Bimetric MOND gravity (BIMOND)

Moti Milgrom (Weizmann)

GReCO June 2013

MOND introduced

A theory of dynamics (gravity/inertia) involving a new constant a_0 (beside $G, ...$)

$a_0 \to 0$: The Newtonian limit

$$\mathrm{MOND\ limit}: \; a_0 \to \infty, \;\; G \to 0, \;\; \Omega_0 \equiv G a_0 \ \mathrm{fixed}:$$

Scale invariance: $(t, \mathbf{r}) \to \lambda(t, \mathbf{r})$

Nonrelativistic theories

Nonlinear Poisson equation:

$$\vec{\nabla} \cdot [\mu(|\vec{\nabla}\phi|/a_0)\vec{\nabla}\phi] = 4\pi G\rho$$

The deep-MOND limit is conformally invariant

Quaslinear MOND (QUMOND):

$$\Delta\phi^N = 4\pi G\rho, \qquad \Delta\phi = \vec{\nabla} \cdot [\nu(|\vec{\nabla}\phi^N|)\vec{\nabla}\phi^N]$$

Kepler-like laws in galaxies

1. Speeds along an orbit around any bounded mass, M, become asymptotically independent of the size of the orbit. For circular orbits $V(r \to \infty) \to V_\infty(M)$. Kepler's 3rd law $T^2 = K(M)r^3 \Rightarrow T = \bar{K}(M)r$. (H)

2. $V_\infty(M) = (MGa_0)^{1/4}$, which dictates the dependence of $\bar{K}$ on the central mass. (H-B)

3. A mass discrepancy appears when we cross $a = a_0$. In disc galaxies, the transition from 'baryon dominance' to 'DM dominance' occurs always around the radius where $V^2(r)/r = a_0$. (H-B)

4. Quasi-isothermal systems, which well model many galactic systems, have mean surface densities $\bar{\Sigma} \lesssim \Sigma_M \equiv a_0/2\pi G$. (B)

5. Quasi-isothermal, or deep-MOND, systems of mass M, have characteristic velocity dispersion $\sigma \sim (MGa_0)^{1/4}$. (B)

6. The external field in which a system is falling affects its intrinsic dynamics: the external-field effect.

7. Disc galaxies behave as if they have both disc and spherical ‘DM’ components of predictable properties.

8. MOND endows self gravitating systems with an increased, but limited stability.

9. The incremental acceleration MOND predicts (that attributed to ‘DM’) can never much exceed a_0. (H)

10. The central surface density of ‘dark halos’ is $\lesssim \Sigma_M$. (H)

MOND phenomenology

- Essentially follow from only the basic tenets of MOND
- Are independent as phenomenological laws–e.g., if interpreted as effects of DM
- Pertain separately to properties of the "DM" alone, of the baryons alone, or to relations between the two
- Revolve around a_0 in different roles

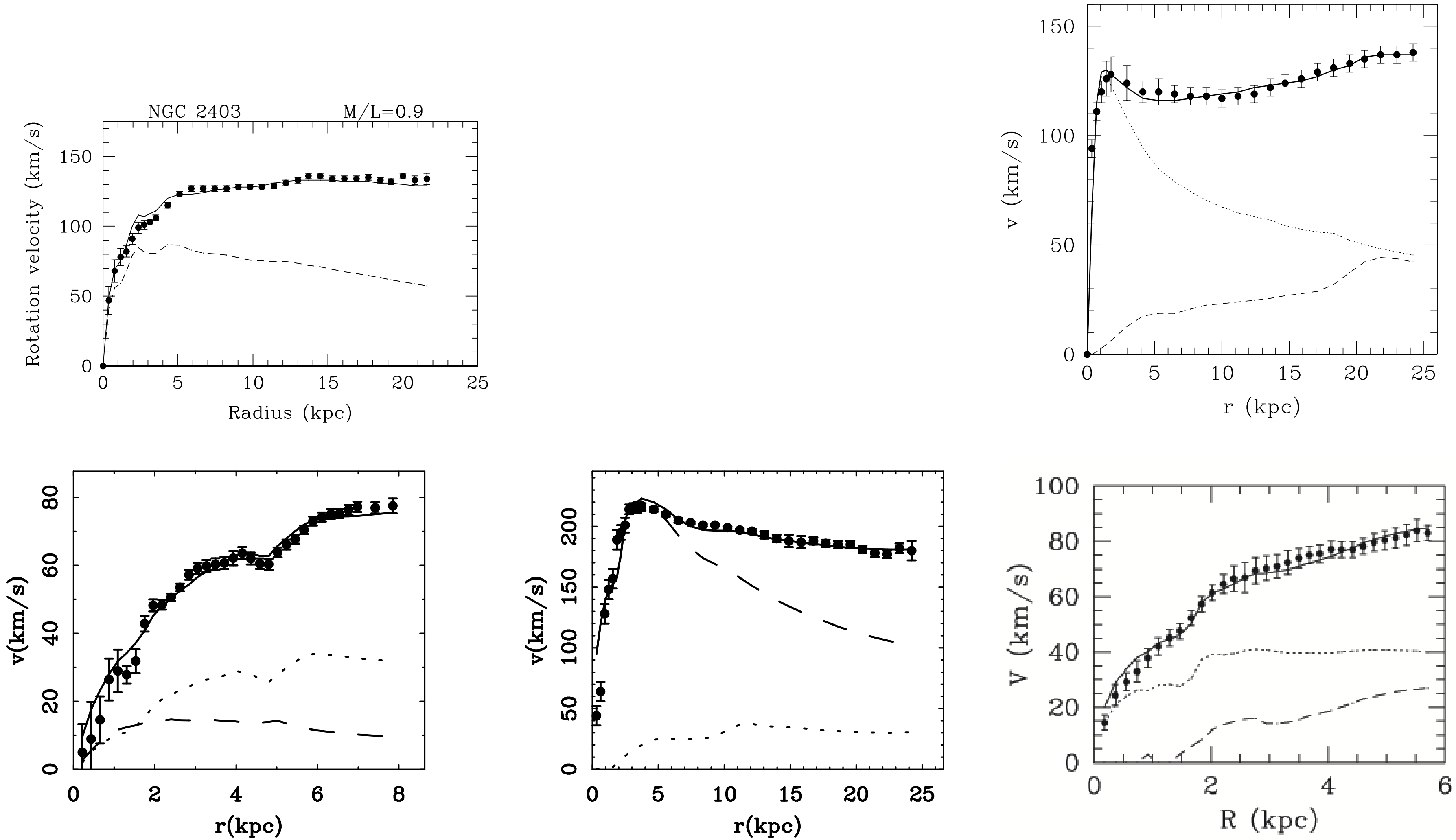
$a_0 =?$

$$\bar{a}_0 \equiv 2\pi a_0 \approx cH_0 \approx c^2(\Lambda/3)^{1/2}$$

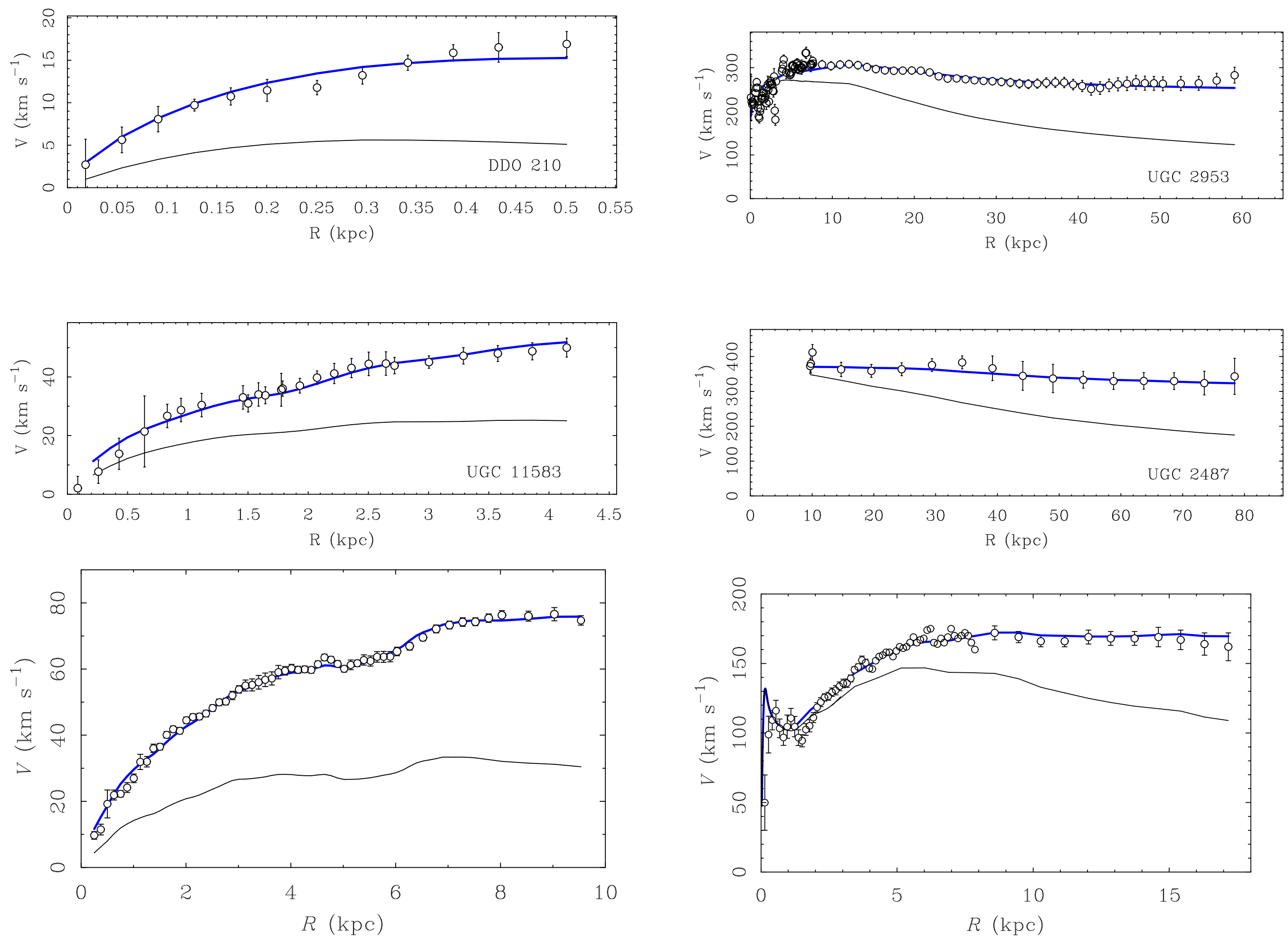
$$\ell_M \equiv c^2/a_0 \approx 2\pi D_H$$

Strong-field deep-MOND limit? $MG/R_s^2 < a_0 \Rightarrow R_s > D_H$

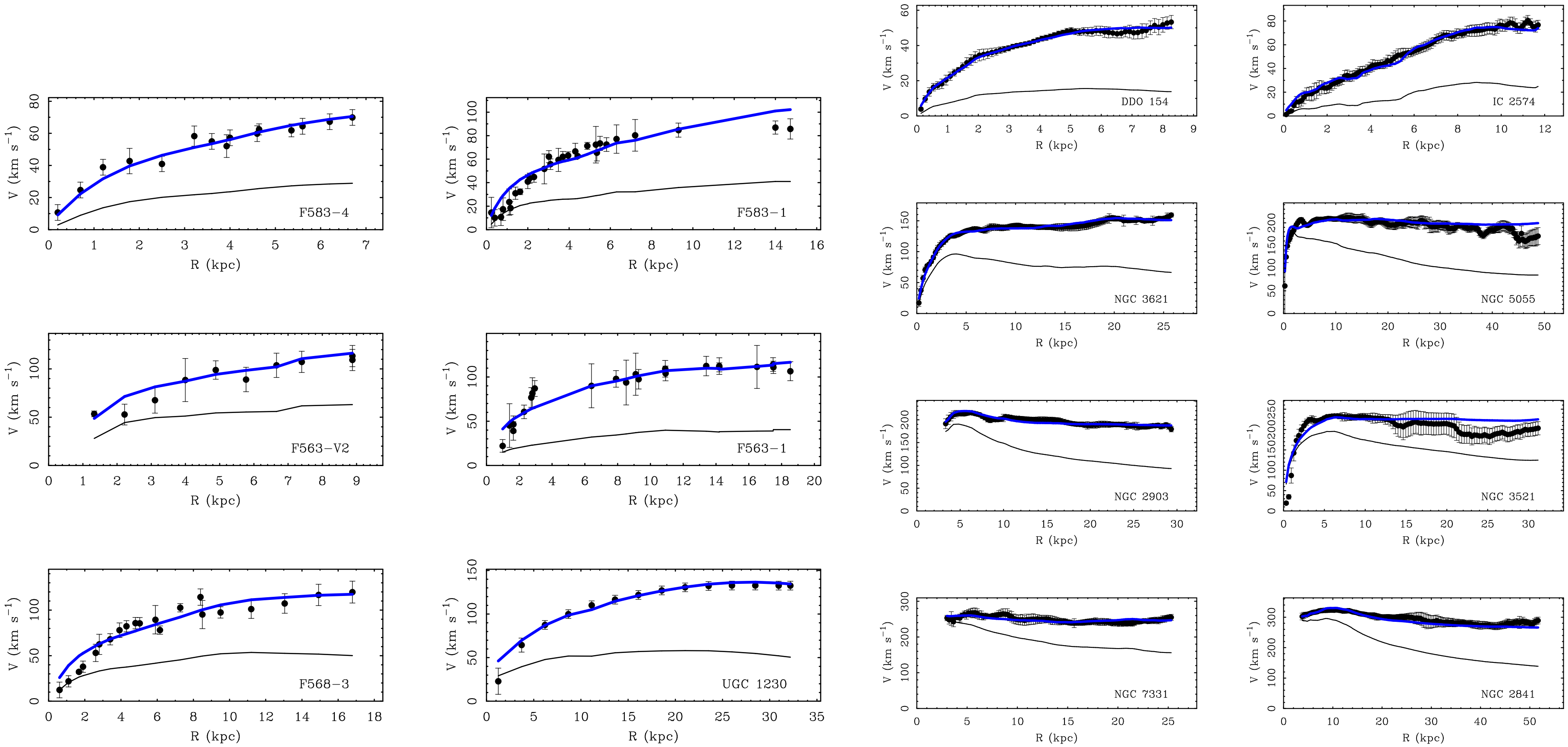
Rotation Curves of Disc Galaxies



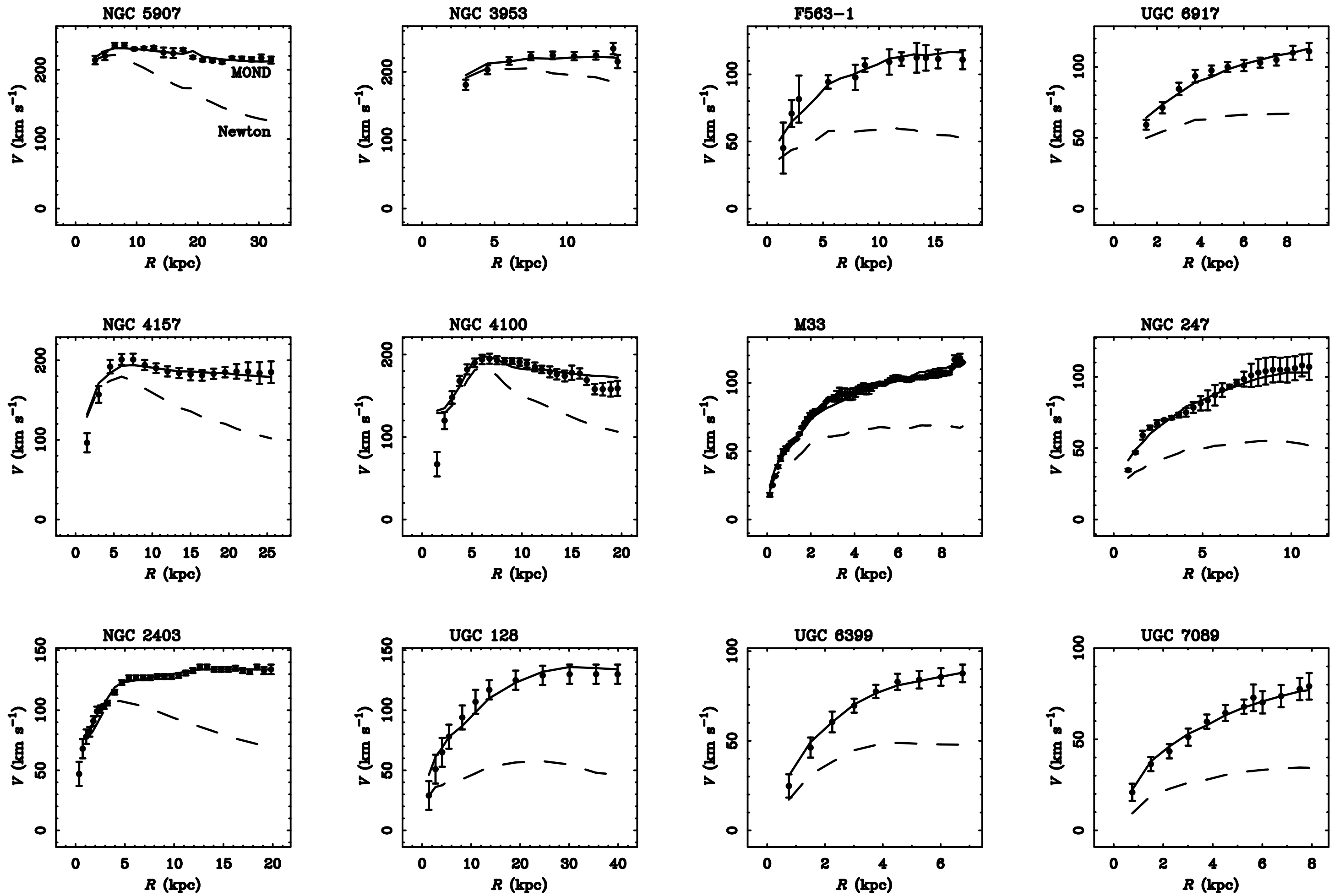
From Sanders 2005 and Sanders and McGaugh 2002



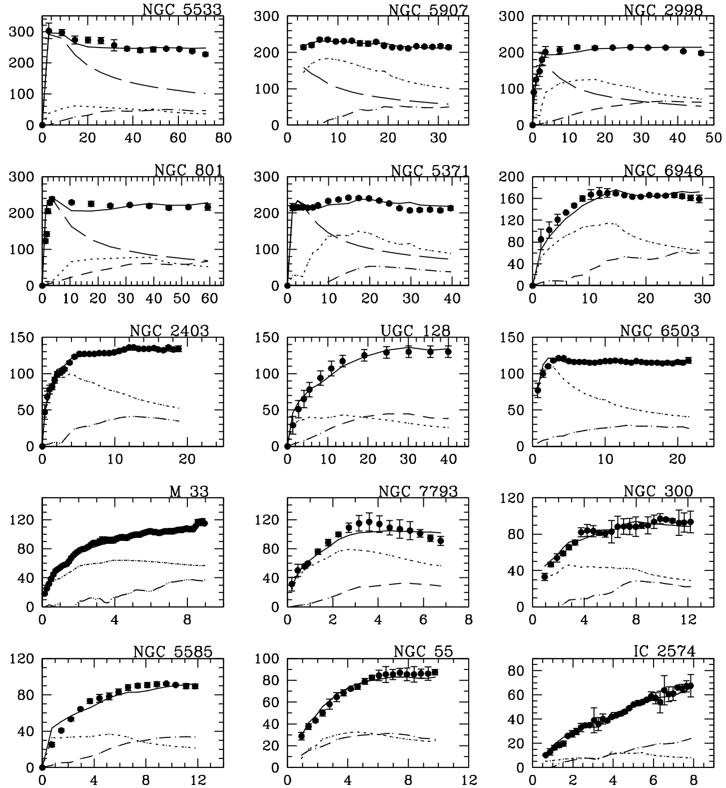
From review by Famaey and McGaugh 2012

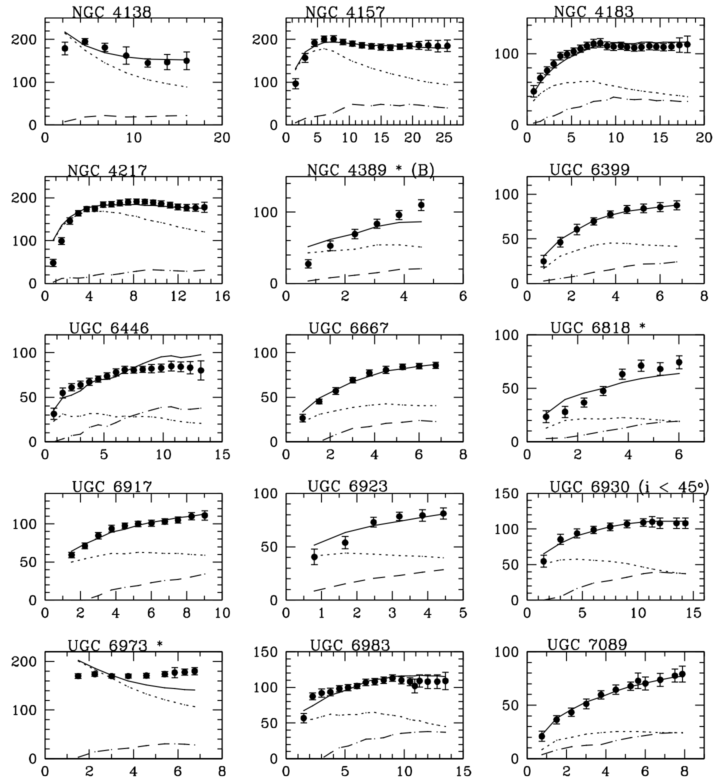


From review by Famaey and McGaugh 2012


NGC 5907
NGC 3953
F563-1
UGC 6917
NGC 4157
NGC 4100
M33
NGC 247
NGC 2403
UGC 128
UGC 6399
UGC 7089
MOND
Newton
V (km s^-1)
R (kpc)

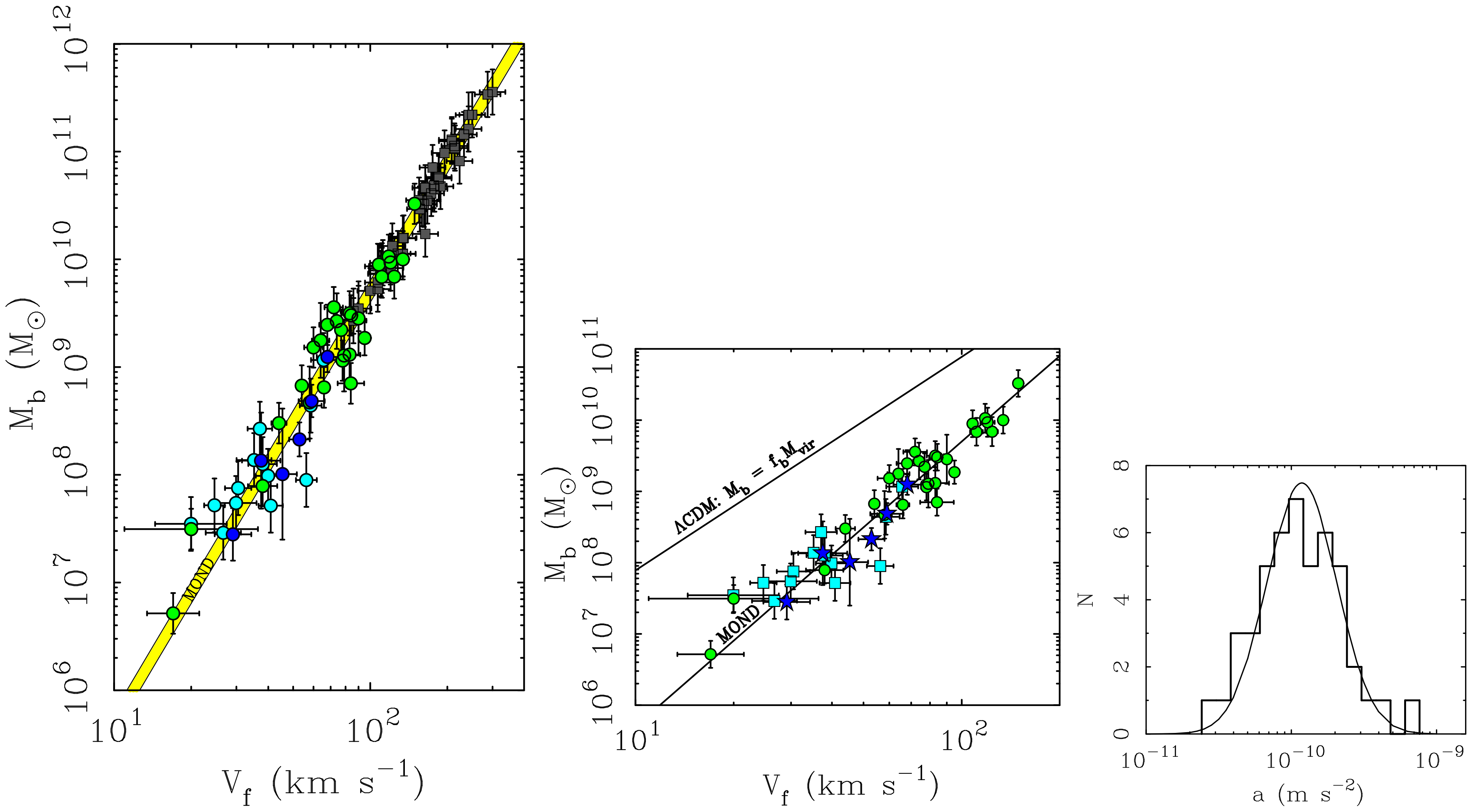
McGaugh



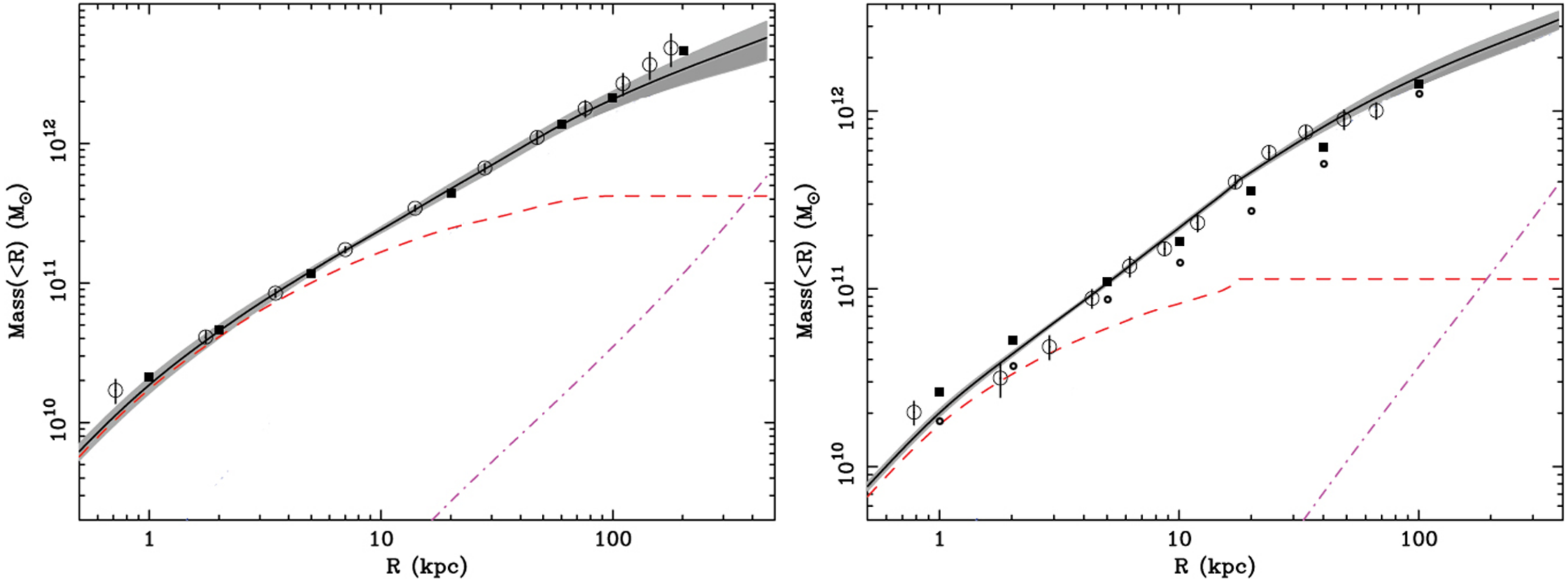


from Sanders and McGaugh 2002

Mass-asymptotic-speed relation–McGaugh 2011

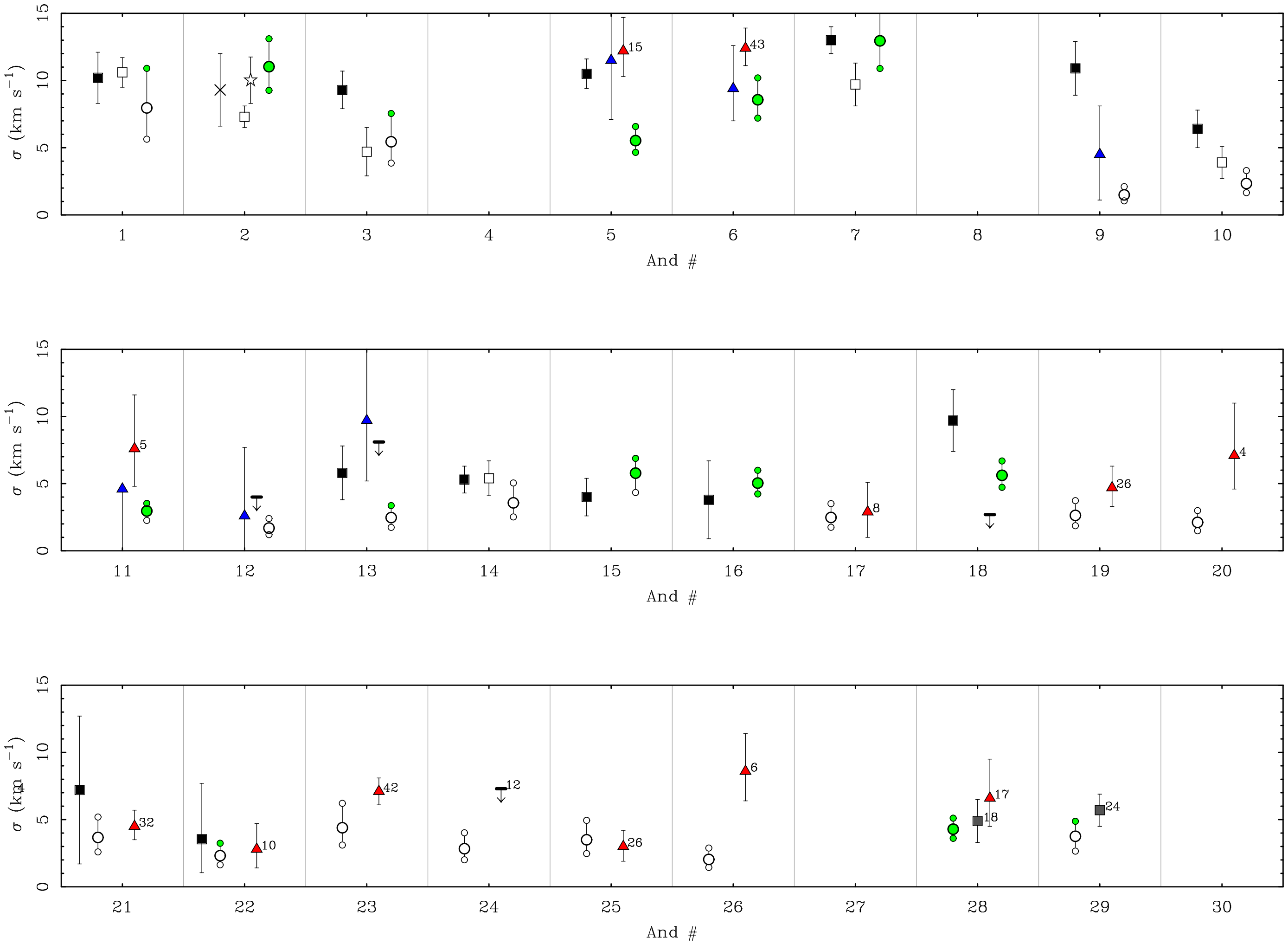


x-ray Ellipticals, tested over an acceleration range $\sim 10a_0 - 0.1a_0$



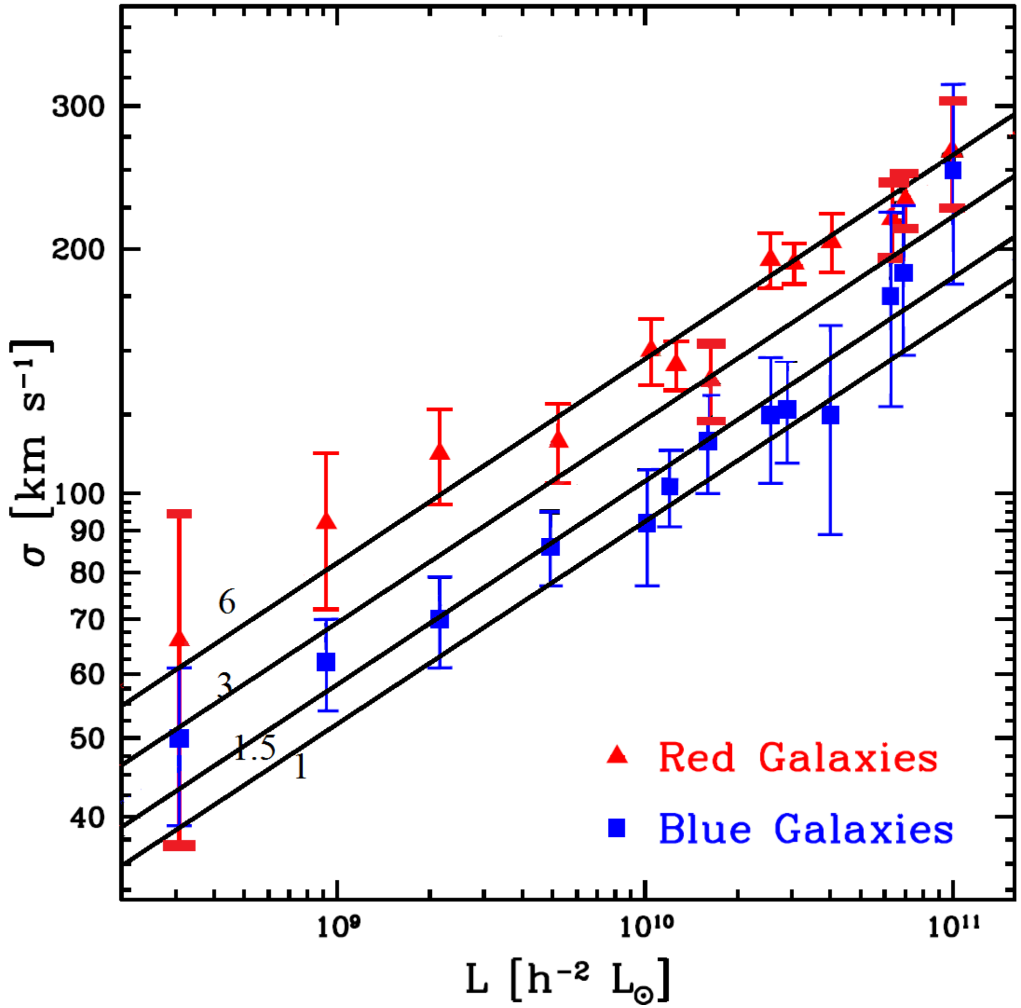
Baryon (dashed) and dynamical masses (grey band and large circles) from Humphrey et al. 2011,2012; MOND points (squares and small rings) from Milgrom 2012

Andromeda satellites–internal dynamics



McGaugh and Milgrom 2013.

Galaxy-galaxy lensing



Data from Brimioulle et al. 2013, analysis from Milgrom 2013.

Relativistic MOND theories

- TeVeS (Sanders, Bekenstein, Babichev, Deffayet, & Esposito-Farese)
- MOND aether theories (Zlosnik, Ferreira, & Starkman, Blanchet & Marsat)
- Polarizable medium (Blanchet, Blanchet & Le Tiec)
- Nonlocal theories (Soussa & Woodard, Deffayet, Esposito-Farese & Woodard)
- BIMOND (Milgrom)

Proof of concept, and good description of lensing, but is any of these the ultimate theory???

BIMOND

$$I = -\frac{1}{16\pi G}\int [g^{1/2}R + \hat{g}^{1/2}\hat{R} + 2v_{g\hat{g}}\ell_M^{-2}\mathcal{M}]d^4x + I_M(g_{\mu\nu},\psi_i) + \hat{I}_M(\hat{g}_{\mu\nu},\chi_i)$$

$$\ell_M \equiv c^2/a_0$$

$\mathcal{M}$ a dimensionless scalar a function of (quadratic) scalars of

$$\ell_M C^\alpha_{\beta\gamma}, \qquad C^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \hat{\Gamma}^\alpha_{\beta\gamma}$$

$$Q^{\beta\gamma\mu\nu}_{\alpha\lambda}(g_{\mu\nu},\hat{g}_{\mu\nu})\ell_M^2 C^\alpha_{\beta\gamma}C^\lambda_{\mu\nu}$$

$$\Upsilon_{\mu\nu} = C^\gamma_{\mu\lambda}C^\lambda_{\nu\gamma} - C^\gamma_{\mu\nu}C^\lambda_{\lambda\gamma}$$

$$\Upsilon = g^{\mu\nu}\Upsilon_{\mu\nu}, \quad \hat{\Upsilon} = \hat{g}^{\mu\nu}\Upsilon_{\mu\nu}$$

A class of theories

- $R \Rightarrow \beta R, \qquad \hat{R} \Rightarrow \alpha \hat{R}$: $(\alpha = \beta = 1)$

- Choice of scalar argument(s) Other examples:

$$g^{\mu\nu} C^{\gamma}_{\mu\lambda} C^{\lambda}_{\nu\gamma}, \quad g^{\mu\nu} C^{\gamma}_{\mu\nu} C^{\lambda}_{\lambda\gamma}, \quad g_{\mu\nu} \hat{g}^{\alpha\beta} C^{\mu}_{\alpha\beta} \hat{g}^{\lambda\delta} C^{\nu}_{\lambda\delta}, \quad \hat{g}_{\alpha\lambda} g^{\beta\mu} g^{\gamma\nu} C^{\alpha}_{\beta\gamma} C^{\lambda}_{\mu\nu}$$

  Also, e.g., $g/\hat{g}$

- Form of $\mathcal{M}$

Field equations

$$G_{\mu\nu} + S_{\mu\nu} = -8\pi G \mathcal{T}_{\mu\nu}$$

$$\hat{G}_{\mu\nu} + \hat{S}_{\mu\nu} = -8\pi G \hat{\mathcal{T}}_{\mu\nu}$$

$$S_{\mu\nu} = [Q(C^2)\{C\}^{\lambda}_{\mu\nu}]_{;\lambda} + N(C^2)\{CC\}_{\mu\nu} + a_0^2 P(C^2) g_{\mu\nu}$$

Q and N derivatives of $\mathcal{M}$; P has both $\mathcal{M}$ and $\mathcal{M}'$

Matter equations of motion as in GR (geodesic motion, etc.)

Limits

- The high-acceleration limit; the decoupling limit

  $\mathcal{M} \to \mathcal{M}(\infty) = const$: we get two uncoupled copies of GR with a CC $\propto a_0^2 P(\infty) \propto a_0^2 \mathcal{M}(\infty)$

  The limit may not be expandable in a_0 (ℓ_M^{-1})

  Implications for the solar system, binary pulsar, etc.

- Metric equality: GR with a CC: $\Lambda \sim \mathcal{M}(0) a_0^2$

  For example, a double Schwarzschild solution

The weak-field limit

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}, \qquad h_{\mu\nu},\ \hat{h}_{\mu\nu} \ll 1$$

$$I \approx -\int d^4x[E(h^{+}_{\mu\nu}) + h^{+}_{\mu\nu}\mathcal{T}^{+\mu\nu} + 4a_0^2\bar{\mathcal{M}}(h^{-}_{\mu\nu}) + h^{-}_{\mu\nu}\mathcal{T}^{-\mu\nu}]$$

$$h^{\pm}_{\mu\nu} \equiv h_{\mu\nu} \pm \hat{h}_{\mu\nu}, \qquad \mathcal{T}^{\pm\mu\nu} \equiv \mathcal{T}^{\mu\nu} \pm \hat{\mathcal{T}}^{\mu\nu}$$

$$\Sigma_{\mu\nu} \equiv [\bar{\mathcal{M}}'(z)\bar{S}^{\lambda}_{\mu\nu}]_{,\lambda} = 4\pi G\mathcal{T}^{-}_{\mu\nu}$$

$$\bar{S}^{\lambda}_{\mu\nu} \equiv C^{\lambda}_{\mu\nu} - \frac{1}{2}\delta^{\lambda}_{\mu}C_{\nu} - \frac{1}{2}\delta^{\lambda}_{\nu}C_{\mu} + \frac{1}{2}\eta_{\mu\nu}(C^{\lambda} - \bar{C}^{\lambda})$$

$$z = \frac{1}{8a_0^2}[h^{-\nu\rho}{}_{,}{}^{\gamma}(h^{-}_{\nu\rho,\gamma} - 2h^{-}_{\nu\gamma,\rho}) - h^{-}{}_{,}{}^{\gamma}(h^{-}_{,\gamma} - 2h^{-\rho}_{\gamma,\rho})]$$

The nonrelativistic limit

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\phi\delta_{\mu\nu}, \qquad \hat{g}_{\mu\nu} = \eta_{\mu\nu} - 2\hat{\phi}\delta_{\mu\nu}$$

$$\tilde{\phi} = \phi + \hat{\phi}, \qquad \bar{\phi} = (\phi - \hat{\phi})/2$$

$$\Delta\tilde{\phi} = 4\pi G(\rho + \hat{\rho}), \qquad \vec{\nabla} \cdot \{\tilde{\mu}(|\vec{\nabla}\bar{\phi}|/a_0)\vec{\nabla}\bar{\phi}\} = 4\pi G(\rho - \hat{\rho})$$

MOND limit: $\tilde{\mu}(x \ll 1) \propto x$

$$\Rightarrow$$

$$z = (\vec{\nabla}\phi - \vec{\nabla}\hat{\phi})^2/a_0^2, \qquad \bar{\mathcal{M}}'(z \ll 1) \propto z^{1/2}$$

$$\Rightarrow$$

Scale invariance of the WFL $h^{-}_{\mu\nu}$ sector for a system of masses: E.g., bending angle is independent on impact parameter for deep MOND

Matter-Twin matter interactions

- No MOND for $\rho = \hat{\rho}$
- Full MOND when $\hat{\rho} = 0$
- No interaction in Newtonian regime for $\beta = 1$ $[\tilde{\mu}(\infty) = 2]$: $\phi = \phi_N$, $\hat{\phi} = \hat{\phi}_N$
- Repulsion in the MOND regime
- Light bending as in GR, but with the MOND potential

Deep-MOND limit

$$a_0 \to \infty, \quad G \to 0, \quad \Omega_0 \equiv a_0 G \ \text{ fixed}$$

$$\phi = -\hat{\phi} = \bar{\phi}, \qquad \vec{\nabla} \cdot [|\vec{\nabla}\bar{\phi}|\vec{\nabla}\bar{\phi}] = 4\pi\Omega_0(\rho - \hat{\rho})$$

- Conformal invariance
- M and TM behave as having opposite gravitational masses: natural in conformal theories

$$\mathbf{a} = -\vec{\nabla}\bar{\phi}, \qquad \hat{\mathbf{a}} = \vec{\nabla}\bar{\phi}$$

- Two-body force: $\mathbf{F} = -\frac{2}{3}(\Omega_0)^{1/2}[(M \pm m)^{3/2} - M^{3/2} - m^{3/2}]\frac{\mathbf{r}}{r^2}$

MOND, cosmology, and cosmological "dark matter"

MOND will surely produce cosmological effects of DM

My own feeling is that the understanding of MOND's origin and of cosmology will come together

Present theories are not built in this way.

Cosmology

The twin sector: Nature of twin matter (interactions, etc.)? Matter content (baryogenesis, nucleosynthesis)? Big bang? Inflation? seed fluctuations?

Symmetric cosmology: standard FRW with CC $\sim a_0^2/c^2$ and $\tilde{G} = G/\beta$. Constraints on $\tilde{G} - G \Rightarrow \ \beta \approx 1$.

MOND appears only due to differences in the matter-twin-matter inhomogeneities

Back reaction from

$$S_{\mu\nu} = [Q(C^2)\{C\}^{\lambda}_{\mu\nu}]_{;\lambda} + N(C^2)\{CC\}_{\mu\nu} + a_0^2 P(C^2) g_{\mu\nu}$$

General (Clifton and Zlosnik) .

Fluctuations and structure formation

$$\mathbf{u} = \delta\mathbf{v} + \delta\hat{\mathbf{v}}, \quad \bar{\mathbf{u}} = \delta\mathbf{v} - \delta\hat{\mathbf{v}}, \quad \epsilon = (\delta\rho + \delta\hat{\rho})/\rho_b, \quad \bar{\epsilon} = (\delta\rho - \delta\hat{\rho})/\rho_b$$

$$\delta p = v_s^2(\rho_b)\delta\rho$$

Comoving coordinates

$$\dot{\epsilon} + a^{-1}\nabla \cdot \mathbf{u} = 0$$

$$\dot{\mathbf{u}} + (\dot{a}/a)\mathbf{u} = -a^{-1}\vec{\nabla}\delta\tilde{\phi} - v_s^2 a^{-1}\vec{\nabla}\epsilon$$

$$\Delta\delta\tilde{\phi} = 4\pi G a^2 \rho_b \epsilon$$

$$\dot{\bar{\epsilon}} + a^{-1}\nabla \cdot \bar{\mathbf{u}} = 0$$

$$\dot{\bar{\mathbf{u}}} + (\dot{a}/a)\bar{\mathbf{u}} = -2a^{-1}\vec{\nabla}\bar{\phi} - v_s^2 a^{-1}\vec{\nabla}\bar{\epsilon}$$

$$\nabla(\tilde{\mu}|\vec{\nabla}\bar{\phi}/aa_0|\vec{\nabla}\bar{\phi}) = 4\pi G a^2 \rho_b \bar{\epsilon}$$

Decoupling. Shepherding. Segregation. Modes are coupled. Interlacing cosmic webs. Back reaction on cosmology. Jeans criterion (gravity dominance)

Observability of twin matter

Nearby TM bodies: Tidal effects, warps, etc.?

$\sim$ Matter bodies in voids?

If only in matter voids

Lensing: No strong lensing ($\beta = 1$).

$$\frac{r_E}{R_M} = \left(\frac{4a_0 d_{ls} d_l}{c^2 d_s}\right)^{1/2} \approx \left(\frac{2 d_{ls} d_l}{\pi d_s D_H}\right)^{1/2}$$

Weak lensing in the MOND regime is repulsive.

Matters of principle

Still much to check: waves, causality, ghosts, stability, etc.