Lensing in a clumpy Universe

Séminaire du $\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$

Pierre Fleury

Institut d'Astrophysique de Paris
PhD student with Jean-Philippe Uzan

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Outline

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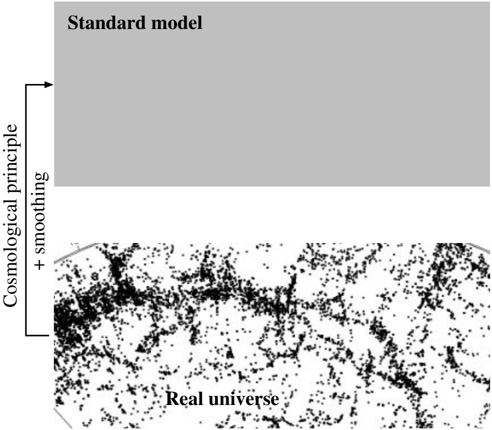
1. Fluid limit and observations in cosmology
2. Geometric optics in curved spacetime
3. Observations in a Swiss-cheese universe
4. Stochastic lensing

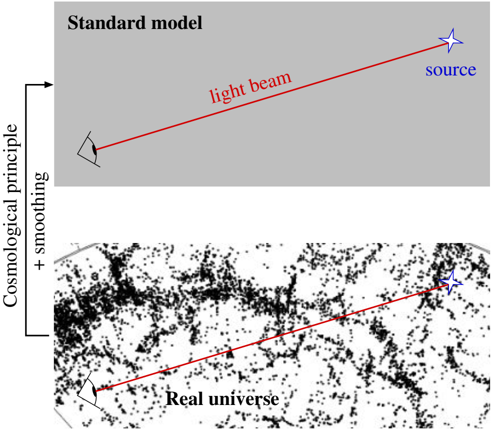
The fluid limit in cosmology



In standard cosmology:

- matter is described as a fluid (continuous medium);

The fluid limit in cosmology



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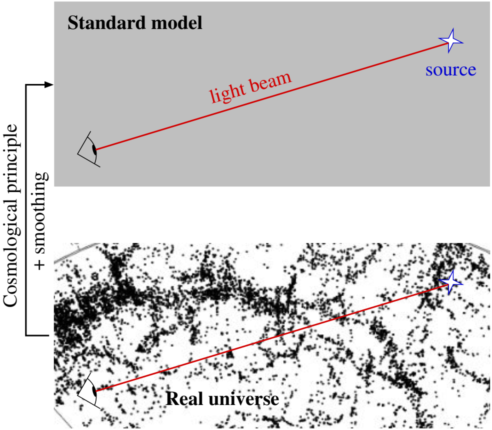

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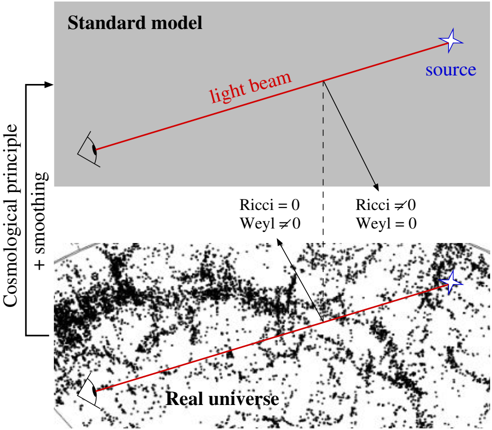
The fluid limit in cosmology



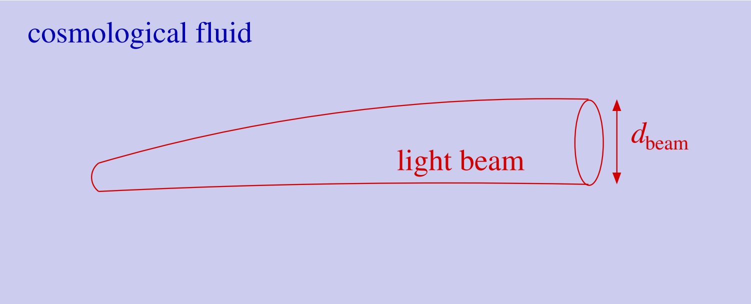
In standard cosmology:

- matter is described as a fluid (continuous medium);
- we assume that light propagates through this fluid;
- observations are interpreted in this context;
- it raises the so-called *Ricci-Weyl paradox*.

General Question [Zel'dovich 1964…Clarkson et al. 2011]

Is the fluid limit a good framework to interpret observations?

Which observations are concerned?



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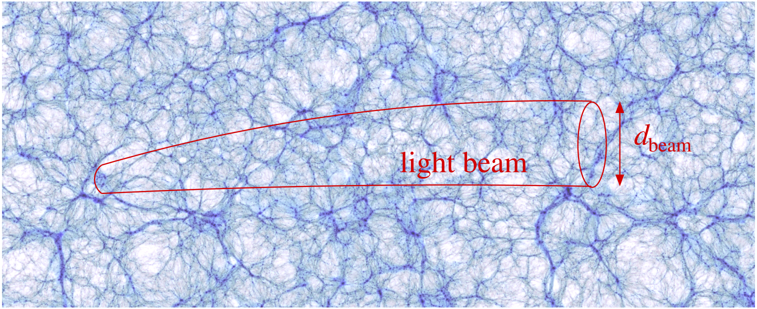

- The fluid limit should be valid if $d_{\text{inhom}} \ll d_{\text{beam}}$;

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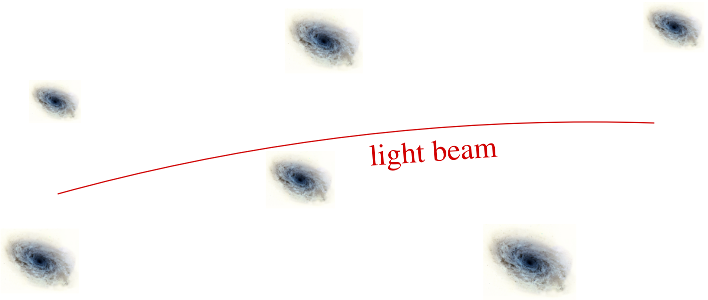

- The fluid limit should be valid if $d_{\text{inhom}} \ll d_{\text{beam}}$;
- It is questionable if $d_{\text{inhom}} \gg d_{\text{beam}}$...

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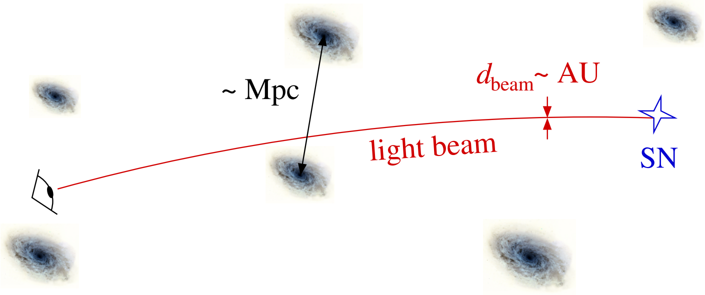
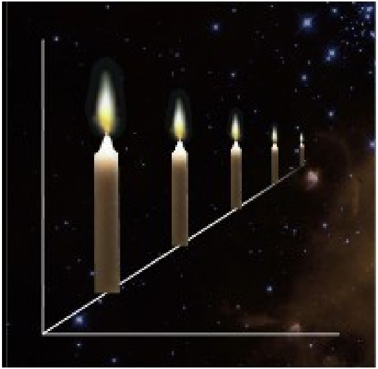


- The fluid limit should be valid if $d_{\text{inhom}} \ll d_{\text{beam}}$;
- It is questionable if $d_{\text{inhom}} \gg d_{\text{beam}}$...
- ...which is the case for astronomical observations (e.g. supernovae).

Hubble diagram and luminosity-redshift relation

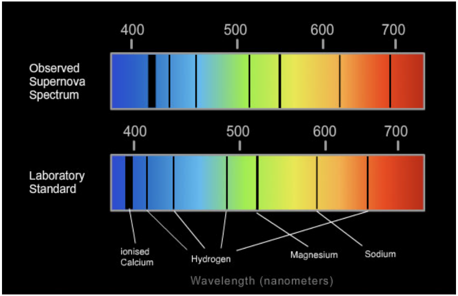


A light source is characterized by:

1. its luminosity distance

$$D_\mathrm{L} \equiv \sqrt{\frac{L}{4\pi F_\mathrm{obs}}},$$

Hubble diagram and luminosity-redshift relation



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$$z \equiv \frac{\nu_{\rm obs} - \nu_{\rm em}}{\nu_{\rm em}}.$$

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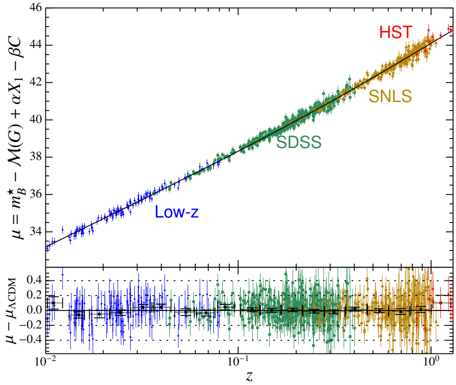

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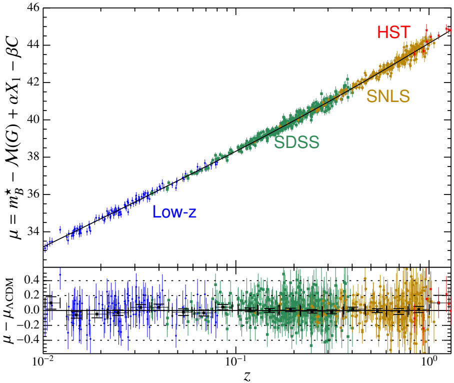

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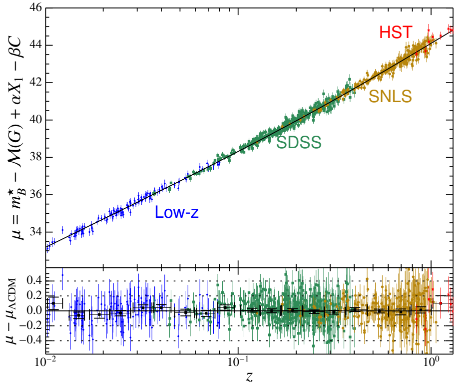
If light travels through a **homogeneous** and **isotropic** universe, then

$$D_\mathrm{L}(z) = (1+z) f_K \left(\int_0^z \frac{\mathrm{d}\zeta}{H_0 \sqrt{\Omega_{\Lambda 0} + \Omega_{K0}(1+\zeta)^2 + \Omega_{\mathrm{m}0}(1+\zeta)^3}} \right)$$

Hubble diagram and luminosity-redshift relation



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If light travels through a **more realistic** universe, then

$$D_\mathrm{L}(z) = ??$$

The questions we want to answer

Technically

How to model the impact of the small-scale inhomogeneity (i.e. clumpiness) of the Universe on light propagation?

Observationally

How does it change the interpretation of the Hubble diagram?

Philosophically?

Why does the standard ΛCDM model work so well?

Outline

Light rays

- Light rays are **null geodesics**. If v denotes an affine parameter along the ray, the wave 4-vector $k^\mu = \mathrm{d}x^\mu/\mathrm{d}v$ reads

$$\frac{\mathrm{D}k^\mu}{\mathrm{d}v} = 0 \qquad \text{and} \qquad k^\mu k_\mu = 0.$$

- The **frequency** ν measured by an observer with 4-velocity u^μ is

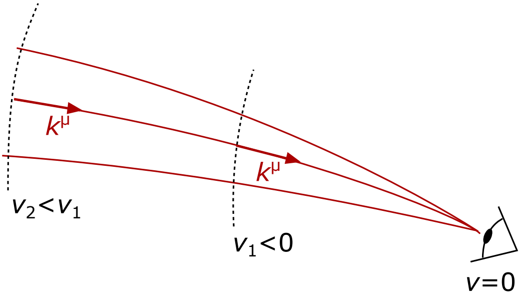
$$2\pi\nu = -u^\mu k_\mu.$$

- The **redshift** z is defined by

$$z \equiv \frac{\nu_{\text{obs}} - \nu_{\text{em}}}{\nu_{\text{em}}}, \qquad \text{i.e.} \qquad 1 + z = \frac{(u^\mu k_\mu)_{\text{em}}}{(u^\mu k_\mu)_{\text{obs}}}.$$

Light beams, Jacobi matrix

Computing angular/luminosity distances requires to consider light *beams* (bundles of null geodesics).



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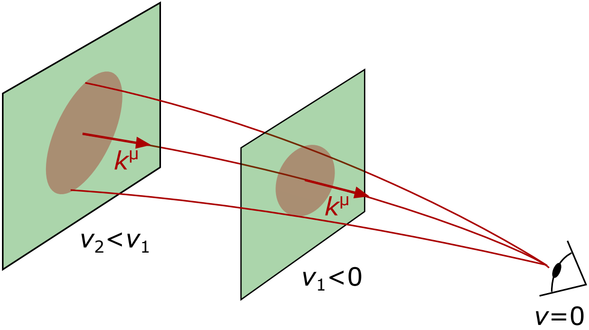

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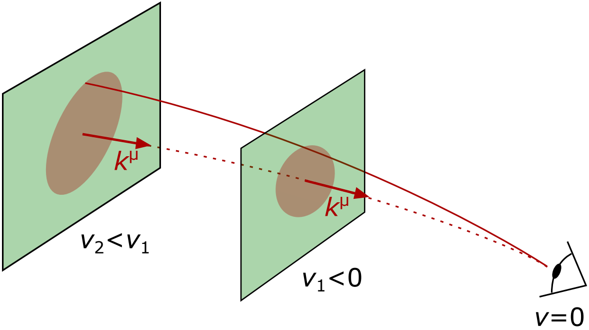

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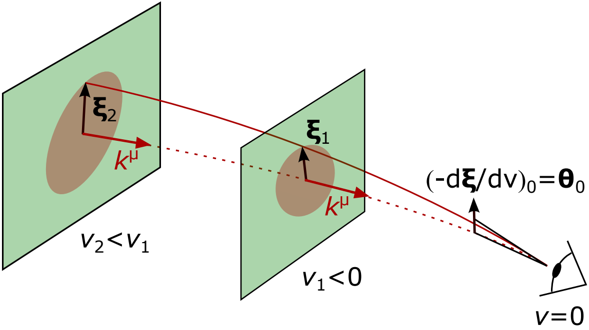


The **Jacobi matrix** is defined as

$$\boxed{\mathcal{D}^A{}_B(v) \equiv \frac{\mathrm{d}\xi^A(v)}{\mathrm{d}\dot{\xi}^B(0)} = -\frac{\mathrm{d}\xi^A(v)}{\mathrm{d}\theta^B(0)}} \qquad A, B \in \{1, 2\}.$$

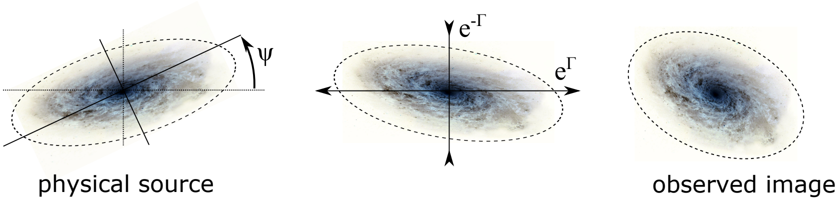
Geometrical meaning of the Jacobi matrix

First note that

$$\det(\mathcal{D}^A{}_B) = \frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}^2\boldsymbol{\theta}_0} \overset{v=v_s}{=} \frac{\text{source's physical area}}{\text{observed angular size}} \equiv D_\mathrm{A}^2 = \left[\frac{D_\mathrm{L}}{(1+z)^2}\right]^2,$$

more generally

$$(\mathcal{D}^A{}_B) = \underbrace{D_\mathrm{A}}_{\text{distance}} \underbrace{\begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix}}_{\text{rotation}} \underbrace{\exp\begin{pmatrix} -\Gamma_1 & \Gamma_2 \\ \Gamma_2 & \Gamma_1 \end{pmatrix}}_{\text{shear}}.$$



Evolution of the Jacobi matrix

From the geodesic deviation equation one derives the **Sachs equation**

$$\boxed{\frac{\mathrm{d}^2\mathcal{D}^A{}_B}{\mathrm{d}v^2} = \mathcal{T}^A_C\mathcal{D}^C{}_B,}$$

where $\mathcal{T}_{AB}$ is the **optical tidal matrix**

$$\mathcal{T}_{AB} \equiv -R_{\mu\nu\rho\sigma}\, s^\mu_A k^\nu s^\rho_B k^\sigma.$$

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$$\mathcal{T}_{AB} \equiv -R_{\mu\nu\rho\sigma}\, s^\mu_A k^\nu s^\rho_B k^\sigma .$$

which can be decomposed into a Ricci part and a Weyl part

$$(\mathcal{T}_{AB}) = \begin{pmatrix} \mathcal{R} & 0 \\ 0 & \mathcal{R} \end{pmatrix} + \begin{pmatrix} -\mathrm{Re}\,\mathcal{W} & \mathrm{Im}\,\mathcal{W} \\ \mathrm{Im}\,\mathcal{W} & \mathrm{Re}\,\mathcal{W} \end{pmatrix},$$

where

$$\mathcal{R} \equiv -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu, \qquad \mathcal{W} \equiv -\frac{1}{2} C_{\mu\nu\rho\sigma}\, (s^\mu_1 - \mathrm{i} s^\mu_2) k^\nu (s^\rho_1 - \mathrm{i} s^\rho_2) k^\sigma .$$

Evolution of the angular distance

The Sachs equation imply in particular

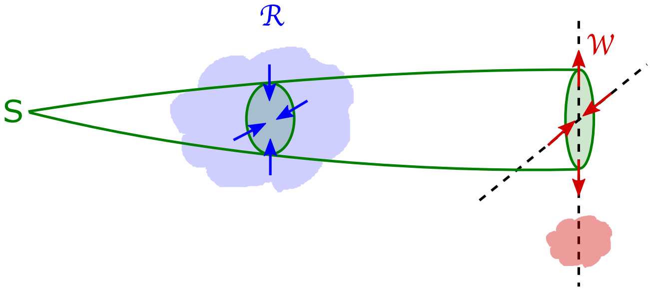
$$\frac{\mathrm{d}^2 D_\mathrm{A}}{\mathrm{d}v^2} = (\mathcal{R} - |\sigma|^2) D_\mathrm{A},$$

$$\frac{\mathrm{d}\, D_\mathrm{A}^2 \sigma}{\mathrm{d}v} = D_\mathrm{A}^2 \mathcal{W},$$

where σ is the shear rate.

Ricci focuses directly, and Weyl indirectly.

Summary



- Ricci focuses due to diffuse matter inside the beam, as

$$\mathcal{R} = -4\pi G T_{\mu\nu} k^{\mu} k^{\nu} = -4\pi G \omega^2 (\rho + p).$$

- Weyl distorts and focuses mostly due to matter outside the beam.

Outline

Swiss-cheese models in brief

[Einstein & Straus 1945]

Construction

1. start from a homogeneous and isotropic model;

Swiss-cheese models in brief

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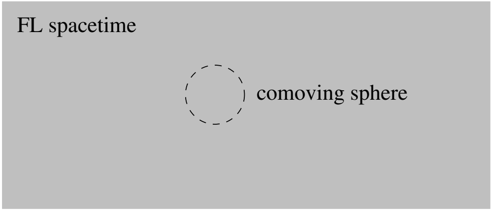
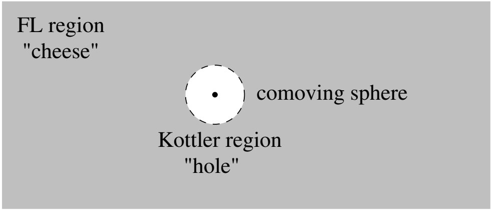


Construction

1. start from a homogeneous and isotropic model;
2. pick a comoving sphere;

Swiss-cheese models in brief

[Einstein & Straus 1945]



Construction

1. start from a homogeneous and isotropic model;
2. pick a comoving sphere;
3. concentrate the matter it contains at the center;

Swiss-cheese models in brief

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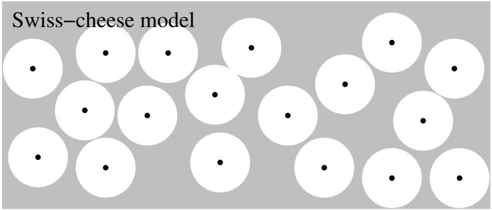

Construction

1. start from a homogeneous and isotropic model;
2. pick a comoving sphere;
3. concentrate the matter it contains at the center;
4. do it again, without overlapping holes.

Swiss-cheese models in brief

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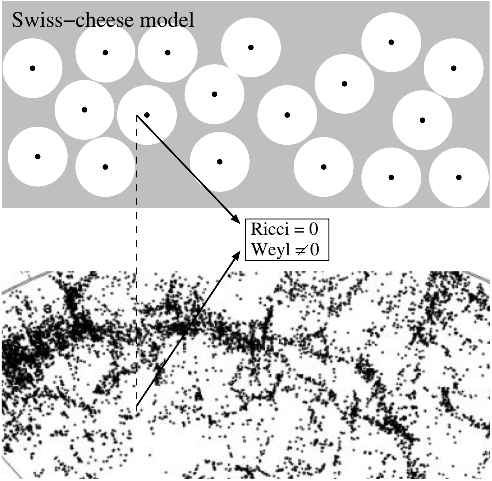


Construction

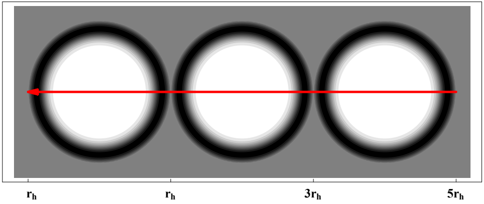
1. start from a homogeneous and isotropic model;
2. pick a comoving sphere;
3. concentrate the matter it contains at the center;
4. do it again, without overlapping holes.

Main advantage

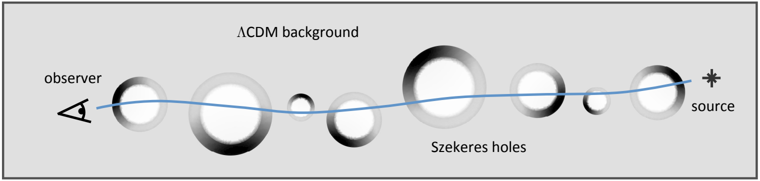
The Ricci-Weyl issue is directly addressed by breaking the fluid approximation.

Other types of Swiss-cheese models

- Lemaître-Tolman-Bondi (LTB) interior [Marra et al. 2007, Brouzakis et al. 2007, Biswas & Notari 2008, Clifton & Zuntz 2009, Valkenburg 2009, Szybka 2011, Bolejko 2011, Flanagan et al. 2012, Lavinto et al. 2013. . .];



- Szekeres interior [Bolejko & Célérier 2010, Peel et al. 2014, . . .]

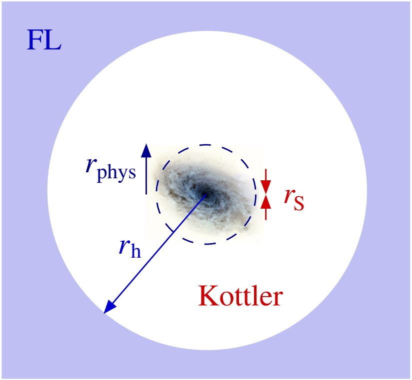


Such models solutions **do not** address the Ricci-Weyl issue.

Orders of magnitude

For us, the masses inside the holes represent bound objects, such as

- galaxies ($M \sim 10^{11} M_\odot$);
- clusters of galaxies ($M \sim 10^{15} M_\odot$).



Type	galaxy	cluster
$r_{\rm S}$ (pc)	10^{-2}	100
$r_{\rm phys}$ (kpc)	10	1000
$r_{\rm h}$ (Mpc)	1	20
ε	10^{-8}	10^{-6}

where

$$\varepsilon \equiv \frac{r_{\rm S}}{r_{\rm h}}$$

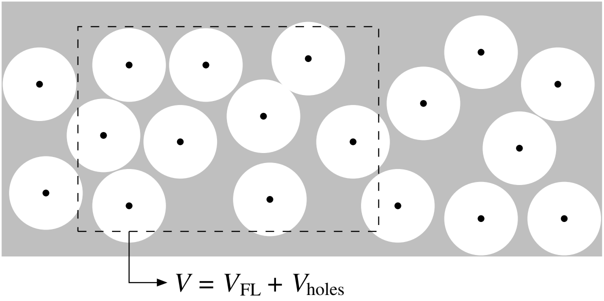
The smoothness parameter

The amount of holes in a given Swiss cheese is quantified by

$$\bar{\alpha} \equiv \lim_{V\to\infty} \frac{V_{\mathrm{FL}}}{V} = \lim_{V\to\infty} \left(1 - \frac{V_{\mathrm{holes}}}{V}\right)$$

called the **smoothness parameter**. Hence,

- $\bar{\alpha} = 1$ refers to a perfectly smooth (FL) universe;
- $\bar{\alpha} = 0$ is a universe entirely filled with clumps and holes.

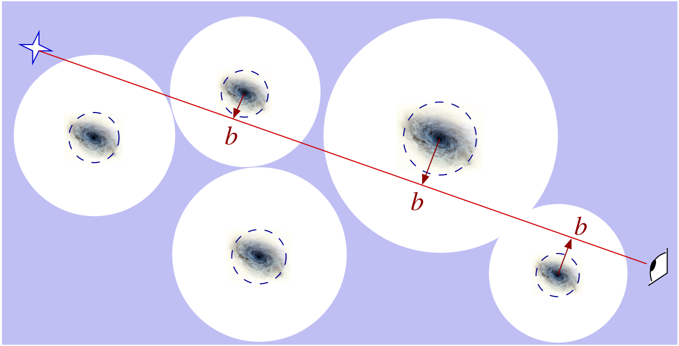


The opacity hypothesis

- We consider the clumps inside holes as **effectively opaque**; thus we only allow for impact parameters b such that

$$\boxed{b > r_{\text{phys}}.}$$

- This is observationally justified in the case of galaxies (signal/noise).



Light propagation in a Swiss cheese

[Kantowski 1969, Dyer & Roeder 1973, Fleury 2014a]

- The $z(v)$ relation is almost unaffected (negligible Rees-Sciama effect);
- Weyl lensing in the holes is very weak if $r_{\mathrm{phys}} \gg r_{\mathrm{S}}$;
- Ricci lensing is effectively reduced,

$$\boxed{\mathcal{R}_{\mathrm{eff}} \approx \alpha \mathcal{R}_{\mathrm{FL}}} \qquad \text{with} \qquad \alpha = \frac{\Delta v_{\mathrm{cheese}}}{\Delta v_{\mathrm{tot}}} \in [0,1] \approx \bar{\alpha}$$

Light propagation in a Swiss cheese

[Kantowski 1969, Dyer & Roeder 1973, Fleury 2014a]

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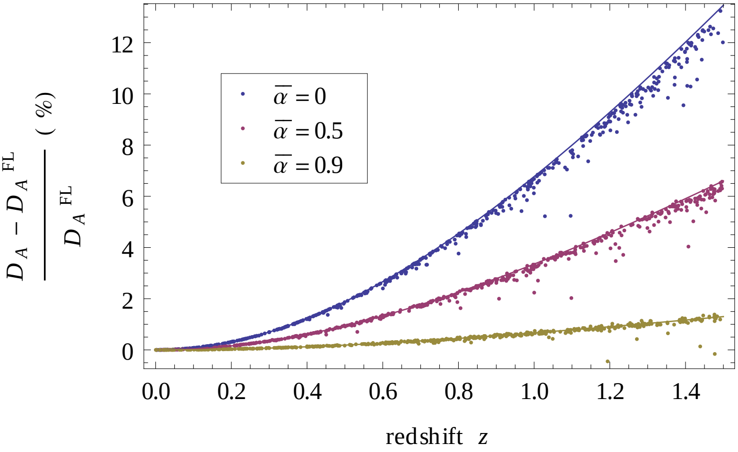
$$\boxed{\mathcal{R}_{\text{eff}} \approx \alpha \mathcal{R}_{\text{FL}}} \qquad \text{with} \qquad \alpha = \frac{\Delta v_{\text{cheese}}}{\Delta v_{\text{tot}}} \in [0, 1] \approx \bar{\alpha}$$

This leads to the Kantowski-Dyer-Roeder distance equation

$$\frac{\mathrm{d}^2 D_{\text{A}}}{\mathrm{d}v^2} = -4\pi G \alpha \rho (1+z)^2 D_{\text{A}}.$$

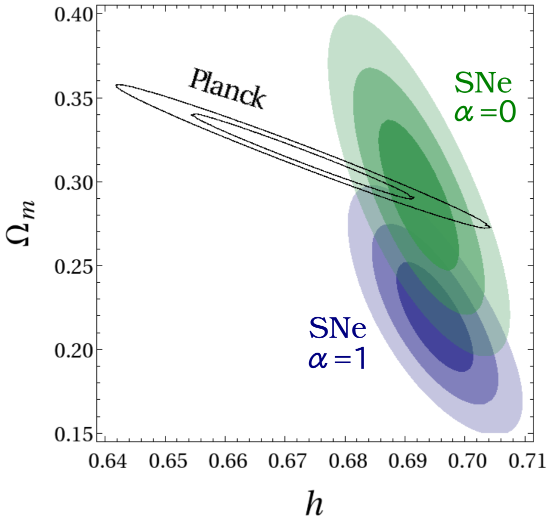
Light effectively propagates in an underdense homogeneous universe.

Numerical illustration



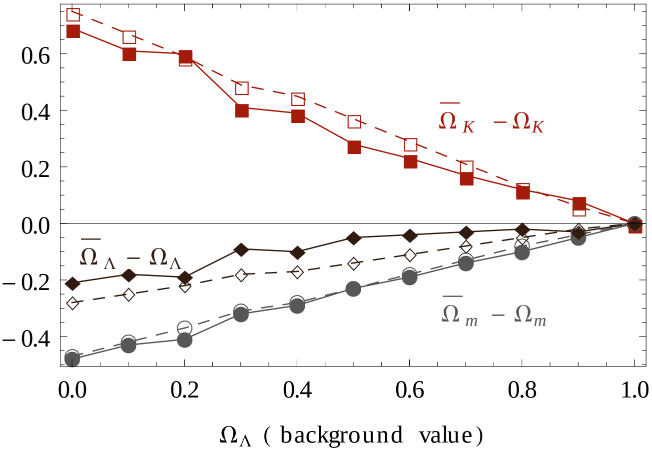
Relieving the tension between SNe and the CMB

[Fleury et al. 2013b]



Dependence in the cosmological constant

[Fleury et al. 2013a]



Summary

Results

- The Hubble diagram is biased in a clumpy universe,
- not enough to kill Λ,
- but enough to explain the tension between SNe and the CMB.
- Why does ΛCDM work so well? Because of Λ!

We need to go further

- How to measure α?
- How to efficiently estimate the shear?
- Swiss-cheese models do not allow for the large-scale structure.

Outline

The idea

The idea

Treating lensing as a diffusion process.

The Sachs-Langevin equations

Consider the equation for the Jacobi matrix as

$$\ddot{\boldsymbol{\mathcal{D}}} = \left(\begin{bmatrix} \langle \mathcal{R} \rangle & 0 \\ 0 & \langle \mathcal{R} \rangle \end{bmatrix} + \underbrace{\begin{bmatrix} \delta\mathcal{R} & 0 \\ 0 & \delta\mathcal{R} \end{bmatrix}}_{\text{Ricci fluct.}} + \underbrace{\begin{bmatrix} -\mathcal{W}_1 & \mathcal{W}_2 \\ \mathcal{W}_2 & \mathcal{W}_1 \end{bmatrix}}_{\text{Weyl fluct.}} \right) \boldsymbol{\mathcal{D}}$$

where $\delta\mathcal{R}$, $\mathcal{W}_A$ are **white noises**, with

$$\begin{aligned} \langle \delta\mathcal{R}(v_1)\delta\mathcal{R}(v_2) \rangle &= C_{\mathcal{R}}(v_1)\delta(v_1 - v_2) \\ \langle \mathcal{W}_A(v_1)\mathcal{W}_B(v_2) \rangle &= C_{\mathcal{W}}(v_1)\delta_{AB}\delta(v_1 - v_2) \\ \langle \delta\mathcal{R}(v_1)\mathcal{W}_A(v_2) \rangle &= 0 \end{aligned}$$

The covariance amplitudes C_X represent

$$C_X \sim (\text{fluctuations of } X)^2 \times \text{correlation interval } \Delta v$$

The Fokker-Planck equation

The probability density function $p(\boldsymbol{\mathcal{D}}, \dot{\boldsymbol{\mathcal{D}}}; v)$ satisfies

$$\frac{\partial p}{\partial v} = -\dot{\mathcal{D}}_{AB} \frac{\partial p}{\partial \mathcal{D}_{AB}} - \langle \mathcal{R} \rangle \, \mathcal{D}_{AB} \frac{\partial p}{\partial \dot{\mathcal{D}}_{AB}} + \frac{1}{2} \left(C_{\mathcal{R}} \, \delta_{AE} \delta_{CF} + C_{\mathcal{W}} \, \delta_{AC} \delta_{EF} - C_{\mathcal{W}} \, \varepsilon_{AC} \varepsilon_{EF} \right) \mathcal{D}_{EB} \mathcal{D}_{FD} \frac{\partial^2 p}{\partial \dot{\mathcal{D}}_{AB} \partial \dot{\mathcal{D}}_{CD}},$$

with a drift term and a diffusion term.

- It generates evolutions equations for the moments of $p(\boldsymbol{\mathcal{D}}, \dot{\boldsymbol{\mathcal{D}}}; v)$.
- Order-n moments form a closed system (no hierarchy).
- Everything is contained in the functions $\langle \mathcal{R} \rangle (v)$, $C_{\mathcal{R}}(v)$, $C_{\mathcal{W}}(v)$.

Moments of the angular distance distribution

- Correction to the average distance: if D_0 is the distance for $\delta\mathcal{R} = \mathcal{W} = 0$,

$$
\begin{aligned}
\delta_{D_\mathrm{A}} &\equiv \frac{\langle D_\mathrm{A} \rangle - D_0}{D_0} \\
&\approx -\int_0^v \mathrm{d}v_1 \int_0^{v_1} \mathrm{d}v_2 \int_0^{v_2} \mathrm{d}v_3 \left[\frac{D_0^2(v_3)}{D_0(v_1) D_0(v_2)} \right]^2 2 C_{\mathcal{W}}(v_3)
\end{aligned}
$$

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$$\approx -\int_0^v \mathrm{d}v_1 \int_0^{v_1} \mathrm{d}v_2 \int_0^{v_2} \mathrm{d}v_3 \left[\frac{D_0^2(v_3)}{D_0(v_1) D_0(v_2)} \right]^2 2C_\mathcal{W}(v_3)$$

- Variance of the distance:

$$\frac{\mathrm{d}^3}{\mathrm{d}x^3} \left(\frac{\sigma_{D_\mathrm{A}}}{D_0} \right)^2 - 2D_0^6 (C_\mathcal{R} - 2C_\mathcal{W}) \left(\frac{\sigma_{D_\mathrm{A}}}{D_0} \right)^2 \approx 2C_\mathcal{R} D_0^6 + 6 \int \mathrm{d}x \left[\frac{\mathrm{d}^2 \delta_{D_\mathrm{A}}}{\mathrm{d}x^2} \right]^2 ,$$

with the variable x defined as $\mathrm{d}x = D_0^{-2} \mathrm{d}v$.

Application to Swiss-cheese models

- We use Swiss-cheese models as a *benchmark* of the method.
- $\langle \mathcal{R} \rangle$, $C_{\mathcal{R}}(v)$, $C_{\mathcal{W}}(v)$, can be calculated analytically, e.g.

$$C_{\mathcal{W}} = \frac{3}{2}(1-\alpha)H_0^2\Omega_{\mathrm{m0}}(1+z)^6 \left\langle \frac{(2GM)^{4/3}}{\left\langle (2GM)^{1/3} \right\rangle r_{\mathrm{phys}}^2} \right\rangle$$

Notice. For galaxies $C_{\mathcal{W}} \sim H_0^3$; for stars $C_{\mathcal{W}} \sim 10^{10} H_0^3$!

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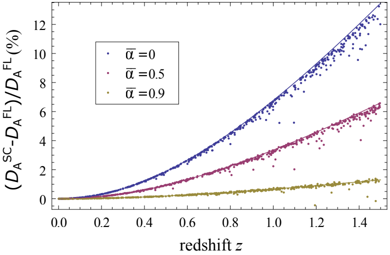

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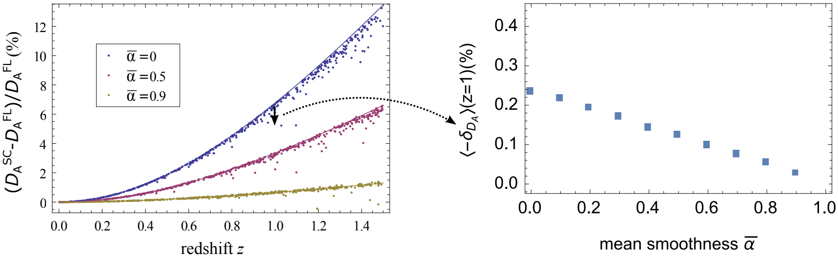

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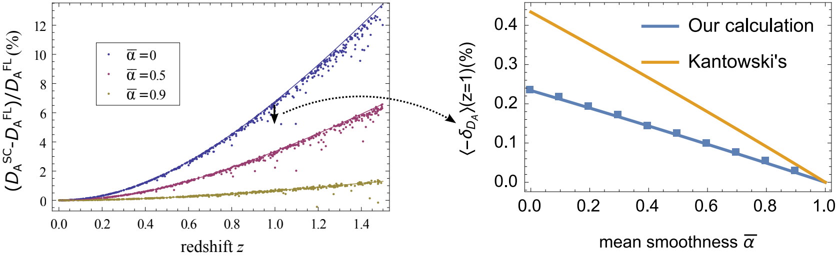

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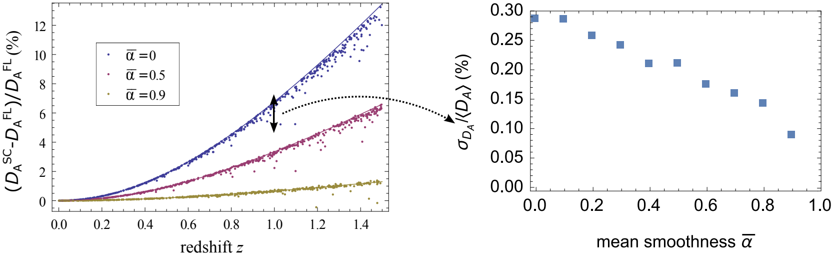

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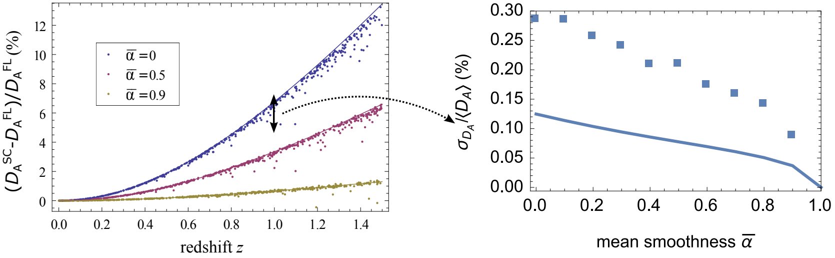

The problem of nongaussianity

- The Fokker-Planck approach assumes **gaussian** noises.
- It is motivated by the central limit theorem.
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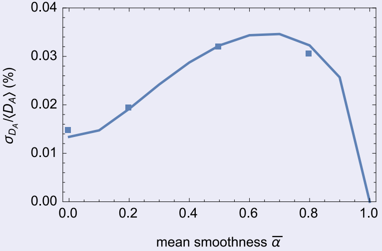
Two arguments

1. Reducing the nongaussianity (by increasing r_{phys}) gives a much better agreement between theory and ray-tracing.



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Two arguments

1. Reducing the nongaussianity (by increasing r_{phys}) gives a much better agreement between theory and ray-tracing.
2. Direct simulations of the Sachs-Langevin equation (J. Larena) give
   - the theoretical result for a Gaussian noise;
   - the ray-tracing result for the appropriate nongaussian noise,
   - nongaussian $\rightarrow$ Gaussian as the integration step is reduced.

Concluding thoughts

On stochastic lensing

- A promising framework, because simple and flexible.
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- The role of clumpiness on lensing is not fully understood yet.
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More generally

- The role of clumpiness on lensing is not fully understood yet.
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What's next?

- Write my thesis.
- Observational/numerical constraints on α? (I need you!)
- Apply stochastic lensing to realistic models.
- Maths: how to extend Fokker-Planck to nongaussian noises?

Merci pour votre attention.

Light propagation in Bianchi I

Bianchi spacetimes are models of homogeneous but possibly anisotropic universes. Bianchi I is one of them:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + X^2(t)\mathrm{d}x^2 + Y^2(t)\mathrm{d}y^2 + Z^2(t)\mathrm{d}z^2.$$

Bianchi I is well known, but is optical properties have not been comprehensively studied.

Result [Fleury et al. 2014c]

- An exact expression for the Jacobi matrix $\mathcal{D}^A{}_B$ in Bianchi I

$$\boldsymbol{\mathcal{D}}(\eta_\mathrm{s} \leftarrow \eta_\mathrm{o}) = a(\eta_\mathrm{s})(\boldsymbol{\mathcal{E}}^{-1})^T \int_{\eta_\mathrm{s}}^{\eta_\mathrm{o}} \tilde{\omega}^{-1} \boldsymbol{\mathcal{E}}^T \boldsymbol{\mathcal{E}} \, \mathrm{d}\eta.$$

- Ideas of new observables to constrain anisotropy.

Stability and causality of scalar-vector models

- Recent interest in inflationary/dark energy models involving both scalar and vector fields.
- Motivated by the large-scale anomalies of the CMB [Planck 2013], or large-scale/primordial magnetic fields [Bonvin et al. 2013,...].
- Problem: there is a priori an infinite amount of models to tests.

Goal

Select the scalar-vector models that are physically acceptable, i.e.

1. stable (Hamiltonian by below),
2. causal (hyperbolic equations of motion).

Result [Fleury et al. 2014b]

Among a very large (but not comprehensive) class of models involving a scalar ϕ and a vector A^μ, the acceptable Lagrangians read

$$\mathcal{L} = -\frac{1}{2} f_0[\phi, (\partial\phi)^2] - \frac{1}{4} f^2(\phi) F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} g(\phi) F^{\mu\nu} \tilde{F}_{\mu\nu}.$$

Affine parameter-redshift relation

Effect of one hole on the redshift:

- the hole grows (its frontier is comoving), hence $\Phi(r_{\rm in}) < \Phi(r_{\rm out})$;
- this gravitational redshift, similar to the integrated Sachs-Wolfe (ISW) effect adds to the usual cosmological redshift.

The exact formula for the redshift is [Dyer 1975, Fleury 2014a]

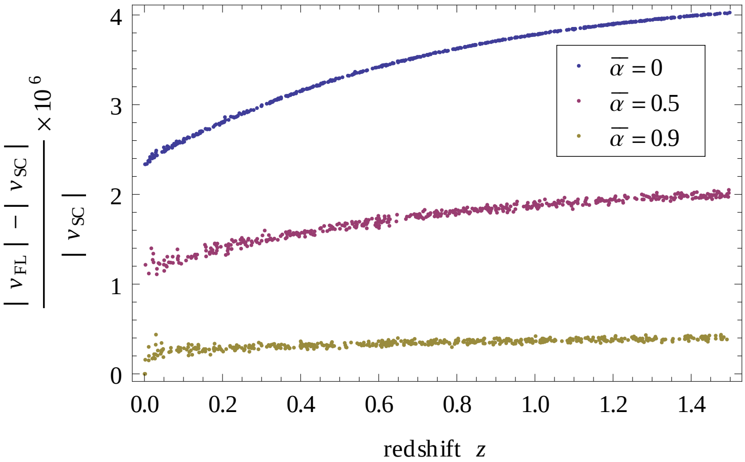
$$(1+z)_{\rm in\to out} = \frac{A_{\rm out}}{A_{\rm in}} \frac{1+\sqrt{1-A_{\rm in}/\gamma^2}\sqrt{1-A_{\rm in}\,(b/r_{\rm in})^2}}{1-\sqrt{1-A_{\rm out}/\gamma^2}\sqrt{1-A_{\rm out}\,(b/r_{\rm out})^2}},$$
$$= \frac{a_{\rm out}}{a_{\rm in}}\,[1+\mathcal{O}(\varepsilon)]$$

Conclusion

The Swiss-cheese inhomogeneities do not change the $z(v)$ relation.

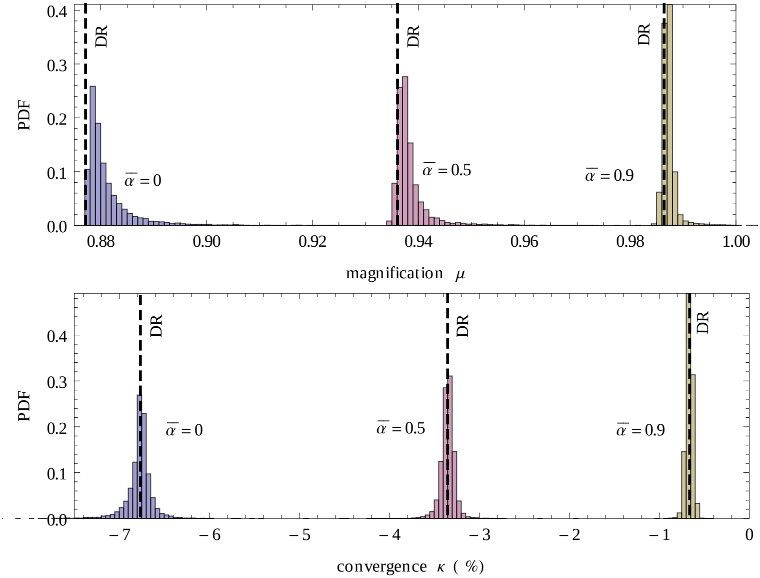
Numerical illustrations

$z(v)$ relation



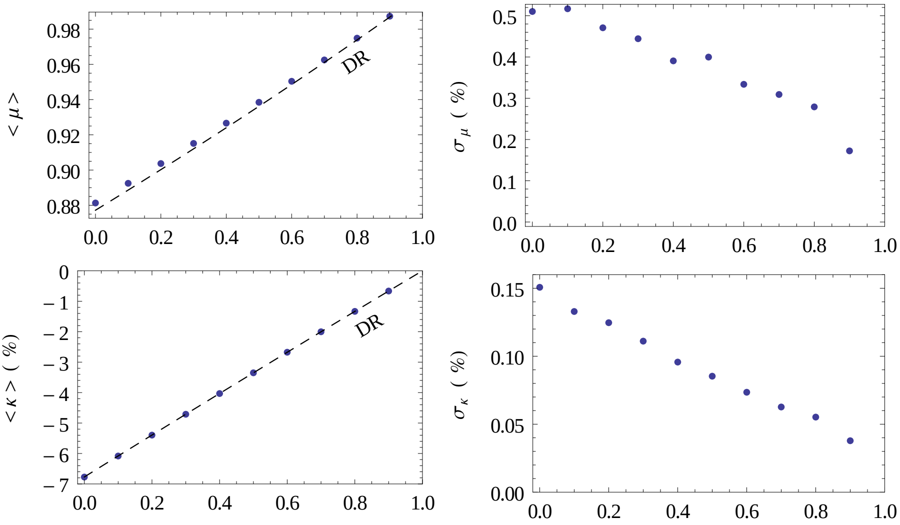
Weak lensing in a Swiss-cheese universe

Magnification and convergence: PDFs



Weak lensing in a Swiss-cheese universe

Magnification and convergence: mean and standard deviation



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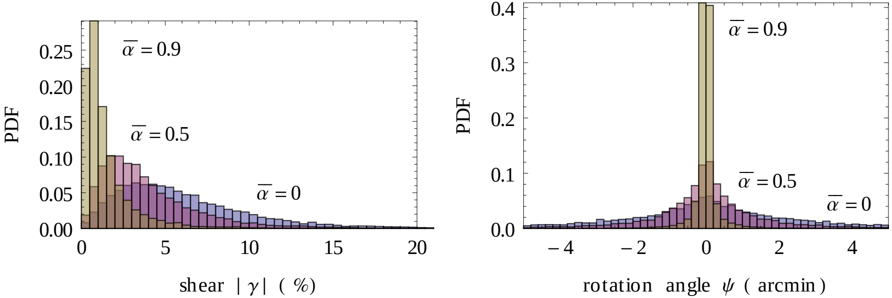
Shear and rotation: PDFs



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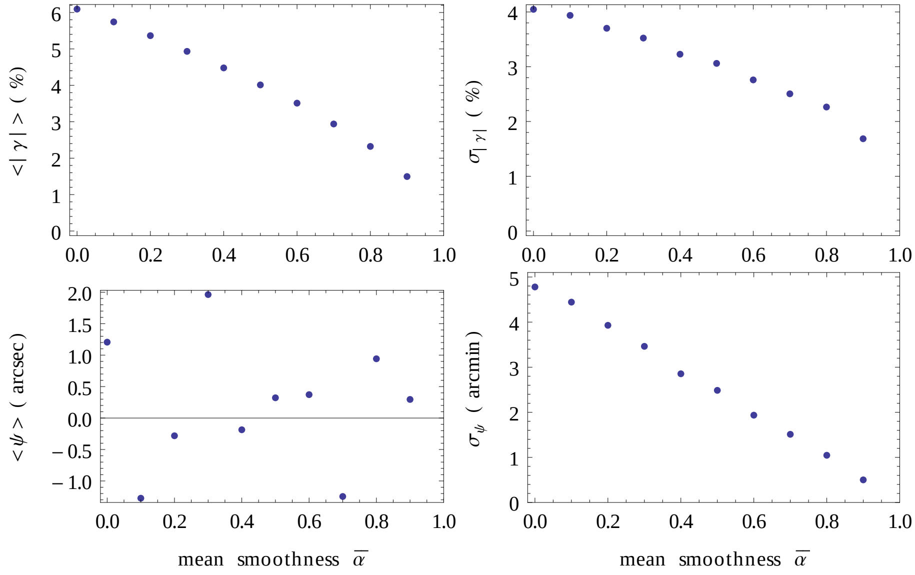
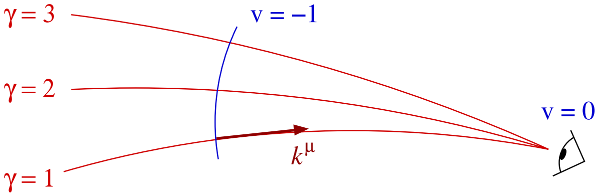


Light beams

A light beam is a **bundle of null geodesics** $\{x^\mu(v, \gamma)\}$.



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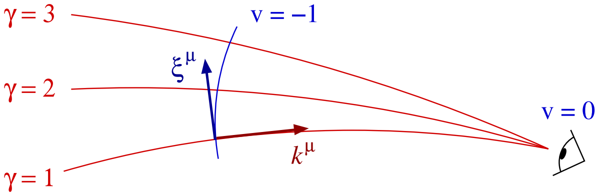


The relative behaviour of two neighbouring geodesics is described by their **separation vector**

$$\xi^\mu \equiv \frac{\partial x^\mu}{\partial \gamma},$$

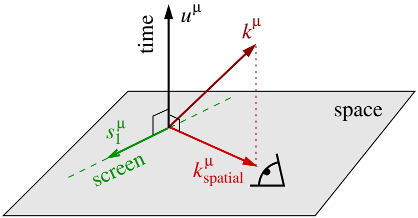
which reads

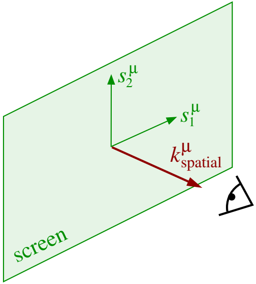
$$k^\mu \xi_\mu = 0, \qquad \text{and} \qquad \boxed{\frac{\mathrm{D}^2 \xi^\mu}{\mathrm{d}v^2} = R^\mu{}_{\nu\alpha\beta}\, k^\nu k^\alpha \xi^\beta}$$

The Sachs basis

An observer with 4-velocity u^μ sees the light beam by projecting it on a **screen**, spanned by two vectors $(s^\mu_A)_{A\in\{1,2\}}$ with

$$s^\mu_A u_\mu = s^\mu_A k_\mu = 0, \qquad g_{\mu\nu} s^\mu_A s^\nu_B = \delta_{AB}.$$





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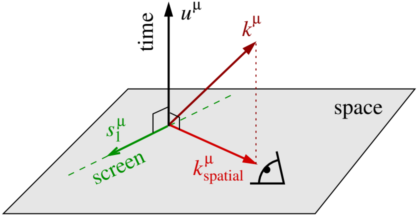
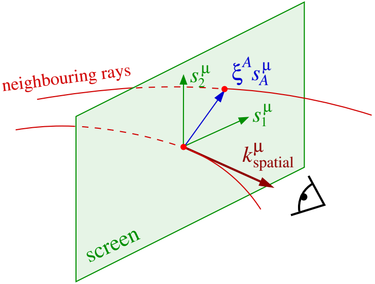
Projection of the separation vector on the screen: $\boxed{\xi_A \equiv \xi_\mu s_A^\mu}$





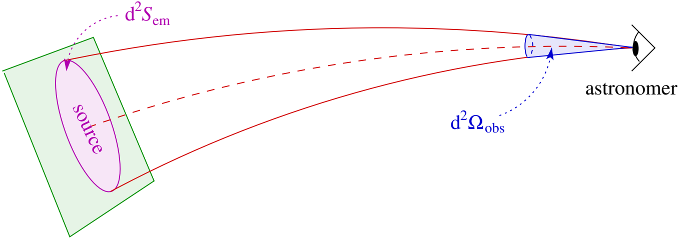
Angular distance

The angular distance D_{A} is defined by

$$D_{\mathsf{A}} \equiv \sqrt{\frac{\mathrm{d}^2 S_{\mathrm{em}}}{\mathrm{d}^2\Omega_{\mathrm{obs}}}}.$$

It is related to the Jacobi map by

$$\boxed{D_{\mathsf{A}} = \sqrt{|\det \boldsymbol{\mathcal{D}}\,|}}$$



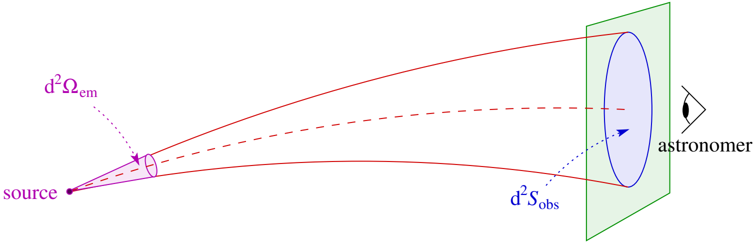
Luminosity distance

The luminosity distance is defined by

$$L_{\rm source} = F_{\rm obs} \times 4\pi\, D_{\rm L}^2 \implies D_{\rm L} = (1+z)\sqrt{\frac{\mathrm{d}^2 S_{\rm obs}}{\mathrm{d}^2\Omega_{\rm em}}}.$$

It is related to the Jacobi map by

$$\boxed{D_{\rm L} = (1+z)^2 D_{\rm A} = (1+z)^2\sqrt{|\det \boldsymbol{\mathcal{D}}\,|}}$$



A new tool: the Wronski matrix

To deal more easily with a patchwork a spacetimes, we extend the Jacobi matrix formalism for arbitrary initial conditions

$$\text{if } \boldsymbol{\xi}(v_0) = \mathbf{0} \text{ then} \qquad \boldsymbol{\xi}(v) = \boldsymbol{\mathcal{D}}(v \leftarrow v_0) \cdot \dot{\boldsymbol{\xi}}(v_0)$$

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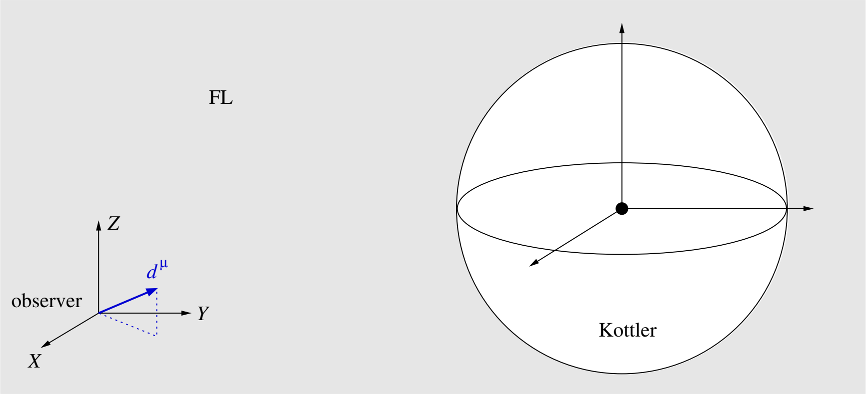
The Wronski matrix $\mathcal{W}$ satifies a Chasles-like relation

$$\boxed{\mathcal{W}(v_3 \leftarrow v_1) = \mathcal{W}(v_3 \leftarrow v_2) \cdot \mathcal{W}(v_2 \leftarrow v_1)}$$

Application to the Swiss-cheese

$$\mathcal{W}(v_{\rm em} \leftarrow 0) = \mathcal{W}_{\rm FL}(v_{\rm em} \leftarrow v_{\rm in}^{(1)}) \cdot \mathcal{W}_{\rm K}(v_{\rm in}^{(1)} \leftarrow v_{\rm out}^{(1)}) \cdot \mathcal{W}_{\rm FL}(v_{\rm out}^{(1)} \leftarrow v_{\rm in}^{(2)}) \cdots \mathcal{W}_{\rm K}(v_{\rm in}^{(N)} \leftarrow v_{\rm out}^{(N)}) \cdot \mathcal{W}_{\rm FL}(v_{\rm out}^{(N)} \leftarrow 0).$$

Simulating a cosmological observation



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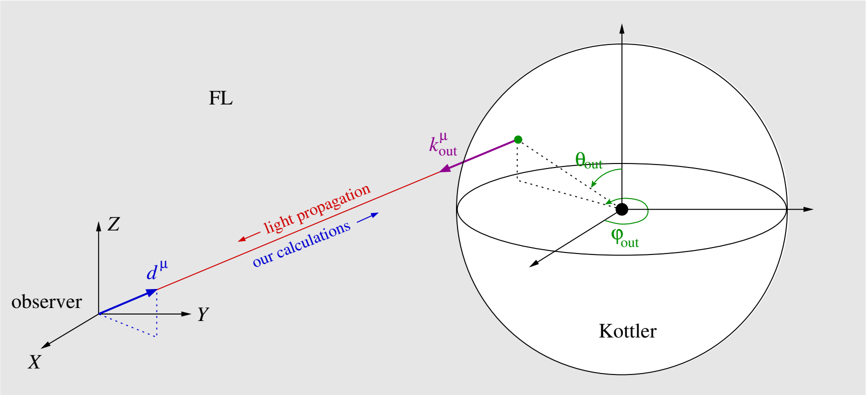

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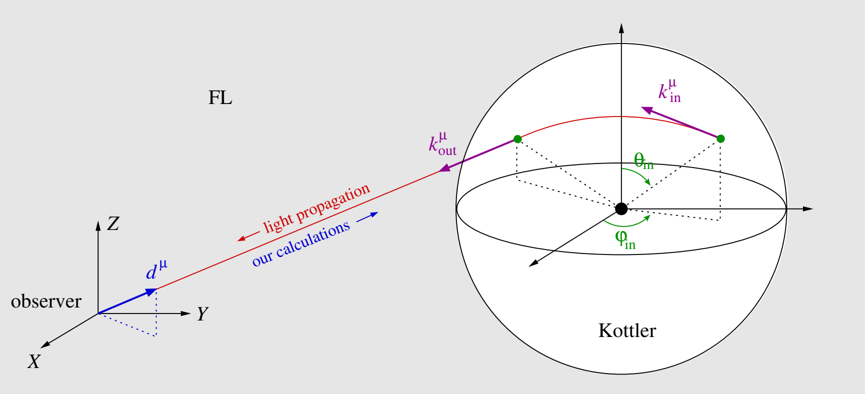

Simulating a cosmological observation