The spin-2 sector and its interactions

Based on work in collaboration with:

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arXiv: 1311.7009, 1408.5131, 1409.7692, 1410.7774, 1411.4780 + work in progress

Johannes Noller, University of Oxford

IAP, Paris, GReCO Seminar, 09/02/15

Consistent field theories

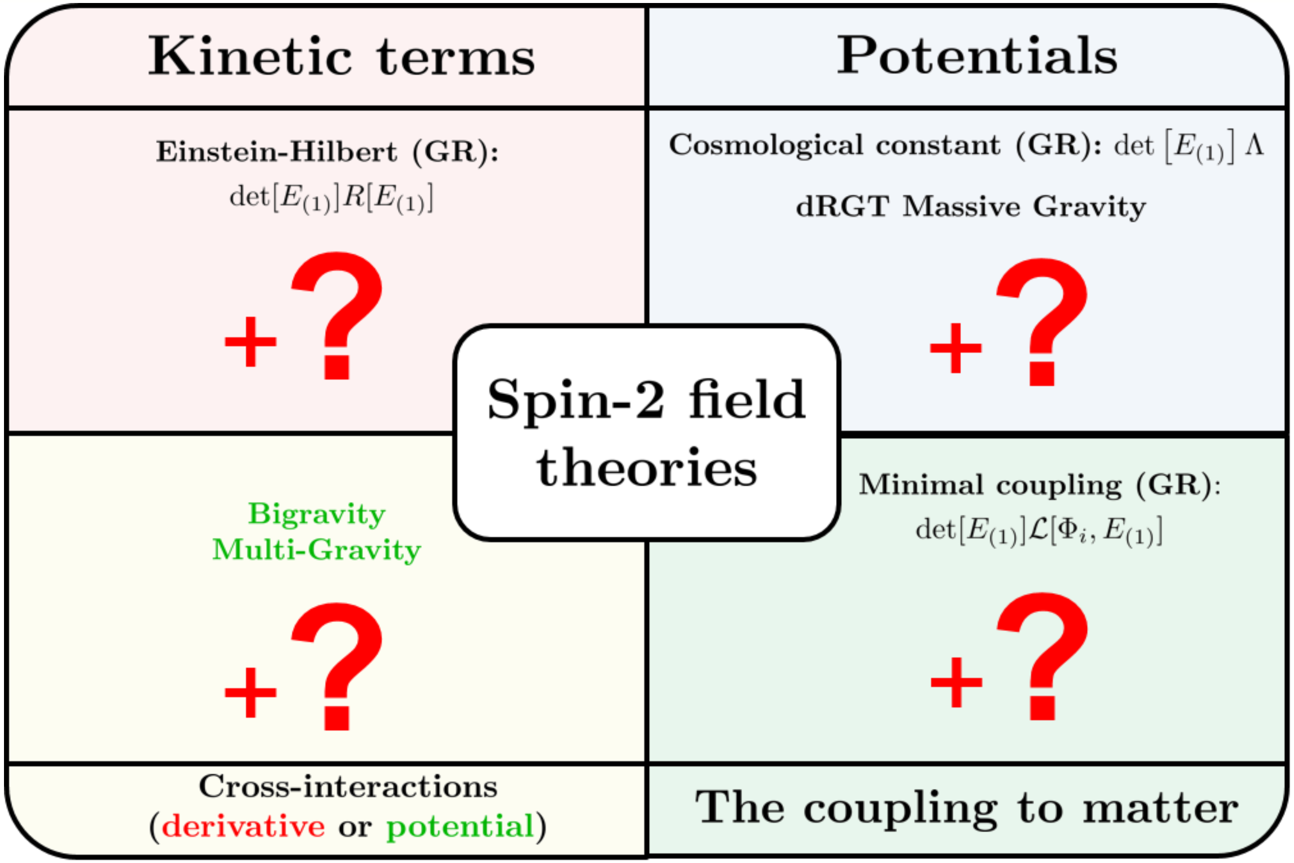
Spin	**Fields**
0	Higgs ϕ
1/2	leptons, quarks ψ^i
1	photons, W- and Z-bosons, gluons A^μ
2	‘graviton’ $g_{\mu\nu}$

Spin-2 field theories

Kinetic terms	Potentials
Cross-interactions (derivative or potential)	The coupling to matter

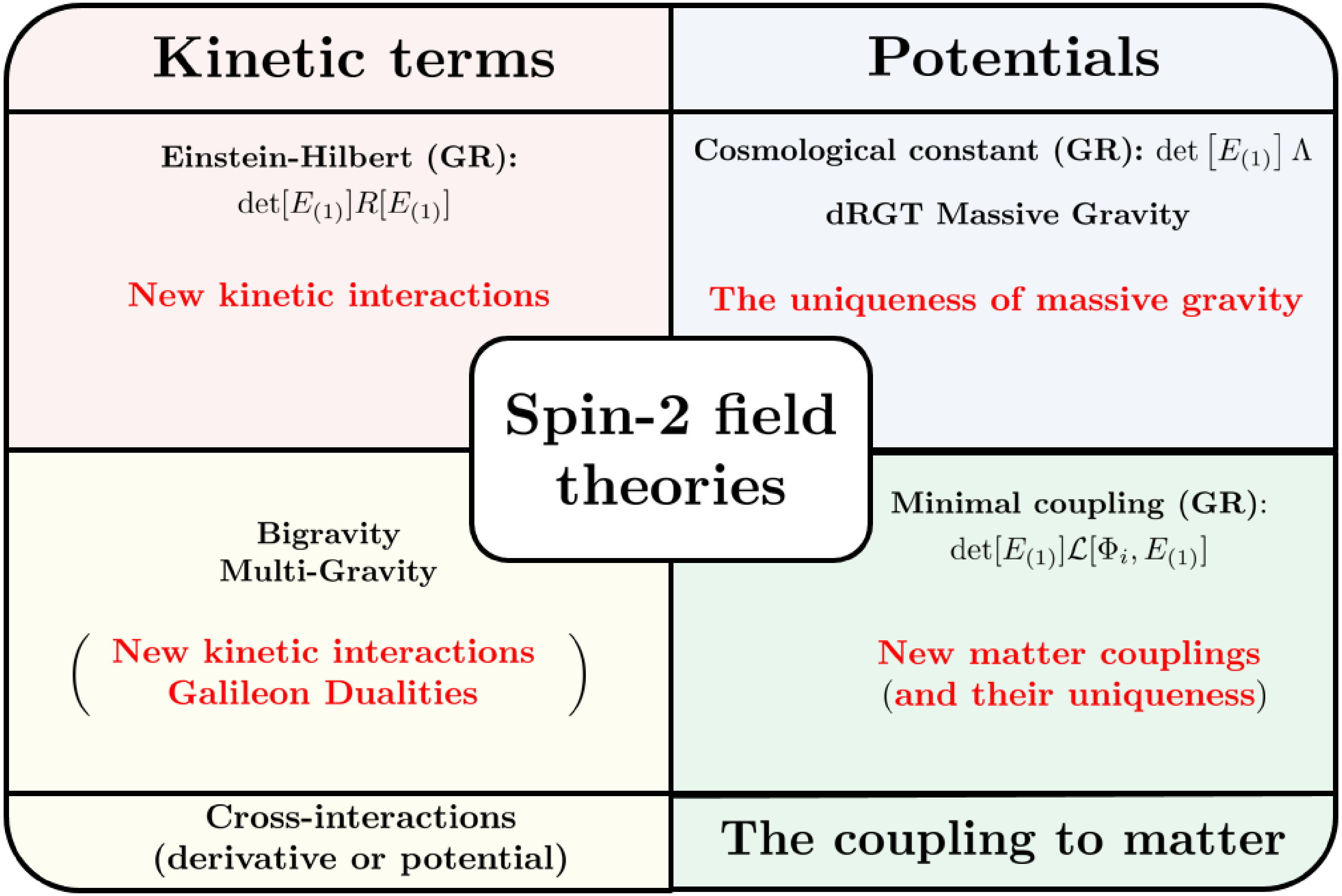
Spin-2 field theories

Spin-2 field theories



cf. *Akrami, Alexandrov, Babichev, Bernard, Bonifacio, Burrage, Comelli, Crisostomi, Deffayet, de Felice, de Rham, Enander, Fasiello, Ferreira, Gabadadze, Gao, Gumrukcuoglu, Hassan, Hinterbichler, Heisenberg, Kaloper, Keltner, Kimura, Koivisto, Koyama, Lagos, Mirbabayi, Matas, Melville, Mourad, Mukohyama, Nesti, Niz, Ondo, Padilla, Pilo, Pirtskhalava, Renaux-Petel, Ribeiro, Rosen, Sandstad, Scargill, Schmidt-May, Shang, Solomon, Steer, Tanaka, Tasinato, Tolley, von Strauss, Yamashita, Yamauchi, Zahariade . . . + many, many more!*

Spin-2 field theories

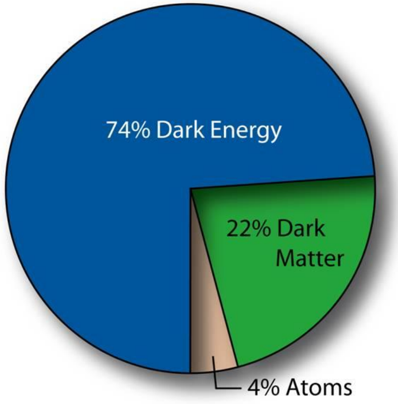


Why bother?

- Consistent theory building blocks

- CC problem I: Relevant operators modify IR (Massive Gravity).
- CC problem II: Degravitation (Massive Gravity), Partial Masslessness (?), ...
- CC problem III: New *dof* may provide self-acceleration.
- Technical naturalness.
- Irrelevant operators can provide Vainshtein screening.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{M_{\rm P}^2} T_{\mu\nu} - \Lambda_0 g_{\mu\nu}$$



What is a consistent EFT?

$$\mathcal{L}_\pi = \underbrace{-\frac{1}{2}\partial_\mu\pi\partial^\mu\pi}_{} + \underbrace{\frac{1}{\Lambda_{DL}}\Box\pi\partial_\mu\pi\partial^\mu\pi + \frac{1}{\Lambda_{(2)}}(\Box\pi)^3}_{\text{Irrelevant operators}} + \ldots + \underbrace{\frac{1}{\Lambda_{(M)}}\pi T^\mu_\mu}_{\text{Matter coupling}}$$

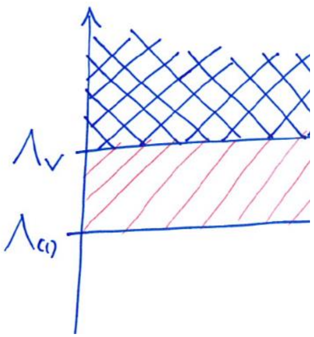
- Linear theory is free of instabilities.
- Non-linear (NL) interactions are ghost-free in the decoupling limit (DL) corresponding to the least-suppressed NL operators. In other words, there is a valid NL regime.
- As a consequence the healthy *dof* of the theory have 2nd order *eoms* in the DL.
- New interactions contribute non-vanishingly in DL.

Required

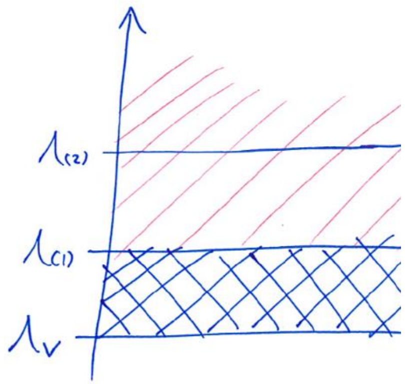
- The cutoff of the theory $\Lambda_c \gg \Lambda_{DL}$.
- Ghost-freedom at all scales. (ADM analysis)

Desirable

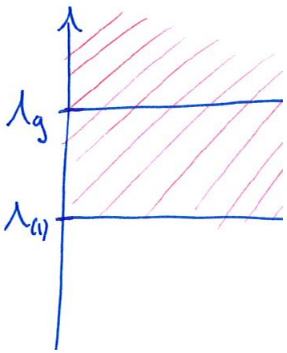
What is a consistent EFT?



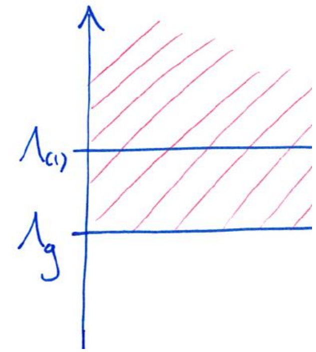
Cutoff scale $\Lambda_{(1)}$



Vainshtein scale Λ_V



Ghosts $\Lambda_g \gg \Lambda_{(1)}$

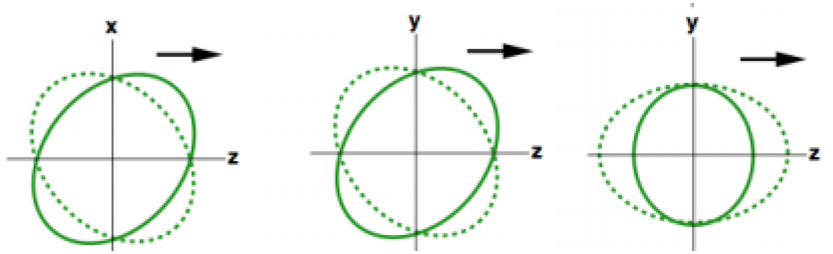


Ghosts $\Lambda_g \lesssim \Lambda_{(1)}$

$$\mathcal{L}_\pi = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \underbrace{\frac{1}{\Lambda_{DL}}\Box\pi\partial_\mu\pi\partial^\mu\pi + \frac{1}{\Lambda_{(2)}}(\Box\pi)^3}_{\text{Irrelevant operators}} + \ldots + \underbrace{\frac{1}{\Lambda_{(M)}}\pi T^\mu_\mu}_{\text{Matter coupling}}$$

Building consistent theories: An example

A healthy vector has 2 *dof* (massless) or 3 *dof* (massive): helicities $+1, -1, 0$



Maxwell terms and Proca theory:

$$\mathcal{L} = -\frac{1}{4}F_{ab}F^{ab} + m^2 A_a A^a$$
$$= -\frac{1}{2}(\partial_a A_b \partial^b A^a - \partial_b A_a \partial^b A^a) + m^2 A_a A^a$$

Broken gauge symmetry $\delta A_a = \partial_a \Lambda$ can be restored via $A_a \to A_a + \partial_a \phi$

What healthy interactions can we have for a vector field?
Systematically investigate $\mathcal{L}_{d,n}$

Building consistent theories: An example

General Lagrangian $\mathcal{L}_{2,2}$:

$$\begin{aligned}\mathcal{L}_{2,2} &= C_1\partial_a A_b \partial^b A^a + C_2 \partial_b A_a \partial^b A^a + C_3 \partial_a A^a \partial_b A^b \\ &\to C_1\partial_a A_b \partial^b A^a + C_2 \partial_b A_a \partial^b A^a\end{aligned}$$

Restore (linear) gauge symmetry/project out *dof*s via $A_a \to A_a + \partial_a \phi$:

$$\mathcal{L} \to C_1\partial_a A_b \partial^b A^a + C_2 \partial_b A_a \partial^b A^a + 2(C_1 + C_2)\partial_b\partial_a\phi\partial^b A^a + (C_1 + C_2)\partial_b\partial_a\phi\partial^b\partial^a\phi$$

Equations of motion:

$$\begin{aligned}\mathcal{E}_\phi &: \quad 2(C_1 + C_2)\partial_b\partial^b\partial_a A^a + 2(C_1 + C_2)\partial_b\partial^b\partial_a\partial^a\phi = 0, \\ \mathcal{E}_A &: \quad -2C_2\partial_a\partial^a A_b - 2(C_1 + C_2)\partial_a\partial^a\partial_b\phi - 2C_1\partial_a\partial_b A^a = 0\end{aligned}$$

$$\boxed{C_1 = -C_2}$$

Building consistent theories: An example

General Lagrangian $\mathcal{L}_{2,2}$:

$$\mathcal{L}_{2,2} = C_1(\partial_a A_b \partial^b A^a - \partial_b A_a \partial^b A^a) \propto F_{ab}F^{ab}$$

Equations of motion:

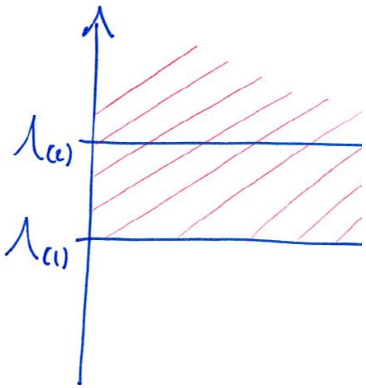
$$\mathcal{E}_A \quad : \quad \partial_a \partial^a A_b - \partial_a \partial_b A^a = 0$$

All dependence on helicity-0 mode ϕ drops out and we recover the (linear) gauge symmetry for free at this order.

The Maxwell term is the unique consistent $\mathcal{L}_{2,2}$ term for a vector.

All consistent interactions at arbitrary orders can be found this way.

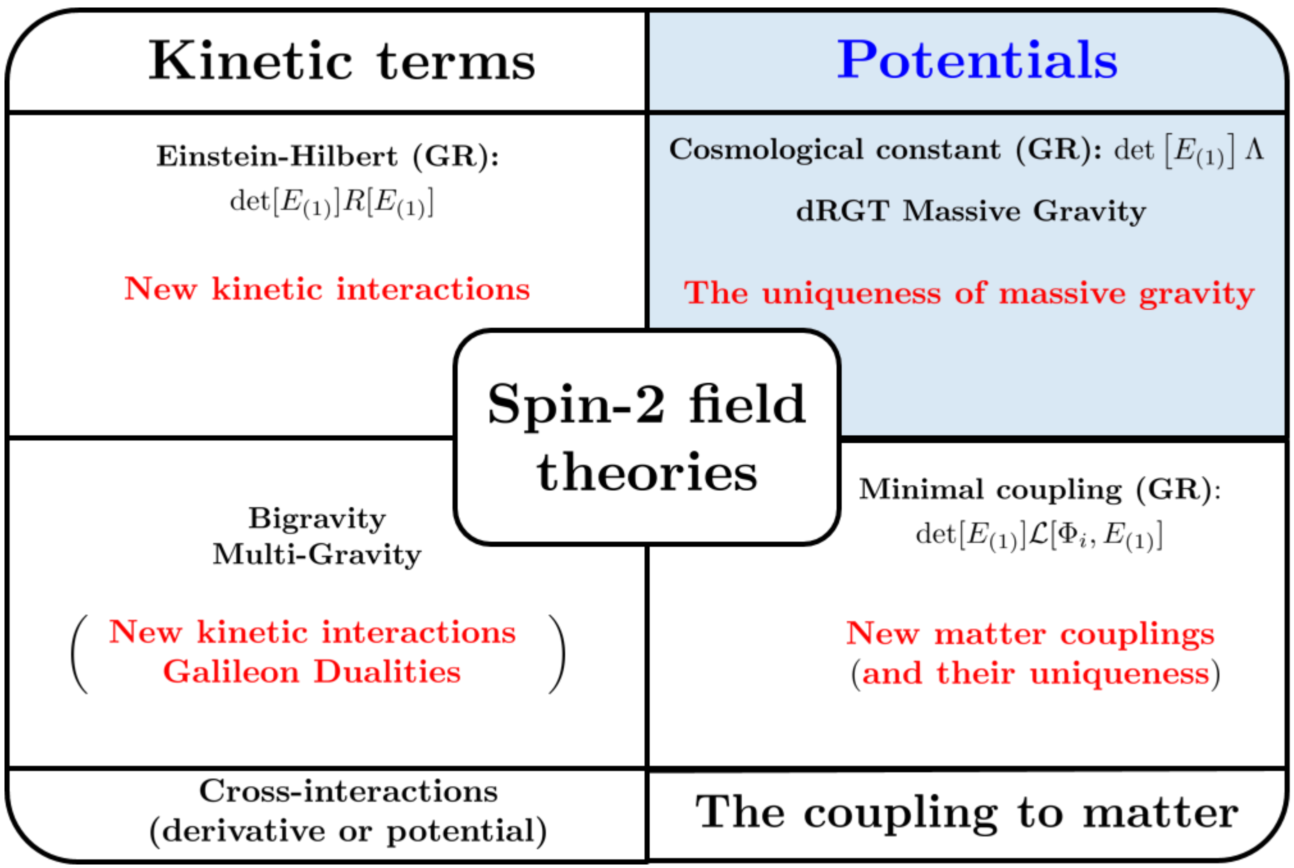
Consistent spin-2 theories



- Construct most general local, Lorentz-invariant action.
- Project out *dofs* and order interactions by energy scales.
- Impose 2nd order *eoms* order-by-order from below.
- Stop once non-vanishing interaction term is reached.

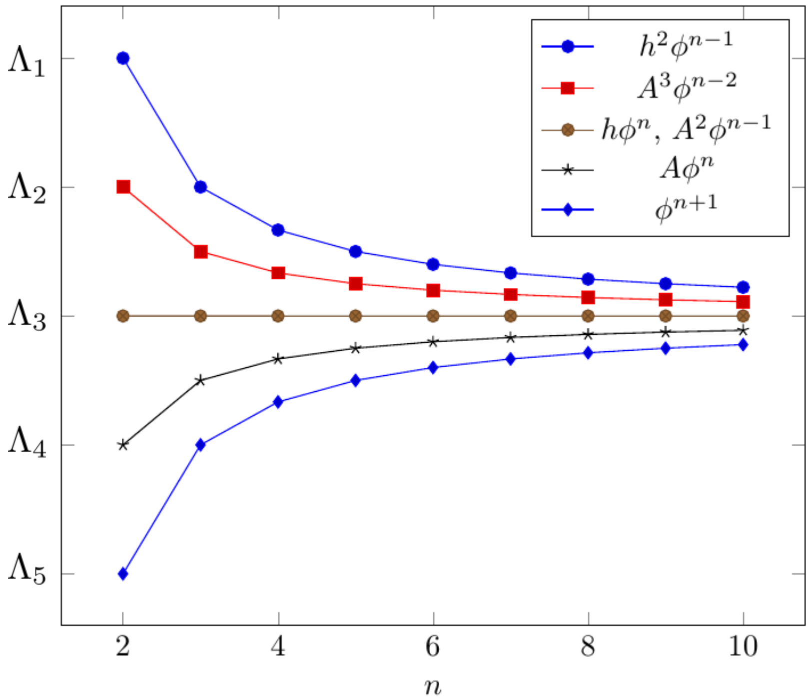
Spin-1 field theories	**Spin-2 field theories**
Degrees of freedom: $\phi, A_a \;\; (\pm 1, 0)$	**Degrees of freedom:** $\phi, A_a, h_{ab} \;\; (\pm 2, \pm 1, 0)$
Scales: m	**Scales:** M_{Pl}, m
Stückelberg replacement:	**Stückelberg replacement:**
$A_a \to A_a + \partial_a \phi$	$h^{ab} \to h^{ab} + \partial^a A^b + 2\partial^a \partial^b \phi + \partial^b A^a,$ $h^{ab} \to h^{ab} + \partial^a A^b + \partial^b A^a + 2\partial^b \partial^a \phi$ $- \; \partial^a A^c \partial^b A_c - \partial^b A^c \partial_c \partial^a \phi$ $- \; \partial^a A^c \partial_c \partial^b \phi - \partial_c \partial^b \phi \partial^c \partial^a \phi.$

Potential interactions: dRGT (and beyond?)



Potential terms I: The road to dRGT

$$S = \int d^4x\sqrt{-g}R - \frac{m^2}{4}\int d^4x\sqrt{-g}V(g,h), \qquad g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$



$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda} = m\left(\frac{M_P}{m}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2},$$

Potential terms I: The road to dRGT

$$S = \int d^4x\sqrt{-g}R - \frac{m^2}{4}\int d^4x\sqrt{-g}V(g,h), \qquad g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

General potential: $V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots,$

$$\begin{aligned}
V_2(g,h) &= b_1\langle h^2\rangle + b_2\langle h\rangle^2, \\
V_3(g,h) &= c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
V_5(g,h) &= f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle + f_6\langle h^2\rangle\langle h\rangle^3 + \\
&\quad f_7\langle h\rangle^5, \\
V_6(g,h) &= g_1\langle h^6\rangle + g_2\langle h^5\rangle\langle h\rangle + g_3\langle h^4\rangle\langle h^2\rangle + g_4\langle h^4\rangle\langle h^1\rangle^2 + g_5\langle h^3\rangle^2 + g_6\langle h^3\rangle\langle h^2\rangle\langle h^1\rangle + \\
&\quad g_7\langle h^3\rangle\langle h\rangle^3 + g_8\langle h^2\rangle^3 + g_9\langle h^2\rangle^2\langle h\rangle^2 + g_{10}\langle h^2\rangle\langle h\rangle^4 + g_{11}\langle h\rangle^6, \\
&\vdots
\end{aligned}$$

$$\begin{aligned}
h^{ab} &\rightarrow h^{ab} + \partial^a A^b + \partial^b A^a + 2\partial^b\partial^a\phi - \partial^a A^c\partial^b A_c \\
&- \partial^b A^c\partial_c\partial^a\phi - \partial^a A^c\partial_c\partial^b\phi - \partial_c\partial^b\phi\partial^c\partial^a\phi.
\end{aligned}$$

Potential terms I: The road to dRGT

$$S = \int d^4x\sqrt{-g}R - \frac{m^2}{4}\int d^4x\sqrt{-g}V(g,h), \qquad g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

2 free coefficients

Fierz-Pauli

$$V_2(g,h) = b_1\langle h^2\rangle + b_2\langle h\rangle^2,$$

Wess-Zumino

$$V_3(g,h) = c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3,$$

$$V_4(g,h) = d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4,$$

0-polynomials

$$V_5(g,h) = f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle + f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5,$$

$$V_6(g,h) = g_1\langle h^6\rangle + g_2\langle h^5\rangle\langle h\rangle + g_3\langle h^4\rangle\langle h^2\rangle + g_4\langle h^4\rangle\langle h^1\rangle^2 + g_5\langle h^3\rangle^2 + g_6\langle h^3\rangle\langle h^2\rangle\langle h^1\rangle + g_7\langle h^3\rangle\langle h\rangle^3 + g_8\langle h^2\rangle^3 + g_9\langle h^2\rangle^2\langle h\rangle^2 + g_{10}\langle h^2\rangle\langle h\rangle^4 + g_{11}\langle h\rangle^6,$$

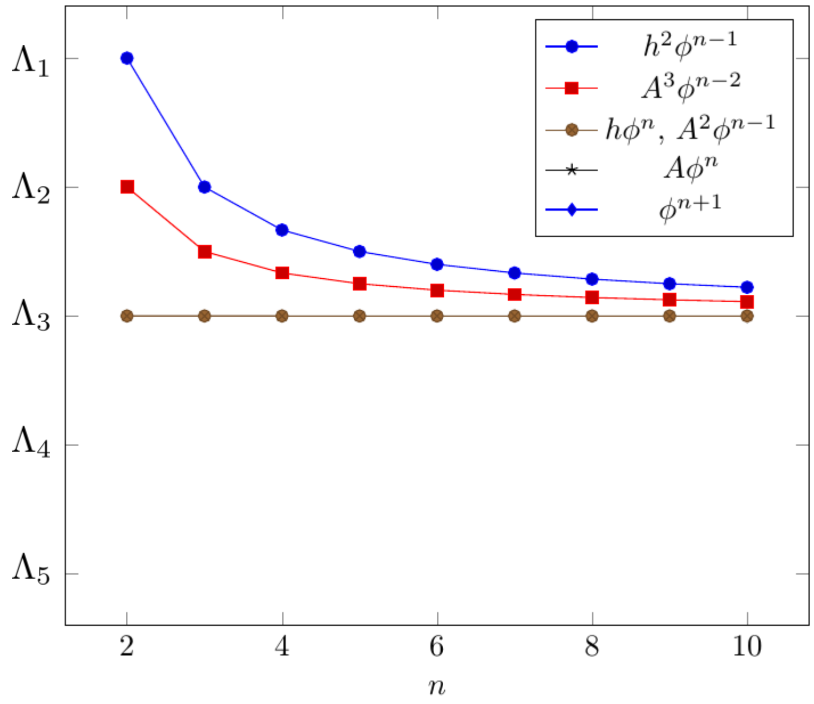
$\vdots$

de Rham, Gabadadze ’10; de Rham, Gabadadze, Tolley ‘10; Hassan and Rosen ‘11

Potential terms II: Strong coupling scales

Scale of interactions Λ:

$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda} = m\left(\frac{M_P}{m}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2},$$



Λ_3 **Decoupling/Scaling limit :** $\quad m \to 0, \quad M_{Pl} \to \infty, \quad \Lambda_3$ fixed

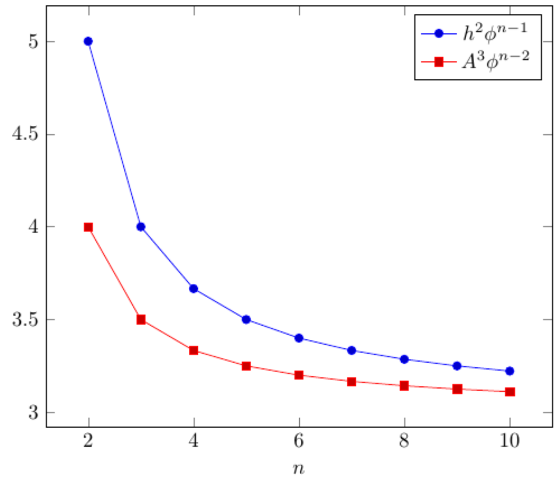
Potential terms II: Strong coupling scales

The 'minimal model': $c_3 = \frac{1}{6}, d_5 = -\frac{1}{48}$

$$S_{min} = -M_{Pl}^2 \int d^4x\sqrt{-g}\left(R + 2m^2(\mathrm{Tr}\sqrt{g^{-1}\eta} - 3)\right).$$

Consider $h^2\Pi^n$ **interactions** $\rightarrow$ **find non-vanishing pieces for all** n**:**

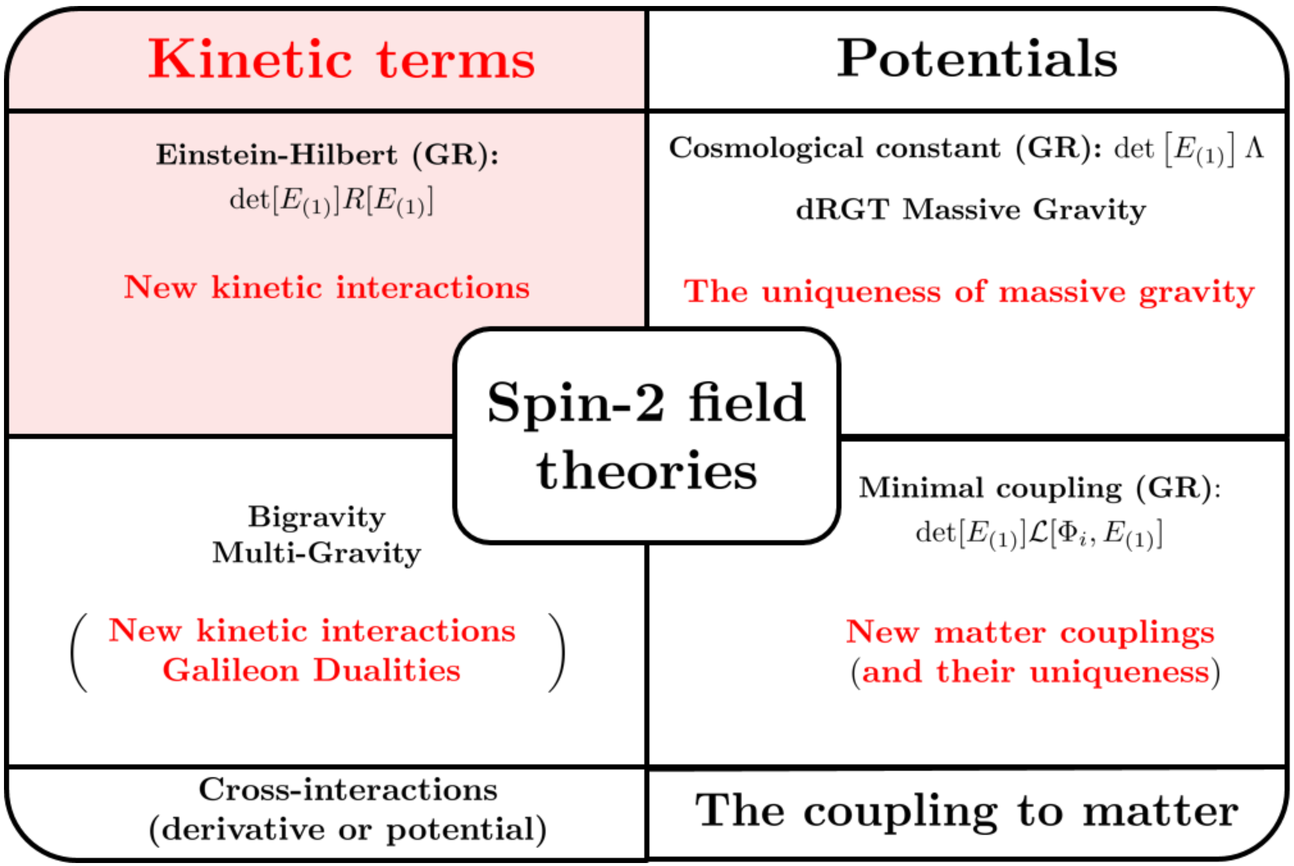
$$\sum_{n=2}^{\infty} c_n n\left[h^2(\delta - \Pi)^2(-2\Pi + \Pi^2)^{n-2}\right] = \frac{1}{4}\sum_{m=1}^{\infty}\frac{1}{2^m}\left[h^2\Pi^m\right]$$



The maximal strong coupling scale for massive gravity is $\Lambda_{3+\epsilon}$

Renaux-Petel '14, Bonifacio, JN '14

Derivative interactions: GR and beyond



Kinetic terms I: GR

Generic Quadratic action up to TD:

$$\mathcal{L}_{2,2} = A_1 \partial_b H^c{}_c \partial^b H^a{}_a + A_2 \partial^b H^a{}_a \partial_c H_b{}^c + A_3 \partial_b H_{ac} \partial^c H^{ab} + A_4 \partial_c H_{ab} \partial^c H^{ab}$$

Imposing 2nd order *eom*s:

$$A_1(\partial_b H^c{}_c \partial^b H^a{}_a - 2\partial^b H^a{}_a \partial_c H_b{}^c + 2\partial_b H_{ac} \partial^c H^{ab} - \partial_c H_{ab} \partial^c H^{ab})$$

This uniquely singles out linearised Einstein-Hilbert

Kinetic terms I: GR

Generic Quadratic action up to TD:

$$\mathcal{L}_{2,2} = A_1 \partial_b H^c{}_c \partial^b H^a{}_a + A_2 \partial^b H^a{}_a \partial_c H_b{}^c + A_3 \partial_b H_{ac} \partial^c H^{ab} + A_4 \partial_c H_{ab} \partial^c H^{ab}$$

Imposing 2nd order *eom*s:

$$A_1 (\partial_b H^c{}_c \partial^b H^a{}_a - 2\partial^b H^a{}_a \partial_c H_b{}^c + 2\partial_b H_{ac} \partial^c H^{ab} - \partial_c H_{ab} \partial^c H^{ab})$$

This uniquely singles out linearised Einstein-Hilbert

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^D x \sqrt{-g} R + \frac{m^2 M_{\rm Pl}^2}{2} \int \mathrm{d}^D x \sqrt{-g} \sum_n \alpha_n \bar{\mathcal{L}}_n^{(\rm m)} + \Lambda_{\rm der}^2 \int \mathrm{d}^D x \mathcal{L}_{\rm der}$$

Take $\Lambda_{\rm der}^2 = M_{Pl} \Lambda_3$ for convenience.

Non-linear nature of diff symmetry requires non-linear completion.

$$\begin{aligned} H^{ab} \quad \rightarrow \quad & H^{ab} + \partial^a A^b + \partial^b A^a + 2\partial^b \partial^a \phi - \partial^a A^c \partial^b A_c \\ - \quad & \partial^b A^c \partial_c \partial^a \phi - \partial^a A^c \partial_c \partial^b \phi - \partial_c \partial^b \phi \partial^c \partial^a \phi. \end{aligned}$$

Kinetic terms II: $\mathcal{L}_{2,n}$

Quadratic order:	A_1,						
Cubic order:	A_1,	B_1,	B_2,				
Quartic order:	A_1,	B_1,	B_2,	C_1,	C_2,		
Quintic order:	A_1,	B_1,	B_2,	C_1,	C_2,	D_1,	D_2

- Non-linear completion of linearised EH.

$$\begin{aligned} \mathcal{L}_{2,3}^{(A_1)} &= \tfrac{1}{2} H^a{}_a \partial_c H^d{}_d \partial^c H^b{}_b + H^{ab} \partial_c H_a{}^c \partial_d H_b{}^d + H^a{}_a \partial_b H^{bc} \partial_d H_c{}^d \\ &- H^a{}_a \partial^c H^b{}_b \partial_d H_c{}^d - H^{ab} \partial_c H_{bd} \partial^d H_a{}^c - \tfrac{1}{2} H^a{}_a \partial_d H_{bc} \partial^d H^{bc} \end{aligned}$$

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Quadratic order:	A_1,						
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- Non-linear completion of linearised EH.
- New ghost-free interaction - vanishes in DL, no fully ghost-free non-linear completion (ADM).

$$\epsilon_{\mu_1 \ldots \mu_{n+1}} \epsilon^{\nu_1 \ldots \nu_{n+1}} (\partial^{\mu_1} H^{\mu_2}_{\nu_2})(\partial_{\nu_1} H^{\mu_3}_{\nu_3}) H^{\mu_4}_{\nu_4} \ldots H^{\mu_{n+1}}_{\nu_{n+1}} = \epsilon\epsilon H^{n-2} \partial H \partial H$$

Hinterbichler '14; de Rham, Matas, Tolley '13

Kinetic terms II: $\mathcal{L}_{2,n}$

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- Non-linear completion of linearised EH.
- New ghost-free interaction - vanishes in DL, no fully ghost-free non-linear completion (ADM).
- New interactions that vanish in DL - induces ghost at higher scales.

$$\begin{aligned}\mathcal{L}_{2,3}^{(B_2)} &= H^{ab}\partial_a H^{cd}\partial_b H_{cd} - H^{ab}\partial_a H^c{}_c\partial_b H^d{}_d + 2H^{ab}\partial_b H^d{}_d\partial_c H_a{}^c \\ &- 3H^{ab}\partial_c H_a{}^c\partial_d H_b{}^d - 2H^{ab}\partial_b H_a{}^c\partial_d H_c{}^d + 3H^{ab}\partial_c H_{bd}\partial^d H_a{}^c\end{aligned}$$

de Rham, Matas, Tolley '13; de Rham, Matas, JN in progress

Kinetic terms II: $\mathcal{L}_{2,n}$

Quadratic order:	A_1,						
Cubic order:	A_1,	B_1,	B_2,				
Quartic order:	A_1,	B_1,	B_2,	C_1,	C_2,		
Quintic order:	A_1,	B_1,	B_2,	C_1,	C_2,	D_1,	D_2

- Non-linear completion of linearised EH.
- New ghost-free interaction - vanishes in DL, no fully ghost-free non-linear completion (ADM).
- New interactions that vanish in DL - induces ghost at higher scales.
- Dependence on higher-order coefficients vanishes due to dimensionally-dependent identities.

$$\epsilon_{\mu_1\ldots\mu_{n+1}}\epsilon^{\nu_1\ldots\nu_{n+1}}(\partial^{\mu_1}H^{\mu_2}_{\nu_2})(\partial_{\nu_1}H^{\mu_3}_{\nu_3})H^{\mu_4}_{\nu_4}\ldots H^{\mu_{n+1}}_{\nu_{n+1}} = \epsilon\epsilon H^{n-2}\partial H\partial H$$

$$\mathcal{V}^{(i)}_{2,n} = f(H^n)\epsilon_{\mu_1\ldots\mu_{D+i}}\epsilon^{\nu_1\ldots\nu_{D+i}}(\partial^{\mu_1}H^{\mu_2}_{\nu_2})(\partial_{\nu_1}H^{\mu_3}_{\nu_3})H^{\mu_4}_{\nu_4}\ldots H^{\mu_{D+i}}_{\nu_{D+i}}$$

de Rham, Matas, JN in progress; JN in progress

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Cubic order:	A_1,	B_1,	B_2,				
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- Non-linear completion of linearised EH.
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- New interactions that vanish in DL - induces ghost at higher scales.
- Dependence on higher-order coefficients vanishes due to dimensionally-dependent identities.

4-parameter family of solutions in the DL.

Only A_1 contributes non-vanishingly in this limit, however.

de Rham, Matas, JN in progress; JN in progress

Kinetic terms II: $\mathcal{L}_{4,n}$

Cubic order:	$B_1,$	$B_2,$	$\dots$			
Quartic order:	$B_1,$	$B_2,$	$\dots$	$C_1,$	$C_2,$	$\dots$

- Quadratic order vanishes identically, B_1 is diff-invariant.
- New ghost-free interactions - vanish due to dimensionally-dependent identities.
- New interactions that vanish in DL - induces ghost at higher scales.
- Dependence on higher-order coefficients vanishes due to dimensionally-dependent identities.

Many-parameter family of solutions, but no contributions in DL.

Cubic order $: \epsilon\epsilon H^{n-2}\partial^2 H\partial^2 H, \epsilon\epsilon H^{n-3}\partial H\partial H\partial^2 H$

de Rham, Matas, JN in progress; JN in progress

The coupling to other fields

Spin-2 field theories

Kinetic terms	**Potentials**
Einstein-Hilbert (GR): $\det[E_{(1)}]R[E_{(1)}]$ **New kinetic interactions**	**Cosmological constant (GR):** $\det\left[E_{(1)}\right]\Lambda$ **dRGT Massive Gravity** **The uniqueness of massive gravity**
Bigravity **Multi-Gravity** (**New kinetic interactions** **Galileon Dualities**)	**Minimal coupling (GR):** $\det[E_{(1)}]\mathcal{L}[\Phi_i, E_{(1)}]$ **New matter couplings** (**and their uniqueness**)
Cross-interactions (derivative or potential)	**The coupling to matter**

The coupling to matter

Vielbein formulation:

$$g_{(i)\,\mu\nu} = E_{(i)\,\mu}{}^{A}\, E_{(i)\,\nu}{}^{B}\, \eta_{AB}$$

Consistent (potential) interaction terms:

$$\mathcal{S}^{(i_1 i_2 i_3 i_4)}_{\text{potential}} \quad = \quad \int \tilde{\epsilon}_{A_1 A_2 \cdots A_D}\, \tilde{\epsilon}^{\mu_1 \mu_2 \cdots \mu_D}\, E_{(i_1)\,\mu_1}{}^{A_1}\, E_{(i_2)\,\mu_2}{}^{A_2} E_{(i_3)\,\mu_3}{}^{A_3} E_{(i_4)\,\mu_4}{}^{A_4}\, d^4x$$

Bigravity:

$$E_{(1)\,\mu}{}^{A} \text{ and } E_{(2)\,\mu}{}^{A} \Longrightarrow \mathcal{S}^{(1111)}, \mathcal{S}^{(1112)}, \mathcal{S}^{(1122)}, \mathcal{S}^{(1222)}, \mathcal{S}^{(2222)}$$

Massive Gravity:

$$E_{(2)\,\mu}{}^{A} = \delta_{\mu}{}^{A}$$

Nibbelink, Peloso, Sexton '06; Hanada, Shinoda, Shiraishi '08; Hinterbichler, Rosen '12; Deffayet, Mourad, Zahariade '12, Hassan, Schmidt-May, von Strauss '12; Banados, Deffayet, Pino '13, Mourad, Steer '14, ..

The coupling to matter

The weak equivalence principle:

$$\mathcal{S}_{\text{matter}} = \int d^4x \sqrt{-g^{(M)}} \mathcal{L}\left[\Phi_i, g^{(M)}_{\mu\nu}\right]$$

The 'matter metric':

$$g^{(M)} = g^{(M)}\left[E_{(1)\mu}{}^{A}, \ldots, E_{(N)\mu}{}^{A}, \eta_{AB}\right]$$

'Cosmological constant' type terms:

$$\int d^4x \sqrt{-g^{(M)}} \iff \mathcal{S}_{\text{potential}}$$

de Rham, Heisenberg, Ribeiro '14, JN, Melville '14

The coupling to matter

The general consistent matter metric:

$$g^{(M)}_{\mu\nu} = \sum_{i=1}^{N} \alpha^2_{(ii)} E^{(i)}{}_\mu{}^A E^{(i)}{}_\nu{}^B \eta_{AB}$$
$$+ \sum_{i,j=1,\dots,N}^{i<j} \alpha_{(ii)}\alpha_{(jj)} \left(E^{(i)}{}_\mu{}^A E^{(j)}{}_\nu{}^B + E^{(j)}{}_\mu{}^A E^{(i)}{}_\nu{}^B \right) \eta_{AB}$$

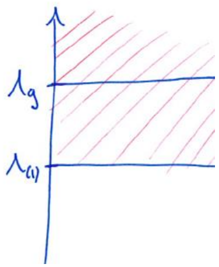
Original metrics | Cross-terms + ST symmetry | Ghost-freedom

The matter vielbein:

$$g^{(M)}_{\mu\nu} = E^{(M)}{}_\mu{}^A E^{(M)}{}_\nu{}^B \eta_{AB}, \qquad E^{(M)}{}_\mu{}^A = \sum_{i=1}^{N} \alpha_{(ii)} E^{(i)}{}_\mu{}^A .$$

JN, Melville '14

The coupling to matter



$$\mathcal{L}_{\rm matter} = -\frac{1}{2}\sqrt{-g_{(M)}}\left(g^{ab}_{(M)}\partial_a\chi\partial_b\chi + M^2\chi^2\right)$$

In the Λ_3 decoupling limit the 'matter contribution' to $\mathcal{E}_\phi$ is

$$\mathcal{E}_\phi^{\rm matter} = \partial_a\left(\sqrt{-g_{(M)}}T^{bc}\underbrace{\left(\Gamma^d_{bc}((\alpha+\beta)\delta^a_d - \beta\partial_d\partial^a\phi) + \beta\partial_d\partial^a\partial_c\phi\right)}_{\equiv 0}\right)$$

Hence there is no ghost in this decoupling limit. Beyond DL_{Λ_3}, however, we have

$$\mathcal{H} \supset \frac{1}{m^2 M_{\rm Pl}^2}\frac{\alpha^2\beta^2}{(\alpha+\beta)^2}(\partial_i\chi)^2 p_\chi^2 N^2$$

de Rham, Heisenberg, Ribeiro '14

Einstein-Jordan frames for these couplings $\Leftrightarrow$ Kinetic terms

JN '14

Conclusions

Spin-2 field theories

Kinetic terms

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New kinetic interactions

Potentials

Cosmological constant (GR): $\det\left[E_{(1)}\right]\Lambda$

dRGT Massive Gravity

The uniqueness of massive gravity

Cross-interactions (derivative or potential)

Bigravity
Multi-Gravity

(New kinetic interactions
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Minimal coupling (GR):
$\det[E_{(1)}]\mathcal{L}[\Phi_i, E_{(1)}]$

New matter couplings
(and their uniqueness)

Thank you!