Testing General Relativity using Supermassive Black Hole Binaries

Ed Porter - APC, Paris
IAP, 20/04/2015

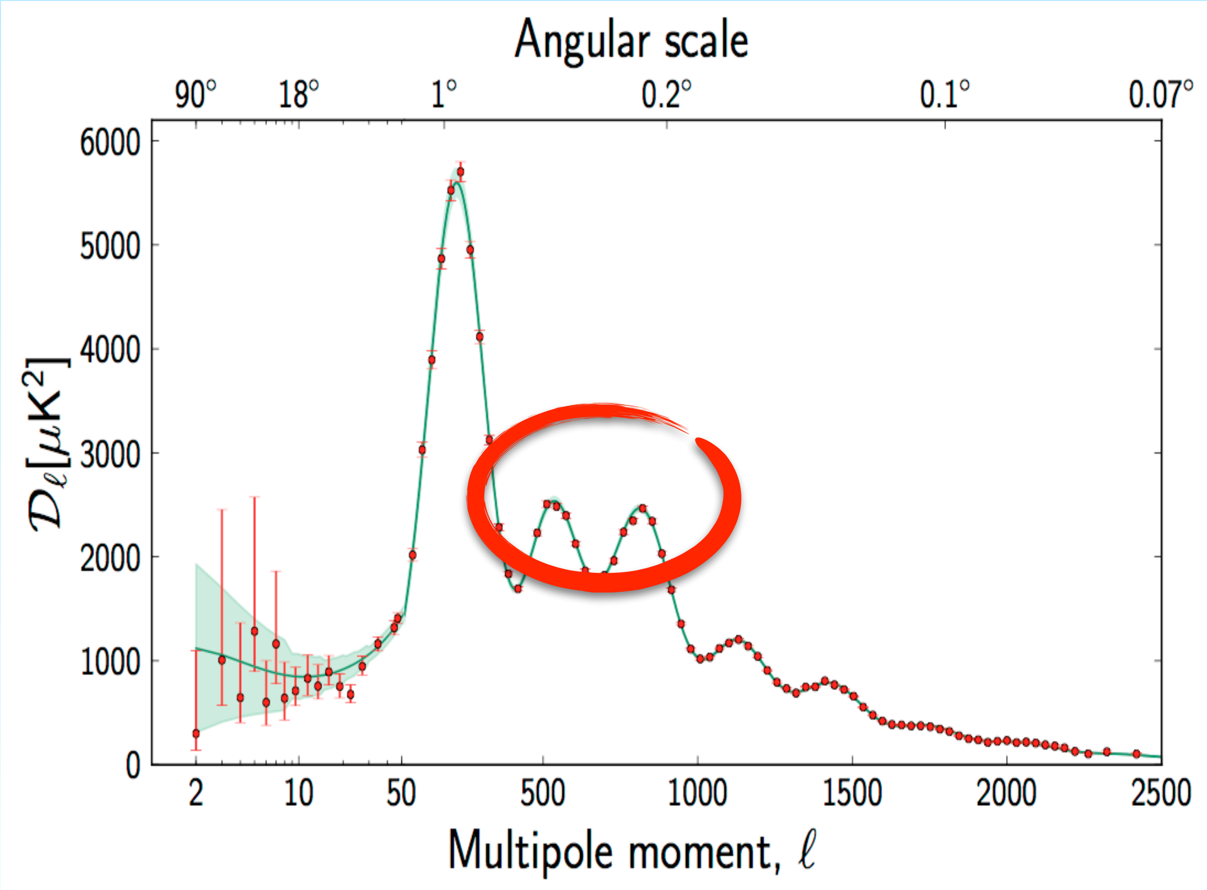
Introduction

1. Why alternatives to GR?
2. Current constraints
3. What can we do with GWs?
4. ppE framework
5. eLISA
6. Generic test results

Alternative Theories of Gravity

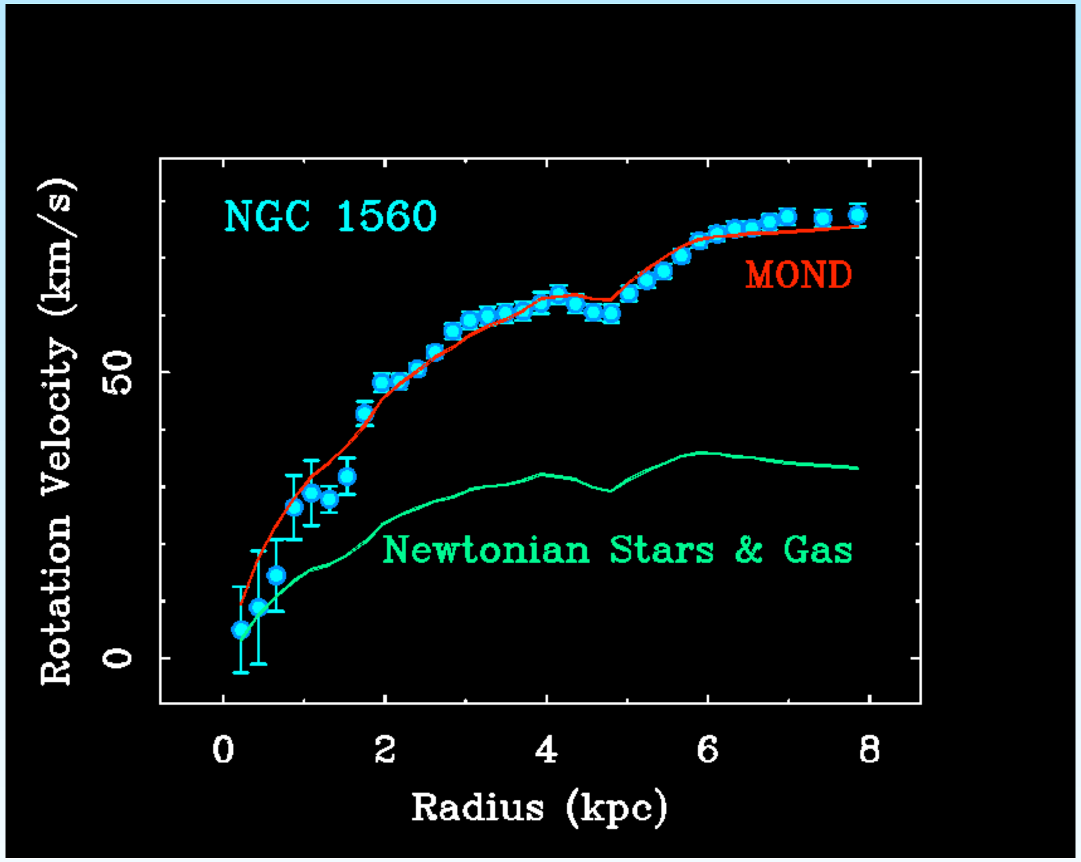
Cosmology

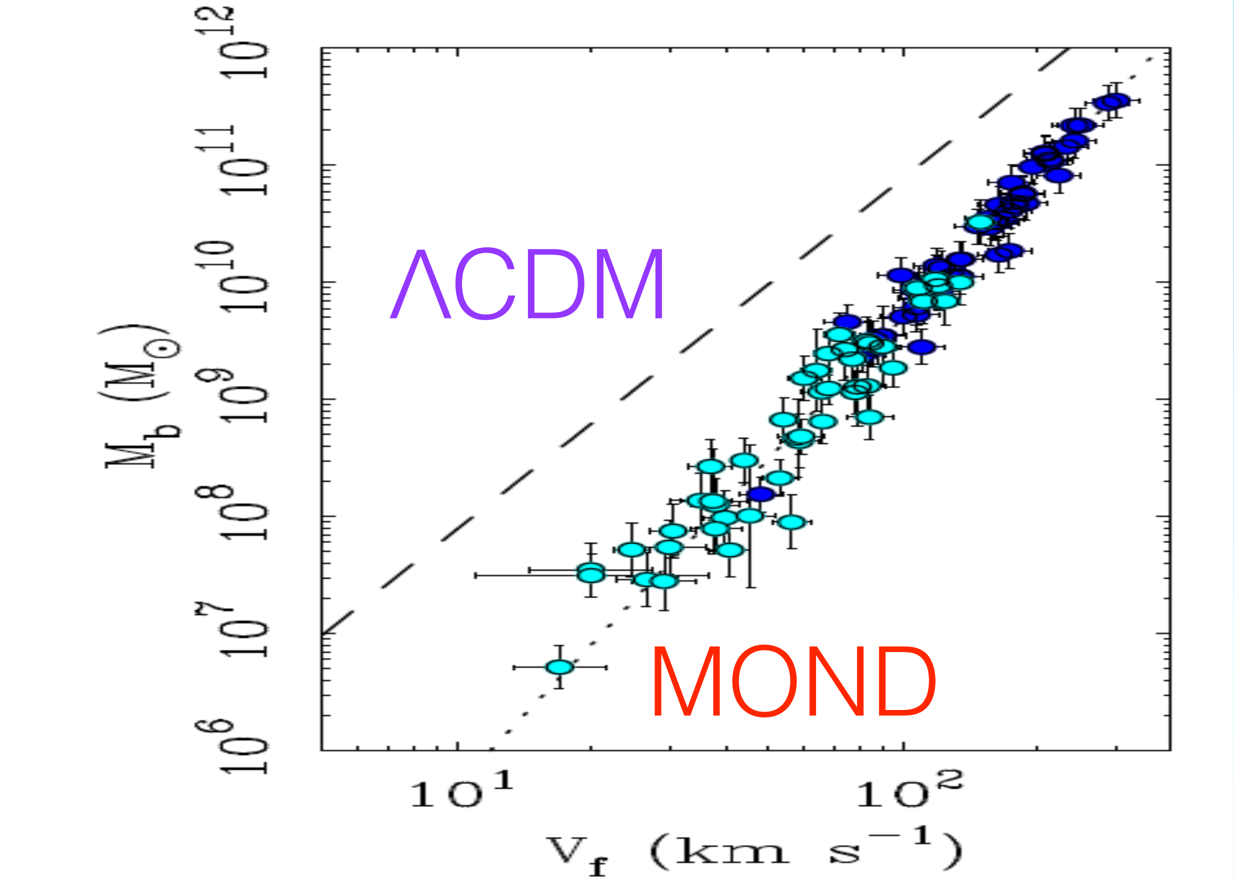
- Quantum theories of gravity
- Explain inflation, dark energy and dark matter
- Why 2nd and 3rd multipole peaks have the same height



Galactic Scales

- ΛCDM does not explain galactic rotation curves
- Problems with the baryonic Tully-Fisher relation
- MOND works on this scale, but this scale only





Current Constraints

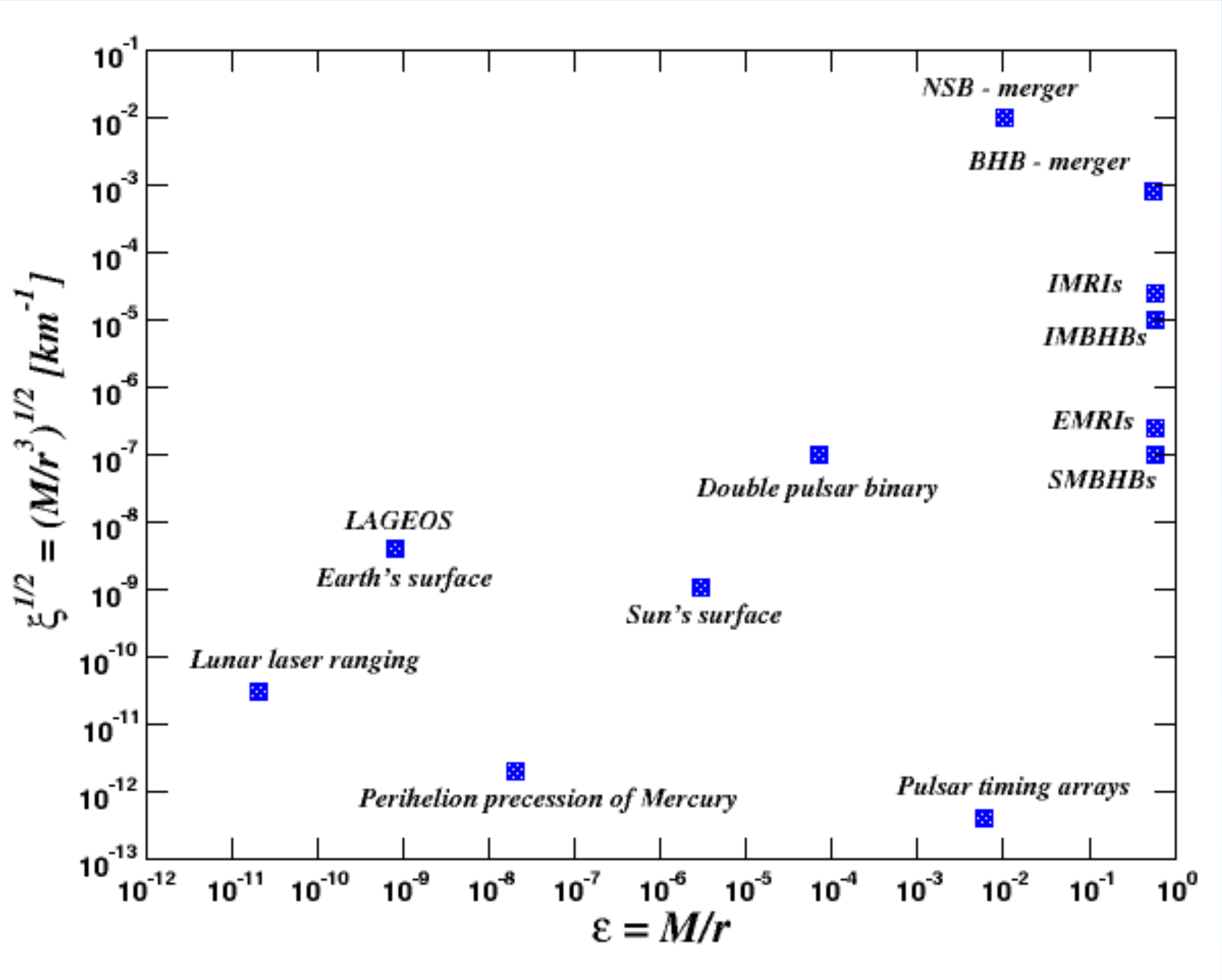
How do you constrain?

Two methods :

1) Top down - take alternative model with known action
- calculate the modified GW emission
- test new waveform against data
- compare with GR waveforms
- needs to be repeated for all theories

2) Generic - assume nothing about underlying theories
- construct phenomenological test
- construct generic non-GR waveform
- Won't tell you what the theory is...
- ...but will confirm deviations from GR

Current Constraints



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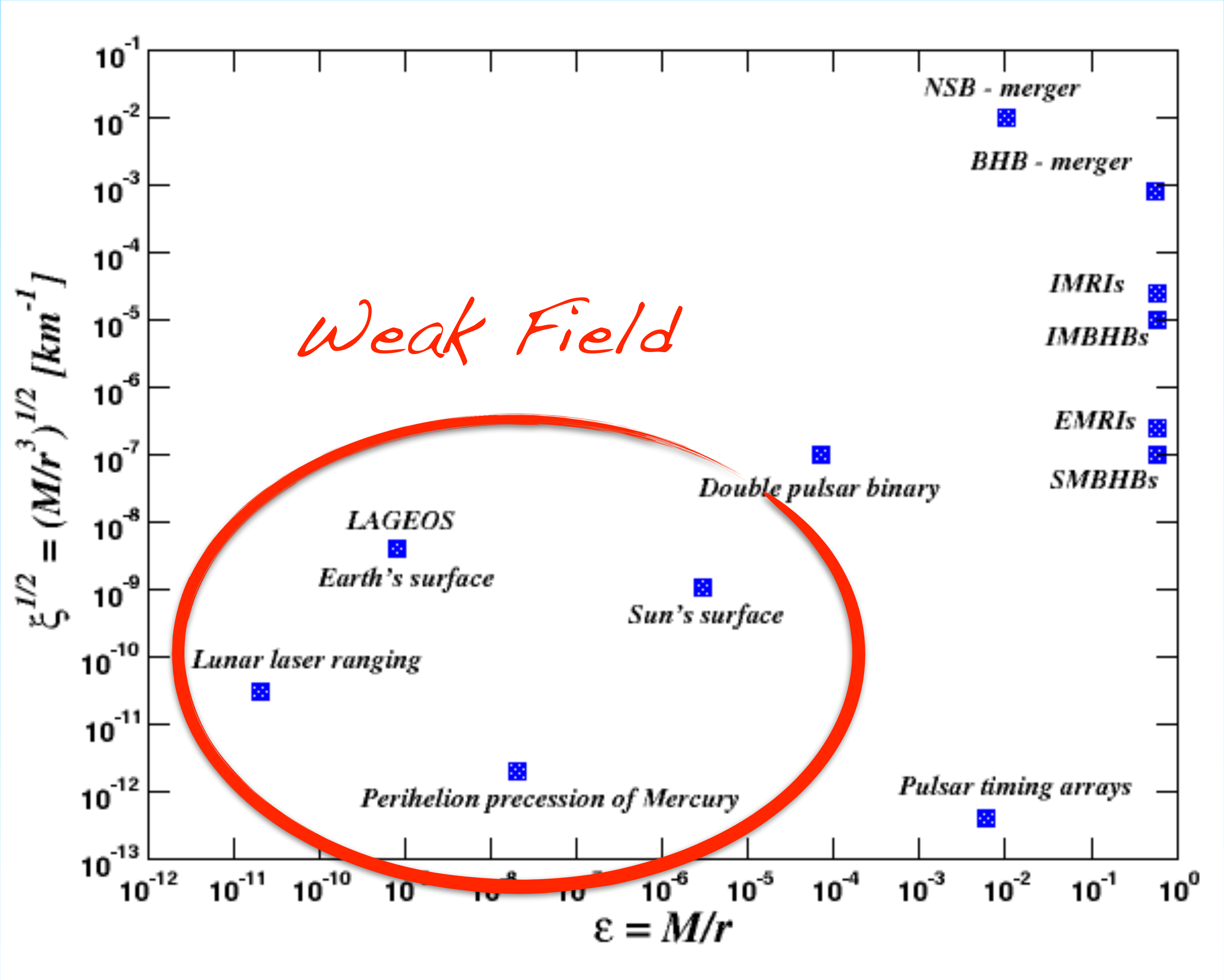

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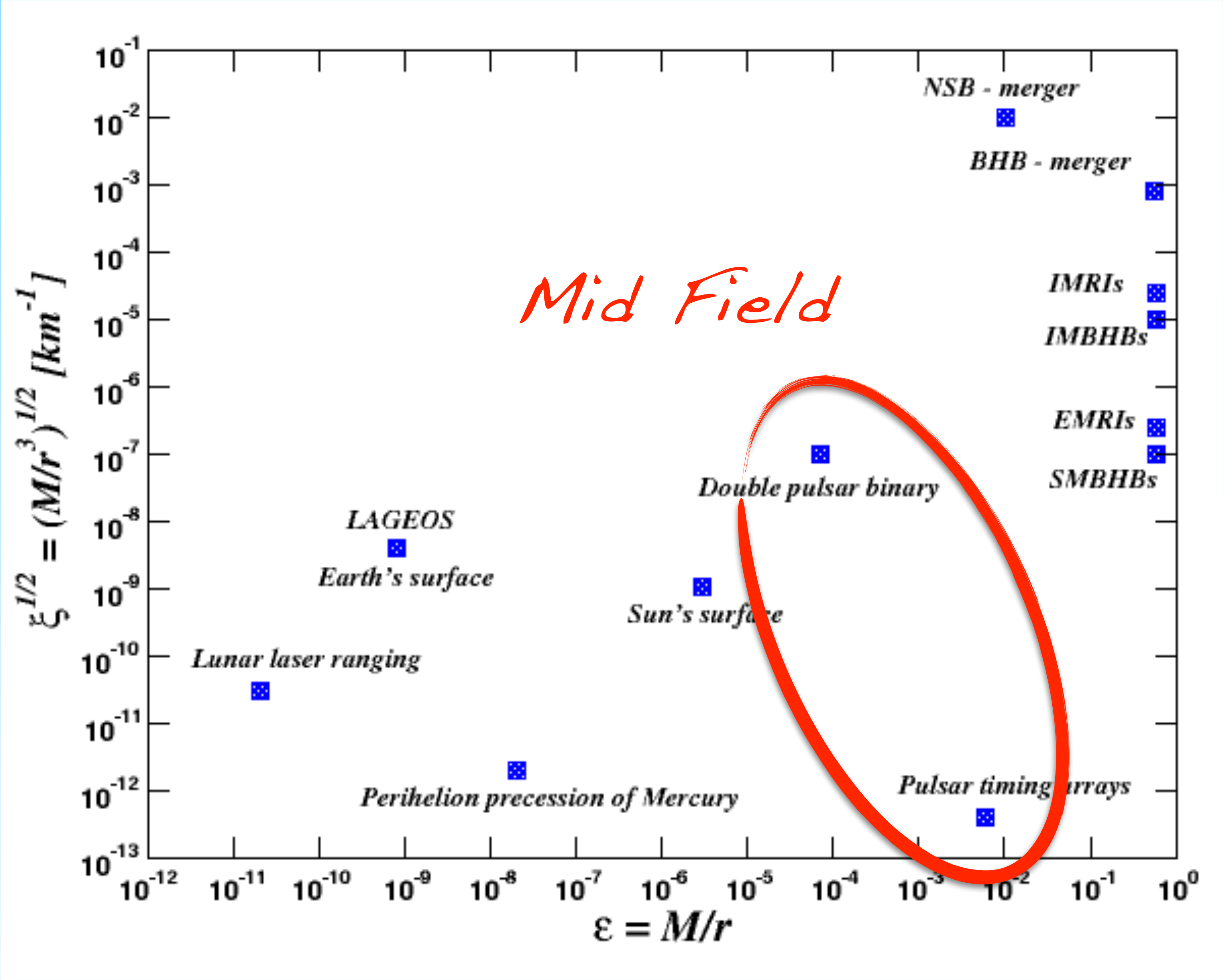

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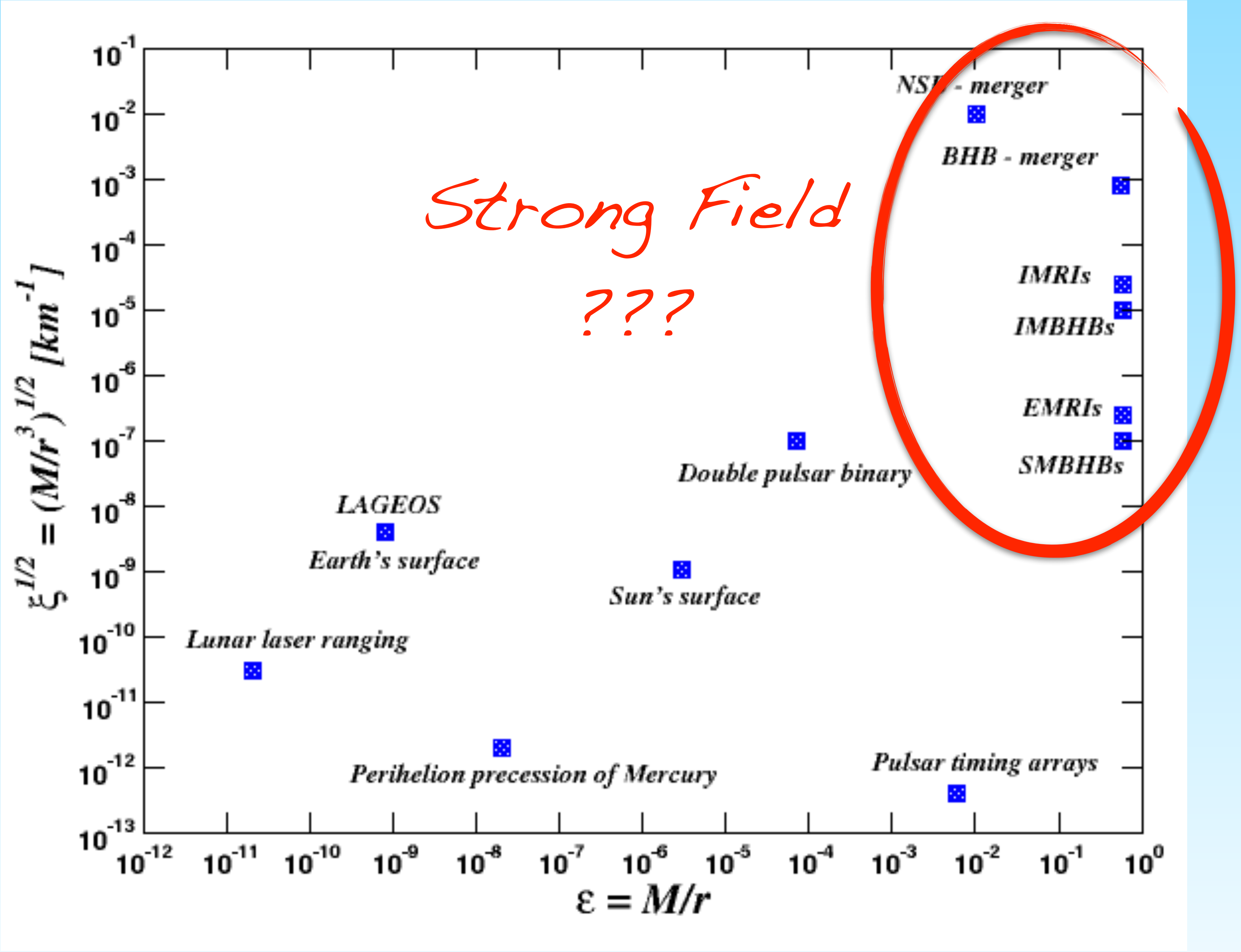

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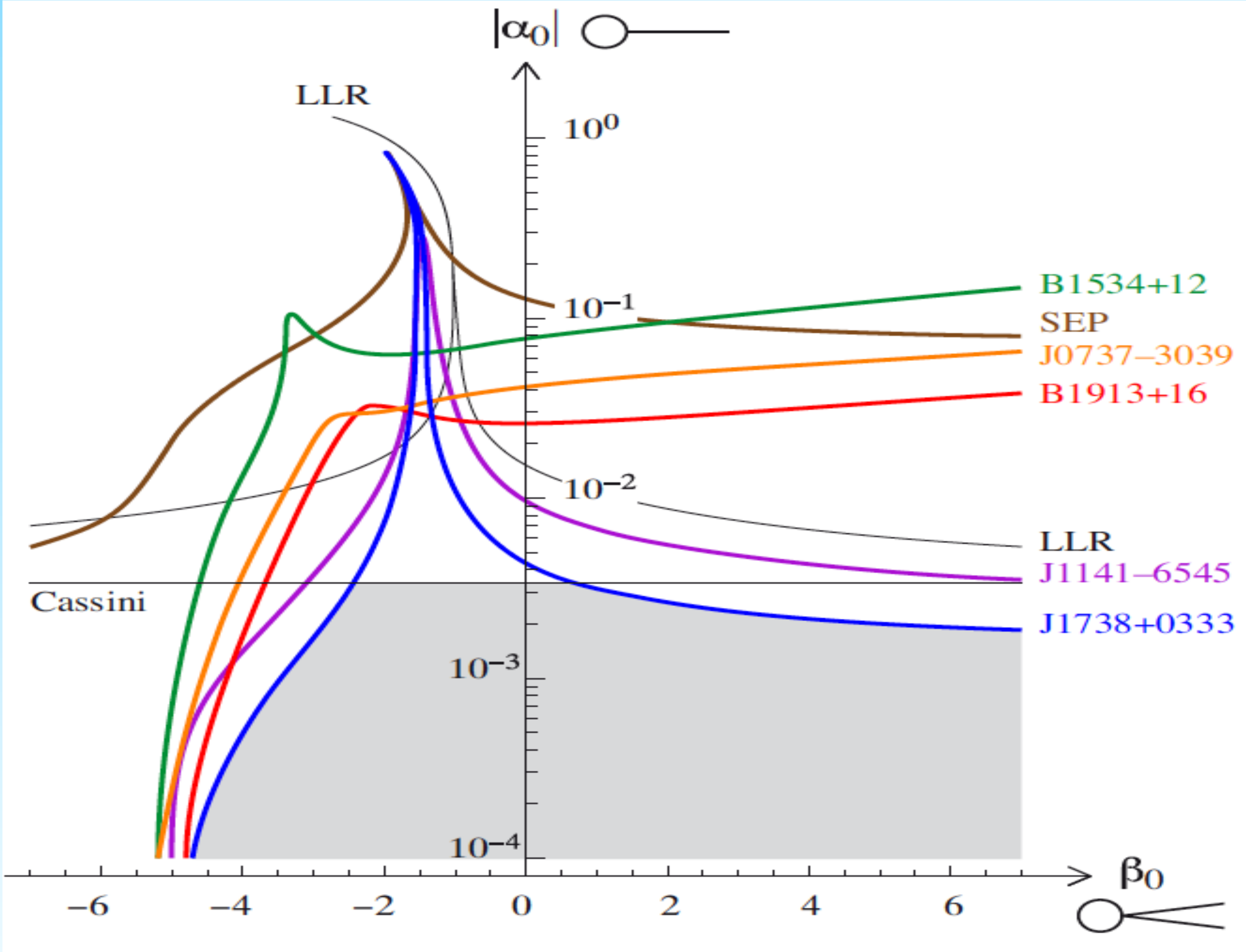

P. Freire et al, MNRAS (2012)

ppE Framework

Yunes & Pretorius (2009)

ppE Framework

- Devise a generic method of testing deviations from GR
- Introduce properly motivated parameters that can measure deviations from GR
- Incorporate
  - Metric theories of gravity
  - Weak field consistency
  - Strong field inconsistency

ppE Framework

- Modify the GR waveform in the SPA

$$\tilde{h}_{\rm ppE}(f) = \tilde{h}_{\rm GR}(f)\,(1+\alpha u^a)\,e^{i\beta u^b}$$

- where $u = \mathcal{M}\pi f$ and $(a,b) = i(-1/3) \qquad i \in \mathbb{Z}$

- GR : $(\alpha, a, \beta, b) = (0, a, 0, b)$
- Brans-Dicke : $(\alpha, a, \beta, b) = (0, a, \beta_{BD}, -7/3)$
- Massive Graviton : $(\alpha, a, \beta, b) = (0, a, \beta_{MG}, -1)$
- Chern-Simons : $(\alpha, a, \beta, b) = (\alpha_{CS}, 1, 0, b)$

Time Domain ppE Framework

Huywler, Porter & Jetzer (2015)

Time Domain Waveform

- SPA was important when $t_{FFT} \sim t_{WF}$
- Now numerical FFT takes 3-8% of generation time
- SPA breaks down at high masses even for LIGO/Virgo
- Unclear of consequence for eLISA
- Certain matching issues encountered in the past

Time Domain Waveform

- GW sources are detected using matched filtering
- Matched filters are mostly sensitive to frequency evolution
- Assume $\alpha = a = 0$
- Our goal is a phase function of the type

$$\Phi_{\rm NGR}^{(\pm)}(\Theta; b, \beta) = \Phi_{\rm GR}(\Theta) \pm \Phi_c(\Theta; b, \beta)$$

Time Domain Waveform

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Theory **Coupling Constant**

Time Domain Waveform

- In the LFA, the GW response is

$$h(t) = h_+(\xi(t))F^+(t) + h_\times(\xi(t))F^\times(t)$$

- The 2PN phase and frequency are given by

$$\omega(\Theta) = \frac{c^3}{8GM}\left[\Theta^{-3/8} + \omega_1\Theta^{-5/8} + \omega_{1.5}\Theta^{-6/8} + \omega_2\Theta^{-7/8}\right],$$

$$\Phi(\Theta) = \Phi_C - \frac{1}{\eta}\left[\Theta^{5/8} + \Phi_1\Theta^{3/8} + \Phi_{1.5}\Theta^{2/8} + \Phi_2\Theta^{1/8}\right]$$

- where $\Theta(t) = \frac{\eta c^3}{5GM}(t_c - t)$

Time Domain Waveform

- *To obtain a correspondence between the SPA and time domain waveforms, we begin with*

$$\Psi_{\rm NGR}(b,\beta;\,u) = \Psi_{\rm GR}(u) \mp \beta\, u^b$$

- *Analytically inverting the SPA phase gives*

$$\Phi_{\rm NGR}(\Theta) = \Phi_{\rm GR}(\Theta) \pm \frac{1}{\eta}\sum_i \kappa_i \Theta^{\frac{5-2i}{8}}$$

- *The question is what set of* $\{\kappa_i\}$ *give a time domain waveform compatible with the SPA?*

Time Domain Waveform

- We can write the SPA phase as

$$\Psi(u) = 2\left[\frac{1}{\mathcal{M}} t(u)\, u - \Phi(u)\right] - \frac{\pi}{4}$$

- The frequency derivative of the orbital phase is

$$\frac{d\Phi}{du}(u) = \frac{dt}{du}(u)\,\frac{d\Phi}{dt}[t(u)] = \frac{dt}{du}\frac{1}{\mathcal{M}}u$$

- meaning the frequency derivative of the SPA phase is

$$\frac{d\Psi}{du} = 2\frac{1}{\mathcal{M}}\,t(u)$$

- This allows us to write

$$t(u) = \frac{1}{2}\mathcal{M}\frac{d\Psi}{du} \qquad u(t) = \mathcal{M}\frac{d\Phi}{dt}$$

Time Domain Waveform

- Truncate corrections at 2PN order
- We need to solve

$$u[\Theta(u)]_{2\mathrm{PN}} = u\left(1 + \sum_{k=0}^{4} u^{k/3}\mathcal{A}_k\right) = u$$

- for $\kappa_i(b, \beta)$
- such that

0	0.5	1	1.5	2 PN

$$b \in \{-5/3, -4/3, -1, -2/3, -1/3\}$$

$$i \in \{0, 1/2, 1, 3/2, 2\}$$

Time Domain Waveform

b	-5/3	-4/3	-1	-2/3	-1/3
κ_0	16β	0	0	0	0
$\kappa_{1/2}$	0	$8\beta\eta^{1/5}$	0	0	0
κ_1	$-16\beta\Phi_1$	0	$4\beta\eta^{2/5}$	0	0
$\kappa_{3/2}$	$-\frac{32}{3}\beta\Phi_{3/2}$	$-\frac{32}{5}\beta\Phi_1\eta^{1/5}$	0	$2\beta\eta^{3/5}$	0
κ_2	$16\beta\left(\frac{4}{5}\Phi_1^2 - \frac{1}{3}\Phi_2\right)$	$-\frac{64}{15}\beta\Phi_{3/2}\eta^{1/5}$	$-\frac{12}{5}\beta\Phi_1\eta^{2/5}$	0	$\beta\eta^{4/5}$

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Time Domain Waveform

- Sampson, Yunes & Cornish (2010) demonstrated that only the dominant terms are required

$$\Psi_{\rm NGR}(b,\beta;\,u) = \Psi_{\rm GR}(u) \mp \beta\, u^{b}$$

$$\Phi^{(\pm)}_{\rm NGR}(b,\beta;\,\Theta) = \Phi_{\rm GR}(\Theta) \pm 2^{-1-3b}\beta\,\eta^{3b/5}\Theta^{-3b/8}$$

- where the coupling constant is manifestly positive

$$\omega^{(\pm)}_{\rm NGR}(b,\beta;\Theta) = \omega_{\rm GR}(\Theta) \pm 2^{-4-3b}\frac{3\beta}{5}\frac{c^3}{GM}\,\eta^{3b/5+1}\Theta^{-3b/8-1}$$

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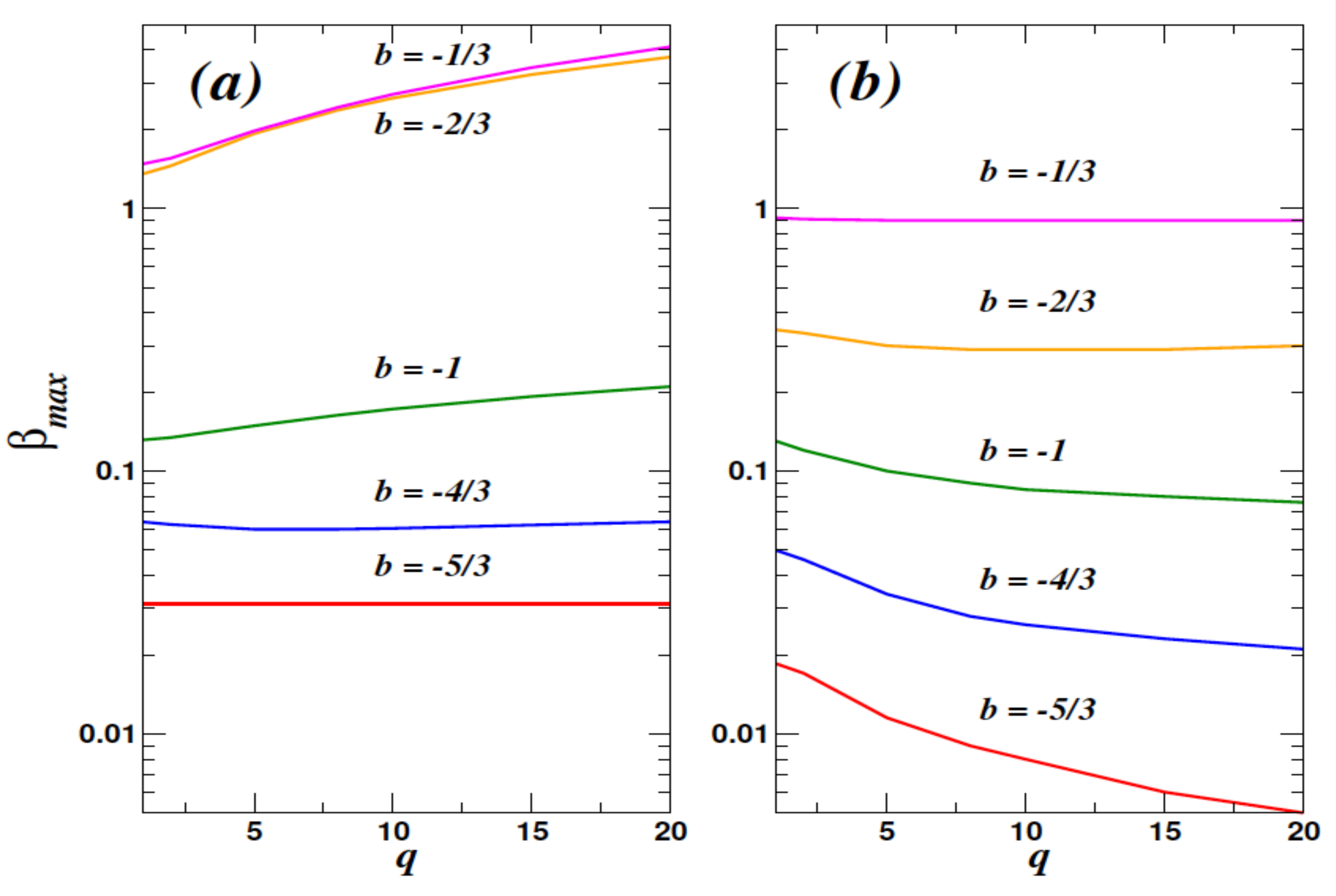
Coupling Constant Limits

- Need to set reasonable limits on non-GR coefficients
- In principle, the limits are unconstrained after 1PN order
- Need priors for Bayesian inference
- Need to specially treat 0.5PN non-GR coefficient
- Chose limit to be 50% of GR coefficient, i.e.

$$\max_i \left| \frac{\kappa_i}{\Phi_i} \right| < 0.5$$

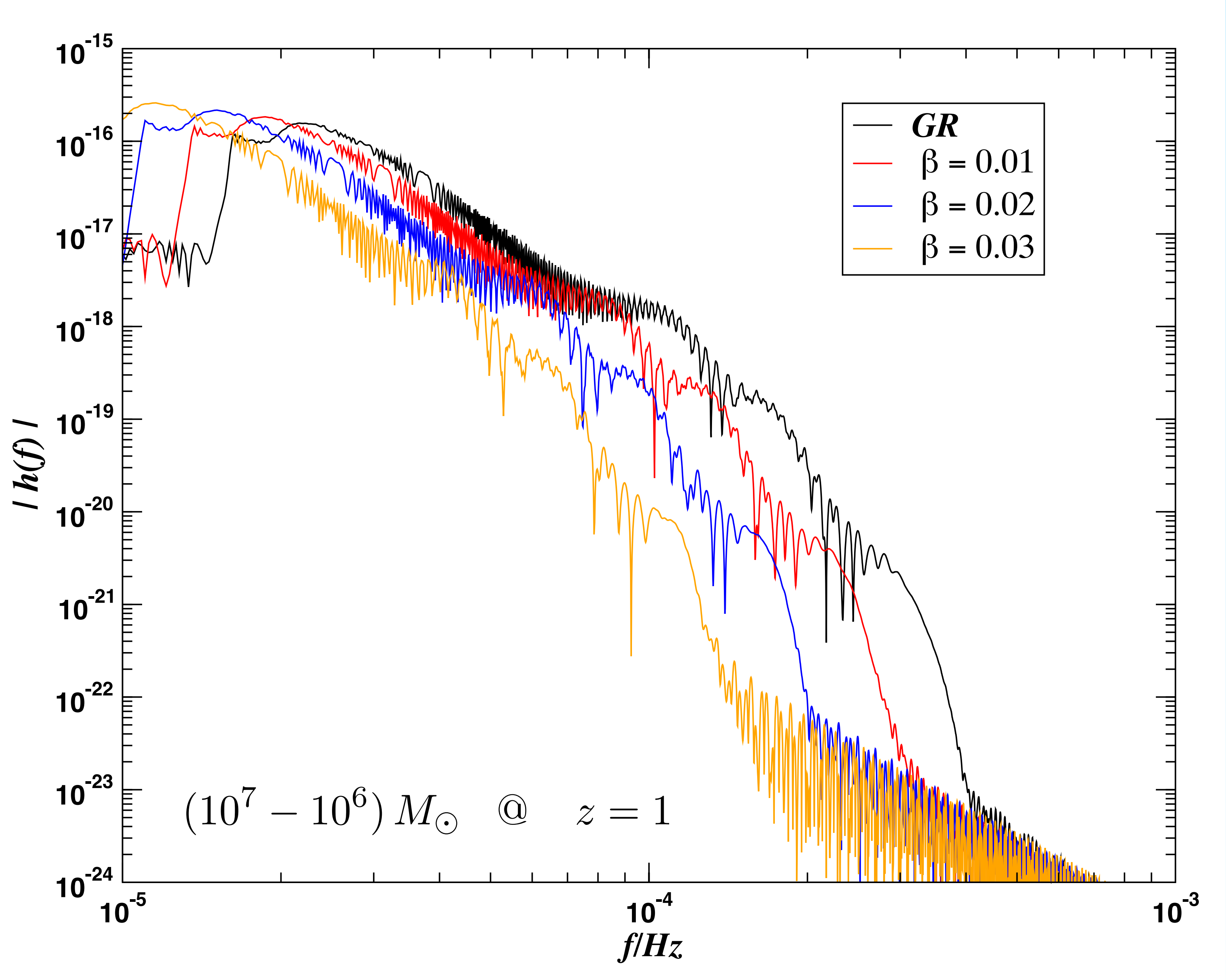
Coupling Constant Limits

$\beta > 0$ $\beta < 0$



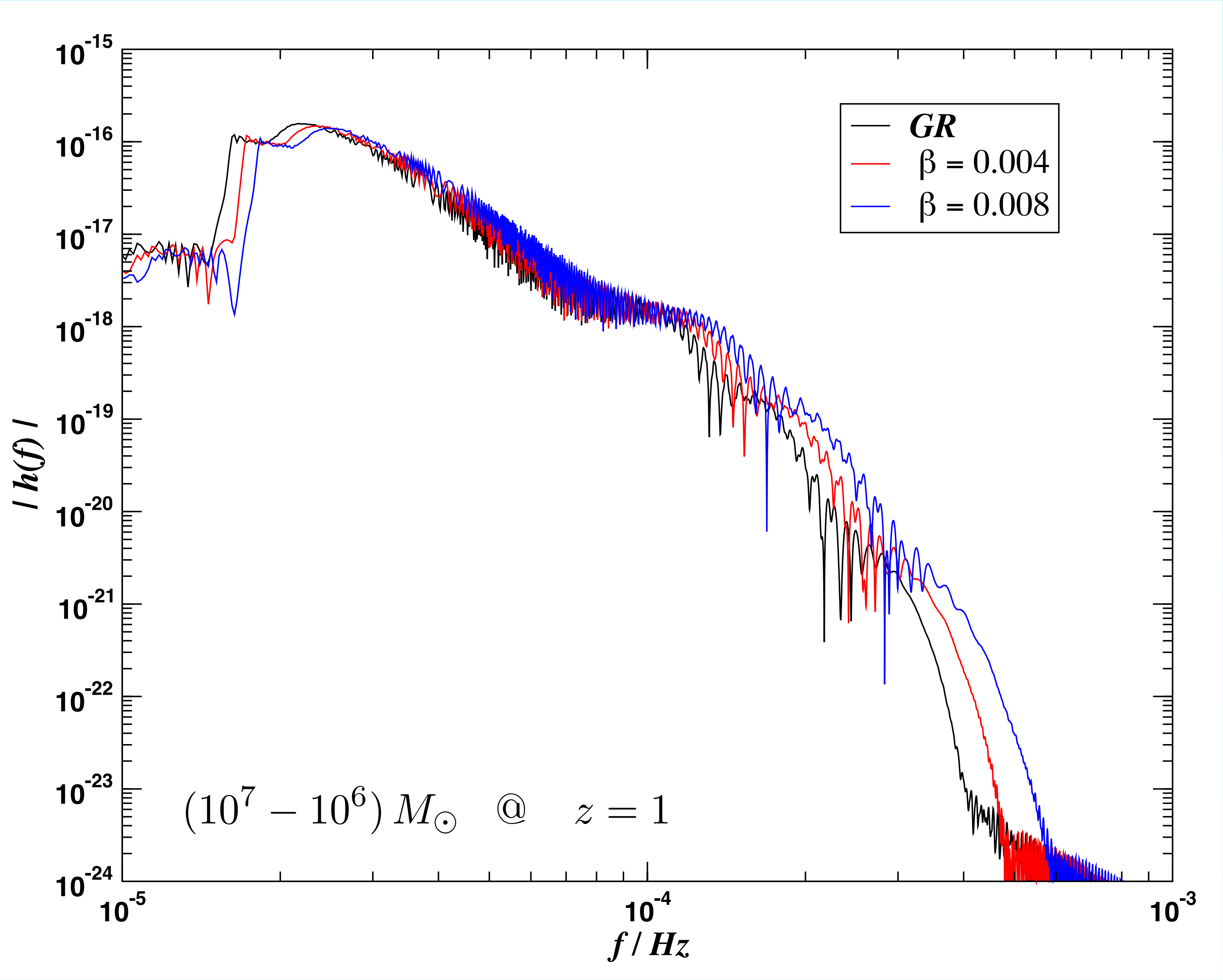
Non-GR Waveforms

Positive Coupling Constants



Non-GR Waveforms

Positive Coupling Constants



eLISA

eLISA

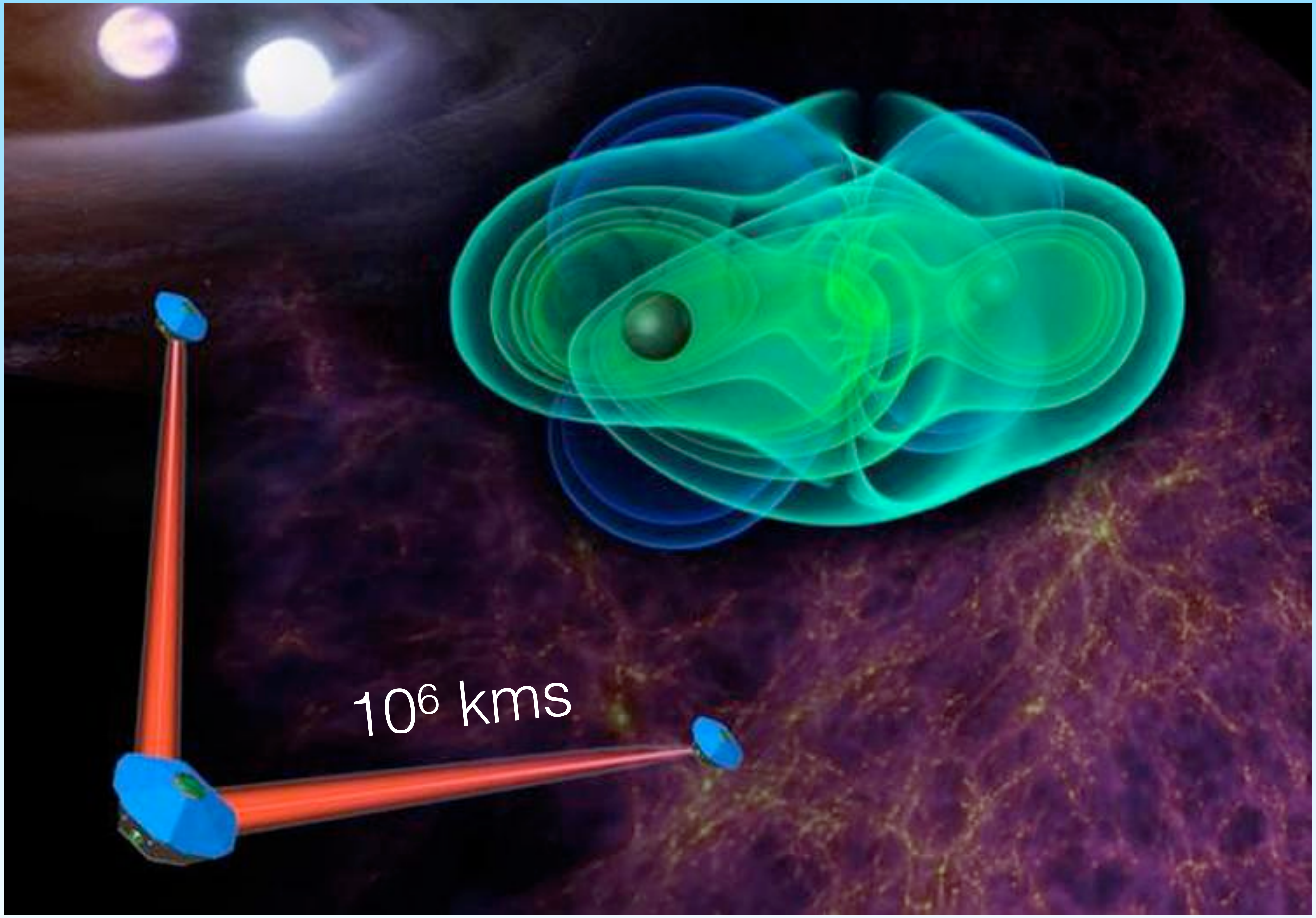
Mission concept currently proposed within the ESA Cosmic Vision L3 program under the theme "The Gravitational Wave Universe"

Estimated launch date is 2034

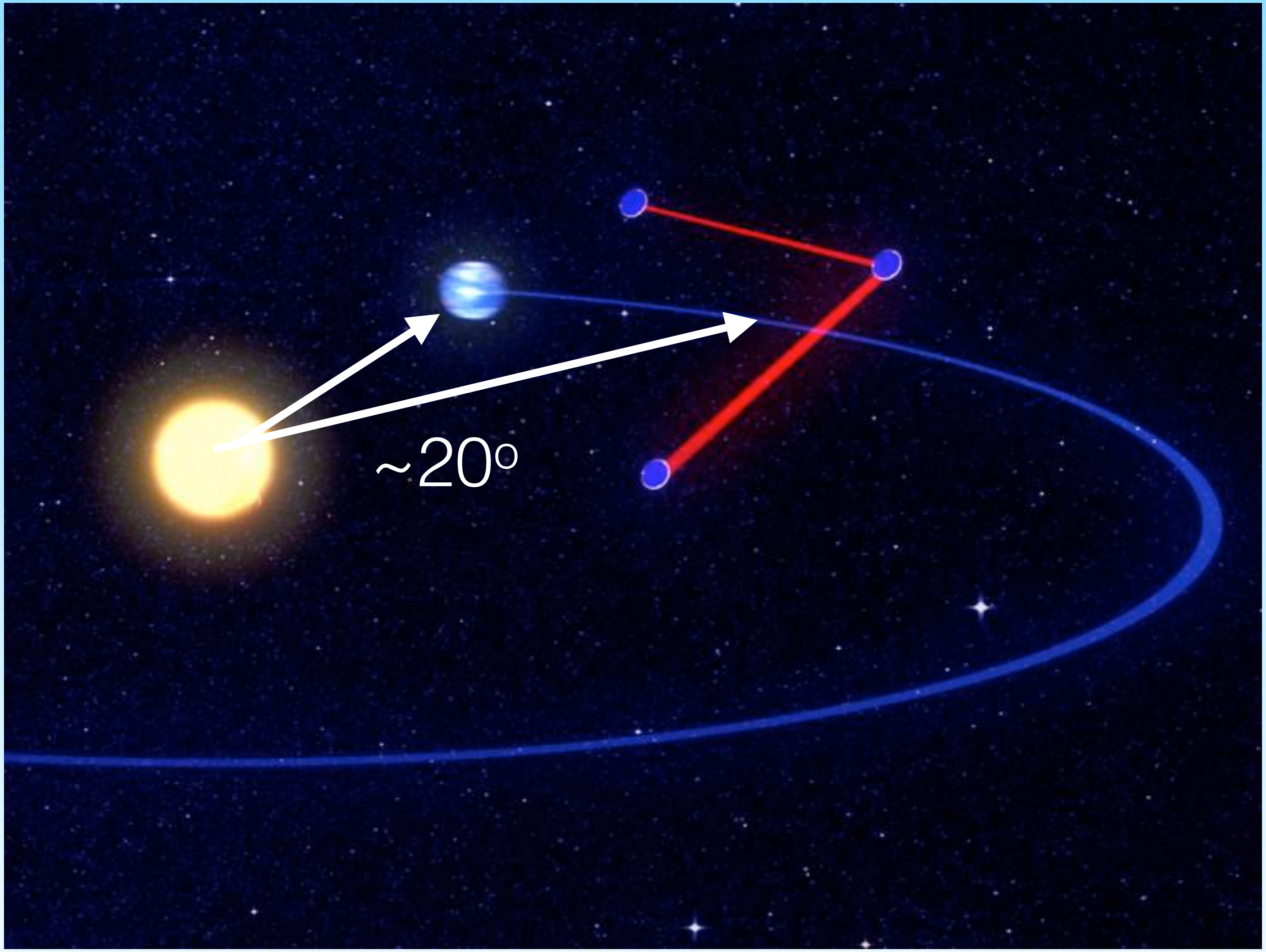
Mission configuration expected to be fixed by 2020

Later this year, ESA will lauch LISA-Pathfinder, a technology demonstration mission for eLISA

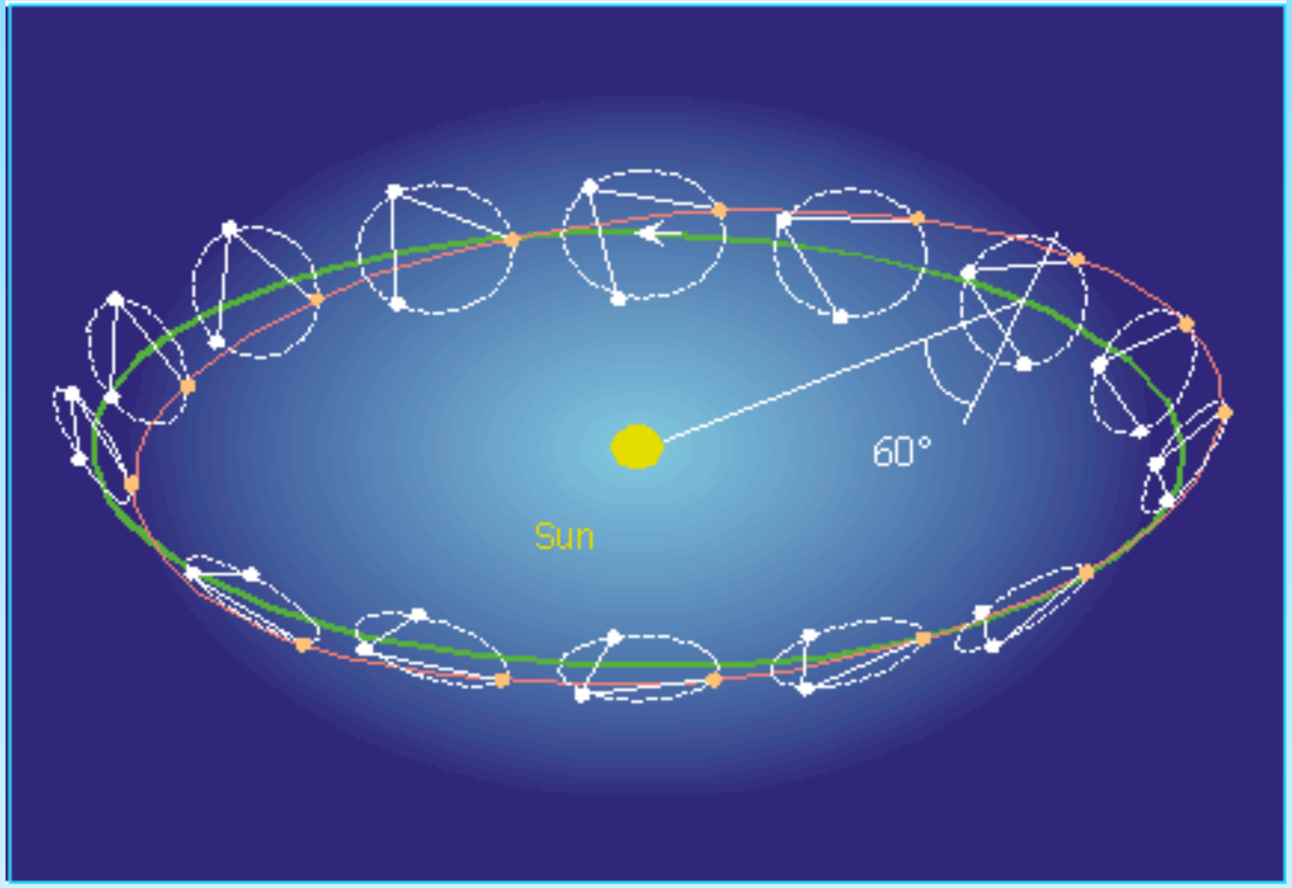
LISA to eLISA



LISA to eLISA



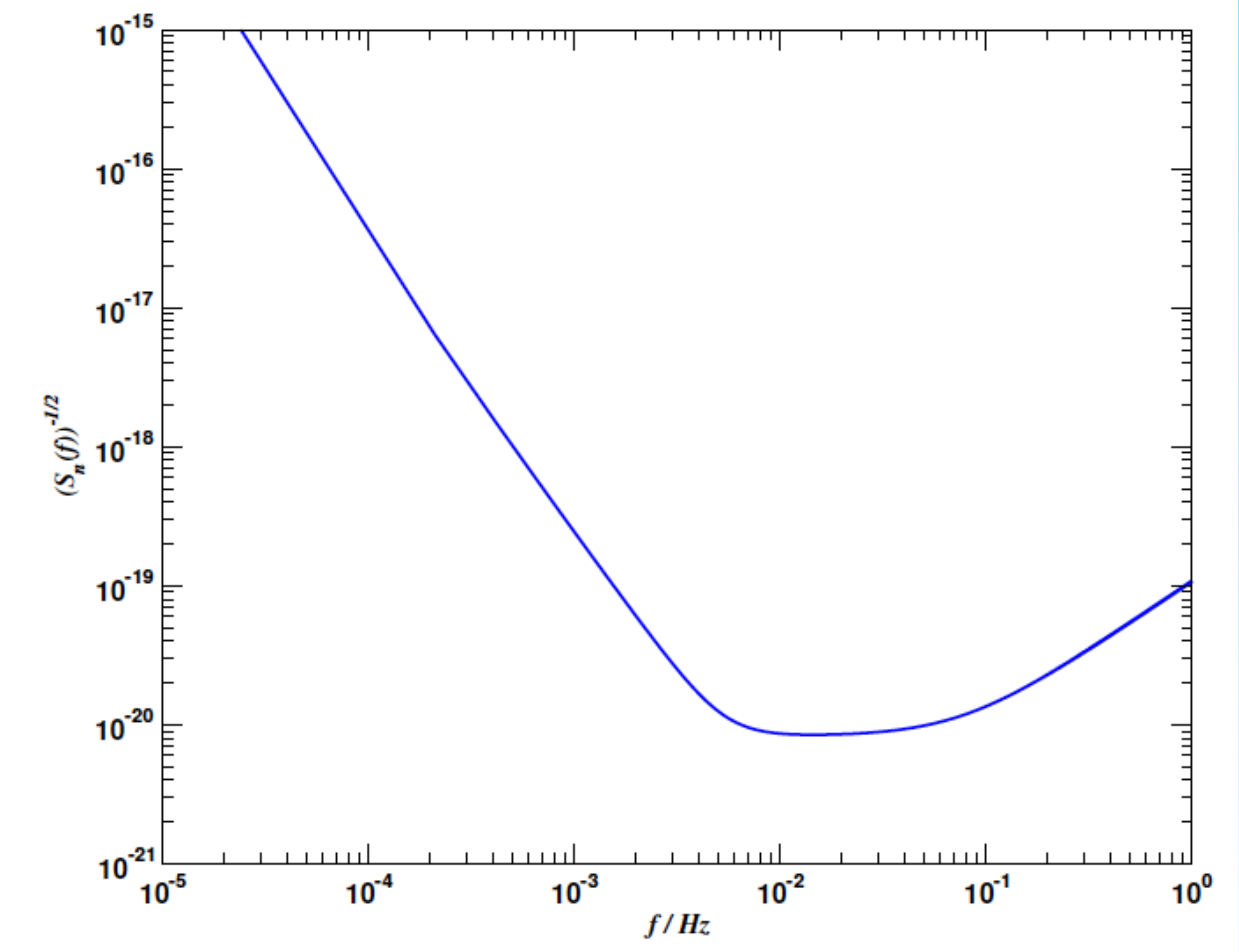
LISA to eLISA



Space-craft travel on ballistic orbits

Induces a Doppler motion which is important for sky position resolution

eLISA

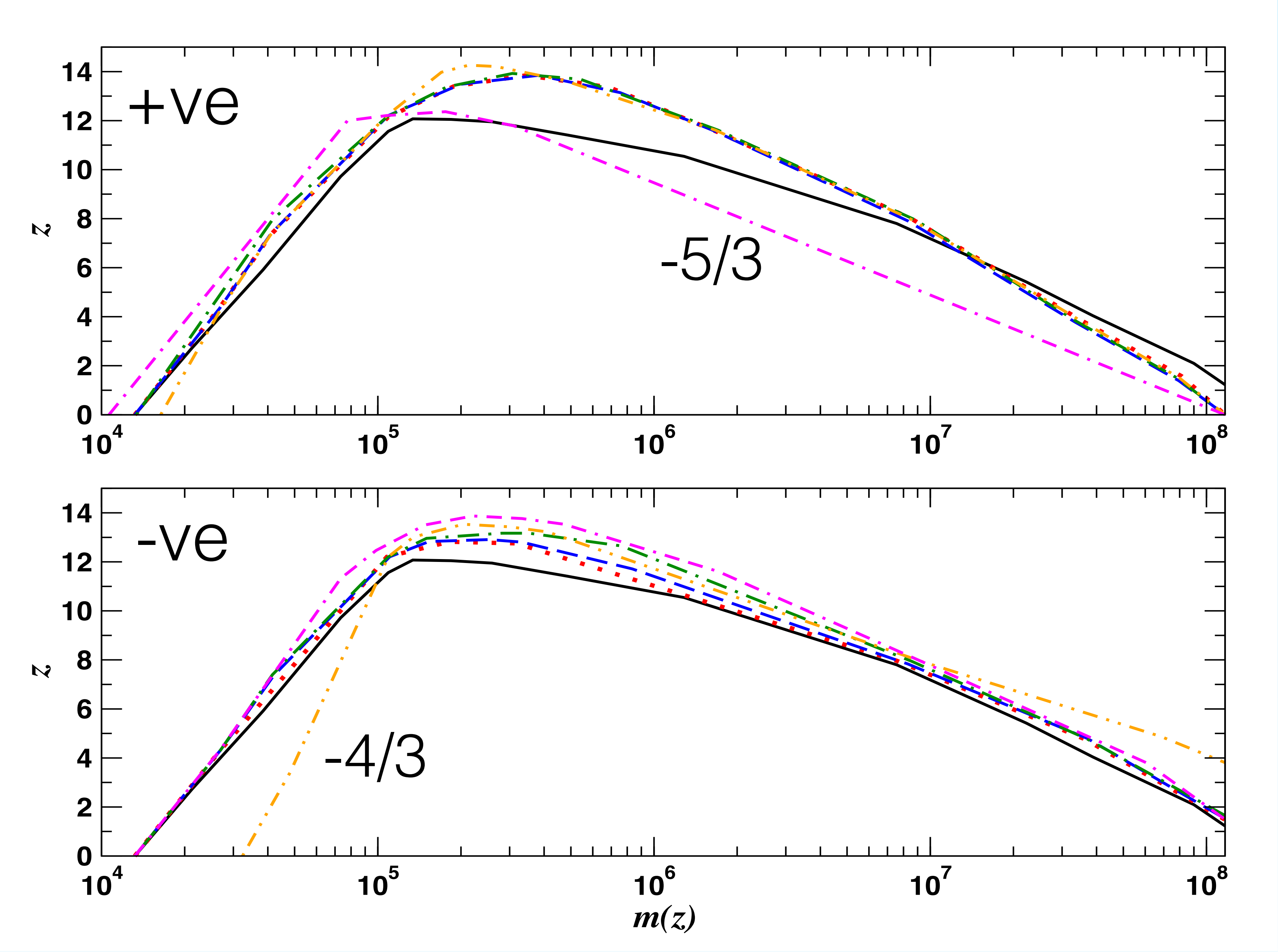


10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
$(S_n(f))^{-1/2}$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$f\,/\,Hz$

Results

Detection Horizon

$$\rho \geq 10$$



Bayesian Inference

- Inject a HHC non-GR signal with Gaussian noise

$$s(t) = h_{NGR}(t; \lambda^{\mu}, b, \beta) + n(t)$$

- Search with HHC GR templates

$$h(t; \lambda^{\mu})$$

- Start with correct GR physical parameters

- Two possible results -
  1) GR templates detected signals above SNR threshold
  2) Recovered parameters are within 2σ of true values

Bayesian Priors

1) $m(z) \leq 1.163 \times 10\, M_\odot$

2) $1 \leq q \leq 100$

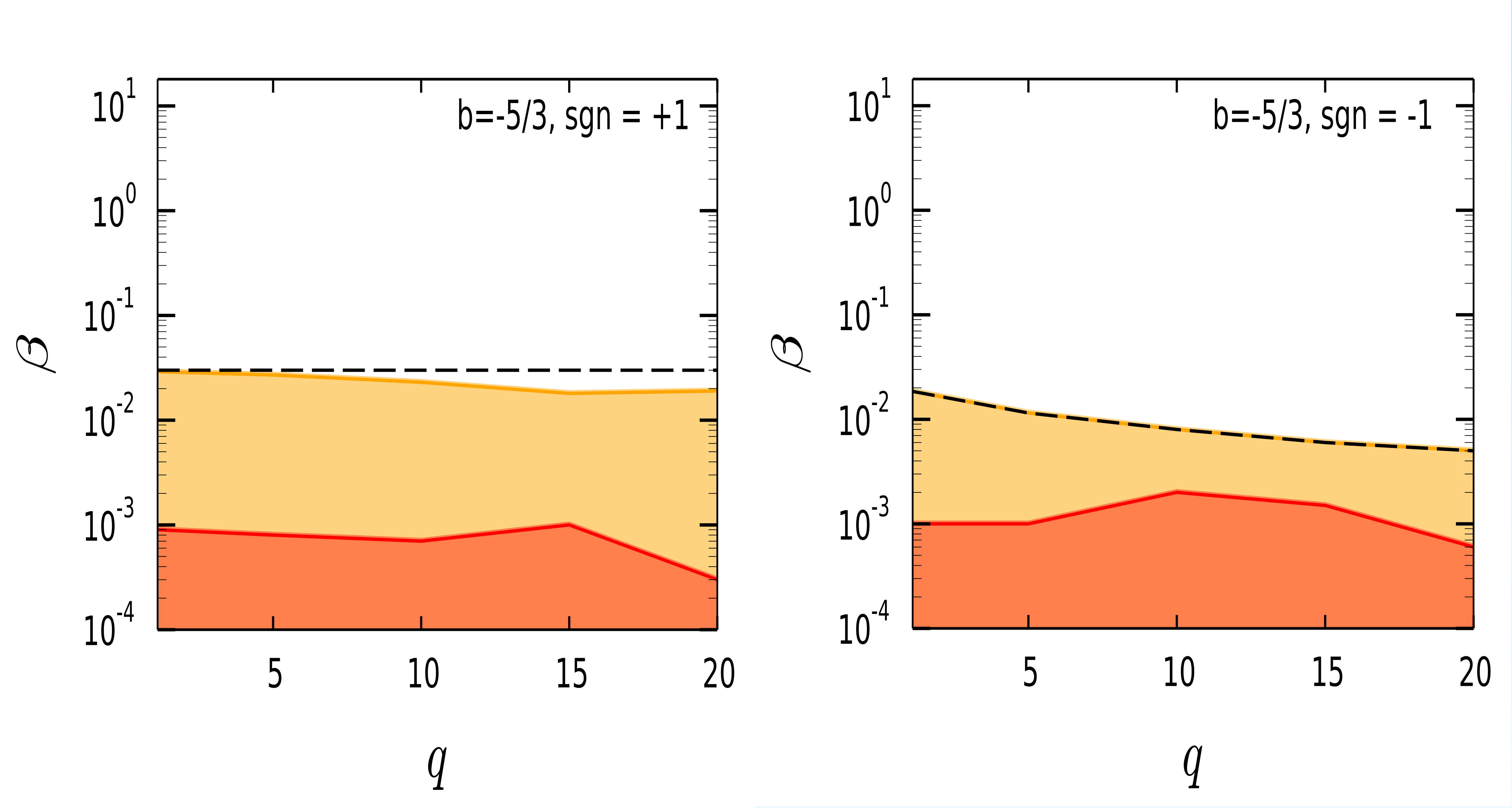
3) $7.7 \times 10^{-4} \leq D_L/Gpc \leq 110$

4) $0.2 \leq t_c/yrs \leq 0.99$

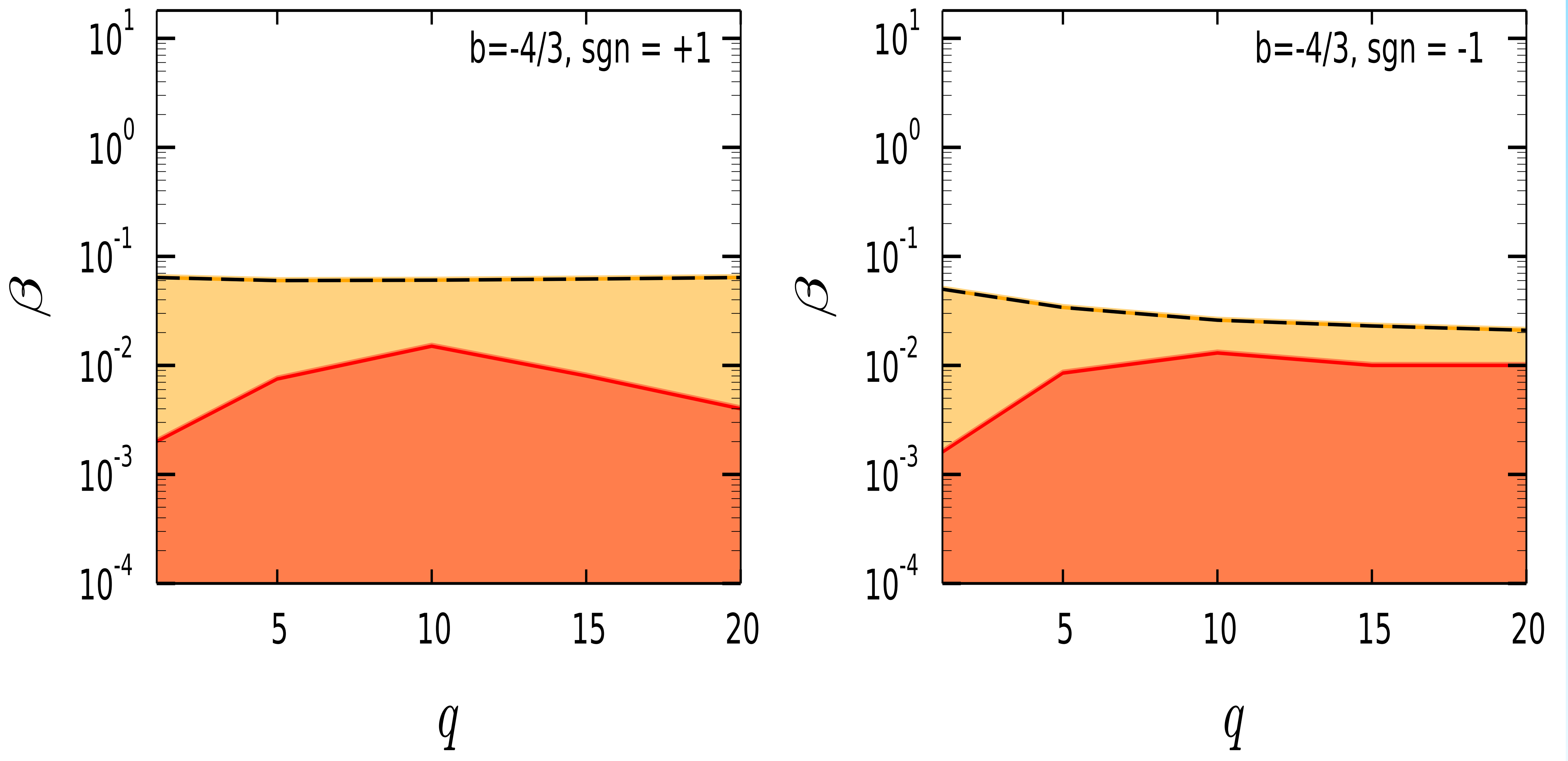
5) All other parameters have open ranges

6) Coupling constant prior given by previous figure

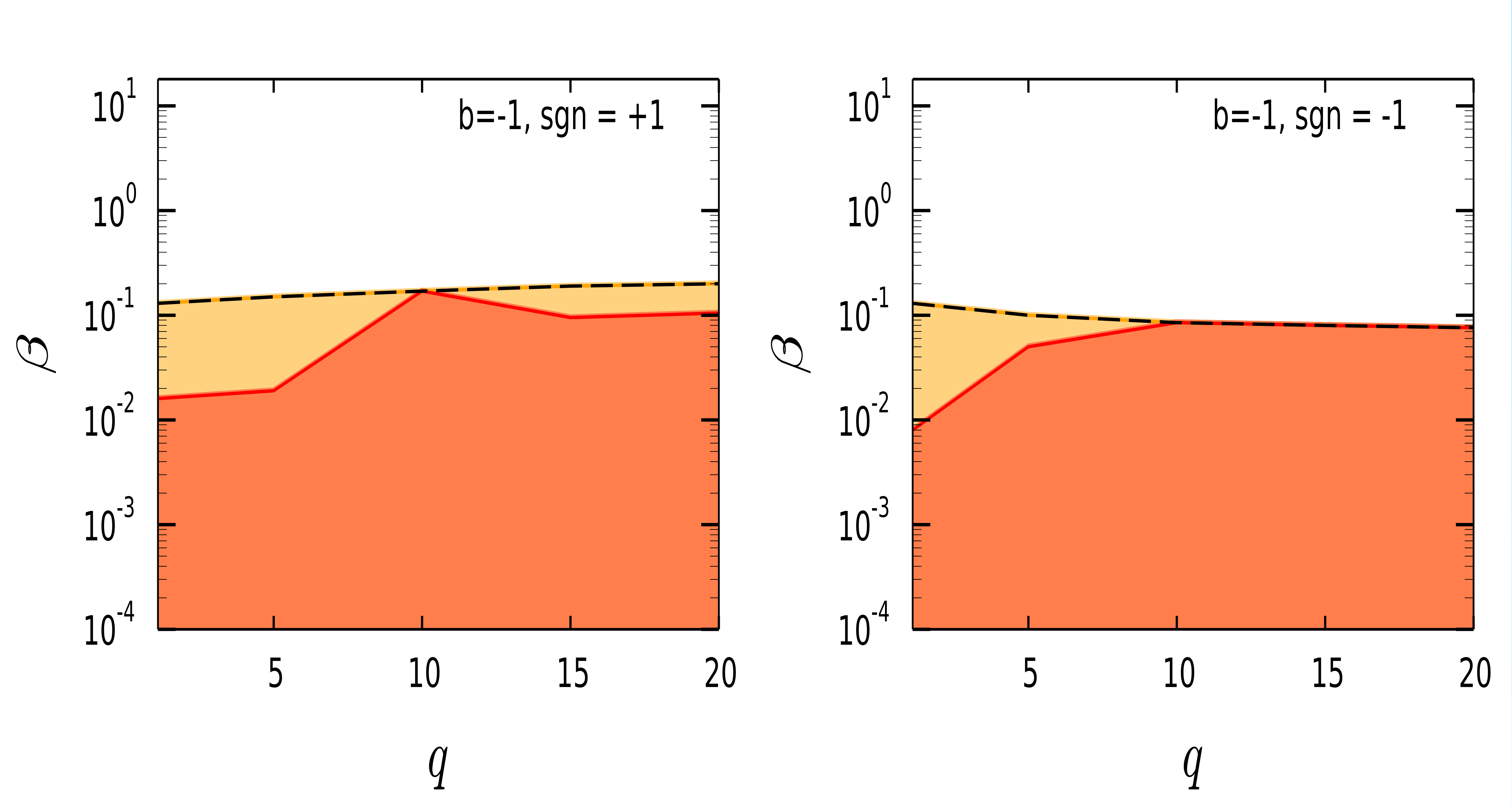
b=-5/3 (0 PN)



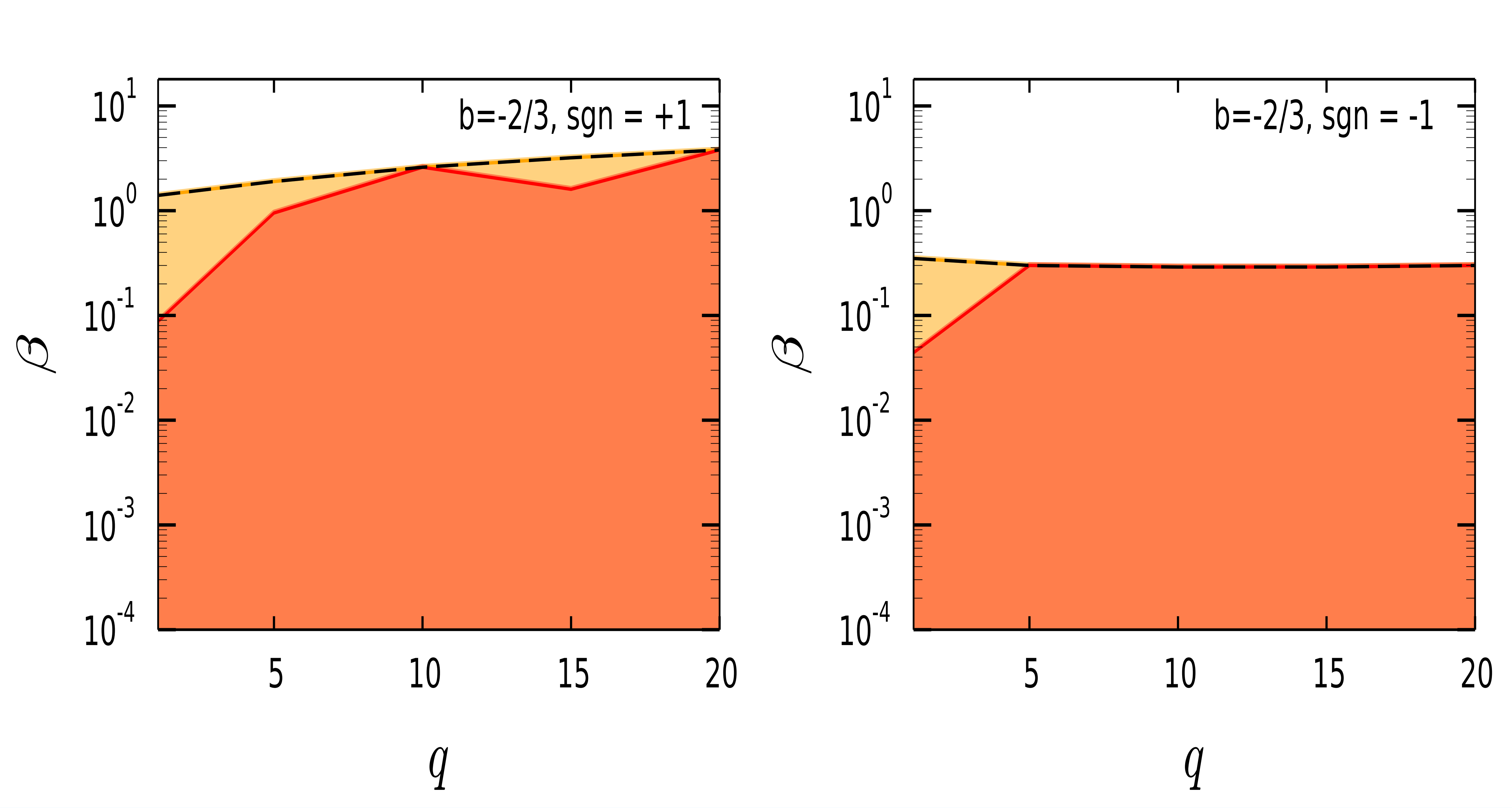
b=-4/3 (0.5 PN)



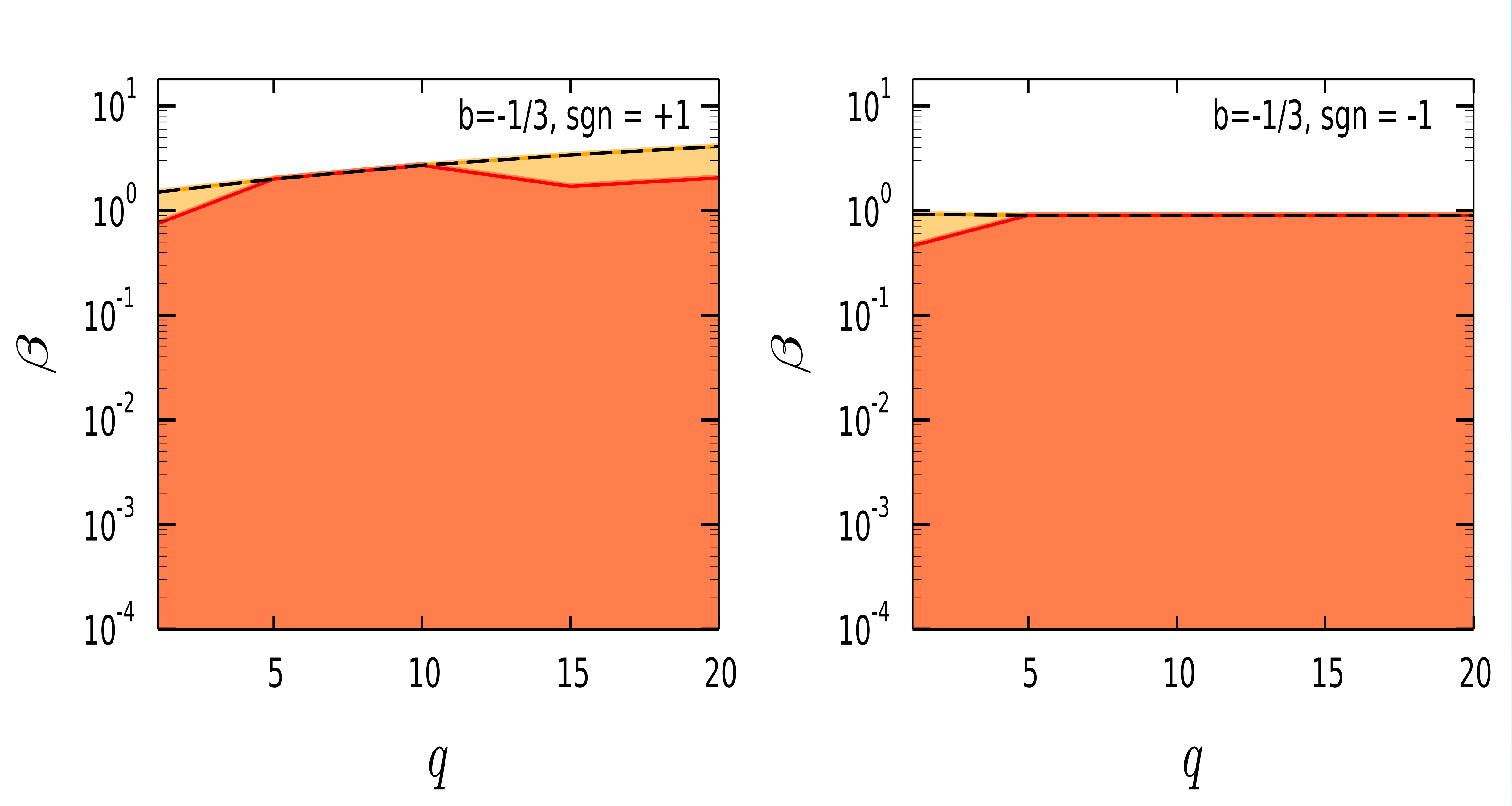
b=-1 (1 PN)



b=-2/3 (1.5 PN)



b=-1/3 (2 PN)



Conclusion

- Heavy investment in alternative theories of GR
- ppE provides a framework for generic tests
- Developed a generic time domain ppE waveform
- Current eLISA configuration not optimal for constraining alternative theories
- Effect of alternative configurations underway