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A NEW WAY TO COUNT DEGREES OF FREEDOM IN DRGT MASSIVE GRAVITY RECENT DEVELOPMENTS IN MASSIVE GRAVITY

ILECENT DEVELOT MENTS IN MASSIVE GRAVIT

George Zahariade

APC, Paris C. Deffayet, J. Mourad, GZ, 1207.6338, 1208.4493

January 14, 2013

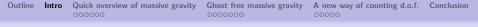
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1 Introduction

- QUICK OVERVIEW OF MASSIVE GRAVITY
 - Quadratic theory
 - Tension between theory and observation
 - Non-linear theories and the BD ghost
- **3** Non-linear ghost free massive gravity
 - The mass terms
 - Motivation of the specific form
 - A (not completely equivalent) vierbein reformulation
- 4 A NEW WAY OF COUNTING DEGREES OF FREEDOM
 - Constraints arising from local Lorentz invariance breaking
 - Constraints arising from diffeomorphism invariance breaking
 - Additional constraint

5 CONCLUSION

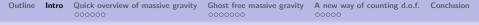
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• **Cosmological motivations:** adding a mass term to the graviton modifies gravity on large scales of order 1/m

$$V(r) \propto rac{1}{r}$$
 becomes $V(r) \propto rac{e^{-mr}}{r}$

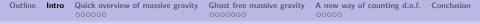
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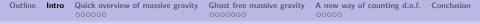
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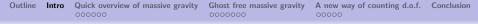


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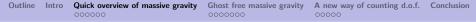
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- ! NOT SO EASY

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General relativity: highly non-linear theory of a massless spin-two

$$S = M_P^2 \int d^4x \sqrt{-g}R$$

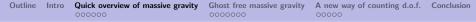


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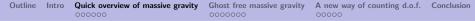
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- ! IMPOSSIBLE
- Introduction of an auxiliary non-dynamical metric $f_{\mu
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QUADRATIC THEORY (MASSLESS)

• Quadratic order action $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$S^{(2)}=-rac{M_P^2}{2}\int d^4x \; h^{\mu
u} {\cal E}^{
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u} h_{
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where

$$\begin{aligned} \mathcal{E}^{\rho\sigma}_{\mu\nu} &\equiv -\frac{1}{2} \Big(\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} \Box - 2 \delta^{(\sigma}_{(\mu} \partial_{\nu)} \partial^{\rho)} &+ \eta^{\rho\sigma} \partial_{\mu} \partial_{\nu} \\ &- \eta_{\mu\nu} \eta^{\rho\sigma} \Box + \eta_{\mu\nu} \partial^{\rho} \partial^{\sigma} \Big) \end{aligned}$$

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• The fixed background metric $\eta_{\mu\nu}$ plays the role of the auxiliary metric $f_{\mu\nu}$

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- The fixed background metric $\eta_{\mu\nu}$ plays the role of the auxiliary metric $f_{\mu\nu}$
- Possible mass terms

$$h_{\mu\nu}h^{\mu\nu}$$
 and h^2

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$$= m^{2}(h_{\mu\nu} - \gamma\eta_{\mu\nu}h)$$

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• Second order equations of motion for a priori 10 degrees of freedom i.e. 10×2 functions of the spatial coordinates can be set as initial conditions

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- Second order equations of motion for a priori 10 degrees of freedom i.e. 10×2 functions of the spatial coordinates can be set as initial conditions
- 4 constraint equations $\partial^{\mu}h_{\mu\nu} \gamma \partial_{\nu}h = 0$ (Bianchi identity)

Ghost free massive gravity A new way of counting d.o.f. Conclusion

FIERZ-PAULI THEORY $\gamma = 1$ (1939)

ADDITIONAL CONSTRAINT EQUATION

- Constraints $\partial^{\mu}h_{\mu\nu} \partial_{\nu}h = 0$ imply $\partial^{\mu}\partial^{\nu}h_{\mu\nu} \Box h = 0$
- Trace of the equations of motion $2(\partial^{\mu}\partial^{\nu}h_{\mu\nu}-\Box h)=-3m^{2}h$
- \rightarrow Additional constraint h = 0

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- Naively $10-5=5=2\times 2+1$ degrees of freedom
- ! Can be seen explicitly by considering plane wave solutions $h_{\mu\nu} = \epsilon_{\mu\nu} e^{ik_{\rho}x^{\rho}}$ of the equations of motion $\Box h_{\mu\nu} - m^2 h_{\mu\nu} = 0$

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GHOST-FREE QUADRATIC THEORY

• If $\gamma \neq 1$ there is no additional constraint

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GHOST-FREE QUADRATIC THEORY

- If $\gamma \neq 1$ there is no additional constraint
- Equation of motion of h

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 - Even when $\gamma \in]1/4, 1[$ a ghost appears...

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RECOVERING GR AT SMALL SCALES

• Fierz-Pauli theory is not in agreement with experiment (cf. light bending)

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RECOVERING GR AT SMALL SCALES

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 $F.P. \rightarrow G.R.$ when $m \rightarrow 0$

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 Non-linearities may save us by screening the effects of the massive graviton at solar system scales (Vainshtein mechanism)

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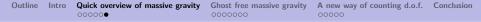
- Non-linearities may save us by screening the effects of the massive graviton at solar system scales (Vainshtein mechanism)
- We need a non-linear theory which reduces to Fierz-Pauli at quadratic order

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NON-LINEAR COMPLETIONS OF FIERZ-PAULI THEORY

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$$m^2 \int d^4x \sqrt{-g} V(g^{-1}f)$$

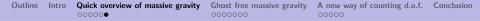


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- Boulware and Deser (1972) showed explicitly that in some non-linear theories of massive gravity there was an inevitable ghost-like instability
- Due to the absence of a fifth constraint as in Fierz-Pauli theory

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DRGT MASSIVE GRAVITY

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 - Vielbein and multi-vielbein reformulation by Hinterbichler, Rosen

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THE MASS TERMS

THE DRGT ACTION

$$S_{dRGT} = M_P^2 \int d^4 x \sqrt{-g} R - M_P^2 m^2 \sum_{n=0}^{3} \alpha_n \int d^4 x \sqrt{-g} E_n(\sqrt{g^{-1}f})$$

where $f_{\mu\nu}$ is non-dynamical and

$$E_0(A) = 1$$

$$E_1(A) = Tr(A)$$

$$E_2(A) = \frac{1}{2}(Tr(A^2) - Tr(A)^2)$$

$$E_3(A) = \frac{1}{6}(Tr(A)^3 - 3Tr(A)Tr(A^2) + Tr(A^3))$$

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 \rightarrow 3 parameter family of non-trivial theories

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MOTIVATION BEHIND THIS AWKWARD FORM

• General non-linear theory of massive gravity

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MOTIVATION BEHIND THIS AWKWARD FORM

- General non-linear theory of massive gravity
- \rightarrow BD ghost \implies higher order equations of motion for the scalar mode in the decoupling limit

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 - Presence of Galileon terms in the decoupling limit action

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VIERBEIN REFORMULATION

• Unpleasant presence of a matrix square root in the action

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EXISTENCE OF MATRIX SQUARE ROOTS

A necessary and sufficient condition for a real matrix A to admit a real square root is the following: for every negative eigenvalue λ , the number of identical Jordan blocks (in the Jordan decomposition of A) associated with λ must be even

 \rightarrow The existence of the matrix square root is not automatic

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- \rightarrow The existence of the matrix square root is not automatic
 - Introducing vierbein variables for each of the two metrics

$$g_{\mu\nu} = \eta_{AB} E^{A}{}_{\mu} E^{B}{}_{\nu} \quad \text{and} \quad g^{\mu\nu} = \eta^{AB} e_{A}{}^{\mu} e_{B}{}^{\nu}$$
$$f_{\mu\nu} = \eta_{AB} L^{A}{}_{\mu} L^{B}{}_{\nu} \quad \text{and} \quad f^{\mu\nu} = \eta^{AB} \ell_{A}{}^{\mu} \ell_{B}{}^{\nu}$$

VIERBEIN REFORMULATION

• Unpleasant presence of a matrix square root in the action

EXISTENCE OF MATRIX SQUARE ROOTS

A necessary and sufficient condition for a real matrix A to admit a real square root is the following: for every negative eigenvalue λ , the number of identical Jordan blocks (in the Jordan decomposition of A) associated with λ must be even

- $\rightarrow\,$ The existence of the matrix square root is not automatic
 - Introducing vierbein variables for each of the two metrics

$$g_{\mu\nu} = \eta_{AB} E^{A}{}_{\mu} E^{B}{}_{\nu} \quad \text{and} \quad g^{\mu\nu} = \eta^{AB} e_{A}{}^{\mu} e_{B}{}^{\nu}$$
$$f_{\mu\nu} = \eta_{AB} L^{A}{}_{\mu} L^{B}{}_{\nu} \quad \text{and} \quad f^{\mu\nu} = \eta^{AB} \ell_{A}{}^{\mu} \ell_{B}{}^{\nu}$$

• Symmetry condition $e_A{}^{\mu}L_{B\mu} = e_B{}^{\mu}L_{A\mu}$ implies

$$\sqrt{g^{-1}f}^{\mu}{}_{\nu} = e_{A}{}^{\mu}L^{A}{}_{\nu}$$

Ghost free massive gravity ○○●●○○○

A new way of counting d.o.f. Conclusion

VIERBEIN REFORMULATION

The vierbein action

$$S_{dRGT} = M_P^2 \int \Omega^{AB} \wedge E_{AB}^* - M_P^2 m^2 \sum_{n=0}^3 \beta_n \int L^{A_1} \wedge \cdots \wedge L^{A_n} \wedge E_{A_1 \dots A_n}^*$$

where

$$E_{A_{1}...A_{n}}^{*} \equiv \frac{1}{(4-n)!} \varepsilon_{A_{n+1}...A_{4}} E^{A_{n+1}} \wedge \cdots \wedge E^{A_{4}}$$

$$\Omega^{AB} \equiv d\omega^{AB} + \omega^{A}{}_{C} \wedge \omega^{CB} \quad \text{(Curvature two-form)}$$

$$dE^{A} + \omega^{A}{}_{B} \wedge E^{B} = 0 \quad \text{(Spin connection)}$$

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? Choosing vierbeins obeying the symmetry condition

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An incomplete equivalence

• Imposing the symmetric vierbein condition has been claimed to be possible in general (not just perturbatively)

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An incomplete equivalence

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- ! BUT reliance on some sort of "generalized polar decomposition"

FIRST RESULT

An invertible matrix M can be decomposed as $M = \Lambda.S$ with Λ being a Lorentz transformation matrix and S a symmetric matrix if and only if the matrix $\eta M^t \eta M$ admits a real square root which can be written as a product of η by a symmetric matrix

An incomplete equivalence

Second result

The symmetric vierbein condition can be imposed via local Lorentz transformations if and only if

- (i) the matrix $g^{-1}f$ admits a real square root γ
- (ii) $f\gamma$ is symmetric

An incomplete equivalence

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 - ? Relationship between hypotheses (*i*) and (*ii*) i.e. is (*i*) sufficient for the result to hold?

An incomplete equivalence

Second result

The symmetric vierbein condition can be imposed via local Lorentz transformations if and only if

(i) the matrix
$$g^{-1}f$$
 admits a real square root γ

(ii) $f\gamma$ is symmetric

- ? Relationship between hypotheses (*i*) and (*ii*) i.e. is (*i*) sufficient for the result to hold?
- ! YES in dimensions 2, 3, 4

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Hyp: $g^{-1}f$ has real square roots

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- HYP: $g^{-1}f$ has real square roots

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- HYP: $g^{-1}f$ has real square roots

 - Negative eigenvalues: only five possible Jordan forms

$$\begin{array}{rcl} J_1 &=& diag(-u,-u,-v,-v) \\ J_2 &=& diag(-u,-u,v,w) & (\text{only one which can occur}) \\ J_3 &=& diag\left(\begin{pmatrix} -u & 0 \\ 0 & -u \end{pmatrix}, \pm \begin{pmatrix} v+iw & 0 \\ 0 & v-iw \end{pmatrix} \right) \\ J_4 &=& diag\left(\begin{pmatrix} -u & 0 \\ 0 & -u \end{pmatrix}, \begin{pmatrix} v & 1 \\ 0 & v \end{pmatrix} \right) \\ J_5 &=& diag\left(\begin{pmatrix} -u & 1 \\ 0 & -u \end{pmatrix}, \begin{pmatrix} -u & 1 \\ 0 & -u \end{pmatrix} \right) \right) \end{array}$$

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D=4 example

- HYP: $g^{-1}f$ has real square roots

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 - Negative eigenvalues: only five possible Jordan forms

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ightarrow J_2 : existence a real square root with the desired symmetry __ $_{
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Ghost free massive gravity A new way of counting d.o.f. Conclusion

THE VIERBEIN ACTION AS A STARTING POINT

THE VIERBEIN ACTION

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where L^A is the non-dynamical one-form dx^A and

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The vierbein action as a starting point

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! Theory with a priori 16 degrees of freedom

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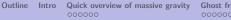
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EQUATIONS OF MOTION

Einstein three-form

$${\cal G}_{A}\equiv -rac{1}{2}\Omega^{BC}\wedge E^{*}_{ABC}\equiv {\cal G}_{A}{}^{B}E^{*}_{B}$$

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Ghost free massive gravity

EQUATIONS OF MOTION

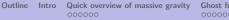
• Einstein three-form

$$G_A \equiv -rac{1}{2}\Omega^{BC} \wedge E^*_{ABC} \equiv G_A{}^B E^*_B$$

• Mass term three-form (analogous to the energy-momentum three-form appearing in the presence of matter)

$$t_A \equiv \frac{1}{2} \sum_{n=0}^{3} \beta_n L^{A_1} \wedge \cdots \wedge L^{A_n} \wedge E^*_{AA_1 \dots A_n} \equiv t_A{}^B E^*_B$$

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 \rightarrow Equations of motion: $G_A = t_A$ or, in components, $G_{AB} = t_{AB}$

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LOCAL LORENTZ SYMMETRY BREAKING

 Invariance of the kinetic term under local Lorentz transformations $G_{[AB]} = 0$ so $t_{[AB]} = 0$

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LOCAL LORENTZ SYMMETRY BREAKING

- Invariance of the kinetic term under local Lorentz transformations $G_{[AB]} = 0$ so $t_{[AB]} = 0$
- 6 constraints which, in principle, eliminate 6 degrees of freedom

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 - Only β_0 and β_1 are non-zero
 - Only β_0 and β_3 are non-zero
 - The β_n satisfy a specific relation such that the mass term $\propto \det(E^A_{\mu} - \kappa L^A_{\mu})$

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 - The β_n satisfy a specific relation such that the mass term $\propto \det(E^{A}_{\mu} - \kappa L^{A}_{\mu})$
 - Counter-example: if only β_0 and β_2 an antisymmetric vierbein condition is also compatible with the constraints (4回) (注) (注) (注) (注)

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DIFFEOMORPHISM INVARIANCE BREAKING

• Invariance of the kinetic term under diffeomorphisms $DG_A = 0$ so $Dt_{\Delta} = 0$ where

$$DF_A \equiv dF_A + \omega_A{}^B \wedge F_B$$

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 - Specific cases
 - Only β_0 and β_1 are non-zero

$$\omega^{B}{}_{A\mu}e_{B}{}^{\mu}=0$$

• Only β_0 and β_2 are non-zero (with the symmetry condition enforced)

$$\omega^{B}{}_{C\mu}e_{B}{}^{\mu}e_{A}{}^{C}+\omega^{B}{}_{A\mu}e_{C}{}^{\mu}e_{B}{}^{C}-\omega^{B}{}_{A\mu}e_{B}{}^{\mu}e_{C}{}^{C}=0$$

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Additional constraint

• As in Fierz-Pauli theory, we trace over the equations of motion and try to get rid of second derivatives

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- $\rightarrow L^A \wedge G_A = L^A \wedge t_A$ is a new constraint!
 - ! But here we enforced the symmetry condition manually
 - Other cases not treatable in the same manner...

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Recovering Fierz-Pauli

• With the additional constraint, 6 - 1 = 5 degrees of freedom naively

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$$E^{A} = dx^{A} + E^{A}_{(1)}$$
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$$E^{A} = dx^{A} + E^{A}_{(1)}$$
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• Symmetry condition verified because $t_{(1)}^{AB} \propto \text{Tr}(e^{(1)})\eta^{AB} - e^{AB}_{(1)}$

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- Symmetry condition verified because $t_{(1)}^{AB} \propto \text{Tr}(e^{(1)})\eta^{AB} e^{AB}_{(1)}$
- Relationship to the metric formalism well defined

$$g_{\mu
u} = \eta_{\mu
u} + h_{\mu
u}$$

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Recovering Fierz-Pauli

• Change of variable formulas

$$E_{AB}^{(1)} = \frac{h_{AB}}{2}$$

$$e_{(1)}^{AB} = -\frac{h^{AB}}{2}$$

$$\omega_{ABC}^{(1)} = \frac{1}{2}(\partial_B h_{AC} - \partial_A h_{BC})$$

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CONSTRAINTS

 $Dt_A = 0$ becomes $\partial^{\mu} h_{\mu\nu} = 0$ while $m^A \wedge G_A = m^A \wedge t_A$ becomes h = 0 and we recover the Fierz-Pauli constraints

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 \rightarrow The constraints are independent

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CONCLUSION AND FUTURE DIRECTIONS

 We studied dRGT theory in the vierbein formalism and clarified some ambiguities

CONCLUSION AND FUTURE DIRECTIONS

- We studied dRGT theory in the vierbein formalism and clarified some ambiguities
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