Decoherence in quantum field theory

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[A. Giraud, J.S. : submitted to PRL]

- Motivations and set-up
- Nonequilibrium QFT: methods
- Results
- Conclusions and outlook

MOTIVATIONS

Quantum decoherence

- Measurement theory
- Ultracold atoms
- Quantum optics
- Quarks-gluons plasma physics
- Early-Universe cosmology
  - Topological defects
  - Quantum-to-classical transition of primordial density fluctuations
  - Decay of inflaton (preheating)
  - ...
- Neutrino physics
- ...

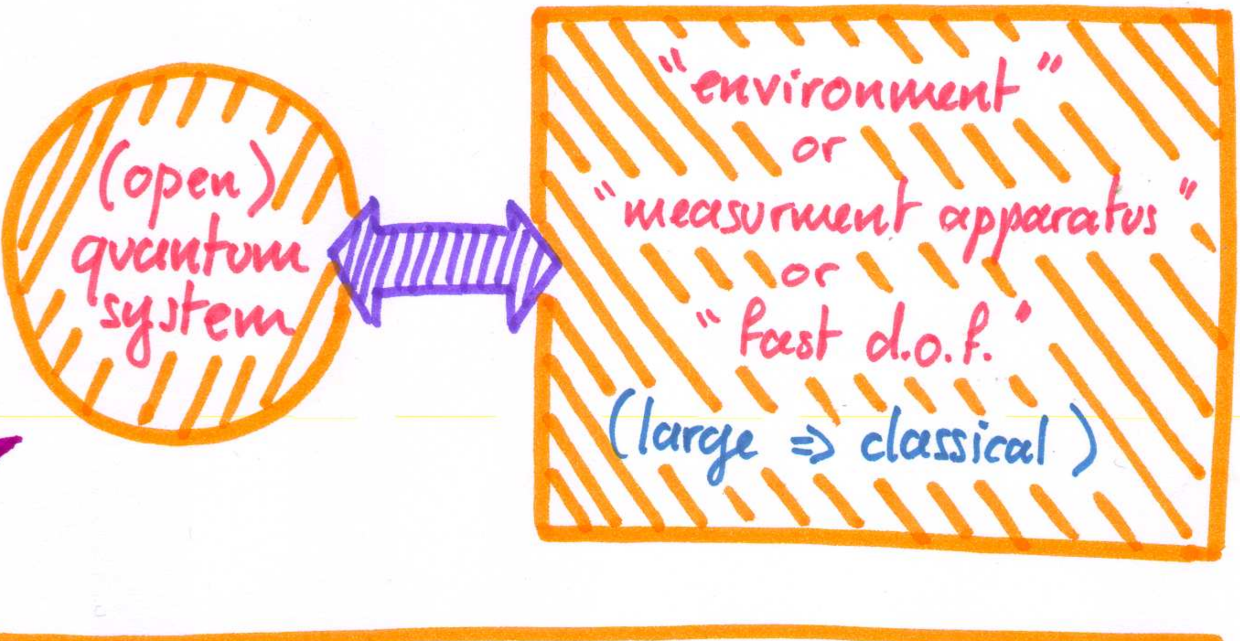
The standard picture

Decoherence: dynamical diagonalization of the density matrix (in a given basis)

ex:

$$\rho(t=0) = |\psi\rangle\langle\psi| \rightsquigarrow \rho(t) = \sum_n p_n |n\rangle\langle n|$$

pure state $\qquad\qquad$ statistical mixture

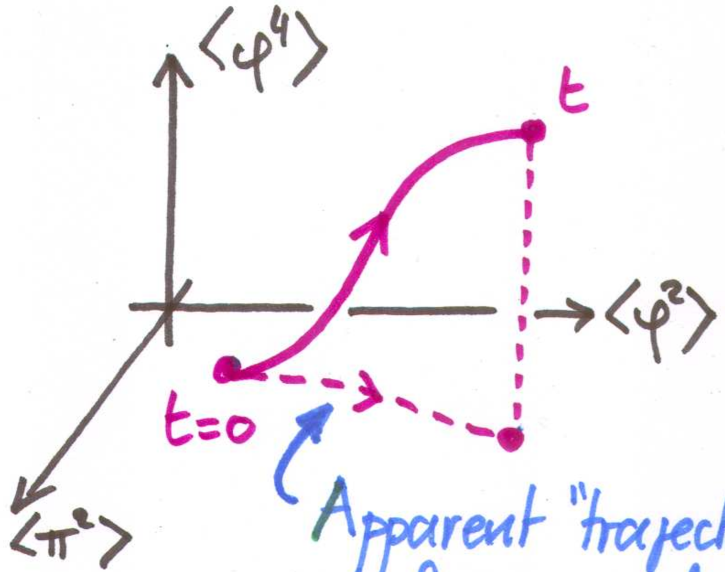


Loss of purity / coherence results from the interaction with an "environment" or from some kind of coarse graining ⇒ **NON-UNITARY EVOLUTION**

Decoherence for a closed system

- Reconstructing the actual state of the system requires one to measure all independent correlation functions.
- For systems with large (infinite) number of d.o.f., this is practically impossible.

→ The inability to reconstruct the actual state of the system due to restricted data may result in apparent decoherence.



Information spreads in the space of correlation functions.

[e.g. Ballan ('00); Campo, Parentani ('05) ...]

Theoretical description

Decoherence : a nonequilibrium quantum process.

Analytical descriptions

- exactly solvable (simple) models
- neglect the dynamics of the environment (e.g. assume a thermal bath)
- neglect back-reaction of the system
- weak coupling techniques

Numerical work

- solve open quantum dynamics obtained by integrating out the environment d.o.f.

Real-time quantum dynamics

Nonequilibrium QFT : methods

⚠ **Standard approximation schemes in QFT fail out of equilibrium.**

- SECULARITY : $e^{-\gamma t} \approx 1 - \gamma t + \ldots$

⇒ Need for (infinite) resummations

- UNIVERSALITY : late-time thermalization (effective loss of memory)

⇒ Need for non-linear approximation schemes

Two-particle-irreducible (2PI) techniques

[e.g. Serreau @ SEWM 08 hep-ph/0410330]

$$\Gamma_{2PI}[\phi, G] = S[\phi] + \frac{i}{2} \mathrm{Tr} \ln G^{-1} + \frac{i}{2} \mathrm{Tr}\, G_0^{-1} G + \Gamma_2[\phi, G]$$

$$\left.\frac{\delta \Gamma_{2PI}}{\delta G}\right|_{\bar{G}} = 0 \quad \Leftrightarrow \quad \boxed{\bar{G}^{-1} = G_0^{-1} - \Sigma[\bar{G}]}$$

Self-adjusting propagator

2PI techniques : State of art

⇨ Solve the secularity and universality problems of nonequilibrium QFT. [Berges, Cox ('01)]

Applications

- Thermalization from first principles
  [Berges, Borsányi, J.S. ('03); Cooper, Dawson, Mihaila ('03)]
- Nonperturbative $1/N$ expansion
  [Aarts, Ahrensmeier, Baier, Berges, J.S. ('02)]
- Dynamics at nonperturbatively high densities
  e.g. preheating after inflation [Berges, J.S. ('03)
  Smit, Tranberg ('04)]
- Expanding geometries [Tranberg ('08)]
- ⋮

Formal developments

- 2PI renormalization theory
  [Van Hees, Knoll ('02); Blaizot, Iancu, Reinosa ('03)
  Berges, Borsányi, Reinosa, J.S ('05) ...]
- Gauge field theories [Reinosa, J.S. ('06)]
- ⋮

Decoherence in QFT : The model

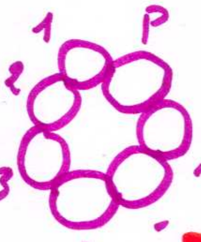
- O(N) scalar field φ_a , $a = 1 \dots N$

$$S[\varphi] = \int d^4x \left[\frac{1}{2} \partial_\mu \varphi_a \partial^\mu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right]$$

- 2PI $1/N$ - expansion at NLO [Berges ('02)]
[Aarts, Ahrensmeier, Baier, Berges, J.S. ('02)]

$$\Gamma_2[G] = \Gamma_2^{LO}[G] + \Gamma_2^{NLO}[G] + \dots$$

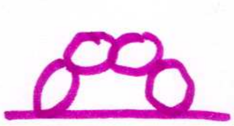
 $\sim \frac{\lambda}{N} N^2 \sim \lambda N$

$\sum_P$ 

$\sim \frac{\lambda^P}{N^P} N^P \sim N^0$

Gaussian dynamics : no "decoherence"

Non-Gaussian

$\Sigma[G] = 2i \frac{\delta \Gamma_2}{\delta G} =$  +  + ... +  + ...

LO (mass correction)

NLO (scattering, memory effects, ...)

Equations of motion

$$G(x,y) = \langle T\varphi(x)\varphi(y)\rangle = \underbrace{F(x,y)}_{\text{statistical}} - \frac{i}{2}\varepsilon(x^0-y^0)\underbrace{\rho(x,y)}_{\text{spectral}}$$

$$[\Box + M^2(x)]\,F(x,y) = -\int_0^{x^0} dz\,\Sigma_\rho(x,z)F(z,y) + \int_0^{y^0} dz\,\Sigma_F(x,z)\rho(z,y)$$

$$[\Box + M^2(x)]\,\rho(x,y) = -\int_{y^0}^{x^0} dz\,\Sigma_\rho(x,z)\rho(z,y)$$

(local) mass corrections

Nonlocal terms = memory / scatterings

$$\Sigma_F(x,y) = \frac{\lambda}{3N}\left[F(x,y)\,I_F(x,y) - \frac{1}{4}\rho(x,y)\,I_\rho(x,y)\right]$$

$$\Sigma_\rho(x,y) = \frac{\lambda}{3N}\left[F(x,y)\,I_\rho(x,y) + \rho(x,y)\,I_F(x,y)\right]$$

$$I_F(x,y) = \Pi_F(x,y) - \int_0^{x^0} dz\,I_\rho(x,z)\Pi_F(z,y) + \int_0^{y^0} dz\,I_F(x,z)\Pi_\rho(z,y)$$

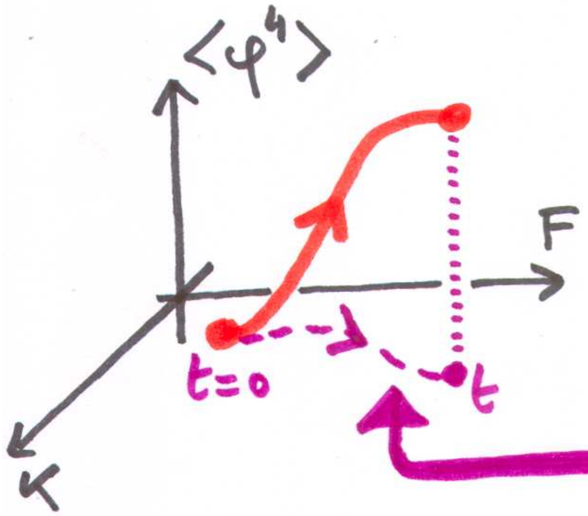
$$I_\rho(x,y) = \Pi_\rho(x,y) - \int_{y^0}^{x^0} dz\,I_\rho(x,z)\Pi_\rho(z,y)$$

$$\Pi_F(x,y) = -\frac{\lambda}{6}\left[F^2(x,y) - \frac{1}{4}\rho^2(x,y)\right]\ ;\quad \Pi_\rho(x,y) = -\frac{\lambda}{3}F(x,y)\rho(x,y)$$

The picture

Assume one only measures equal-time 2-point functions : $F_p(t)=\langle \varphi_p^\dagger(t)\varphi_p(t)\rangle$; $K_p(t)=\langle \Pi_p^\dagger(t)\Pi_p(t)\rangle$

$$R_p(t)=\frac{1}{2}\langle \Pi_p^\dagger(t)\varphi_p(t)+\varphi_p^\dagger(t)\Pi_p(t)\rangle$$



The least-biased (effective) density matrix compatible with the known data :

$$\rho_{eff}(t)=\prod_p \rho_p(t)$$

$$\rho_p(t)\propto \exp\left[\kappa_p(t)\left(F_p(t)\Pi_p^\dagger\Pi_p+K_p(t)\varphi_p^\dagger\varphi_p+R_p(t)\left(\varphi_p^\dagger\Pi_p+\Pi_p^\dagger\varphi_p\right)\right)\right]$$

$\kappa_p(t)=-\dfrac{\ln(1+1/n_p(t))}{2n_p(t)+1}$; $n_p(t)+\dfrac{1}{2}=\sqrt{F_p(t)K_p(t)-R_p^2(t)}$

N.B. : $\boxed{\mathrm{tr}\,\rho_p^2(t)=\dfrac{1}{2n_p(t)+1}}$: a measure of purity

The Gaussian density matrix

Parametrization:

$0 \leq \phi < 2\pi$

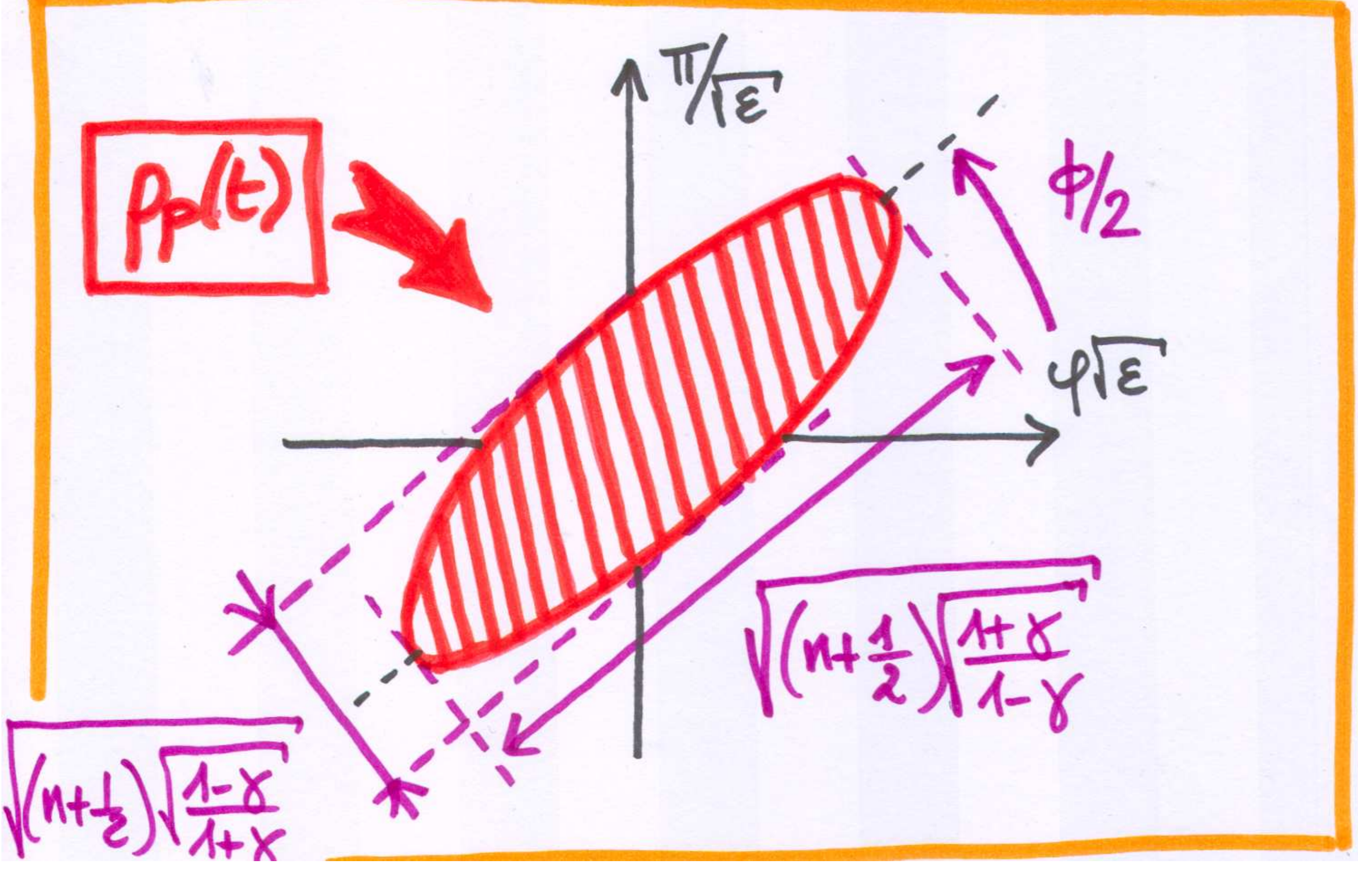
$0 \leq \gamma < 1$

$$\varepsilon F = \left(n + \frac{1}{2}\right) \frac{1 - \gamma \cos\phi}{\sqrt{1-\gamma^2}}$$

$$R = -\left(n + \frac{1}{2}\right) \gamma \frac{\sin\phi}{\sqrt{1-\gamma^2}}$$

$$K/\varepsilon = \left(n + \frac{1}{2}\right) \frac{1 + \gamma \cos\phi}{\sqrt{1-\gamma^2}}$$

$\boxed{\varepsilon}$ = energy scale : fixes the basis
we take $\varepsilon_p(t) = \sqrt{p^2 + M^2(t)}$



Observables

- $\mathrm{tr}\,\rho_p^2(t) = \dfrac{1}{2n_p(t)+1} \leq 1$

PURITY (basis-indep.)

Pure state $(n=0)$

- $S_p(t) = -\mathrm{tr}\,\rho_p(t)\ln\rho_p(t)$

$$= (n_p(t)+1)\ln(n_p(t)+1) - n_p(t)\ln n_p(t)$$

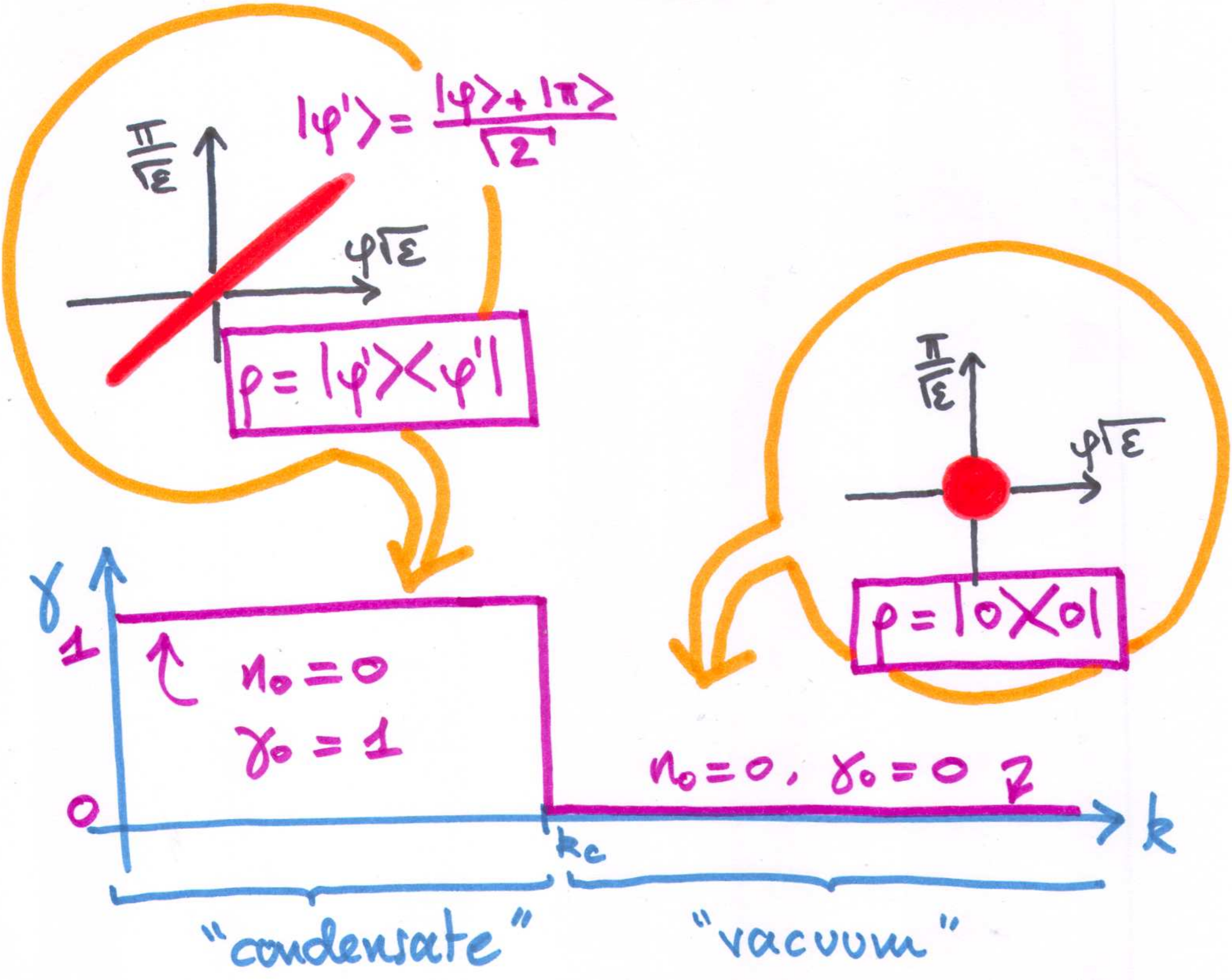
ENTROPY = **MISSING INFORMATION** (basis-indep.)

- $\gamma_p(t)$: squeezing parameter

COHERENCE in two-modes coherent states basis [Campo, Parentani ('05)]

Initial conditions

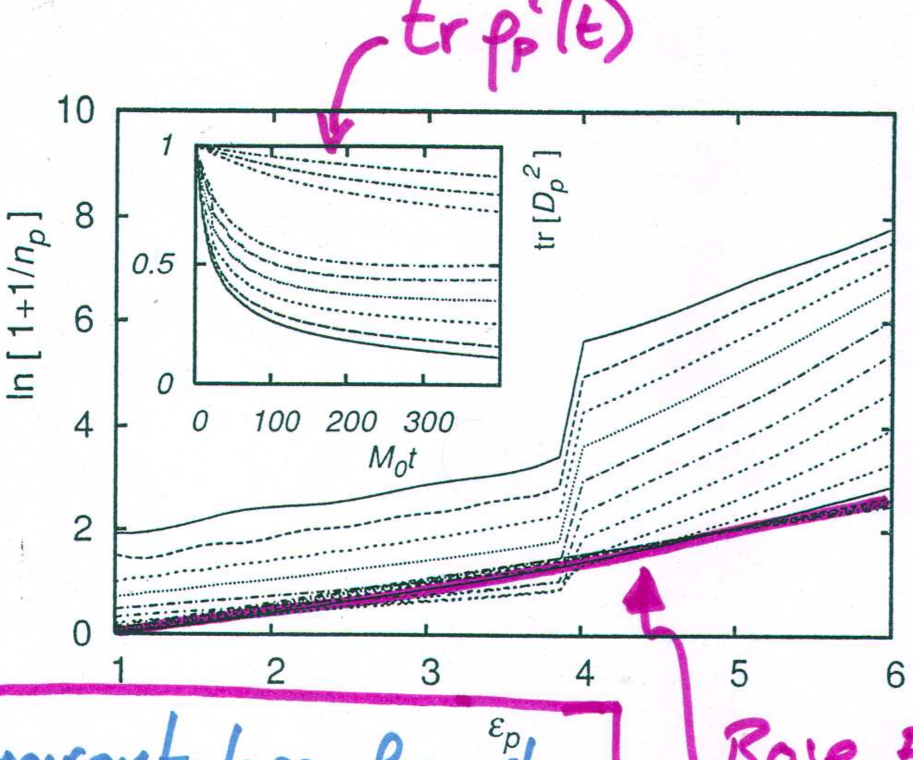
We assume a Gaussian initial state



N.B.: No environment, no (thermal) bath ...

$\Rightarrow$ **Loss of quantum purity / coherence is to be caused by vacuum quantum fluctuations !!**

Results



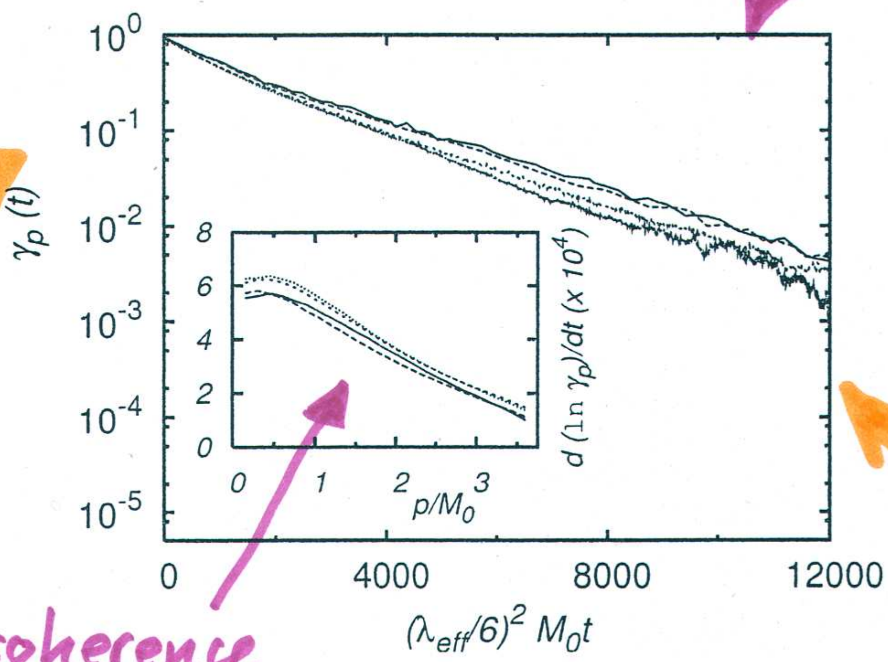
Apparent loss of purity and thermalization

Bose-Einstein distribution!

The pure state looks mixed and even (later) thermal to the observer who has restricted information.

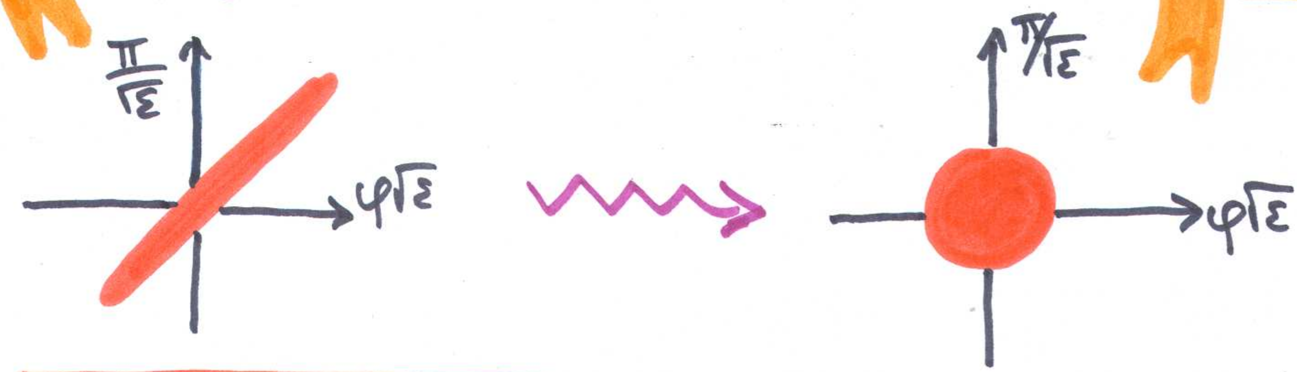
Results

Decoherence



$k = 0^+$

decoherence rate



$\Rightarrow$ Loss of quantum purity/coherence induced by quantum fluctuations

Classical scaling

$$F, R, K \sim \frac{1}{\sqrt{1-\gamma^2}} \underset{\gamma \to 1}{\gg} \rho \sim 1$$

Classical (statistical) fluctuations regime

$$S[\varphi] = \int d^4x \left(\frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!N}\varphi^4 \right)$$

$$= \eta \int d^4x \left(\frac{1}{2}(\partial\varphi')^2 - \frac{m^2}{2}\varphi'^2 - \frac{\lambda\eta}{4!N}\varphi'^4 \right)$$

$\boxed{\varphi = \sqrt{\eta}\,\varphi'}$

effective coupling

⇒ In the classical (strong) field regime, the results should only depend on

$$\boxed{\lambda_{\text{eff}} = \frac{\lambda}{\sqrt{1-\gamma_0^2}}}$$

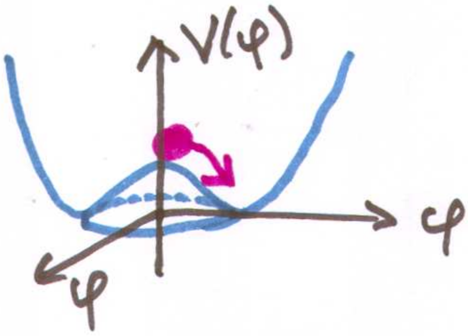
⇒ Loss of purity/coherence can be described by means of classical statistical field theory

Conclusions ...

- effective loss of quantum purity / coherence due to restricted knowledge on the system.
- induced by pure quantum fluctuations.
- can be described by classical statistical (stochastic) field theory.

... and Outlook

- finite temperature (in progress)
- phase transition



- expanding geometries
- fermionic d.o.f.

...

[Special thanks to LUNA for color pencils]