Non-Gaussianity of superhorizon curvature perturbations beyond δN-formalism

Resceu, University of Tokyo

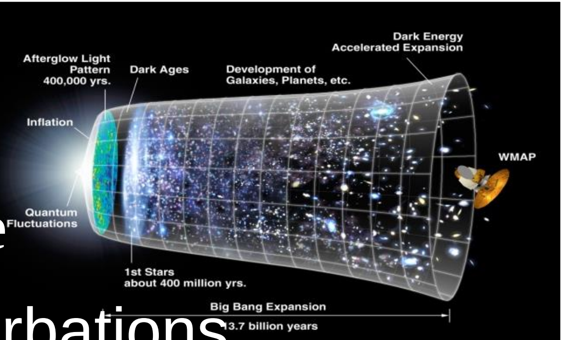
Yuichi Takamizu

Collaborator: Shinji Mukohyama (IPMU,U of Tokyo),
Misao Sasaki & Yoshiharu Tanaka (YITP,Kyoto U)

Ref In progress & JCAP01 013 (2009）

◆ Introduction

● Inflationary scenario in the early universe



- Generation of primordial density perturbations
- **Seeds** of large scale structure of the universe

□ The primordial fluctuations generated from Inflation are one of *the most interesting prediction of quantum theory of fundamental physics.*

However, we have *known little* information about Inflation

□ More accurate observations give more information about primordial fluctuations

- PLANCK (2009-)

Will detect second order primordial perturbations

◆ Non-Gaussianity from Inflation

$$\boxed{\zeta(\mathrm{x}) = \zeta_G(\mathrm{x}) + \frac{3}{5} f_{NL} \zeta_G^2(\mathrm{x})}$$

Komatsu & Spergel (01)

◆ Introduction

● Non-Gaussianity from *inflation*

Bispectrum　　Power spectrum

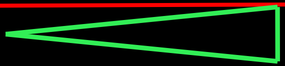
$$f_{NL}(\vec{k}) \approx \frac{B_{\mathcal{R}}}{P_{\mathcal{R}}^2}\frac{\square k_i^3}{\sum k_i^3}$$

Mainly two types of bispectrum:

■ WMAP 7-year

■ Local type $\zeta(x) = \zeta_G(x) + \frac{3}{5} f_{NL}^{\text{local}} \zeta_G^2(x)$

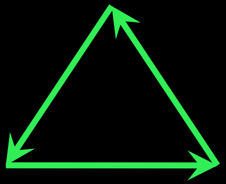
$$\boxed{-10 < f_{\rm NL}^{\rm local} < 74}$$



・・・Multi-field, Curvaton

■ Equilateral type $f_{NL}^{\text{equil}}(k_1,k_2,k_3) = \frac{10}{3}\frac{\mathcal{A}_{NL}}{\sum_i k_i^3}$

$$\boxed{-214 < f_{\rm NL}^{\rm equil} < 266}$$



$$\mathcal{A}_{NL} \propto \frac{1}{8}\sum_i k_i^3 - \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3$$

with $K \equiv k_1 + k_2 + k_3.$

・・・Non-canonical kinetic term

Cf) Orthogonal type: generated by higher derivative terms (Senatore etal 2010)

Introduction

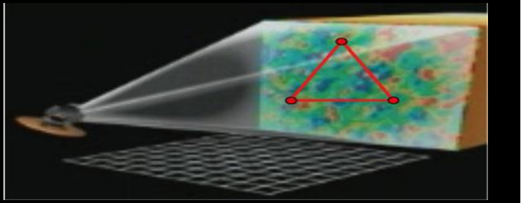
✓ If we detect non-Gaussianity, what we can know about Inflation ?

- *Slow-rolling ?*
- *Single field or multi-field ?*
- *Canonical kinetic term ?*
- *Bunch-Davies vacuum ?*

- ***Theoretical side***
  - Standard single slow-roll scalar $f_{\rm NL} = O(10^{-2})$
  - Many models predicting Large Non-Gaussianity
    （Multi-fields, DBI inflation and Curvaton） $f_{\rm NL} \gg O(1)$
- ***Observational side***

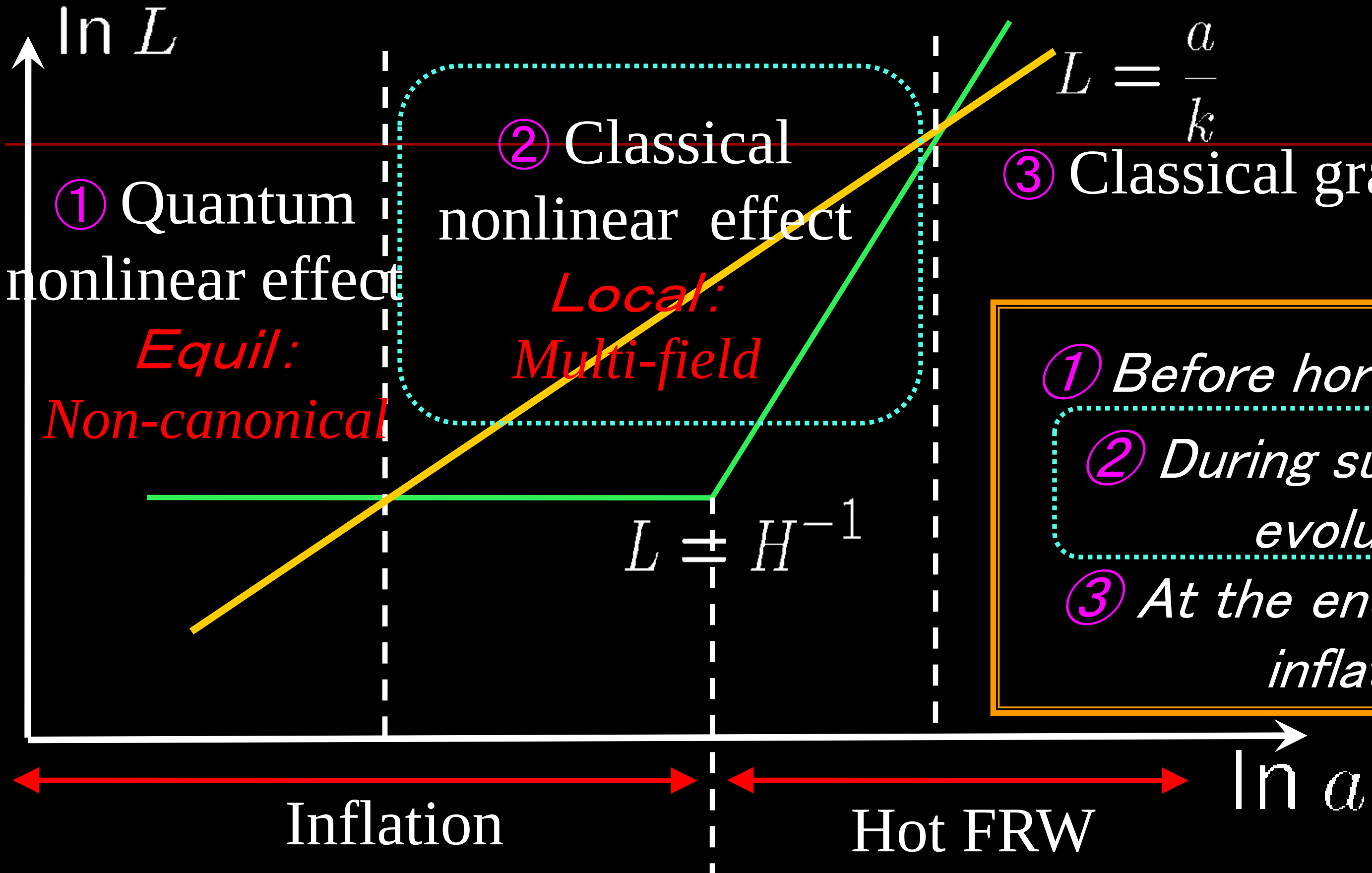
PLANCK   2009-

Can detect within $|f_{\rm NL}| \gtrsim 5$



- *Non-Gaussianity (nonlinearity) will be one of powerful tool to discriminate many possible inflationary models with the current and future precision observations*

◆ Three stages for generating Non-Gaussianity



$\ln L$

① Quantum nonlinear effect

Equil: Non-canonical

② Classical nonlinear effect

Local: Multi-field

$L = \frac{a}{k}$

③ Classical gravity effect

$L = H^{-1}$

$\ln a$

Inflation

Hot FRW

① Before horizon crossing

② During superhorizon evolution

③ At the end of or after inflation

● ***δN-formalism***

(Starobinsky 85, Sasaki & Stewart 96)

● *Nonlinear perturbations on superhorizon scales*

- *Spatial gradient approach* : $\boxed{\epsilon = 1/(HL)}$ Salopek & Bond (90)
  - ~~Spatial derivatives are small compared to time derivative~~
  - Expand Einstein eqs in terms of small parameter ε, and can solve them for nonlinear perturbations iteratively

◆ *δN formalism* *(Separated universe)*

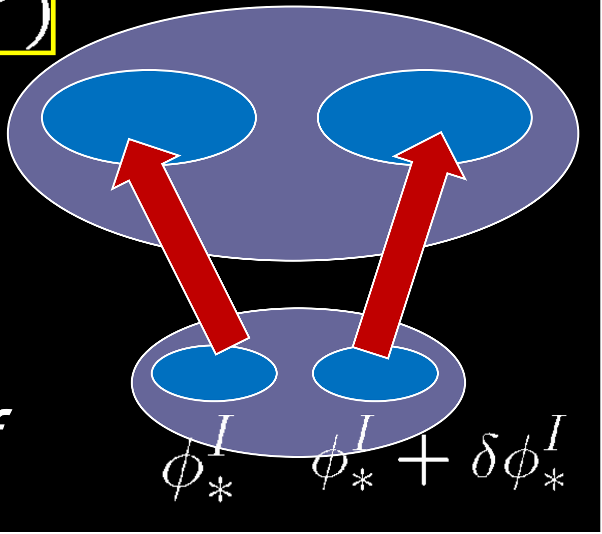
(Starobinsky 85, Sasaki & Stewart 96)

$$\boxed{\zeta(t,\mathrm{x}) = \delta N \equiv N(t,\mathrm{x}) - N_0(t)}$$

$$\boxed{O(\epsilon^0)}$$

$$\delta N = N_I^* \delta\phi_*^I + \frac{1}{2} N_{IJ}^* \delta\phi_*^I \delta_*^J + \cdots$$

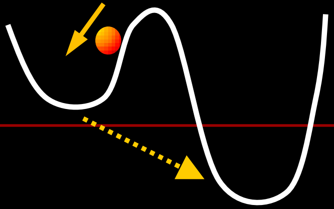
Curvature perturbation = Fluctuations of the local e-folding number



◇ *Powerful tool for the estimation of NG*

◆ *Temporary violating of slow-roll condition*

- e.g. Double inflation, false vacuum inflation



◆ *δN formalism* $O(\epsilon^0)$

- Adiabatic (Single field) curvature perturbations on superhorizon are *Constant* $\zeta(t,\mathbf{x}) = \mathrm{const}$
- Independent of Gravitational theory (Lyth, Malik & Sasaki 05)
- Ignore the **decaying mode** of curvature perturbation

◆ *Beyond δN formalism* $O(\epsilon^2)$

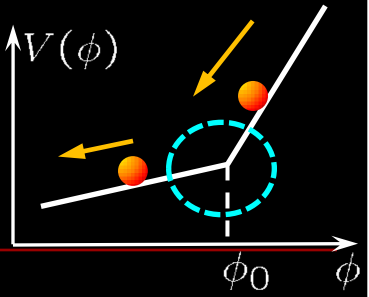
- $O(\epsilon^2)$ is known to play a crucial role in this case
- This order correction leads to a **time dependence** on superhorizon
- **Decaying modes** cannot be neglected in this case
- **Enhancement** of curvature perturbation in the **linear theory**

[Seto et al (01), Leach et al (01)]

◆ Example

● Starobinsky's model (92)

$$V(\phi)=\begin{cases} V_0+A_+(\phi-\phi_0) & \text{for } \phi>\phi_0\\ V_0+A_-(\phi-\phi_0) & \text{for } \phi<\phi_0\end{cases}$$



● Leach, Sasaki, Wands & Liddle (01)

- **There is a stage at which slow-roll conditions are violated**
- **Enhancement of the curvature perturbation near superhorizon**

- Linear theory
- The $O(\epsilon^2)$ in the expansion

Initial: $u - v \ll u$ ⇒ Final: u

Growing mode decaying mode

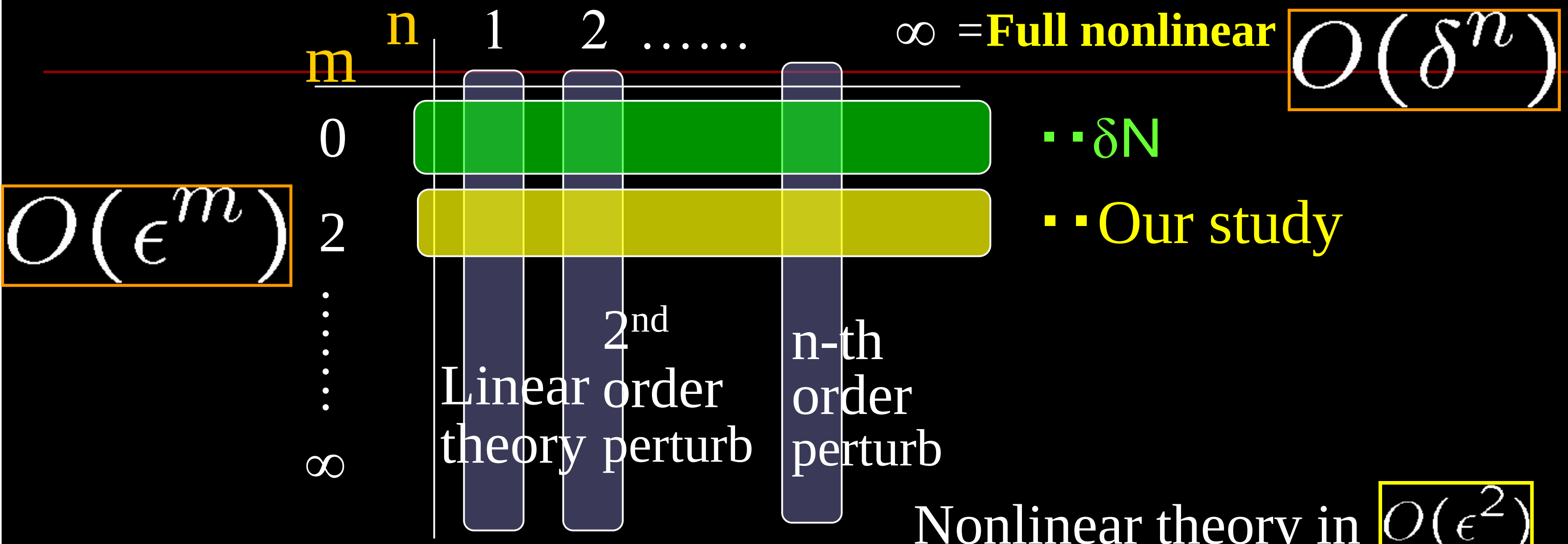
⇒ $\boxed{\mathcal{R}(0)=\alpha^{\text{Lin}}\mathcal{R}(\eta_*)}$

$\boxed{\text{at } \eta=0 \text{ (Final)}}$ $\boxed{\text{at } \eta_* \text{ (initial)}}$

$$\alpha^{\text{Lin}}\simeq 1+\overset{\substack{O(\epsilon^2)\\ \text{Decaying mode}}}{D(\eta_*)}+\overset{\substack{O(\epsilon^2)\\ \text{Growing mode}}}{F(\eta_*)}$$

◆ *Enhancement of curvature perturbation near* $\boxed{\phi=\phi_0}$

● *Nonlinear perturbations on superhorizon scales up to Next-leading order in the expansion*



∞ =**Full nonlinear** $O(\delta^n)$

$O(\epsilon^m)$

··δN

··Our study

Linear theory perturb

2nd order perturb

n-th order perturb

Nonlinear theory in $O(\epsilon^2)$

Simple result !

$$\mathcal{R}_c^{\mathrm{NL}''} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{NL}'} + \frac{c_s^2}{4}K^{(2)}[\mathcal{R}_c^{\mathrm{NL}}] = O(\epsilon^4)$$

Linear theory

$$\mathcal{R}_c^{\mathrm{Lin}''} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}'} + k^2 c_s^2 \mathcal{R}_c^{\mathrm{Lin}} = 0$$

System : $$\boxed{I = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G_N} + P(X,\phi)\right],}$$

Y.T and S. Mukohyama
JCAP01 (2009)

$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

- *Single scalar field with general potential & kinetic term* including K- inflation & DBI etc

◆ Scalar field in a perfect fluid form

$$T_{\mu\nu} = 2P_X\partial_\mu\phi\partial_\nu\phi + Pg_{\mu\nu} = (\rho + P)u_\mu u_\nu + Pg_{\mu\nu},$$

$$\boxed{\rho(X,\phi) = 2P_X X - P, \quad u_\mu = -\frac{\partial_\mu\phi}{\sqrt{X}}.}$$

● *The difference between a perfect fluid and a scalar field*

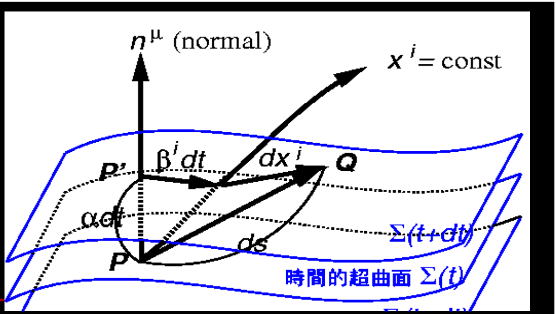
$$\underline{\delta P = c_s^2\delta\rho + \rho\Gamma\delta\phi,} \qquad \boxed{c_s^2 = \frac{P_X}{2P_{XX}X + P_X}, \quad \Gamma = \frac{1}{\rho}\left(P_\phi - c_s^2\rho_\phi\right).}$$

For a perfect fluid（adiabatic）

$$\delta P = c_s^2\delta\rho \quad c_s^2 = \dot{P}/\dot{\rho}$$

The propagation speed of perturbation（speed of sound）

● ADM decomposition



$$ds^2 = (-\alpha^2 + \beta_k \beta^k) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

- *Gauge degree of freedom*

EOM: Two constraint equations ①Hamiltonian & Momentum constraint eqs

- *EOM for* γ_{ij} *: dynamical equations*

In order to express as a set of first-order diff eqs,
Introduce the extrinsic curvature

$$K_{ij} = -\frac{1}{2\alpha}(\partial_t \gamma_{ij} - D_i \beta_j - D_j \beta_i)$$

② Evolution eqs for K_{ij}, γ_{ij}

- *Conservation law* $T^{\mu\nu}{}_{;\mu} = 0$ ③ Energy momentum conservation eqs

Further decompose,

② Four Evolution eqs

- Spatial metric $\gamma_{ij} = a^2 \psi^4 \tilde{\gamma}_{ij}$; $\det(\tilde{\gamma}_{ij}) = 1$
- Extrinsic curvature $K_{ij} = \frac{\gamma_{ij}}{3} K + \psi^4 a^2 \tilde{A}_{ij}$ ← Traceless part

Gradient expansion

Small parameter: $\boxed{\epsilon = 1/(HL)}$ $L \sim O(1/\epsilon)$ $\partial_i \psi = \psi \times O(\epsilon)$

◆ Background is the flat FLRW universe

$$\boxed{H^2 = \frac{\kappa^2}{3}\rho_0, \qquad \frac{2}{a^3}\partial_t\left(a^3 P_{0X}\partial_t\phi_0\right) - P_{0\phi} = 0,}$$

$P_{0X} \equiv P_X(X_0, \phi_0),\ P_{0\phi} \equiv P_\phi(X_0, \phi_0)$ $X_0 \equiv (\partial_t\phi_0)^2$

◆ Basic assumptions

$$\beta^i = O(\epsilon), \qquad v^i = O(\epsilon), \qquad \partial_t\tilde{\gamma}_{ij} = O(\epsilon)$$

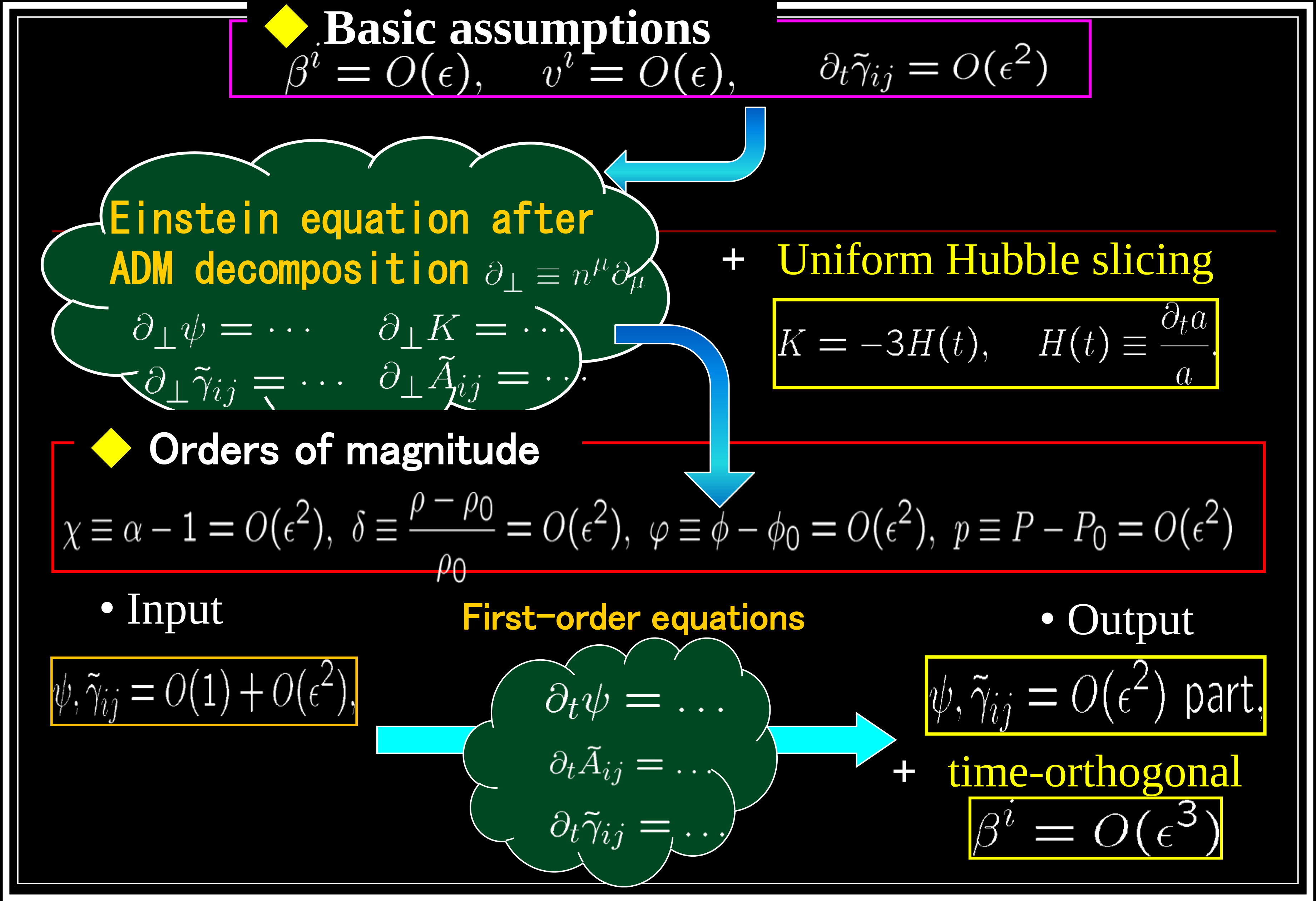
◆ Since the flat FLRW background is recovered in the limit $\epsilon \to 0$

$\boxed{\partial_t\tilde{\gamma}_{ij} = O(\epsilon)}$ ⇒ ● Assume a stronger condition $\boxed{\partial_t\tilde{\gamma}_{ij} = O(\epsilon^2)}$

- ➢ It can simplify our analysis
- ➢ Absence of any decaying at leading order
- ➢ Can be justified in most inflationary model with $N \gg 60$

◆ Under general cond (Hamazaki 08) $\partial_t\tilde{\gamma}_{ij} = O(\delta_c) = O(\epsilon^0)$ δ_c : Amplitude of decaying mode

$\boxed{\delta \propto \frac{1}{a^6} + O(\epsilon^2)}$: decaying mode of density perturbation



◆ Basic assumptions

$$\beta^i = O(\epsilon), \quad v^i = O(\epsilon), \quad \partial_t\tilde{\gamma}_{ij} = O(\epsilon^2)$$

Einstein equation after ADM decomposition $\partial_\perp \equiv n^\mu\partial_\mu$

$$\partial_\perp\psi = \cdots \quad \partial_\perp K = \cdots$$

$$\partial_\perp\tilde{\gamma}_{ij} = \cdots \quad \partial_\perp\tilde{A}_{ij} = \cdots$$

+ Uniform Hubble slicing

$$\boxed{K = -3H(t), \quad H(t) \equiv \frac{\partial_t a}{a}.}$$

◆ Orders of magnitude

$$\chi \equiv \alpha - 1 = O(\epsilon^2), \ \delta \equiv \frac{\rho - \rho_0}{\rho_0} = O(\epsilon^2), \ \varphi \equiv \phi - \phi_0 = O(\epsilon^2), \ p \equiv P - P_0 = O(\epsilon^2)$$

- Input

$$\boxed{\psi, \tilde{\gamma}_{ij} = O(1) + O(\epsilon^2),}$$

First-order equations

$$\partial_t\psi = \dots$$

$$\partial_t\tilde{A}_{ij} = \dots$$

$$\partial_t\tilde{\gamma}_{ij} = \dots$$

- Output

$$\boxed{\psi, \tilde{\gamma}_{ij} = O(\epsilon^2) \text{ part},}$$

+ time-orthogonal

$$\boxed{\beta^i = O(\epsilon^3)}$$

● General solution

$R^{(0)} = \left[(L^{(0)})^4 f^{(0)}\right]$: **Ricci scalar of 0th spatial metric**

● *Density perturbation & scalar fluctuation*

$$\delta = \frac{R^{(0)}}{2\rho_0\kappa^2 a^2} + O(\epsilon^4),$$

➢ **Density perturb is determined by 0th Ricci scalar**

$$\varphi = -\frac{\partial_t\phi_0}{6(\rho_0+P_0)\kappa^2 a^3}\left(R^{(0)}\int_{t_0}^{t} a(t')dt' + \tilde{C}^{(2)}\right) + O(\epsilon^4),$$

Scalar decaying mode

● *Curvature perturbation*

$$\psi = \left(L^{(0)} + L^{(2)}\right)\left(1 + \frac{1}{2}\int_{t_0}^{t} H(t')\chi(t')dt'\right) + O(\epsilon^4),$$

Constant (δN)

$$p = \rho_0(c_{s0}^2\delta + \Gamma_0\pi)$$

$$\chi = -\frac{1}{6(\rho_0+P_0)\kappa^2 a^2}\left[\left(1 + 3c_{s0}^2 - \frac{\rho_0\Gamma_0\partial_t\phi_0}{(\rho_0+P_0)a}\int_{t_0}^{t} a(t')dt'\right)R^{(0)} - \frac{\rho_0\Gamma_0\partial_t\phi_0}{(\rho_0+P_0)a}\tilde{C}^{(2)}\right]$$

➢ **Its time evolution is affected by the effect of p (speed of sound etc), through solution of χ**

● General solution

$$\gamma_{ij} = a^2\boxed{\psi^4}\tilde{\gamma}_{ij};\ \det(\tilde{\gamma}_{ij}) = 1$$

$$\boxed{\tilde{\gamma}_{ij}} = f_{ij}^{(0)} + f_{ij}^{(2)} - 2\left(F_{ij}^{(2)}\int_{t_*}^{t}\frac{dt'}{a^3(t')}\int_{t_*}^{t'}a(t'')dt'' + C_{ij}^{(2)}\int_{t_*}^{t}\frac{dt'}{a^3(t')}\right) + O(\epsilon^4),$$

$$\tilde{A}_{ij} = \frac{1}{a^3}\left(F_{ij}^{(2)}\int_{t_0}^{t}a(t')dt' + C_{ij}^{(2)}\right) + O(\epsilon^4),$$ ← Gravitational wave

where $F_{ij}^{(2)}(x^k) \equiv \frac{1}{(L^{(0)})^4}R_{ij}\left[(L^{(0)})^4 f^{(0)}\right] - \frac{1}{3}f_{ij}^{(0)}R\left[(L^{(0)})^4 f^{(0)}\right]$

$$\boxed{f_{ij}^{(0)} = \delta_{ij}}$$

- Growing GW (Even if spacetime is flat , 2nd order of GW is generated by scalar modes)

● The number of degrees of freedom

☐ The `constant` of integration $\boxed{L^{(0)}, f_{ij}^{(0)}, C^{(2)} \text{ and } C_{ij}^{(2)}}$

depend only on the spatial coordinates and satisfy constraint eq

$$\boxed{\left(L^{(0)}\right)^6 \partial_i C^{(2)} = 6 f_{(0)}^{jk}\tilde{D}_j^{(0)}\left[\left(L^{(0)}\right)^6 C_{ki}^{(2)}\right].}$$

Scalar 1(G) + 1(D) & Tensor 1(G) + 1(D)=(6-1)+(6-1)-3- 3 gauge remain

under $\beta^i = O(\epsilon^3)$ $\boxed{x^i \to \bar{x}^i = f^i(x^i) = O(1).}$

◆ Nonlinear curvature perturbation

Complete gauge fixing $\boxed{\tilde{\gamma}_{ij} \to \delta_{ij} \; (t \to \infty)}$

Focus on only Scalar-type mode
(e.g. curvature perturbation)

Define nonlinear curvature perturbation $\boxed{\psi^4 = e^{2\zeta}}$

Uniform Hubble &
time-orthogonal gauge

$$K = -3H(t), \beta^i(t, x^i) = 0$$

Gauge
transformation

Comoving & time-
orthogonal gauge

$$\varphi_c(t, x^i) = \beta^i_c(t, x^i) = 0$$

$$\boxed{\zeta_c = \zeta_H - \frac{H}{\dot{\phi}}\varphi_H + O(\epsilon^3)}$$

In this case, the same as linear !

$$\varphi_H = O(\epsilon^2)$$

● Comoving curvature
perturbation

Time dependence

$$\boxed{\zeta_c = \ell^{(0)} + \tilde{\ell}^{(2)} + f_K(t)\, K^{(2)} + f_C(t)\, \tilde{C}^{(2)} + O(\epsilon^4)}$$

δ N

$$\ell^{(0)} = 2 \ln L^{(0)} \qquad \tilde{\ell}^{(2)} = 2L^{(2)}/L^{(0)}$$

$$K^{(2)}[\ell^{(0)}] = R\left[(L^{(0)})^4 f^{(0)}\right] = -2(2\Delta\ell^{(0)} + \delta^{ij}\partial_i\ell^{(0)}\partial_j\ell^{(0)})e^{-2\ell^{(0)}}$$

: Ricci scalar of 0th spatial metric

◆ Nonlinear curvature perturbation

Linear theory

$$\gamma_{ij} = a^2(\eta)(\delta_{ij} + 2H_L^{\mathrm{Lin}} Y \delta_{ij} + 2H_T^{\mathrm{Lin}} Y_{ij})$$

Traceless

$$\mathcal{R}^{\mathrm{Lin}} = \left(H_L^{\mathrm{Lin}} + \frac{H_T^{\mathrm{Lin}}}{3} \right) Y$$

$$(\Delta + k^2) Y_k = 0 \quad Y_{ij} = k^{-2} \left[\partial_i \partial_j - \frac{1}{3} \delta_{ij} \Delta \right] Y$$

Correspondence to our notation

$$\zeta = H_L^{\mathrm{Lin}} Y \quad \tilde{\gamma}_{ij} = \delta_{ij} + 2H_T^{\mathrm{Lin}} Y_{ij}$$

$$H_L Y = \zeta$$

Constraint eq

$$\psi^6 \partial_i \tilde{C}^{(2)} = 6 \partial^j \left(\psi^6 C_{ji}^{(2)} \right)$$

$$H_T Y = E \equiv -\frac{3}{4} \Delta^{-1} \left[\partial^i \psi^{-6} \partial^j \psi^6 (\ln \tilde{\gamma})_{ij} \right]$$

$$\tilde{\gamma}_{ij} = f_{ij}^{(0)} + f_{ij}^{(2)} - \underline{2F_{ij}^{(2)}} A(t) - \underline{2C_{ij}^{(2)}} B(t) + O(\epsilon^4) \quad F_{ij}^{(2)} \; K_{ij}^{(2)} \quad C_{ij}^{(2)} \; \tilde{C}_{ij}^{(2)}$$

Without non-local operators

$$E_c = 3H^{(2)} + \frac{K^{(2)}}{4} A(t) + \frac{\tilde{C}^{(2)}}{4} B(t) + O(\epsilon^4), \Rightarrow \mathcal{R}^{\mathrm{NL}} \equiv \zeta + \frac{E}{3}$$

Some integrals

◆ Nonlinear second-order differential equation

Variable: $z = \frac{a}{H}\left(\frac{\rho + P}{c_s^2}\right)^{\frac{1}{2}}$ $\quad d\eta = \frac{dt}{a(t)}$ $\quad \boxed{\mathcal{R}_c^{NL} = \eta_c + \frac{E_c}{3}}$

$$\boxed{\mathcal{R}_c^{\mathrm{NL}}(\eta) = \ell^{(0)} + \ell^{(2)} + H^{(2)} + \frac{1}{4}\big[F(\eta) - F_*\big]K^{(2)} + \big[D(\eta) - D_*\big]C^{(2)}}$$

η_* (initial)

Integrals: $D(\eta) = 3\mathcal{H}_* \int_\eta^0 \frac{z^2(\eta_*)}{z^2(\eta')} d\eta'$ $\quad F(\eta) = \int_\eta^0 \frac{d\eta'}{z^2(\eta')} \int_{\eta_*}^{\eta'} z^2 c_s^2(\eta'') d\eta''$

satisfies $\boxed{\mathcal{R}_c^{\mathrm{NL}\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{NL}\prime} + \frac{c_s^2}{4} K^{(2)}[\mathcal{R}_c^{\mathrm{NL}}] = O(\epsilon^4)}$

$$\mathcal{R}_c^{\mathrm{Lin}\prime\prime} + 2\frac{z'}{z}\mathcal{R}_c^{\mathrm{Lin}\prime} - c_s^2 \Delta[\mathcal{R}_c^{\mathrm{Lin}}] = 0$$

Ricci scalar of 0th spatial metric

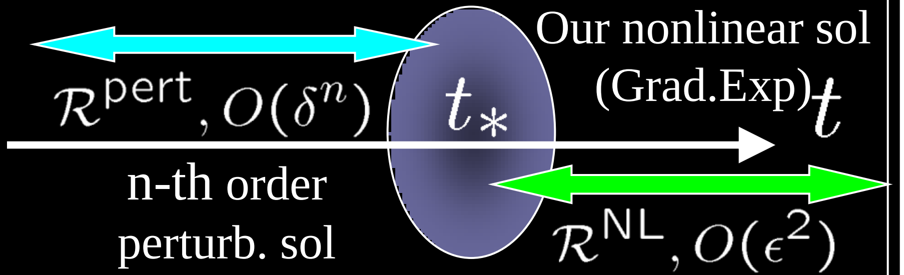
● *Natural extension of well-known linear version*

Decaying mode $D'' + 2\frac{z'}{z}D' = 0$ $\quad \boxed{O(\epsilon^2)}$

Growing mode $F'' + 2\frac{z'}{z}F' + c_s^2 = 0$ $\quad \boxed{O(\epsilon^2)}$

◆ Matching conditions

In order to determine the initial cond.



$\mathcal{R}^{\rm pert}, O(\delta^n)$

t_*

Our nonlinear sol (Grad.Exp)

t

n-th order perturb. sol

$\mathcal{R}^{\rm NL}, O(\epsilon^2)$

$$\mathcal{R}_c^{\rm exact}(\eta) = \mathcal{R}_c^{\rm pert}(\eta) + O(\delta^{n+1}) = \mathcal{R}_c^{\rm NL}(\eta) + O(\epsilon^4)$$

Interest k after Horizon crossing following

$$\left(\frac{k}{\mathcal{H}_*}\right)^2 = \left(\frac{k}{a_* H_*}\right)^2 = (k\eta_*)^2 \ll 1$$

2-order diff. eq → Value & derivative can determine that uniquely

$$\boxed{\begin{aligned} \mathcal{R}_c^{\rm NL}\Big|_{\eta=\eta_*} &= \mathcal{R}_c^{\rm pert}\Big|_{\eta=\eta_*} + O(\epsilon^4, \delta^{n+1}), \\ (\mathcal{R}_c^{\rm NL})'\Big|_{\eta=\eta_*} &= (\mathcal{R}_c^{\rm pert})'\Big|_{\eta=\eta_*} + O(\epsilon^4, \delta^{n+1}). \end{aligned}}$$

● Final result

(at late times $\eta = 0$)

$$\boxed{\mathcal{R}_c^{\rm NL}(0) = \mathcal{R}_c^{\rm pert}(\eta_*) + \frac{D_*}{3\mathcal{H}_*}(\mathcal{R}_c^{\rm pert})'(\eta_*) - \frac{F_*}{4}K^{(2)}[\mathcal{R}_c^{\rm pert}(\eta_*)] + O(\epsilon^4, \delta^{n+1})}$$

◆ Matched Nonlinear sol to linear sol

Approximate Linear sol around horizon crossing @ $\eta = \eta_*$

$$\mathcal{R}_c^{\mathrm{pert}}(\eta_*) \to \mathcal{R}_c^{\mathrm{Lin}}(\eta_*)\,, \quad \mathcal{R}_c^{\mathrm{pert}\prime}(\eta_*) \to \mathcal{R}_c^{\mathrm{Lin}\prime}(\eta_*)$$

- Final result

$$\mathcal{R}_c^{\mathrm{NL}}(0) = u^{(0)} - (1-\alpha^{\mathrm{Lin}})u^{(0)} - F_*\left[\Delta u^{(0)} + \frac{K^{(2)}[u^{(0)}]}{4}\right] + O(\epsilon^4)\,.$$

δN

Enhancement in Linear theory

Nonlinear effect

$$\Delta\, u^{(0)} + \frac{K^{(2)}[u^{(0)}]}{4} = -2\left(\delta^{ij}\partial_i u^{(0)}\partial_j u^{(0)} - 4u^{(0)}\Delta u^{(0)}\right) + O((u^{(0)})^3)$$

- In Fourier space, calculate Bispectrum as

$$f_{NL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{5}{6\sum_i k_i^3}\left[\sum_{i\neq j,j\neq k,k\neq i} G^*(k_i)\,G(k_j)\,H(k_k)\left\{5(k_i^2+k_j^2)-k_k^2\right\}k_k^3\right].$$

$$G(k) = \alpha^{\mathrm{Lin}} = 1 - \frac{D_*}{3\mathcal{H}_*}\frac{\mathcal{R}'}{\mathcal{R}}(\eta_*) - k^2F_*,\quad H(k) = F_*/2$$

◆ *Summary* & *Remarks*

- We develop a theory of non-linear cosmological perturbations on ~~superhorizon scales for a scalar field with a general potential & kinetic terms~~
- We employ the **ADM formalism** and the spatial **gradient expansion** approach to obtain general solutions valid up through second-order $O(\epsilon^2)$
- This Formulation can be applied to k-inflation and DBI inflation to investigate superhorizon evolution of non-Gaussianity beyond δN-formalism
- Show the **simple 2nd order diff eq** for **nonlinear variable:** $\boxed{\mathcal{R}^{\mathrm{NL}} \equiv \zeta + \frac{E}{3}}$
- We formulate a general method to match a n-th order perturbative sol
- **Calculate the bispectrum** using the solution including the nonlinear terms
- Can applied to Non-Gaussianity in temporary violating of slow-roll cond
- Extension to the models of multi-scalar field

◆ Application

- There is a stage at which slow-roll conditions are violated

● *Linear analysis*

(Starobinsky's model)

$$V(\phi) = \begin{cases} V_0 + A_+(\phi - \phi_0) & \text{for } \phi > \phi_0 \\ V_0 + A_-(\phi - \phi_0) & \text{for } \phi < \phi_0 \end{cases}$$

at η_* (initial)・・crossing

$$\mathcal{R}_c(\eta_*) = \alpha u(\eta_*) + \beta v(\eta_*) = u(\eta_*)$$

at $\eta = 0$ (Final) $\lim_{\eta \to 0} v(\eta) = 0$

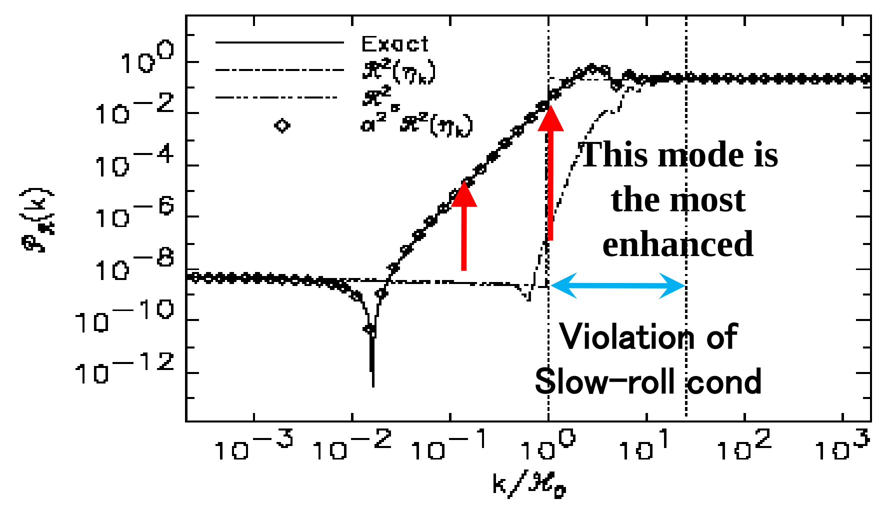
$$\Rightarrow \quad \mathcal{R}_c(0) = \alpha u(0) \simeq \alpha \mathcal{R}_c(\eta_*)$$

$$\alpha \simeq 1 + D(\eta_*) - F(\eta_*)$$

Can be amplified if

$$z \ll z_* \rightarrow D_*, F_* \gg 1$$

$$\ddot{\phi} \approx -3H\dot{\phi} \quad \text{(fast rolling)}$$



Leach, Sasaki, Wands & Liddle (01)

● ***go up to nonlinear analysis !***