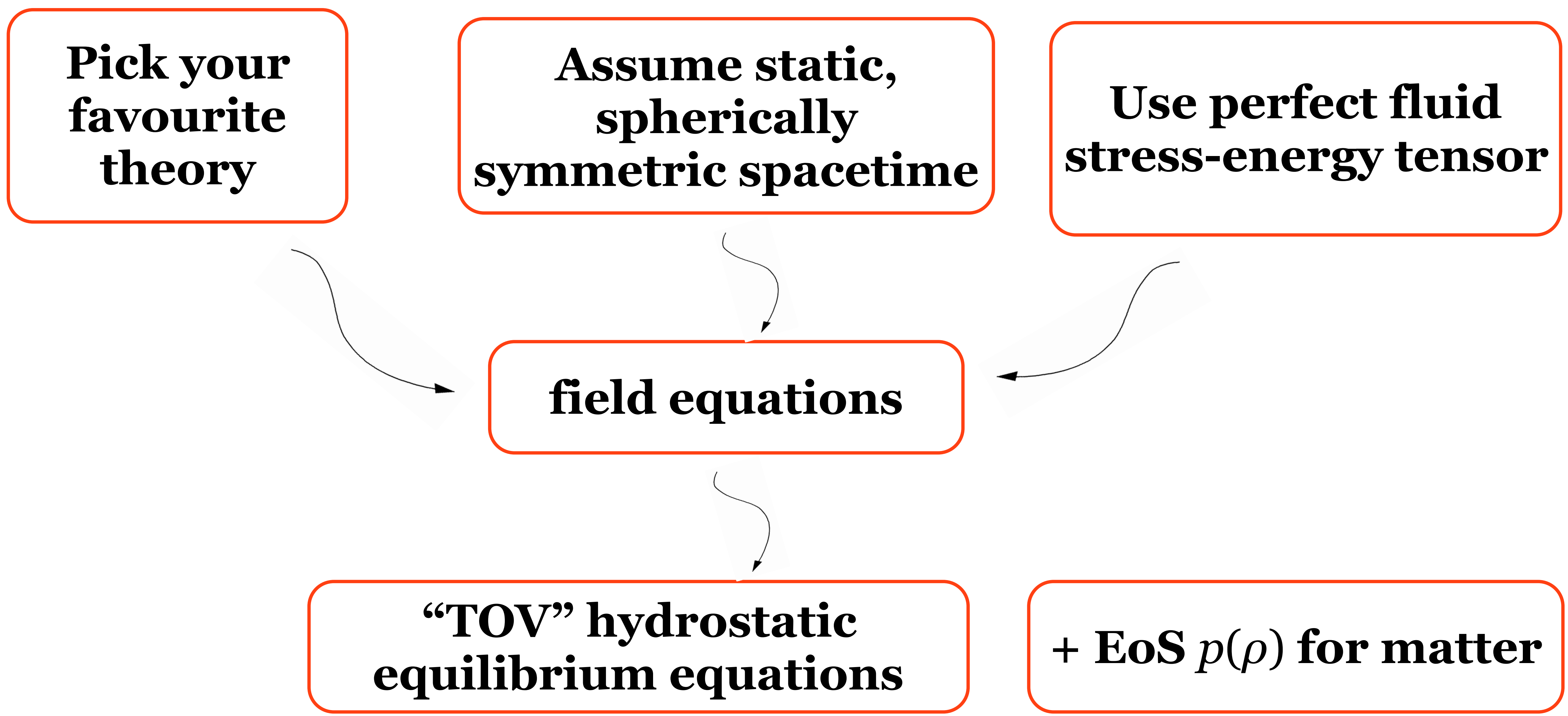


$$M \sim M_{\rm P} \left(\frac{M_{\rm P}}{m_n}\right)^2 \approx 1.5\, M_\odot$$

$$R \sim \lambda_n \frac{M_{\rm P}}{m_n} \approx 10^6\, {\rm cm}$$

$$f_{\rm spin} \lesssim 1\, {\rm kHz}$$

Recipe for building neutron stars



Recipe for building neutron stars

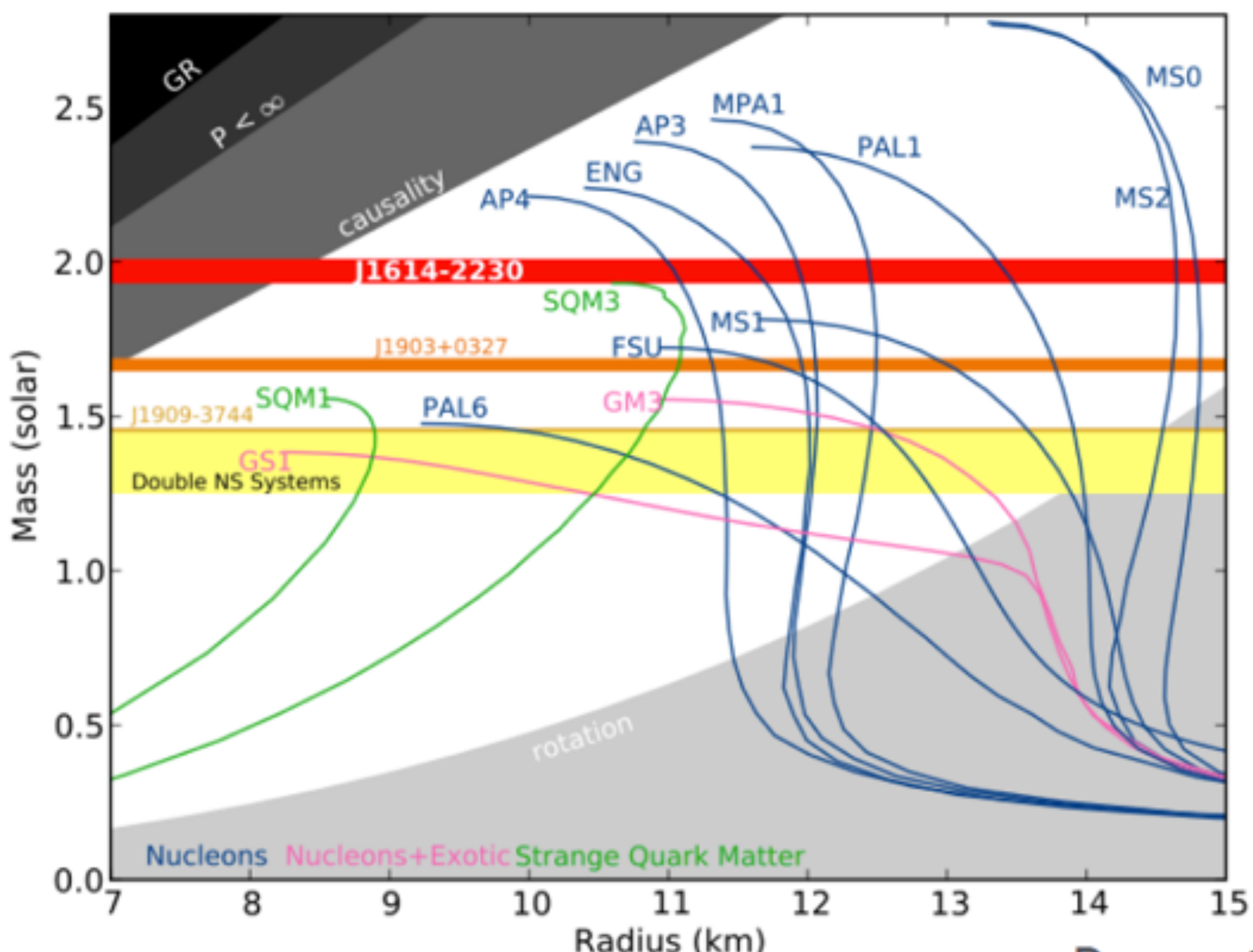
Done many times!

[Berti et al. 2015]

Theory	Structure NR	Structure SR	Structure FR	Collapse	Sensitivities	Stability	Geodesics
Extra scalar field							
Scalar-Tensor	[109–114]	[112, 115, 116]	[117–119]	[120–127]	[128]	[129–139]	[118, 140]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[141–153]	[154]	[155]	[156, 157]	?	[158, 159]	?
Quadratic gravity							
Gauss-Bonnet	[160]	[160]	[77]	?	?	?	?
Chern-Simons	$\equiv$ GR	[25, 40, 161–163]	?	?	[162]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[164, 165]	?	?	[166]	[43, 44]	[158]	?
Khronometric/ Hořava-Lifshitz	[167]	?	?	?	[43, 44]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[168, 169]	?	?	?	?	?	?
Galileon	[170]	[170]	?	[171, 172]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[173–177]	?	?	?	–	?	?
Eddington-Born-Infeld	[178–184]	[178, 179]	?	[179]	–	[185, 186]	?

NR = non-rotating, SR = slow rotation, FR = fast rotation

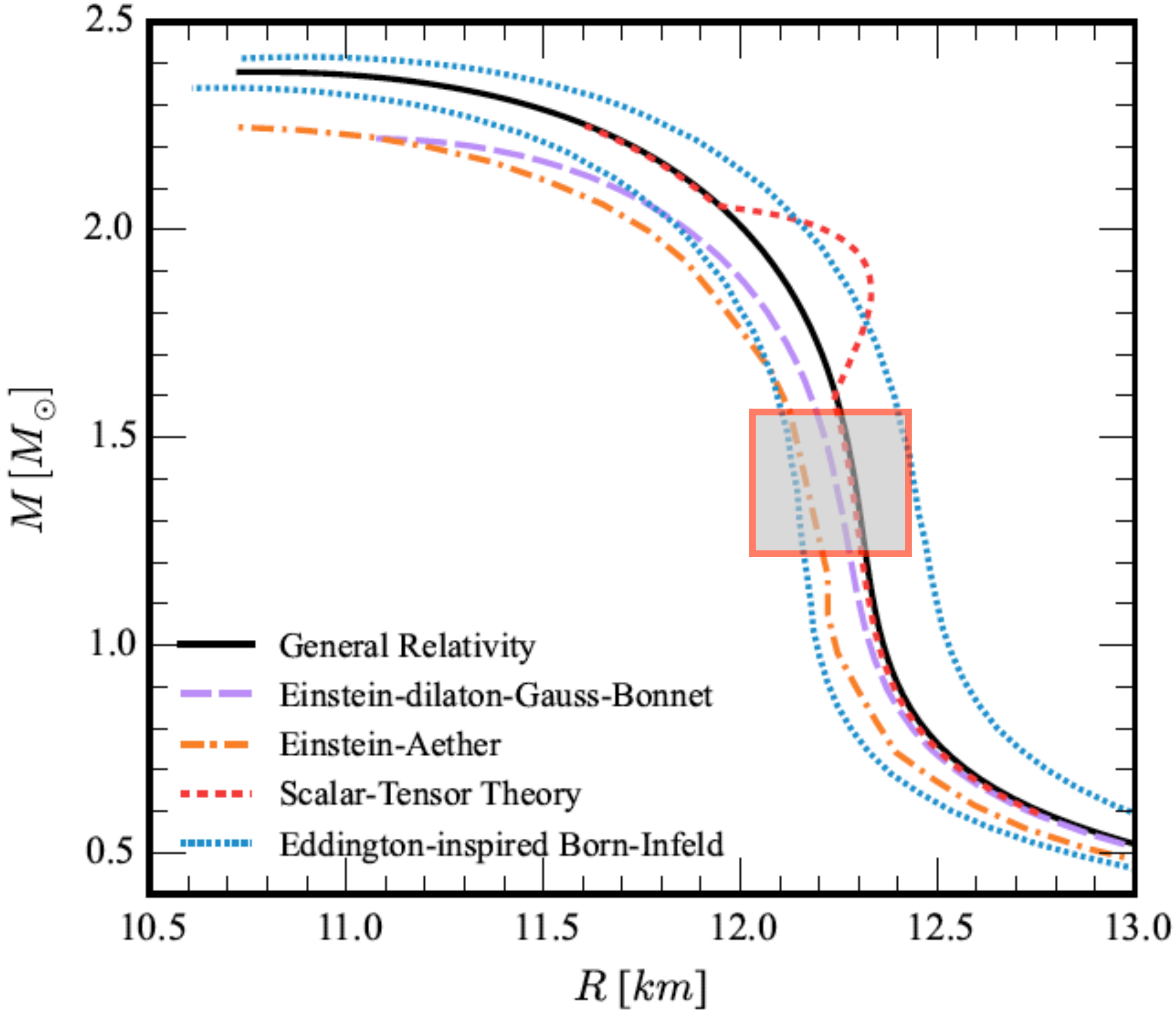
“*Matter*-gravity” degeneracy



Demorest+, 1010.5788

Fixed gravity theory (GR), varying EoS

“Matter-*gravity*” degeneracy



Fixed EoS (APR), varying gravity

(different theories can cause similar deviations from GR)

A different approach

- *Do not commit to any particular "favourite" theory of gravity.*
- Parametrize deviations from GR in a PPN theory-manner.
- At the same time allow for strong gravity, by going beyond "simple" PN expansions.
- Examples from the literature:

  "Bumpy" or "quasi-Kerr" BH metrics,

  "post-Friedmannian" cosmology,

  "post-Einsteinian" GW waveforms.

Strategy for a "post-TOV" formalism

- This is a formalism for *relativistic stars in spherical symmetry* (i.e. no rotation), *not* a fully-fledged theory of gravity.

- **Main idea:**
  augment the General Relativistic TOV stellar structure equations by adding 1PN and 2PN corrections with *arbitrary coefficients*.
  These terms are to be built out of the available parameters:

$$p, \ \ \rho, \ \ \Pi, \ \ m, \ \ r \qquad\qquad \text{matter energy density:} \ \ \epsilon = \rho(1+\Pi)$$

- The hydrostatic equations for the pressure $p(r)$ and mass function $m(r)$ take the symbolic form:

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\rm GR} + \{\text{ PN corrections }\}, \quad \frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\rm GR} + \{\text{ PN corrections }\}$$

Looking for post-TOV corrections

- **1PN order**: these can be extracted from the existing PPN theory:

$$\Lambda_1 \sim \Pi,\ \frac{m}{r},\ \frac{r^3 p}{m}$$

- **2PN** : use dimensional analysis (excluding the presence of dimensional coupling constants):

General 2PN term (dimensionless): $\Lambda_2 \sim \Pi^\theta (r^2 p)^\alpha (r^2 \rho)^\beta \left(\frac{m}{r}\right)^{2-2\alpha-\beta-\theta}$

- Limits on { α, β, θ }: (i) avoid divergence at the stellar center/surface and (ii) assume field equations *linear* in the stress-energy tensor:

$$\{\,\text{geometry}\,\} \sim 8\pi T^{\mu\nu}$$

$$\{\,\text{geometry}\,\} \sim (\epsilon + \tau p)^n$$

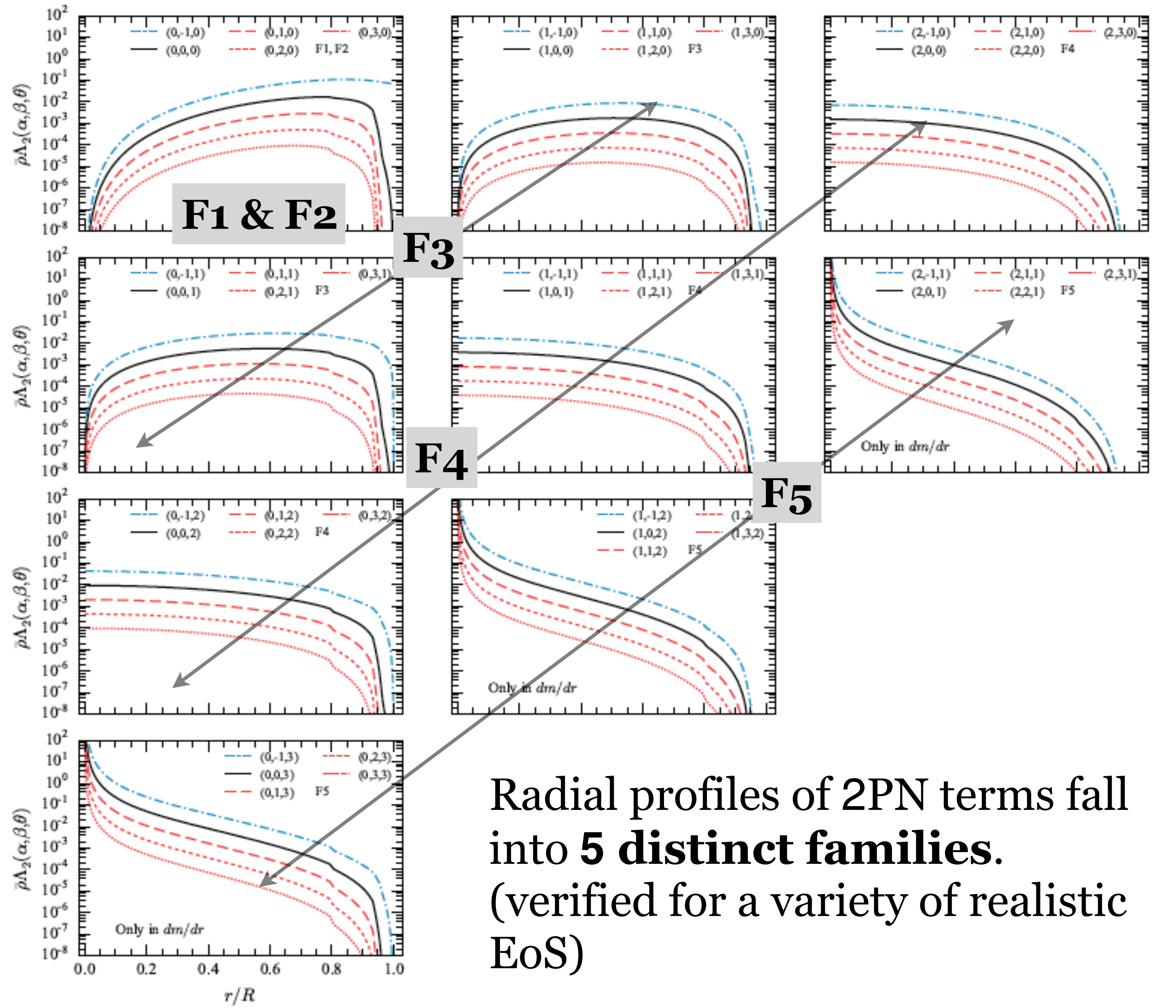
$$\tau, n = \mathcal{O}(1)$$

$$\begin{cases} \beta \geq -1 \\ 0 \leq \theta \leq 2 \text{ or } 3 \\ 0 \leq \alpha \leq 2-\theta \text{ or } 3-\theta \end{cases}$$

In principle, there are infinite 2PN terms ...

It's all about Families!

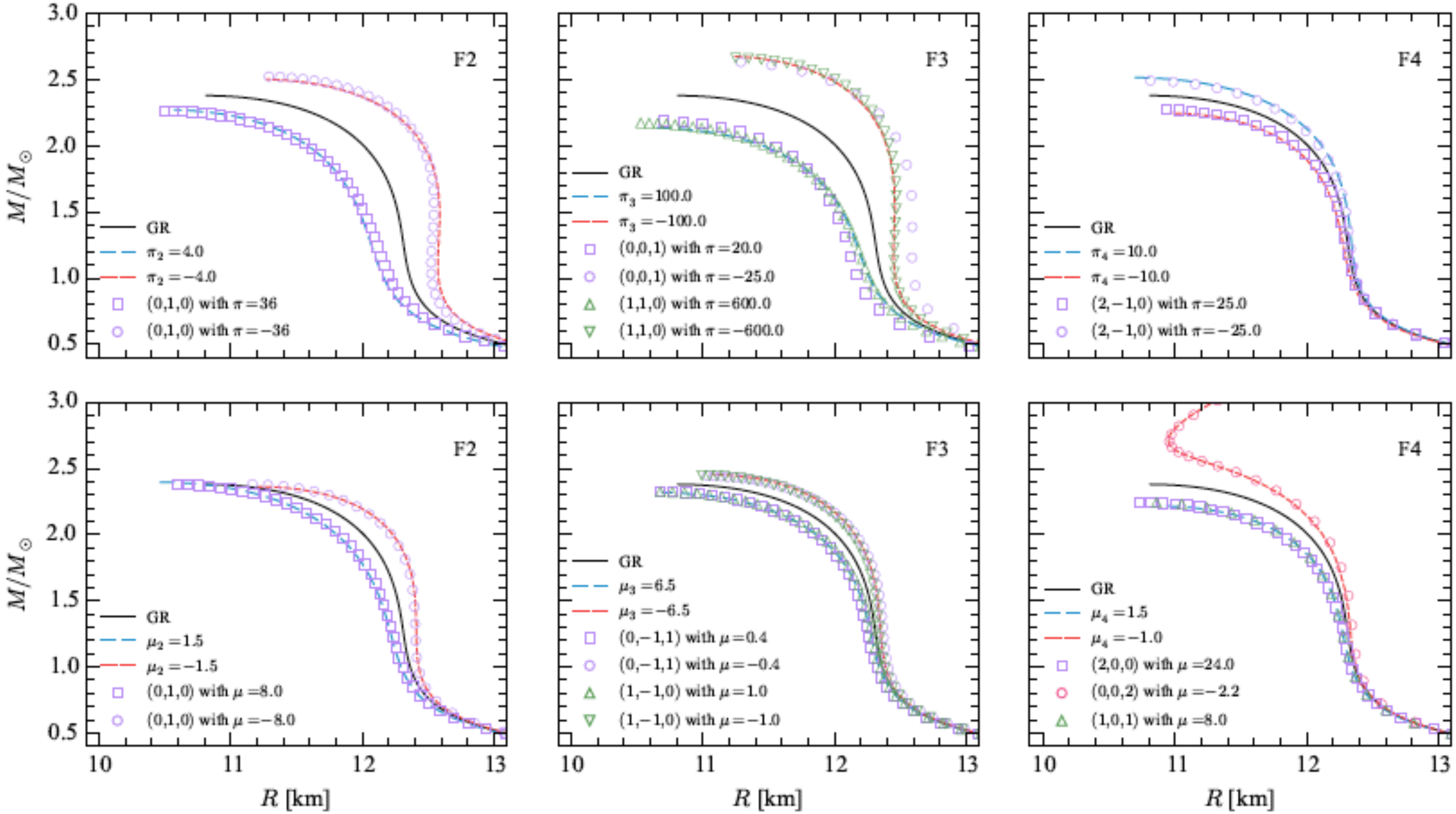
Family	2PN term	(α, β, θ)
F1	$m^3/(r^5\rho)$	$(0, -1, 0)$
F2	$(m/r)^2$	$(0, 0, 0)$
F2	$rm\rho$	$(0, 1, 0)$
F3	$mp/(r\rho)$	$(1, -1, 0)$
F3	$r^2 p$	$(1, 0, 0)$
F3	$\Pi m^2/(r^4\rho)$	$(0, -1, 1)$
F3	$\Pi m/r$	$(0, 0, 1)$
F3	$r^2\Pi\rho$	$(0, 1, 1)$
F4	$r^3p^2/(\rho m)$	$(2, -1, 0)$
F4	$r^6p^2/(m^2)$	$(2, 0, 0)$
F4	$\Pi p/\rho$	$(1, -1, 1)$
F4	$\Pi r^3 p/m$	$(1, 0, 1)$
F4	$\Pi^2 m/(r^3\rho)$	$(0, -1, 2)$
F4	Π^2	$(0, 0, 2)$
F5	$\Pi r^4p^2/(\rho m^2)$	$(2, -1, 1)$
F5	$\Pi r^7p^2/m^3$	$(2, 0, 1)$
F5	$\Pi^2 rp/m\rho$	$(1, -1, 2)$
F5	$\Pi^2 r^4 p/m^2$	$(1, 0, 2)$
F5	$\Pi^3/(r^2\rho)$	$(0, -1, 3)$
F5	$\Pi^3 r/m$	$(0, 0, 3)$



Radial profiles of 2PN terms fall into **5 distinct families**. (verified for a variety of realistic EoS)

Families: *M*-*R* self-similarity

Remarkably, the 2PN terms of the same family lead to self-similar M-R curves when added as post-TOV corrections (results shown assume APR but have been verified for other EoS too).



Post-TOV structure equations (I)

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\rm GR} - \frac{\rho m}{r^2}\,(\mathcal{P}_1 + \mathcal{P}_2)$$

$$\frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\rm GR} + 4\pi r^2 \rho\,(\mathcal{M}_1 + \mathcal{M}_2)$$

- 1PN-order corrections:

$$\mathcal{P}_1 = \delta_1 \frac{m}{r} + 4\pi \delta_2 \frac{r^3 p}{m}$$

$$\mathcal{M}_1 = \delta_3 \frac{m}{r} + \delta_4 \Pi$$

- Current PPN limits: $|\delta_i| \ll 1 \quad \rightarrow \quad |\mathcal{P}_1|,\ |\mathcal{M}_1| \ll 1$

1PN terms can be ignored

Post-TOV structure equations (II)

$$\frac{dp}{dr} \approx \left(\frac{dp}{dr}\right)_{\rm GR} - \frac{\rho m}{r^2} \mathcal{P}_2 \qquad \frac{dm}{dr} \approx \left(\frac{dm}{dr}\right)_{\rm GR} + 4\pi r^2 \rho \mathcal{M}_2$$

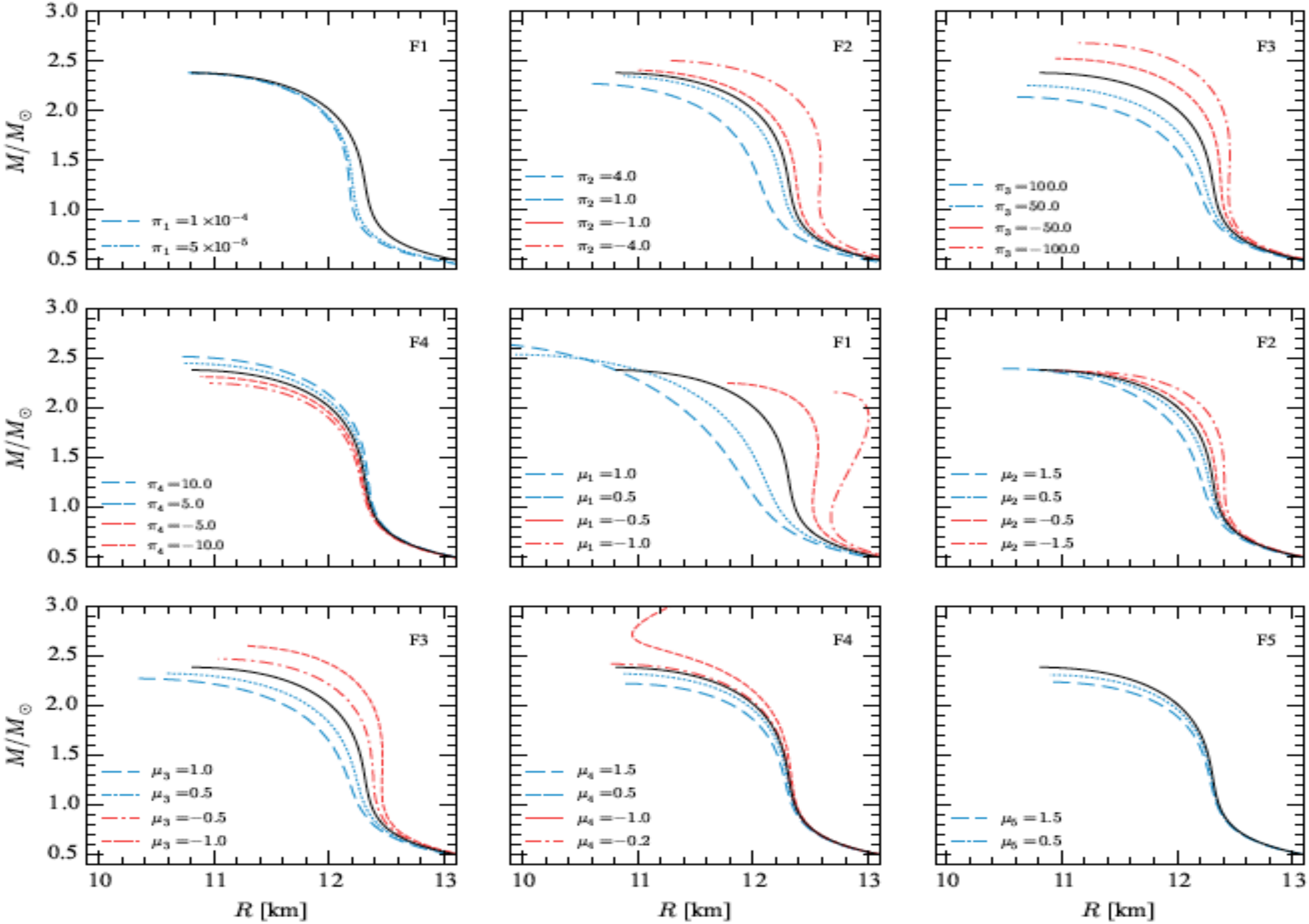
- 2PN-order corrections:
  - use *just one representative term per family*
  - self-similarity: all other terms are accounted for by varying the corresponding coefficient

$\{\pi_i, \mu_i\} \leftrightarrow F_i$ family

$$\mathcal{P}_2 = \pi_1 \frac{m^3}{r^5 \rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 r^2 p + \pi_4 \frac{\Pi p}{\rho}$$

$$\mathcal{M}_2 = \mu_1 \frac{m^3}{r^5 \rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \Pi^3 \frac{r}{m}$$

post-TOV: sample of *M*-*R* curves



EoS: APR

Post-TOV as "effective GR"

- The post-TOV equations can be mapped onto an *effective* GR formulation:

$$\nabla_\nu T^{\mu\nu}_{\rm eff} = 0, \qquad T^{\mu\nu}_{\rm eff} = (\epsilon_{\rm eff} + p) u^\mu u^\nu + p g^{\mu\nu}$$

$$\frac{dp}{dr} = -\frac{1}{2}\left(\epsilon_{\rm eff} + p\right)\frac{d\nu}{dr} \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon_{\rm eff}$$

- *Gravity-shifted effective* EoS: $p = p(\epsilon_{\rm eff}), \quad \epsilon_{\rm eff} = \epsilon + \rho \mathcal{M}_2$

- Effective *interior* metric:

$$g_{\mu\nu} = {\rm diag}[\,-e^{\nu(r)},\ (1 - 2m(r)/r)^{-1}, r^2, r^2 \sin^2\theta\,]$$

Exterior metric (I)

- Our scheme also allows the construction of an exterior metric.
- Set all fluid parameters to zero: $p = \epsilon = \rho = \Pi = 0$
- The resulting post-TOV equations are: $\begin{cases} \dfrac{d\nu}{dr} = \left(\dfrac{d\nu}{dr}\right)_{\mathrm{GR}} + 2(\pi_2 - \mu_2)\dfrac{m^3}{r^4} \\ \dfrac{dm}{dr} = 4\pi\mu_1 \dfrac{m^3}{r^3} \end{cases}$
- Integrating and keeping the leading post-TOV terms:

$$m(r) \approx M_\infty \left(1 - 2\pi\mu_1 \frac{M_\infty^2}{r^2}\right) \qquad \Rightarrow \qquad M \approx \underbrace{M_\infty}_{\text{ADM mass}} \left(1 - 2\pi\mu_1 \frac{M_\infty^2}{R^2}\right)$$

$$\nu(r) \approx \log\left(1 - \frac{2M_\infty}{r}\right) - \frac{2\chi}{3}\frac{M_\infty^3}{r^3}$$

Schwarzschild mass $M \equiv m(R)$

Exterior metric (II)

- We assume the *same* effective metric form as in the interior:

$$g_{\mu\nu} = \mathrm{diag}[\, -e^{\nu(r)},\ (1-2m(r)/r)^{-1}, r^2, r^2 \sin^2\theta \,]$$

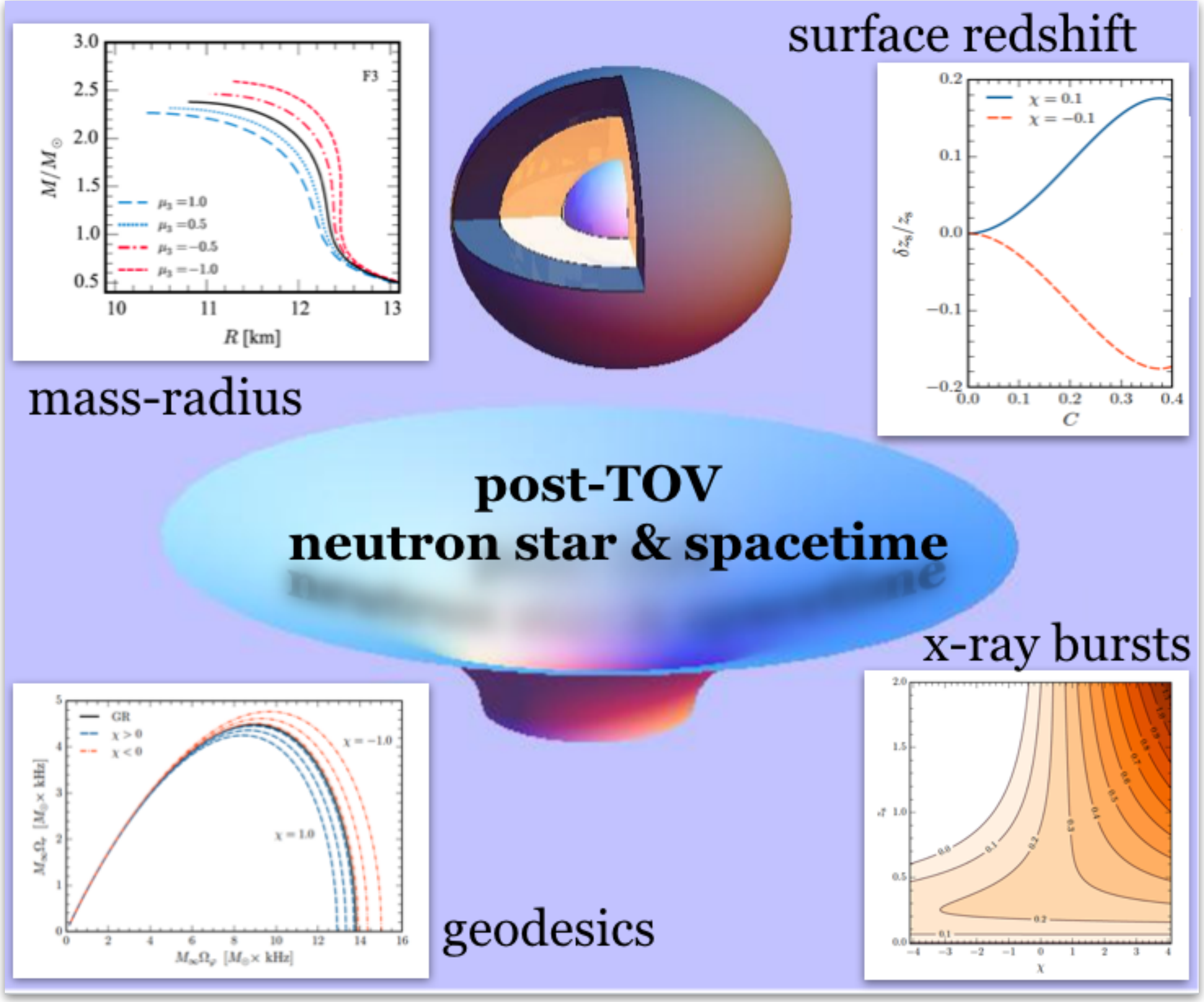
- The exterior metric takes a *post-Schwarzschild* form:

$$g_{tt}(r) \approx -\left(1 - \frac{2M_\infty}{r}\right) + \frac{2\chi}{3}\frac{M_\infty^3}{r^3}$$

$$g_{rr}(r) \approx \left(1 - \frac{2M_\infty}{r}\right)^{-1} - 4\pi\mu_1 \frac{M_\infty^3}{r^3}$$

where $\chi = \pi_2 - \mu_2 - 2\pi\mu_1$

post-TOV: astrophysics



Surface redshift (I)

- The first "observable" we can construct is surface redshift (of absorption & emission lines). This is defined in the usual way:

$$z_s = \frac{f_s}{f_\infty} - 1$$

compactness $C = M_\infty / R$

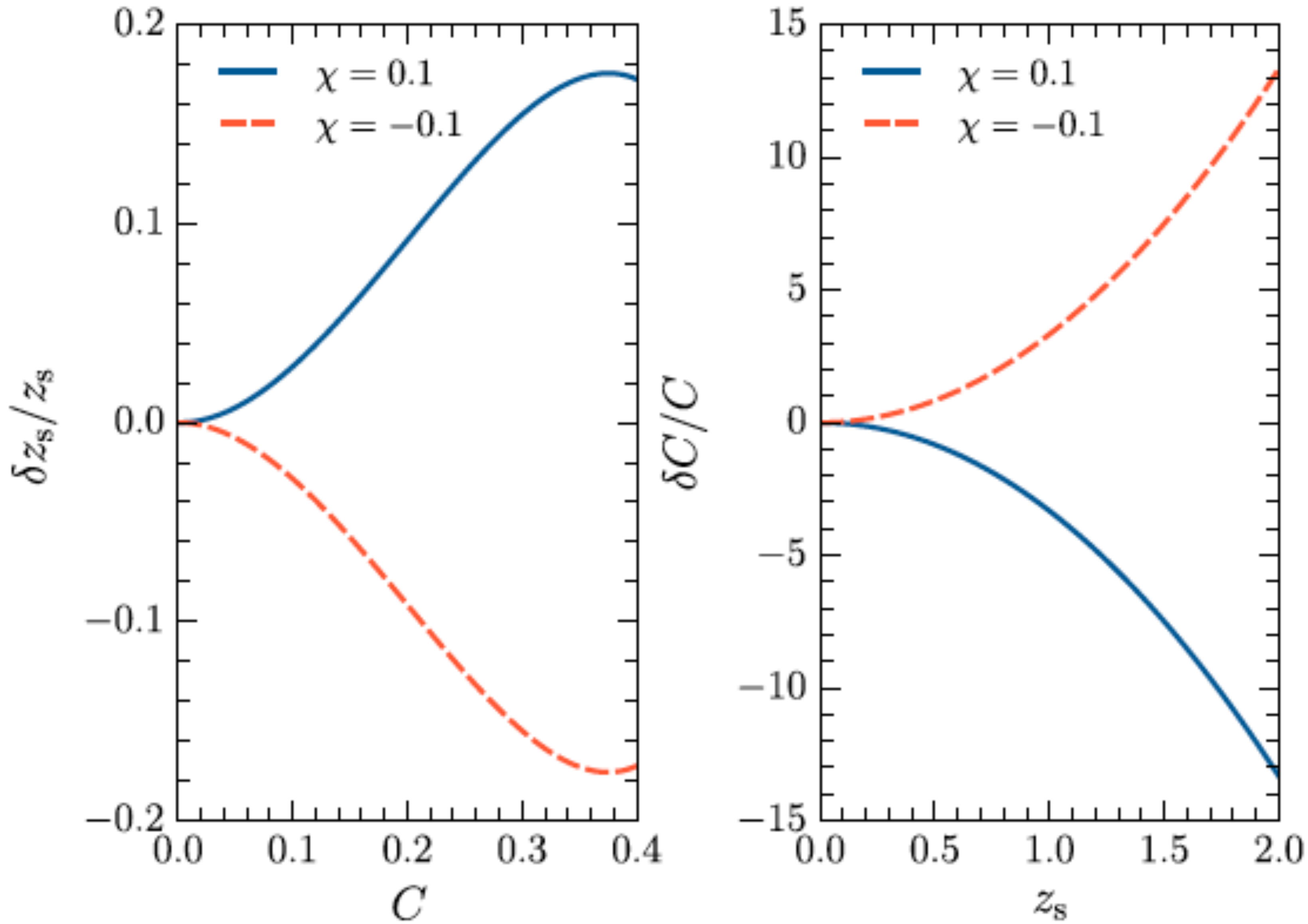
- For any static spacetime:

$$\frac{f_\infty}{f_s} = \left[\frac{g_{tt}(R)}{g_{tt}(\infty)} \right]^{1/2} \Rightarrow \begin{cases} z_s(C) = (1-2C)^{-1/2} - 1 + \dfrac{\chi}{3} C^3 \\ \\ C(z_s) = \dfrac{1}{2}\left[1 - (1+z_s)^{-2}\right]\left(1 - \dfrac{\chi}{3} z_s^2\right) \end{cases}$$

GR part

Surface redshift (II)

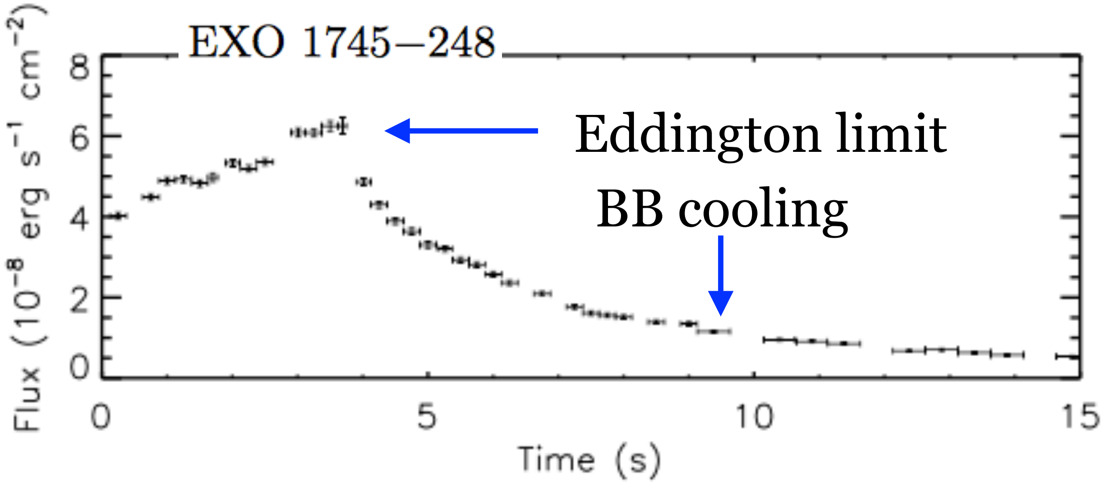
- The redshift depends *only* on $\chi = \pi_2 - \mu_2 - 2\pi\mu_1 \;\Rightarrow$ degeneracy!



Thermonuclear bursts (I)

- These are X-ray flashes produced by nuclear detonation of accreted matter on the surface of a neutron star.

- We follow the recipe of Psaltis (2008).



all relations as in GR

- The *observed* flux & apparent radius:

$$L_\infty = 4\pi D^2 F_\infty = \sigma_{\rm SB} S_{\rm app} \bar{T}_\infty^4$$

$$R_{\rm app} \equiv \left(\frac{S_{\rm app}}{4\pi}\right)^{1/2} = D\left(\frac{F_\infty}{\sigma_{\rm SB}\bar{T}_\infty^4}\right)^{1/2}$$

- Relation between the "colour" and effective BB temperature:

$$\underbrace{\bar{T}_\infty}_{\text{extracted from BB spectrum}} = \underbrace{f_c}_{\text{"colour correction"}} \sqrt{-g_{tt}(R)}\, T_{\rm eff}$$

$$L_s = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4$$

conserved number of emitted photons: $L_\infty = -g_{tt}(R) L_s$

Thermonuclear bursts (II)

- The second key observable is the Eddington flux ("touchdown luminosity"):

$$L_{\rm E}^{\infty} = 4\pi D^2 F_{\rm E}^{\infty} = \underbrace{\frac{4\pi}{\kappa}}_{\text{opacity of matter (Thomson scattering)}} \frac{R^2}{(1+z_s)^2}\, g_{\rm eff} \quad \Rightarrow \quad g_{\rm eff} = \kappa \sigma_{\rm SB} \frac{F_{\rm E}^{\infty}}{F_{\infty}} \left(\frac{\bar{T}_{\infty}}{f_c} \right)^4 (1+z_s)^4$$

- *Only the effective surface "g" takes a non-GR form:*

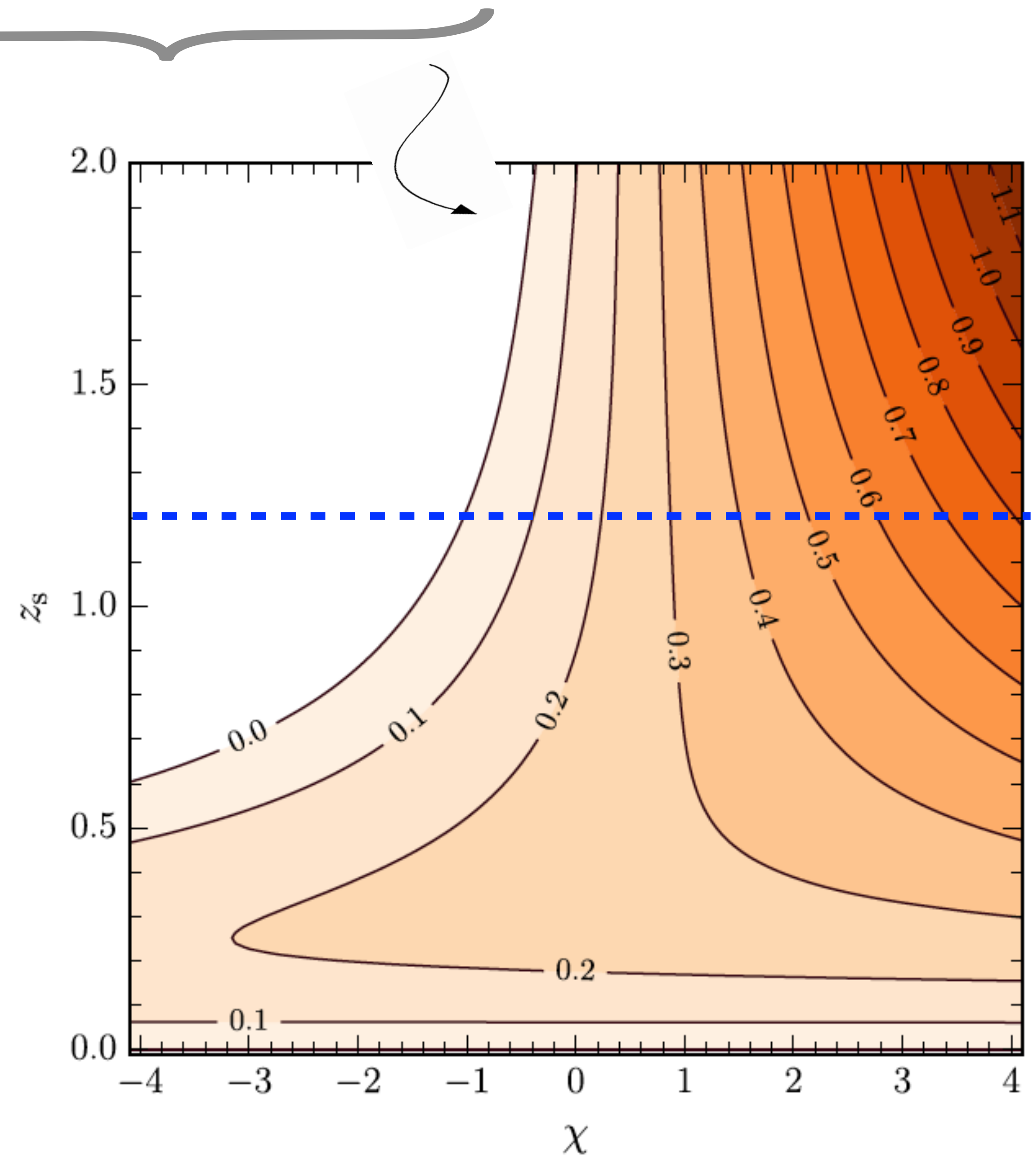
$$g_{\rm eff} = \frac{1}{2\sqrt{g_{rr}(R)}} \frac{g'_{tt}(R)}{g_{tt}(R)} \quad \Rightarrow \quad g_{\rm eff} = \frac{z_s}{2R} \frac{(2+z_s)}{(1+z_s)} \left(1 + \frac{2}{3} \chi z_s^2 \right)$$

- Combine the above equations to produce a relation:

$$f(\chi, z_s) = \text{ observables}$$

X-ray bursts (III)

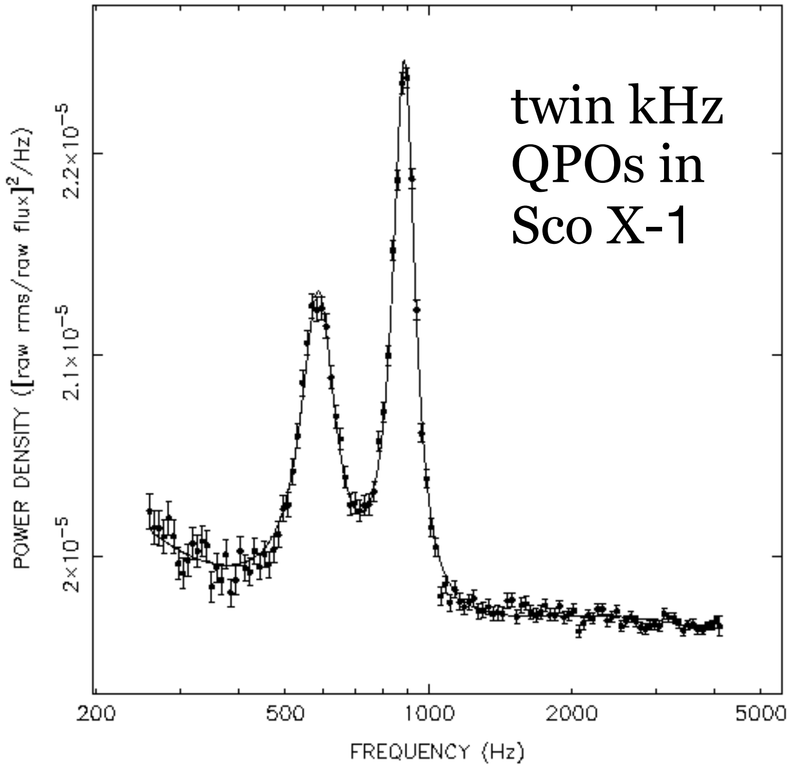
$$\frac{z_s(2+z_s)}{(1+z_s)^4}\left(1+\frac{2}{3}\chi z_s^2\right) = 2\kappa D\frac{F_{\rm E}^{\infty}}{f_c^2}\left(\frac{\sigma_{\rm SB}\bar{T}_{\infty}^4}{F_{\infty}}\right)^{1/2}$$



measure redshift to get χ!

Geodesics & QPOs (I)

- In the most popular models, QPOs from accreting systems are associated with geodesic frequencies (the reality, the situation may not be that simple!)



- Azimuthal frequency :

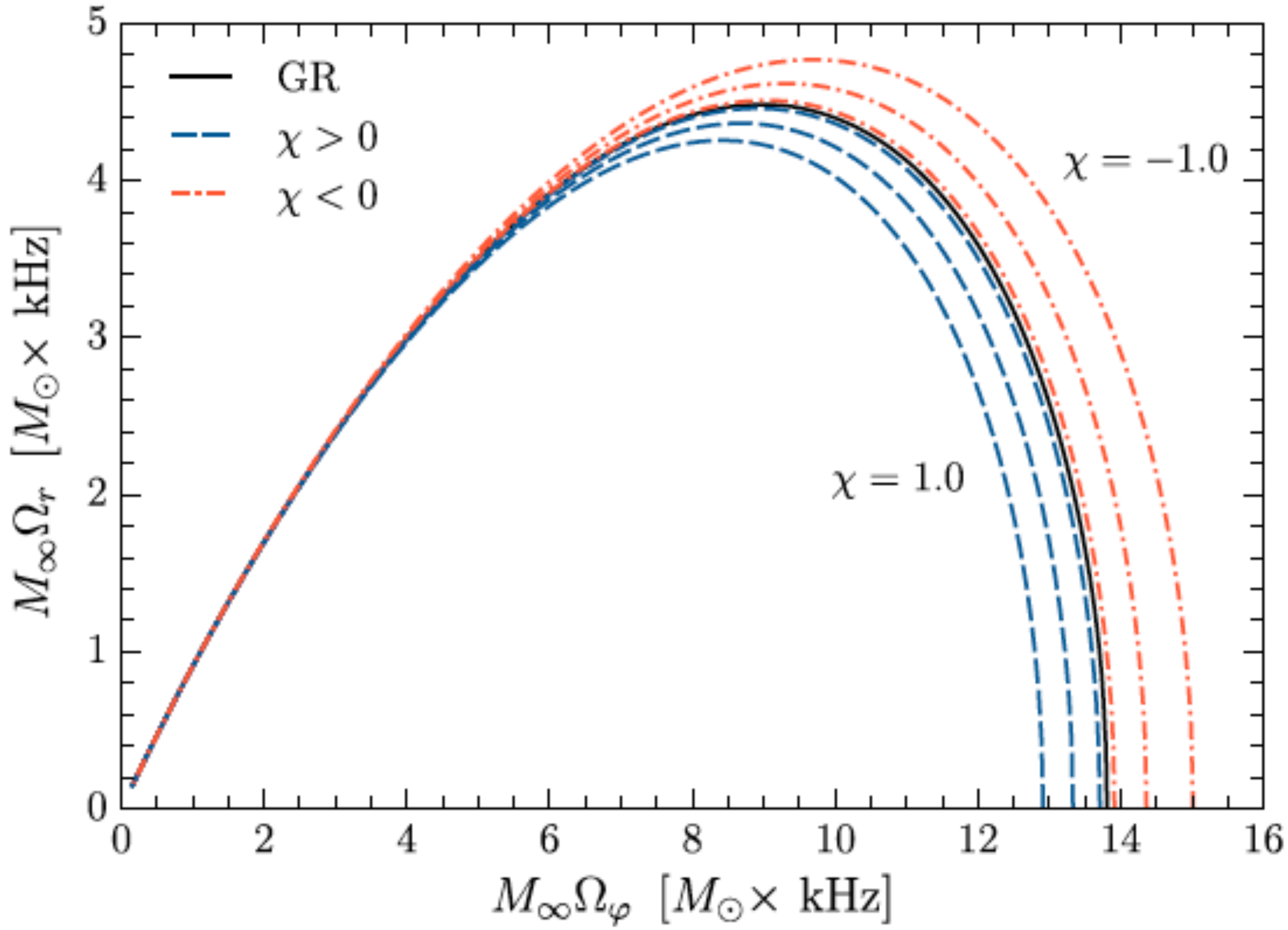
$$\Omega_\varphi^2 = -\frac{g'_{tt}}{g'_{\varphi\varphi}} \approx \frac{M_\infty}{r^3}\left(1+\chi\frac{M_\infty^2}{r^2}\right)$$

- Radial (epicyclic) frequency:

$$\Omega_r^2 = -\frac{g^{rr}}{2\dot{t}^2}V''_{\rm eff}(r) \approx \frac{M_\infty}{r^3}\left(1-\frac{6M_\infty}{r}-\chi\frac{M_\infty^2}{r^2}\right)$$

- ISCO radius: $r_{\rm isco} \approx 6M_\infty\left(1+\frac{19}{324}\chi\right)$

Geodesics & QPOs (II)



post-TOV: how general is it?

- Modified theories of gravity typically include one (or, less frequently, more) additional dynamical degrees of freedom (e.g. a scalar field)

- Our post-TOV formalism, featuring only two equations for $\mathrm{d}p/\mathrm{d}r$, $\mathrm{d}m/\mathrm{d}r$, implies that the extra d.o.f. ψ can be expressed in terms of the matter variables, i.e. $\psi=\psi(p, \rho, \Pi)$, *when building a NS*.

- This is the case for scalar-tensor theory and the same is likely true for other theories with a single extra d.o.f.

- By construction, the post-TOV scheme assumes "small" departures from GR, so it may *not* capture non-perturbative effects like "spontaneous scalarization".

Outlook

- The post-TOV formalism is a toolkit for building relativistic stellar models with small/moderate departures from GR.

- Its parametrised form should (eventually) encompass a large class of modified theories of gravity.

- **Plenty of extensions:**

- Include *dimensional* coupling constants (in progress): more post-TOV terms but families still exist.

- Map formalism onto various alternative theories. Capture non-linear effects (e.g. scalarisation).

- Add *slow rotation*: necessary step for a realistic framework.

- Study I-Q "universal" relations (and perhaps break matter-gravity degeneracy?)