Effects of Non-adiabatic Cosmological Perturbations

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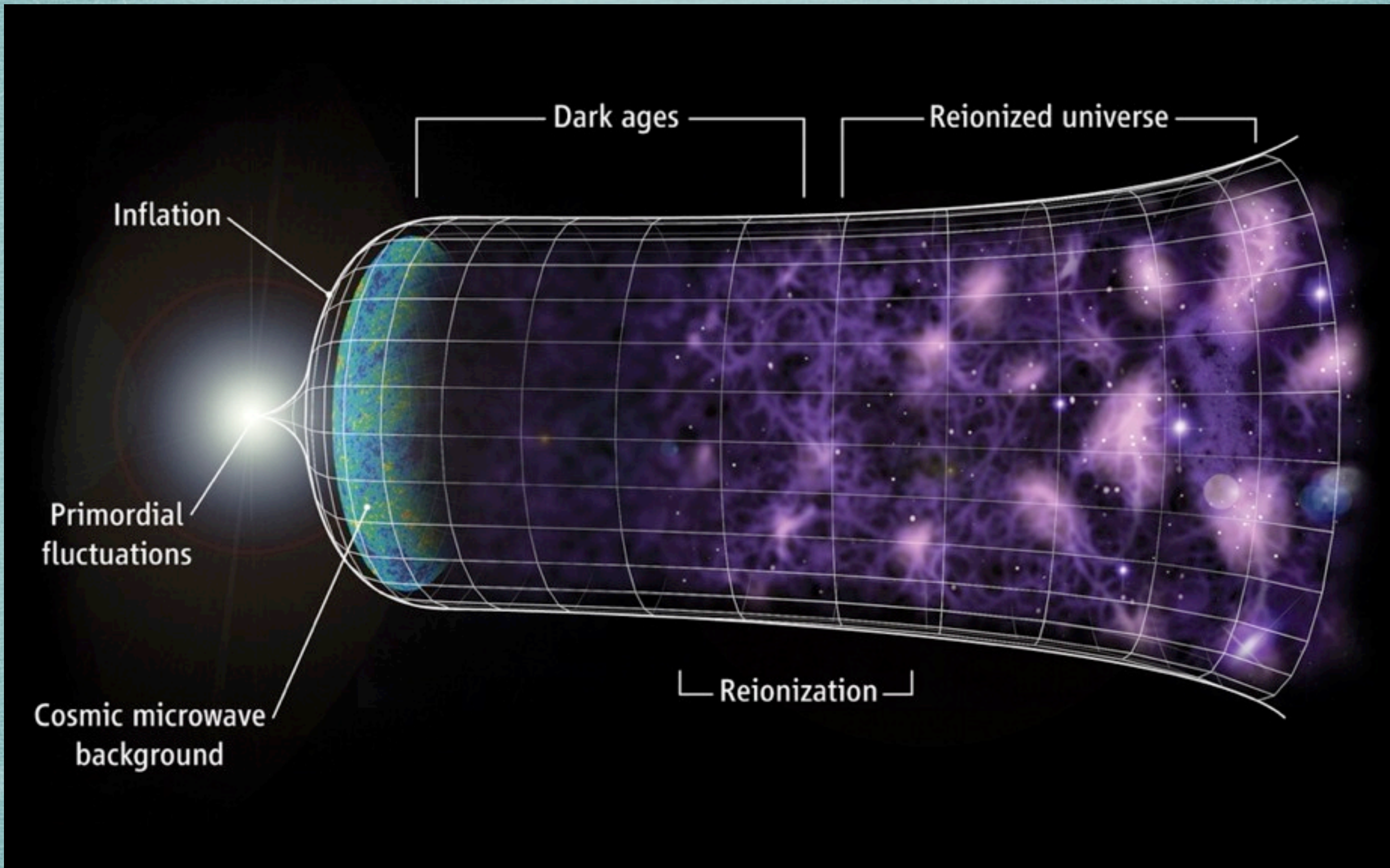
Outline

* Introduction and motivation
* Cosmological perturbation theory
* Non-adiabatic pressure perturbation:
  - in the concordance cosmology
  - in multi-field inflationary systems
* Application: vorticity beyond linear order
* Summary and conclusions

Evolution of the Universe



Dynamics of the universe

- On large scales, universe appears homogeneous and isotropic
- This is an approximation: there exists structure (galaxies, stars, etc..), and CMB anisotropies
- Dynamics of the universe governed by General Relativity
- How to proceed?
  - Fully inhomogeneous solution (*extremely* difficult in principle; impossible in practice?)
  - Make an approximation and expand around a homogeneous solution: **Cosmological Perturbation Theory**

Cosmological perturbation theory

Basic idea: expand around exact homogeneous solution

- Geometry: $$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

  where $g^{(0)}_{\mu\nu}$ is Friedmann-Lemaître-Robertson-Walker spacetime

- Matter: $$\rho(x^i, t) = \rho_0(t) + \delta\rho(x^i, t)$$

- Perturbations are then expanded in a series as

$$\delta\rho(x^i, t) = \sum_n \epsilon^n \frac{\delta\rho_n}{n!} = \delta\rho_1 + \frac{1}{2}\delta\rho_2 + \frac{1}{3!}\delta\rho_3 + \cdots$$

Linear perturbation theory

Truncate the expansion after the first term. The most general linear scalar, vector and tensor perturbations to FLRW are

$$ds^2 = a^2(\eta)\Big[-(1+2\phi_1)d\eta^2 + 2(B_{1,i} - S_{1i})dx^i d\eta + \{(1-2\psi_1)\delta_{ij} + 2E_{1,ij} + 2F_{1(i,j)} + h_{1ij})\}dx^i dx^j \Big]$$

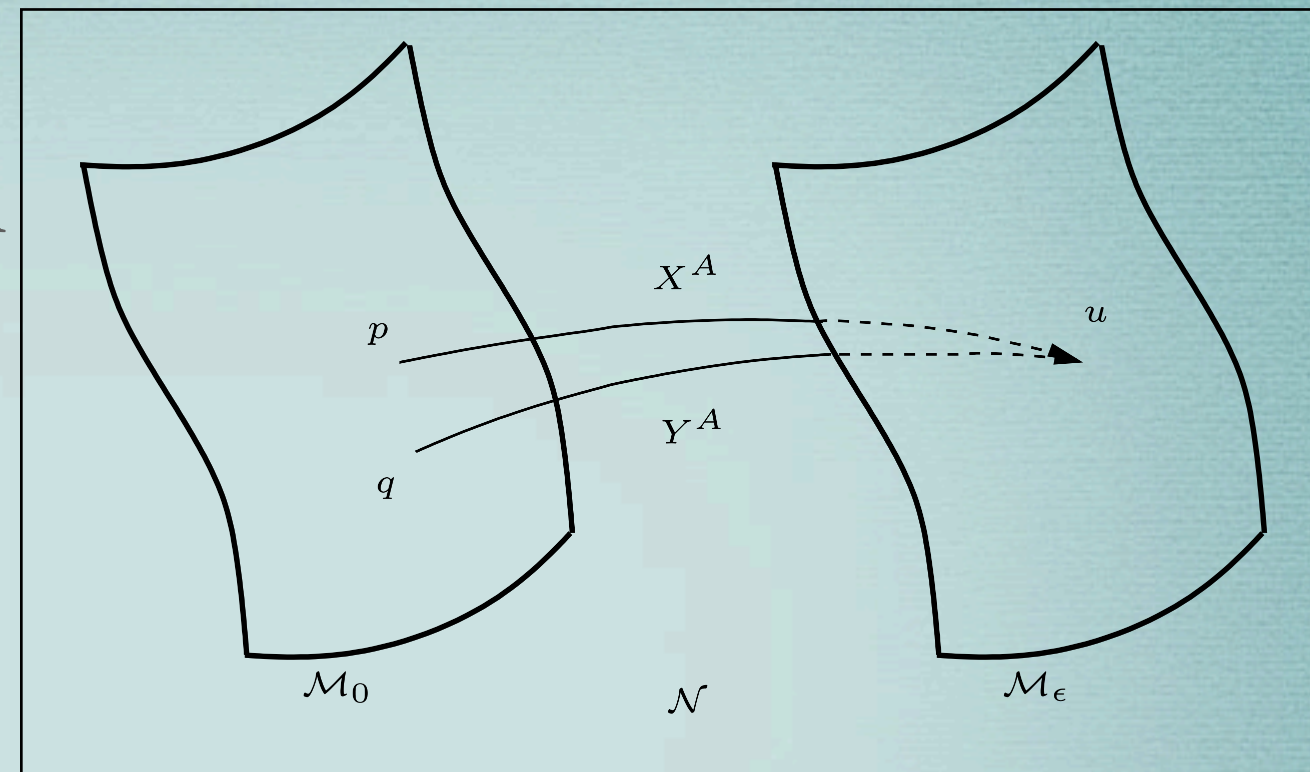
Bardeen (1980), Kodama & Sasaki (1984), Stewart (1990)

- Scalars: ϕ_1 lapse, ψ_1 curvature perturbation, E_1 and B_1 shear
- Vectors, S_{1i} and F_{1i} are divergence-free
- Tensor h_{1ij} is trace-less and divergence-free

At first order scalars, vectors and tensors all decouple.

Covariance

- Splitting of spacetime into background and perturbation introduces spurious coordinate dependence
- GR is covariant and so these gauge modes are not physical



We can remove these gauge dependencies by inspecting the transformation behaviour of quantities and then constructing gauge invariant variables so that the gauge artefacts cancel.

Gauge invariant variables

Consider behaviour under gauge transformation

$$x^\mu \to \widetilde{x^\mu} = x^\mu + \xi_1^\mu \qquad\qquad \xi_1^\mu = (\delta\eta, \delta x^i)$$

* Energy density transforms as

$$\widetilde{\delta\rho_1} = \delta\rho_1 + \rho_0' \delta\eta_1$$

* Curvature perturbations transforms as

$$\widetilde{\psi_1} = \psi_1 - \mathcal{H}\delta\eta_1$$

* Combine the two to get a gauge invariant variable, e.g.,

$$-\zeta_1 \equiv \widetilde{\psi_1}\Big|_{\delta\rho} = \psi_1 + \mathcal{H}\frac{\delta\rho}{\rho_0'}$$

Bardeen (1980), Bardeen, Steinhardt, Turner (1983)

Gauge choice

Equivalently, choose a gauge; popular choices (scalars only):

- Flat gauge $(E_1 = \psi_1 = 0)$
- Longitudinal gauge $(E_1 = B_1 = 0)$
  scalar metric perturbations are the Bardeen potentials $\Phi,\ \Psi$
- Synchronous gauge $(\phi_1 = B_1 = 0)$
- Uniform density gauge $(\delta\rho_1 = E_1 = 0)$
- Comoving gauge $(v_1 = 0 \Rightarrow V_1 \equiv v_1 + B_1 = 0)$
  in scalar field systems,

$$V_1 = -\frac{\delta\varphi_1}{\dot{\varphi}_0} \qquad \text{so} \qquad \delta\varphi_1 = 0$$

e.g. Malik & Wands (2009)

Governing equations

* Evolution equations from energy-momentum conservation

$$\delta\rho_1' + 3\mathcal{H}(\delta\rho_1 + \delta P_1) = (\rho_0 + P_0)(3\psi_1' - \nabla^2 E_1' - v_{1i,}{}^{i})$$

$$V_{1i}' + \mathcal{H}(1 - c_{\rm s}^2)V_{1i} + \left[\frac{\delta P_1}{\rho_0 + P_0} + \phi_1\right]_{,i} = 0$$

* Constraints from field equations

$$3\mathcal{H}(\psi_1' + \mathcal{H}\phi_1) - \nabla^2(\psi_1 + \mathcal{H}E_1') + \mathcal{H}\nabla^2 B_1 = -4\pi G a^2 \delta\rho_1$$

$$\psi_{1,i}' + \frac{1}{4}(\nabla^2 B_{1i} - B^k_{1,ki}) + \mathcal{H}\phi_{1,i} = -4\pi G a^2(\rho_0 + P_0)V_{1i}$$

Extension to second order

* Truncate perturbative expansions after the second term
* Gives, e.g., energy conservation (flat gauge, no tensors)

$$\delta\rho_2' + 3\mathcal{H}(\delta\rho_2 + \delta P_2) + (\rho_0 + P_0) v_{2k,}{}^{k} + 2\Big[(\delta\rho_1 + \delta P_1) v_1^k\Big]_{,k}$$

$$+ 2(\rho_0 + P_0)\Big[(V_1^k + v_1^k)V_{1k}' + v_1^k{}_{,k}\phi_1 + 2v_1^k\phi_{1,k} + 4\mathcal{H}v_1^k(V_{1k} + v_{1k})\Big] = 0$$

cf. first order

$$\delta\rho_1' + 3\mathcal{H}(\delta\rho_1 + \delta P_1) + (\rho_0 + P_0) v_{1i,}{}^{i} = 0$$

* Important difference: beyond linear order, perturbations no longer decouple.

Non-adiabatic pressure perturbation

Introducing the non-adiabatic pressure, for a single fluid

- equation of state $P \equiv P(\rho, S)$, expand to get

$$\delta P = \left.\frac{\partial P}{\partial S}\right|_{\rho} \delta S + \left.\frac{\partial P}{\partial \rho}\right|_{S} \delta \rho$$

e.g. AJC & Malik (2008)

- or rewriting, $\delta P = \delta P_{\rm nad} + c_{\rm s}^2 \delta \rho$

  where

$$\delta P_{\rm nad} = \left.\frac{\partial P}{\partial S}\right|_{\rho} \delta S \quad \text{and} \quad c_{\rm s}^2 = \left.\frac{\partial P}{\partial \rho}\right|_{S} \Rightarrow c_{\rm s}^2 = \frac{\dot{P}_0}{\dot{\rho}_0}$$

NB. barotropic fluid, $P \equiv P(\rho)$, has zero non-adiabatic pressure

For multiple fluids, expand non-adiabatic pressure as

$$\delta P_{\rm nad} \equiv \delta P_{\rm intr} + \delta P_{\rm rel}$$

Kodama & Sasaki (1984)

Intrinsic non-adiabatic pressure

Relative non-adiabatic pressure

* the intrinsic part is then

$$\delta P_{\rm intr} = \sum_{\alpha} \delta P_{{\rm intr},\alpha}$$

where the intrinsic perturbation of each fluid, α, is

$$\delta P_{{\rm intr},\alpha} \equiv \delta P_\alpha - c_\alpha^2 \delta\rho_\alpha$$

* This term is zero for barotropic fluids, or for scalar fields on superhorizon scales.

* the relative entropy perturbation between two fluids is

$$S_{\alpha\beta} = -3H\left(\frac{\delta\rho_\alpha}{\dot{\rho}_\alpha} - \frac{\delta\rho_\beta}{\dot{\rho}_\beta}\right)$$

* this gives

$$\delta P_{\rm rel} = -\frac{1}{6H\dot{\rho}}\sum_{\alpha,\beta}\dot{\rho}_\alpha\dot{\rho}_\beta\left(c_\alpha^2 - c_\beta^2\right)S_{\alpha\beta}$$

$$= \frac{1}{2\dot{\rho}}\sum_{\alpha,\beta}\left(c_\alpha^2 - c_\beta^2\right)\left(\dot{\rho}_\beta\delta\rho_\alpha - \dot{\rho}_\alpha\delta\rho_\beta\right)$$

Note that $S_{\alpha\beta}$, and the non-adiabatic pressure, are gauge invariant, so cannot be 'gauged away'.

Non-adiabatic pressure...

* Should emphasise that single (barotropic) fluid systems have zero non-adiabatic pressure
  - single scalar field, in superhorizon limit can be treated as a barotopic fluid
* Focus on relative entropy/non-adiabatic pressure perturbation
* Study:
  - relative entropy between fluids in the usual cosmic fluid (i.e. baryons, cold dark matter, radiation, neutrinos ...)
  - isocurvature perturbations in multi-field inflation model

… in concordance cosmology

- baryons, CDM have $w_b = w_c = c_b^2 = c_c^2 = 0$
- photons, neutrinos are relativistic: $w_\gamma = w_\nu = c_\gamma^2 = c_\nu^2 = \frac{1}{3}$
- use WMAP7 parameters

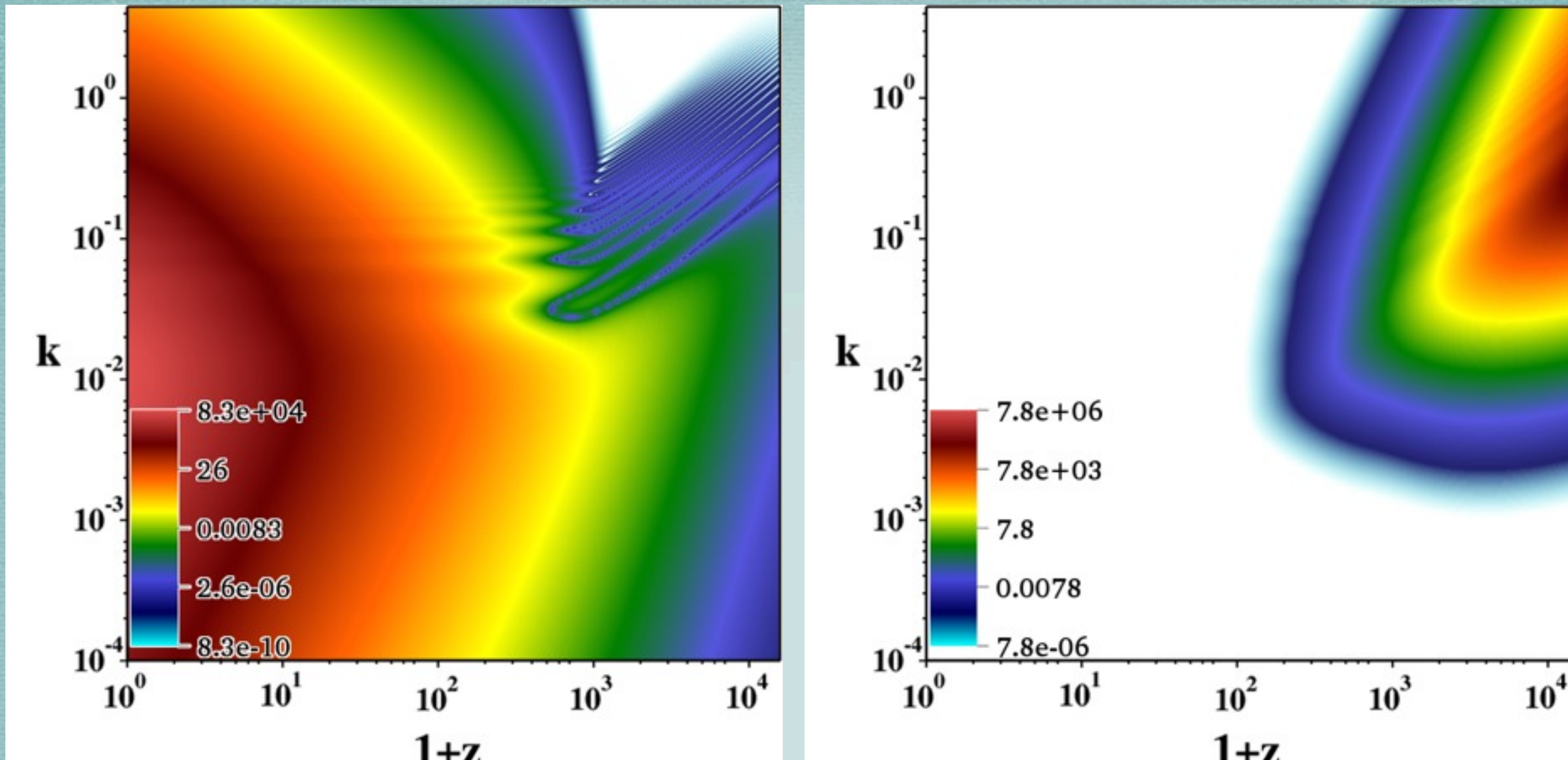
$$\Omega_b h^2 = 2.253 \times 10^{-2},\ \Omega_c h^2 = 0.112,\ \Omega_\Lambda = 0.728,\ h = 0.704$$

- adiabatic initial conditions

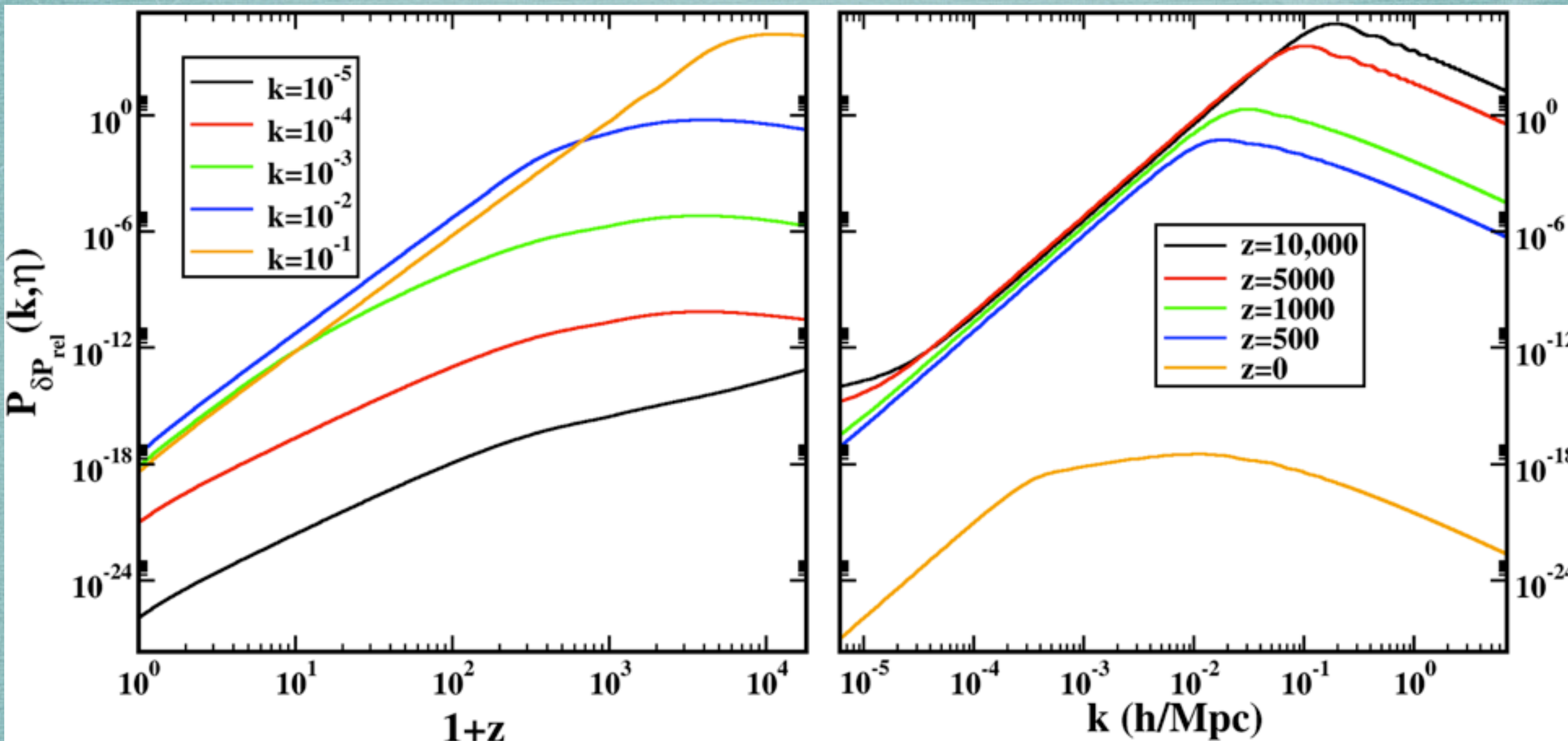
$$\delta_\gamma = \delta_\nu = \frac{4}{3}\delta_b = \frac{4}{3}\delta_c = -\frac{2}{3}Ck^2\eta_i^2$$

i.e. $S_{\alpha\beta} = 0$

- solve using a modified version of CMBFast



Power spectra of baryon density contrast $P_b(k,\eta)$ (left) and the non-adiabatic pressure perturbation $P_{\delta P_{\rm rel}}(k,\eta)$ (right)



$P_{\delta P_{\rm rel}}(k, \eta)$ as a function of redshift for set wavenumber (left); and as a function of wavenumber for set redshift (right).

Brown, AJC & Malik (2011)

… in multi-field inflation

- Consider two field inflation models with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(\dot{\varphi}^2 + \dot{\chi}^2\right) + V(\varphi, \chi)$$

- To compare with comoving curvature perturbation

$$\mathcal{R} = \frac{H}{\dot{\varphi}^2 + \dot{\chi}^2}\left(\dot{\varphi}\delta\varphi + \dot{\chi}\delta\chi\right)$$

introduce

$$\mathcal{S} = \frac{H}{\dot{P}}\delta P_{\rm nad}$$

- Alternatively, field rotation

Gordon et. al, (2001)

$$\delta\sigma = \cos\theta\delta\varphi + \sin\theta\delta\chi\,,\ \delta s = -\sin\theta\delta\varphi + \cos\theta\delta\chi$$

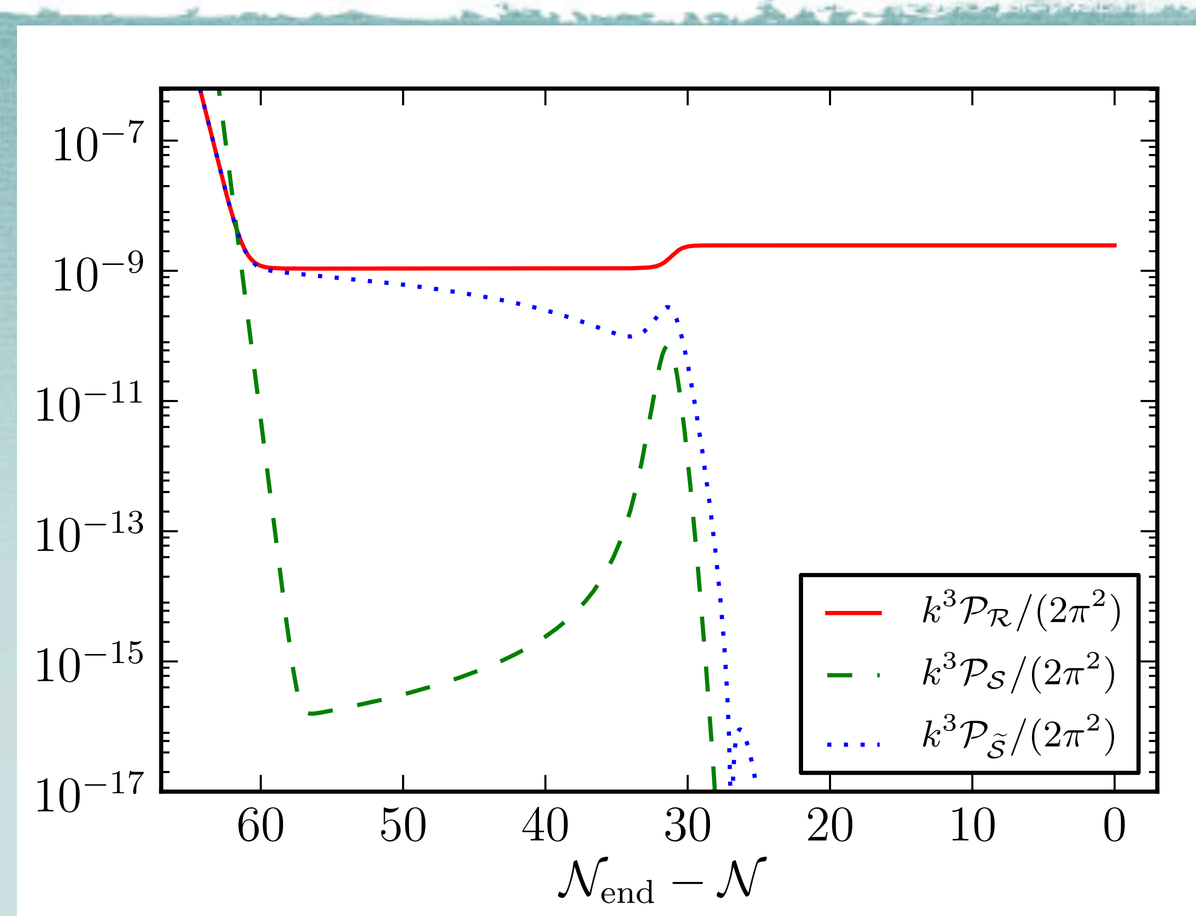
and then

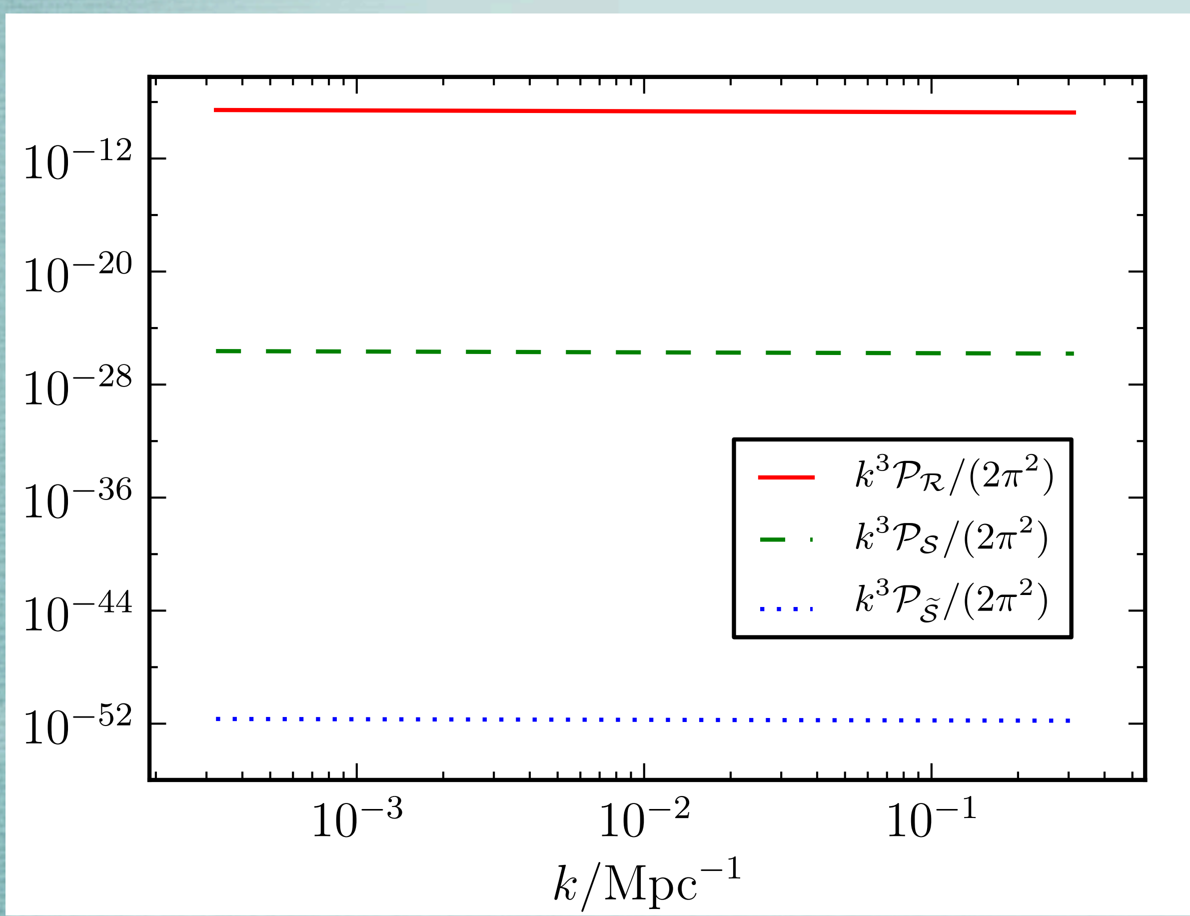
$$\widetilde{\mathcal{S}} = \frac{H}{\dot{\sigma}}\delta s \quad \Rightarrow \quad \widetilde{\mathcal{S}} \sim \mathcal{S}$$

in slow roll, large scale limit

Double quadratic inflation

$$V(\varphi, \chi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{2} m_\chi^2 \chi^2$$





$$m_\chi = 7 m_\varphi$$

$$m_\varphi = 1.395 \times 10^{-6} M_{\rm PL}$$

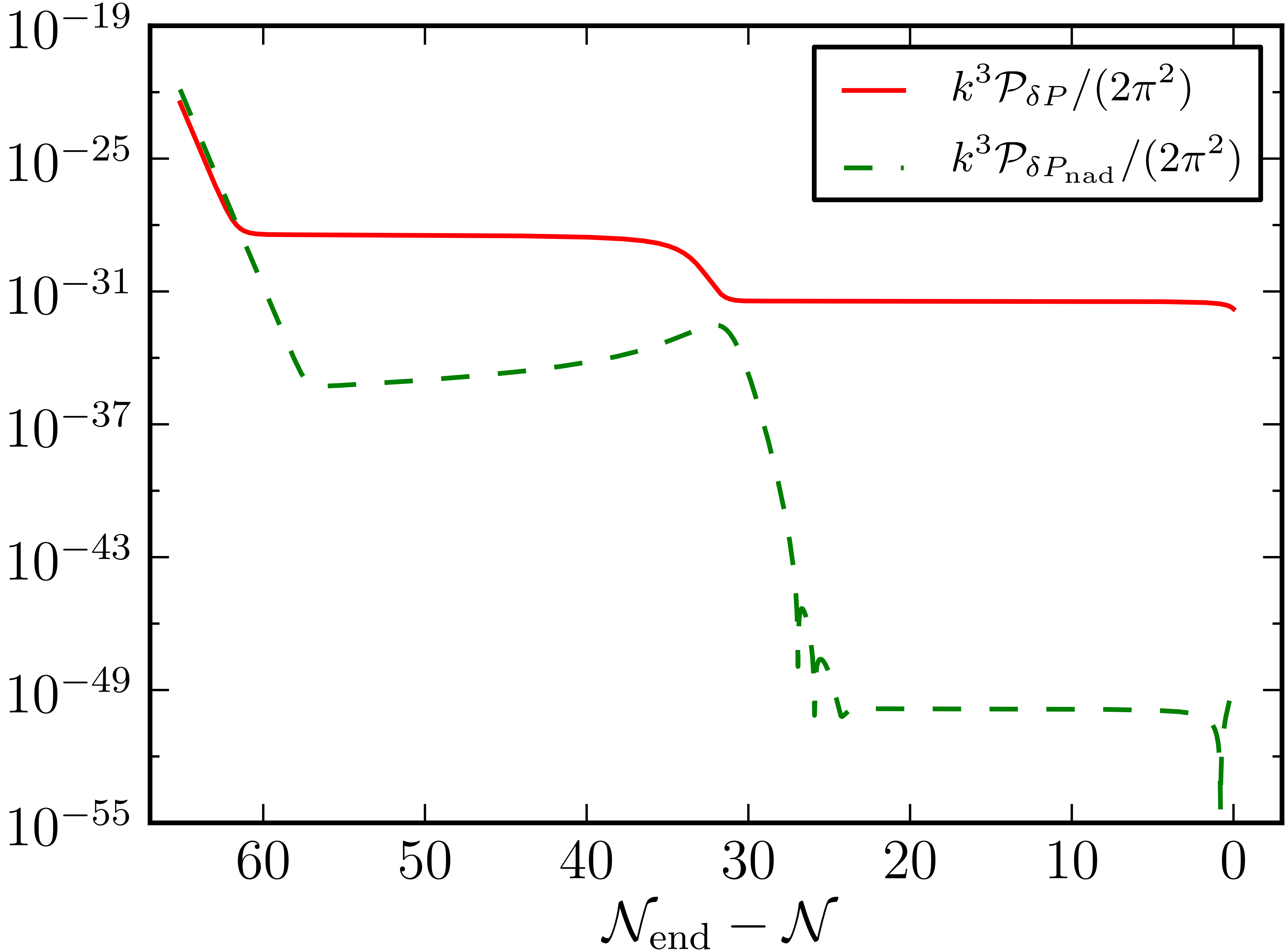
IC's: $\varphi_0 = \chi_0 = 12 M_{\rm PL}$

$\dot{\varphi}_0 , \dot{\chi}_0$ given slow roll values

$$n_{\mathcal{R}} \simeq 0.937$$

http://pyflation.ianhuston.net/

Huston & AJC (2011)

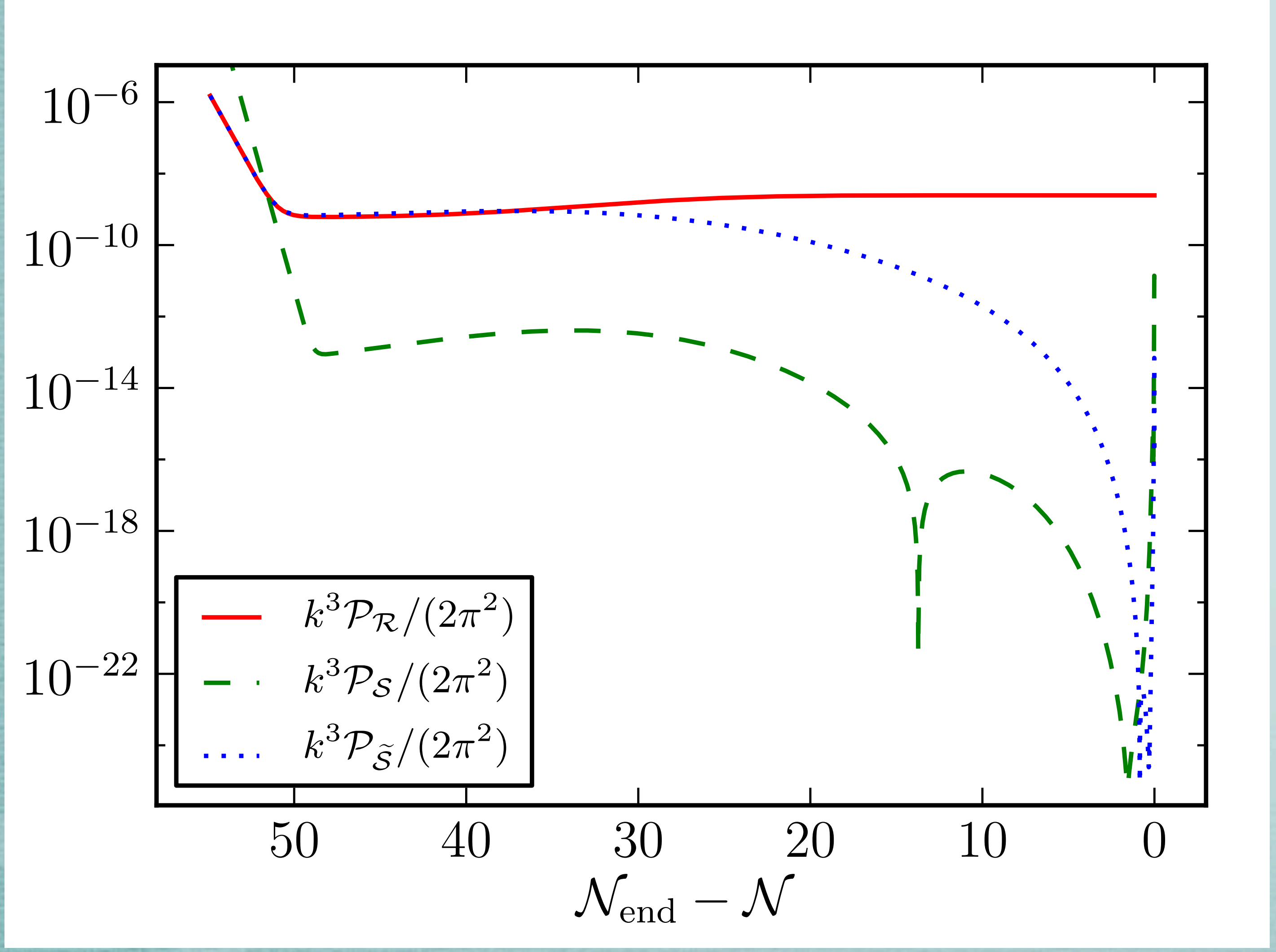


10^{-19}
10^{-25}
10^{-31}
10^{-37}
10^{-43}
10^{-49}
10^{-55}
60
50
40
30
20
10
0
$\mathcal{N}_{\rm end} - \mathcal{N}$
$k^3 \mathcal{P}_{\delta P}/(2\pi^2)$
$k^3 \mathcal{P}_{\delta P_{\rm nad}}/(2\pi^2)$

Double quartic inflation

$$V(\varphi,\chi) = \Lambda^4\left[\left(1-\frac{\chi_0^2}{v^2}\right)^2 + \frac{\varphi^2}{\mu^2} + \frac{2\varphi^2\chi^2}{\varphi_c^2 v^2}\right]$$

Avgoustidis et. al. (2011)



$$v = 0.10 M_{\rm PL}$$

$$\varphi_c = 0.01 M_{\rm PL}$$

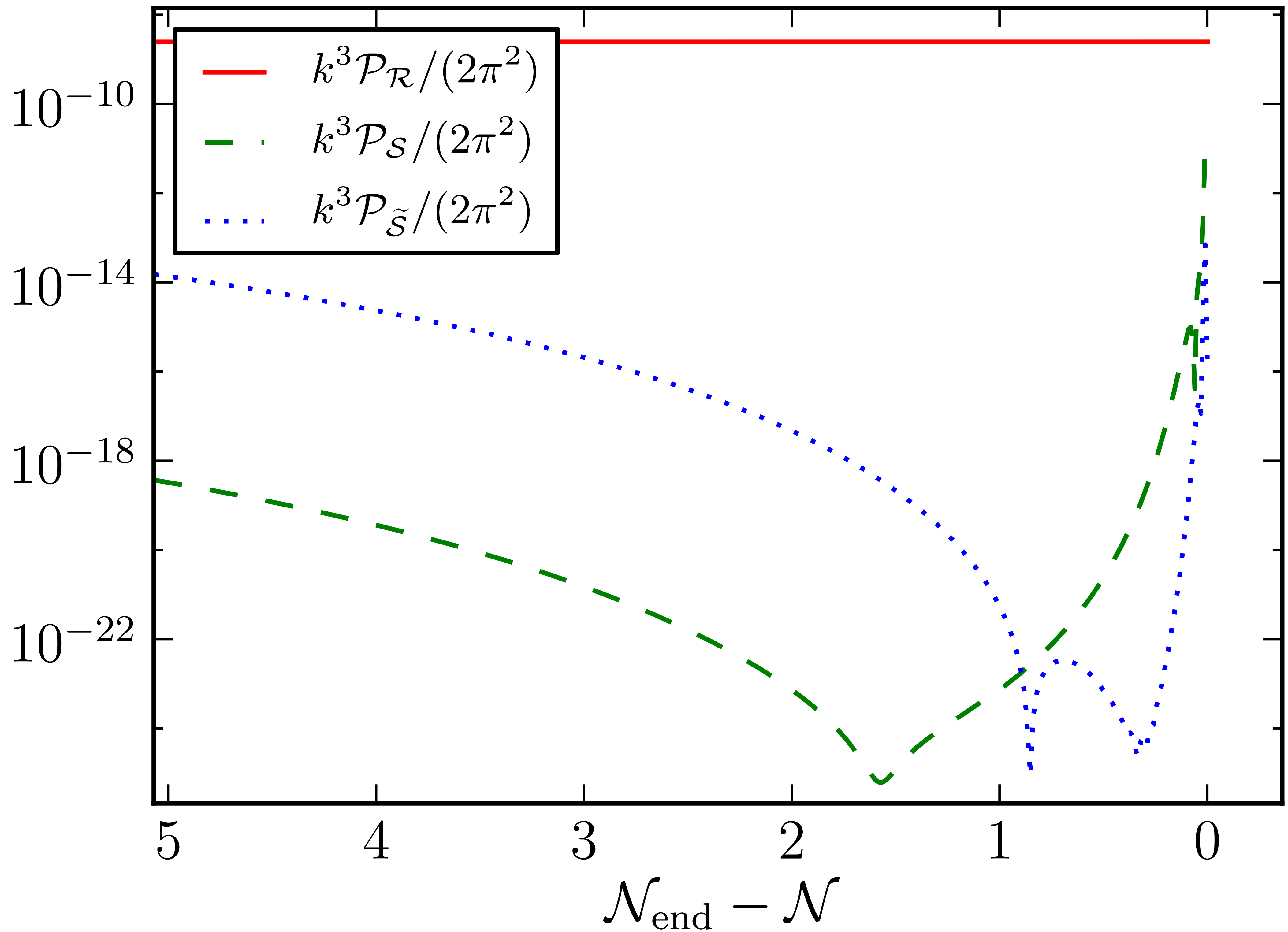
$$\mu = 10^3 M_{\rm PL}$$

IC's: $\varphi_0 = 0.01 M_{\rm PL}$

$$\chi_0 = 1.63 \times 10^{-9} M_{\rm PL}$$

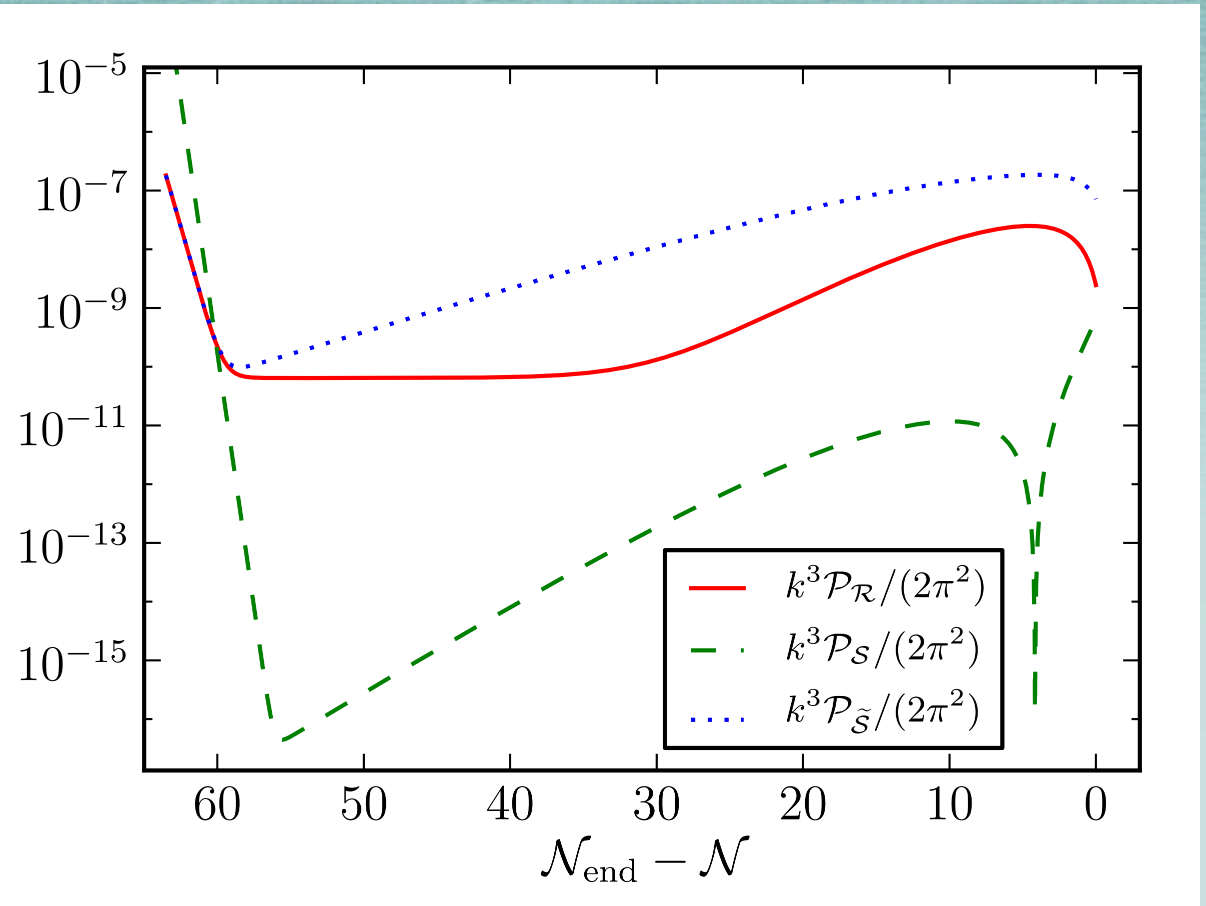
$\dot{\varphi}_0\,,\,\dot{\chi}_0$ given slow roll values

$$n_\mathcal{R} = 0.932$$



$k^3\mathcal{P}_{\mathcal{R}}/(2\pi^2)$
$k^3\mathcal{P}_{\mathcal{S}}/(2\pi^2)$
$k^3\mathcal{P}_{\tilde{\mathcal{S}}}/(2\pi^2)$
10^{-10}
10^{-14}
10^{-18}
10^{-22}
5
4
3
2
1
0
$\mathcal{N}_{\rm end} - \mathcal{N}$

Product exponential



$$V(\varphi,\chi) = V_0\varphi^2 e^{-\lambda\chi^2}$$

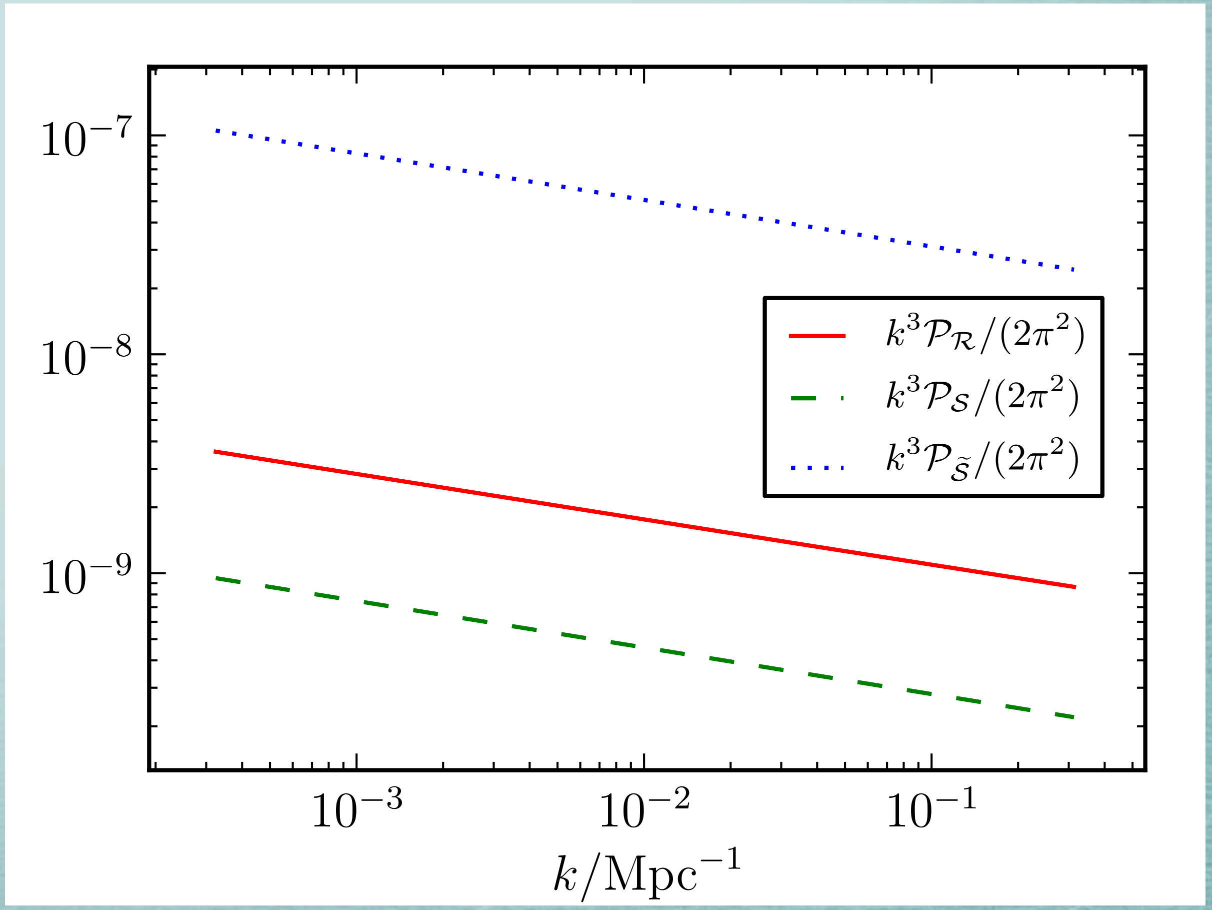
Byrnes , Choi & Hall (2008)

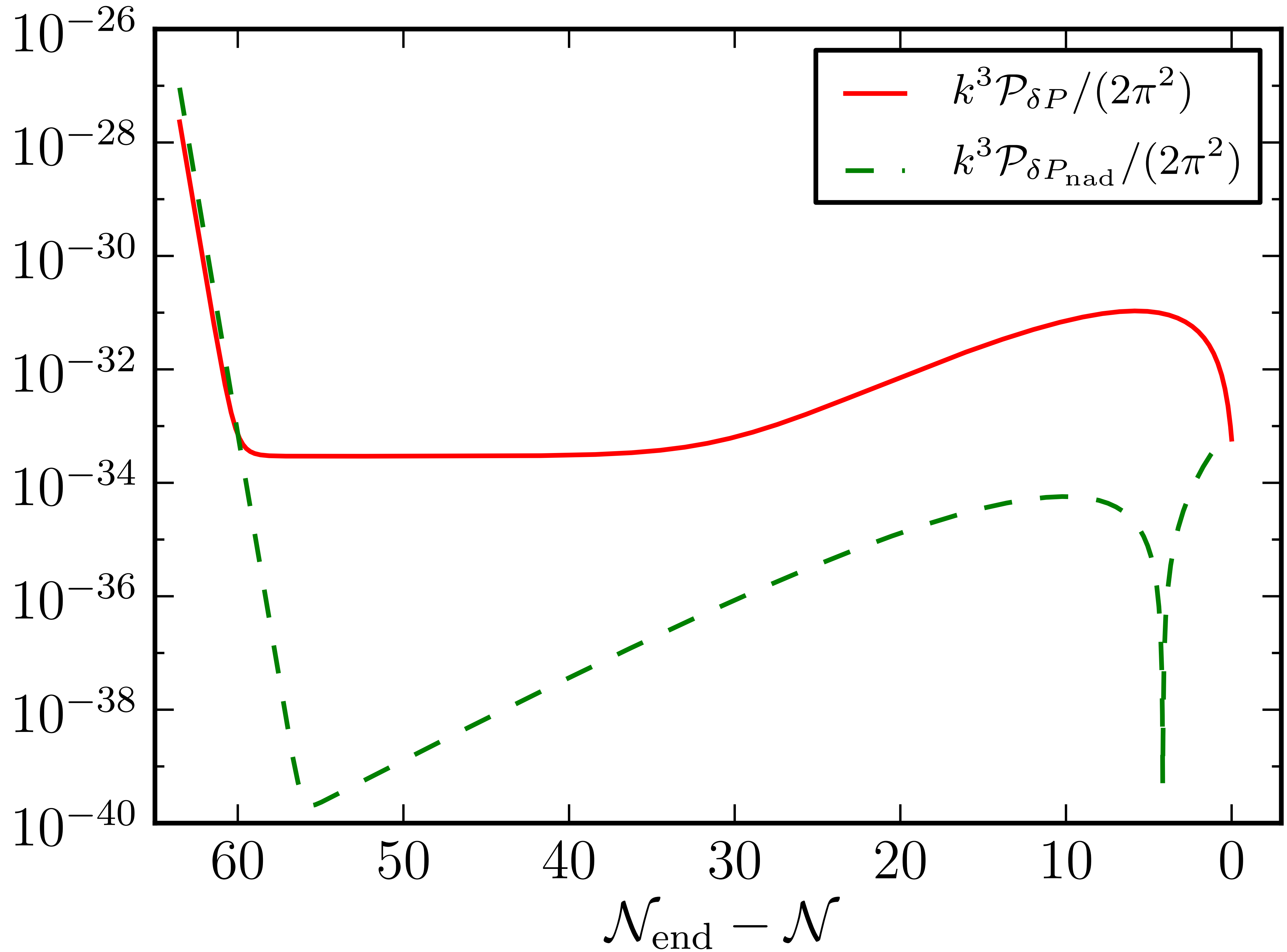
$$\lambda = 0.05 M_{\rm PL}^{-2}$$

WMAP: $V_0 = 5.37\times 10^{-13} M_{\rm PL}^2$

IC's: $\varphi_0 = 18 M_{\rm PL}$

$\chi_0 = 0.001 M_{\rm PL}$





$k^3\mathcal{P}_{\delta P}/(2\pi^2)$
$k^3\mathcal{P}_{\delta P_{\rm nad}}/(2\pi^2)$
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-34}
10^{-36}
10^{-38}
10^{-40}
60
50
40
30
20
10
0
$\mathcal{N}_{\rm end} - \mathcal{N}$

Application: vorticity

* Classical fluid dynamics $\boldsymbol{\omega} \equiv \nabla \times \boldsymbol{v}$
* Euler equation
$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla P$$
* Evolution:
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{1}{\rho^2}\nabla\rho \times \nabla P$$
   - ‘source’ term zero if ∇P and $\nabla\rho$ are parallel
   - i.e. barotropic fluid, no source term
* The inclusion of entropy provides a source for vorticity

Crocco (1937)

Vorticity in cosmology

* Define the vorticity tensor

$$\omega_{\mu\nu} = \mathcal{P}_\mu{}^\alpha \mathcal{P}_\nu{}^\beta u_{[\alpha;\beta]}$$

* projection tensor

$$\mathcal{P}_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

* expand order by order; u^μ fluid four velocity
* first order vorticity:

$$\omega_{1ij} = a(v_1 + B_1)_{[i,j]}$$

Vorticity evolution: first order

* First order vorticity evolves as

$$\omega'_{1ij} - 3\mathcal{H}c_{\rm s}^2\omega_{1ij} = 0$$

Kodama & Sasaki (1984)

* Reproduces well known result that, in radiation domination,

$$|\omega_{1ij}{\omega_1}^{ij}| \propto a^{-2}$$

* i.e. in absence of anisotropic stress, no source term: $\omega_{1ij} = 0$ is a solution to the evolution equation

Vorticity evolution: second order

* Second order vorticity, ω_{2ij}, evolves as

$$\omega'_{2ij} - 3\mathcal{H}c_{\rm s}^2\omega_{2ij} = \frac{2a}{\rho_0 + P_0}\left\{3\mathcal{H}V_{1[i}\delta P_{{\rm nad}1,j]} + \frac{\delta\rho_{1,[j}\delta P_{{\rm nad}1,i]}}{\rho_0 + P_0}\right\}$$

assuming zero first order vorticity.

* For vanishing non-adiabatic pressure, vorticity decays as at first order

Lu et. al. (2009)

* Including entropy gives a non-zero source term

AJC, Malik & Matravers (2009)

* This generalises Crocco's theorem to an expanding framework

'Estimating' the power spectrum

* Work in radiation era, and define the power spectrum as

$$\langle \omega_2^*(\boldsymbol{k}_1, \eta)\omega_2(\boldsymbol{k}_2, \eta)\rangle = \frac{2\pi}{k^3}\delta(\boldsymbol{k}_1 - \boldsymbol{k}_2)\mathcal{P}_\omega(k, \eta)$$

* For the inputs:
  - Can solve linear equation for $\delta\rho_1$; leading order for small $k\eta$

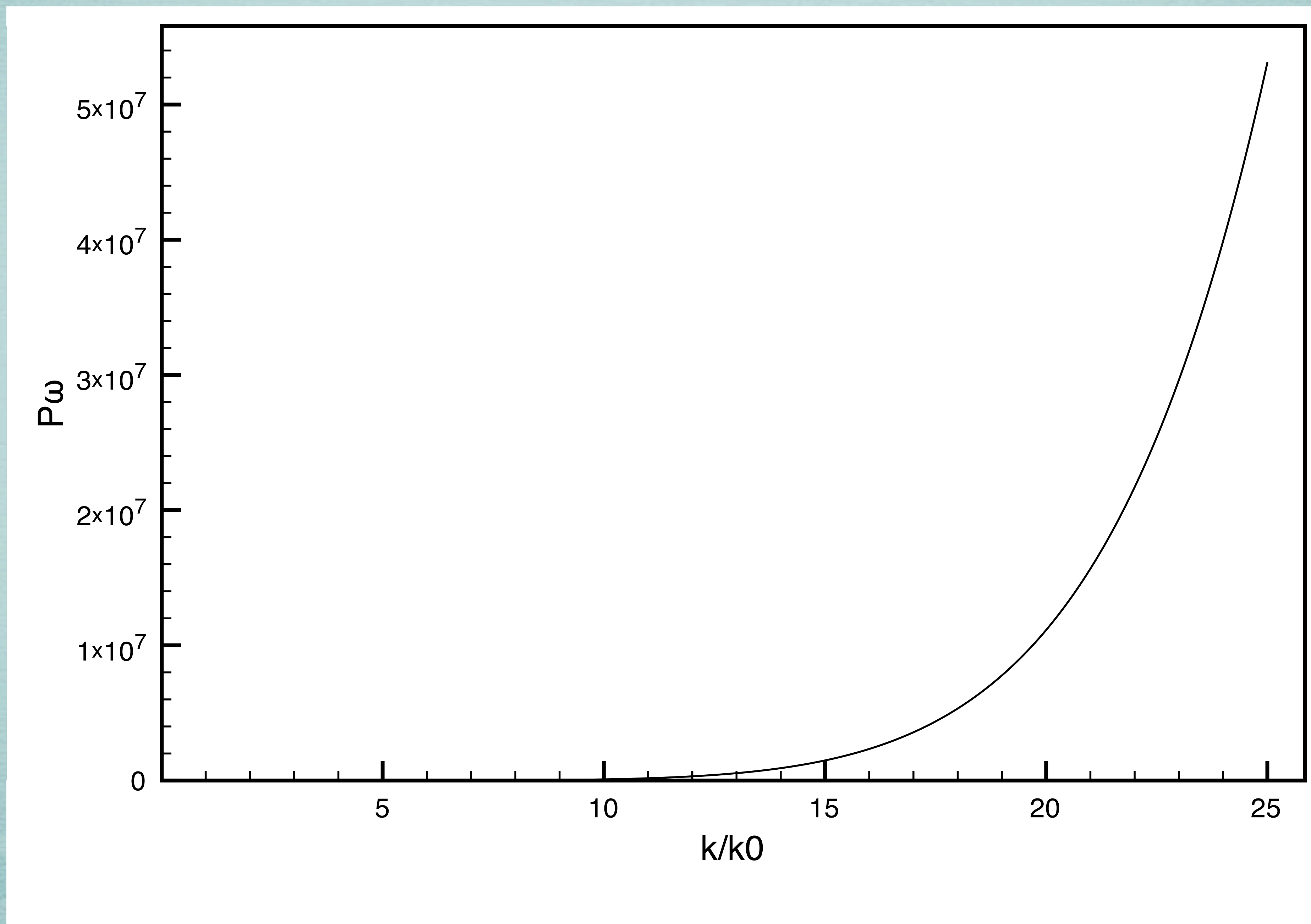
$$\delta\rho_1(k, \eta) = A\left(\frac{k}{k_0}\right)\left(\frac{\eta}{\eta_0}\right)^{-4}$$

  - 'Ansatz' for non-adiabatic pressure

$$\delta P_{\mathrm{nad1}}(k, \eta) = D\left(\frac{k}{k_0}\right)^2\left(\frac{\eta}{\eta_0}\right)^{-5}$$

* These give the spectrum

$$\frac{\mathcal{P}_\omega}{\mathrm{Mpc}^4} \sim 0.87 \times 10^{-2} \left(\frac{k}{k_0}\right)^7 + 3.73 \times 10^{-11} \left(\frac{k}{k_0}\right)^9 - 7.71 \times 10^{-20} \left(\frac{k}{k_0}\right)^{11}$$



AJC, Malik & Matravers (2010)

Observational signatures

* For linear perturbations, B mode polarisation of the CMB only produced by tensor perturbations:
  - scalar perturbations only produce E mode polarisation
  - vectors produce B modes, but decay with expansion
* Second order, vector perturbations produced by first order density and entropy perturbations source B mode polarisation
* Important for current and future CMB polarisation expts
* Could prove important for studying physics of primordial magnetic fields

Fenu et. al. (2011)

Summary

* Non-adiabatic pressure perturbations arise naturally in systems containing more than one fluid/field
  - cosmic fluid containing relativistic/non-relativistic matter
  - multi-field inflationary models
* Vorticity generated at second order in perturbation theory from entropy perturbations
* Future work: consider vorticity spectrum from
  - cosmic fluid — Brown, AJC & Malik (in progress)
  - inflation — Alabidi, AJC & Huston (in progress)

References

Non-adiabatic pressure:

- AJC & Malik, `0809.3518` (general single field)
- Brown, AJC & Malik, `1108.0639` (cosmic fluid)
- Huston & AJC, `1111.6919` (multi-field inflation)

Vorticity:

- AJC, Malik & Matravers, `0904.0940`
- AJC, Malik & Matravers, `1008.4866` (estimate of spectrum)