

Forecast constraints on cosmic string parameters from future gravitational wave experiments

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(RESearch Center for the Early Universe)

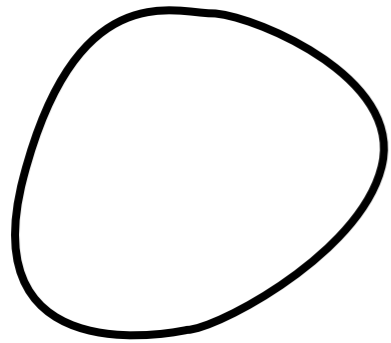
12 July 2012

Based on

S. Kuroyanagi, K. Miyamoto, T. Sekiguchi, K. Takahashi, J. Silk,
PRD 86, 023503 (2012), arXiv:1202.3032 [astro-ph]

Cosmic string ?

One dimensional topological defects
generated in the early universe



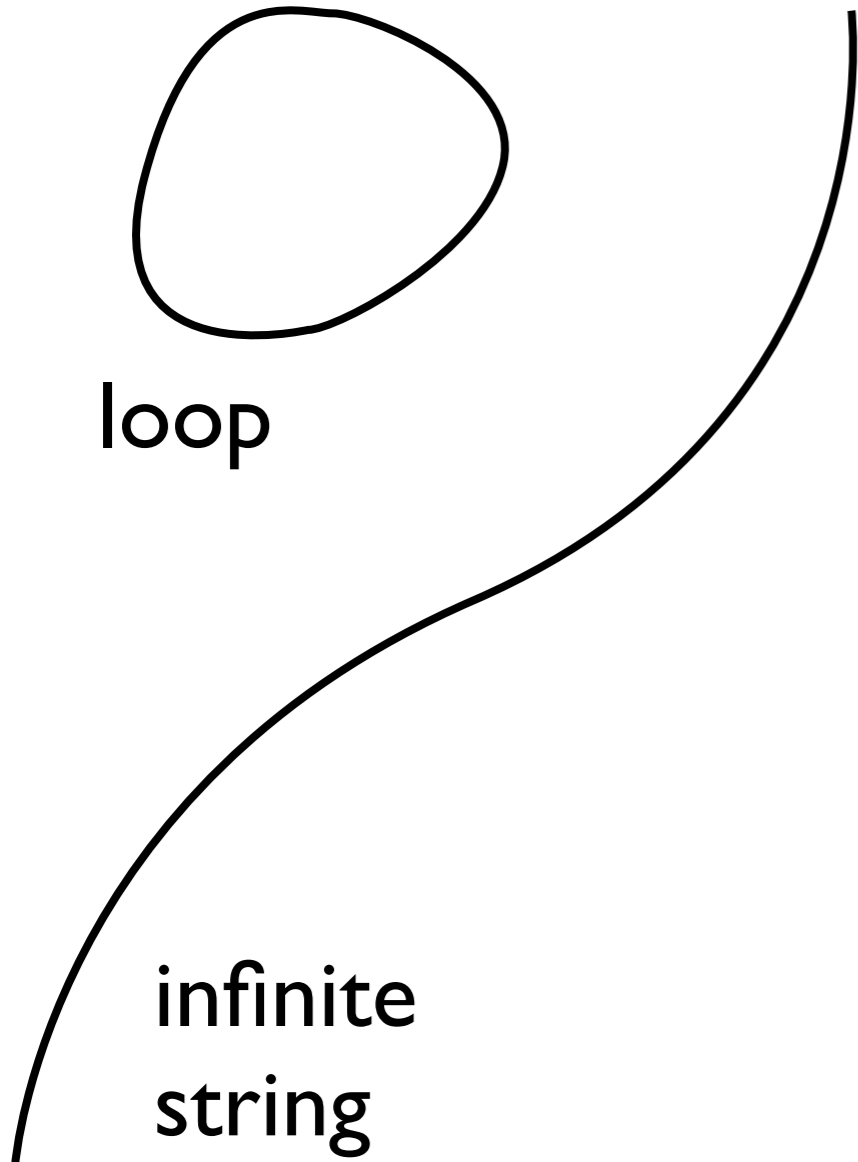
loop

- have infinite length (= no end point)
- can be a form of loop
- affect only through gravitational force

- emits gravitational waves

μ : tension (line density)

$$G \mu = \mu / M_{\text{pl}}^2$$

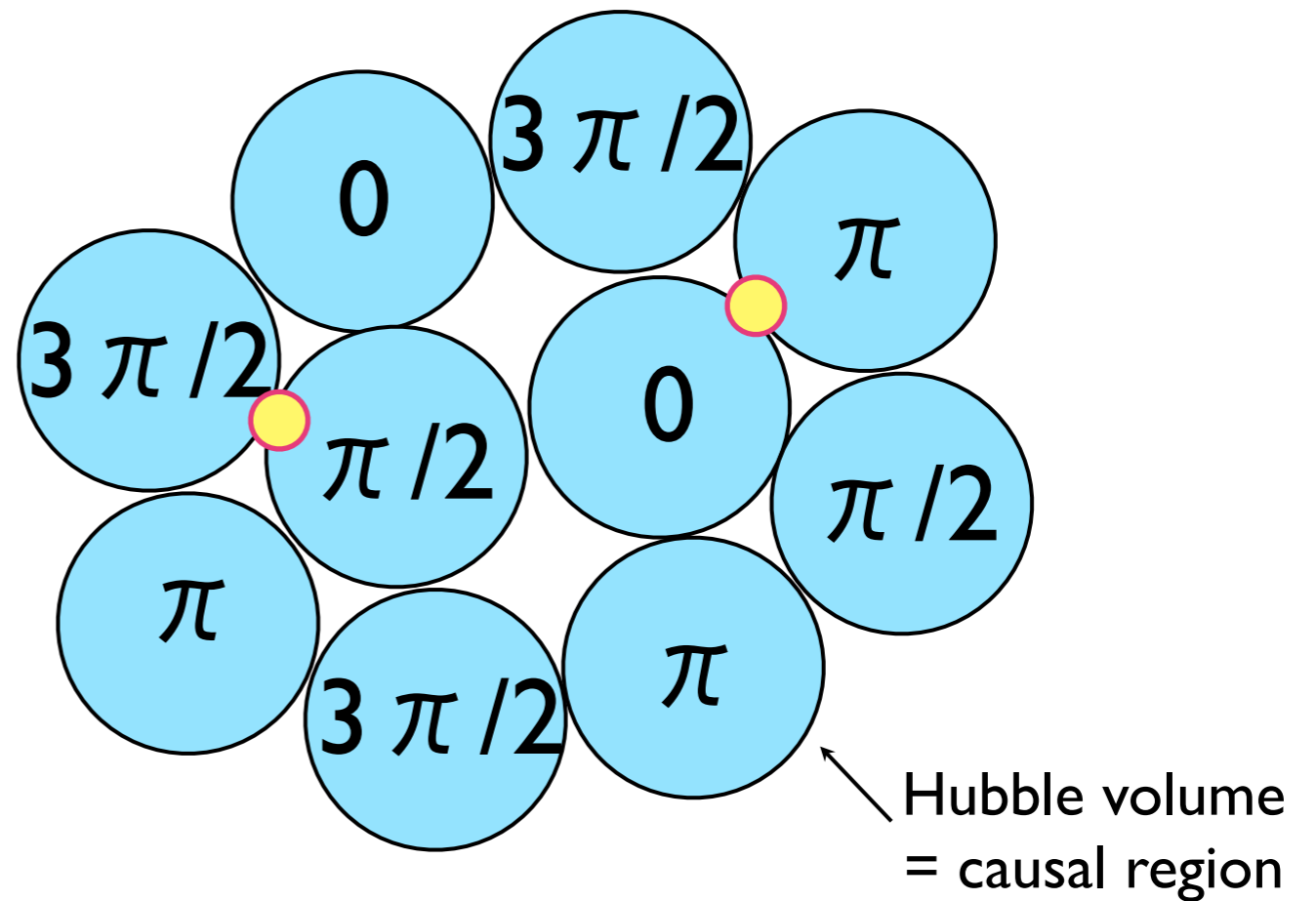
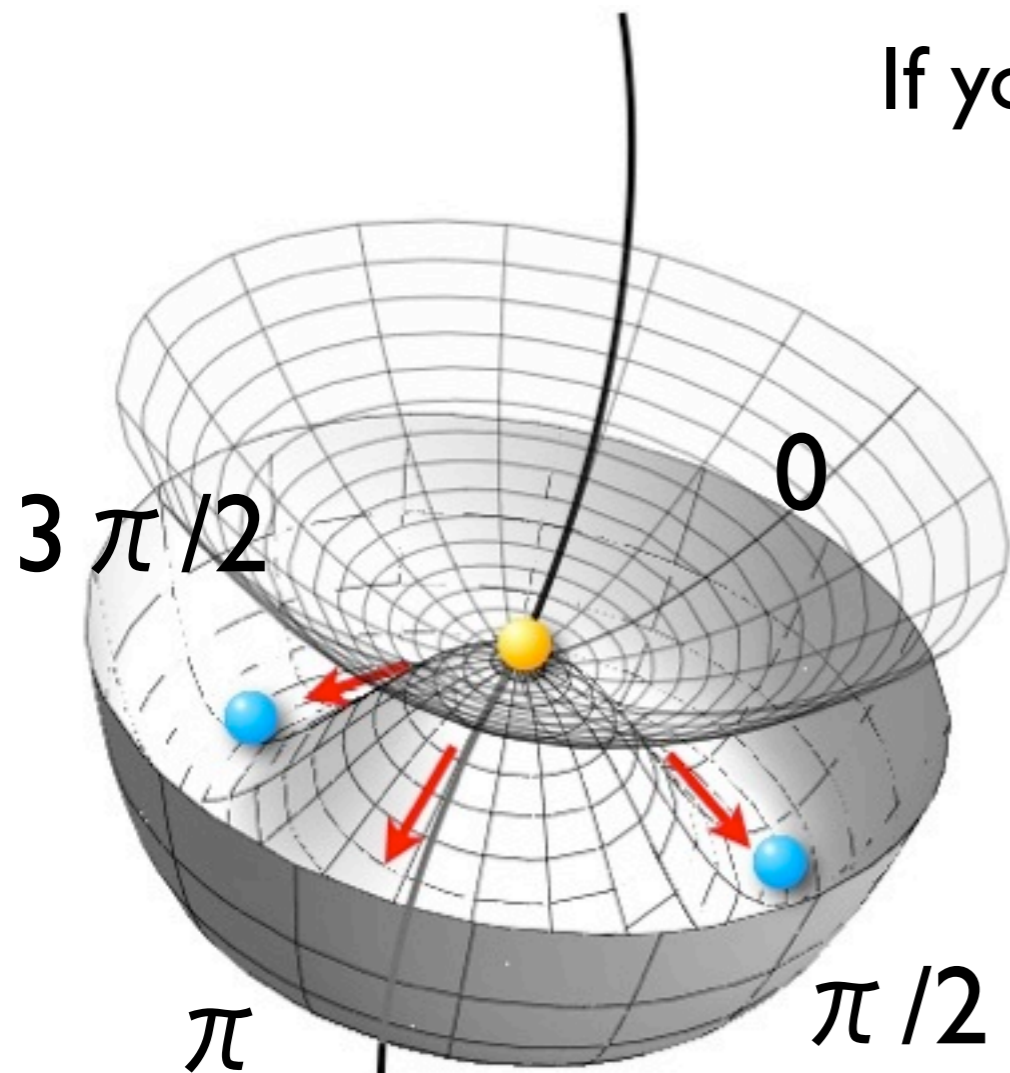


infinite
string

Generation mechanism I: phase transition

The Universe has experienced symmetry breakings.

If you consider $U(1)$ symmetry breaking...



High energy vacuum remains at the center

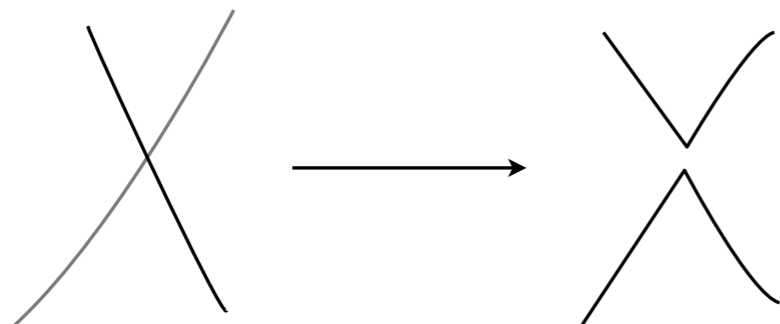
Tension $G \mu \sim$ the energy scale of the phase transition

Generation mechanism 2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation in superstring theory

Difference from phase transition origin

- reconnection probability: p



Phase transition origin: $p=1$

D-string: $p=0.1-1$

F-string: $p=10^{-3}-1$

- can have broad values of $G\mu$

→ **Cosmic strings could give some insight into fundamental physics**

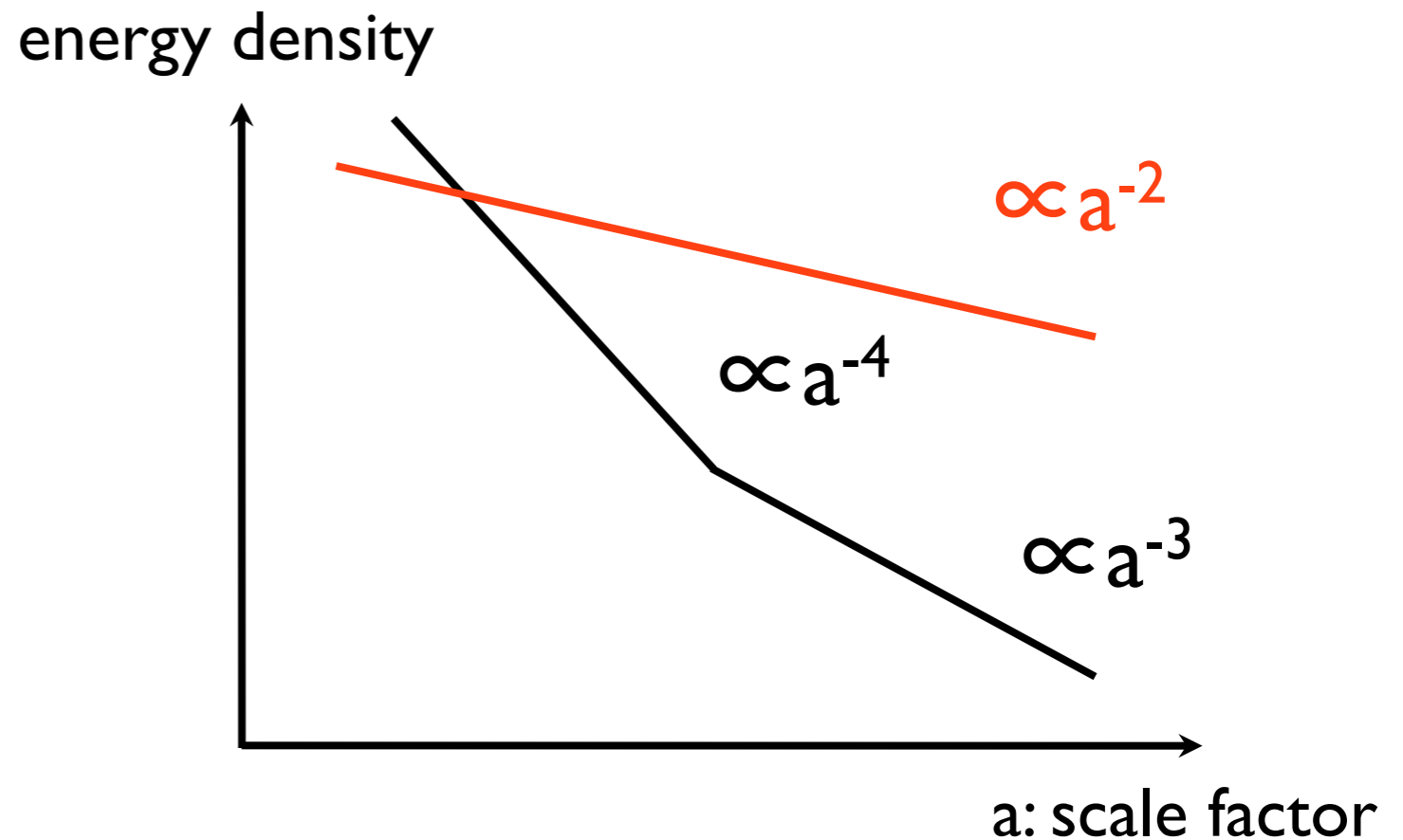
Evolution of cosmic strings



energy density of the cosmic strings $\propto a^{-2}$
 $\sim \frac{\text{line density} \times \text{length}}{\text{volume}}$
 constant $\propto a^1$ $\propto a^{-3}$

radiation $\propto a^{-4}$

matter $\propto a^{-3}$

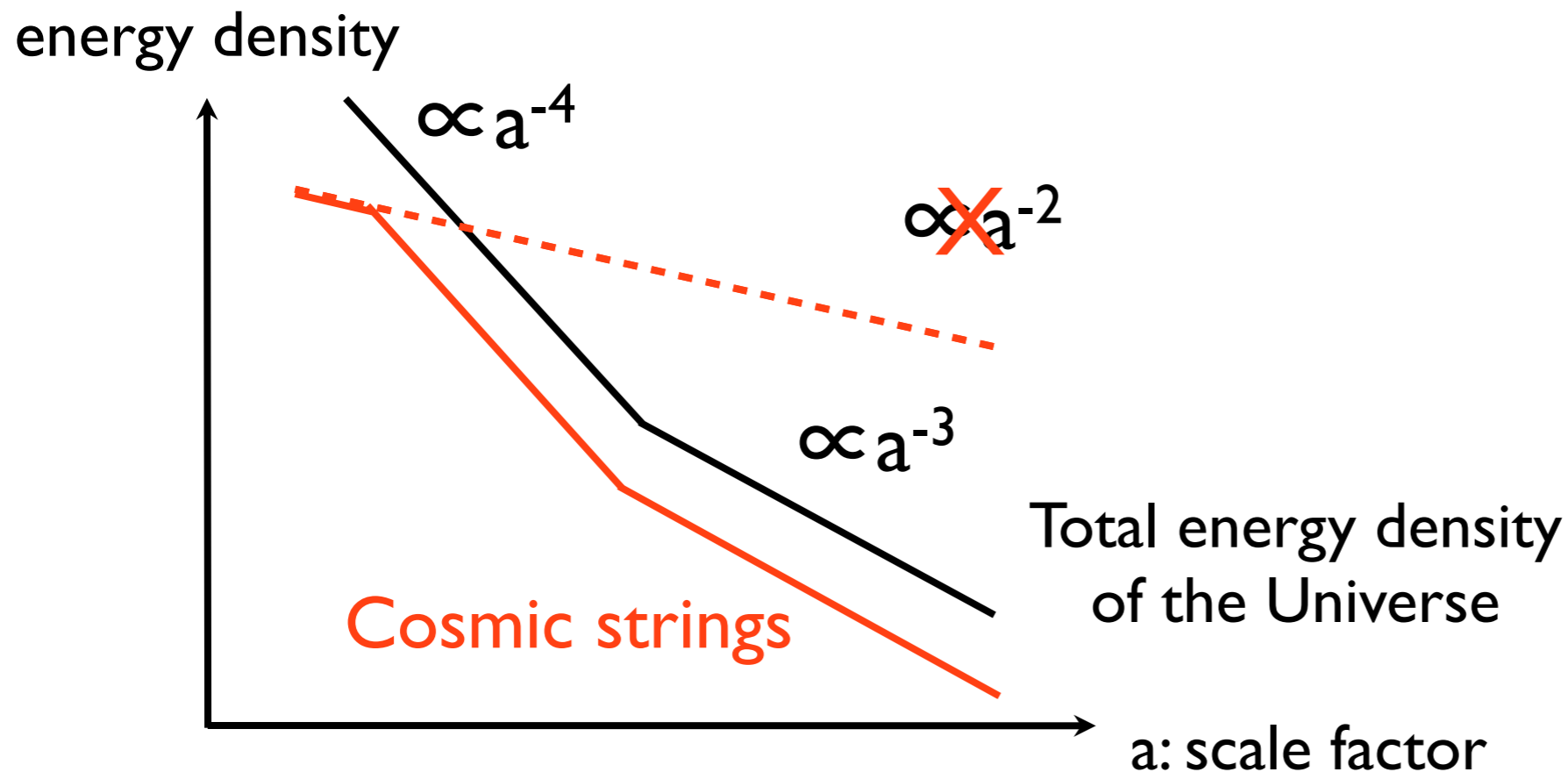
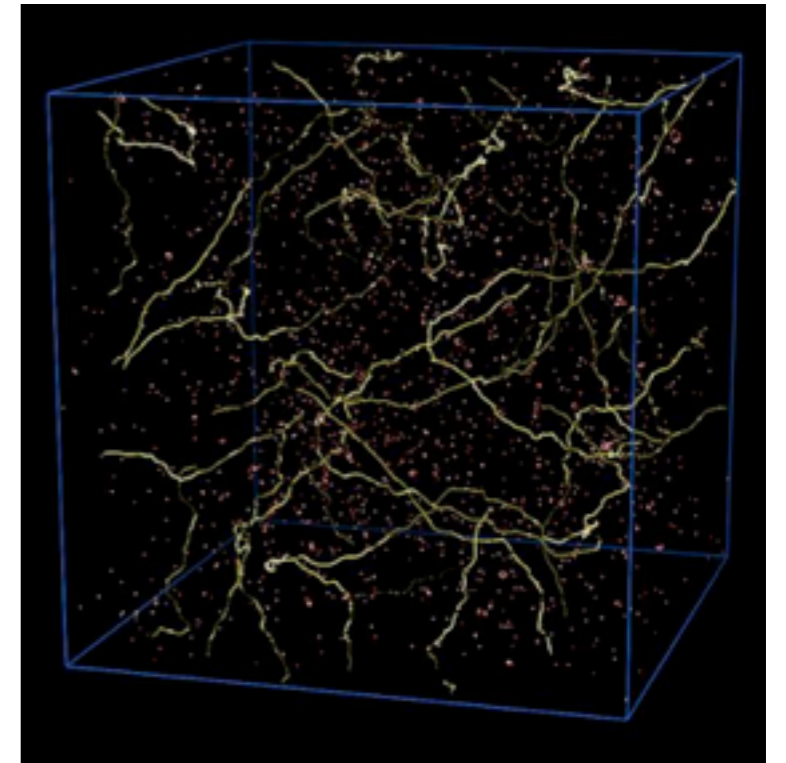


→ easily dominates the energy density of the Universe.
 not allowed by the observational facts?

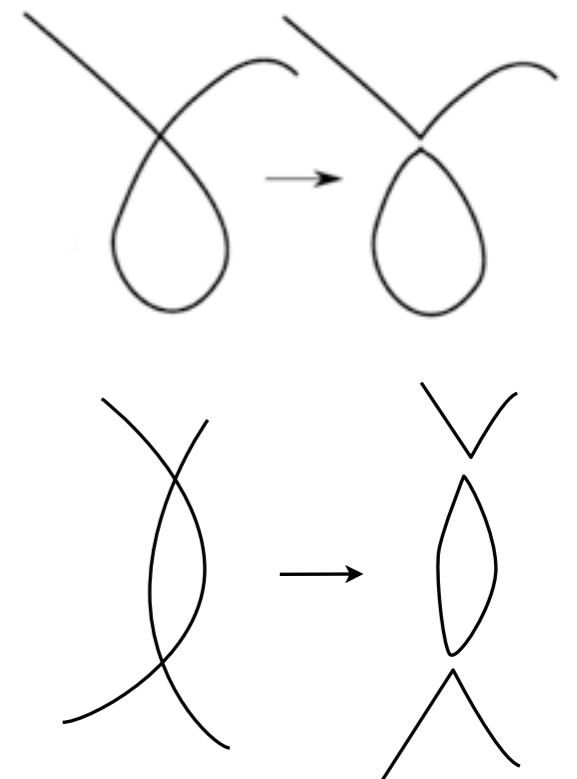
Evolution of cosmic strings

Scaling law

Cosmic String Networks approach a self-similar solution, which always looks same at the Hubble scale.



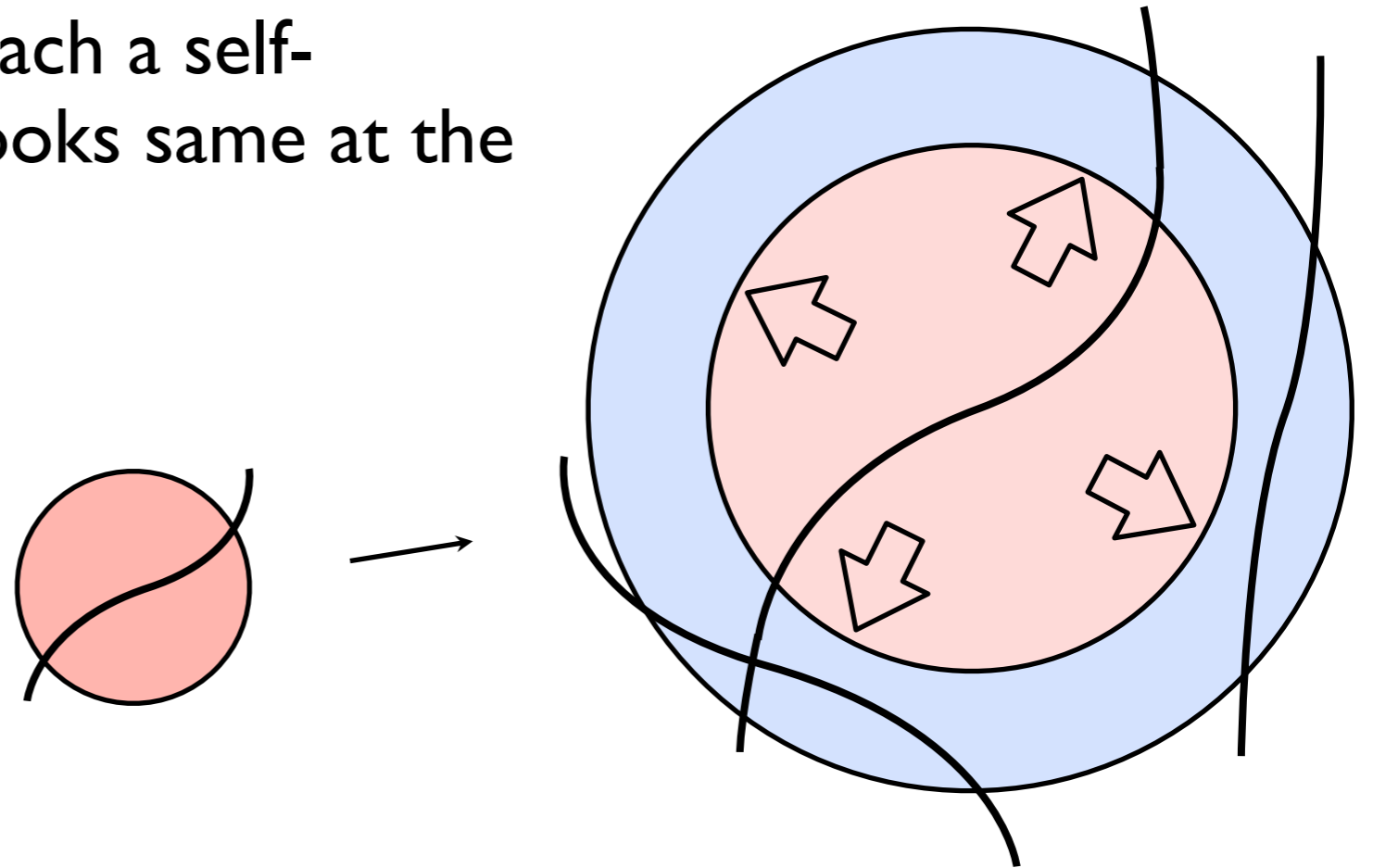
Cosmic strings become loops via reconnection.
Loops lose energy by emitting **gravitational waves**.



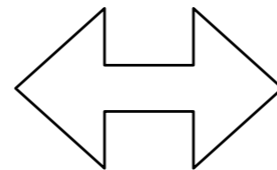
Evolution of cosmic strings

Scaling law

Cosmic String Networks approach a self-similar solution, which always looks same at the Hubble scale.



Increase of infinite string length
by the Hubble expansion

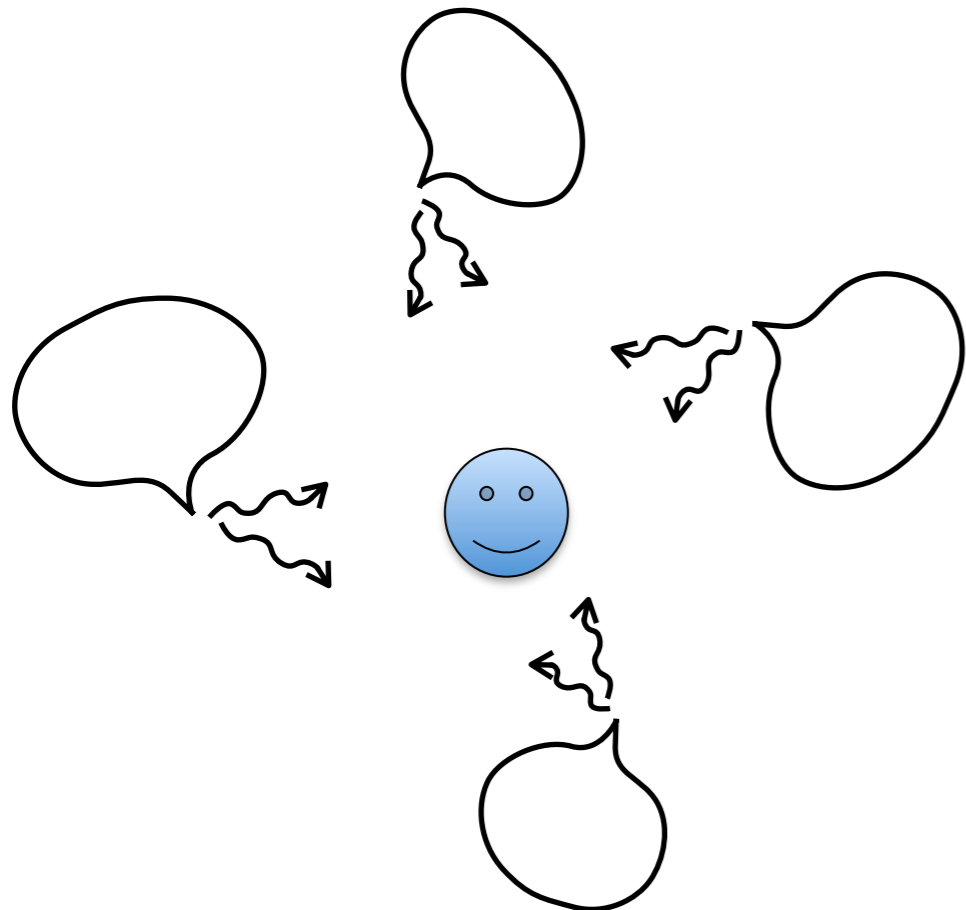
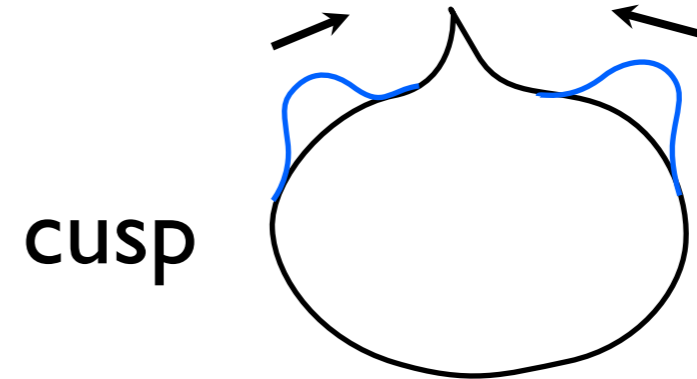
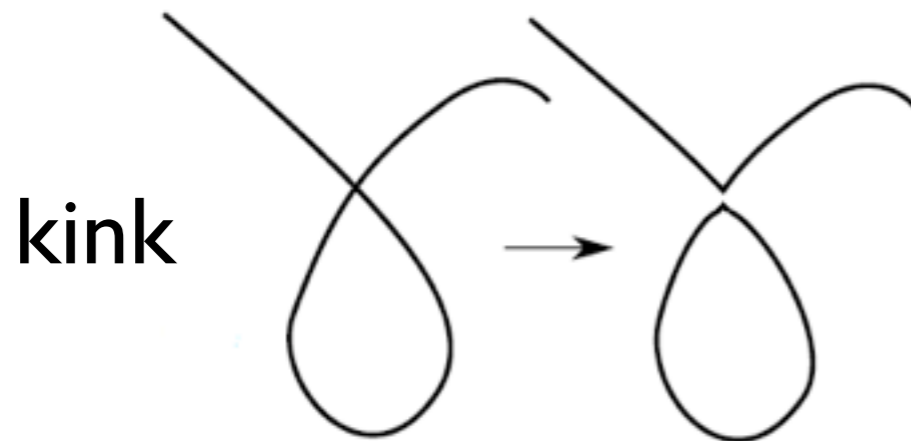


Loss of infinite string length
by generation of loops

Higher reconnection rate
more efficient generation of loops
more energy release by the emission of GWs

Gravitational waves from cosmic strings

Strong GW emission from singular points called **kinks** and **cusps**



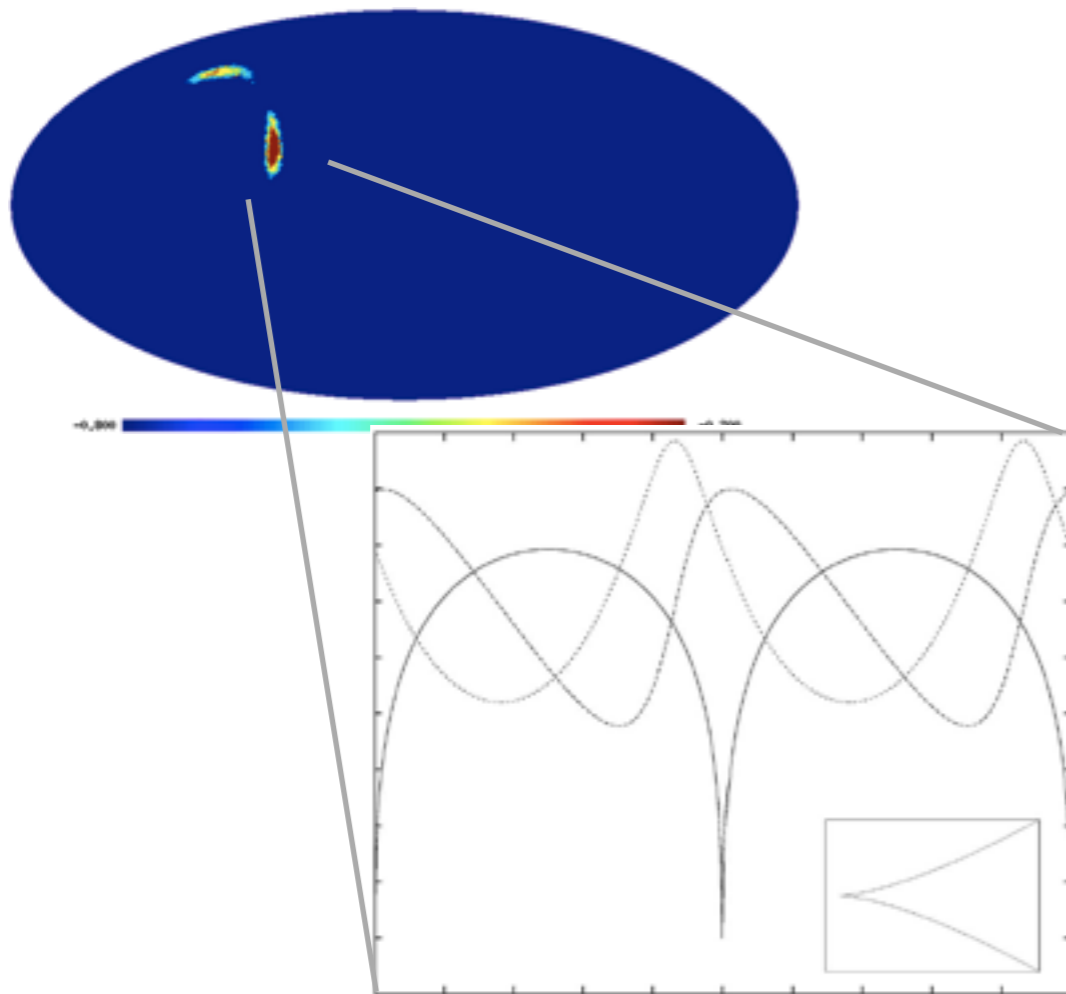
Rare Burst: GWs with large amplitude coming from close loops

Gravitational wave background (GWB): superposition of small GWs coming from the early epoch

Observations of GWs

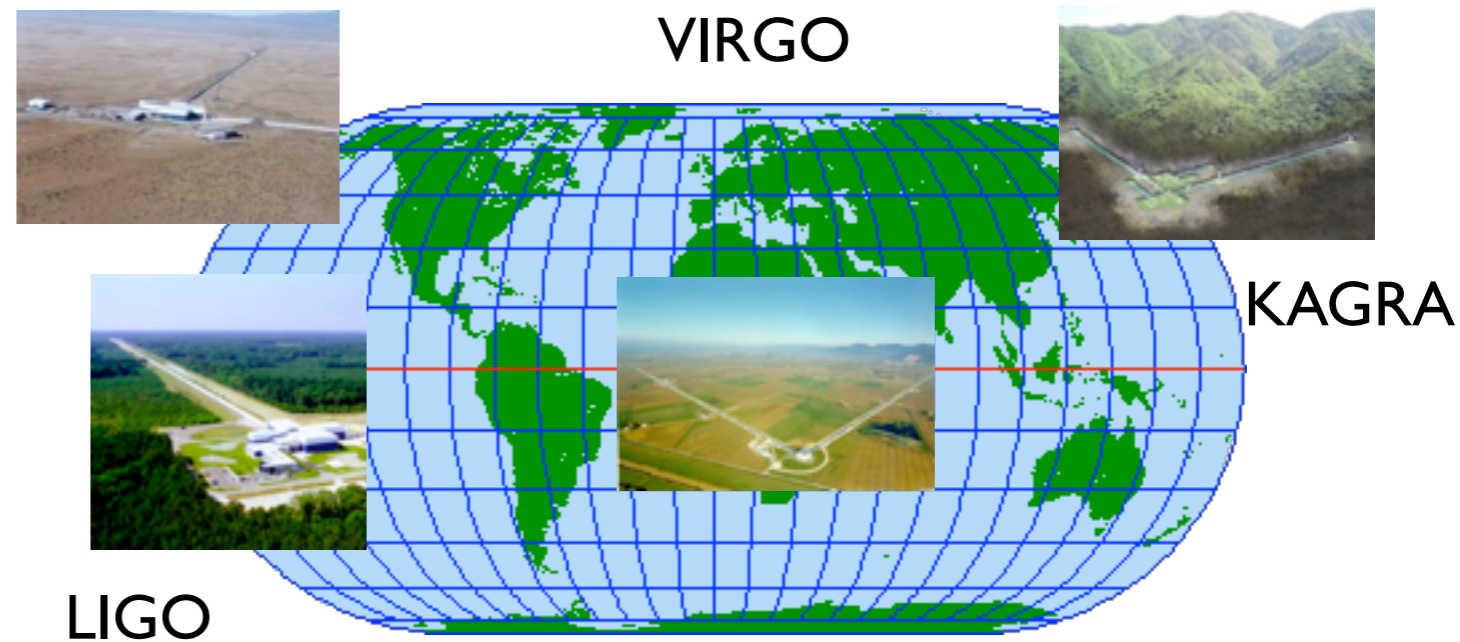
GWs with large amplitude

Burst



GWs with small amplitude
but numerous

GWB



Cross correlation analysis

Cross correlate the signals from two or more detector and extract stable GWs

Gravitational wave experiments

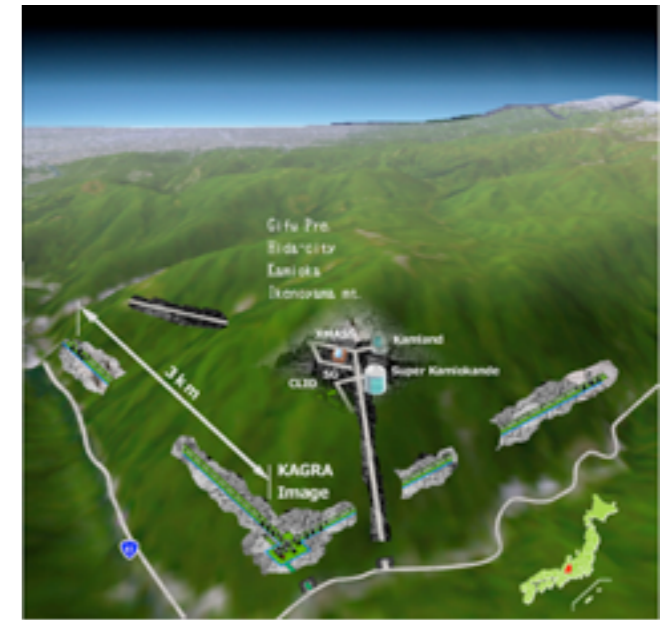
- Direct detection

Ground : **Advanced-LIGO**, **KAGRA**,
Virgo, **IndIGO (2017-)**

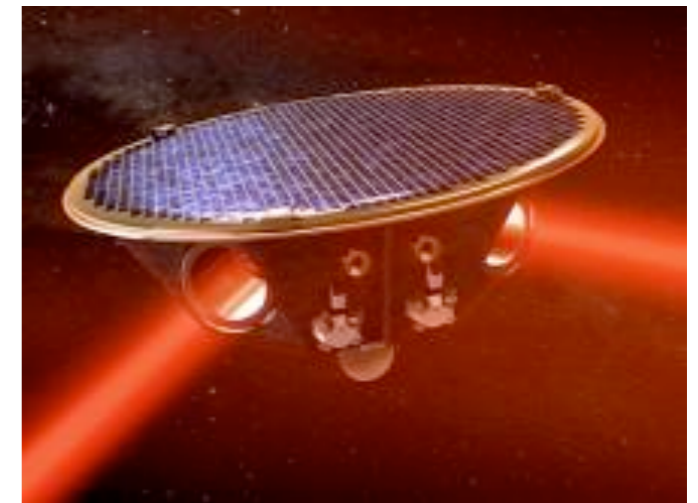
Space : **eLISA/NGO (2022?)**, **DECIGO (2027)**

- Pulsar timing: **SKA (2020)**

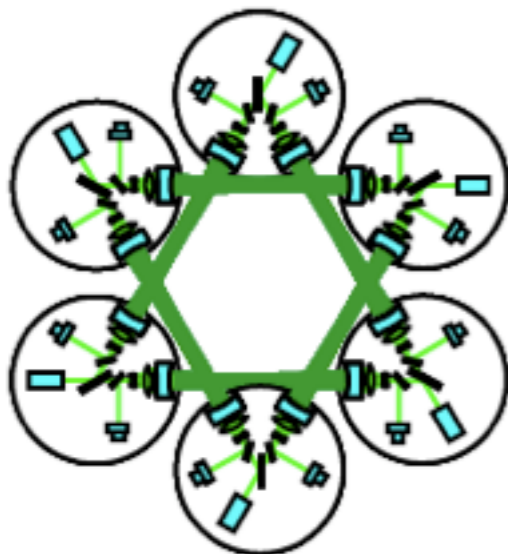
- CMB B-mode polarization: **Planck**, **CMBpol**



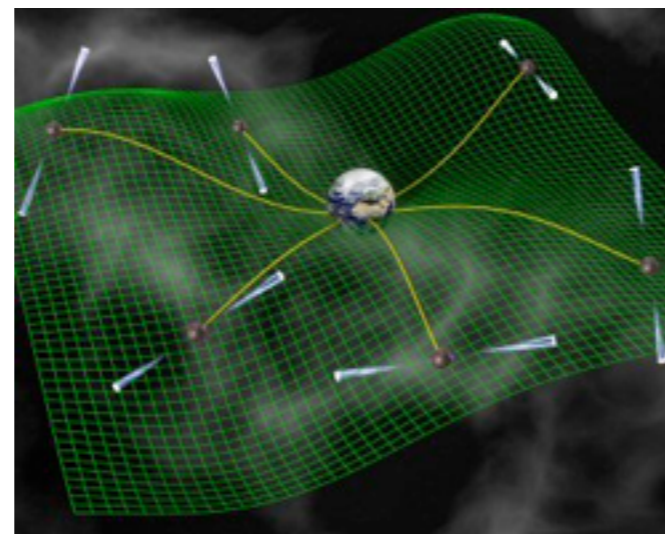
KAGRA image (<http://gwcenter.icrr.u-tokyo.ac.jp/>)



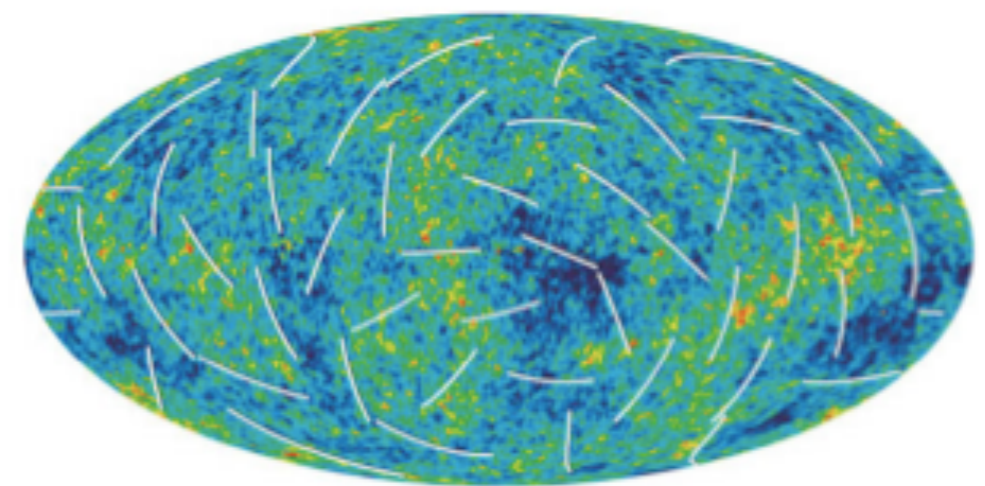
eLISA image (<http://elisa-ngo.org/>)



DECIGO image, S. Kawamura et al, J. Phys.: Conf. Ser. 122, 012006 (2006)



PTA image (NRAO)



WMAP Three Year Polarized CMB Sky (<http://wmap.gsfc.nasa.gov/>)

Current constraints on cosmic string parameters

3 parameters to characterize cosmic string

- $G\mu$ ($= \mu / M_{\text{pl}}^2$) : tension (line density)
- α : initial loop size $L \sim \alpha H^{-1}$
- p : reconnection probability

- CMB temperature fluctuation: $G\mu < \sim 10^{-7}$
 - Gravitational lensing: $G\mu < \sim 10^{-6}$
- for infinite strings

• **Gravitational waves**

Pulsar timing: $G\mu < \sim 10^{-9}$ for loops $\alpha = 0.1, p = 1$

Direct detection (LIGO GWB): $G\mu < \sim 10^{-6}$

**What about future constraints
from Advanced-LIGO ?**

Estimation of the GW burst rate

Initial number density of loops

depends on α and p

$$= \frac{\text{(length of infinite string discarded to loops)}}{\text{(initial length of loops = } \alpha t_i \text{)}}$$

Evolution of infinite strings

- velocity-dependent one-scale model

$$2 \frac{dL}{dt} = 2HL(1 + v^2) + \underline{cv}$$

energy conservation

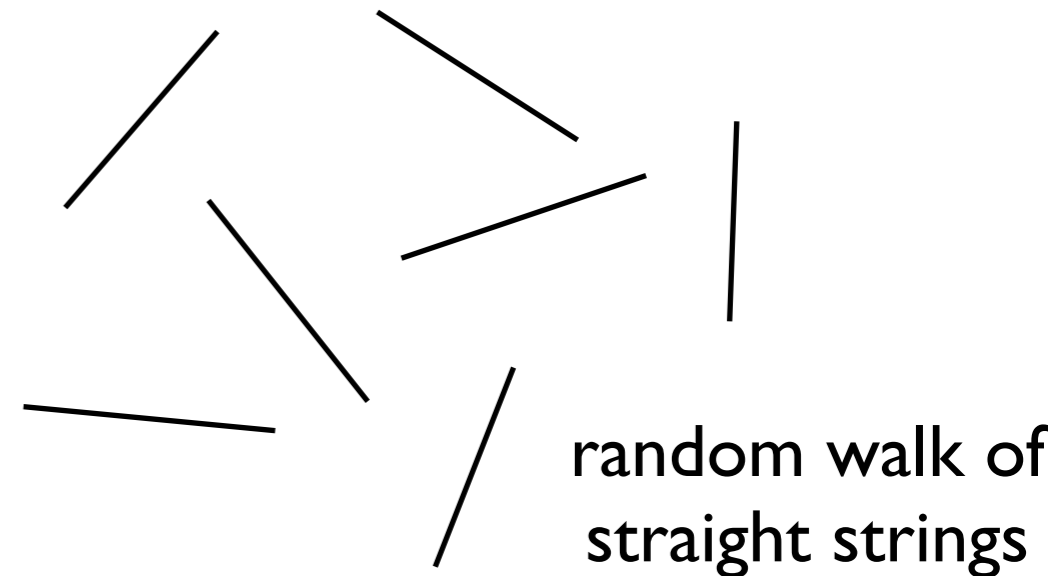
energy discarded to loops

$$\frac{dv}{dt} = (1 - v^2) \left(\frac{k}{R} - 2Hv \right)$$

acceleration due to the curvature of the strings

damping due to the expansion

momentum parameter: $k = \frac{2\sqrt{2}}{\pi} \left(\frac{1 - 8v^6}{1 + 8v^6} \right)$



random walk of straight strings
length L , velocity v

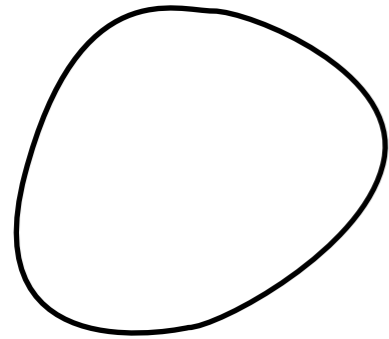
$$\left. \frac{d\rho_{inf}}{dt} \right|_{loop} = cv \frac{\rho_{inf}}{L}$$

for small p : $c \rightarrow cp$

Estimation of the GW burst rate

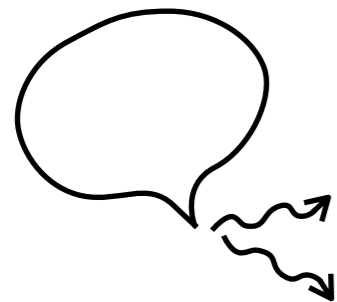
loop evolution

depends on $G\mu$ and α



Initial loop length = αt_i

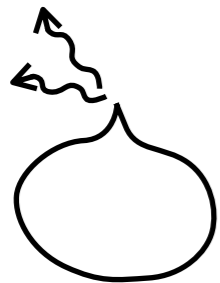
t_i : time when the loop formed



GW power $P = \Gamma G\mu^2$ Γ : numerical constant $\sim 50-100$

From the energy conservation law
(energy of loop at time $t = \mu l$)

$$= (\text{initial energy of the loop} = \mu\alpha t_i) - (\text{energy released to GWs} = P\Delta t)$$



Loop length at time t $l(t, t_i) = \alpha t_i - \Gamma G\mu(t - t_i)$



Lifetime of the loop = $\frac{(\text{initial loop energy})}{(\text{energy release rate per time})}$

$$= \frac{\mu\alpha t_i}{\Gamma G\mu^2} = \frac{\alpha t_i}{\Gamma G\mu}$$

0



Estimation of the GW burst rate

GW burst rate emitted at $t \sim t+dt$ from loops formed at $t_i \sim t_i+dt_i$

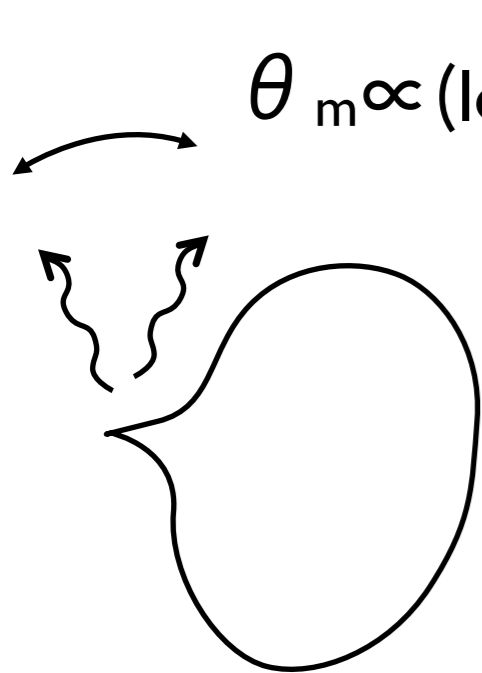
$$\frac{dR}{dt dt_i} = \underbrace{\frac{1}{4} \theta_m(f, z, l)^2}_{\text{Beaming}} \underbrace{\frac{2c}{(1+z)l(t, t_i)}}_{\text{Time interval of GW emission}} \underbrace{\frac{dn}{dt_i}(t, t_i) \frac{dV}{dt}}_{\text{Loop number}} \times \underbrace{\Theta(1 - \theta_m(f, z, l))}_{\text{Beaming}}$$

Beaming

Time interval of GW emission

Loop number

$$\propto (\text{loop length at } t)^{-1}$$



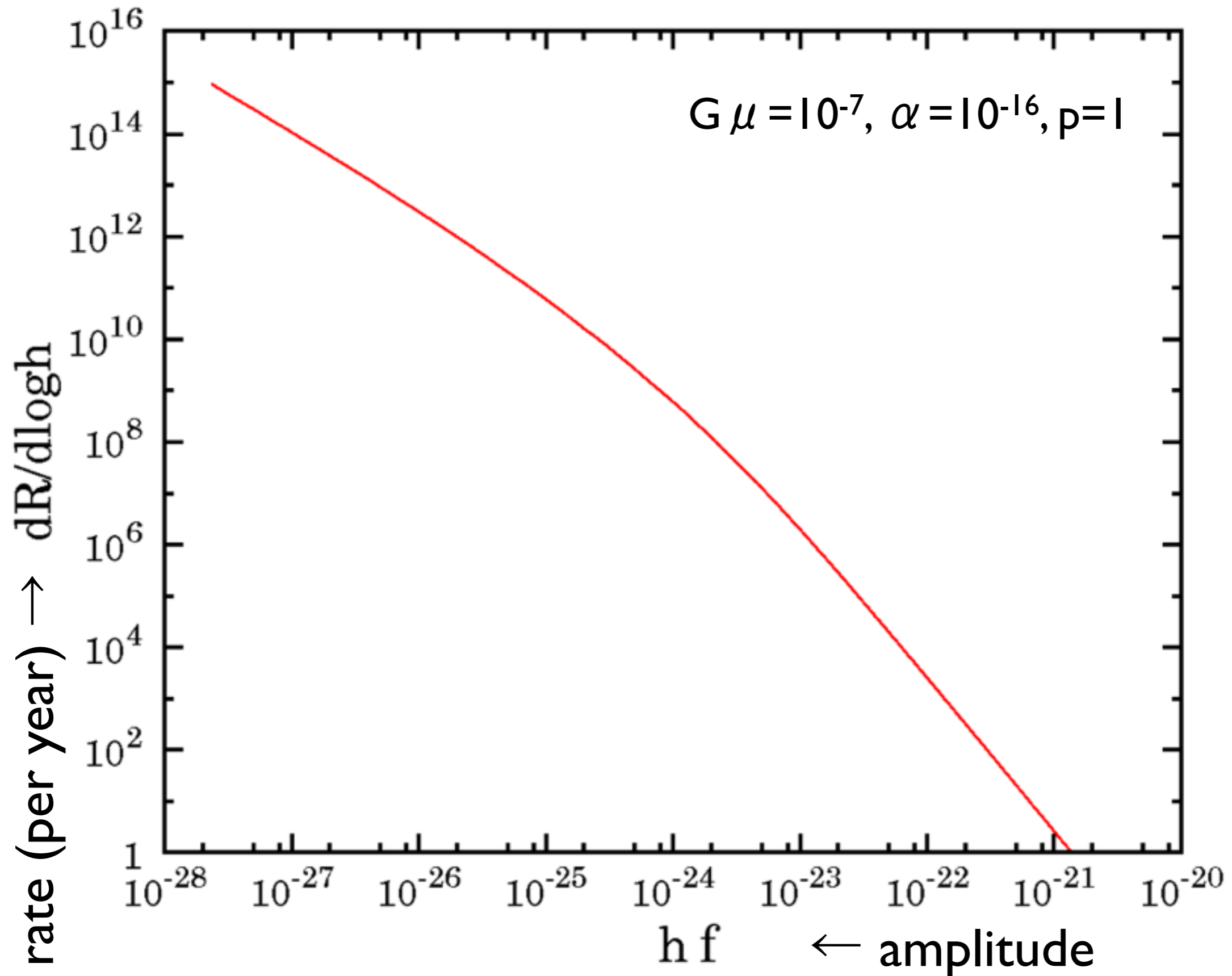
$$\theta_m \propto (\text{loop length at } t)^{1/3}$$

$$l(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$$

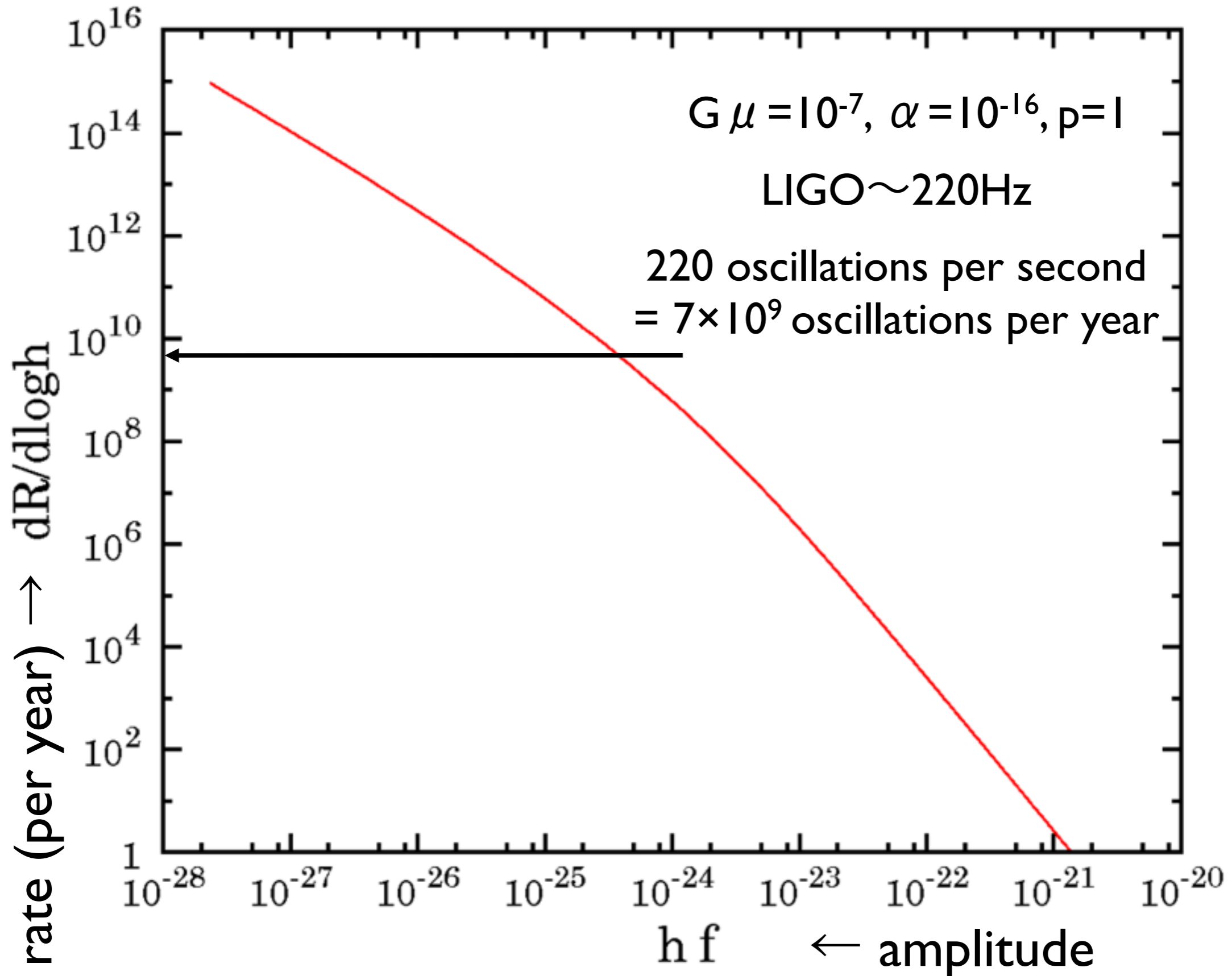
GW amplitude from loop of length l

$$h(f, z, l) \simeq 2.68 \frac{G \mu l}{((1+z)fl)^{1/3} r(z) f}$$

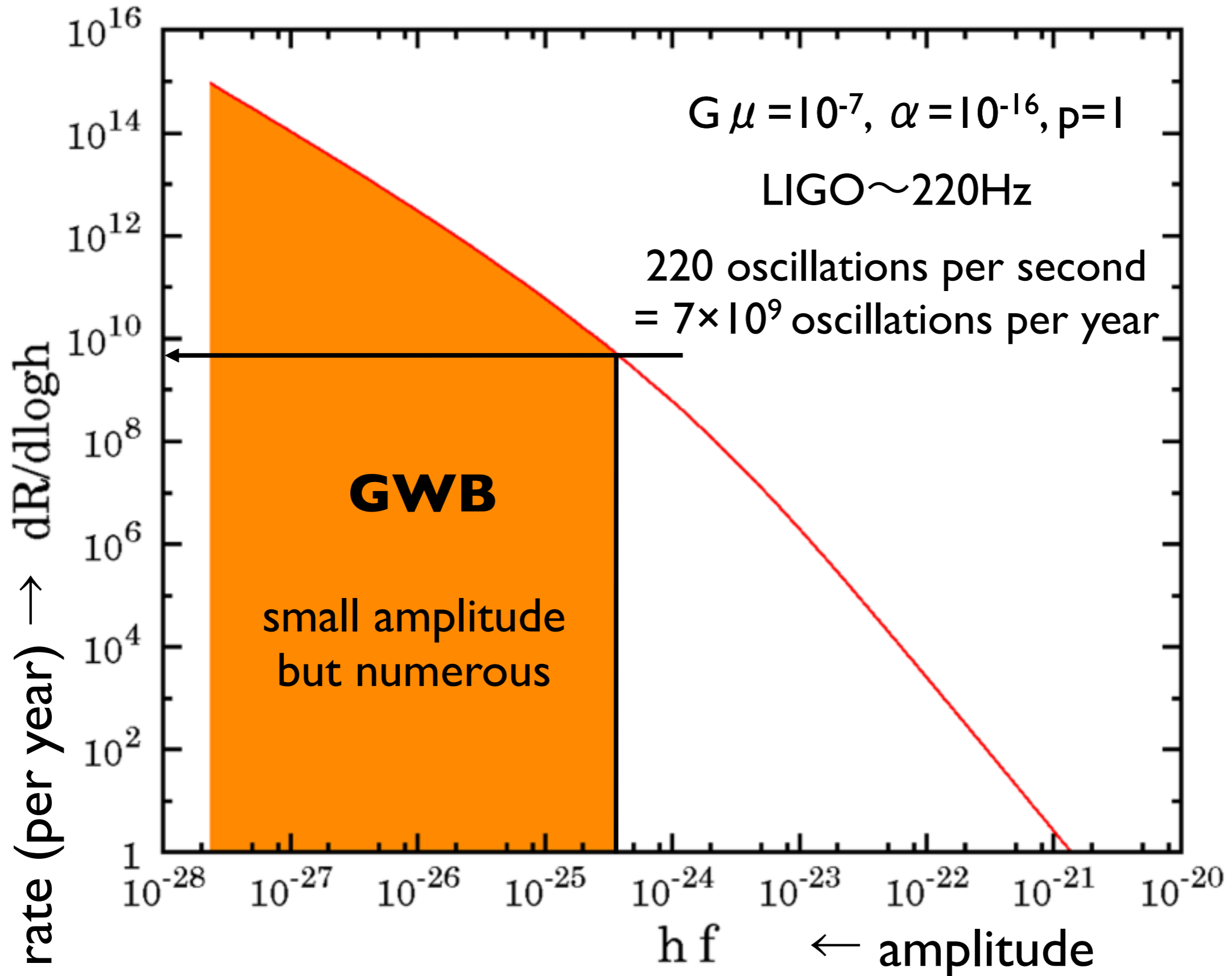
How many cosmic string bursts are coming to the earth per year?
(plotted as a function of the amplitude for the fixed frequency @220Hz)



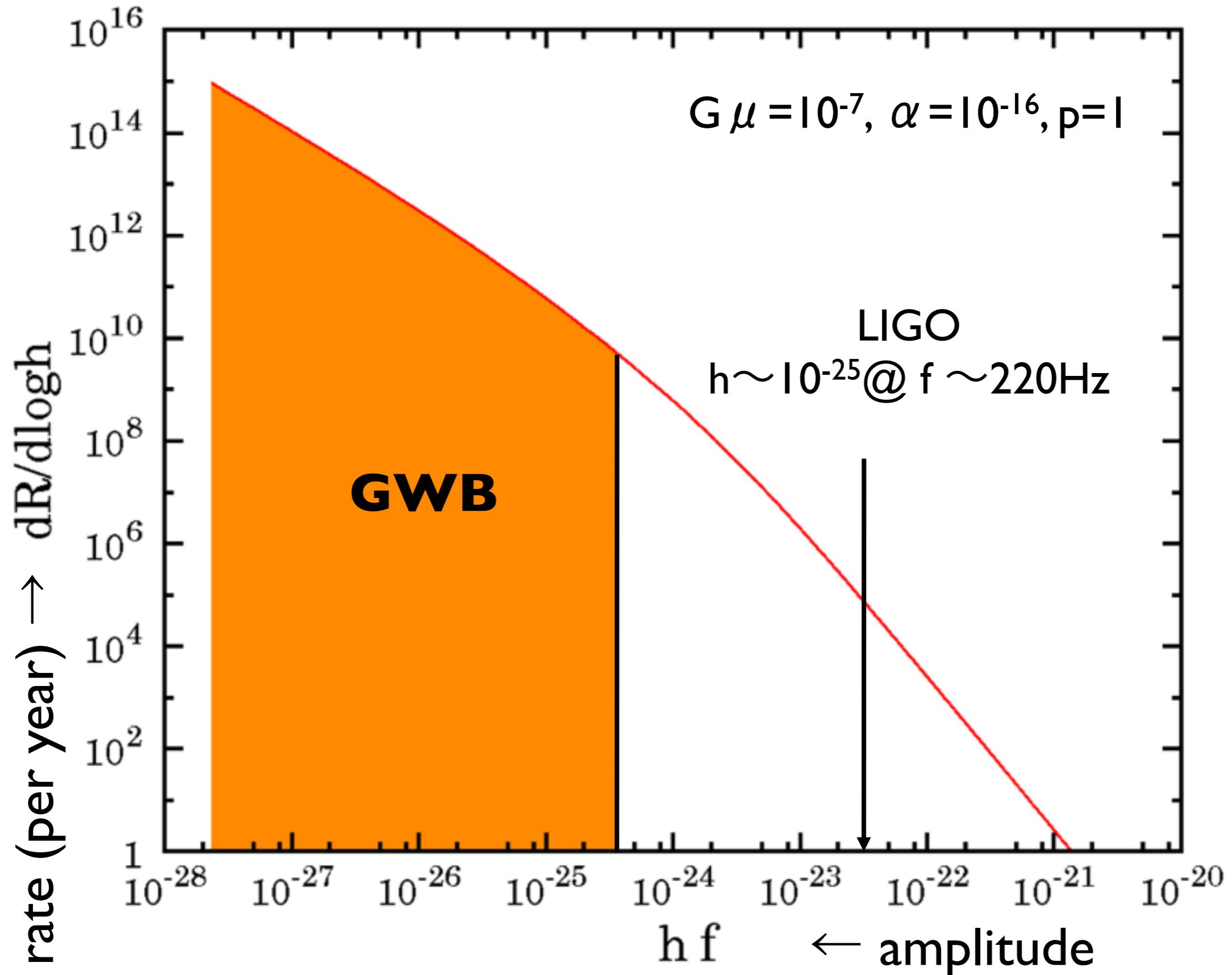
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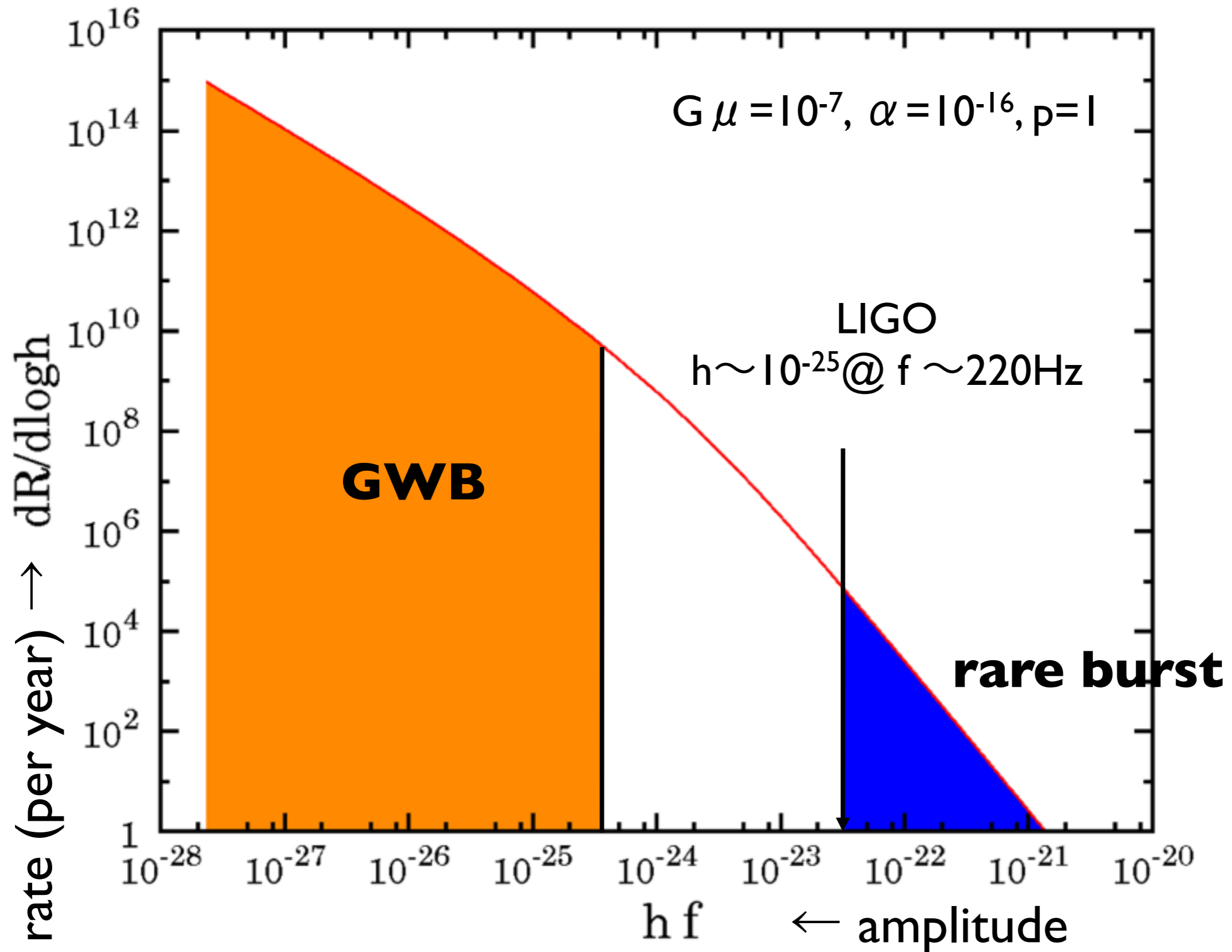
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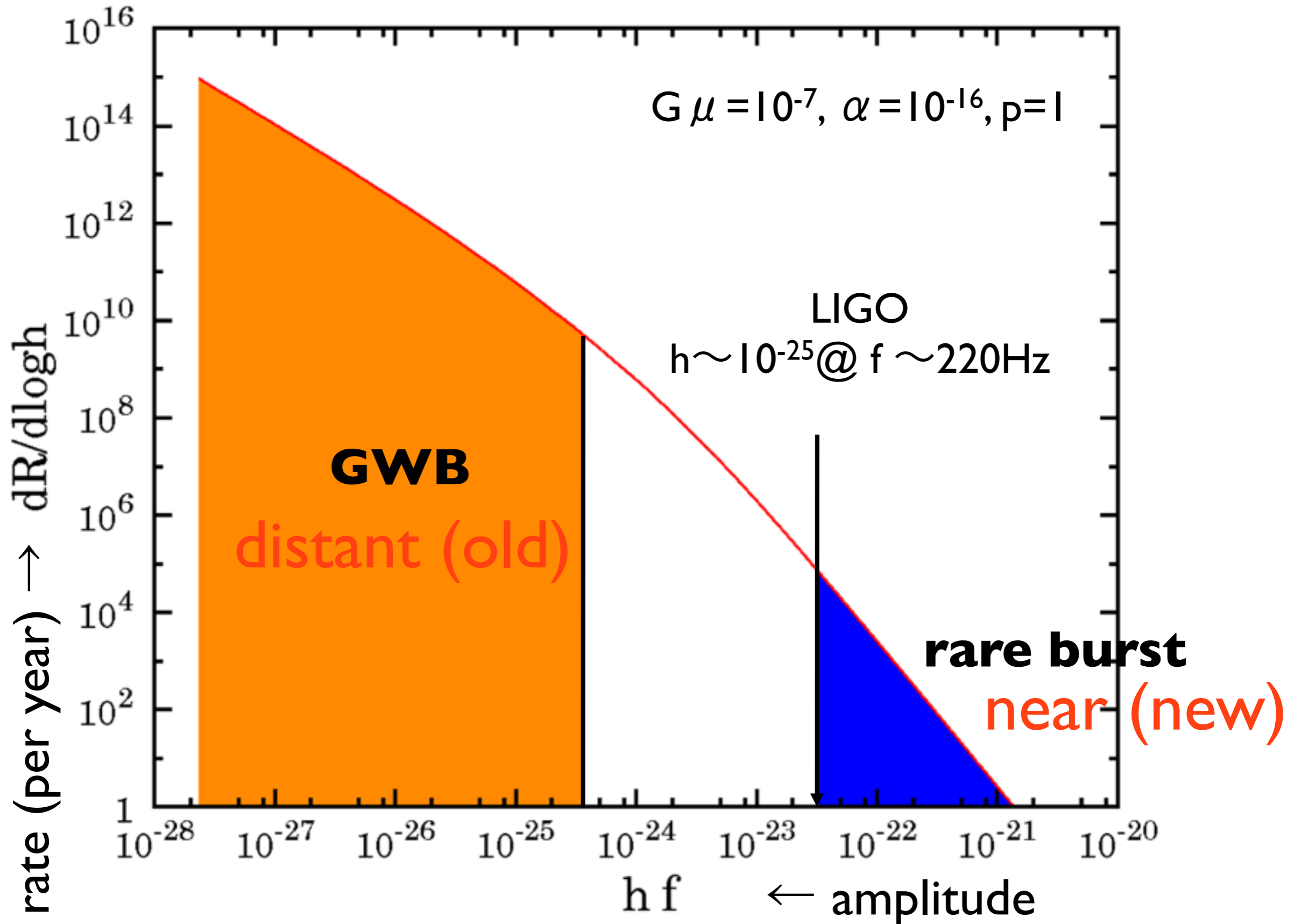
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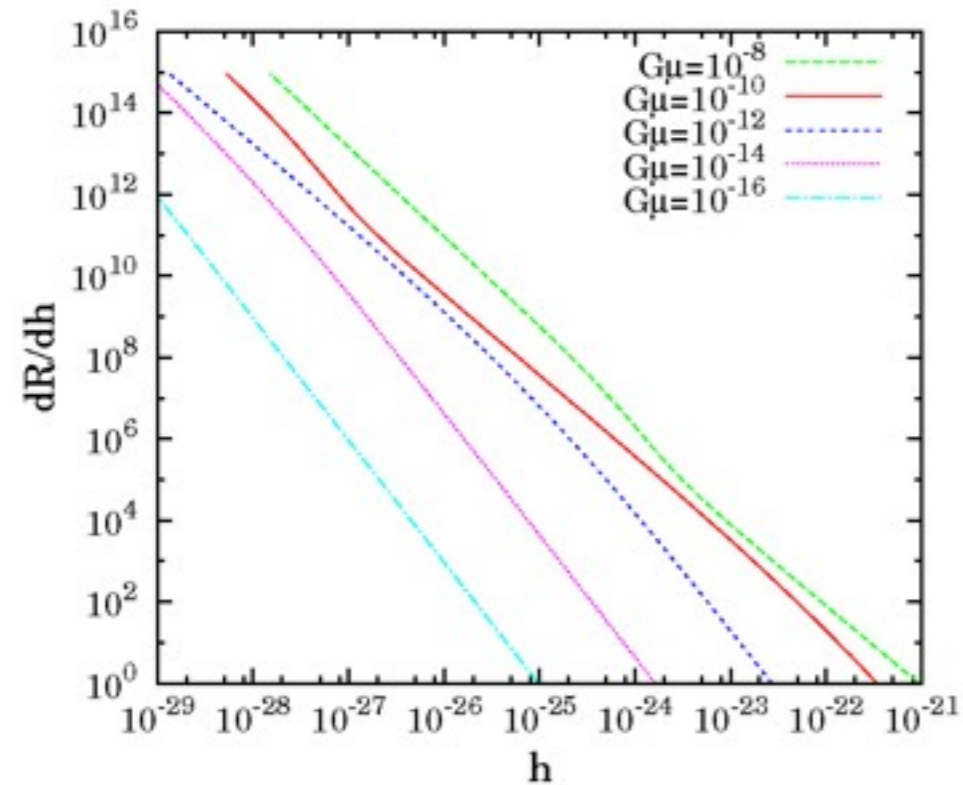


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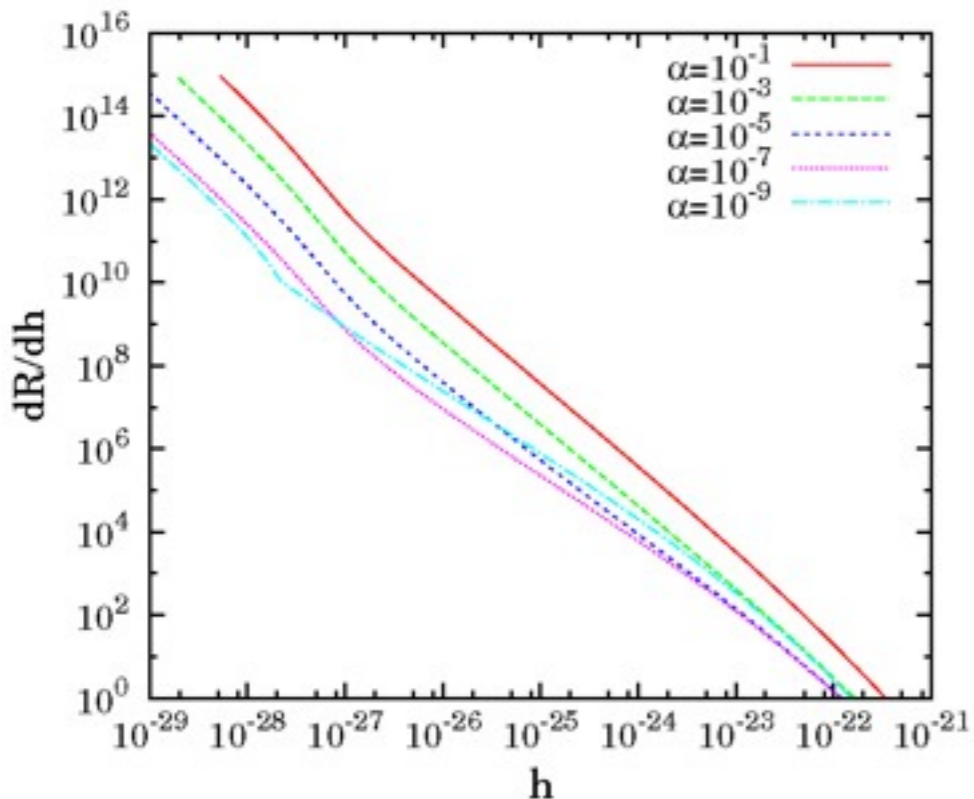
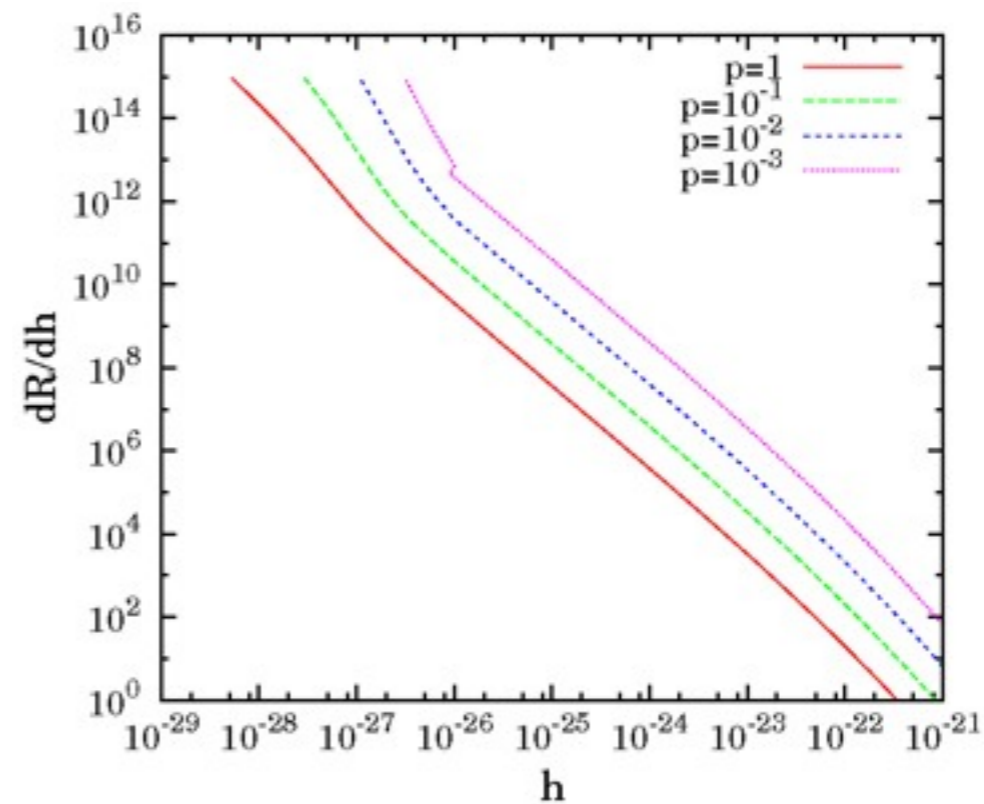


Parameter dependences of the rate

$G\mu$



p



α

The parameter dependences of the large burst (rare burst) and small burst (GWB) are different because they are looking at different epoch of the Universe

→ give different information on cosmic string parameters

Spectrum of the GWB

dependence
on $G\mu$

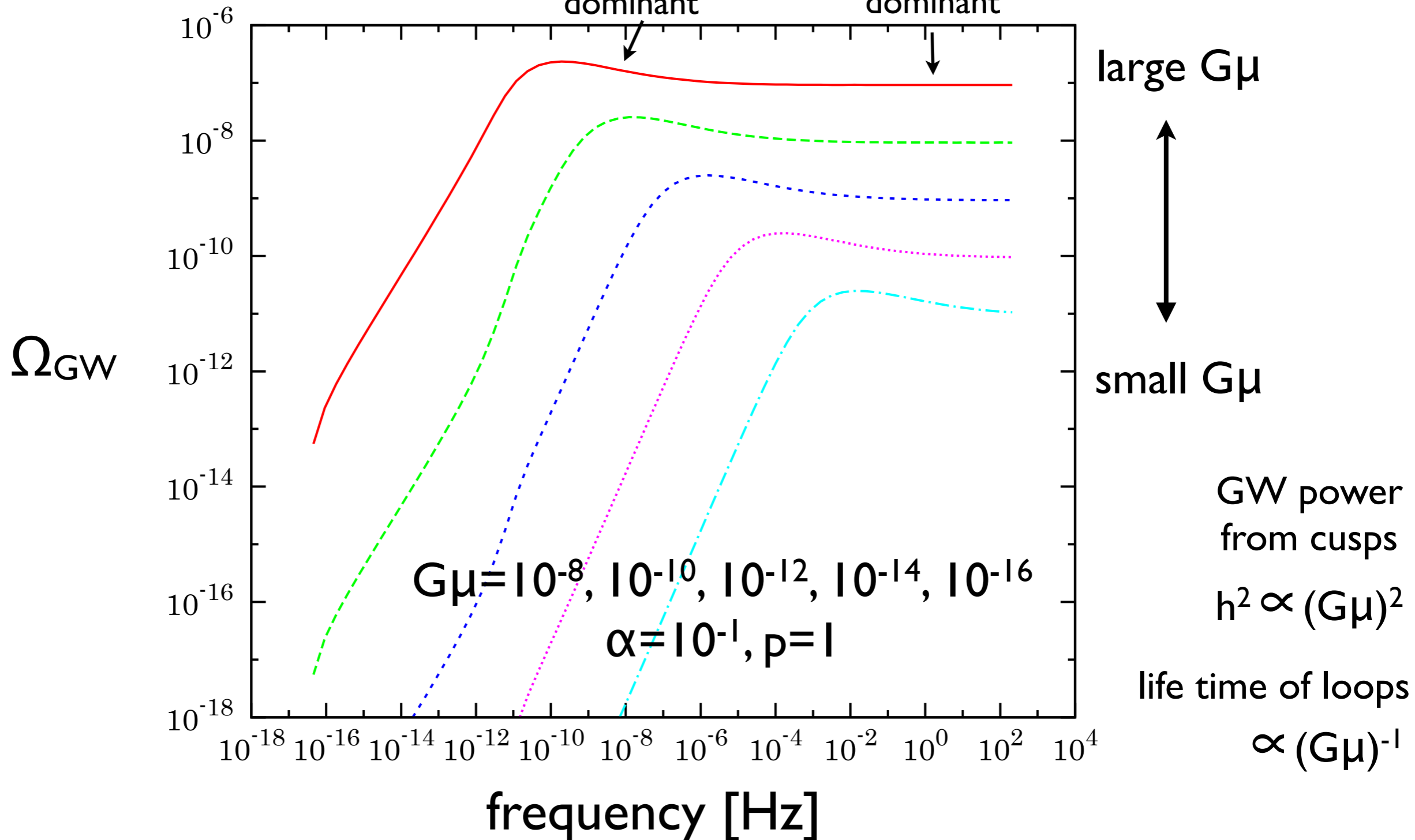
new



old

Matter
dominant

Radiation
dominant



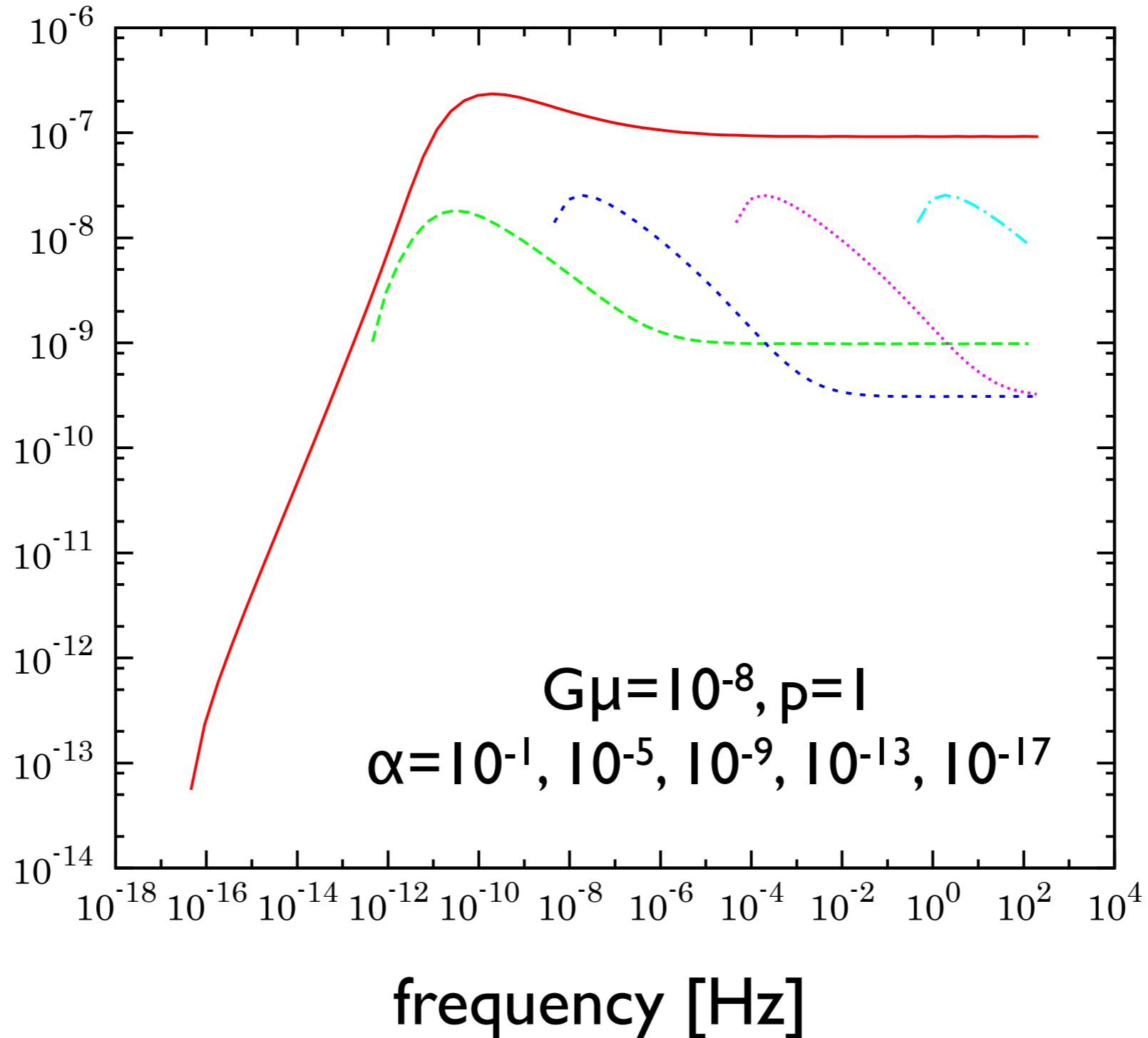
Spectrum of the GWB

dependence
on α

loop size directly corresponds to the frequency of the GW

→ small α

Ω_{GW}



Spectrum of the GWB

dependence
on ρ

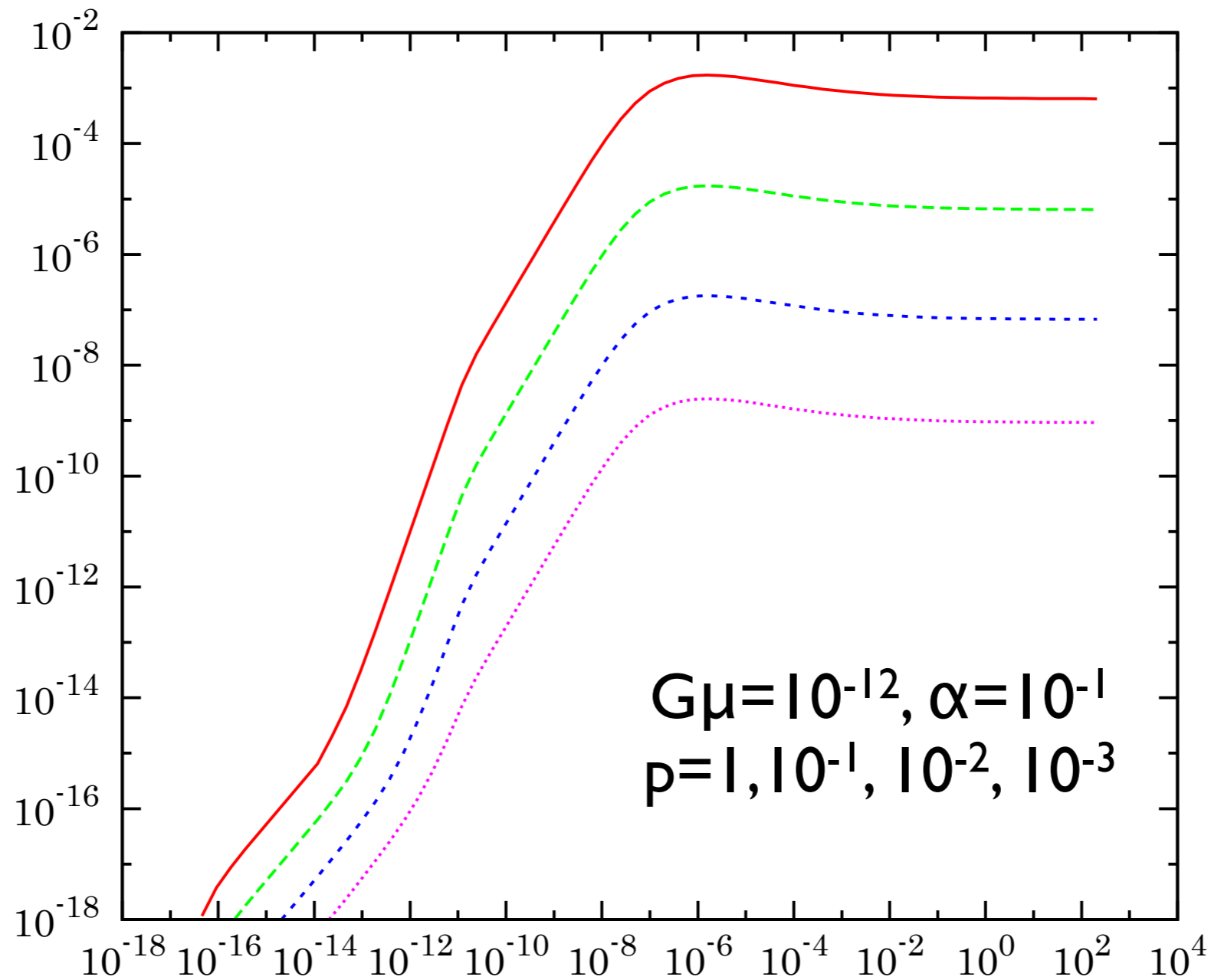
small ρ increases the number density of loops

on ρ

small ρ

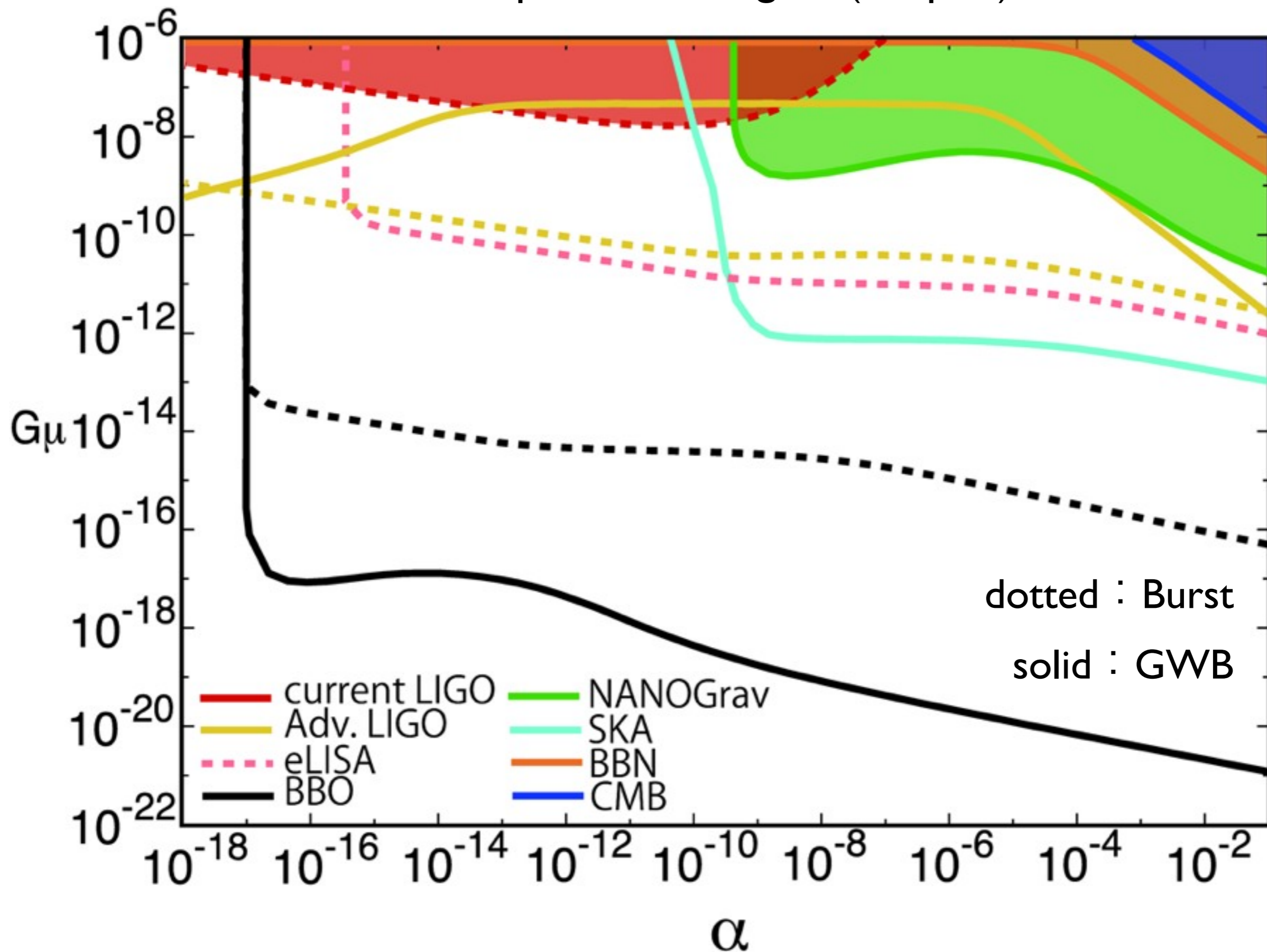
Ω_{GW}

large ρ

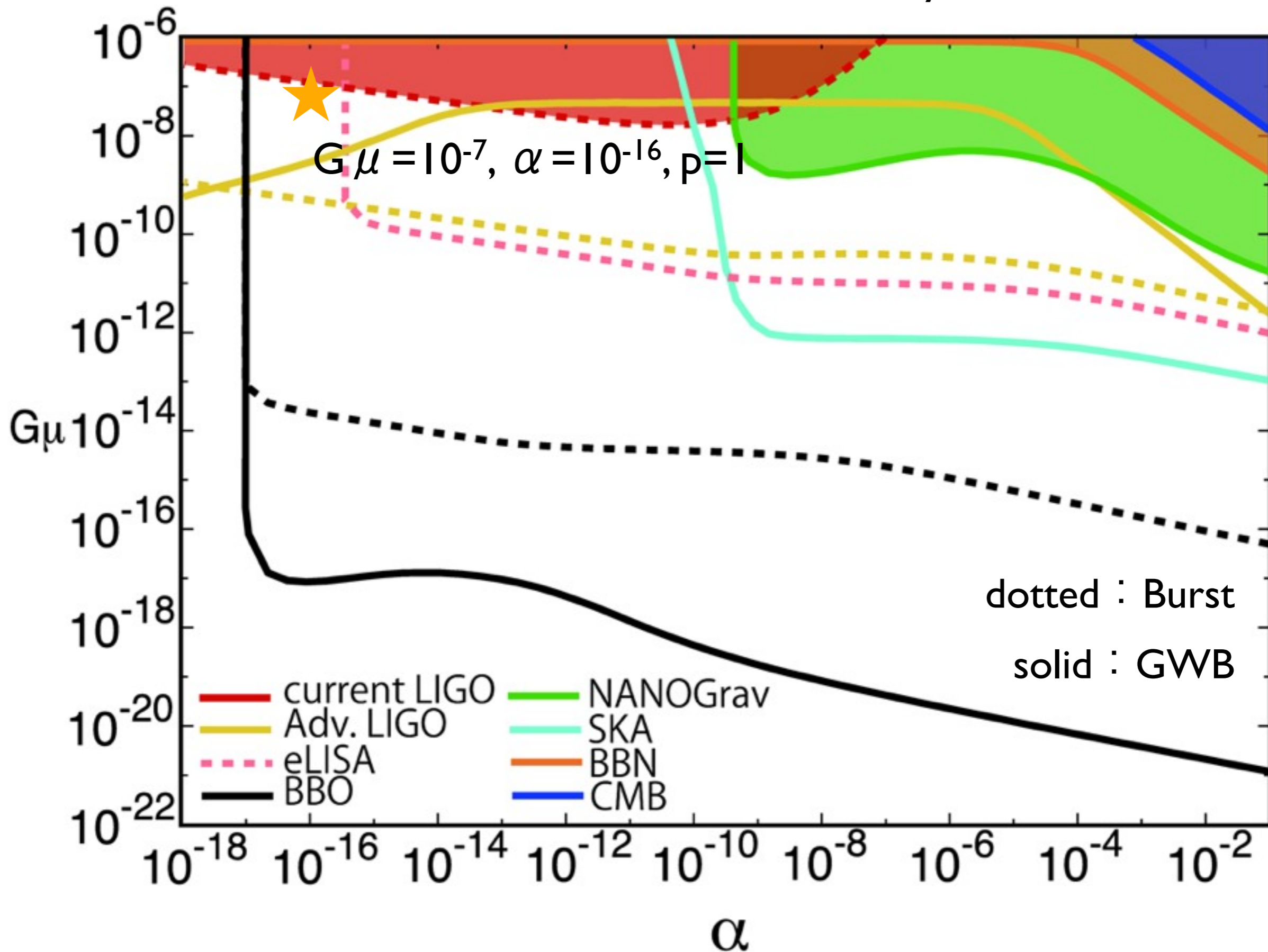


frequency [Hz]

Accessible parameter region (for $p=1$)



What if both bursts and GWB are detected by Advanced-LIGO?

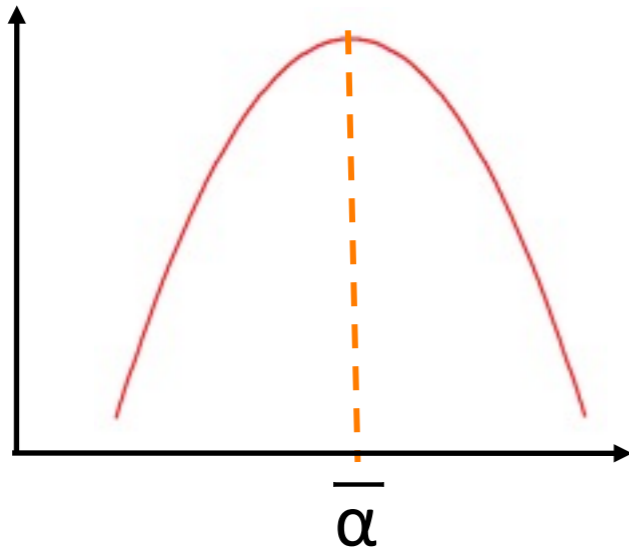


Constraint on parameters

Fisher information matrix

log(Likelihood)

$$\mathcal{F}_{ij} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle$$



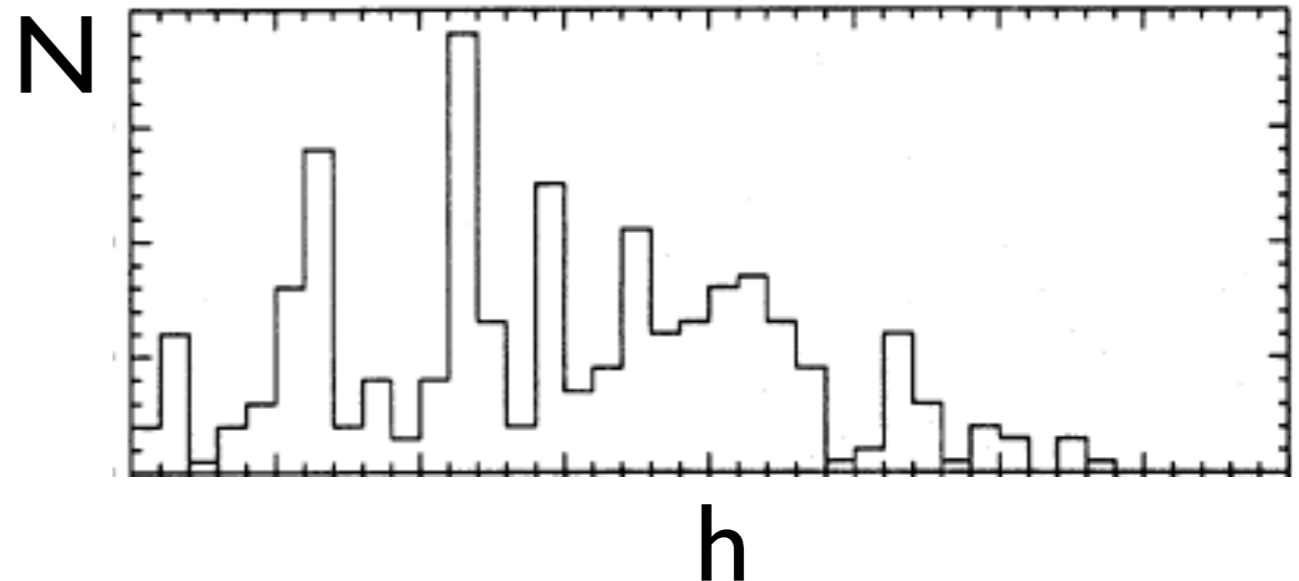
If the likelihood shape is sensitive to the parameter
= easy to estimate the parameter

Burst

Observable : amplitude vs number

N is predictable by the rate dR/dh

$$\mathcal{F}_{ij} \propto \frac{\partial(dR/dh)}{\partial p_i} \frac{\partial(dR/dh)}{\partial p_j}$$

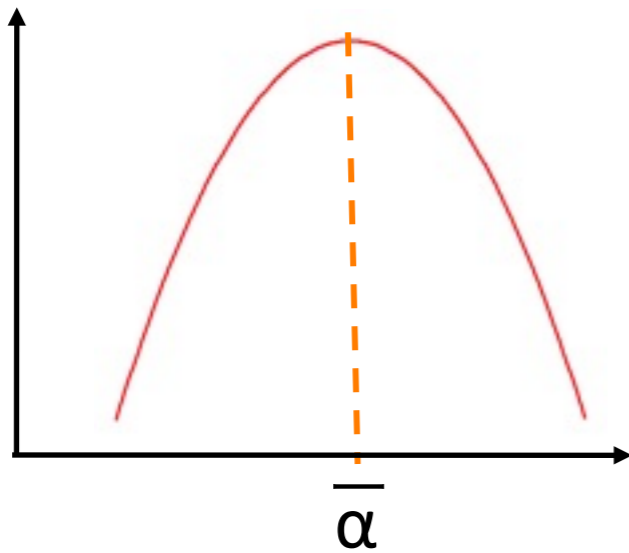


Constraint on parameters

Fisher information matrix

log(Likelihood)

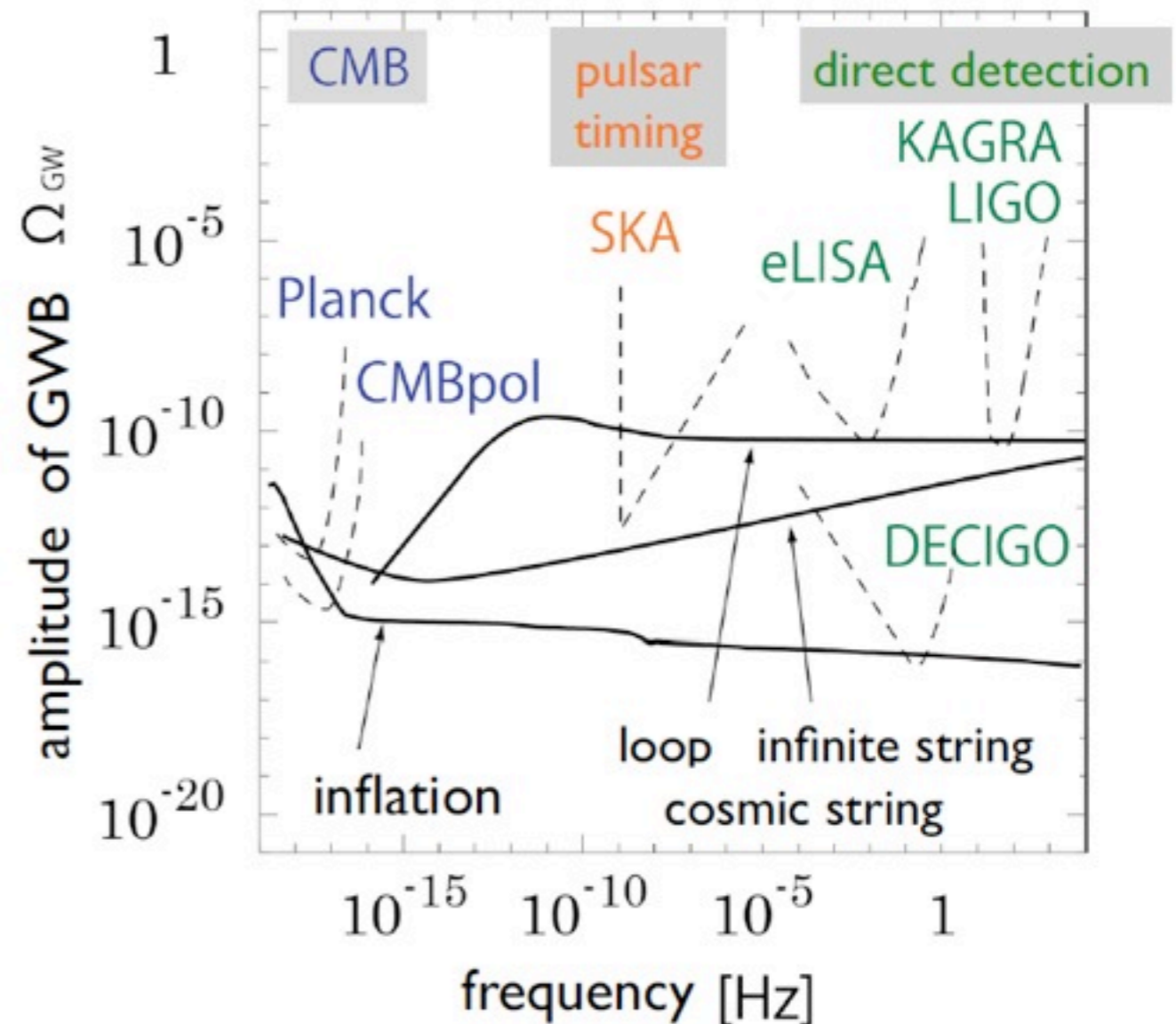
$$\mathcal{F}_{ij} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle$$



If the likelihood shape is sensitive to the parameter
= easy to estimate the parameter

GWB Observable : Ω_{GW}

$$\mathcal{F}_{ij} \propto \frac{\partial \Omega_{GW}}{\partial p_i} \frac{\partial \Omega_{GW}}{\partial p_j}$$

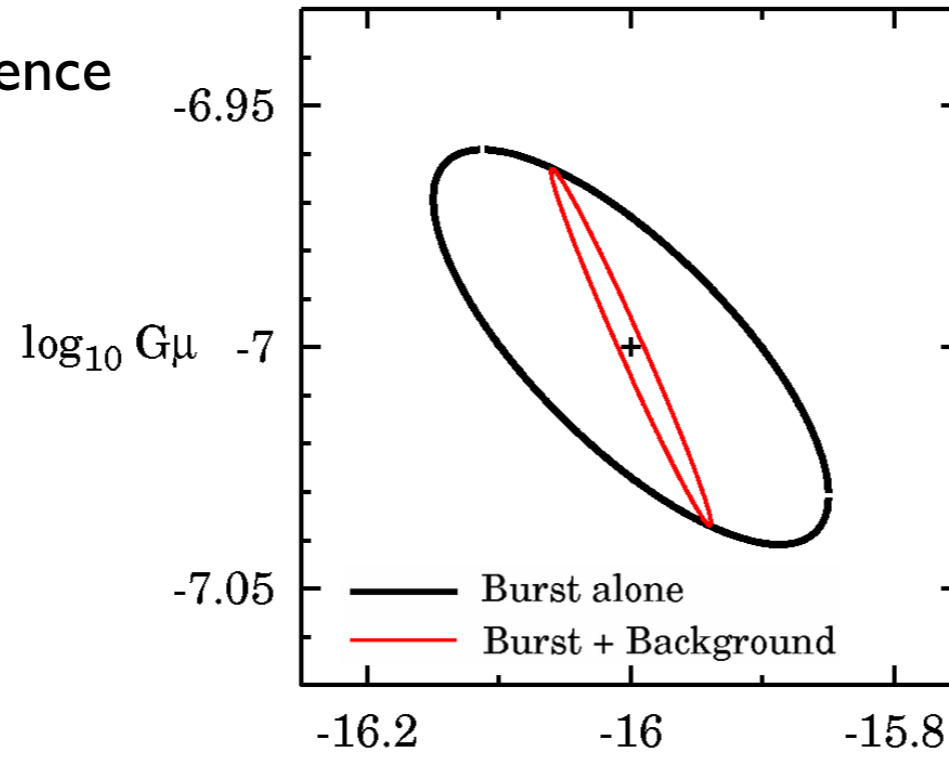


Predicted constraint on parameters

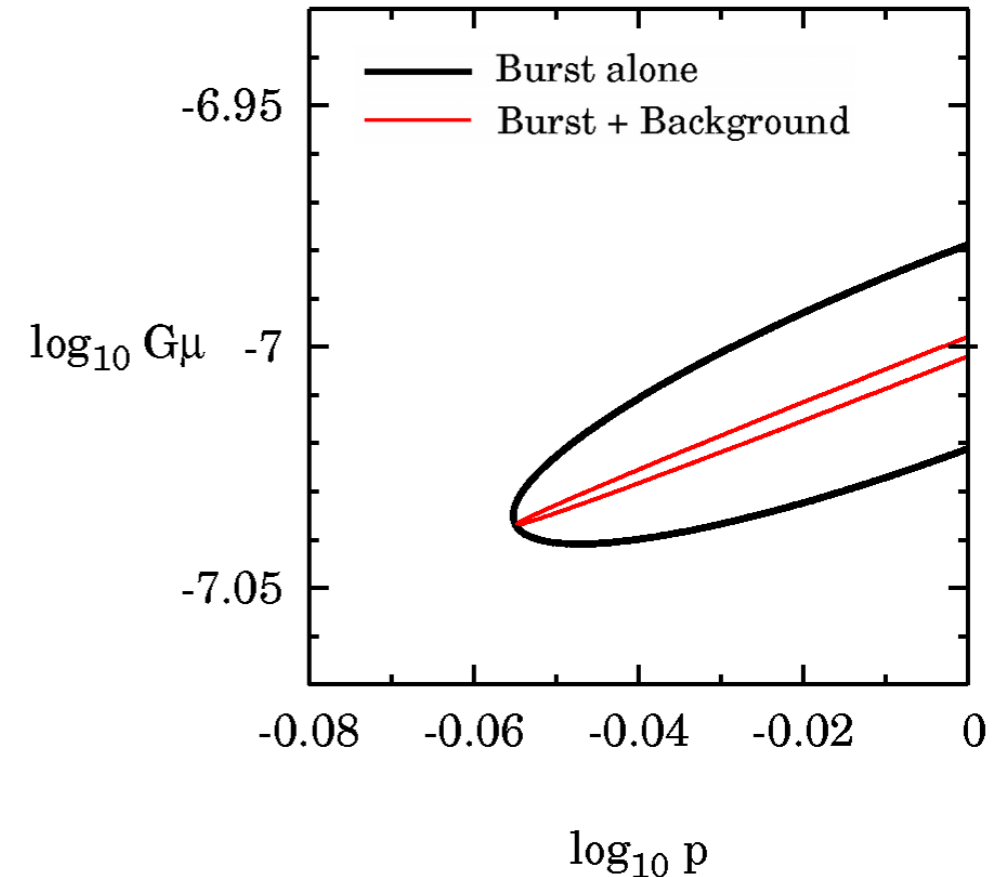
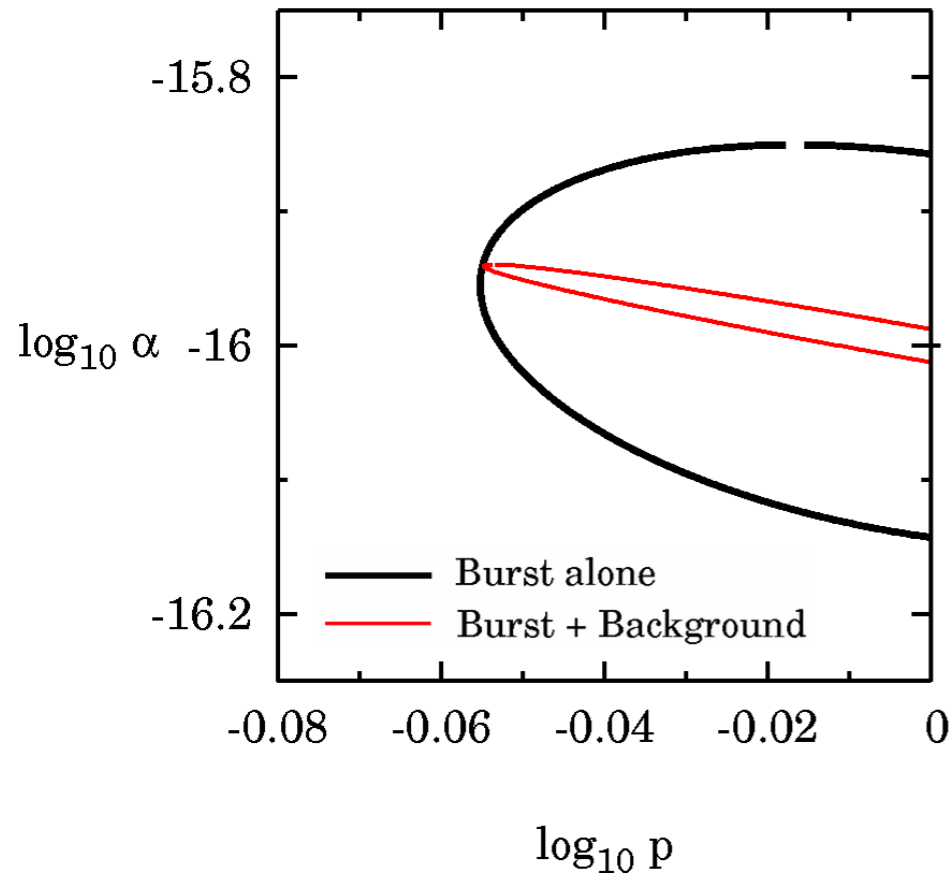
$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year

Kuroyanagi et. al. PRD 86, 023503 (2012)

different parameter dependence
= different constraints on
parameters



black : Burst only
red : Burst + GWB

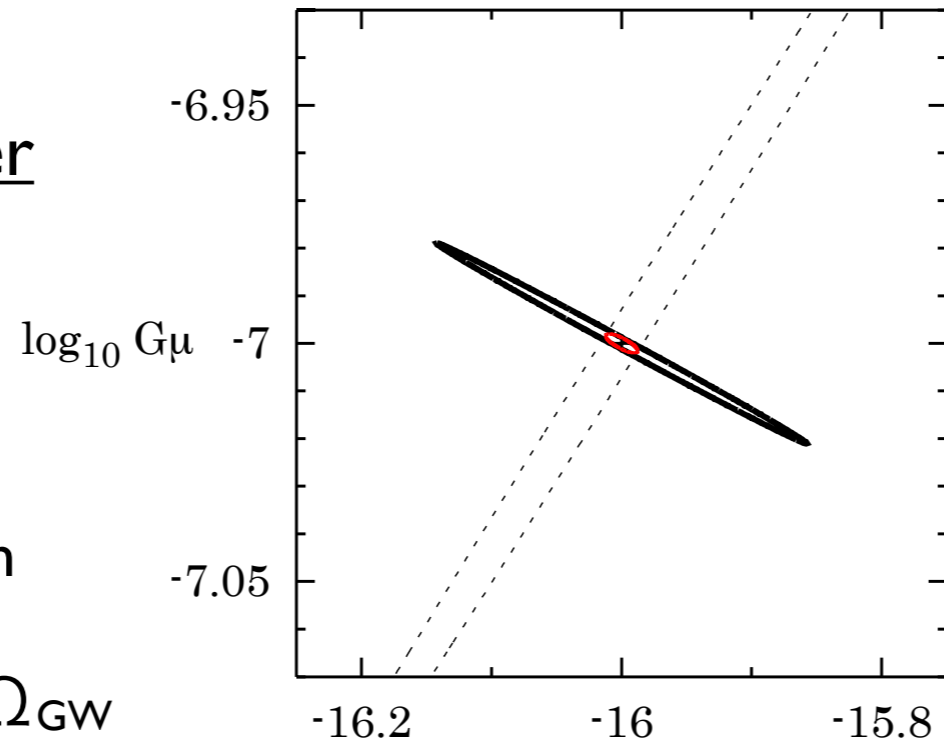


Predicted constraint on parameters

$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year

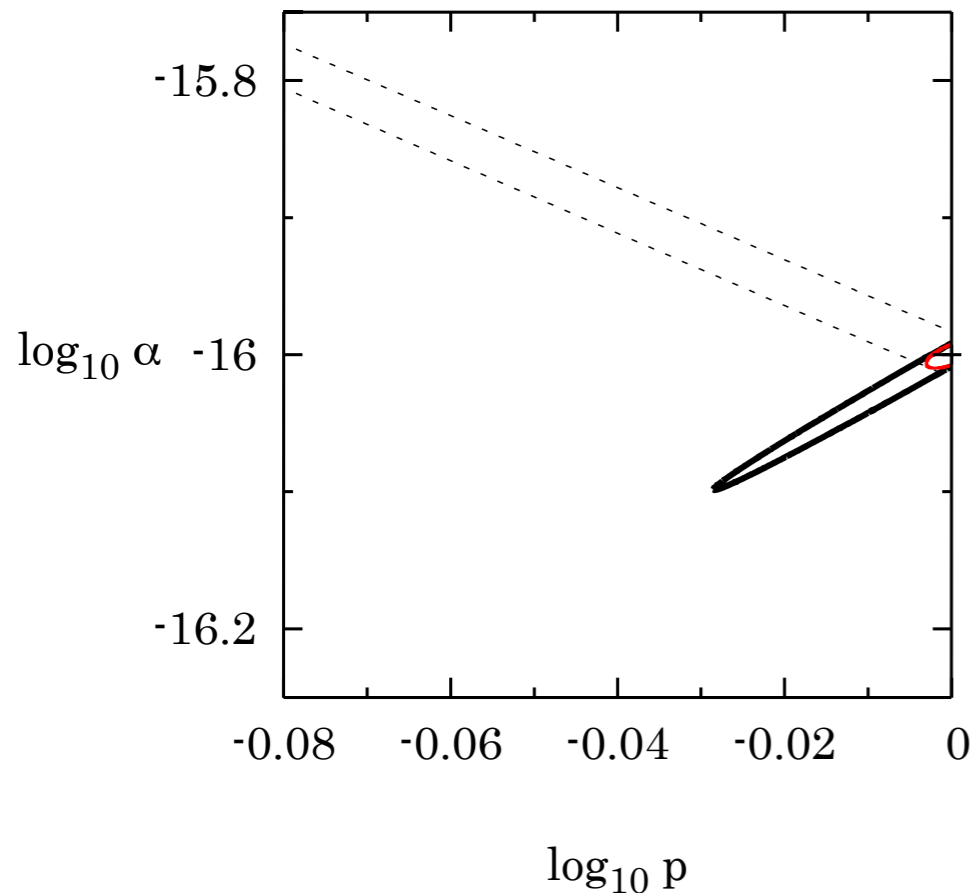
Kuroyanagi et. al. PRD 86, 023503 (2012)

Before marginalized over

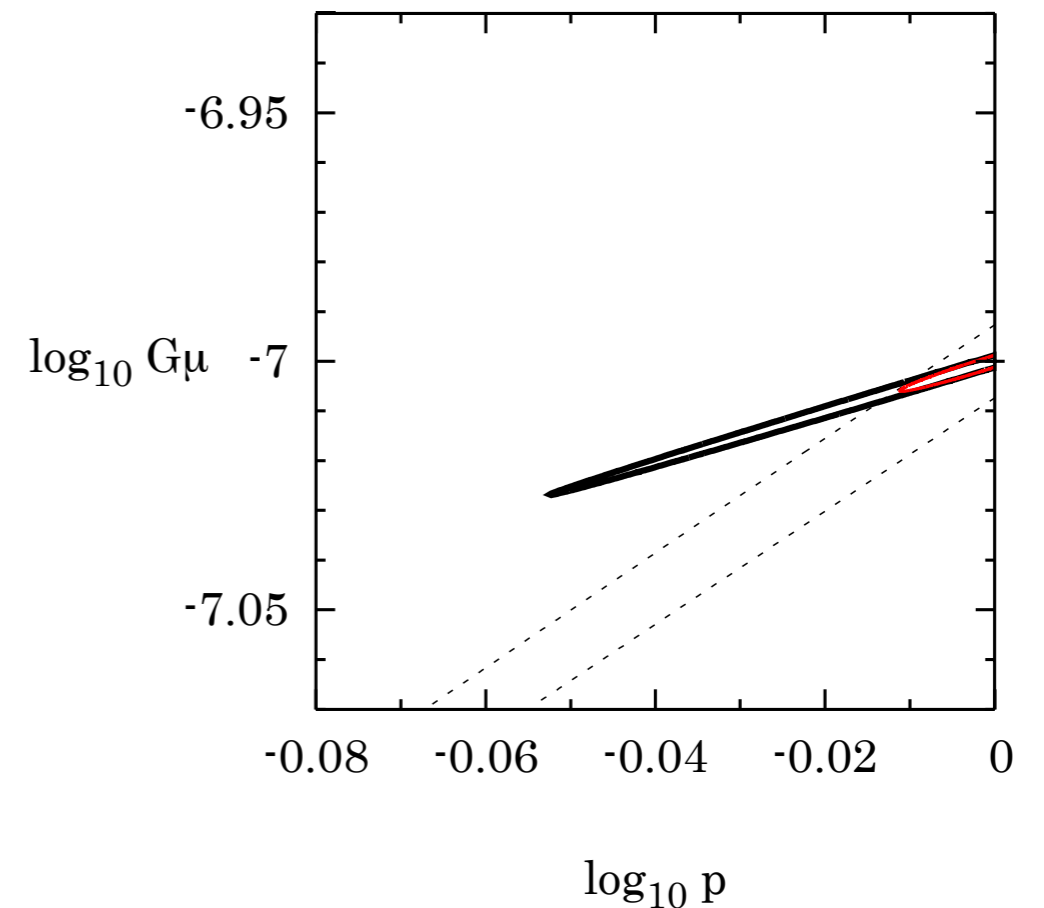


black : Burst only
dotted: GWB only
red : Burst + GWB

Strong degeneracy seen in constraint from GWB since the observable is only Ω_{GW}

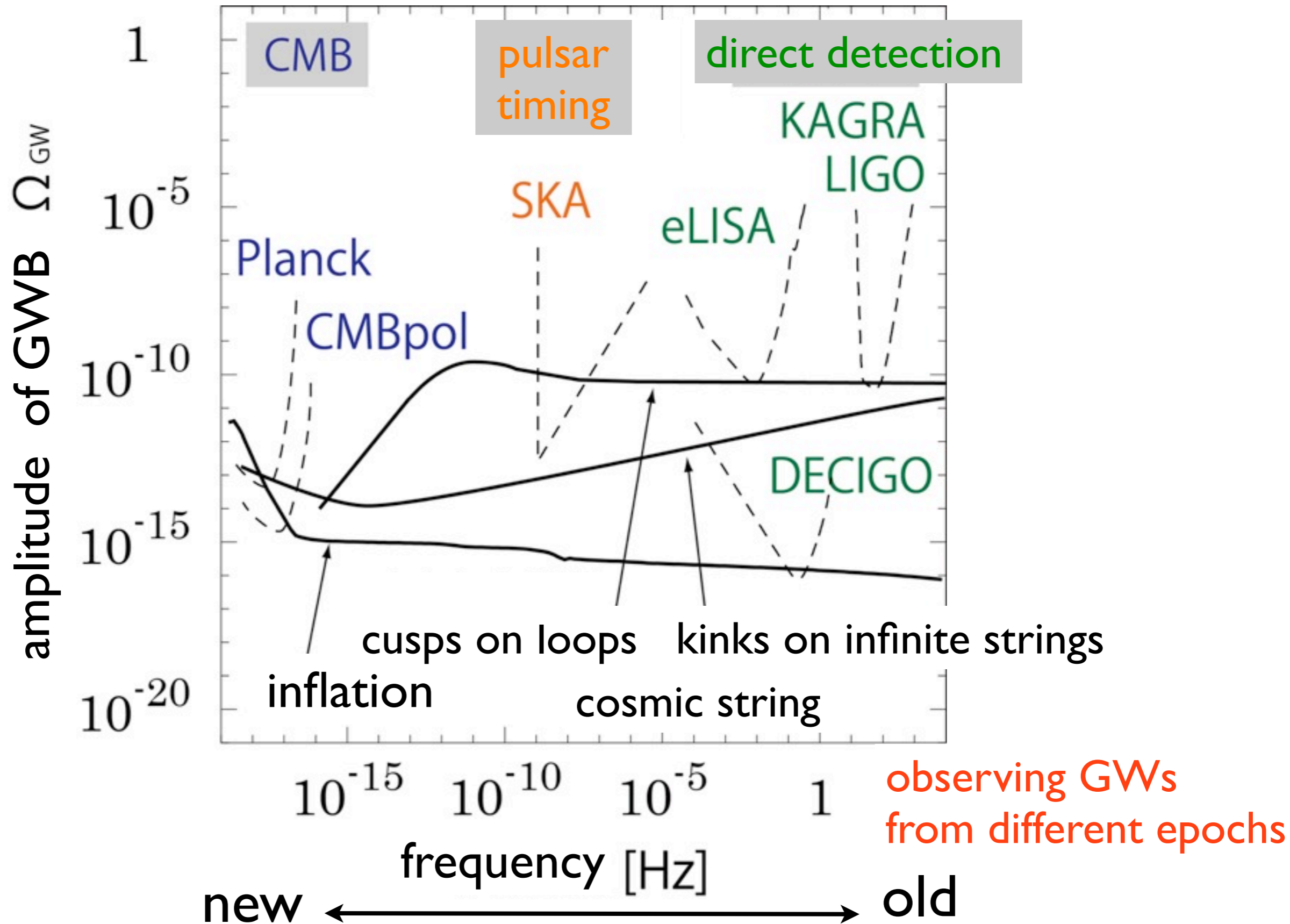


$\log_{10} \alpha$



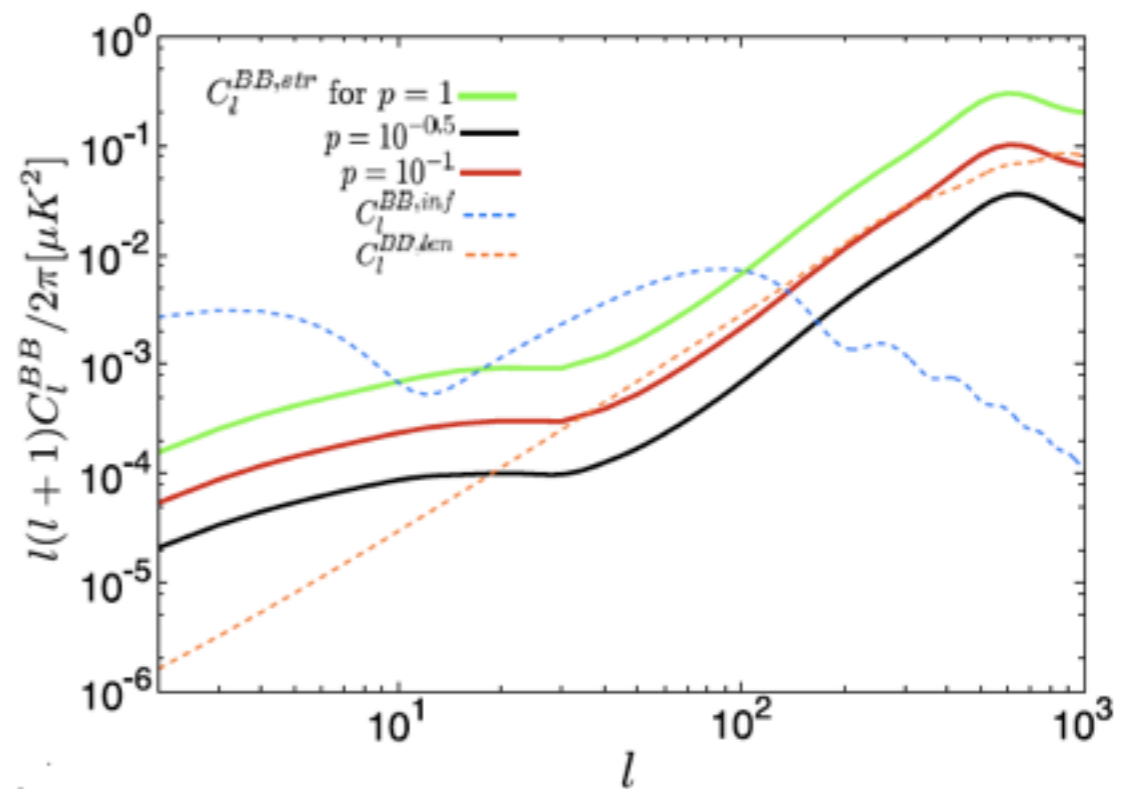
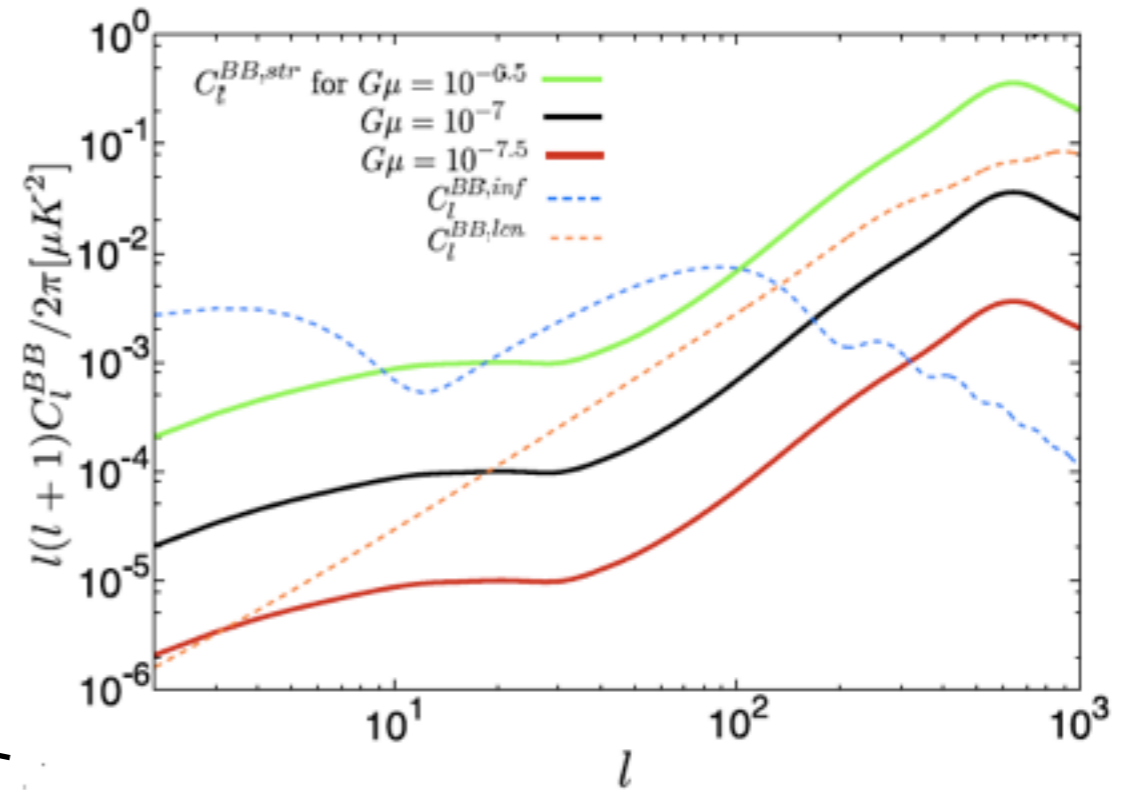
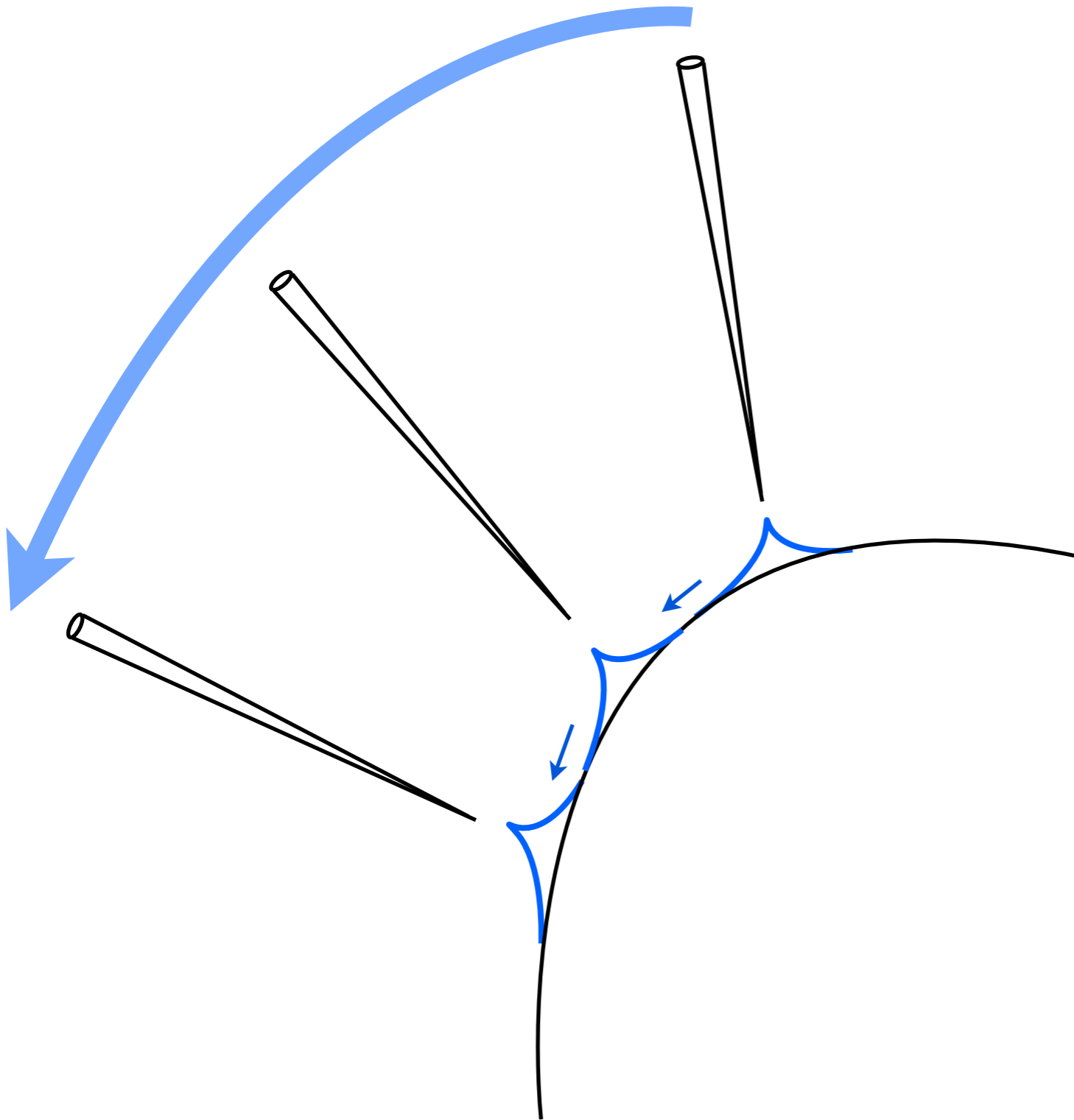
$\log_{10} G\mu$

Constraints from other experiments?



Signal in the CMB B-mode

kink on infinite string can generate large scale GWs

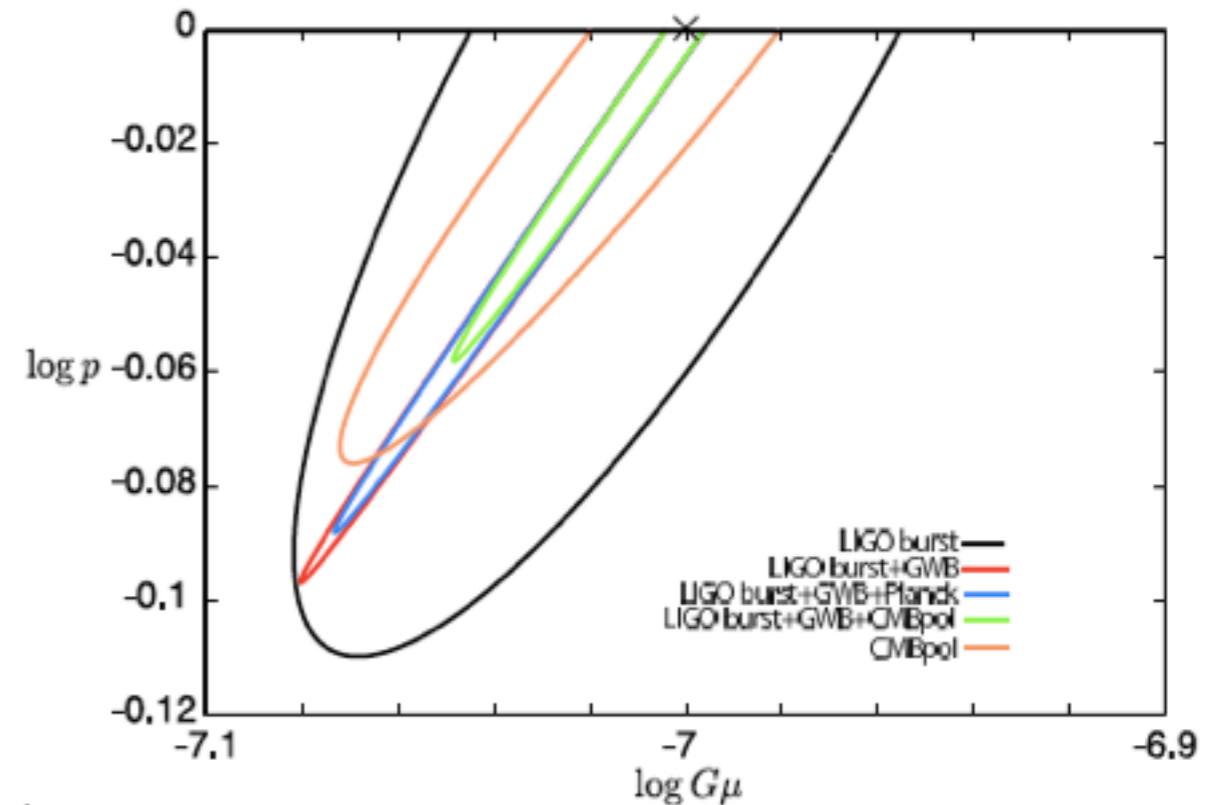
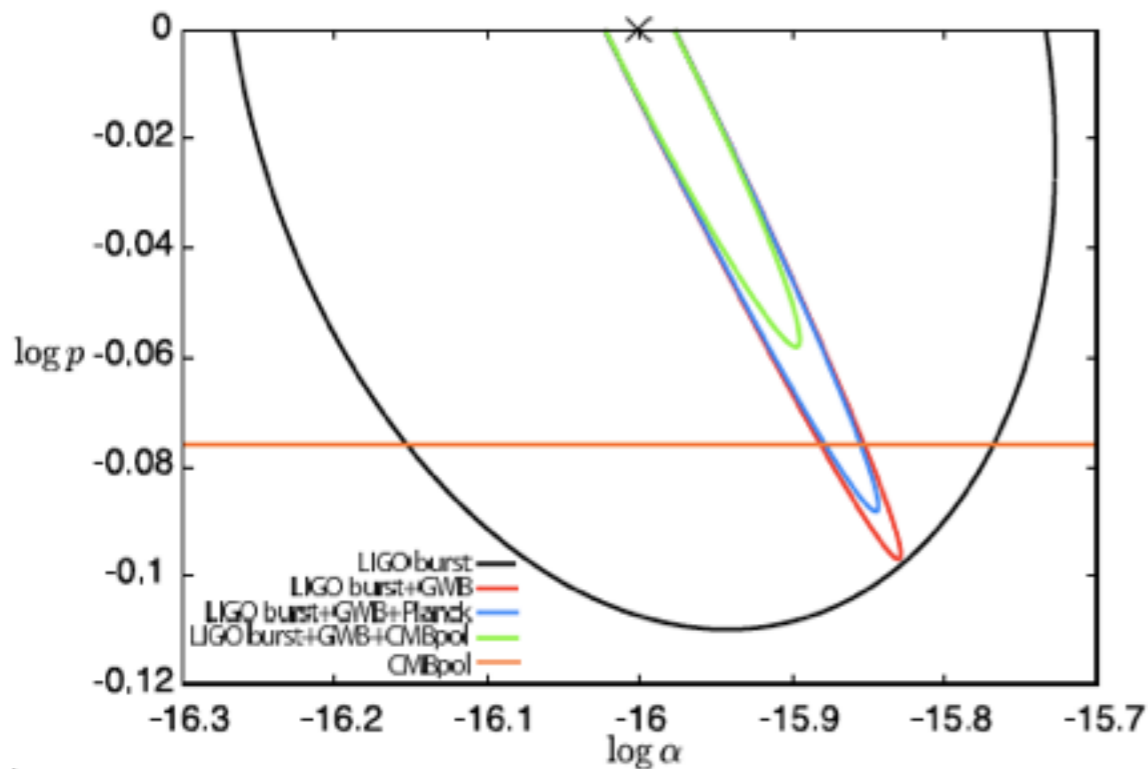
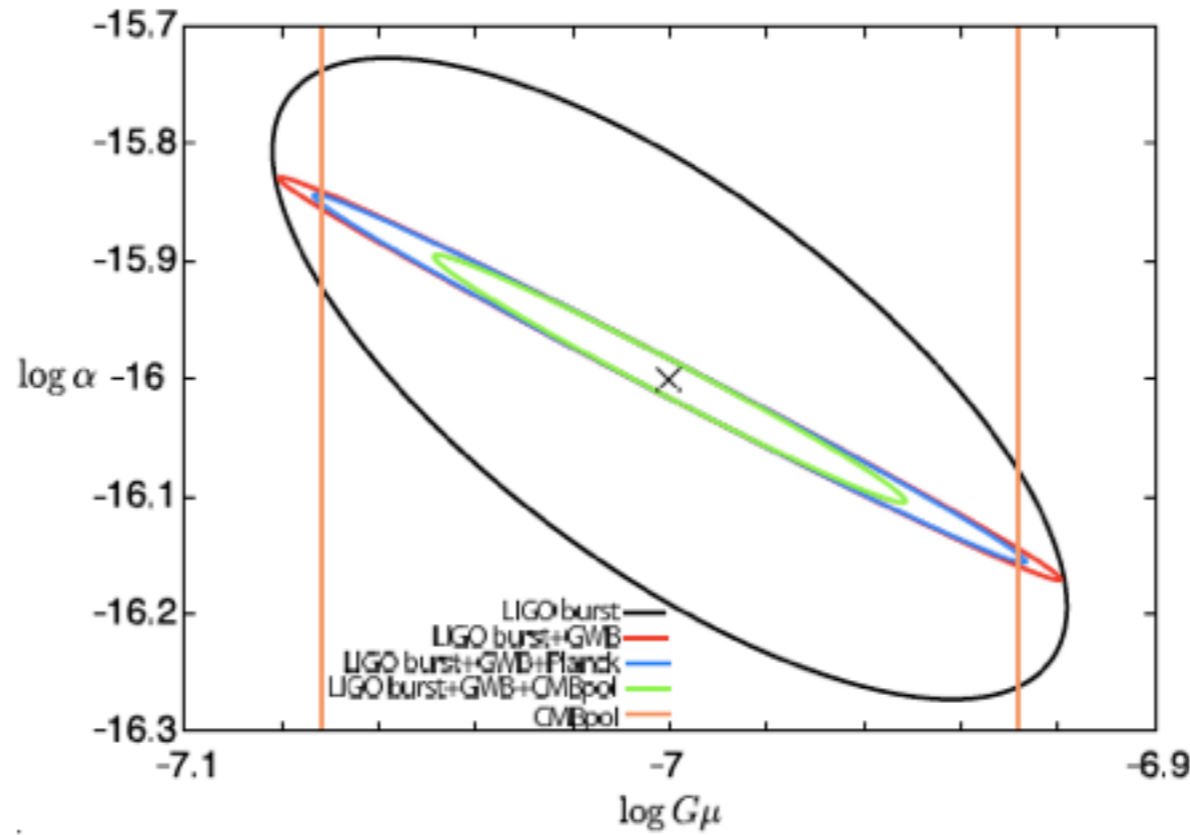


If we combine CMB constraints...

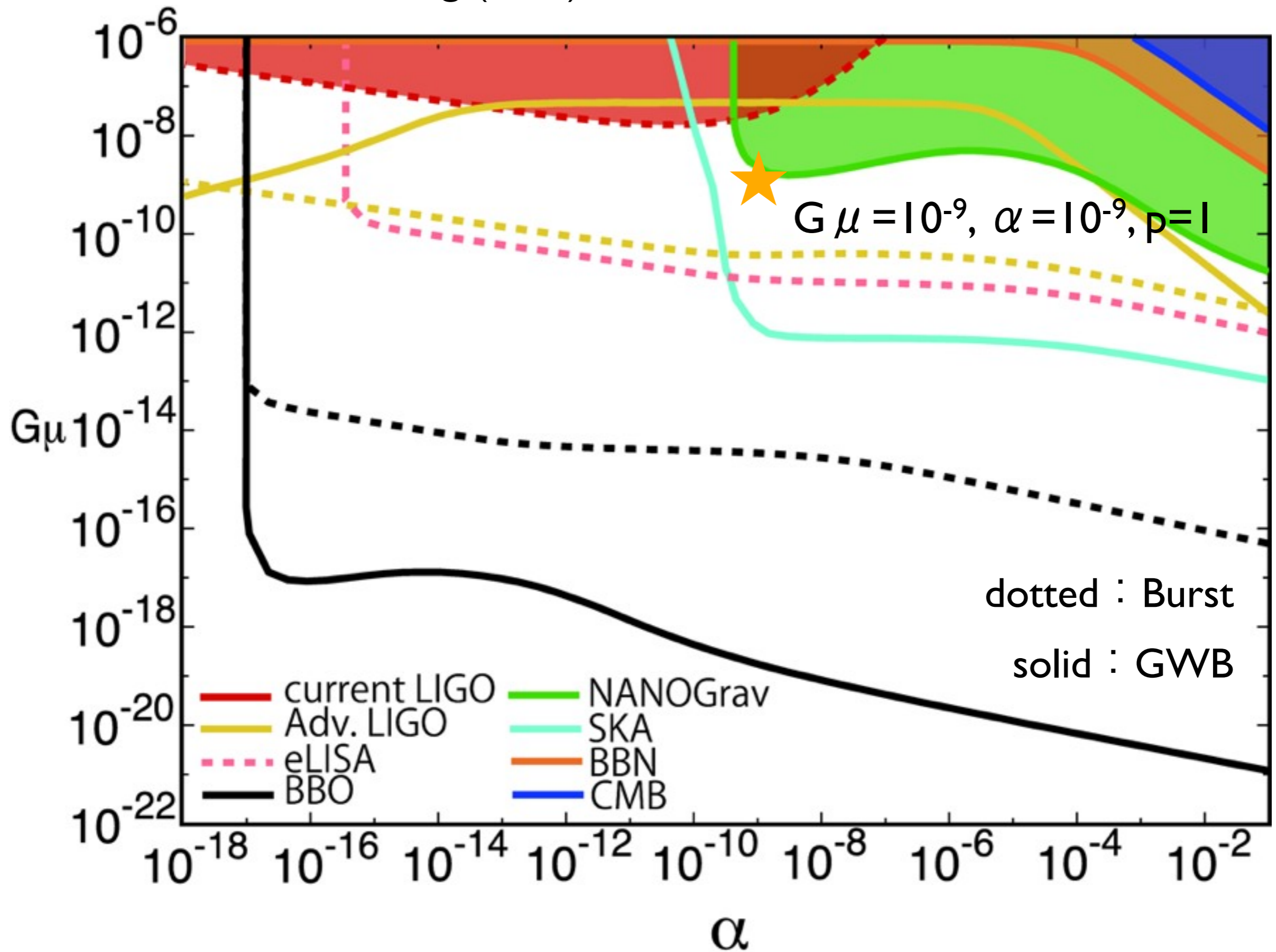
$G\mu = 10^{-7}$, $\alpha = 10^{-16}$, $p=1$
LIGO 3year
+ CMB B-mode

Kuroyanagi et. al. in preparation

black : LIGO Burst only
red : LIGO Burst + GWB
blue: LIGO +Planck
green: LIGO+CMBpol
orange: CMB pol only



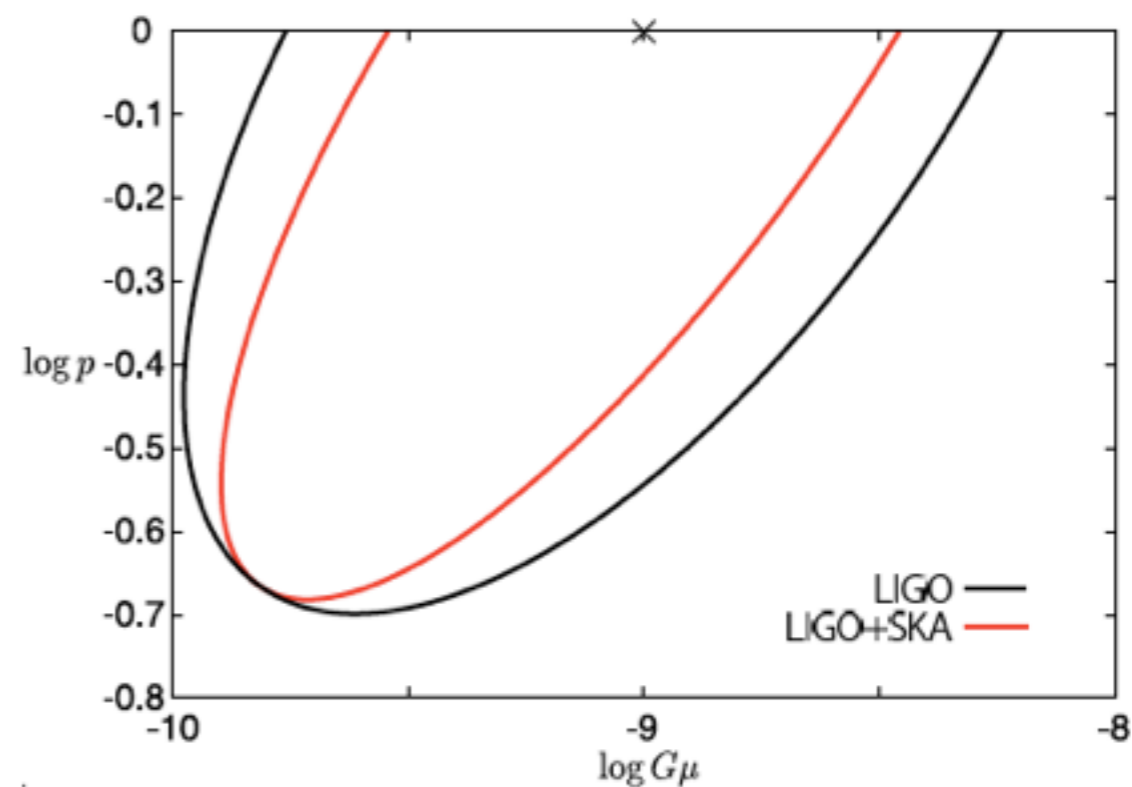
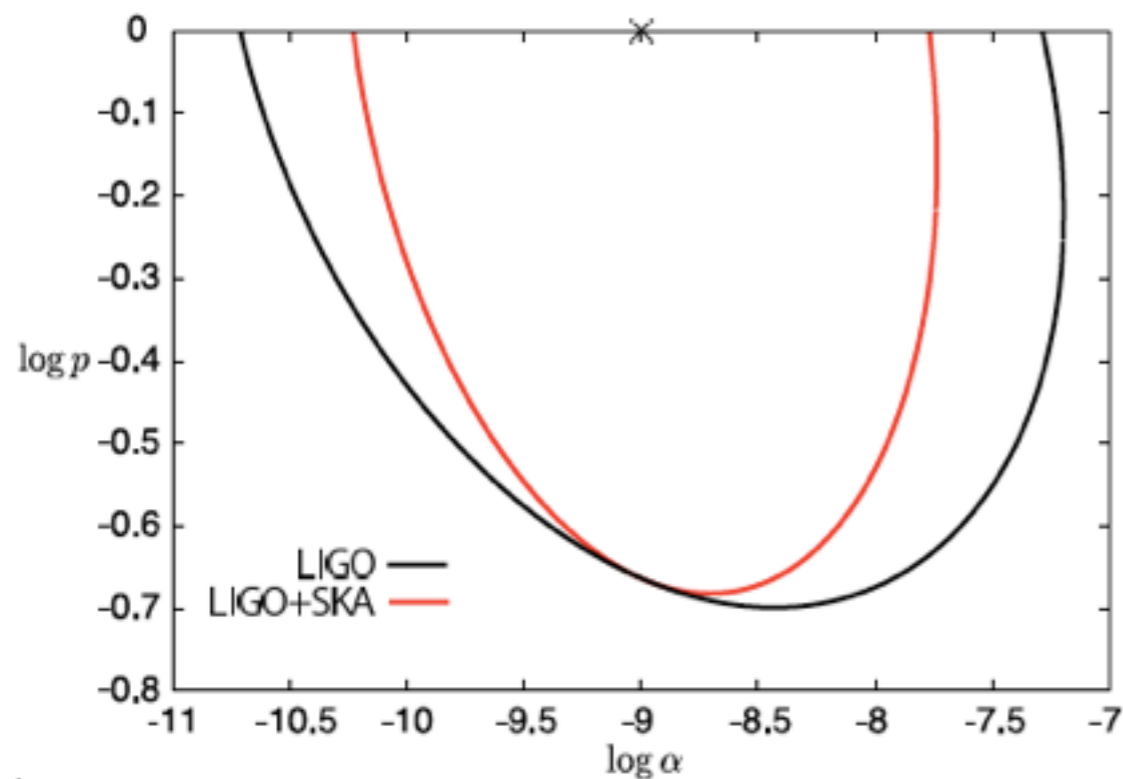
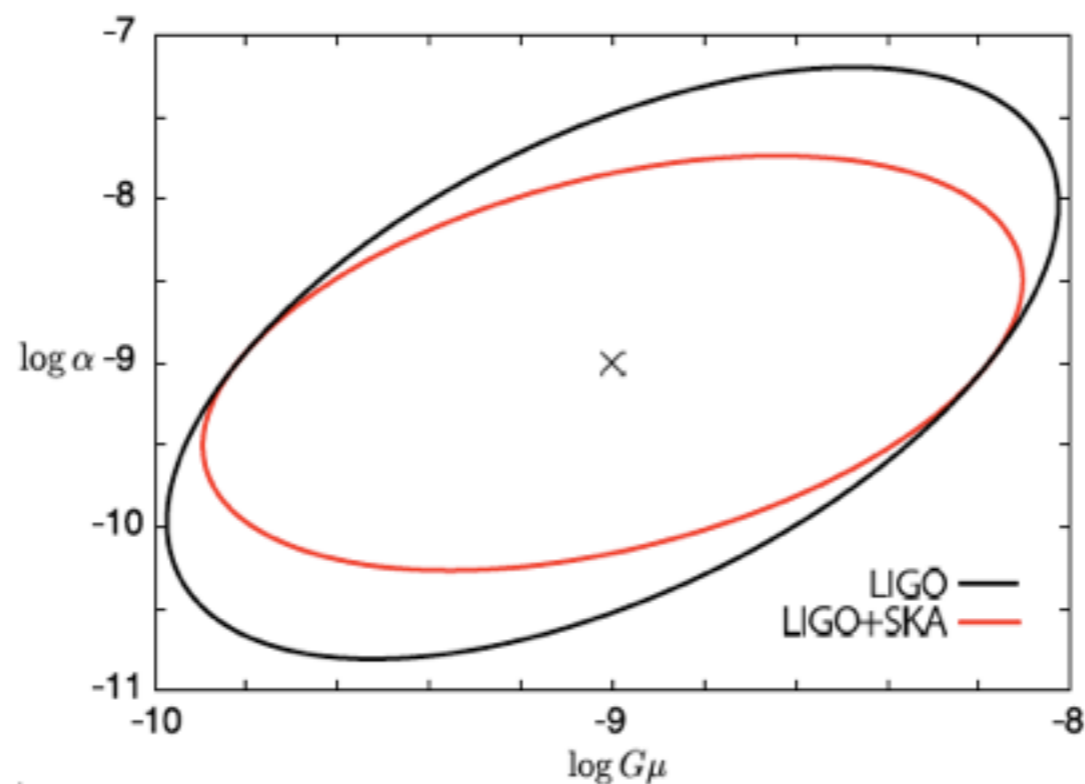
Pulsar timing (SKA) + Advanced-LIGO burst search



Direct detection + Pulsar timing

$G\mu = 10^{-9}$, $\alpha = 10^{-9}$, $p=1$
LIGO 3year (burst only)
+ SKA 10year

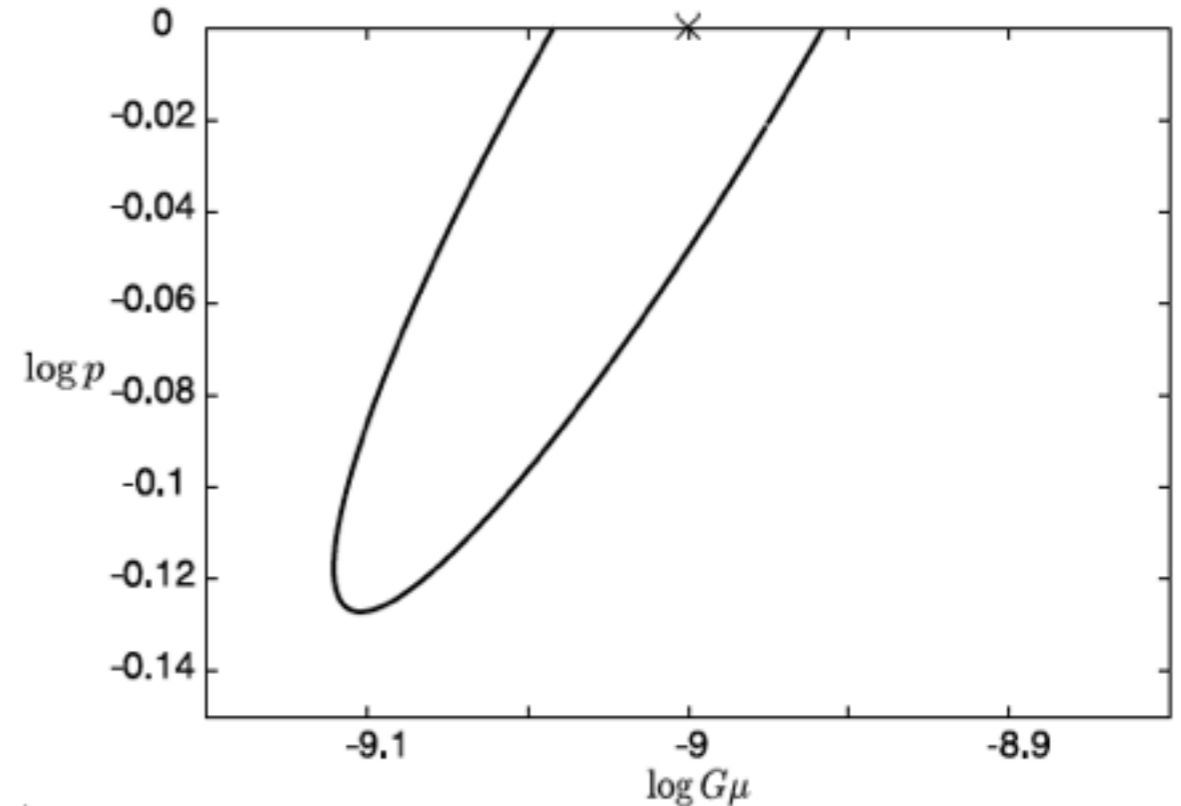
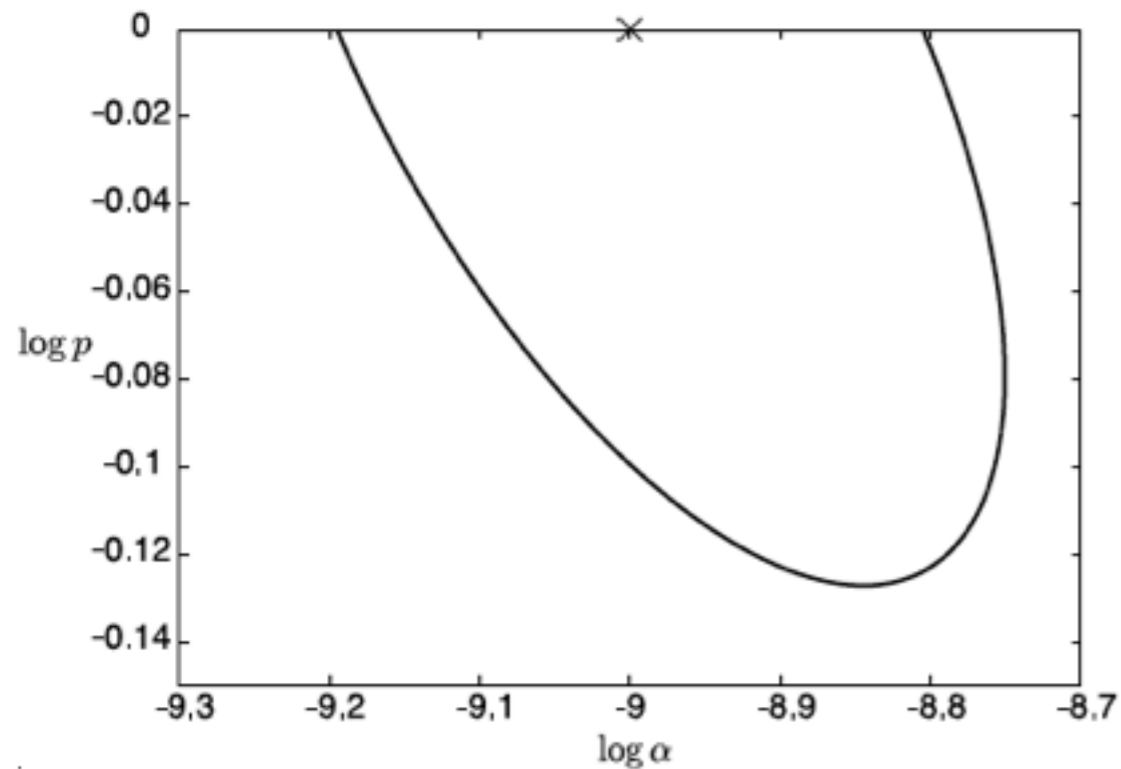
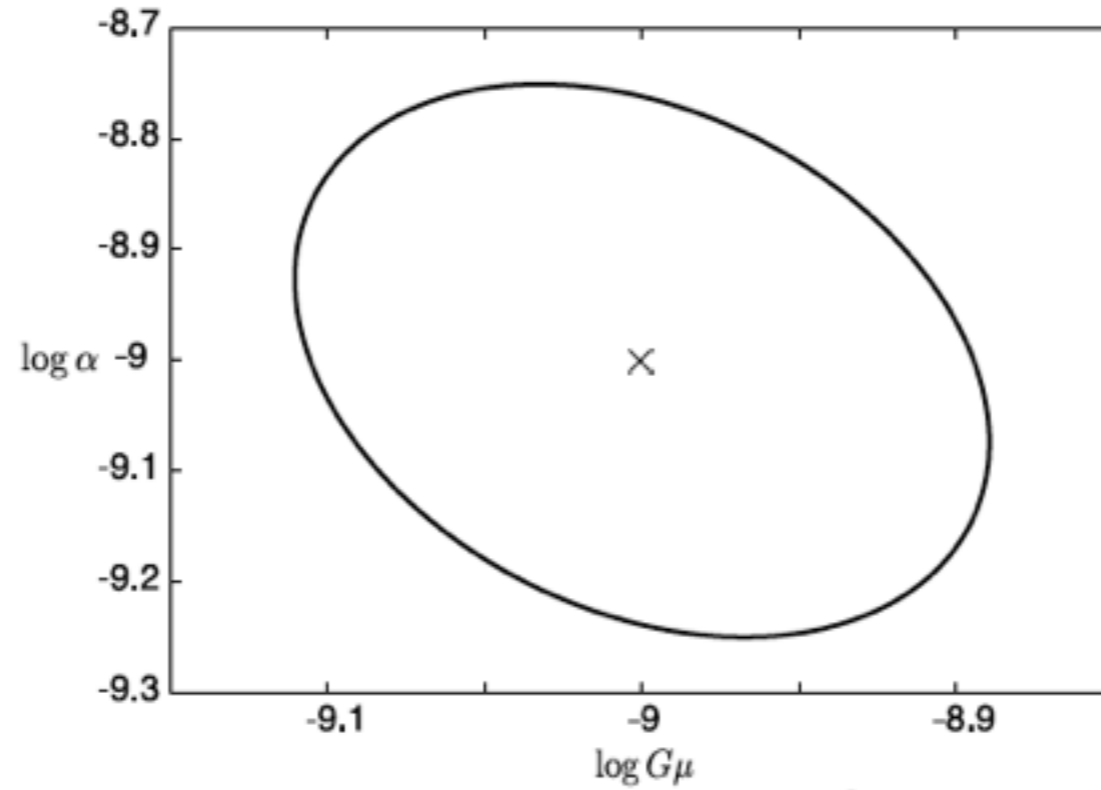
Kuroyanagi et. al. in preparation



Parameter constraint by eLISA

$G\mu = 10^{-9}$, $\alpha = 10^{-9}$, $p=1$
eLISA 3year
(burst only)

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Summary

- Future GW experiments can be a powerful tool to prove cosmic strings.
- It could provide strong constraints on cosmic string parameters. If it is detected, it would determine cosmic string parameters, which can provide us with hints of fundamental physics such as particle physics or superstring theory.
- Two different kinds of GW observation (rare burst and GWB) provide different constraints on cosmic string parameters and lead to better accuracy in determining parameters.
- Combination with CMB or Pulsar timing also helps to get stronger constraints, depending on the value of the parameters.
- Space GW missions are more powerful to prove cosmic strings.