Rolling tachyon in a separated $D - \bar{D}$ system

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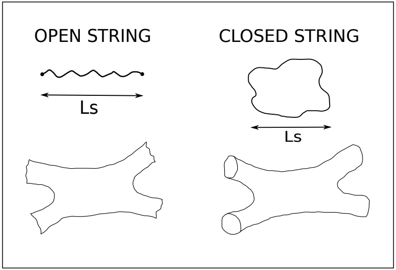
Introducing strings



$$\ell_s = \ell_p \approx 10^{-33} cm$$

Two principal theories

- Bosonic : only bosons
- Supersymmetric : boson and fermion superpartners

Bosonic open string Theory

- Worldsheet (strip) action :

$$S_p = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$$

  with $\mu = 0 \dots D-1$ and $\alpha' = \ell_s^2$
- Second Quantification of field X
  $\Rightarrow$ string mass spectrum :

$$\alpha' m^2 = N - 1$$

- Various space-time fields (N) :
  - tachyon (0) : scalar τ
  - massless (1) : vector A^μ
  - massive ...
- Space-time dimension : $D = 26$.

Supersymmetric string Theory

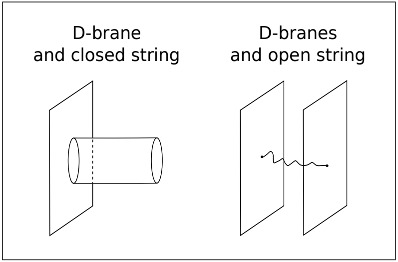
- Add fermion in the worldsheet action :

$$S_p = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{g} \left[g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + \frac{i}{2} \bar{\psi}^\mu \not{\partial} \psi^\nu G_{\mu\nu} \right]$$

- Get five (tachyon free) new theories with $D = 10$:

Type IIA and IIB	$\mathcal{N} = 2$ *non-chiral* and *chiral* $G_{(\mu\nu)}$, $B_{[\mu\nu]}$, Φ + C_p forms + fermion partners
Type I	$\mathcal{N} = 1$ SYM $SO(32)$ $A^\mu \in SO(32)$ + fermion partners
Heterotic $O(32)$	$(\mathcal{N} = 1) \times Ad(O(32))$ sugra + vector supermultiplet
Heterotic $E_8 \times E_8$	$(\mathcal{N} = 1) \times Ad(E_8 \times E_8)$ sugra + vector supermultiplet

Introducing branes

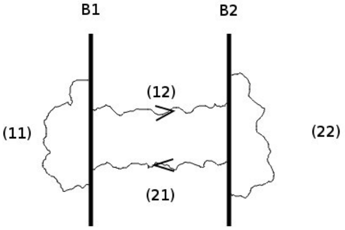


- Dp-Branes are $(p+1)$ dimensional hyperplanes in IIA, IIB and I theories.
  $\Rightarrow$ where open strings end points are attached.
- Dp-Branes emit and absorb closed string.
  $\Rightarrow$ naturally coupled to closed string fields.

Dp-Branes and gauge interaction

- Dp-Branes = charged objects $\Leftrightarrow$ sources for the related gauge (closed string) field C_{p+1}.
- The charge is given by the flux : $Q_p = \oint_{D_p} C_{(p+1)}$
- The anti-brane $\bar{D}p$ has opposite charge $\bar{Q}_p = -Q_p$.

Separated branes and open strings



Mass spectrums :

- in (11) and (22)

  $\Rightarrow \alpha' m^2 = N$

- in (12) and (21)

  $$\Rightarrow \alpha' m^2 = \frac{d^2}{4\pi^2} + N + \begin{cases} -\frac{1}{2} & D\bar{D} \\ 0 & DD \end{cases}$$

Sectors of open-strings

- 2 branes B_1 and B_2 spatially separated admit 4 open-string sectors.
- Sectors denote independent open string states.
  $\Rightarrow$ configurations of end points : (11), (22), (12) and (21).

Mode of string exchange between branes

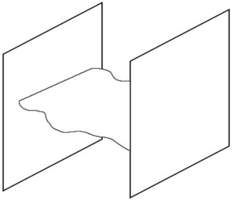


Figure: Open string (tree) channel when $m^2(open) \leq 0$

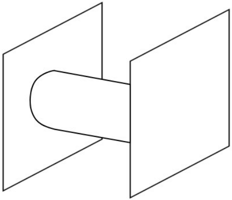


Figure: Closed string (one-loop) channel when $m^2(open) > 0$

- As $d > \sqrt{2\pi^2}$ fundamental open string fields of (12)-type sectors are massive. The exchange of massless closed string dominates.
  $\Rightarrow$ Effective (attrative) coulombian potential.
- As $d \sim \sqrt{2\pi^2}$ fundamental open string fields of (11)-type sectors are massless and (12)-type sectors are quasi-massless.
  $\Rightarrow$ New effective (attrative) potential.

And a tachyon appears !

- When $d < \sqrt{2\pi^2}$ the fundamental (12)-type open string becomes tachyonic and then dominates the dynamics at tree level
  $\Rightarrow$ tachyon in the theory $\Rightarrow$ tachyon condensation.
- It is postulated that this tachyonic phase ends up to the annihilation of the branes.
  $\Rightarrow$ release of energy for reheating.

Conformal field theory

- String Theory can be studied as a field theory on a 2D manifold (the worldsheet), where fields are space-time dimensions X^μ with fermionic partners ψ^μ.
- This is a fundamental property of any 2D manifold that *locally* all metrics are conformally equivalent: $g_{ab}(\sigma) = e^{2\omega(\sigma)}\tilde{g}_{ab}(\sigma)$
- Since conformal transformations in Minkowskian space-time preserves the 'light'-cone, there is no prefered metric on the worldsheet (WS).

Locally, it appears as a natural constraint that field theory on the worldsheet is at least *locally* conformally invariant as well.

- Field theory on WS is then a Conformal Field Theory (CFT).

- The WS action is expressed as a function of the fields and there derivatives and parametrized by external 'coupling constants' (target-space quantities).

$$S = S_{(\mu_i)}[X, \partial X]$$

- Conformal invariance of the WS action now imposes that the coupling constants should not roll under conformal rescalling, say $z \to \ell\, z$:

$$\frac{\partial \mu_i(\ell)}{\partial \ell} = 0$$

- We introduce the beta-function $\beta_\mu = \ell \frac{\partial \mu(\ell)}{\partial \ell}$ such that the previous condition is $\beta_{\mu_i} = 0$.

Conformal invariance imposes that all β-functions of all the coupling constants should vanish

- Vanishing β-functions (a set of equations) determine values of 'coupling constants' such that amplitudes of string propagation are properly defined.
- This suggests upon identification of 'coupling constant' with classical values of target-space fields, that β-functions are equations of motion.
- The effective action is then defined such as to give $\beta_\mu = 0$ as *on-shell* Euler-Lagrange equations.

A drawback of this approach

The effective action is *a priori* only defined around the classical values of the fields (μ_i) used in the CFT.
An other approach called background independent String Field Theory evades this issue, but is much more involved.

Effective action and partition function

- The partition function in QFT is the vacuum amplitude,
  $\Rightarrow$ noted $Z[J]$, with J the diverse sources.
- The open string partition function Z is expanded as:

$$Z = Z_{\text{tree}} + Z_{1-\text{loop}} + \dots$$

- Z_{tree} is defined as the vacuum amplitude on the disk WS:

$$Z_{\text{tree}}[\mu_i] = \int \mathcal{D}X\mathcal{D}g \; e^{-S_{(\mu_i)}[X,g]}$$

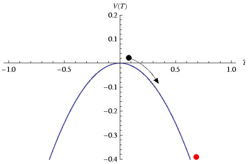
- A crucial identity, with background fields μ_i :

$$Z_{\text{tree}}[\mu_i] \quad = \quad S_{on-shell}[\mu_i]$$

- We more likely consider partition function densities (unintegrated on x-coords) :

$$Z^{NI}_{\text{tree}}[\mu_i] \quad = \quad \mathcal{L}_{on-shell}(\mu_i)$$

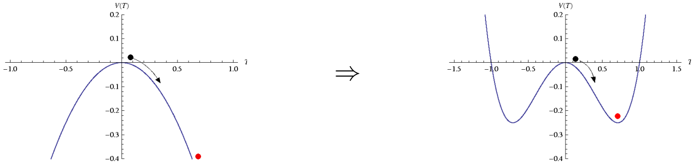
Tachyon in QFT



Presence of tachyon means that we quantify in a bad vacuum

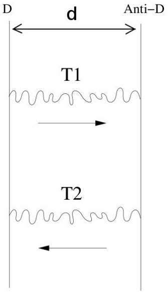
- The vacuum around $T = 0$ is ill-defined.
- Tachyon unstable : it leaves the $T = 0$ position to go to $T \to \infty$
- Perturbation theory breaks down ! This vacuum is not fine for QFT studies !

Tachyon in QFT



Potential must admit some local minimum

- Tachyon should reach some local minimum T_0 at which it could be quantized properly with perturbation theory.
- At this place the tachyon is not anymore tachyonic : $m^2 \geq 0$.



- For $r < \sqrt{2}\pi$, $D - \bar{D}$ system has two tachyons in (12)-type sector :

$$\tau = Te^{i\phi} \quad \text{and} \quad \tau^* = Te^{-i\phi}$$

- tachyon mass : $\alpha' m^2 = \frac{r^2}{4\pi^2\alpha'} - 1/2$

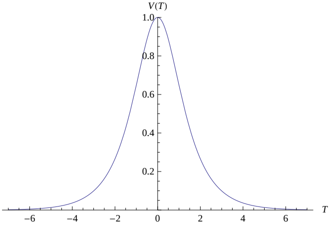
Noticeable example : in trivial flat background at $r = 0$

Effective action for $D3 - \bar{D}3$ in type IIB: Sen [hep-th/0303057]

$$S_{eff} = -2T_3 \int d^{3+1}\sigma V(|\tau|)\sqrt{-\det\left[\eta_{ab} + \partial_a\tau\partial_b\tau^*\right]}$$

with potential : $V(T) = \frac{1}{\cosh\frac{T}{\sqrt{2\alpha'}}}$

Example of tachyon condensation

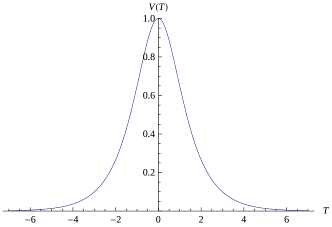


- Around $T = 0$, the tachyon has $m^2 < 0$
- Vacuum unstable $\Rightarrow$ tachyon kicked out.
- At $|T| \to \infty$, the potential becomes flat

Brane annihilation

- As $|T| \to \infty$, sources for closed string, such as graviton, disappear. $\Rightarrow$ D and $\bar{D}$ have annihilated.
- Only two possible vacua remain : $T \to +\infty$ or $T \to -\infty$ $\Rightarrow$ Solitons (domain walls) are expected after annihilation : kink solutions *eg.*

Example of tachyon condensation



- Around $T = 0$, the tachyon has $m^2 < 0$
- Vacuum unstable $\Rightarrow$ tachyon kicked out.
- At $|T| \to \infty$, the potential becomes flat

  $\Rightarrow$ massless field !

  $\Rightarrow$ Stable vacuum, appropriate for QFT !

Brane annihilation

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  $\Rightarrow$ D and $\bar{D}$ have annihilated.
- Only two possible vacua remain : $T \to +\infty$ or $T \to -\infty$
  $\Rightarrow$ Solitons (domain walls) are expected after annihilation : kink solutions *eg.*

Effective action of the separated $D3 - \bar{D}3$ branes

- For $r > 0$ Garousi [hep-th/0710.5469] proposed that homogeneous tachyon in flat trivial background has effective action:

$$S_{eff} = -2T_3 \int d^{3+1}\sigma \; V(T,r) \sqrt{1 - \frac{\dot{T}^2 + \frac{\dot{r}^2}{4}}{1 + \frac{T^2 r^2}{4\pi^2}}}$$

- with the following potential :

$$V(T,r) = \frac{1}{\cosh \frac{T}{\sqrt{2\alpha'}}} \sqrt{1 + \frac{T^2 r^2}{4\pi^2}} \approx 1 + \left(-\frac{1}{2} + \frac{r^2}{4\pi^2} \right) \frac{T^2}{2} + \dots$$

- Computation of eom's shows however that this action is not compatible with rolling tachyon at static distance $\dot{r} = 0$.
- This action is verified only around $T = 0$, it is then not so surprising that it does not concern the whole phase space of the tachyon.

Effective action around rolling tachyon background

- Express a general lagrangian to quadratic order in T:

$$\mathcal{L} = A(r^2) + B(r^2)\left|T\right|^2 + C(r^2)\left|\dot{T}\right|^2 + o(T^4)$$

- Expressing the equations of motion at this order is not difficult. Using $\dot{r} = 0$ and $T = e^{t\sqrt{1/2 - r^2}}$ as imposed solution, we completely fix A, B and C to get:

$$\mathcal{L} = -2 + \sqrt{1 - 2r^2}\left(\left|T\right|^2 - \frac{\left|\dot{T}\right|^2}{\sqrt{1/2 - r^2}}\right) + \dots$$

- We don't have further constraints. At least up to r-dependent field redefinition, this is how effective action should behave at quadratic order.

Computing β-function in CFT on the boundary

Suppose we have the following general Worldsheet action:

$$S = S_{bulk} + \sum_i \mu_i^{(B)} \oint V_i(X^a, \tilde{X}^\mu)$$

In CFT language V_i's are called vertex operator and have conformal dimension h_i. They verify OPE identities:

$$V_i(z)V_j(w) = \sum_k \frac{C_{ij}{}^k}{|z-w|^{h_i+h_j-h_k}} V_k(w)$$

Perturbatively, any amplitude with previous action, involves products between boundary deformations $\mu_i^{(B)} \oint \; V_i(X^a, \tilde{X}^\mu)$:

$$\int \mathcal{D}X \; e^{-S_{bulk} - \sum_i \mu_i^{(B)} \oint \; V_i(X^a, \tilde{X}^\mu)}$$

$$= \int \mathcal{D}X \; e^{-S_{bulk}} \sum_n \frac{1}{n!} \left(\sum_i \mu_i^{(B)} \oint \; V_i(X^a, \tilde{X}^\mu) \right)^n$$

And any products of vertex operators at any order can be expanded in terms of boundary deformations $\oint \; V_i(X^a, \tilde{X}^\mu)$, such that it should look like:

$$\int \mathcal{D}X \; e^{-S_{bulk} - \sum_i \mu_i^{(B)} \oint \; V_i(X^a, \tilde{X}^\mu)} = \int \mathcal{D}X \; e^{-S_{bulk}} \sum_m F_m(\mu_i) \oint \; V_i(X^a, \tilde{X}^\mu)$$

Coefficients in front of such terms are generally divergent, with respect to a UV cut-off ϵ or an IR cut-off L. For instance, if we parametrize the boundary by the real line:

$$\int_{-L}^{+L} \mathrm{d}z \int_{z-L}^{z-\epsilon} \mathrm{d}w \; V_i(z)V_j(w) = \sum_k {C_{ij}}^k (h_i + h_j - h_k - 1) \left(\epsilon^{1-h_i-h_j+h_k} - L^{1-h_i-h_j+h_k}\right)$$

If there exist $k = k_0$ such that $h_i + h_j - h_{k_0} = 1$, we have what is called a resonance and the divergence is logarithmic:

$$\sum_{k \neq k_0} {C_{ij}}^k (h_i + h_j - h_k - 1) \left(\epsilon^{1-h_i-h_j+h_k} - L^{1-h_i-h_j+h_k}\right) + {C_{ij}}^{k_0} \log \frac{L}{\epsilon}$$

- Amplitudes should not be IR nor UV divergent $\Rightarrow$ renormalization procedure.
- In the minimal scheme, we simply add counterterms to the action such to get rid perturbatively of all divergences.
- It is a general result that in that scheme only logarithmic counterterms modify the couplings β-function.

Example

From previous example, we obtain that the β-function for coupling μ_{k_0} is given by:

$$\beta_{k_0} = (1-h_{k_0})\mu_{k_0} + \sum_{\substack{i,j \\ h_i+h_j-h_{k_0}=1}} C_{ij}{}^{k_0}\,\mu_i\,\mu_j + \dots$$

WS action with tachyon

- Let us look at $D0-\bar{D}0$ system separated in X-direction.
- Add the rolling tachyon and distance fields, for static configuration :

$$S = S_{bulk} + \frac{\delta r}{4\pi}\sigma^3 \otimes \oint dt\ \partial_t \tilde{X}$$
$$+\lambda^+ \sigma^+ \otimes \oint dt\ \psi^+ e^{\sqrt{\frac{1}{2}-r^2}X^0 + ir\tilde{X}} + \lambda^- \sigma^- \otimes \oint dt\ \psi^- e^{\sqrt{\frac{1}{2}-r^2}X^0 - ir\tilde{X}}$$
$$+\sum_{n=1}^{\infty} \mu_n \oint dt\ e^{2n\sqrt{\frac{1}{2}-r^2}X^0}$$

with

$$\psi^\pm = \sqrt{\frac{1}{2}-r^2}\psi^0 \pm ir\tilde{\psi}$$
$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Counterterms

We have the following 2n-OPE of order $(\lambda^+\lambda^-)^n$:

$$\left(\sigma^+ \otimes \int_{-L}^{+L} \mathrm{d}t\ \psi^+ e^{\sqrt{\frac{1}{2}-r^2}X^0+ir\tilde{X}}\ \sigma^- \otimes \int_{t-L}^{t-\epsilon} \mathrm{d}u\ \psi^- e^{\sqrt{\frac{1}{2}-r^2}X^0-ir\tilde{X}}\right)^n$$

$$\sim \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes F_{r^2}^{(n)}(L,\epsilon)\ \oint \mathrm{d}t\ e^{2n\sqrt{\frac{1}{2}-r^2}\,X^0}(t)\left[1 + H(\partial_t^{(m)}\tilde{X}, \partial_t^{(m)}X^0)\right]$$

with:

- $F_{r^2}^{(n)}(L,\epsilon)$ some potentially divergent term in L and/or ϵ
- $H_{\epsilon,L,r}(\partial_t^{(m)}\tilde{X}, \partial_t^{(m)}X^0)$ a functionnal of derivatives of X-fields.

Counterterms

Similarly:

$$\left(\sigma^- \otimes \int_{-L}^{+L} \mathrm{d}t\ \psi^- e^{\sqrt{\frac{1}{2}-r^2}X^0 - ir\tilde{X}}\ \sigma^+ \otimes \int_{t-L}^{t-\epsilon} \mathrm{d}u\ \psi^+ e^{\sqrt{\frac{1}{2}-r^2}X^0 + ir\tilde{X}}\right)^n$$

$$\sim \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes F_{r^2}^{(n)}(L,\epsilon)\ \oint \mathrm{d}t\ e^{2n\sqrt{\frac{1}{2}-r^2}\,X^0}(t)\left[1 + H(\partial_t^{(m)}\tilde{X}, \partial_t^{(m)}X^0)\right]$$

And:

$$\int_{-L}^{+L} \mathrm{d}t\ e^{2n\sqrt{\frac{1}{2}-r^2}X^0} \int_{t-L}^{t-\epsilon} \mathrm{d}u\ e^{2m\sqrt{\frac{1}{2}-r^2}X^0}$$

$$= G_{r^2}^{(n,m)}(L,\epsilon)\ \oint \mathrm{d}t\ e^{2(n+m)\sqrt{\frac{1}{2}-r^2}\,X^0}(t)\left[1 + K(\partial_t^{(m)}X^0)\right]$$

β-functions

- Simple vertex operator $\sigma^3 \otimes \oint dt\ \partial_t \tilde{X}$ is not produced from these OPE for $r < \sqrt{2}\pi$.

  $\Rightarrow$ with $h = 1$, its β-function vanishes : $\beta_{\delta r} = 0$

- On the other hand $\oint dt\ e^{2n\sqrt{\frac{1}{2}-r^2}X^0}$ are produced as dominant terms.

  $\Rightarrow$ If the functions F_{r^2} and G_{r^2} are divergent one has to add corresponding counterterms.

- What is the behavior of these coefficient when there is a resonnance, *i.e.* when $2n\sqrt{\frac{1}{2}-r^2} = 1$?

  $\Rightarrow$ Analytically, we find that all logarithmic divergences in F_{r^2} cancel exactly together, for $n = 1$ and $n = 2$. It is also numerically verified at $n = 3$. D. Israel, F.K. [hep-th/1108.5763]

β-functions of μ_p

- We are tempted to extend the previous result to any n, since it is not too surprising.
  - $\Rightarrow$ Indeed this theory can be described, up to some power-like counterterms, with a manifestly supersymmetric action on the WS
- Supersymmetry provides non-renormalisation theorem for any coupling constant.
  - $\Rightarrow$ So $\beta_p = (1 - h_p)\mu_p$
- For $h_p \neq 1$, vanishing β_p implies $\mu_p = 0$.
- For $h_p = 1$, the β-function is zero so one can still choose $\mu_p = 0$ consistently.

β-functions of $\lambda^{\pm}$

- Only one kind of OPE is able to contribute to the β-function of the tachyons $\lambda^{\pm}$, namely:

$$\left(\sigma^3 \otimes \partial_t \tilde{X}(t)\right)^m \ \sigma^{\pm} \otimes \psi^{\pm} e^{\sqrt{\frac{1}{2}-r^2}X^0 \pm ir\tilde{X}}(u)$$

- It is easy and sufficient to compute for $m = 1$:

$$\sigma^3 \otimes \partial_t \tilde{X}(t)\ \sigma^{\pm} \otimes \psi^{\pm} e^{\sqrt{\frac{1}{2}-r^2}X^0 \pm ir\tilde{X}}(u) = \pm i\ \sigma^{\pm} \otimes \frac{2r}{t-u}\psi^{\pm} e^{\sqrt{\frac{1}{2}-r^2}X^0 \pm ir\tilde{X}}(u) + \dots$$

- Since $h_{\pm} = 1$ this leads to the following β-function:

$$\beta_{\pm} = 2r\delta r \lambda^{\pm} + o(\delta r^2)$$

- Then $\beta_{\pm} = 0 \Rightarrow \delta r = 0$

- To summarize, we find:

$$\begin{aligned} C^r_{+-} &= 0 \\ C^+_{r+} = C^-_{r-} &= r \\ C^p_{nm} = C^p_{+-} &= 0 \end{aligned}$$

- β-functions for δr, $\lambda^\pm$ and μ_n vanish at all-order for any $r < \sqrt{17}/6$ provided $\delta r = 0$ and $\mu_n = 0$. This could be extended to all $r < \sqrt{2\pi}$

The rolling tachyon background at constant distance is consistent !

- The theory is expressed in terms of a supersymmetric action with a finite number of power-like counterterms
- Simple conformal dimension-counting shows that any counterterm containing at least one derivative of X-fields has positive power of ϵ :

$$\Delta = 4n^2(\frac{1}{2} - r^2) + m - 1 > 0 \qquad \text{for any} \qquad r < \sqrt{2}\pi$$

$\Rightarrow$ for $m = 0$ only $n \leq 1/\sqrt{2 - 4r^2}$ contribute as counterterms

- The non-vanishing counterterms are of the form:

$$f(r^2)\epsilon^{4n^2(\frac{1}{2}-r^2)-1} \oint dt \; e^{2n\sqrt{\frac{1}{2}-r^2}X^0}$$

- We have showed that rolling tachyon background at constant distance is a solution of the string field equations of motion for all $r < \sqrt{17}/6$.
- On the other hand, this is not consistent with Garousi's action proposal.

  $\Rightarrow$ This suggests that his action has a limited domain of definition in the tachyon field, around $T = 0$.

- Computation of the effective action around the rolling tachyon background is untractable perturbatively so we aim to go in an other direction such to circle this issue.

Further work

- Study Boundary string field theory for separated brane-antibrane system in order to obtain the tachyon potential

  $\Rightarrow$ This would let appear non-trivial tachyon condensation process, *eg.* spatially dependent (soliton: kink solutions)

- On the other hand, we plan to study this static system in a thermodynamical point of view, especially closed strings emitted, to understand what happen physically in such situation.