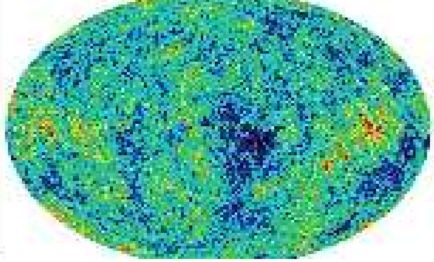
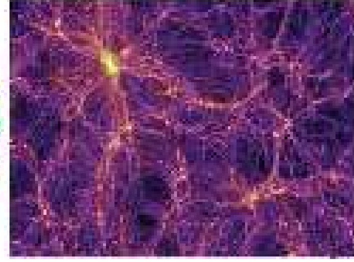
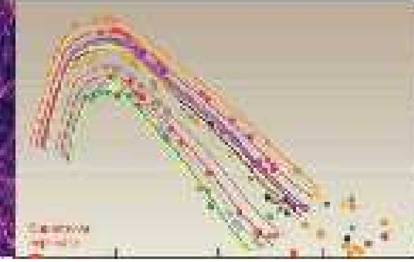
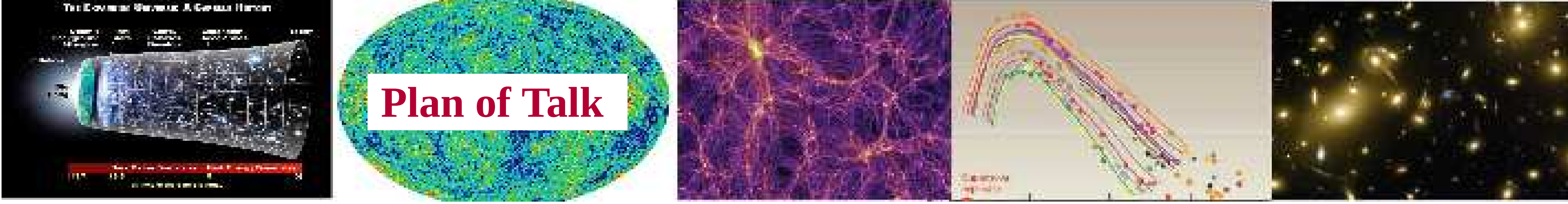






Constraining the Nature of Dark Energy through Cosmological Observations

Ujjaini Alam (LANL)

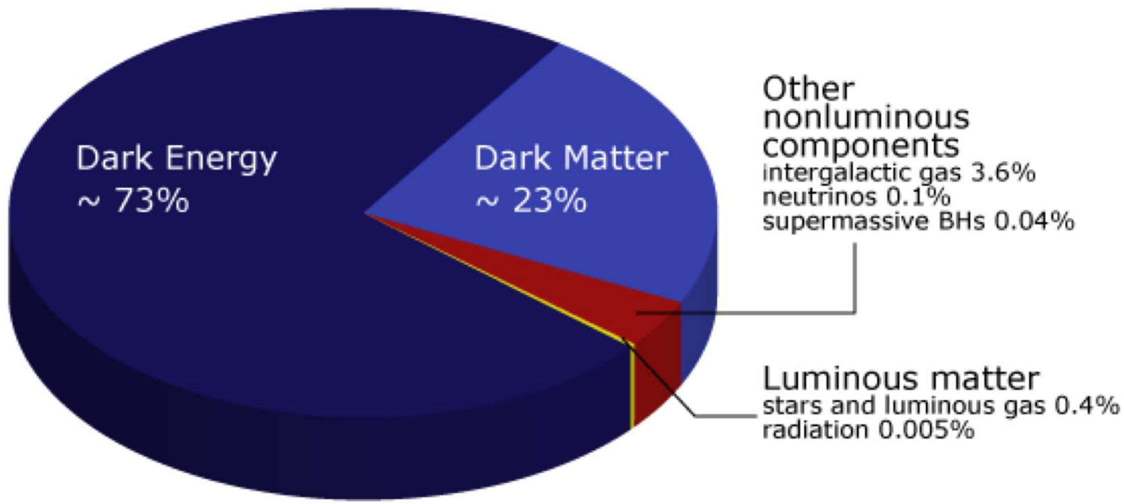
S. Bhattacharya (UChicago), S. Habib, K. Heitmann (ANL), D. Higdon (LANL), Z. Lukic (LBL), T. Holsclaw, H. Lee, B. Sanso (UCSC), V. Sahni (IUCAA), A. A. Starobinsky (Landau I)



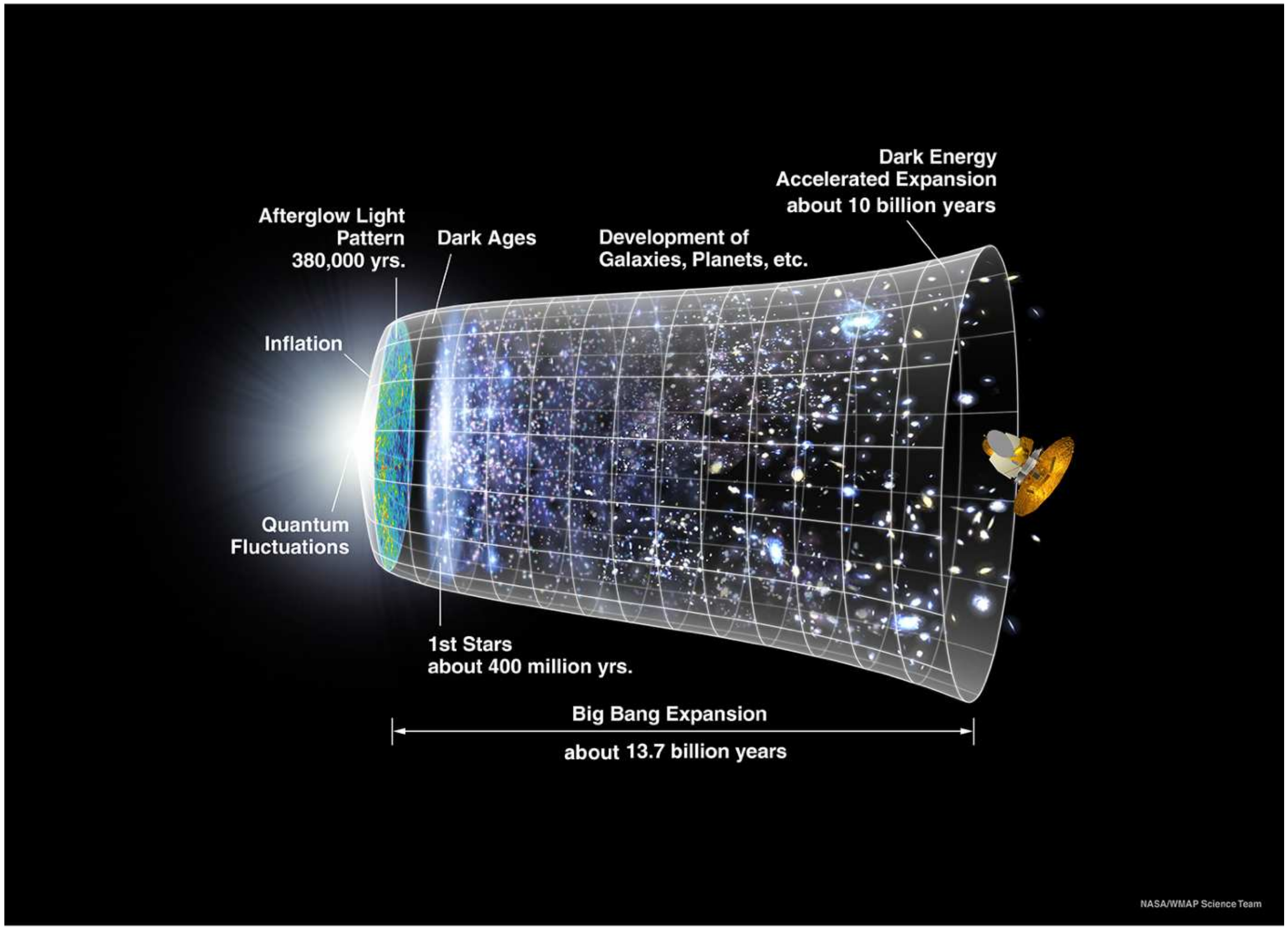
Plan of Talk

- Introduction : Dark Energy
  - Observational Evidence
  - The Cosmological Constant
  - Other Dark Energy Models
- Non-parametric reconstruction of Dark Energy parameters
  - Gaussian Process modeling
  - Current and future constraints
- Constraints from perturbative measurements
  - Perturbations from distance measures
  - Non-standard Dark Energy Models : Early Dark Energy
- Conclusion

Energy Content of the Universe



Expansion of the Universe



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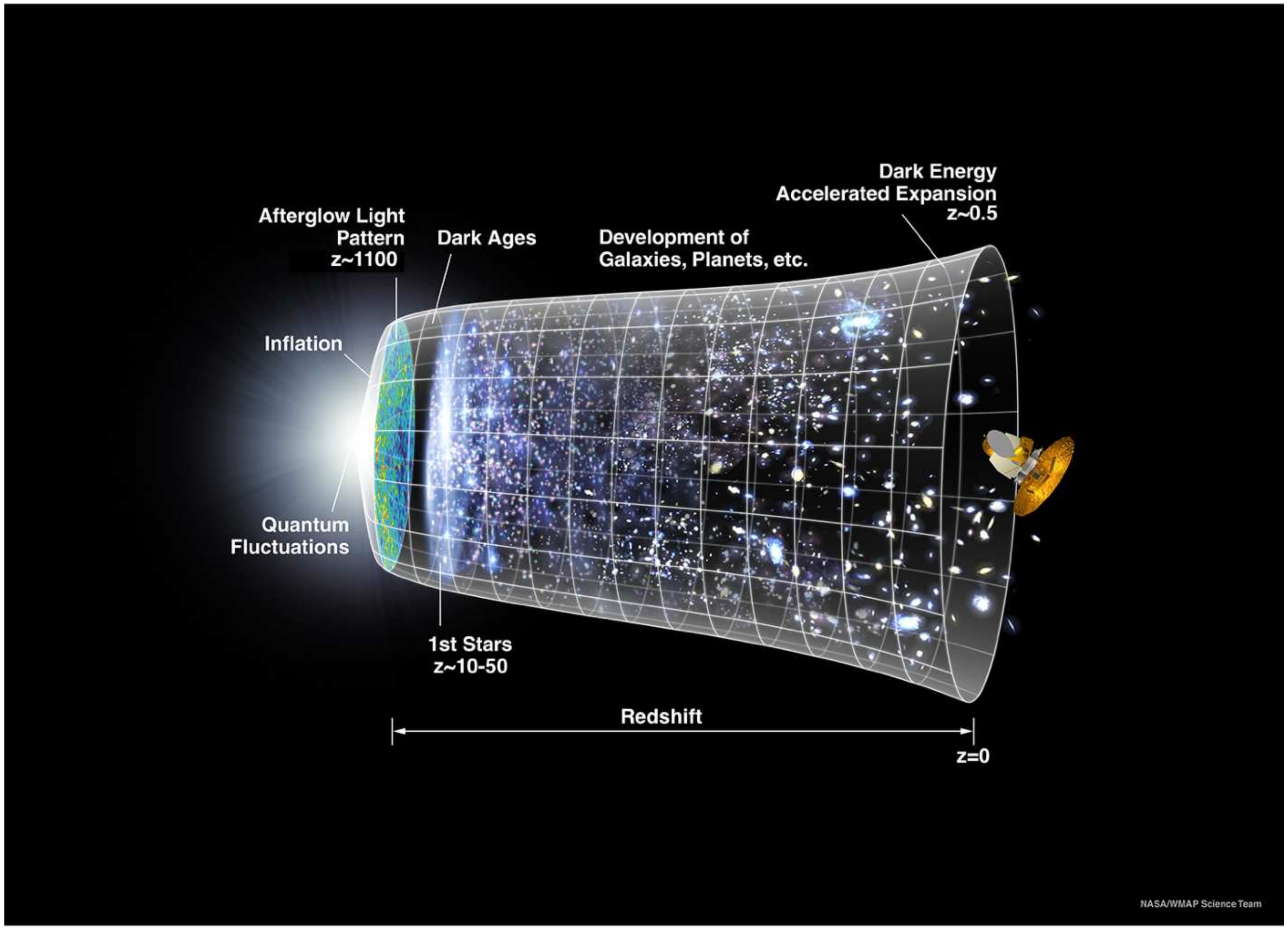
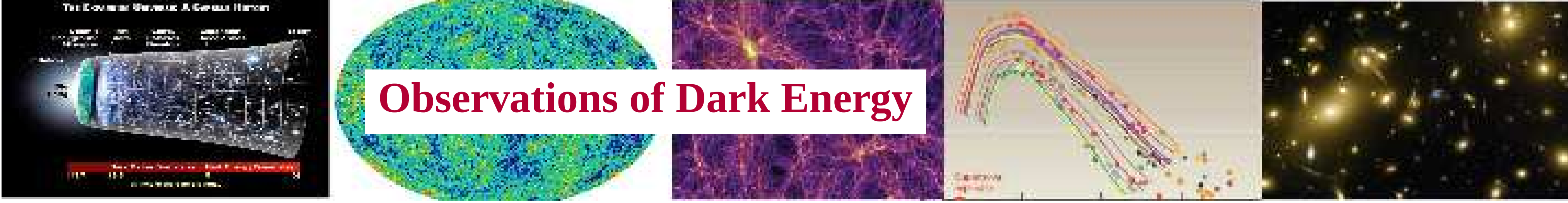
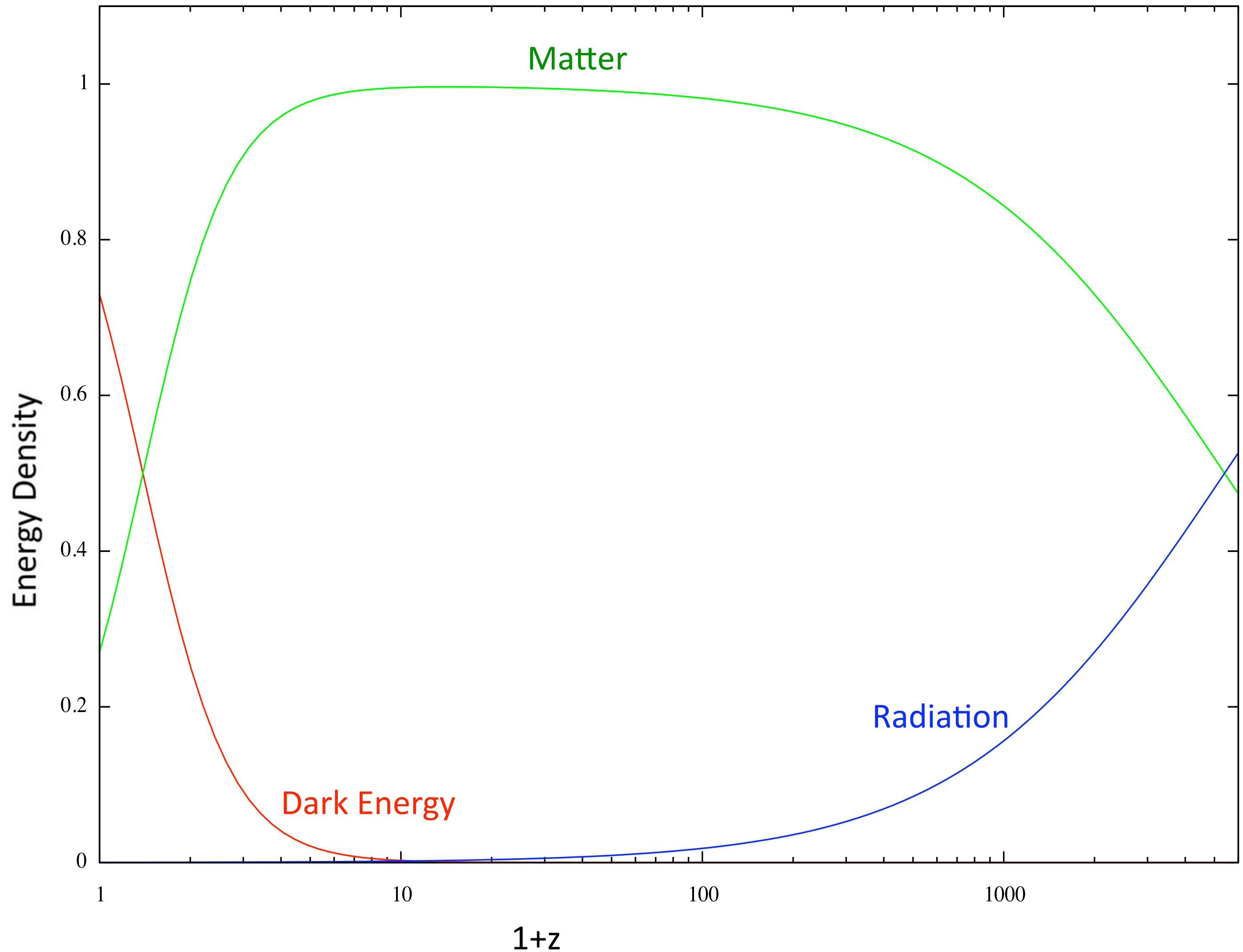
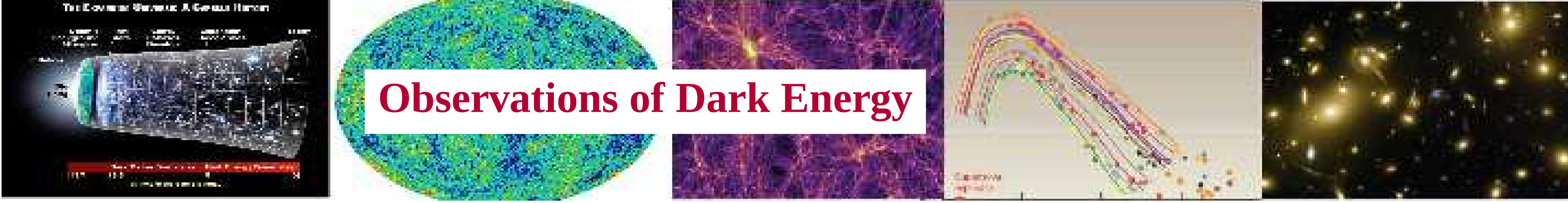


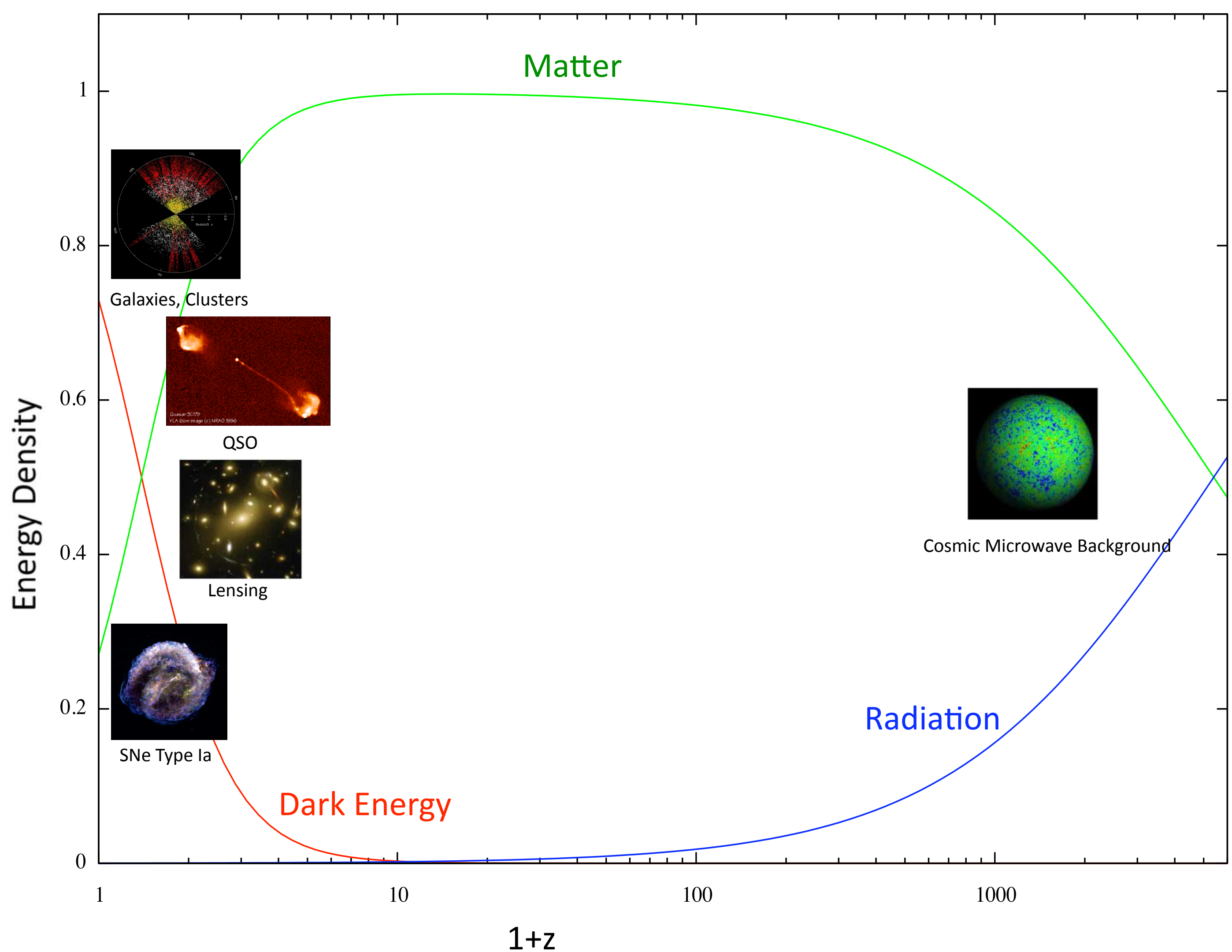


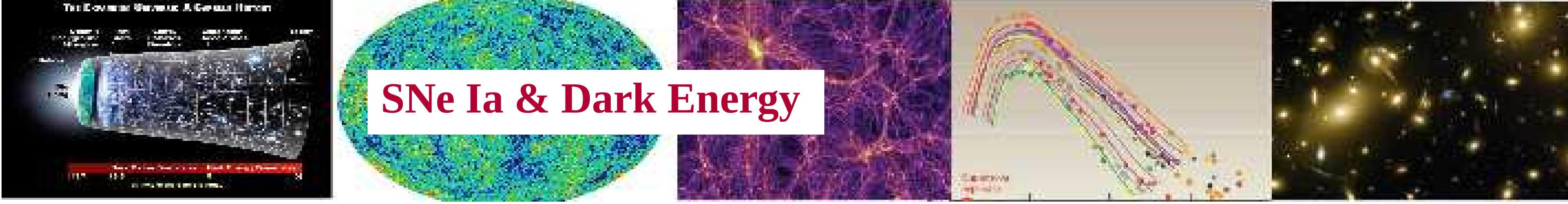
Observations of Dark Energy



Observations of Dark Energy

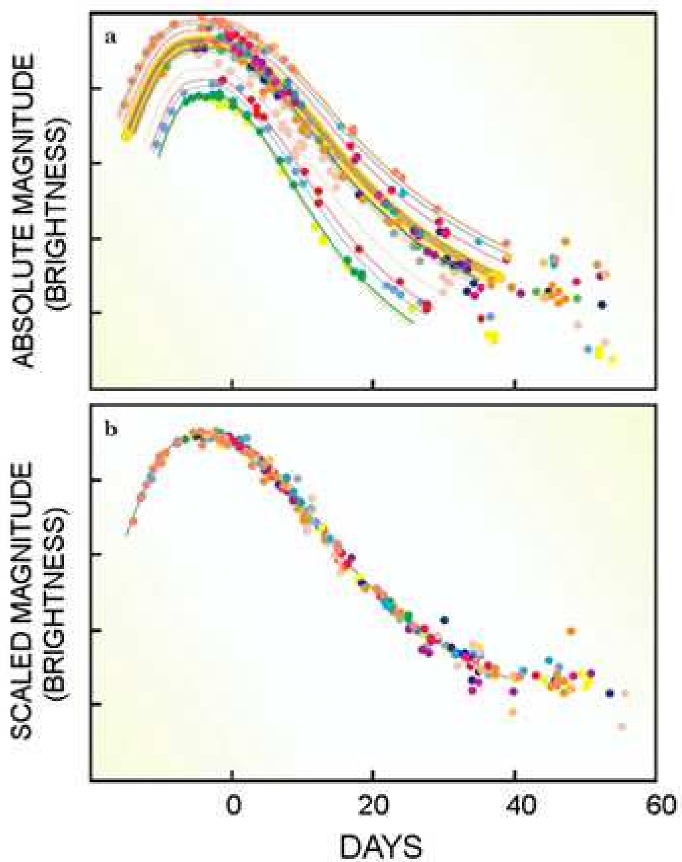


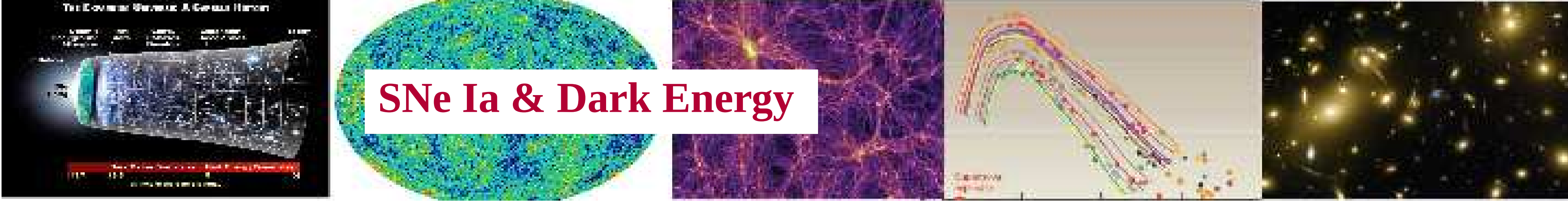




SNe Ia & Dark Energy

SNe Ia $\Rightarrow$ Thermonuclear explosion in C+O white dwarf
Strong correlation between peak magnitude & light curve shape
$\rightarrow$ calibrated candles

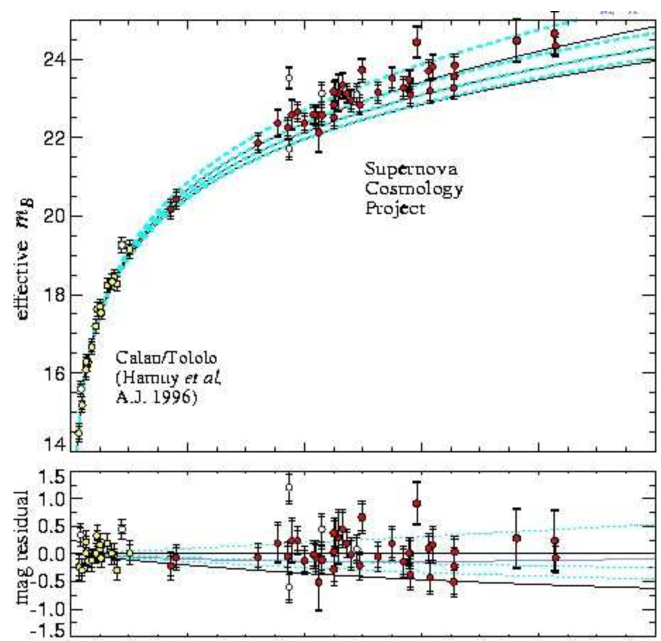




SNe Ia & Dark Energy

SNe Ia ⇒ Thermonuclear explosion in C+O white dwarf
Strong correlation between peak magnitude & light curve shape
→ calibrated candles

High z SNe dimmer than expected (1997-98)
⇒ Expansion of Universe accelerating
⇒ Dominant energy component of Universe has negative pressure
= Dark Energy !!



Theoretical Models of Dark Energy : Cosmological Constant

Cosmological Constant : $R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik} + \Lambda g_{ik}$

– Einstein (1917)

$w = -1 \leftarrow$ May explain accelerated expansion of Universe

Theoretical explanation :

Zero point vacuum fluctuation $< T_{ik} >= \Lambda g_{ik}$

– Zeldovich (1968)

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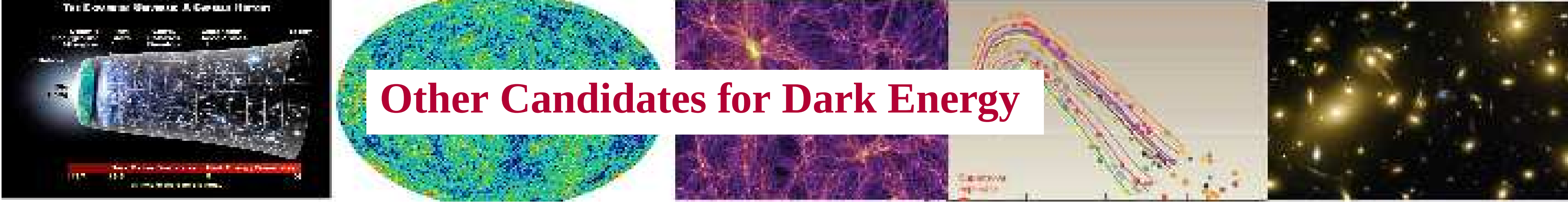
Problems :

- Coincidence Problem : Why now
- Divergence problem : $\Lambda / 8\pi G = < T_{00} >_{\rm vac} \propto \int_0^\infty k^2 \sqrt{k^2 + m^2} dk$
  Planck scale cut-off $\rightarrow < T_{00} >_{\rm vac} \simeq 10^{76} {\rm Gev}^4$
  QCD scale cut-off $\rightarrow < T_{00} >_{\rm vac} \simeq 10^{-3} {\rm Gev}^4$
  Observed value $\rightarrow \rho_\Lambda \simeq 10^{-47} {\rm Gev}^4$



Other Candidates for Dark Energy

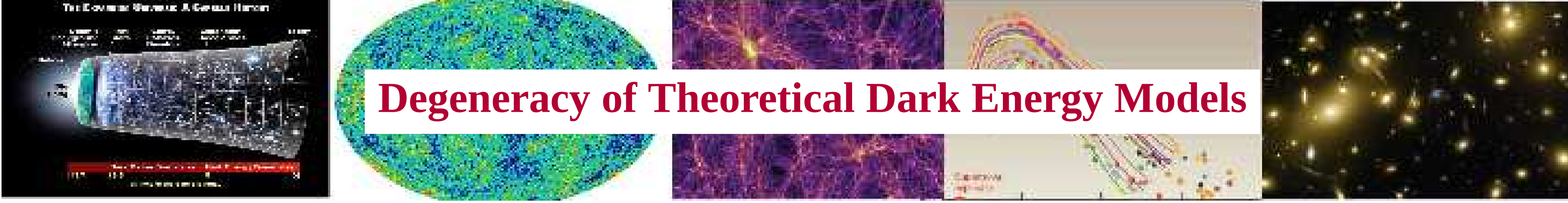
- Quiessence : $-1 < w =$constant $< -1/3$
- Quintessence : $\mathcal{L} = \frac{1}{2}\partial_a\phi\,\partial^a\phi - V(\phi)$

$$V = V_0/\phi^\alpha$$
$$V = V_0 \exp(\lambda\phi^2)/\phi^\alpha$$
$$V = V_0(\cosh\lambda\phi - 1)^p$$

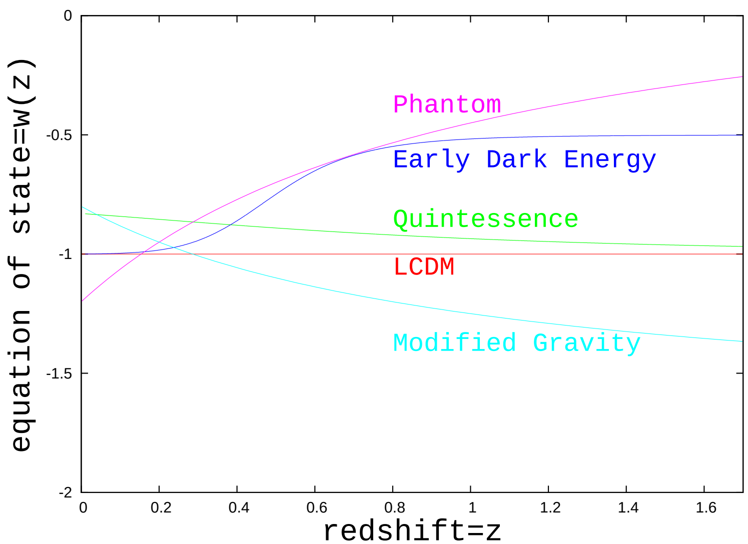
- Phantom fields with $w < -1$, Early Dark Energy Models
- k-essence : $\mathcal{L} = -V(\phi)\sqrt{1 - \partial_a\phi\,\partial^a\phi}$
  (Chaplygin gas : $P = -A/\rho^\alpha$)
- Modified gravity models : $f(r)$ theories, braneworld models....

Observations sensitive to Dark Energy

Distance Measures	Perturbation Measures
SNe Type Ia, BAO, Weak Lensing, GRB	Galaxy Clusters, CMB, Weak Lensing
Measures expansion history of Universe	Measures growth of perturbations
Measurements at low z sensitive to Dark Energy	Some high z measurements not so sensitive
Degenerate between various Dark Energy models	Complementary to distance measures, may break degeneracy
Model-independent analysis easy	Analysis must include assumptions on growth of perturbations



Degeneracy of Theoretical Dark Energy Models



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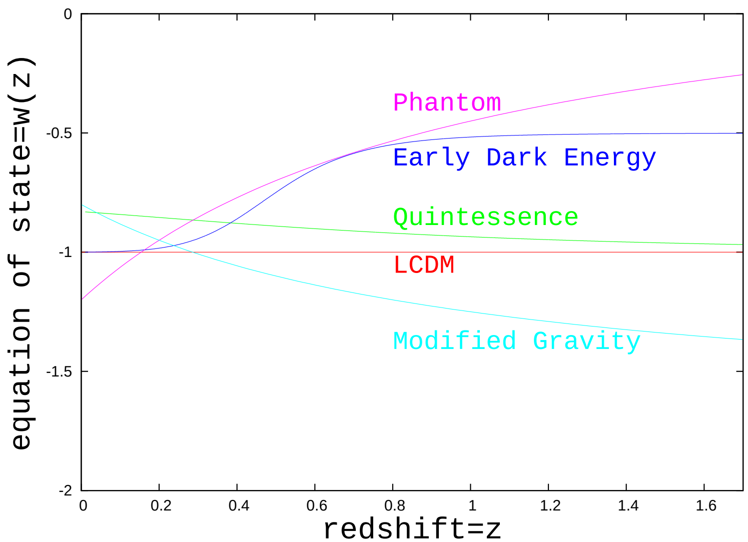

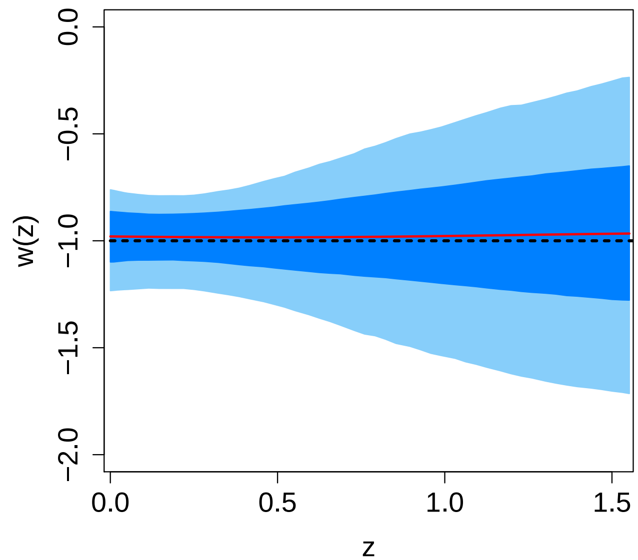

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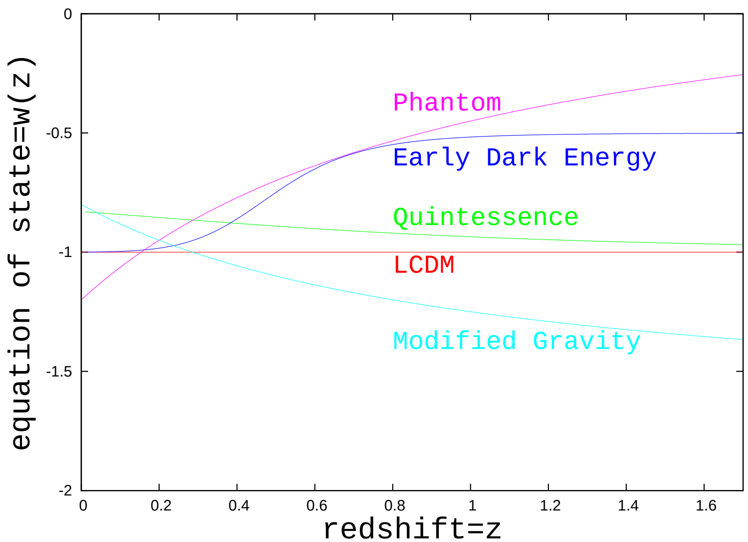
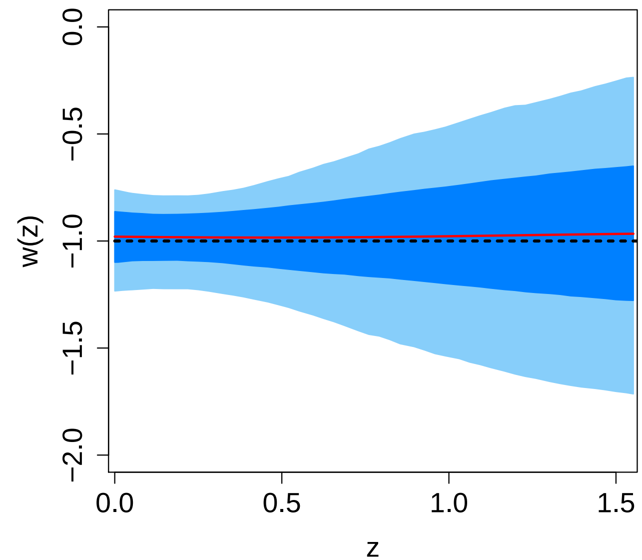




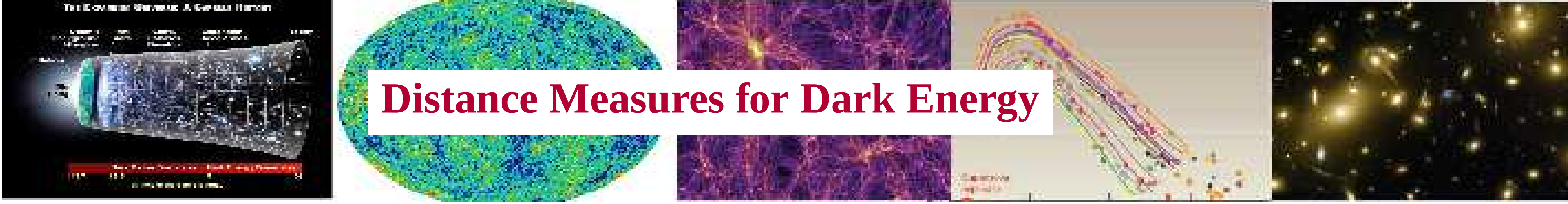
Multiple Degenerate Theoretical Models of Dark Energy

$\Downarrow$

Development of statistical methods to extract maximum information from observations

$+$

Constraints on theoretical models from complementary observations



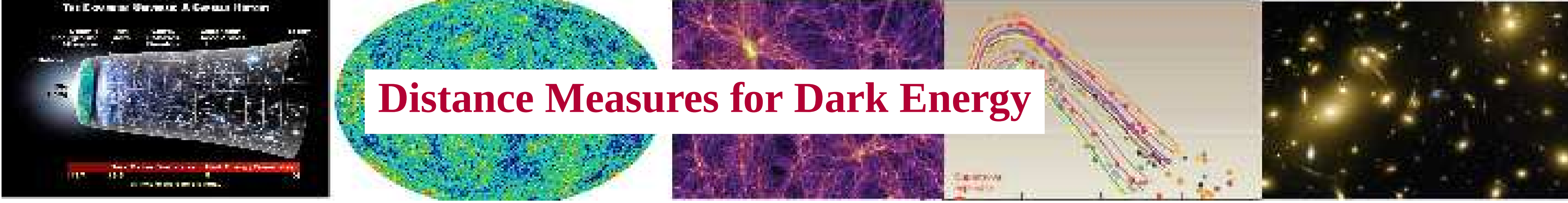
Distance Measures for Dark Energy

$$r(z) = \int_0^z \frac{dz}{h(z)} = \int_0^z \frac{dz}{\sqrt{\tilde{\Omega}_r(1+z)^4 + \Omega_{0\mathrm{m}}(1+z)^3 + \Omega_\Lambda \exp\left[\int_0^z \frac{3(1+w(u))du}{1+u}\right.}}$$

$$\mu_B(z) = \mathcal{M} + 5\log_{10}[(1+z)r(z)] \qquad \leftarrow \mathrm{SNe}$$

$$\frac{d_A(z)}{r_s(z_\star)} = \frac{c}{H_0}\frac{r(z)}{(1+z)r_s(z_\star)}; H(z)r_s(z_\star) \qquad \leftarrow \mathrm{BAO}$$

$$R(z_{\mathrm{CMB}}) = \sqrt{\Omega_{0\mathrm{m}}}\, r(z_{\mathrm{CMB}}) \qquad \leftarrow \mathrm{CMB}$$



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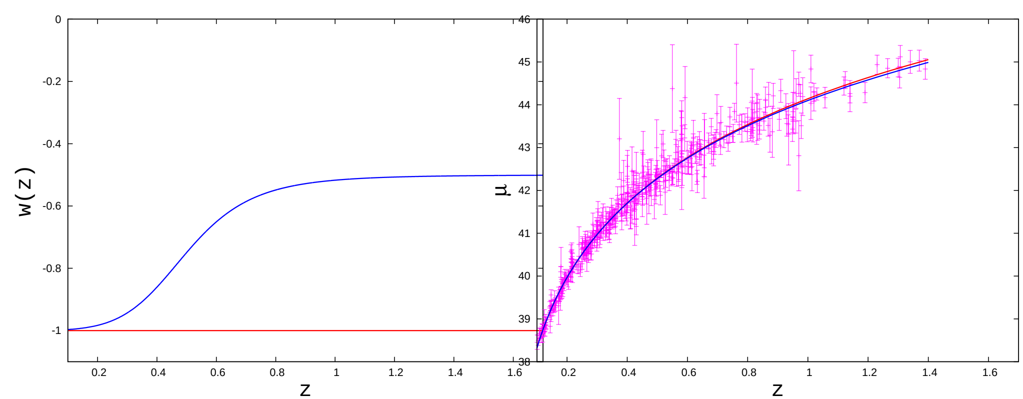

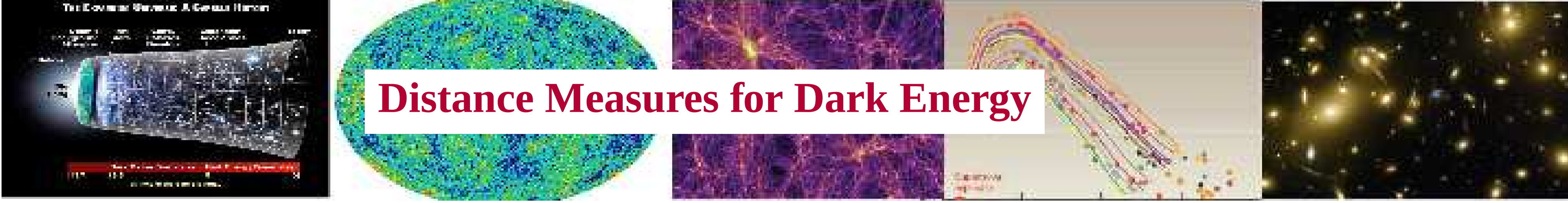

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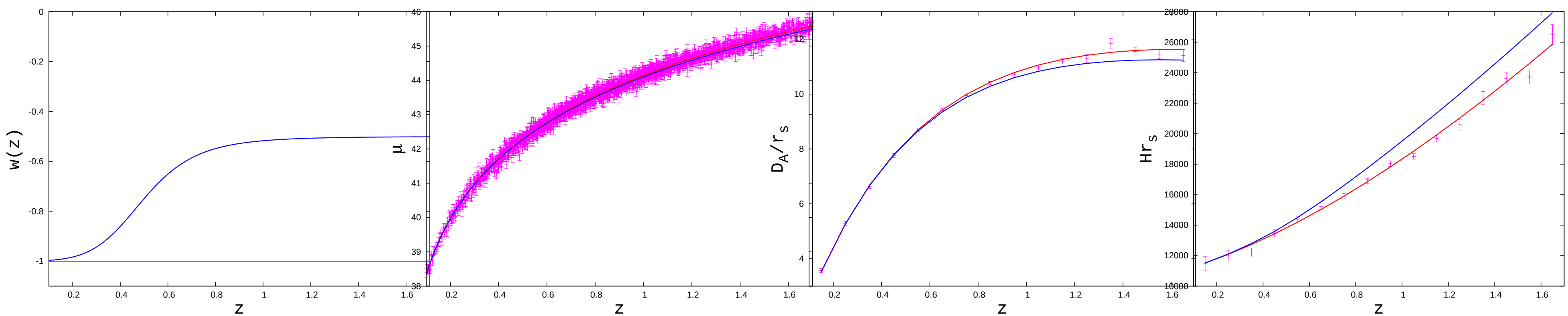

Reconstruction of Dark Energy Parameters

$$p(a|X,y) = \frac{p(y|X,a)p(a)}{p(y|X)} \leftarrow a = \Omega_{0\mathrm{m}}; w_{DE} \text{ or } \rho_{DE}$$

$$p(y|X,a) = \frac{1}{(2\pi\sigma_n^2)^{n/2}} \exp\left[-\frac{1}{2\sigma_n^2}\left(y - f(x;a)\right)^2\right]$$

$$p(y|X) = \int p(y|X,a)p(a)da$$

Fitting functions : $\rho_{DE}(z) = \rho_0 + \rho_1(1+z) + \rho_2(1+z)^2; w_{DE}(z) = w_0 + w_a z/(1+z)$

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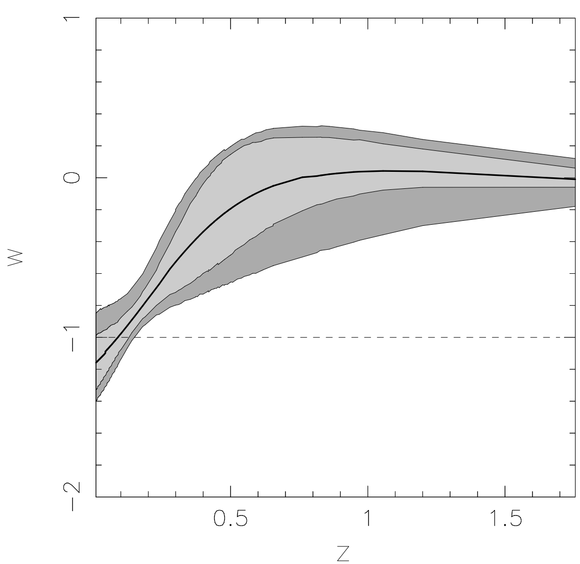
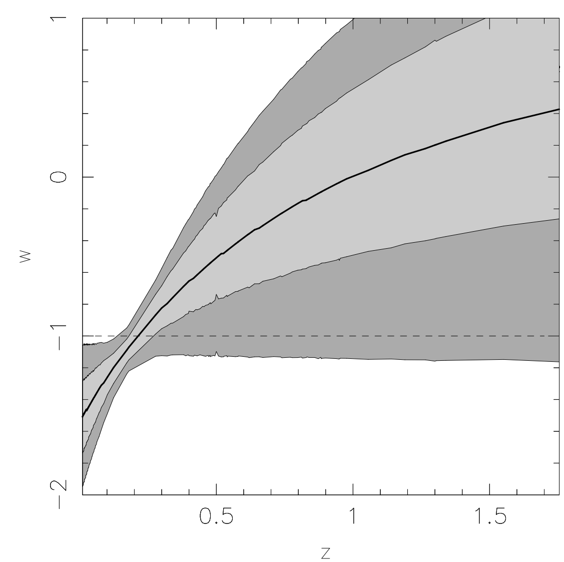
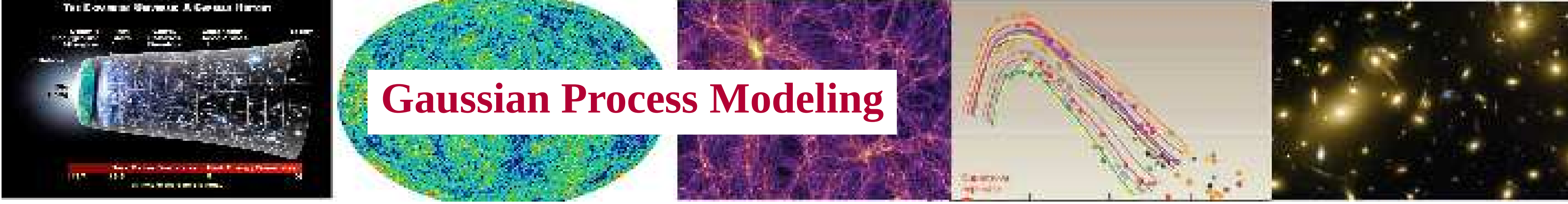
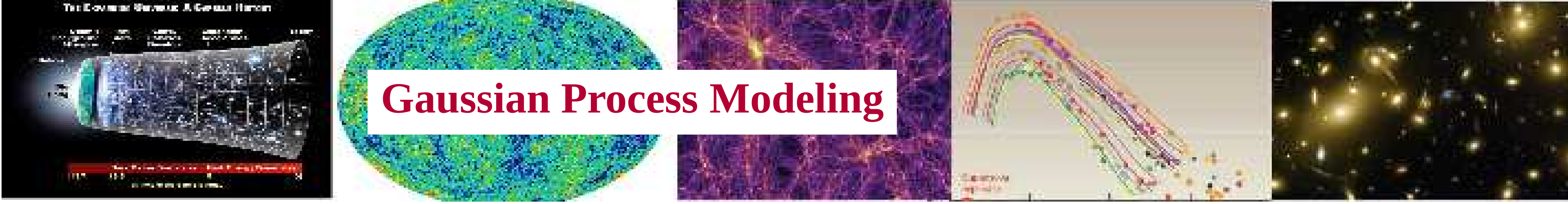






Gaussian Process Modeling

$$
\begin{aligned}
p(y|X) &= \frac{1}{(2\pi\sigma_n^2)^{n/2}} \exp\left[-\frac{1}{2\sigma_n^2}\,(y - f(x))^2\right] \\
f(x) &= \mathcal{GP}(m(x), K(x, x'))
\end{aligned}
$$



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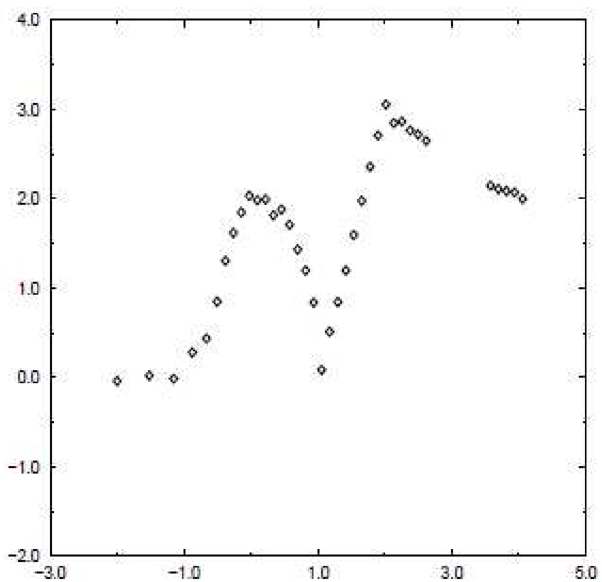
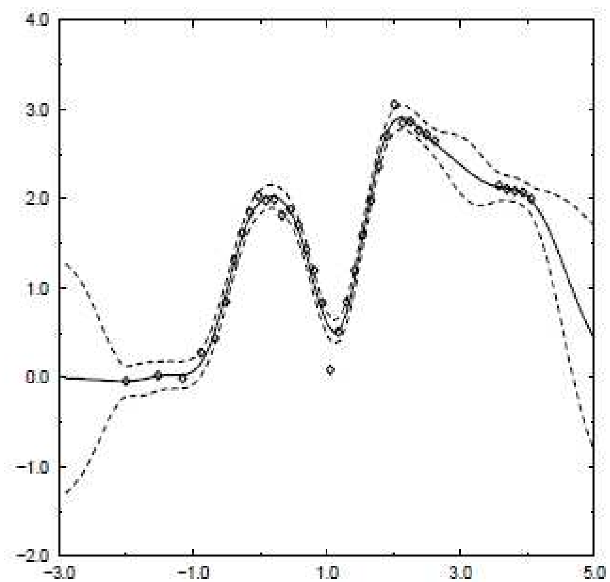


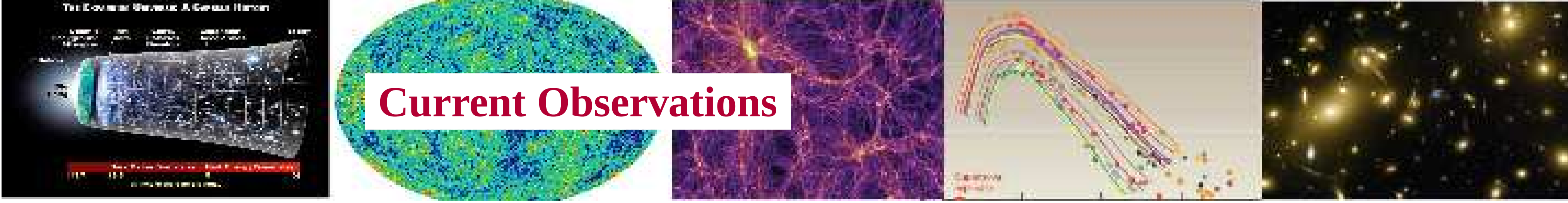


Application to distance measures

- $w(u) \sim \mathcal{GP}(-1, \kappa^2 \rho^{|u-u'|^\alpha})$
- $y(s) \sim \mathcal{GP}\left(-\ln(1+s), \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{u-u'} du du'}{(1+u)(1+u')}\right)$
- Joint GP for $y(s)$ and $w(u)$:

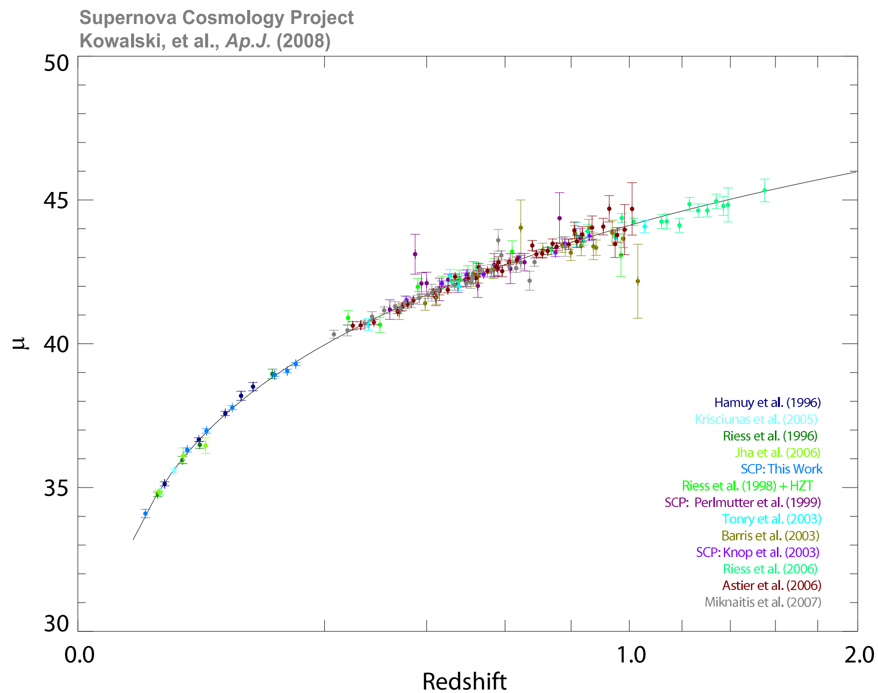
$$\begin{bmatrix} y(s) \\ w(u) \end{bmatrix} \sim \mathrm{MVN}\left[\begin{bmatrix} -\ln(1+s) \\ -1 \end{bmatrix}\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right],$$

- Mean for $y(s)$ given $w(u)$:
  $y(s)|w(u) = -\ln(1+s) + \Sigma_{12}\Sigma_{22}^{-1}(w(u) - (-1))$
- Obtain 2nd integral numerically, compute likelihood



Current Observations

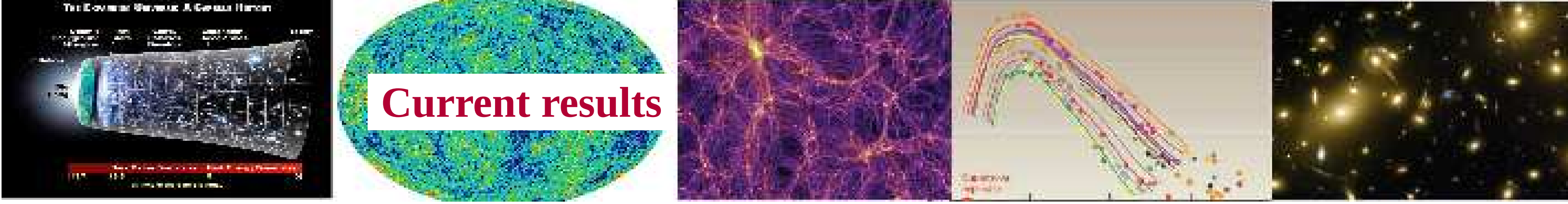
SNe Union2 compilation → 557 SNe, $\sigma_{m_B} \sim 0.15$



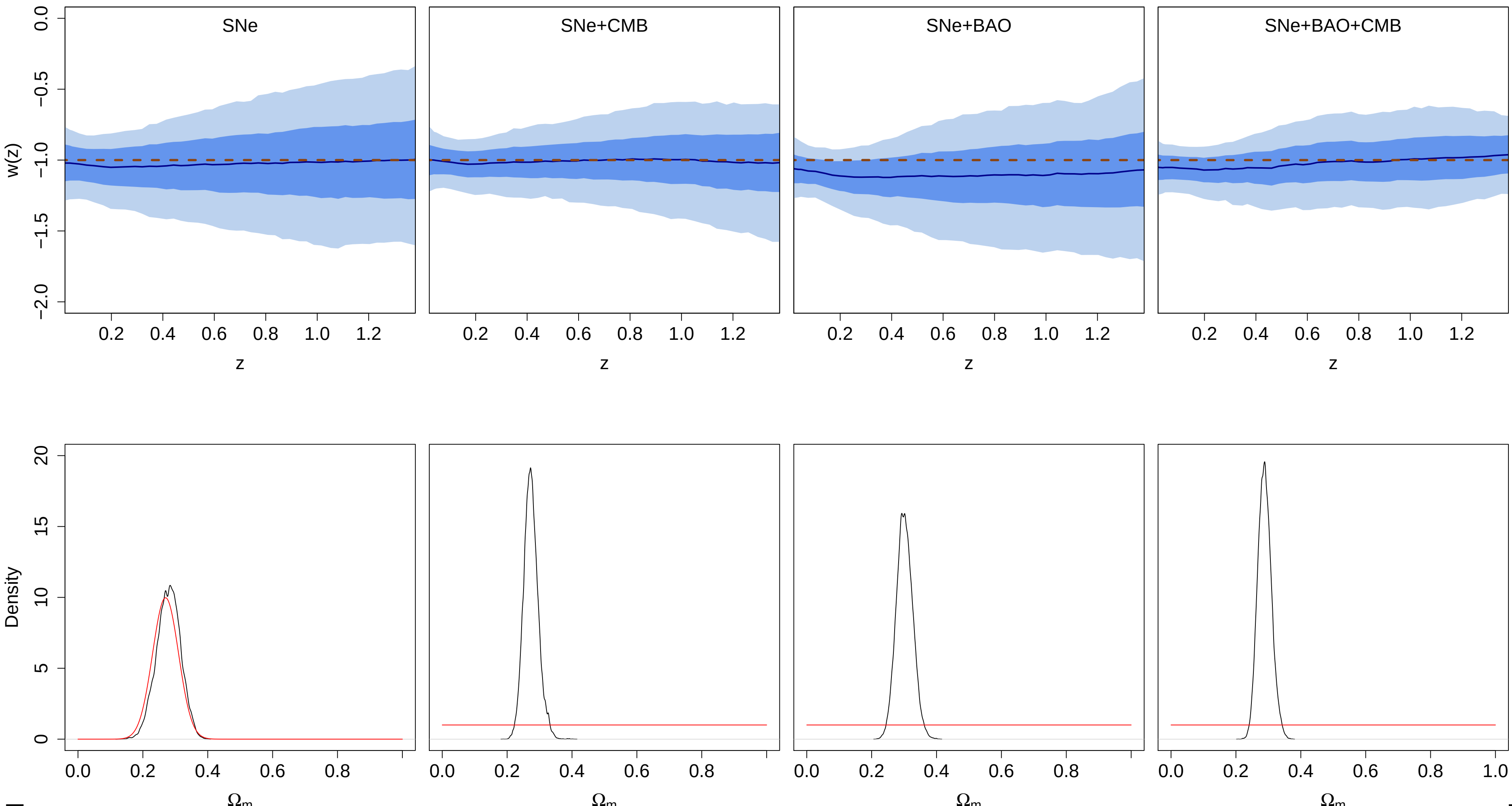
BAO SDSS $\Rightarrow r_s(z_\star)(H(z)/(1+z)^2 d_A^2 cz)^{1/3} = 0.19 \pm 0.0061 (z = 0.2)$

$= 0.11 \pm 0.0036 (z = 0.35)$

CMB WMAP7 $\Rightarrow R = 1.719 \pm 0.019$

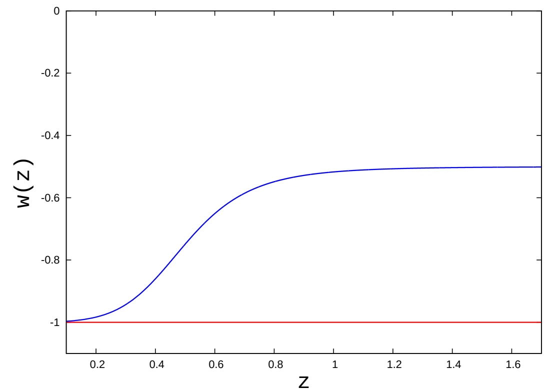


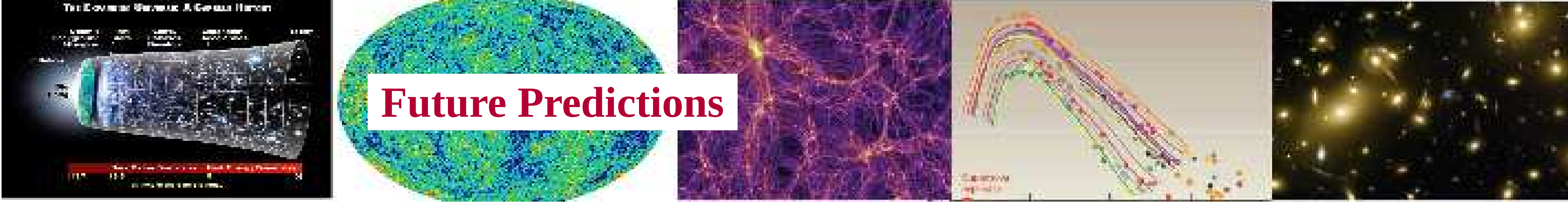
Current results



(Holsclaw, Alam, et.al, 2011)

Future Predictions





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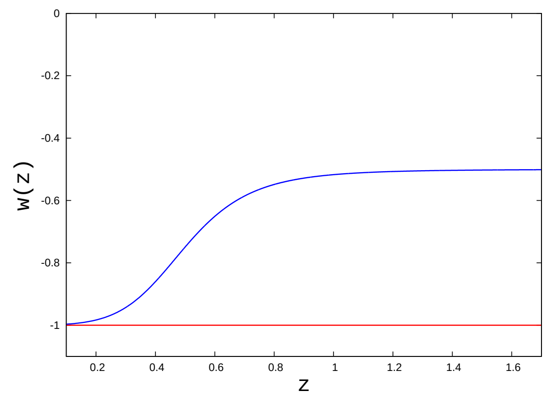
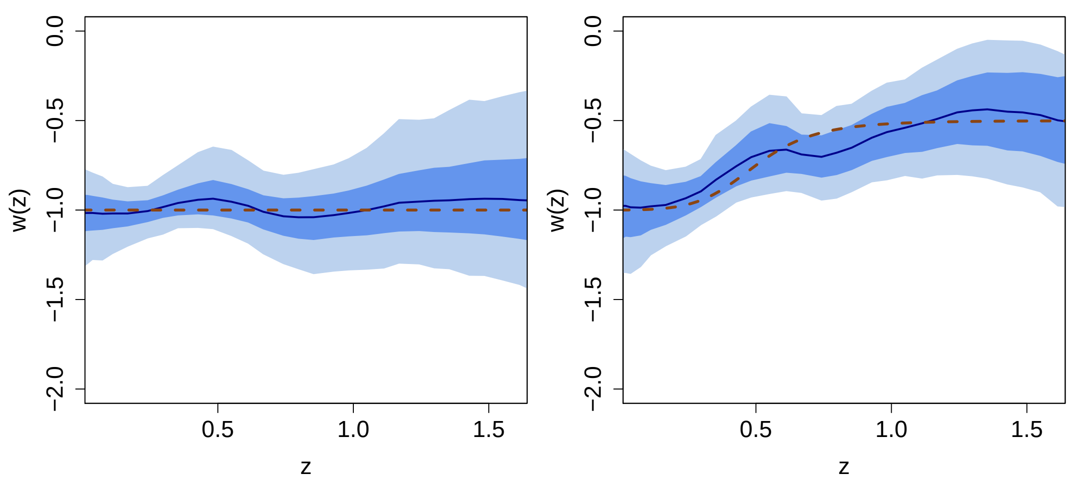
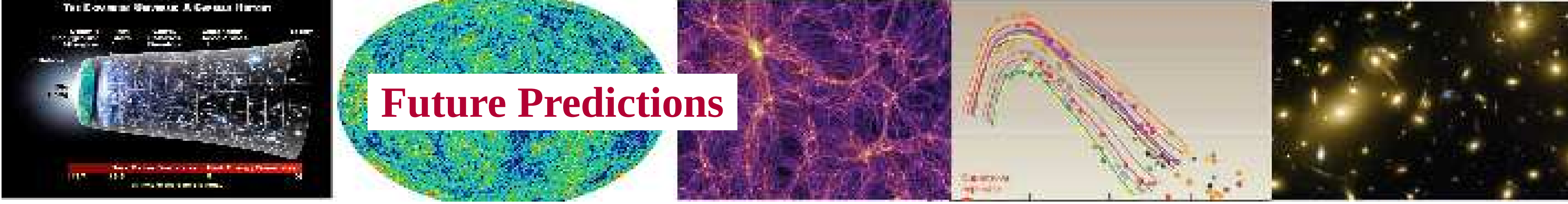


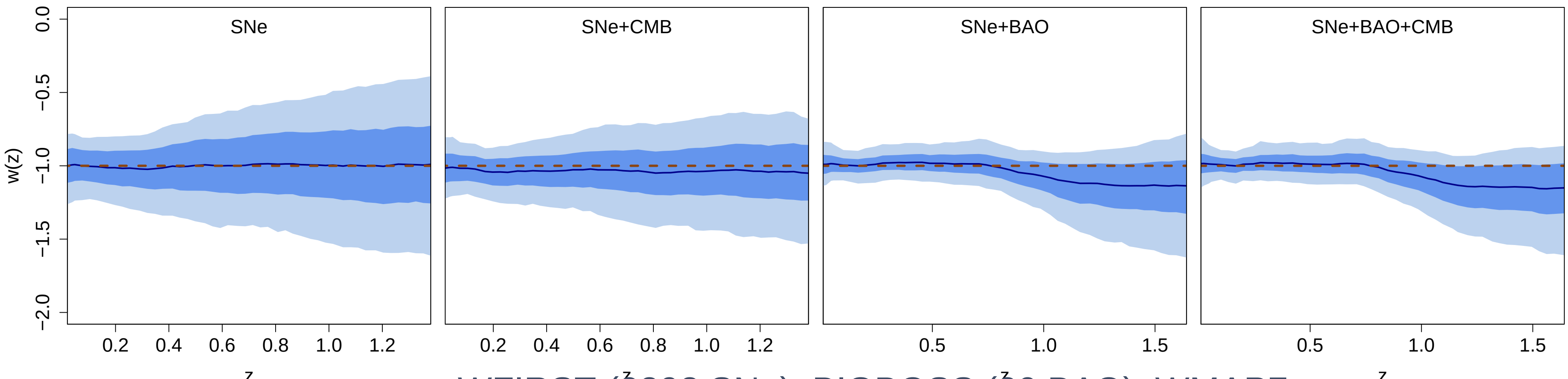
BIGBOSS (20 BAO)



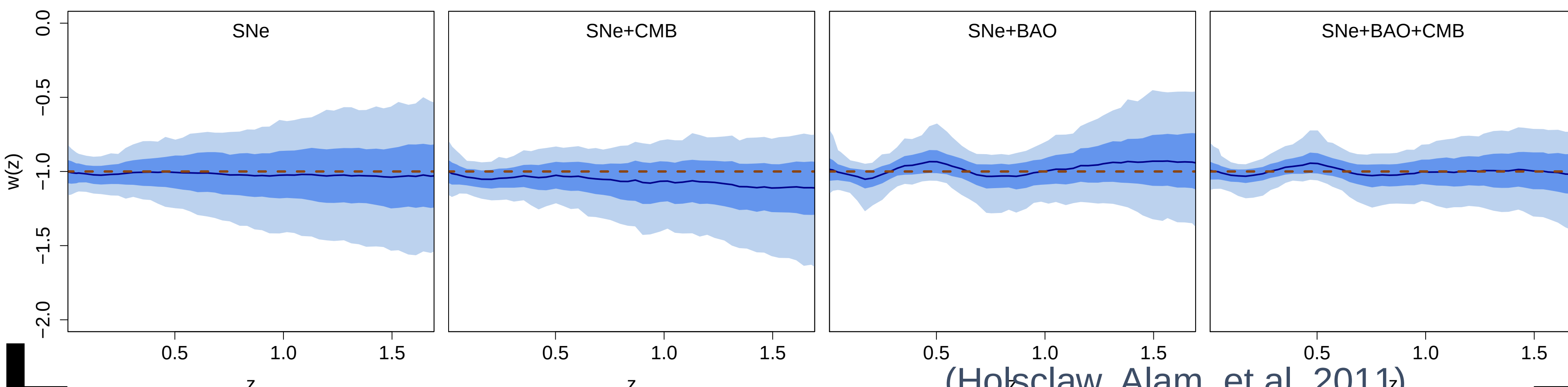


Future Predictions

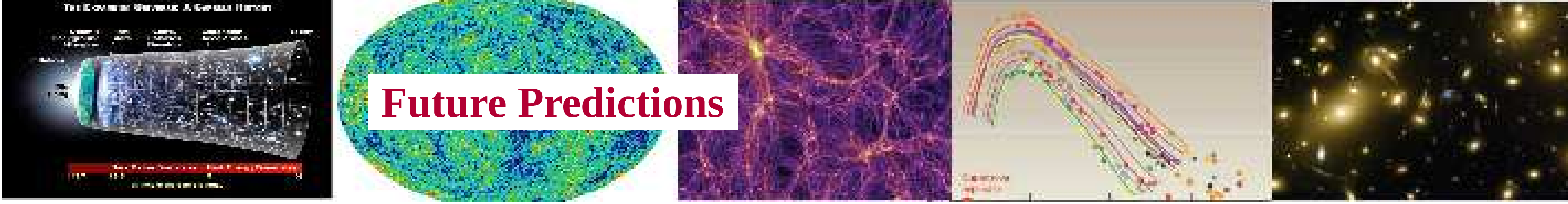
Union2 (557 SNe), BIGBOSS (20 BAO), WMAP7



WFIRST (2298 SNe), BIGBOSS (20 BAO), WMAP7



(Holsclaw, Alam, et.al, 2011)



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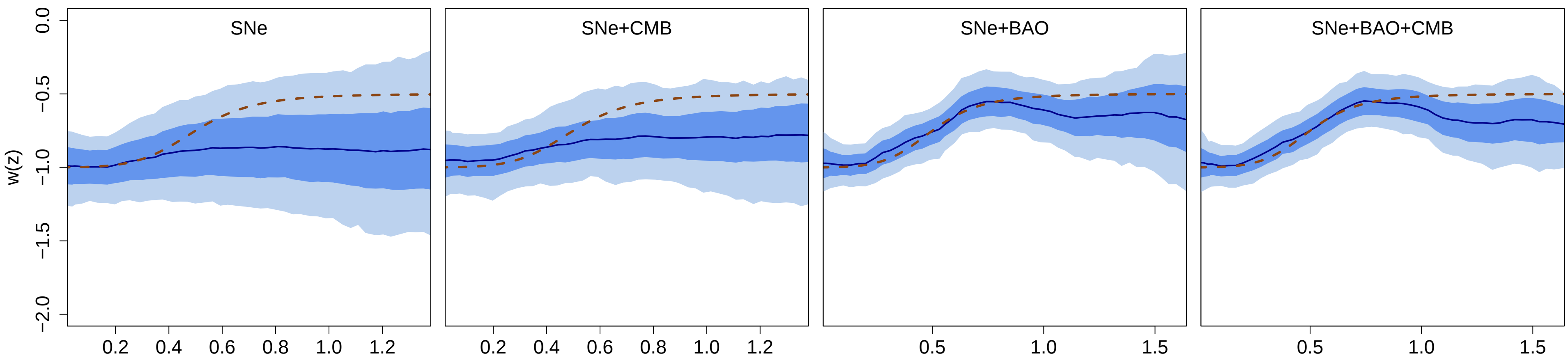

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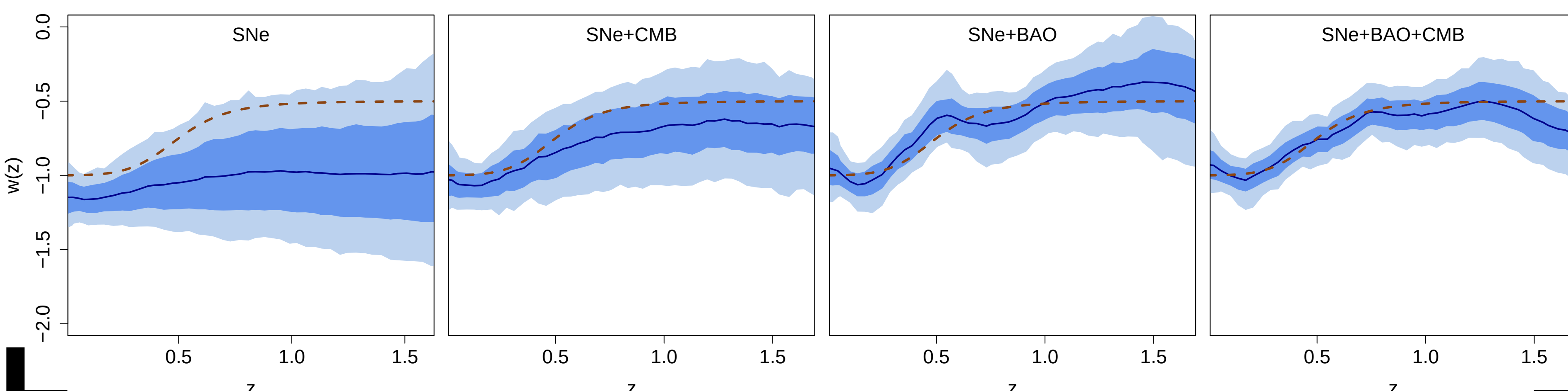

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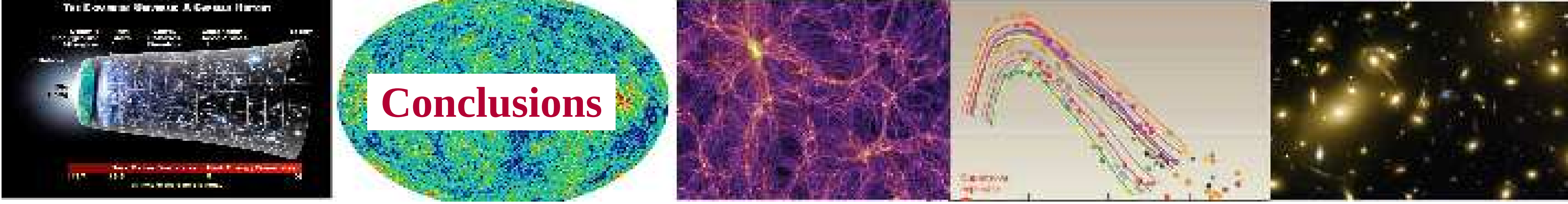

Conclusions

- SNe data alone– degeneracy between $\Omega_{0\mathrm{m}}$ and w_{DE}
- Combination of SNe, BAO, CMB consistent with ΛCDM
- As data quality improves, parametric methods inadequate to find subtle differences in w_{DE}
- Gaussian process modeling provides non-parametric, unbiased estimation of w_{DE}
- GP may provide effective importance of different datasets
- Next step : Effect of systematics on results

Perturbations from Distance measures

$$r(z) = H_0 \int_0^z \frac{dz_1}{H(z_1)} \Leftarrow \mu(z) \propto 5\log_{10} r(z)$$

$$ds^2 = a^2(\eta)\left[(1+2\Psi(\mathbf{x},\eta))d\eta^2 - (1+2\Phi(\mathbf{x},\eta))\delta_{ij}dx^i dx^j\right]$$

$$\ddot{\delta_m} + 2H\dot{\delta_m} - 4\pi G\rho_m\delta_m = 0$$

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$$\Downarrow$$

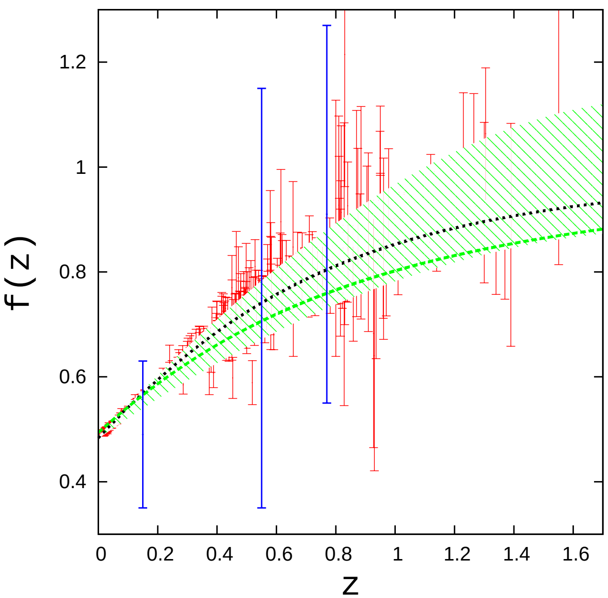
$$\delta_m(r) = 1 + \delta_0' \int_0^r [1+z(r_1)]dr_1 + \frac{3}{2}\Omega_{0\mathrm{m}} \int_0^r [1+z(r_1)] \int_0^{r_1} \delta_m(r_2)dr_2 dr_1$$

$$\delta_m'(r) = \delta_0'[1+z(r)] + \frac{3}{2}\Omega_{0\mathrm{m}}[1+z(r)] \int_0^r \delta_m(r_1)dr_1$$

$$\Downarrow$$

$$\boxed{f(z) = \frac{1+z}{H(z)}\frac{\delta_m'}{\delta_m}}$$

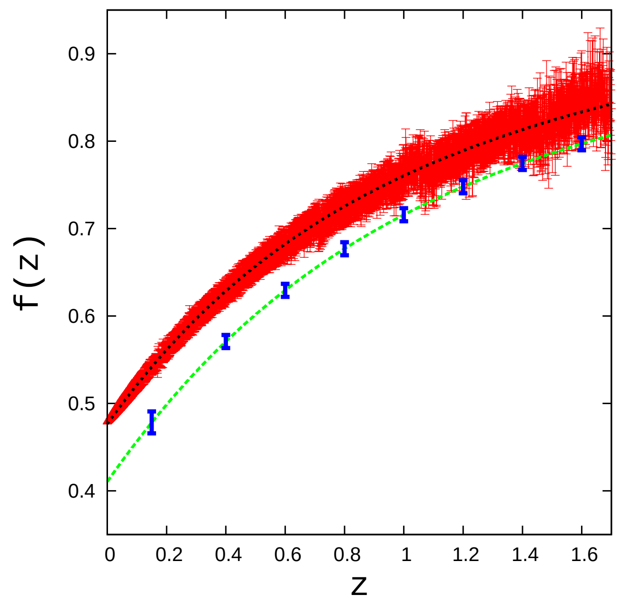
Discriminating physical and geometrical Dark Energy models



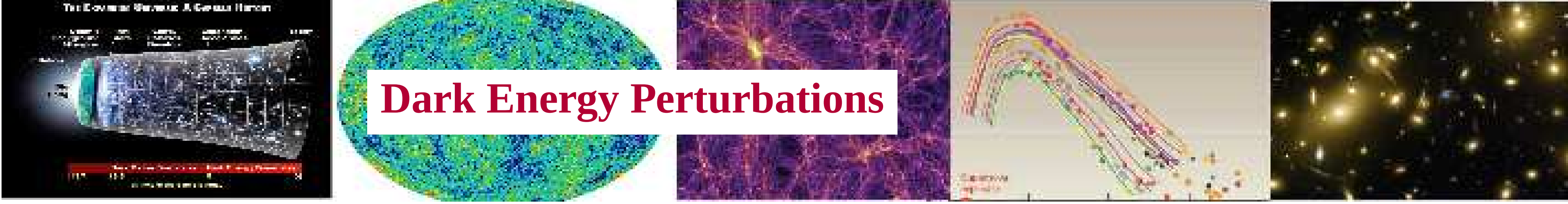
Discriminating physical and geometrical Dark Energy models

Toy Model : DGP with $f(z) = \Omega_m^{0.68}$

SNe data : WFIRST, Cluster data : Euclid



(Alam, Sahni, Starobinsky, 2009)



Dark Energy Perturbations

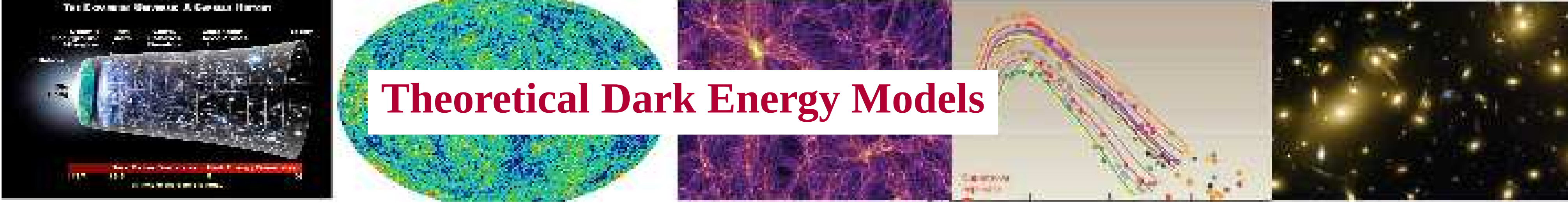
$$ds^2 = a^2(\eta)\left[(1+2\Psi(\mathbf{x},\eta))d\eta^2 - (1+2\Phi(\mathbf{x},\eta))\delta_{ij}dx^i dx^j\right]$$

Linearized Einstein equations for gauge-invariant perturbations of non-interacting components :

$$\delta_i' = -3\mathcal{H}(c_{s,i}^2 - w_i)\delta_i - \left[\frac{9\mathcal{H}^2}{k}(c_{s,i}^2 - c_{a,i}^2) + k\right](1+w_i)v_i - 3(1+w_i)h'$$

$$v_i' = -\mathcal{H}(1-3c_{s,i}^2)v_i - kc_{s,i}^2\frac{\delta_i}{1+w_i} - kA$$

$$\left(w_i = \frac{P_i}{\rho_i},\ c_{s,i}^2 = \frac{\delta P_i}{\delta \rho_i},\ c_{a,i}^2 = \frac{\dot{P}_i}{\dot{\rho}_i}\right)$$

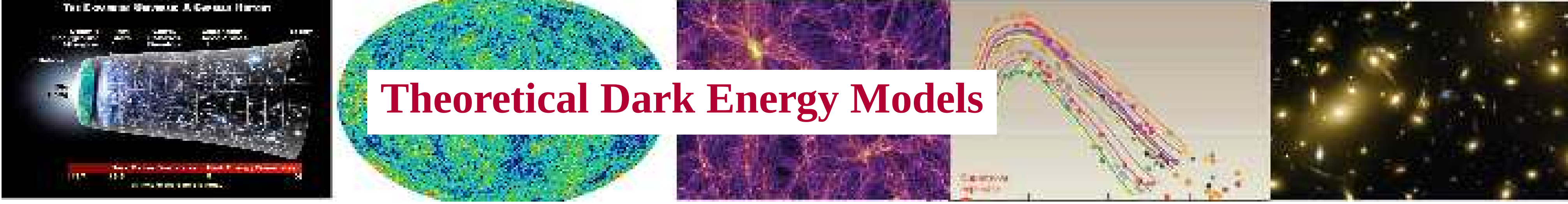


Theoretical Dark Energy Models

- Cosmological Constant : $w = -1$
- Quiessence : $-1 < w =$constant $< -1/3$
- Quintessence : $\mathcal{L} = \frac{1}{2}\partial_a\phi\,\partial^a\phi - V(\phi)$

$$V = V_0/\phi^\alpha$$
$$V = V_0 \exp(\lambda\phi^2)/\phi^\alpha$$
$$V = V_0(\cosh\lambda\phi - 1)^p$$

- Phantom fields with $w < -1$, Early Dark Energy Models
- k-essence : $\mathcal{L} = -V(\phi)\sqrt{1 - \partial_a\phi\,\partial^a\phi}$
  (Chaplygin gas : $P = -A/\rho^\alpha$)
- Modified gravity models : $f(r)$ theories, braneworld models....



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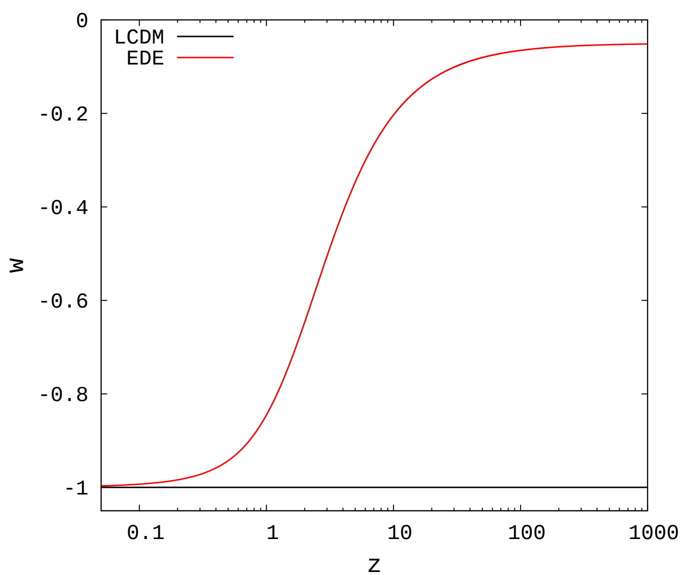
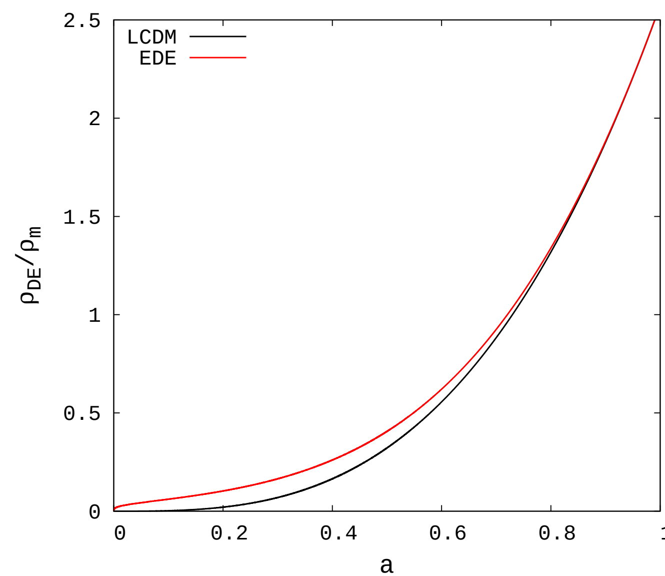
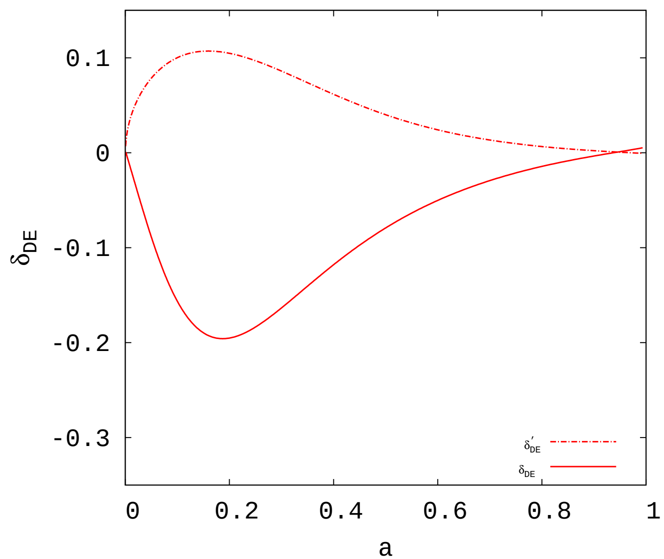
Early Dark Energy

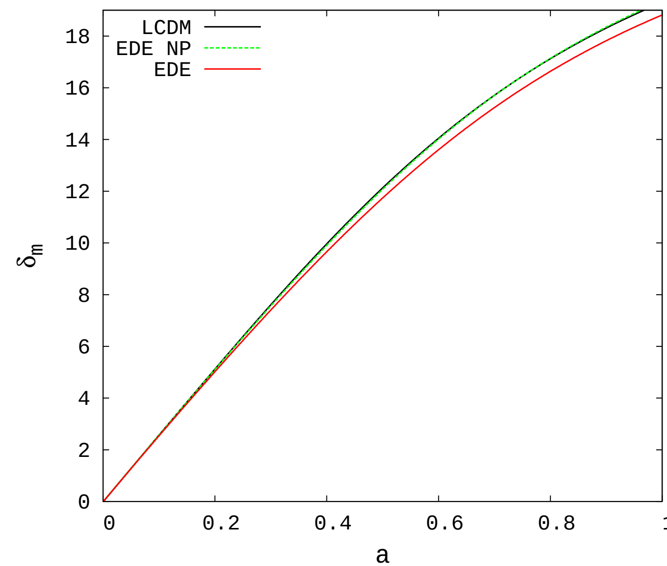




Growth of Perturbations

Low-k



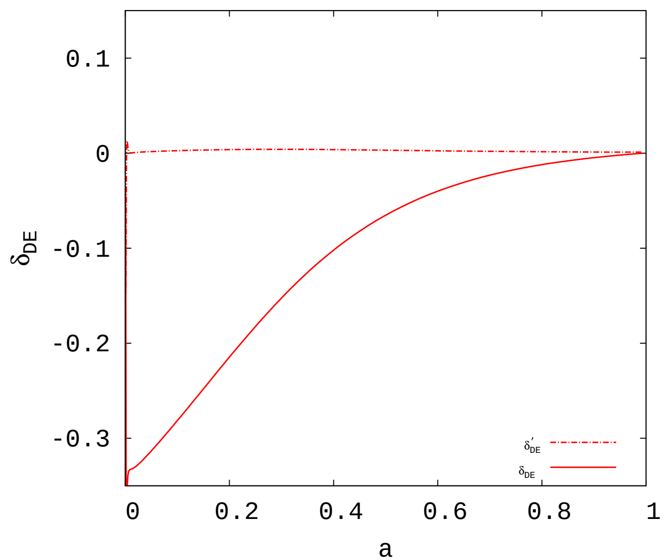


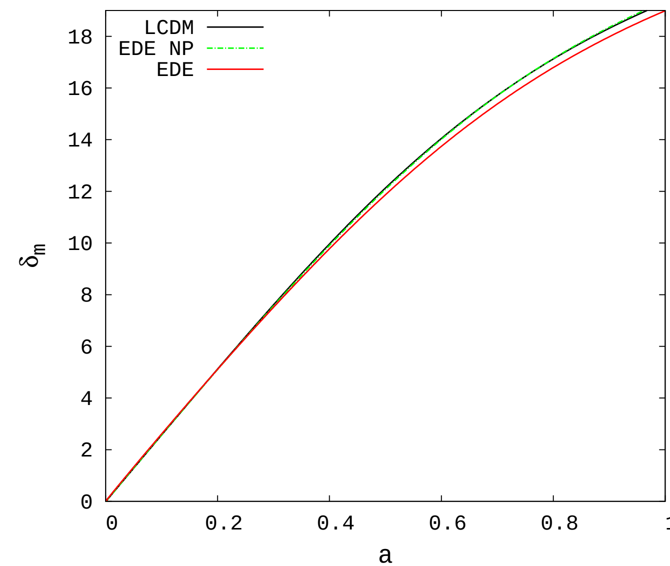
Dark Energy perturbations change rapidly

Matter perturbations lower than ΛCDM

Growth of Perturbations

High-k

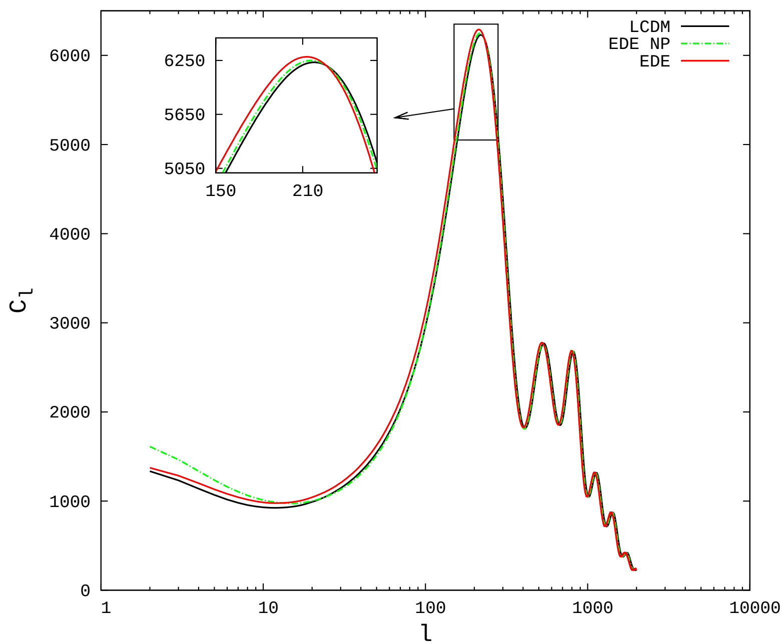


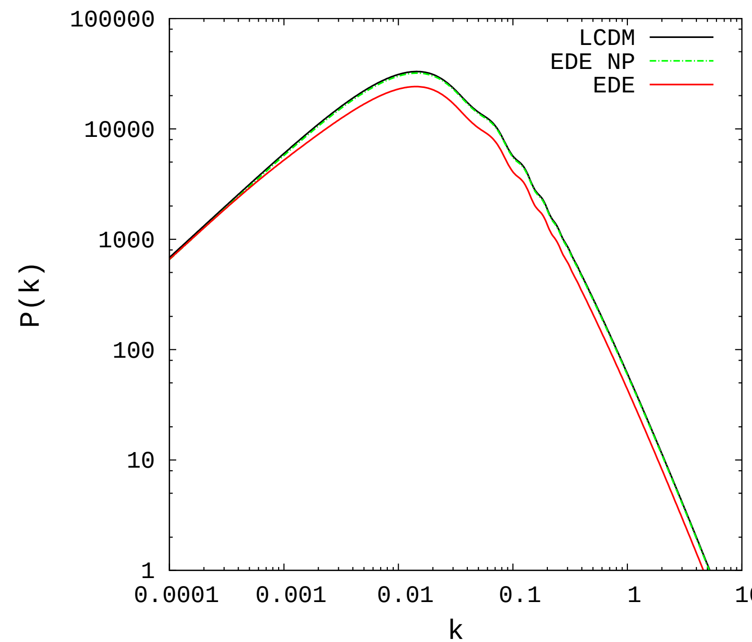


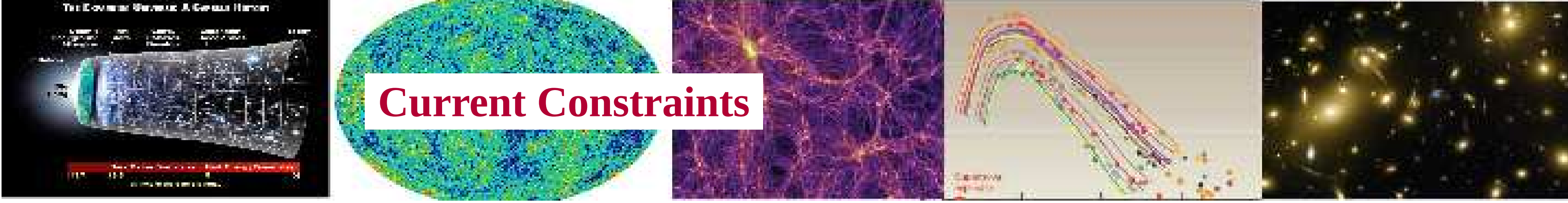
(f)

Dark Energy perturbations change slowly
Matter perturbations slightly higher than in low-k case

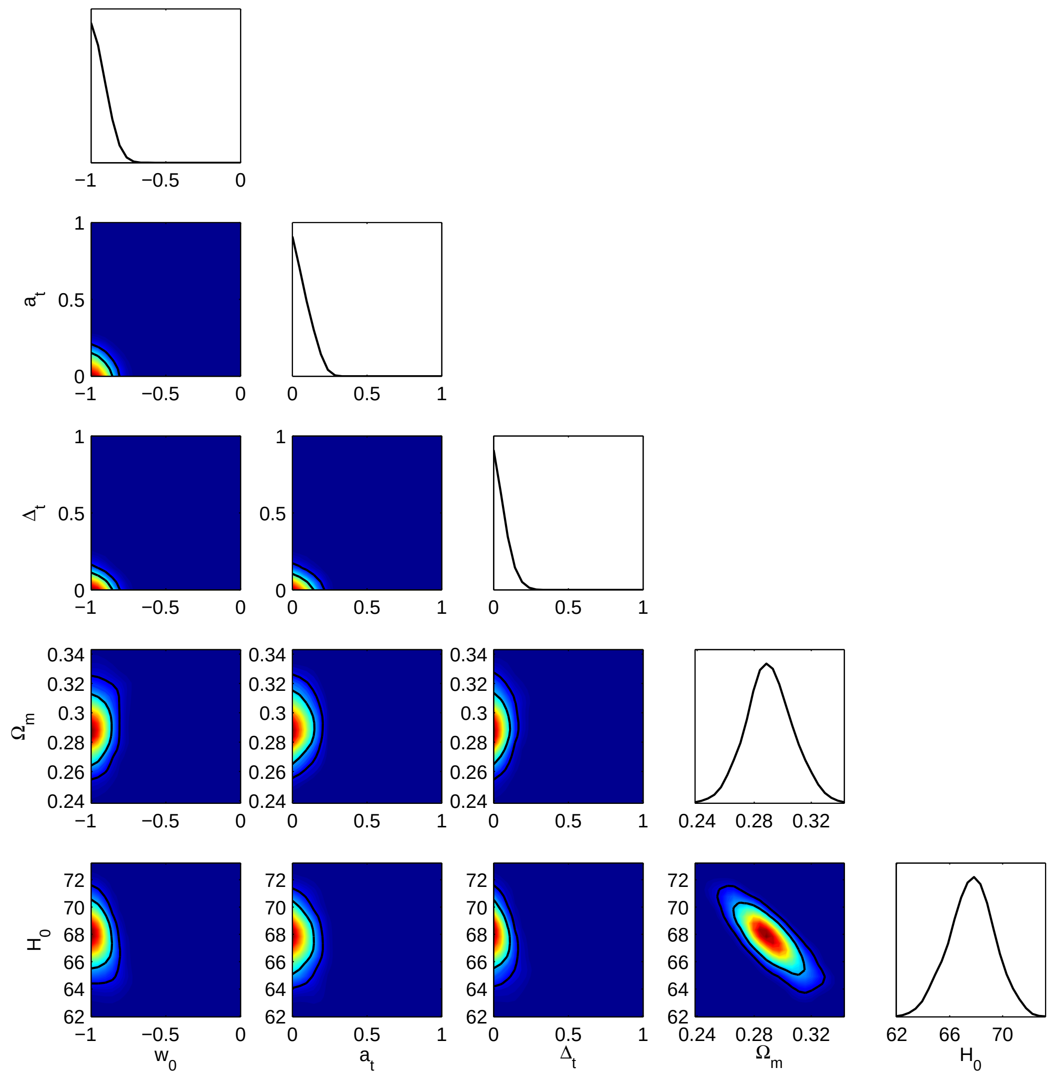
Effect of Dark Energy Perturbations





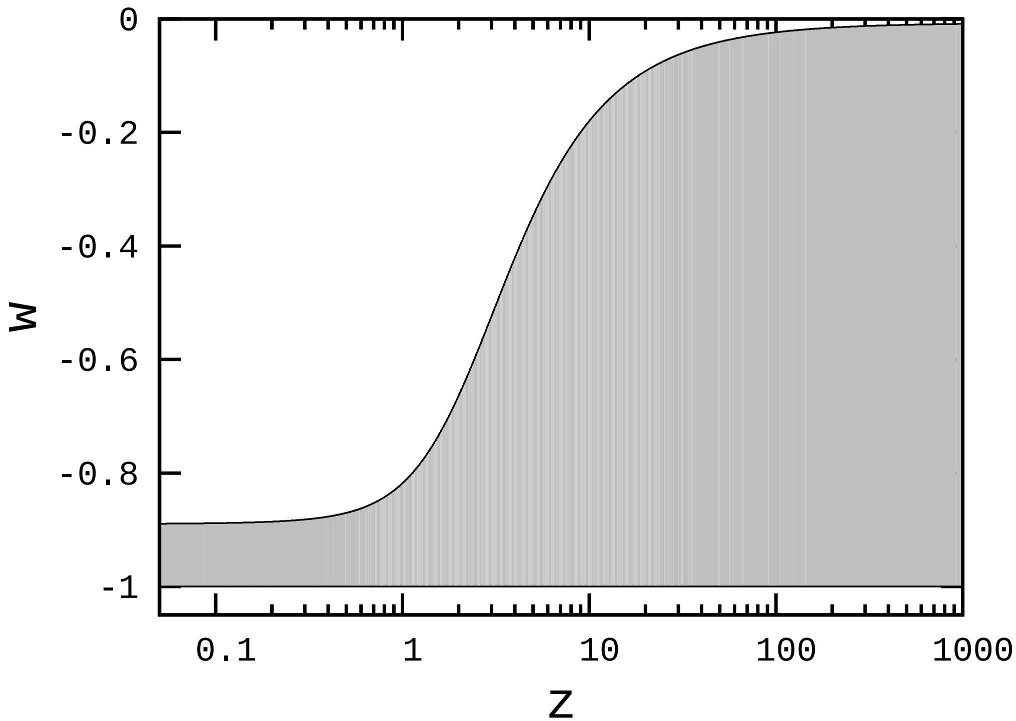


Current Constraints



(Alam, 2010)

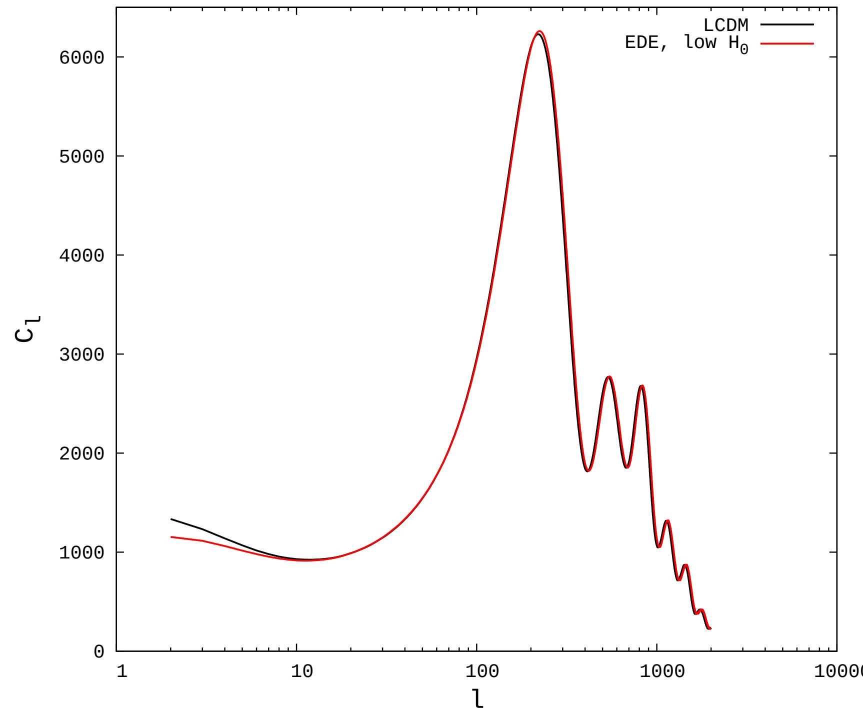
Current Constraints



At 95% CL :
equation of state today $w_0 < -0.9$,
redshift of transition $z_t > 4$,
width of transition $\Delta_t < 0.2$

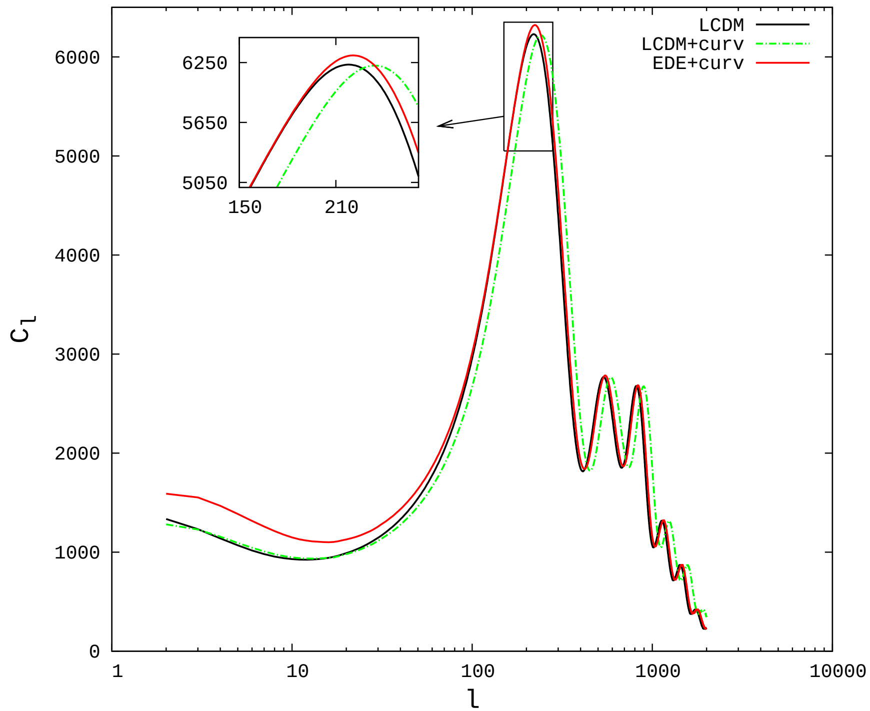
(Alam, 2010)

Degeneracy with H_0



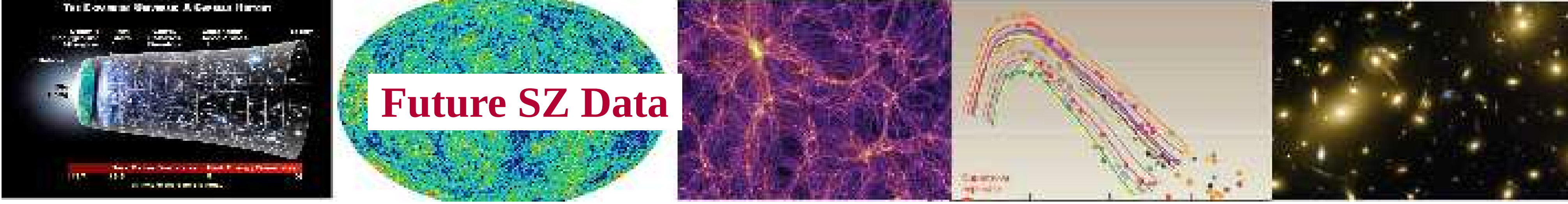
Complementary measurements of H_0 (e.g. SHOES)

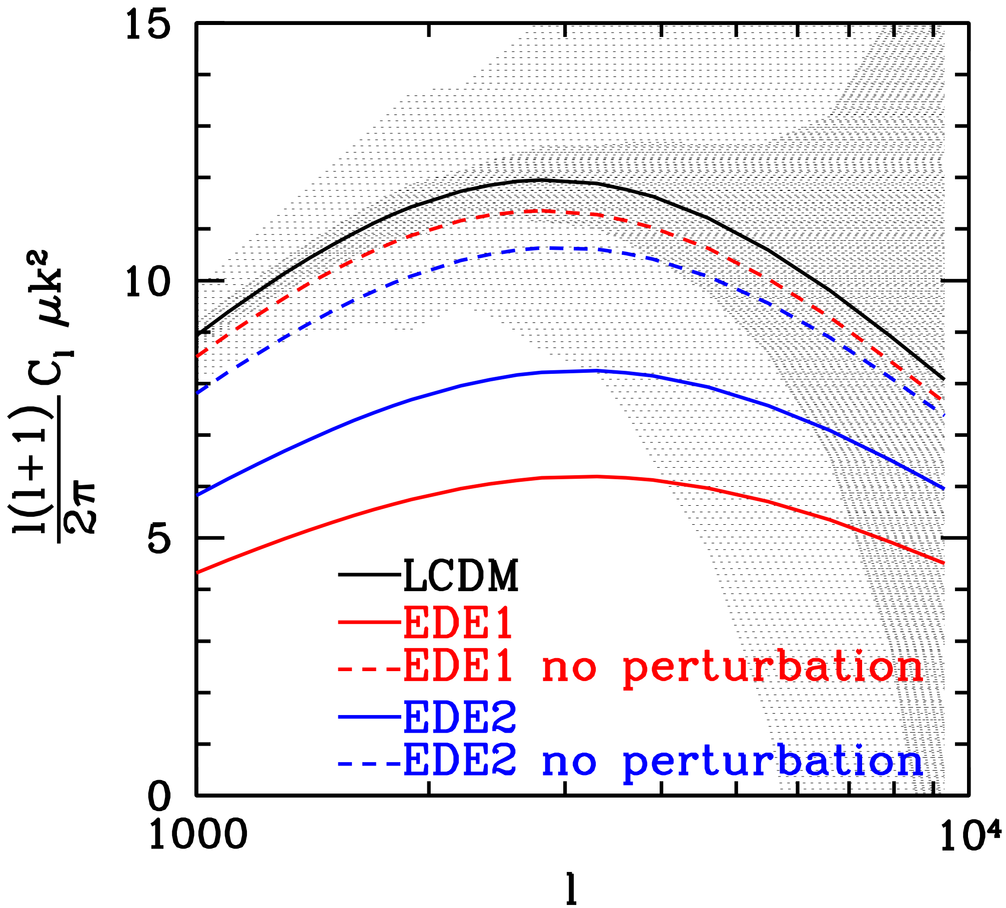
Degeneracy with Ω_κ



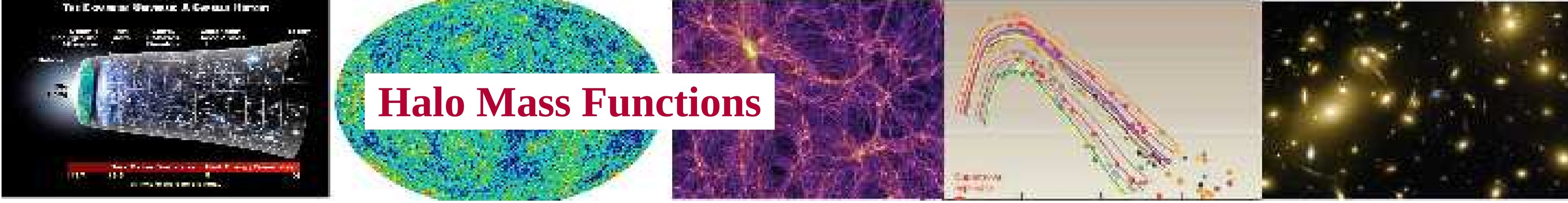
At 95% confidence level :
equation of state today, $w_0 < -0.77$
redshift of transition, $z_t < 1.8$
width of transition, $\Delta_t < 0.35$
$-0.014 < \text{curvature of universe}, \Omega_\kappa < 0.031$

Future SZ Data

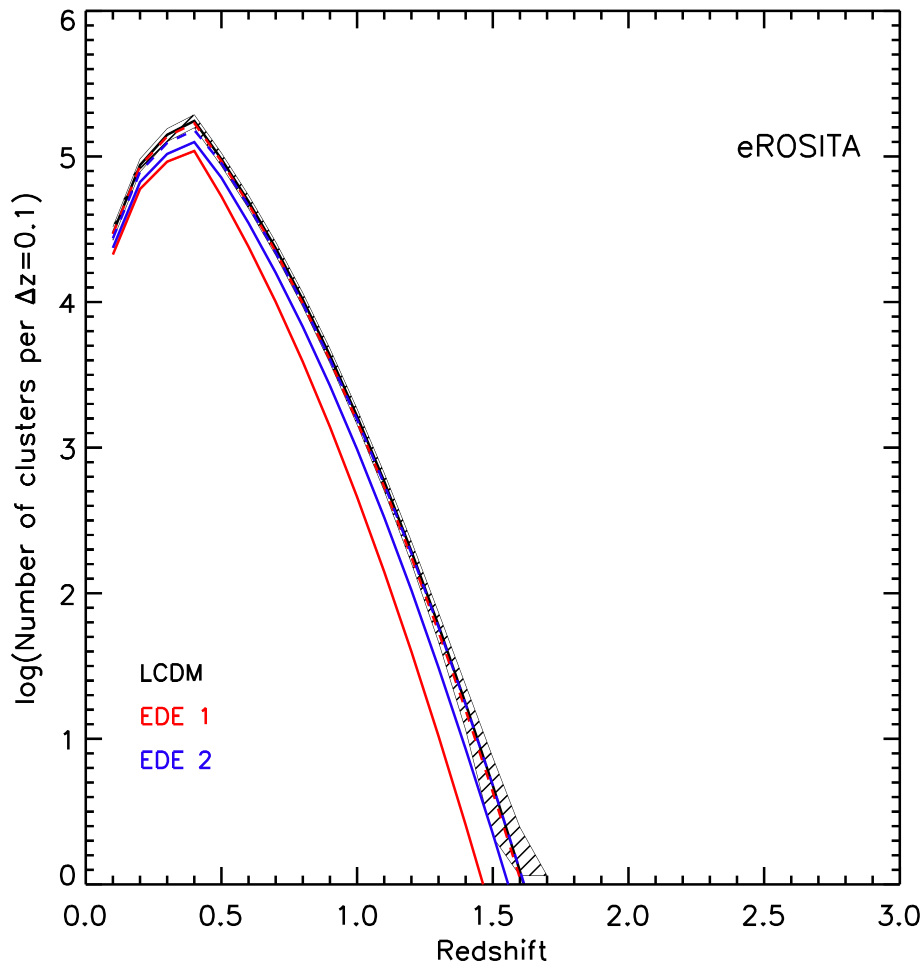




(Alam, Lukic, Bhattacharya, 2011)



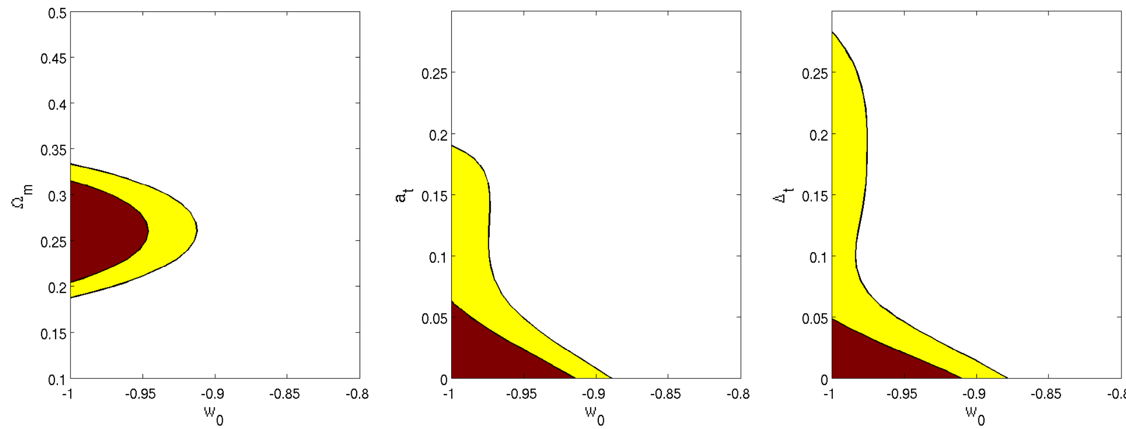
Halo Mass Functions



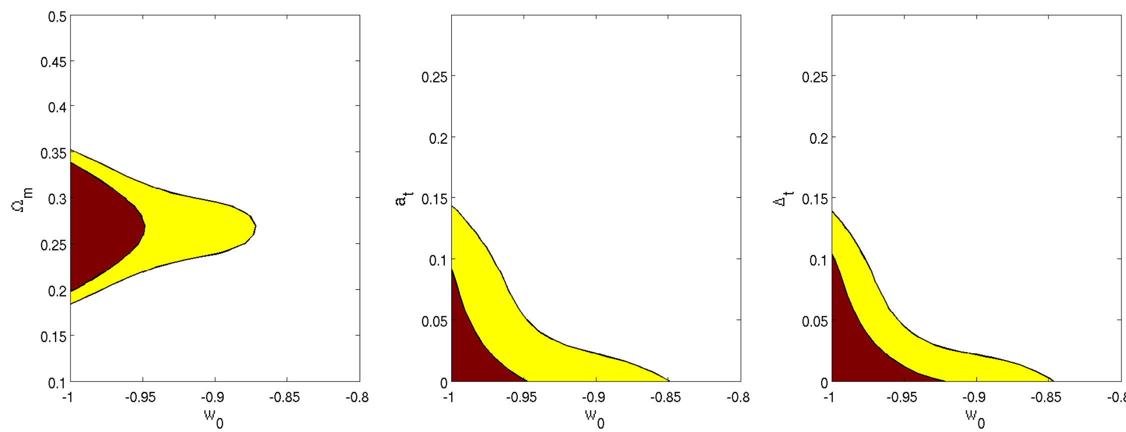
(Alam, Lukic, Bhattacharya, 2011)

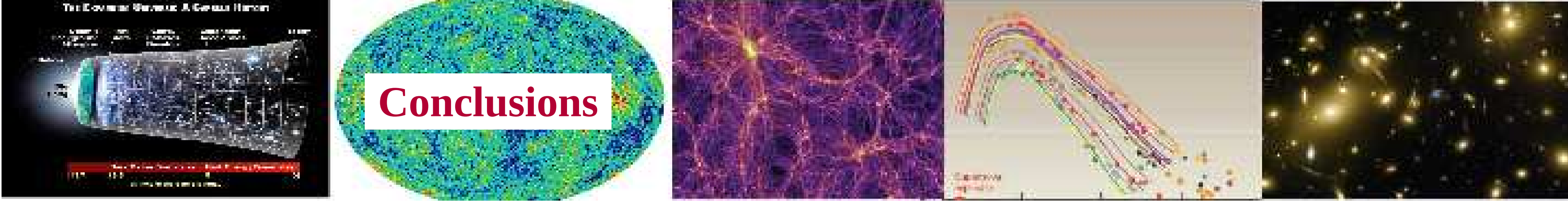
Constraints from Planck+ACT/SPT

Cluster Counts



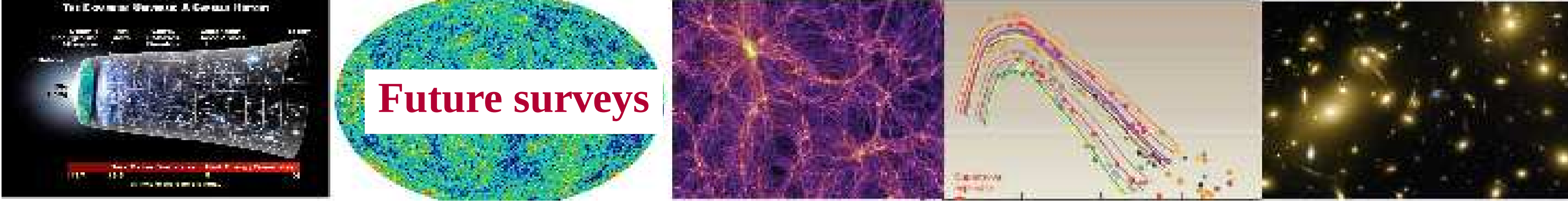
SZ data



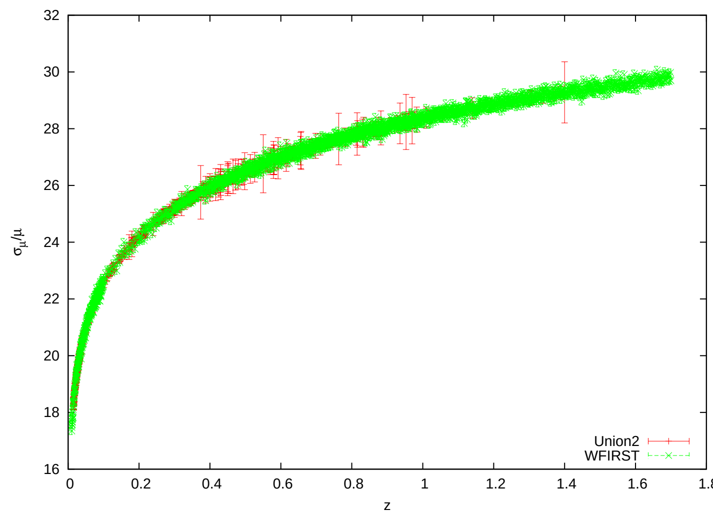


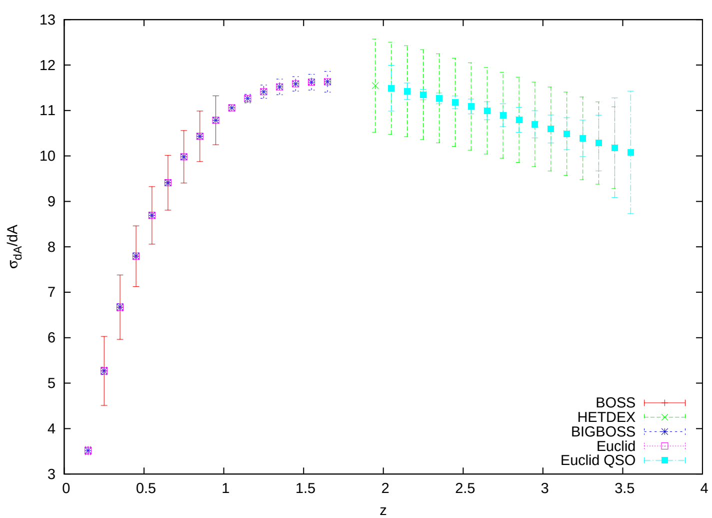
Conclusions

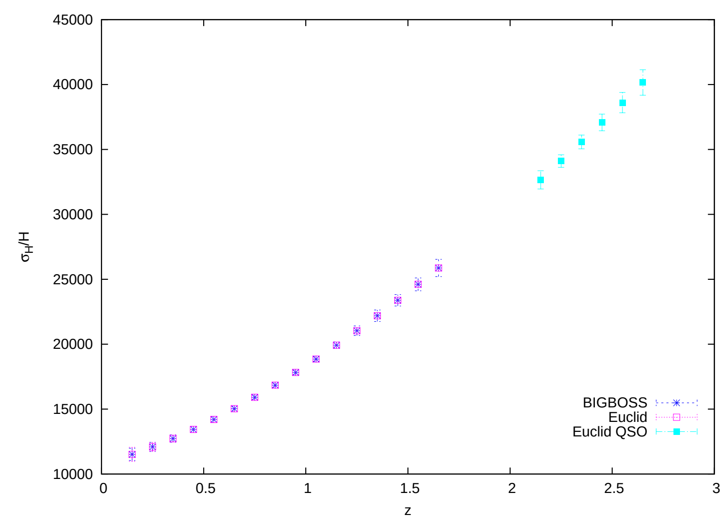
- Dark Energy perturbations have significant effect in certain Dark Energy models
- Current data already rules out a large portion of these early dark energy models
- Future galaxy surveys may put stronger constraints on these models
- Next step : effect on non-linear perturbations

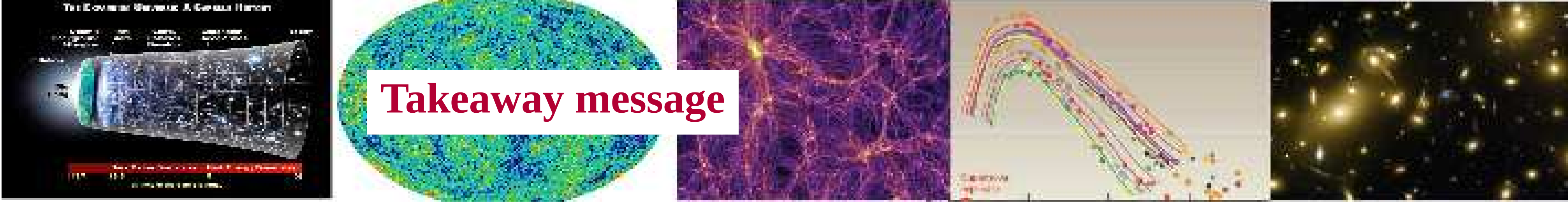


Future surveys









Takeaway message

- A lot of data will be available in the future for Dark Energy
- Distance measures and perturbative measures are complementary and break the degeneracy in Dark Energy models
- More sophisticated statistical methods will be required to extract information from future data
- Biggest roadblock : Systematics in the observations