Realistic Cosmic Strings

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Outline

Topological defects

- **Topological defects**
  - Phase transitions with Spontaneous Symmetry Breaking (SSB)

$$G \xrightarrow{\langle \Phi \rangle} G'$$

  - General predictions
  - Topological properties of G/G'

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- **Different kinds**
  - Monopoles
  - Domain walls
  - Cosmic strings

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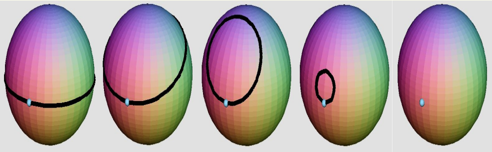
  - Vortex lines
  - $\pi_1(G/G') \neq \mathbf{1}$



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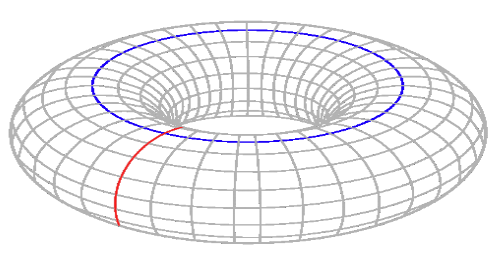
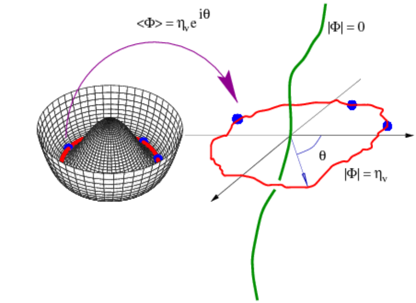
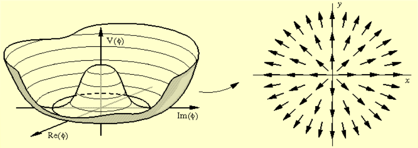


Abelian Higgs model

$$\mathcal{L} = -\left(\partial_\mu + iqA_\mu\right)\phi^*\left(\partial^\mu - iqA^\mu\right)\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \kappa^2(\phi^*\phi - M^2)^2$$



- **Ansatz**
  - Cylindrical coordinates locally along the string (r, θ, z, t)
  - $\phi = f(r)\mathrm{e}^{in\theta}$ with $f \in \mathbb{R}$
  - $A^\mu = A_\theta(r)\tau_{\mathrm{str}}\delta^\theta_\mu$



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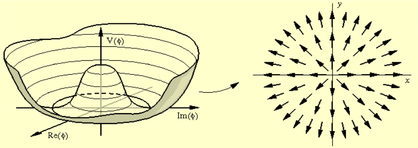
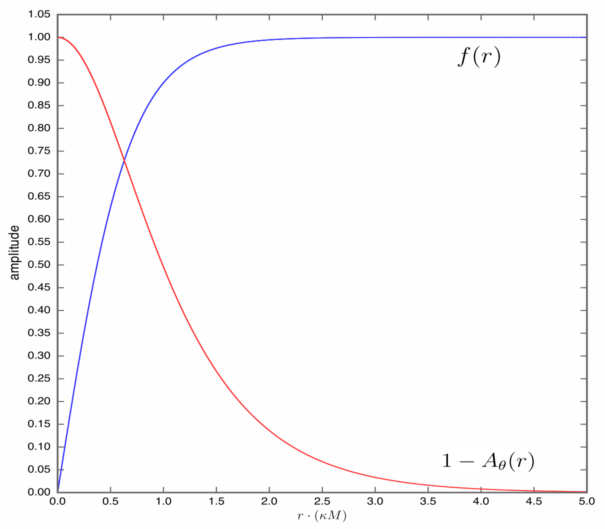


  - $\lim_{r\to\infty} f(r) = M$ and $\lim_{r\to\infty} A^{\mathrm{str}}_\theta(r) = n/g$
  - $f(0) = 0$ and $A_\theta(0) = 0$


1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
amplitude
$f(r)$
$1 - A_\theta(r)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
$r \cdot (\kappa M)$

Macroscopic properties

- **From $T^{\mu\nu}$**
  - Symmetries $\Rightarrow T^{ab}$ with $a, b \in \{z, t\}$
  - Diagonalization $\Rightarrow$ energy per unit length and tension

$$U = \int 2\pi r \mathrm{d}r T^{t't'} \qquad T = -\int 2\pi r \mathrm{d}r T^{z'z'}$$

  - Equation of state $U = f(T)$

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- **Nambu-Goto String**
  - Lorentz-invariant along the worldsheet
  - $\Rightarrow U = T$

Cosmic strings and cosmology

- **Grand Unified Theory (GUT)**
  - Particle physics in one gauge group G_{GUT}, e.g. SO(10) or SU(6)
  - SSB to Standard Model (SM) at $E_{\mathrm{GUT}} \sim (10^{15} - 10^{16})\mathrm{GeV}$

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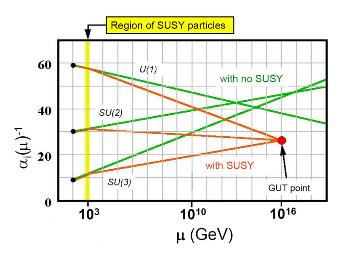
  - Many theoretical motivations, e.g. $q \in \mathbf{Z}/3$ or coupling constants



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  - During SSB from the GUT to the SM
  - Formation of strings at the end of inflation

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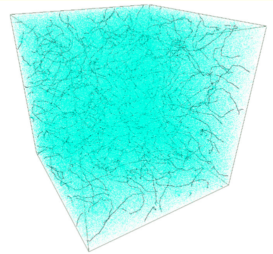
  - During SSB from the GUT to the SM
  - Formation of strings at the end of inflation
  - Almost unavoidable consequence

$$
\mathrm{SU}(6)\begin{cases}
\stackrel{1}{\to} 5\ 1_6 & \begin{cases} \stackrel{2}{\to} 5 \stackrel{1}{\to} G_{\mathrm{SM}} \\ \stackrel{1}{\to} 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1\ 1 \stackrel{2}{\to} G_{\mathrm{SM}} \\ \stackrel{1,2}{\to} G_{\mathrm{SM}} \end{cases} \\
\stackrel{1}{\to} 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1 & \begin{cases} \stackrel{1}{\to} 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1\ 1 \stackrel{2}{\to} G_{\mathrm{SM}} \\ \stackrel{2'}{\to} G_{\mathrm{SM}} \end{cases} \\
\stackrel{1}{\to} 3_{\mathrm{C}}\ 3_{\mathrm{L}}\ 1 & \begin{cases} \stackrel{1}{\to} 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1\ 1 \stackrel{2}{\to} G_{\mathrm{SM}} \\ \stackrel{2'}{\to} G_{\mathrm{SM}} \end{cases} \\
\stackrel{1}{\to} 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1\ 1 \stackrel{2}{\to} G_{\mathrm{SM}} \\
\stackrel{0}{\to} 5 \stackrel{1}{\to} G_{\mathrm{SM}} \\
\stackrel{1}{\to} G_{\mathrm{SM}}.
\end{cases}
$$

Jeannerot, Rocher, Sakellariadou, Phys.Rev., D68:103514, (2003)

- **String Network**
  - Initial conditions from correlation length
  - Numerical simulations



Ringeval, Sakellariadou, Bouchet, JCAP 0702 (2007)

- **Evolution from large to small scales**

- **In the CMB**
  - Good candidate before inflation to seed the LSS formation
  - Power spectrum different from inflationary prediction

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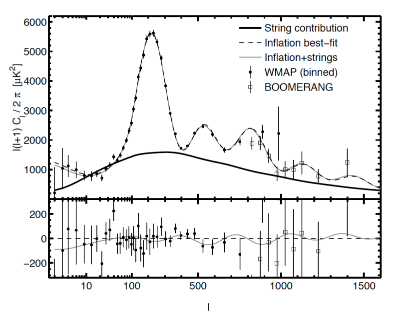
  - Good candidate before inflation to seed the LSS formation
  - Power spectrum different from inflationary prediction
  - No significant observations



Bevis, Hindmarsh, Kunz, Urrestilla, Phys. Rev. Lett. 100 (2008)

Necessity of a realistic implementation

- **Toy model strings**
  - Only scale of formation appears
  - Does not encode the GUT

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- **Toy model strings**
  - Only scale of formation appears
  - Does not encode the GUT
- **Realistic implementation**
  - U as function of the GUT parameters
  - New structure and properties
  - Could differ from toy-models

Outline

SUSY GUT

- **GUT**
  - Gauge group SO(10)
  - 45 gauge field A_μ^a
  - Higgs field content : $\mathbf{\Sigma}$ in **126**, $\overline{\mathbf{\Sigma}}$ in $\overline{\mathbf{126}}$ and $\mathbf{\Phi}$ in **210**

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  - Higgs field content : $\mathbf{\Sigma}$ in **126**, $\overline{\mathbf{\Sigma}}$ in $\overline{\mathbf{126}}$ and $\mathbf{\Phi}$ in **210**
- **Valid with particle physics**
  - Recover the SM at low energy
  - Coupling constant, mass spectrum, proton decay...

- **Supersymmetry (SUSY)**
  - Bosonic part of the supermultiplets
  - F-term theory $\Rightarrow$ D-term identically zero

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  - Bosonic part of the supermultiplets
  - F-term theory $\Rightarrow$ D-term identically zero
- **Potential from F-terms**
  - Superpotential W, singlet, X^3 ($\Rightarrow X^4 \in V$)
  - Potential from F-terms

$$F_X = \frac{\partial W}{\partial X} \quad \text{and} \quad V_X = F^\dagger F$$

- **With SO(10)**

$$W = \frac{m}{2}\mathbf{\Phi}^2 + m_{\Sigma}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \frac{\lambda}{3}\mathbf{\Phi}^3 + \eta\mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}}$$

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$$F_\Phi = m\mathbf{\Phi} + \lambda(\mathbf{\Phi}^2)_{\mathbf{\Phi}} + \eta(\mathbf{\Sigma}\overline{\mathbf{\Sigma}})_{\mathbf{\Phi}}$$

$$\begin{aligned} V_\Phi =& m^2\mathbf{\Phi}\mathbf{\Phi}^\dagger + |\eta|^2(\mathbf{\Sigma}\overline{\mathbf{\Sigma}})_{\mathbf{\Phi}}(\mathbf{\Sigma}\overline{\mathbf{\Sigma}})^\dagger_{\mathbf{\Phi}} + |\lambda|^2(\mathbf{\Phi}^2)_{\mathbf{\Phi}}(\mathbf{\Phi}^2)^\dagger_{\mathbf{\Phi}} \\ &+ \left[\lambda\eta^*(\mathbf{\Phi}^2)_{\mathbf{\Phi}}(\mathbf{\Sigma}\overline{\mathbf{\Sigma}})^\dagger_{\mathbf{\Phi}} + + m\lambda^*\mathbf{\Phi}(\mathbf{\Phi}^2)^\dagger_{\mathbf{\Phi}} + m\eta^*\mathbf{\Phi}(\mathbf{\Sigma}\overline{\mathbf{\Sigma}})^\dagger_{\mathbf{\Phi}}\right] + \text{h.c.} \end{aligned}$$

Hybrid inflation

- **Hybrid inflation (HI)**
  - Inflationary potential from GUT physics
  - Inflaton S, SO(10) singlet
  - Additional term in W

$$W_{\mathrm{HI}} = \kappa S \left(\mathbf{\Sigma} \overline{\mathbf{\Sigma}} - M^2 \right)$$

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$$W_{\rm HI} = \kappa S \left(\mathbf{\Sigma} \overline{\mathbf{\Sigma}} - M^2 \right)$$

- **Generalization**
  - SUSY, HI, SO(10) = example

$$V_S = \kappa^2 (\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)^2$$

$$\begin{aligned} V_{\overline{\Sigma}} =& m_{\Sigma}^2 \mathbf{\Sigma}\mathbf{\Sigma}^\dagger + |\eta|^2 (\mathbf{\Phi}\mathbf{\Sigma})_{\mathbf{\Sigma}} (\mathbf{\Phi}\mathbf{\Sigma})_{\mathbf{\Sigma}}^\dagger + \kappa^2 S S^* \mathbf{\Sigma}\mathbf{\Sigma}^\dagger + \Big[m_{\Sigma} \kappa S^* \mathbf{\Sigma}\mathbf{\Sigma}^\dagger \\ & + \eta\kappa S^* (\mathbf{\Phi}\mathbf{\Sigma})_{\mathbf{\Sigma}} \mathbf{\Sigma}^\dagger + m_{\Sigma} \eta^* \mathbf{\Sigma} (\mathbf{\Phi}\mathbf{\Sigma})_{\mathbf{\Sigma}}^\dagger \Big] + \text{h.c.} \end{aligned}$$

$$V_{\Sigma} = V_{\overline{\Sigma}} (\mathbf{\Sigma} \longleftrightarrow \overline{\mathbf{\Sigma}})$$

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Cosmological evolution

- **SSB scheme**

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$$\mathrm{SO}(10) \xrightarrow{\langle\Phi\rangle} G' \xrightarrow{\langle\Sigma\rangle} G_{\mathrm{SM}} \times \mathbf{Z}_2$$

  - Monopoles form at first step (before inflation)
  - Inflation begins after first step, ends at second step
  - Cosmic strings form at second step (after inflation)

- **First step**
  - $V_\Sigma \sim |\kappa S \mathbf{\Sigma}|^2 \Rightarrow \mathbf{\Sigma} = 0$
  - $V_\Phi = 0 \Rightarrow$ several solutions, associated to different G'
  - Inflaton : $V = \kappa^2 M^4 +$ quant. corr. $\Rightarrow$ inflation begins

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- **Second step**
  - $S \longrightarrow S_{\text{crit}}$
  - $V = 0 \Rightarrow$ several solutions, different gauge symmetries

- **Requirement**
  - Recover the SM at low energy
  - Stability of inflationary valley

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  - Stability of inflationary valley
- **Two possible schemes**
  - $G'_1 = 3_C 2_L 2_R 1_{B-L}$
  - $G'_2 = 3_C 2_L 1_R 1_{B-L}$

Description with singlet of the SM

- **Proper description**
  - Vacuum Expectations Values (VEVs) at the end of SSB
  - No symmetry restoration
  - Simplify the model

- **Branching rules**
  - Description on a (maximal) subalgebra
  - SU(4) ⊂ SU(3)×U(1)
  - Restricted representations w.r.t. the subalgebra

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$$\left\{\begin{array}{l}\mathbf{1}=\mathbf{1}(0)\\ \mathbf{10}=\mathbf{1}(-6)+\mathbf{3}(-2)+\mathbf{6}(2)\\ \mathbf{15}=\mathbf{1}(0)+\mathbf{3}(4)+\overline{\mathbf{3}}(-4)+\mathbf{8}(0)\end{array}\right.$$

- **SO(10) case**
  - $\mathbf{\Sigma}$ in **126**, $\overline{\mathbf{\Sigma}}$ in $\overline{\mathbf{126}}$ and $\mathbf{\Phi}$ in **210**
  - Pati-Salam group : $2_L 2_R 4_C$

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$$\begin{aligned}
\mathbf{\Phi}_p &= \mathbf{\Phi}(1,1,1), & \boldsymbol{\sigma} &= \mathbf{\Sigma}(1,3,\overline{10}),\\
\mathbf{\Phi}_a &= \mathbf{\Phi}(1,1,15), & \overline{\boldsymbol{\sigma}} &= \overline{\mathbf{\Sigma}}(1,3,10),\\
\mathbf{\Phi}_b &= \mathbf{\Phi}(1,3,15), & S &= S(1,1,1).
\end{aligned}$$

- **SSB scheme**
  - $V(\boldsymbol{\Phi}_a, \boldsymbol{\Phi}_b, \boldsymbol{\Phi}_p, \boldsymbol{\sigma}, \overline{\boldsymbol{\sigma}}, S)$
  - Properly describe the SSB scheme

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$$\mathrm{SO}(10) \xrightarrow{\langle\Phi\rangle} G' \xrightarrow{\langle\Sigma\rangle} G_{\mathrm{SM}} \times \mathbf{Z}_2$$

  - $G'_1 = 3_C 2_L 2_R 1_{B-L} \, : \, \mathbf{\Phi}_a \neq 0$
  - $G'_2 = 3_C 2_L 1_R 1_{B-L} \, : \, \mathbf{\Phi}_a \neq 0$, $\mathbf{\Phi}_b \neq 0$ and $\mathbf{\Phi}_p \neq 0$

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Cosmic strings

$$\mathrm{SO}(10) \xrightarrow{\langle\Phi\rangle} G' \xrightarrow{\langle\Sigma\rangle} G_{\mathrm{SM}} \times \mathbf{Z}_2$$

- **String-forming step**
  - From G' to $G_{\mathrm{SM}} \times \mathbf{Z}_2$
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- **Abelian string**
  - Non abelian case left aside
  - No current
  - $\mathrm{U}(1)_{\mathrm{str}}$ and τ_{str}
  - $\mathrm{U}(1)_{\mathrm{str}}$=$\mathrm{U}(1)_{\mathrm{B\text{-}L}}$

- **Ansatz for the string**
  - Cylindrical coordinates locally along the string (r, θ, z, t)
  - $\mathbf{\Sigma} = \overline{\mathbf{\Sigma}} = 0$ in the center of string
  - VEVs defining the SM at infinity

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  - Cylindrical coordinates locally along the string (r, θ, z, t)
  - $\mathbf{\Sigma} = \overline{\mathbf{\Sigma}} = 0$ in the center of string
  - VEVs defining the SM at infinity
- **Coupling with the gauge field**
  - Only $\mathbf{\Sigma}$ and $\overline{\mathbf{\Sigma}}$ charged under $\mathrm{U}(1)_{\mathrm{str}}$
  - Phase in $\mathrm{e}^{\pm in\theta}$

- **Use of SM singlets**
  - (1) Potential at least quadratic in the non singlet terms

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  - (2) They all vanish at infinity
  - (1) + (2) $\Rightarrow$ configuration where they all vanish permitted by the EOM

Tensor formulation

- **Fields**
  - $\mathbf{\Sigma}$: (**126**) fifth rank anti-symmetric tensor Σ_{ijklm}, self dual

$$\Sigma_{ijklm} = \frac{i}{5!}\epsilon_{ijklmabcde}\Sigma_{abcde},$$

  - $\overline{\mathbf{\Sigma}}$ ($\overline{\mathbf{126}}$): fifth rank anti-symmetric tensor $\overline{\Sigma}_{ijklm}$, anti-self-dual

$$\overline{\Sigma}_{ijklm} = -\frac{i}{5!}\epsilon_{ijklmabcde}\overline{\Sigma}_{abcde},$$

  - $\mathbf{\Phi}$ (**210**) fourth rank anti-symmetric tensor Φ_{ijkl}
  - S singlet: complex function

$$W = \frac{1}{2} m \Phi_{ijkl}\Phi_{ijkl} + m_\Sigma \Sigma_{ijklm}\overline{\Sigma}_{ijklm} + \frac{1}{3}\lambda \Phi_{ijkl}\Phi_{klmn}\Phi_{mnij} + \eta \Phi_{ijkl}\Sigma_{ijmno}\overline{\Sigma}_{klmno} + \kappa S(\Sigma_{ijklm}\overline{\Sigma}_{ijklm} - M^2)$$

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- ***F*-terms computation**
  - Taking into account that the components are not independent
  - F_X in the conjugate representation of X

$$(F_\Phi)_{ijkl} = m\Phi_{ijkl} + \lambda \Phi_{[ij|ab}\Phi_{ab|kl]} + \eta \Sigma_{[ij|abc}\overline{\Sigma}_{|kl]abc}$$

- **Group theoretical approach**

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$$\mathbf{1_{210\times 126\times \overline{126}} \ni 126 \times (210 \times \overline{126})_{\overline{126}}}$$

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$$\mathbf{\Phi\Sigma\overline{\Sigma}} = \mathbf{1}_{\mathbf{210}\times\mathbf{126}\times\overline{\mathbf{126}}}$$

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$$\mathbf{1}_{\mathbf{210}\times\mathbf{126}\times\overline{\mathbf{126}}} \ni \mathbf{126}\times(\mathbf{210}\times\overline{\mathbf{126}})_{\overline{\mathbf{126}}}$$

$$\Rightarrow \frac{\partial(\mathbf{\Phi\Sigma\overline{\Sigma}})}{\partial\mathbf{\Sigma}} = (\mathbf{\Phi\overline{\Sigma}})_{\overline{\Sigma}}$$

- **SM singlet formulation**

$$\mathbf{\Phi}_a = a(x^\mu)\langle\mathbf{\Phi}_a\rangle_0 \quad \text{with} \quad \langle\mathbf{\Phi}_a\rangle_0\langle\mathbf{\Phi}_a\rangle_0^\dagger = 1$$

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$$\begin{cases} \mathbf{\Sigma} = \langle\overline{\mathbf{\Sigma}}\rangle^\dagger = \sigma(x^\mu)\langle\boldsymbol{\sigma}\rangle_0, \\ \mathbf{\Phi} = a(x^\mu)\langle\mathbf{\Phi}_a\rangle_0 + b(x^\mu)\langle\mathbf{\Phi}_b\rangle_0 + p(x^\mu)\langle\mathbf{\Phi}_p\rangle_0, \end{cases}$$

$$\begin{cases} \dfrac{p}{\sqrt{4!}} = \Phi_{1234} \\ \dfrac{a}{\sqrt{4!3}} = \Phi_{5678} = \Phi_{5690} = \Phi_{7890} \\ \dfrac{b}{\sqrt{4!6}} = \Phi_{1256} = \Phi_{1278} = \Phi_{1290} = \Phi_{3456} = \Phi_{3478} = \Phi_{3490} \\ \dfrac{1}{\sqrt{5!2^5}}(i)^{(-a-b+c+d+e)}\sigma = \Sigma_{a+1,b+3,c+5,d+7,e+9} \\ \dfrac{1}{\sqrt{5!2^5}}(-i)^{(-a-b+c+d+e)}\overline{\sigma} = \overline{\Sigma}_{a+1,b+3,c+5,d+7,e+9} \end{cases}$$

$$\langle \mathbf{\Phi} \rangle \langle \mathbf{\Phi} \rangle^{\dagger} = pp^* + aa^* + bb^*$$

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$$\langle \mathbf{\Phi\Phi} \rangle_\Phi \langle \mathbf{\Phi} \rangle^\dagger = \underbrace{\frac{1}{9\sqrt{2}}}_{\sqrt{N}} a^2 a^* + \left[\frac{1}{6\sqrt{6}} \left(2b^* bp + b^2 p^* \right) + \frac{1}{9\sqrt{2}} \left(2b^* ba + b^2 a^* \right) \right]$$

$$\langle \mathbf{\Phi} \rangle \langle \mathbf{\Phi} \rangle^\dagger = pp^* + aa^* + bb^*$$

$$\langle \mathbf{\Phi\Phi} \rangle_\Phi \langle \mathbf{\Phi} \rangle^\dagger = \underbrace{\frac{1}{9\sqrt{2}}}_{\sqrt{N}} a^2 a^* + \left[\frac{1}{6\sqrt{6}} \left(2b^* bp + b^2 p^* \right) + \frac{1}{9\sqrt{2}} \left(2b^* ba + b^2 a^* \right) \right]$$

$$\langle \mathbf{\Phi\Phi} \rangle_\Phi \langle \mathbf{\Sigma} \overline{\mathbf{\Sigma}} \rangle_\Phi^\dagger = \underbrace{\frac{a^2 \sigma \sigma^*}{180}}_{N} + \frac{b^2 \sigma \sigma^*}{120} - \frac{ab\sigma\sigma^*}{45\sqrt{2}} - \frac{bp\sigma\sigma^*}{30\sqrt{6}},$$

$$K = -\left|\left(\nabla_\mu - igA_\mu^{\text{str}}\right)\sigma\right|^2 - \left|(\nabla_\mu p)\right|^2 - \left|(\nabla_\mu a)\right|^2 - \left|(\nabla_\mu b)\right|^2 - \left|(\nabla_\mu S)\right|^2 - \frac{\text{Tr}\left({\tau_{\text{str}}}^2\right)}{4} F_{\mu\nu}^{\text{str}} F^{\mu\nu\,\text{str}}$$

Ansatz for the string

- **Ansatz**

$$\begin{cases} p = p(r) \\ a = a(r) \\ b = b(r) \\ \sigma = f(r)\mathrm{e}^{in\theta} \\ A_\mu = A_\theta^{\mathrm{str}}(r)\tau^{\mathrm{str}}\delta_\mu^\theta \\ S = S(r) \end{cases}$$

- **Boundary conditions**
  - $r \to \infty$: $\begin{cases} f, a, b, p, S \to \text{SM VEVs} \\ A_\theta = n \end{cases}$

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  - $r \to \infty$: $\begin{cases} f, a, b, p, S \to \text{SM VEVs} \\ A_\theta = n \end{cases}$

  - $r = 0$: $\begin{cases} f = A_\theta = 0 \\ \dfrac{\mathrm{d}x}{\mathrm{d}r} = 0 \text{ for } a, b, p \text{ and } S \end{cases}$

Outline

Numerical solution

- **Numerical Implementation**
  - Successive overrelaxed method
  - On a finite range lattice
  - Around 2000 configurations studied

- **Range of GUT parameters**

$$W = \frac{m}{2}\mathbf{\Phi}^2 + m_\Sigma \mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \frac{\lambda}{3}\mathbf{\Phi}^3 + \eta\mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2).$$

$$\text{with} \begin{cases} \dfrac{m}{M} = \dfrac{m_\Sigma}{M} \in \{1, 20\} \\ \kappa \in \{10^{-2}, 30\} \\ \eta \in \{10^{-2}, 10\} \\ \lambda \in \{10^{-2}, 1\} \end{cases}$$

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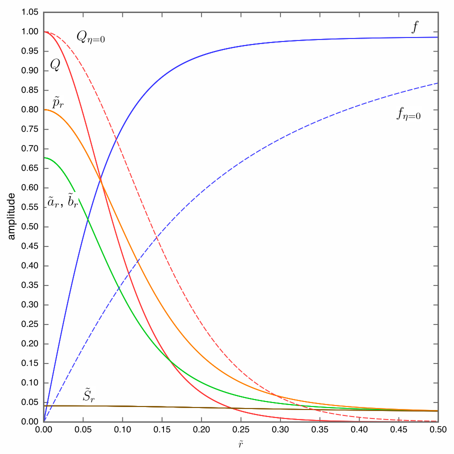
- **Toy-model limit:** $\eta \to 0$

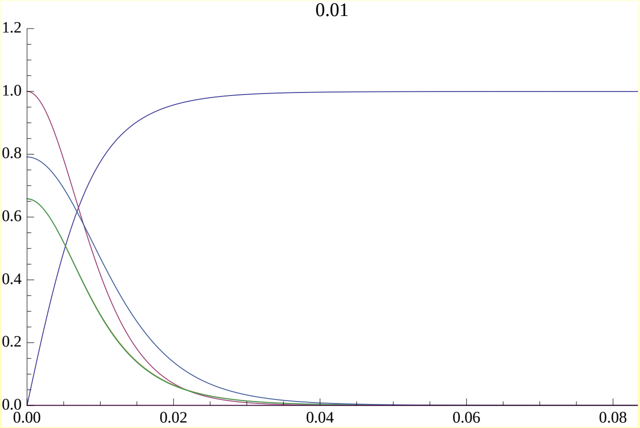
Microscopic structure



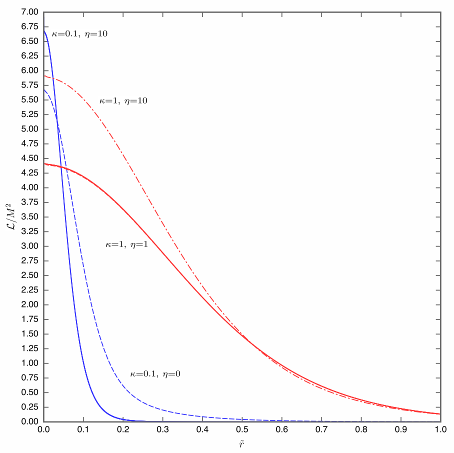
X(r)

- $\eta = 10$, $\kappa \in \{10^{-2}, 30\}$

$$W \supset \eta \mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$$

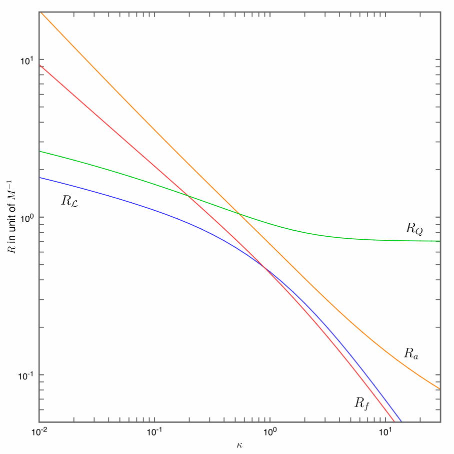


- **Lagrangian density** $W \supset \eta \mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$



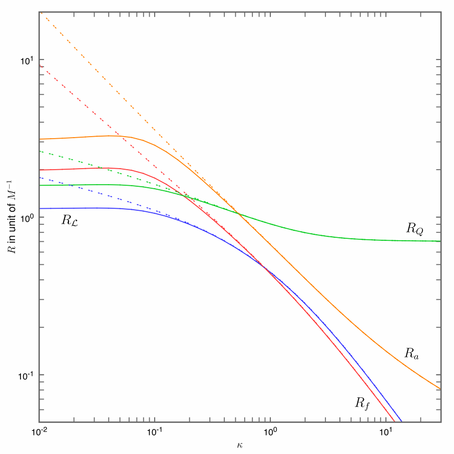
$\mathcal{L}(r)$

- **Characteristic radiuses** $W \supset \eta \mathbf{\Phi} \mathbf{\Sigma} \overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$



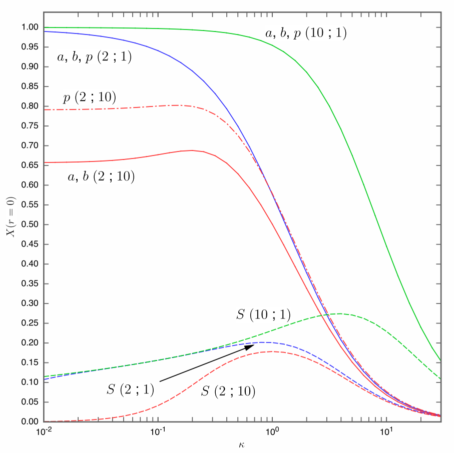
R_X

- **Characteristic radiuses** $W \supset \eta \mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$



R_X

- **Condensation in the string** $W \supset \eta \mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$

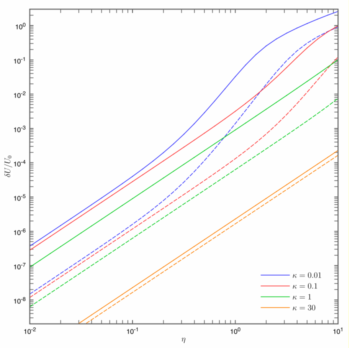


$X(r=0)$ for $(m/M, \eta)$

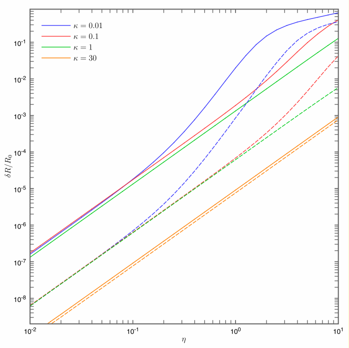
Macroscopic structure

- **Influence of η**

$$W \supset \eta \mathbf{\Phi}\mathbf{\Sigma}\overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$$

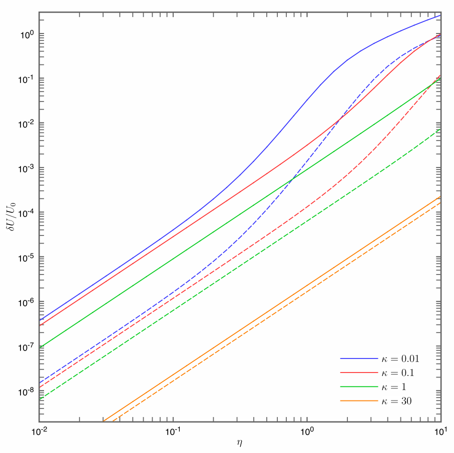


$\delta U/U_0$



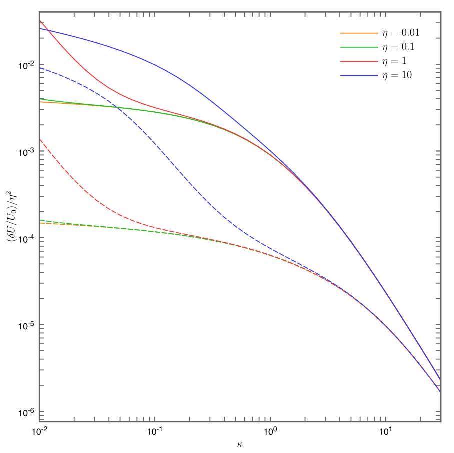
$\delta R/R_0$

- **Influence of η** $$W \supset \eta \mathbf{\Phi} \mathbf{\Sigma} \overline{\mathbf{\Sigma}} + \kappa S(\mathbf{\Sigma}\overline{\mathbf{\Sigma}} - M^2)$$



$$\delta U / U_0$$

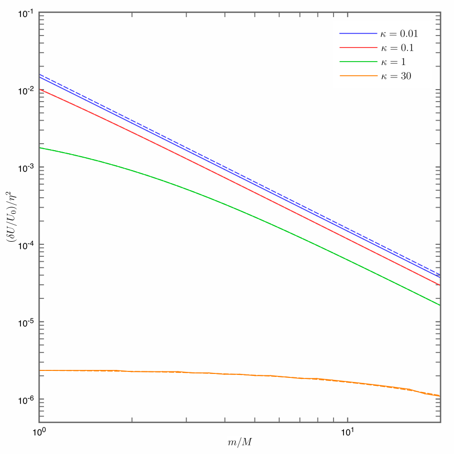
- **Influence of κ** $$W \supset \eta \mathbf{\Phi\Sigma\overline{\Sigma}} + \kappa S(\mathbf{\Sigma\overline{\Sigma}} - M^2)$$



$$\delta U/U_0/\eta^2$$

- **Influence of** $\frac{m}{M} = \frac{m_\Sigma}{M}$

$$W \supset \eta \boldsymbol{\Phi}\boldsymbol{\Sigma}\overline{\boldsymbol{\Sigma}} + \kappa S(\boldsymbol{\Sigma}\overline{\boldsymbol{\Sigma}} - M^2)$$



$$\delta U/U_0/\eta^2$$

Conclusion

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  - Realistic description of cosmic strings
  - Structure and energy per unit length function of the GUT parameters
  - Perturbative description $\Rightarrow$ two different classes of model

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- **Thanks for your attention !**