The first law of binary black hole mechanics

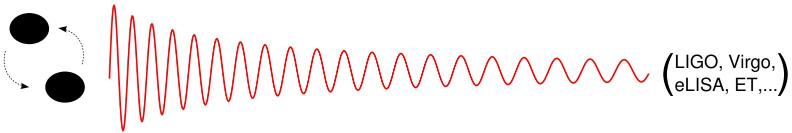
Applications to gravitational-wave source modeling

Alexandre Le Tiec

Maryland Center for Fundamental Physics
University of Maryland, College Park

Based on collaborations with:

E. Barausse, L. Blanchet, A. Buonanno & B. F. Whiting



Outline

① Black hole binaries and gravitational waves

② The first law of binary black hole mechanics

1. Derivation of the first law
2. Consequences of the first law
3. Verification of the first law in PN theory

③ Applications to gravitational-wave source modeling

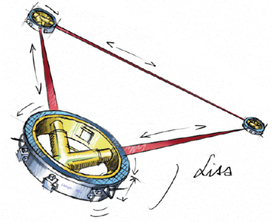
1. High-order PN coefficients in the binding energy
2. Frequency shift of the Schwarzschild ISCO
3. Perturbation theory for comparable mass binaries
4. EOB potentials at linear order in the mass ratio

Outline

Interferometric detectors of gravitational waves (GW)

Virgo (Cascina, Italy)

High frequency band:
$10 \text{ Hz} \lesssim f \lesssim 10^3 \text{ Hz}$

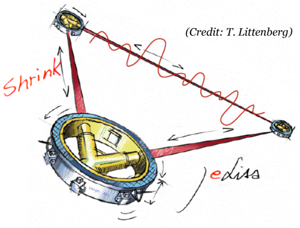


LISA (sketch)

Low frequency band:
$10^{-4} \text{ Hz} \lesssim f \lesssim 10^{-1} \text{ Hz}$

Interferometric detectors of gravitational waves (GW)

Virgo (Cascina, Italy)

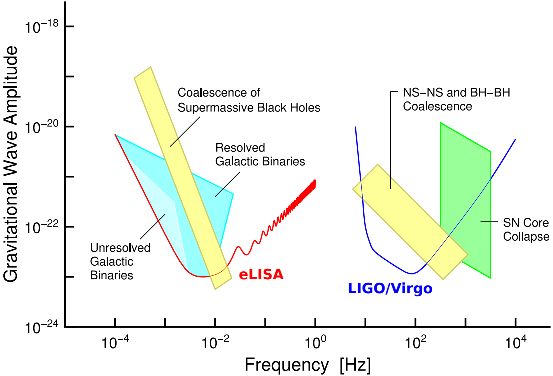


eLISA (sketch)

High frequency band:
$10\ \text{Hz} \lesssim f \lesssim 10^3\ \text{Hz}$

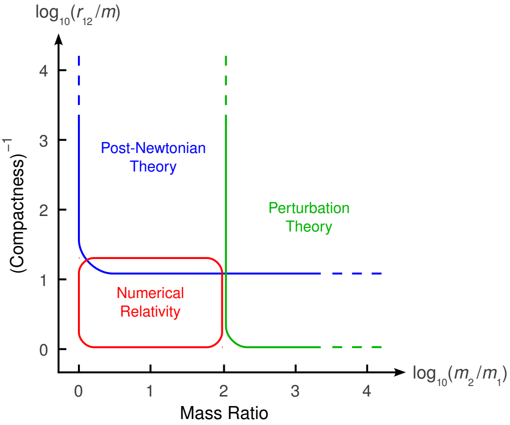
Low frequency band:
$10^{-4}\ \text{Hz} \lesssim f \lesssim 10^{-1}\ \text{Hz}$

Main sources of GW for LIGO/Virgo and eLISA

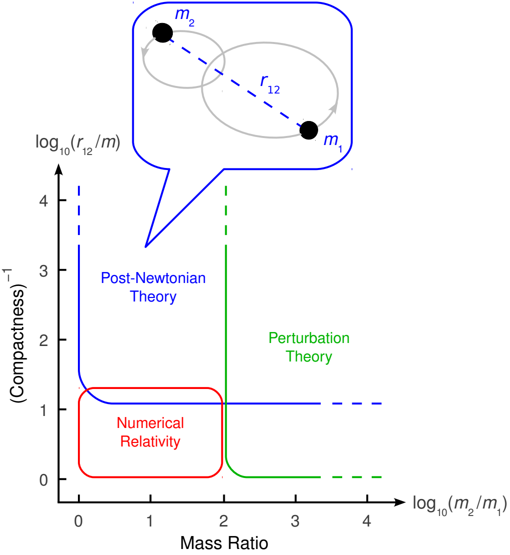


- Binary neutron stars ($2\times \sim 1.4 M_\odot$)
- Stellar mass black hole binaries ($2\times \sim 10 M_\odot$)
- Supermassive black hole binaries ($2\times \sim 10^6 M_\odot$)
- Extreme mass ratio inspirals ($\sim 10 M_\odot + \sim 10^6 M_\odot$)

Methods to compute GW templates for compact binaries



Methods to compute GW templates for compact binaries

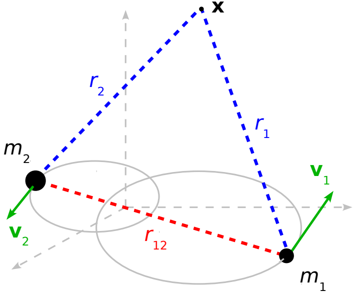


Methods to compute GW templates for compact binaries

Post-Newtonian (PN) theory

Perturbation parameter

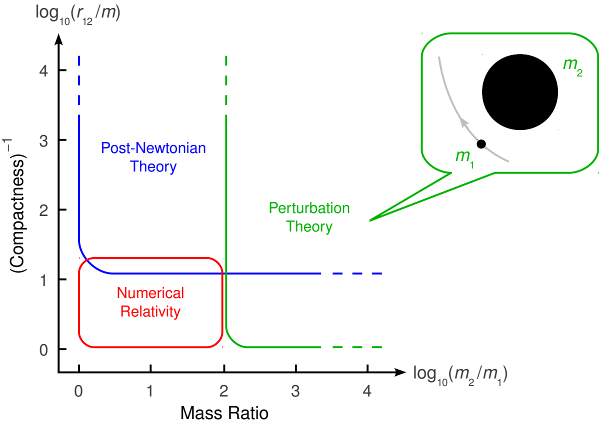
$$\varepsilon_{\rm PN} \sim \frac{{v_{12}}^2}{c^2} \sim \frac{Gm}{r_{12}c^2} \ll 1$$



Example

$$g_{00}(\mathbf{x}) = -1 + \underbrace{\frac{2Gm_1}{r_1 c^2}}_{\text{Newtonian}} + \underbrace{\frac{4Gm_2 {v_2}^2}{r_2 c^4}}_{\text{1PN term}} + \cdots + (1 \leftrightarrow 2)$$

Methods to compute GW templates for compact binaries

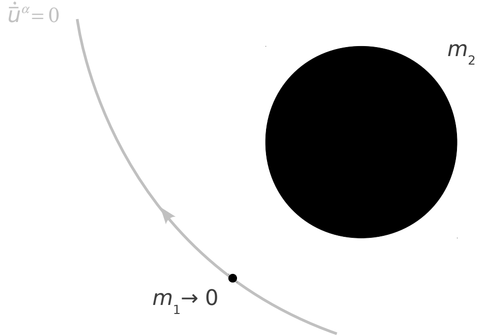


Methods to compute GW templates for compact binaries

Perturbation theory and the gravitational self-force (GSF)

Spacetime metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta}$$



Methods to compute GW templates for compact binaries

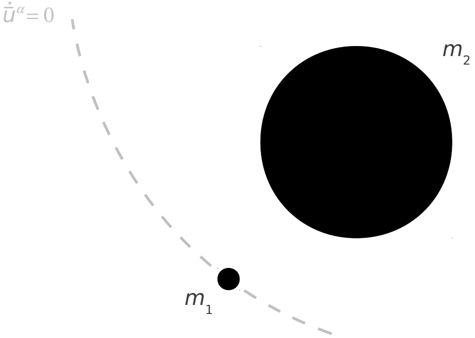
Perturbation theory and the gravitational self-force (GSF)

Spacetime metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta}$$

Perturbation parameter

$$q \equiv \frac{m_1}{m_2} \ll 1$$



Methods to compute GW templates for compact binaries

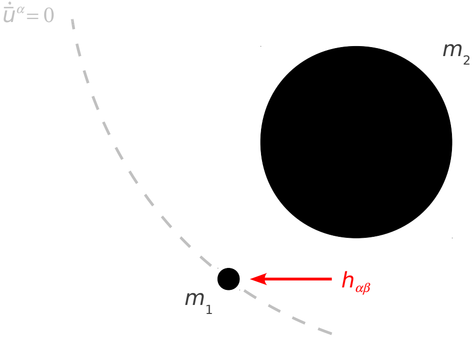
Perturbation theory and the gravitational self-force (GSF)

Spacetime metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

Perturbation parameter

$$q \equiv \frac{m_1}{m_2} \ll 1$$



Methods to compute GW templates for compact binaries

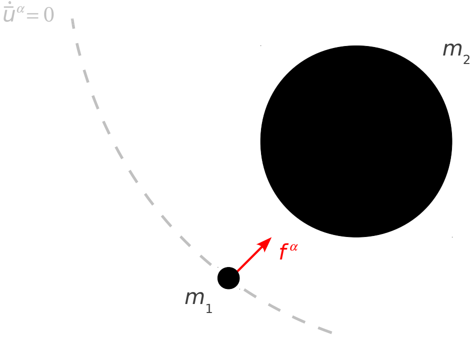
Perturbation theory and the gravitational self-force (GSF)

Spacetime metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

Perturbation parameter

$$q \equiv \frac{m_1}{m_2} \ll 1$$

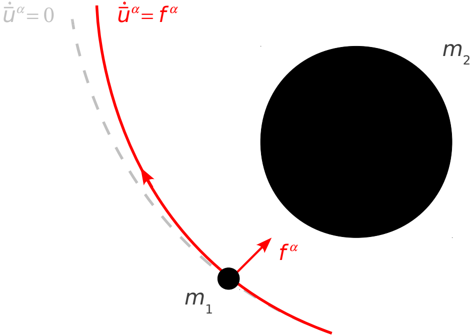


Methods to compute GW templates for compact binaries

Perturbation theory and the gravitational self-force (GSF)

Spacetime metric

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$



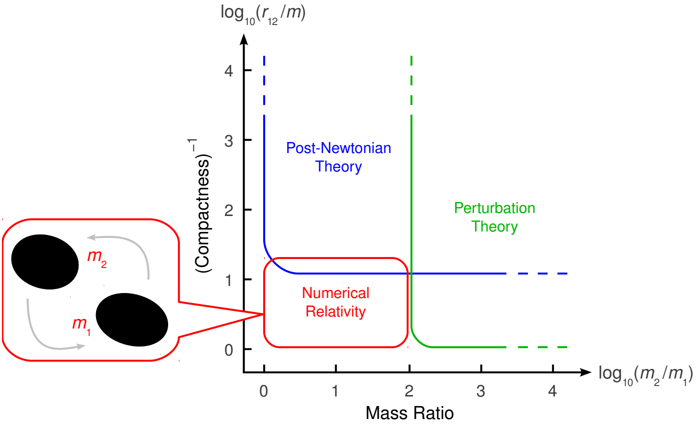
Perturbation parameter

$$q \equiv \frac{m_1}{m_2} \ll 1$$

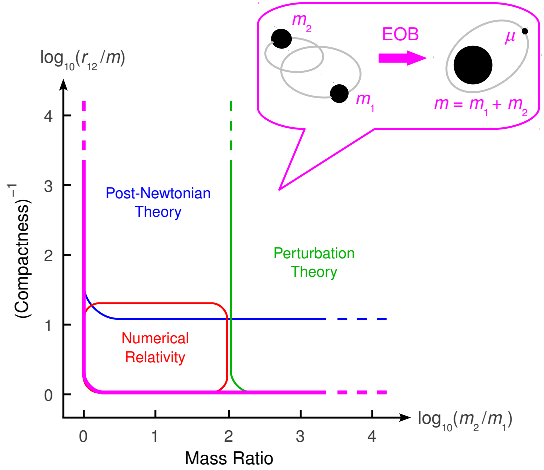
Gravitational self-force

$$\dot{u}^\alpha = f^\alpha = \mathcal{O}(q)$$

Methods to compute GW templates for compact binaries



Methods to compute GW templates for compact binaries



Outline

Outline

① Black hole binaries and gravitational waves

② The first law of binary black hole mechanics

1. Derivation of the first law
2. Consequences of the first law
3. Verification of the first law in PN theory

③ Applications to gravitational-wave source modeling

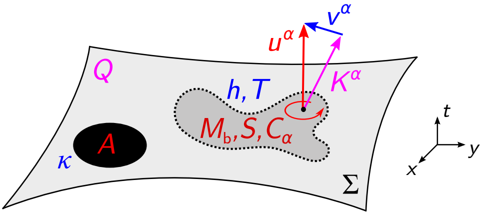
1. High-order PN coefficients in the binding energy
2. Frequency shift of the Schwarzschild ISCO
3. Perturbation theory for comparable mass binaries
4. EOB potentials at linear order in the mass ratio

Generalized first law of mechanics

[Friedman, Uryū & Shibata, PRD (2002)]

- Spacetimes with black holes + perfect fluid matter sources
- One-parameter family of solutions $\{g_{\alpha\beta}(\lambda), u^\alpha(\lambda), \rho(\lambda), s(\lambda)\}$
- Globally defined Killing vector field K^α $\rightarrow$ conserved charge Q

$$\delta Q = \sum_i \frac{\kappa_i}{8\pi}\,\delta A_i + \int_\Sigma \left[\,\bar{h}\,\Delta(\mathrm{d}M_\mathrm{b}) + \bar{T}\,\Delta(\mathrm{d}S) + v^\alpha \Delta(\mathrm{d}C_\alpha)\right]$$



Application to compact binaries on circular orbits

- For circular orbits, the geometry admits a helical Killing vector

$$K^\alpha \to (\partial_t)^\alpha + \Omega\,(\partial_\varphi)^\alpha \quad (\text{when } r \to +\infty)$$

- For asymptotically flat spacetimes [Friedman *et al.*, PRD (2002)]

$$\delta Q = \delta M - \Omega\,\delta J$$

- In the exact theory of GR, helically symmetric spacetimes are not asymptotically flat [Klein, PRD (2004)]
- Asymptotic flatness can be recovered if gravitational radiation can be "turned off":
  - Conformal Flatness Condition
  - Post-Newtonian theory

Application to compact binaries on circular orbits

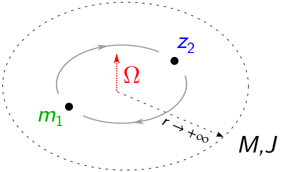
[Le Tiec, Blanchet & Whiting, PRD (2012)]

- Conservative dynamics only → no gravitational radiation
- Non-spinning compact objects modeled as point masses m_A:

$$T^{\alpha\beta} = \sum_{A=1}^{2} \frac{m_A\, z_A}{\sqrt{-g}}\, u_A^\alpha u_A^\beta\, \delta(\mathbf{x} - \mathbf{y}_A)$$

- For two point masses on a circular orbit, the first law becomes

$$\boxed{\delta M - \Omega\, \delta J = z_1\, \delta m_1 + z_2\, \delta m_2}$$



Outline

① Black hole binaries and gravitational waves

② The first law of binary black hole mechanics

1. Derivation of the first law
2. Consequences of the first law
3. Verification of the first law in PN theory

③ Applications to gravitational-wave source modeling

1. High-order PN coefficients in the binding energy
2. Frequency shift of the Schwarzschild ISCO
3. Perturbation theory for comparable mass binaries
4. EOB potentials at linear order in the mass ratio

Physical interpretations of the "redshift observable"

- It measures the redshift of light emitted from the particle [Detweiler, PRD (2008)]:

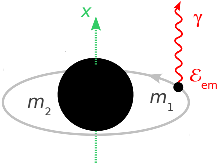
$$\frac{\mathcal{E}_{\text{obs}}}{\mathcal{E}_{\text{em}}} \equiv \frac{(p^\alpha u_\alpha)_{\text{obs}}}{(p^\alpha u_\alpha)_{\text{em}}} = z_1$$

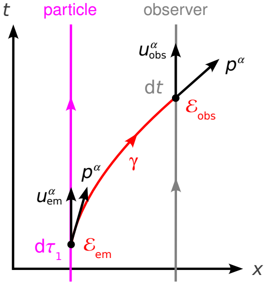
- It is a constant of the motion associated with the helical symmetry:

$$z_1 = -u_1^\alpha K_\alpha$$

- In a gauge such that $K^\alpha \partial_\alpha = \partial_t + \Omega\, \partial_\varphi$ everywhere,

$$z_1 = \left(u_1^t\right)^{-1} = \frac{\mathrm{d}\tau_1}{\mathrm{d}t}$$





First integral associated with the variational law

[Le Tiec, Blanchet & Whiting, PRD (2012)]

- Variational first law: $\delta M - \Omega\,\delta J = z_1\,\delta m_1 + z_2\,\delta m_2$
- Since $\{M, J, z_A\}$ are all functions of $\{\Omega, m_A\}$, we have

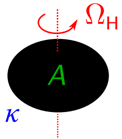
$$\frac{\partial M}{\partial \Omega} = \Omega \frac{\partial J}{\partial \Omega} \quad \text{and} \quad z_A = \frac{\partial (M - \Omega J)}{\partial m_A}$$

- After a few algebraic manipulations, we obtain

$$\boxed{M - 2\Omega J = m_1 z_1 + m_2 z_2}$$

- Alternative derivations based on:
  - Euler's theorem applied to the function $M(J^{1/2}, m_1, m_2)$
  - The combination $M_\mathrm{K} - 2\Omega J_\mathrm{K}$ of the Komar quantities

Analogies with single and binary black holes

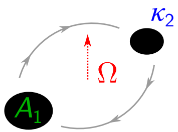


$$\delta M - \Omega_\mathrm{H}\,\delta J = \frac{\kappa\,\delta A}{8\pi}$$

[Bardeen *et al.* (1973)]

$$M - 2\Omega_\mathrm{H} J = \frac{\kappa A}{4\pi}$$

[Smarr (1973)]

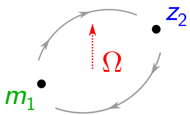


$$\delta M - \Omega\,\delta J = \sum_{i=1}^{2} \frac{\kappa_i\,\delta A_i}{8\pi}$$

[Friedman *et al.* (2002)]

$$M - 2\Omega J = \sum_{i=1}^{2} \frac{\kappa_i A_i}{4\pi}$$

[Le Tiec *et al.* (2012)]



$$\delta M - \Omega\,\delta J = \sum_{i=1}^{2} z_i\,\delta m_i$$

$$M - 2\Omega J = \sum_{i=1}^{2} z_i m_i$$

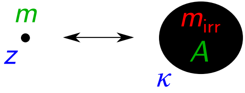
Analogies with single and binary black holes

- Irreducible mass of a BH [Christodoulou & Ruffini, PRL (1971)]

$$m_{\mathrm{irr}} = \sqrt{\frac{A}{16\pi}} \quad \Longrightarrow \quad \frac{\kappa}{8\pi}\,\delta A = (4\kappa m_{\mathrm{irr}})\,\delta m_{\mathrm{irr}}$$

- We notice the formal analogies

$$m \longleftrightarrow m_{\mathrm{irr}} \quad \text{and} \quad z \longleftrightarrow 4\kappa m_{\mathrm{irr}}$$



- For large separations, we recover the correct limits

$$z \longrightarrow 1 \quad \text{and} \quad \kappa \longrightarrow (4m_{\mathrm{irr}})^{-1}$$

- Extension of the first law to spinning point particles needed

Outline

① Black hole binaries and gravitational waves

② The first law of binary black hole mechanics

1. Derivation of the first law
2. Consequences of the first law
3. Verification of the first law in PN theory

③ Applications to gravitational-wave source modeling

1. High-order PN coefficients in the binding energy
2. Frequency shift of the Schwarzschild ISCO
3. Perturbation theory for comparable mass binaries
4. EOB potentials at linear order in the mass ratio

A short history of the conservative PN dynamics

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \underbrace{-\frac{Gm}{r^2}\mathbf{n} + \frac{\mathbf{A}_{1\mathrm{PN}}}{c^2} + \frac{\mathbf{A}_{2\mathrm{PN}}}{c^4}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{2.5\mathrm{PN}}}{c^5}}_{\text{dissipative}} + \underbrace{\frac{\mathbf{A}_{3\mathrm{PN}}}{c^6}}_{\text{conservative}} + \underbrace{\frac{\mathbf{A}_{3.5\mathrm{PN}}}{c^7}}_{\text{dissipative}} + \cdots$$

	Total mass M (and J)	Redshift observable z_A
2PN	[Damour & Deruelle (1982)]	[Detweiler (2008)]
3PN	[Jaranowski & Schäfer (1999)] [Damour *et al.* (2000)] [Blanchet & Faye (2000)] [Itoh & Futamase (2003)] [Foffa & Sturani (2011)]	[Blanchet, Detweiler, Le Tiec & Whiting (2010a)]
4PN + $5\mathrm{PN}_{\mathrm{log}}$	[Blanchet *et al.* (2010b)] [Le Tiec *et al.* (2012)]	[Blanchet *et al.* (2010b)] [Le Tiec *et al.* (2012)]

Verification of the first law in PN theory

[Le Tiec, Blanchet & Whiting, PRD (2012)]

- The PN results for $M(\Omega, m_A)$, $J(\Omega, m_A)$ and $z_A(\Omega, m_A)$ are expressed in terms of

$$m \equiv m_1 + m_2\,, \quad \nu \equiv m_1 m_2/m^2 \equiv \mu/m\,, \quad \text{and} \quad x \equiv (m\Omega)^{2/3}$$

- For instance, the binding energy $E \equiv M - m$ reads

$$E = -\frac{1}{2}\,\mu\,x\left\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \cdots + \frac{448}{15}\,\nu\,x^4 \ln x + \cdots\right\}$$

- The first law is satisfied up to 3PN order included, as well as by the 4PN+5PN logarithmic terms:

$$\frac{\partial M}{\partial \Omega} = \Omega \frac{\partial J}{\partial \Omega} \quad \text{and} \quad z_A = \frac{\partial (M - \Omega J)}{\partial m_A}$$

Outline

Putting the first law at work using GSF results

- In the extreme mass ratio limit $\nu \to 0$, the redshift observable can be expanded as

$$z_1 = \sqrt{1-3x} + \nu\, z_{\mathrm{GSF}}(x) + \mathcal{O}(\nu^2)$$

- The self-force contribution $z_{\mathrm{GSF}}(x)$ is known numerically
- The first law provides relationships between the binding energy, angular momentum, and redshift observable:

$$E \leftrightarrow J\,, \quad E \leftrightarrow z_1\,, \quad \text{and} \quad J \leftrightarrow z_1$$

- These can be used to gain information about E and J beyond the test-mass limit

Outline

High-order PN coefficients in the redshift observable

[Blanchet, Detweiler, Le Tiec & Whiting, PRD (2010)]

- The numerical GSF data is fitted to a PN series of the type

$$z_{\mathrm{GSF}} = \sum_{n \geqslant 0} \alpha_n \, x^{n+1} + \ln x \sum_{n \geqslant 4} \beta_n \, x^{n+1}$$

- The exact values of all analytically known PN coefficients α_n and β_n are used
- The best fit yields for the unkown higher order coefficients:

PN order	Coeff.	Value
4	α_4	$+53.43220(5)$
5	α_5	$-37.72(1)$
6	α_6	$+123(2)$
6	β_6	$-311.9(5)$

High-order PN coefficients in the binding energy

[Le Tiec, Blanchet & Whiting, PRD (2012)]

$$
\begin{aligned}
\frac{E}{\mu} = -\frac{x}{2}\bigg\{1 &+ \left(-\frac{3}{4} - \frac{\nu}{12}\right) x \\
&+ \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right) x^2 \\
&+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right) x^3 \\
&+ \left(-\frac{3969}{128} + e_4(\nu) + \frac{448}{15}\nu \ln x\right) x^4 \\
&+ \left(-\frac{45927}{512} + e_5(\nu) + \left[-\frac{4988}{35} - \frac{656}{5}\nu\right]\nu \ln x\right) x^5 \\
&+ \left(-\frac{264627}{1024} + e_6(\nu) + e_6^{\ln}(\nu) \ln x\right) x^6 \bigg\}
\end{aligned}
$$

High-order PN coefficients in the binding energy

[Le Tiec, Blanchet & Whiting, PRD (2012)]

$$\begin{aligned}\frac{E}{\mu} = -\frac{x}{2}\Bigg\{1 &+ \left(-\frac{3}{4} - \frac{\nu}{12}\right) x \\ &+ \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right) x^2 \\ &+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right) x^3 \\ &+ \left(-\frac{3969}{128} + 153.8803(1)\,\nu + \frac{448}{15}\nu\ln x\right) x^4 \\ &+ \left(-\frac{45927}{512} - 55.13(3)\,\nu + \left[-\frac{4988}{35} - \frac{656}{5}\nu\right]\nu\ln x\right) x^5 \\ &+ \left(-\frac{264627}{1024} + 588(7)\,\nu - 1144(2)\,\nu\ln x\right) x^6 + \mathcal{O}(\nu^2)\Bigg\}\end{aligned}$$

Outline

Binding energy and total angular momentum

[Le Tiec, Barausse & Buonanno, PRL (2012)]

- In the extreme mass ratio limit $\nu \to 0$:

$$z_1 = \sqrt{1-3x} + \nu\, z_{\mathrm{GSF}}(x) + \mathcal{O}(\nu^2)$$
$$\frac{E}{\mu} = \left(\frac{1-2x}{\sqrt{1-3x}} - 1\right) + \nu\, E_{\mathrm{GSF}}(x) + \mathcal{O}(\nu^2)$$

- The first law provides a relationship $E \leftrightarrow z_1$, which implies

$$E_{\mathrm{GSF}}(x) = \frac{1}{2}\, z_{\mathrm{GSF}}(x) - \frac{x}{3}\, z'_{\mathrm{GSF}}(x) + f(x)$$

- The self-force contribution $E_{\mathrm{GSF}}(x)$ is known numerically, even for highly relativistic circular orbits
- A similar result holds for the angular momentum $J/(m\mu)$

GSF correction to the Schwarzschild ISCO frequency

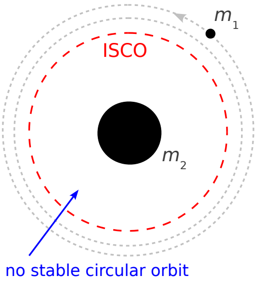
- The orbital frequency of the Schwarzschild ISCO is shifted under the effect of the conservative self-force:

$$m\Omega_{\text{ISCO}} = \underbrace{6^{-3/2}}_{\text{Schwarz. result}} \left\{ 1 + \underbrace{\nu\, C_\Omega}_{\text{conservative GSF effect}} + \mathcal{O}(\nu^2) \right\}$$

- A stability analysis of slightly eccentric orbits near the ISCO yields [Barack & Sago, PRL (2009)]

$$C_\Omega^{\text{BS}} = 1.2512(4)$$

- Strong-field benchmark used for comparison with PN/NR/EOB



GSF correction to the Schwarzschild ISCO frequency

- The angular frequency of the minimum energy circular orbit (MECO) is solution of

$$\left.\frac{\partial E}{\partial \Omega}\right|_{\Omega_{\text{MECO}}} = 0$$

- Hamiltonian system: ISCO $\Leftrightarrow$ MECO [Buonanno *et al.* (2003)]
- Our result for the energy $E_{\text{GSF}}(x)$ yields [Le Tiec *et al.* (2012)]

$$C_\Omega = \frac{1}{2} + \frac{1}{4\sqrt{2}}\left\{\frac{1}{3}\,z''_{\text{GSF}}(1/6) - z'_{\text{GSF}}(1/6)\right\}$$

- Using accurate numerical self-force data for $z_{\text{GSF}}(x)$, we find

$$C_\Omega = 1.2510(2) \qquad \left[C_\Omega^{\text{BS}} = 1.2512(4)\right]$$

Outline

① Black hole binaries and gravitational waves

② The first law of binary black hole mechanics

1. Derivation of the first law
2. Consequences of the first law
3. Verification of the first law in PN theory

③ Applications to gravitational-wave source modeling

1. High-order PN coefficients in the binding energy
2. Frequency shift of the Schwarzschild ISCO
3. Perturbation theory for comparable mass binaries
4. EOB potentials at linear order in the mass ratio

ADM mass, Bondi mass, and binding energy

- Conservation of mass-energy

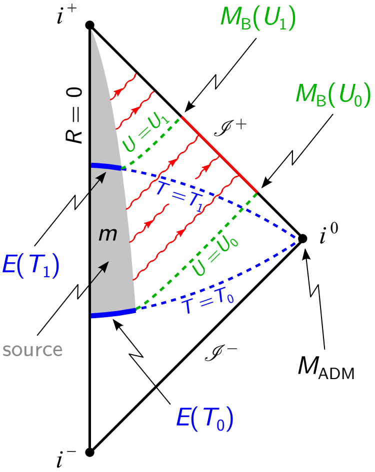
$$M_{\mathrm{ADM}} = M_{\mathrm{B}}(U) + \int_{-\infty}^{U} \mathcal{F}(U')\,\mathrm{d}U'$$

- Bondi-Sachs mass loss formula

$$\frac{\mathrm{d}M_{\mathrm{B}}}{\mathrm{d}U} = -\mathcal{F}(U)$$

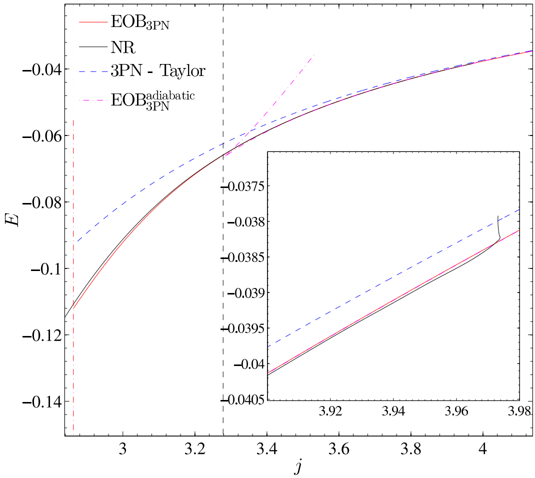
- Binding energy of the binary

$$E(T) = M_{\mathrm{B}}(U) - m$$



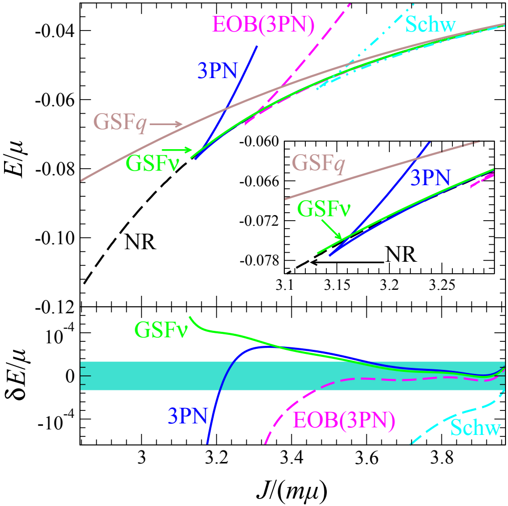
NR/EOB comparison for an equal mass binary

[Damour, Nagar, Pollney & Reisswig, PRL (2012)]



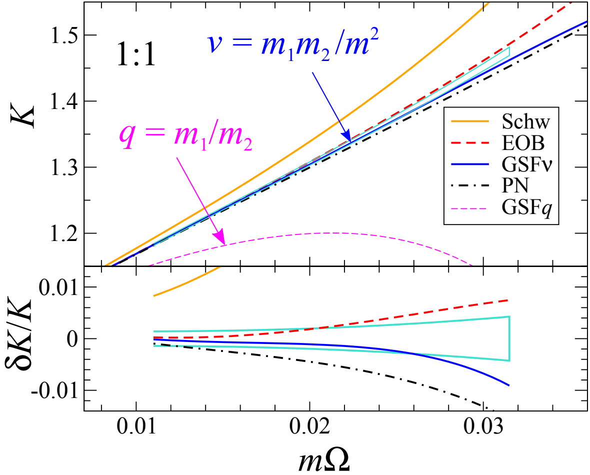
NR/GSF comparison for an equal mass binary

[Le Tiec, Barausse & Buonanno, PRL (2012)]



Periastron advance in black hole binaries

[Le Tiec, Mroué *et al.*, PRL (2011)]



Why do the GSFν results perform so well?

- In perturbation theory, one traditionally expands as

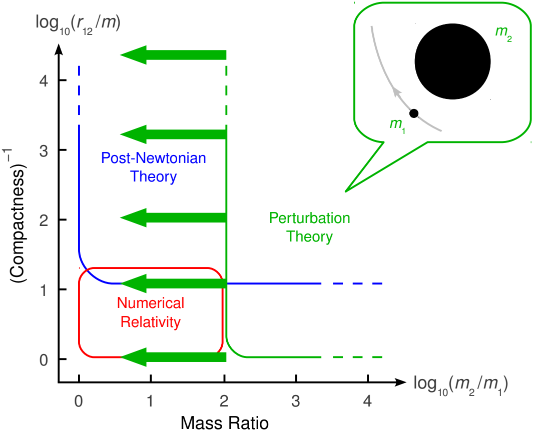
$$\text{GSF}q: \quad \sum_{k=0}^{k_{\max}} A_k(m_2\Omega)\, q^k \quad \text{where} \quad q \equiv m_1/m_2 \in [0,1]$$

- However, the relations $K(\Omega; m_A)$, $E(\Omega; m_A)$, and $J(\Omega; m_A)$ must be symmetric under exchange $m_1 \longleftrightarrow m_2$
- Hence, a better-motivated expansion is

$$\text{GSF}\nu: \quad \sum_{k=0}^{k_{\max}} B_k(m\Omega)\, \nu^k \quad \text{where} \quad \nu \equiv m_1 m_2/m^2 \in [0,1/4]$$

- In a PN expansion, we have $B_n = \mathcal{O}\left(1/c^{2n}\right) = n\text{PN} + \cdots$
- Previously noticed for dissipative GSF [Detweiler & Smarr (1979)]

Perturbation theory for comparable mass binaries



Outline

The Effective-One-Body (EOB) model

[Buonanno & Damour, PRD (1999)]

- Motion of a test-particle of mass $\mu = m_1 m_2 / m = \nu\, m$ in a static and spherically symmetric effective metric

$$\mathrm{d}s^2_{\text{eff}} = -A(r;\nu)\,\mathrm{d}t^2 + B(r;\nu)\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

- Reduces to the Schwarzschild metric of a black hole of mass $m = m_1 + m_2$ in the limit $\nu \to 0$

- Potentials A and B determined by mapping the effective Hamiltonian of μ to the known PN Hamiltonian of the binary
- Additional free parameters are calibrated to NR simulations

EOB potentials at linear order in the mass ratio

[Barausse, Buonanno & Le Tiec, PRD (2012)]

- 3PN expansion in powers of $u \equiv m/r$ of the potential A:

$$A(u;\nu) = 1 - 2u + 2\,\nu\,u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu\,u^4 + \mathcal{O}(u^5)$$

- Mass ratio expansion in powers of ν of the potential A:

$$A(u;\nu) = 1 - 2u + \nu\,A_{\rm GSF}(u) + \mathcal{O}(\nu^2)$$

- Using the 1st law we can compute the exact GSF contribution:

$$A_{\rm GSF}(u) = \sqrt{1-3u}\,z_{\rm GSF}(u) - u\left(1 + \frac{1-4u}{\sqrt{1-3u}}\right)$$

- We also computed the exact GSF contribution to the potential B using results for the periastron advance

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
    - New high-order PN coefficients in binding energy E

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results
  - EOB potentials at linear order in the mass ratio

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results
  - EOB potentials at linear order in the mass ratio
- Some directions for future research include:

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results
  - EOB potentials at linear order in the mass ratio
- Some directions for future research include:
  - Extending the first law to spinning point particles

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results
  - EOB potentials at linear order in the mass ratio
- Some directions for future research include:
  - Extending the first law to spinning point particles
  - A better understanding of the links black hole $\leftrightarrow$ point mass

Summary and prospects

- The first law uncovers deep relations between local and global physical quantites in binary black hole spacetimes
- It holds up to very high orders in post-Newtonian theory
- Formal relations between black holes and point masses in GR
- Numerous applications to GW source modeling:
  - New high-order PN coefficients in binding energy E
  - E and J at leading order beyond the test-mass results
  - EOB potentials at linear order in the mass ratio
- Some directions for future research include:
  - Extending the first law to spinning point particles
  - A better understanding of the links black hole $\leftrightarrow$ point mass
  - Using perturbation theory to model comparable-mass binaries

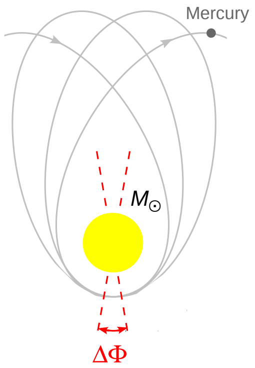
EXTRA SLIDES

Relativistic perihelion advance of Mercury

- Observed anomalous precession of Mercury's perihelion of $\sim$ 43" /cent.
- Accounted for by the leading-order relativisic angular advance per orbit

$$\Delta\Phi_{\rm GR} = \frac{6\pi G M_\odot}{c^2 a\,(1-e^2)}$$

- One of the first successes of Einstein's general theory of relativity
- Relativisic periastron advance of $\sim\,^\circ$/yr now measured in binary pulsars



Periastron advance in black hole binaries

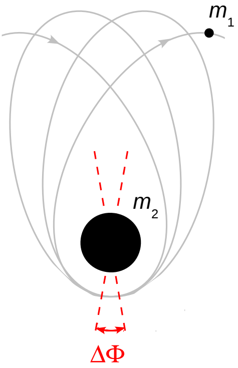
- Conservative part of the dynamics only
- Generic non-circular orbit parametrized by the two invariant frequencies

$$\Omega_r = \frac{2\pi}{P}\,, \quad \Omega_\varphi = \frac{1}{P}\int_0^P \dot{\varphi}(t)\,\mathrm{d}t$$

- Periastron advance per radial period

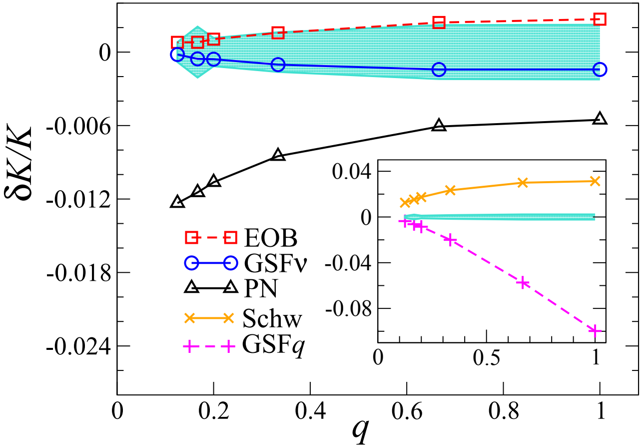
$$K \equiv \frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\Delta\Phi}{2\pi}$$

- In the circular orbit limit $e \to 0$, the relation $K(\Omega_\varphi)$ is coordinate invariant



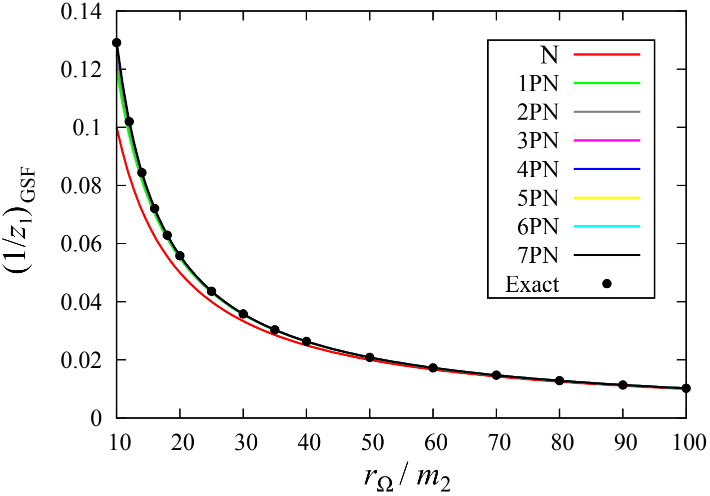
Periastron advance in black hole binaries

[Le Tiec, Mroué *et al.*, PRL (2011)]



GSF/PN comparison based on the redshift observable

[Blanchet, Detweiler, Le Tiec & Whiting, PRD (2010)]



GSF/PN comparison based on the redshift observable

[Blanchet, Detweiler, Le Tiec & Whiting, PRD (2010)]