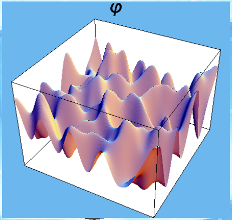
Jean-François Dufaux **A.P.C.**

Gravitational Waves from Preheating after Inflation: Overview and Recent Results



JFD - Phys.Rev.Lett.103(2009)

JFD, D. Figueroa, J. Garcia-Bellido - to appear

P. Brax, JFD, S. Mariadassou - in preparation

OUTLINE

- Introduction
    - ↪ Gravity waves (GW) from the early universe
    - ↪ Preheating after Inflation
- GW from preheated scalar fields
    - ↪ Methods
    - ↪ Results
- GW from the non-perturbative decay of SUSY flat directions
- GW from preheated gauge fields
- Conclusions

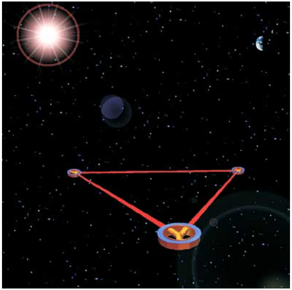
Gravitational Waves from the Early Universe

Interact very weakly and are not absorbed
⇒ Direct probe of high-energy phenomena producing them

Target for several high-sensitivity interferometric experiments (LIGO, VIRGO, LISA, ET, BBO, DECIGO, ...) in frequency range 10^{-5} to 10^{3} Hz

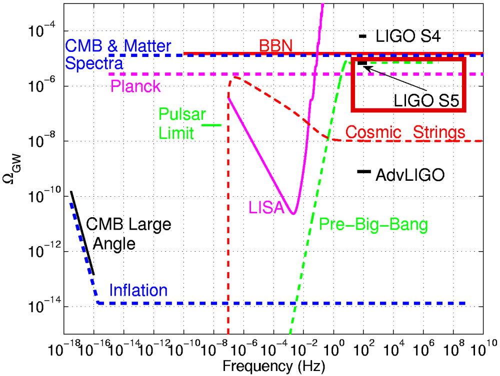
Typical wavelength when produced: $\lambda_p < H_p^{-1}$ (post-inflationary source)
↪ Frequency today: $f_0 = (a_p/a_0) * (2\pi/\lambda_p) < 10^3\,\mathrm{Hz} \quad \Rightarrow \quad \boldsymbol{\rho_p^{1/4} < 10^{11}}$ **GeV**

Possible sources: inflation, preheating, phase transitions, cosmic strings, ...



A milestone achieved by ground interferometers

[Ligo and Virgo collaborations, Nature 460 (2009)]



Cosmological GW (extra radiation component) may affect expansion rate of the universe during BBN and at last scattering

↪ Constrained by abundance of light elements and CMB

For the first time, LIGO reached a sensitivity below these bounds

Preheating after Inflation

Inflation
$\phi(t)$: condensate
no particle
$\hookrightarrow$ **CMB, ...**

$\longrightarrow$

Reheating
Particle-genesis
Thermalization
$\hookrightarrow$ **????**

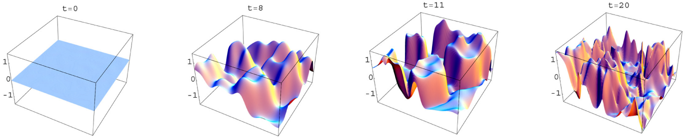
$\longrightarrow$

Hot Big Bang
Thermal bath
of particles
$\hookrightarrow$ **BBN, ...**

Two kinds of decay channels for the inflaton:

- Perturbative: $\phi(t) =$ collection of particles decaying independently
  $\hookrightarrow$ Slow decay, faster thermalization
  Valid if the inflaton couples very weakly to all other fields
- Non-perturbative: coherent and collective effects $\Rightarrow$ **Preheating**
  $\hookrightarrow$ Explosive decay, slower thermalization
  If one non-perturbative decay channel exists, it dominates the dynamics

Preheating is a violent and highly inhomogeneous process



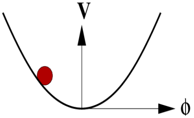
$\Rightarrow$ Production of gravity waves, carrying relic information about this epoch

Preheating after chaotic inflation [Kofman, Linde, Starobinsky], ...

Model:

$$V = m^2\,\phi^2 + g^2\,\phi^2\,\chi^2$$

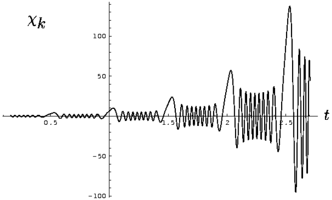
$$q = \frac{g^2\Phi^2}{m^2} >> 1$$



Quantum fluctuations of χ:

$$\ddot{\chi}_k + \omega_k^2(t)\,\chi_k = 0$$

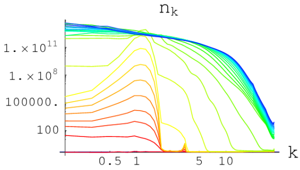
with $\omega_k^2(t) = k^2 + g^2\,\phi^2(t)$



Particle production ($\dot{\omega}_k > \omega_k^2$) in very small intervals around $\phi(t) = 0$

Some modes enter in resonance with inflaton oscillations(parametric resonance)

Explosive production of particles with occupation numbers $n_k \sim 1/g^2$. Followed by long, turbulent-like evolution towards thermal equilibrium.



Non-linear classical random fields ⇒ Lattice Simulations

GW from preheated scalar fields: Methods

[JFD, Bergman, Felder, Kofman, Uzan '07]

$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right]$ with $\partial_i h_{ij} = h_{ii} = 0$ (TT)

$$h_{ij}'' + 2\,\frac{a'}{a}\,h_{ij}' - \nabla^2 h_{ij} = 16\pi G\, a^2\, \Pi_{ij}^{TT} \tag{1}$$

In Fourier space: $\Pi_{ij}^{TT}(\vec{k}) = \left(P_{il}P_{jm} - \frac{1}{2}\,P_{ij}P_{lm}\right)\Pi_{lm}(\vec{k})$ with $P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Analytics: Solve Eq.(1) in Fourier space with Green function. With informations about the statistical properties of the source, calculate $\rho_{\rm gw} \propto \langle h_{ij}' h_{ij}' \rangle$ with ensemble average.

Numerics: Solve Eq.(1) together with evolution equations for the source (scalar fields) and scale factor. Calculate $\rho_{\rm gw}$ with spatial and time averages. Apply TT projection in Fourier space when GW spectra are output.

Spectrum today:

$$\mathbf{h^2\,\Omega_{\rm gw}(f)} = \left(\frac{\mathbf{h^2}}{\rho_{\mathbf{c}}}\,\frac{\mathbf{d}\rho_{\rm gw}}{\mathbf{d\ln f}}\right)_{\mathbf{0}} = \left(\frac{1}{\rho_{\rm tot}}\,\frac{d\rho_{\rm gw}}{d\ln k}\right)_p \left(\frac{a_p}{a_*}\right)^{1-3w} \left(\frac{g_0}{g_*}\right)^{1/3} h^2\,\Omega_{\rm rad}$$

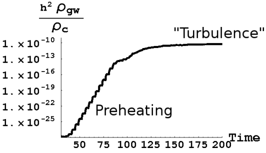
NB: Equation of state jumps quickly towards $w = 1/3$ during preheating

[Podolsky et al '05], **[JFD et al '06]**

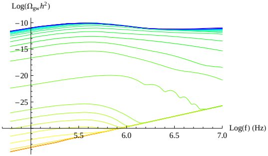
GW from preheated scalar fields: Results

[JFD, Bergman, Felder, Kofman, Uzan '07]

Evolution of ρ_{gw} with time



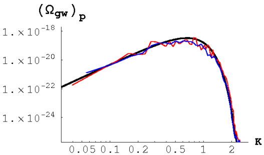
Growth of GW spectrum with time



Linear stage of preheating:

Gaussian random fields and analytical evolution of the source

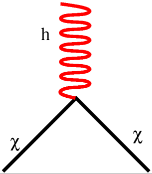
⇒ Check of the lattice results



Turbulence and thermal bath

Scalar fields with $\chi_k(t) \propto \mathrm{Exp}[i\omega_k t]$
GW production *forbidden* by helicity conservation

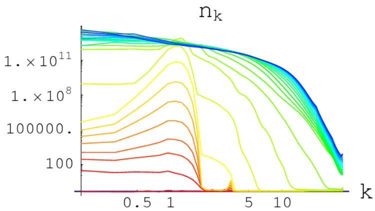
Different for *massless* vector fields



Most GW produced in intermediate non-linear and non-perturbative stage

The spectra of particles produced by preheating are usually strongly peaked around some typical (comoving) momentum k_*

$R_* = a/k_*$: Characteristic physical size of the source inhomogeneities



Typical frequency and amplitude of GW when produced:

$$f_p \approx \frac{1}{R_*} \quad , \quad \left(\frac{\rho_{\rm gw}}{\rho_{\rm tot}}\right)_p \approx (R_* \, H)_p^2$$

Similar to GW from bubble collisions in 1st-order phase transition

In a given model of preheating, one can calculate k_* analytically
$\Rightarrow$ Estimate how the GW spectrum depends on the parameters

Peak frequency and amplitude of GW spectrum today:

$$f_* \approx \frac{1}{(R_* \, H)_p} \left(\frac{\rho_{\rm tot}^{1/4}}{10^{11}\,{\rm GeV}}\right) 10^3\,{\rm Hz} \quad , \quad h^2\,\Omega_{\rm gw}^* \approx 10^{-6}\,(R_*\,H)_p^2$$

GW from Preheating after Hybrid Inflation

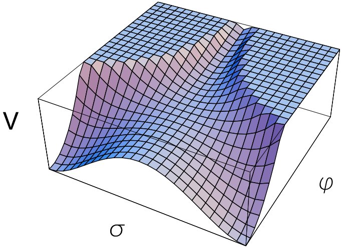
$$V = \frac{\lambda}{4}\left(|\sigma|^2 - v^2\right)^2 + \frac{g^2}{2}\phi^2|\sigma|^2 + V_{\rm sr}(\phi)$$

Inflation when $\phi > \phi_c = \sqrt{\lambda}\, v/g$

When $\phi < \phi_c$, (ϕ, σ) roll to minimum $(0, v)$

Fluctuations: $\delta\ddot{\sigma}_k + \left(k^2 + \partial^2 V\right)\delta\sigma_k = 0$

$m_\sigma^2 = \partial^2 V < 0$: "Tachyonic Preheating"



Key parameters: λ, g^2, v and $V_c = \dot{\phi}_c/(m\,\phi_c)$ $\quad(\tilde{V}_c = 10^3 g^3)$

Three regimes of GW production: **[JFD, Felder, Kofman, Navros '08]**

- Significant V_c $(> \tilde{V}_c)$ and $g^2 \geq \lambda$: preheating onset by inflaton velocity

$$f_* \sim \lambda^{1/4}\, V_c^{1/3}\, 10^{11}\,\mathrm{Hz} < 10^3\,\mathrm{Hz} \;\Rightarrow\; \lambda \text{ as low as } 10^{-30}$$

- Negligible V_c $(< \tilde{V}_c)$ and $g^2 \geq \lambda$: preheating onset by quantum diffusion

$$f_* \sim g\,\lambda^{1/4}\, 10^{11}\,\mathrm{Hz} < 10^3\,\mathrm{Hz} \;\Rightarrow\; \lambda \sim g^2 < 10^{-11}$$

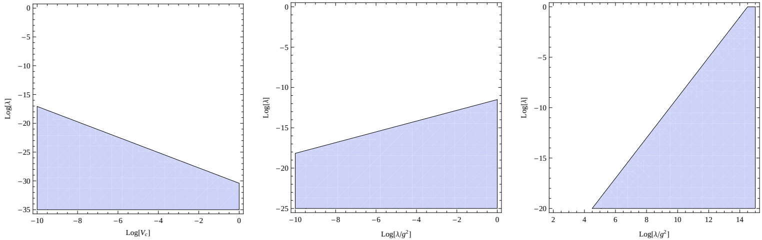
- $g^2 << \lambda$: successive inflaton oscillations and GW production

$$f_* \sim g\,\lambda^{-1/4}\, 10^{10}\,\mathrm{Hz} < 10^3\,\mathrm{Hz} \;\Rightarrow\; \lambda \sim 1\,,\; g^2 < 10^{-15}$$

Peak frequency and amplitude of the GW spectrum today:

$$f_* \sim f(\lambda, g^2, V_c)\, 10^{10}\ \mathrm{Hz} \quad , \quad h^2\, \Omega^*_{\mathrm{gw}} \sim g(\lambda, g^2, V_c)\ (v/M_P)^2$$

Regions of the parameter space leading to observable GW



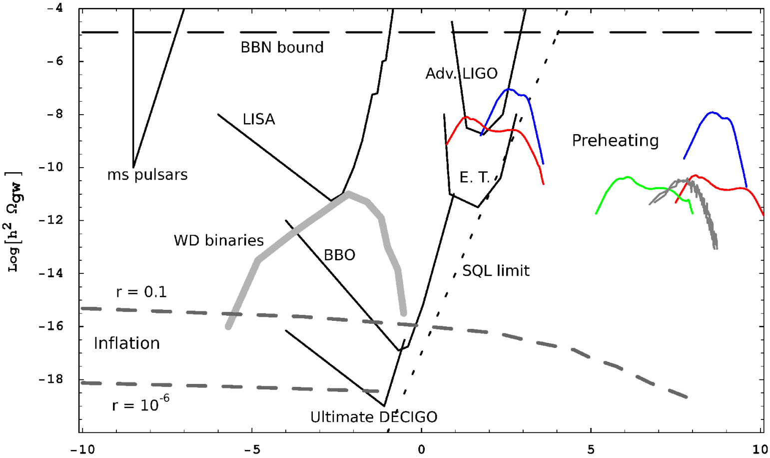
Particle-physics models of hybrid inflation with very small couplings:

- In SUSY models (D-term, P-term, ...), $g^2 \sim \lambda < 10^{-11}$ allows to satisfy bounds on cosmic strings **[Endo et al '03], [Kallosh, Linde '03], ...**
- **[Barnaby, Cline '06]** argues that the end of brane / anti-brane inflation in a warped throat can be modeled by a 4-D hybrid potential with $g^2 \ll \lambda$
- If σ is a moduli field, $m \sim \sqrt{\lambda} v \sim$ TeV and $v \sim M_P \Rightarrow \lambda \sim 10^{-30}$
"Super-natural inflation" **[Randall, Soljacic, Guth '95]**,
"Thermal inflation" **[Lyth, Stewart '95]**

GW from Preheating Vs Observations: Status

Preheating after chaotic inflation: $f_* \sim 10^6 - 10^9$ Hz $\quad (\rho_{\rm inf}^{1/4} \sim 10^{15}$ GeV)

Preheating after hybrid inflation: GW cover a wide range of frequencies and amplitudes. Can be observable, but requires very small coupling constants



[JFD, Felder, Kofman, Navros '08]

Preheating after Hilltop Inflation

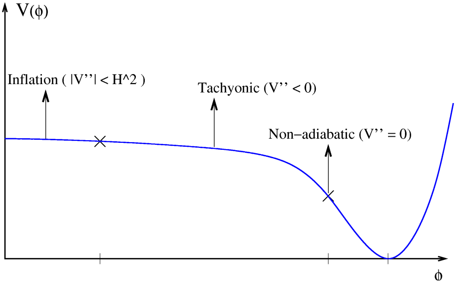
[Brax, JFD, Mariadassou], in progress

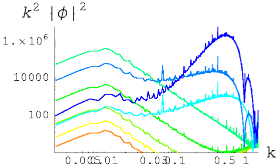
Ex.: $V = \left(M^2 - \phi^4/\Lambda^2\right)^2$

$\delta\ddot{\phi}_k + \left(k^2 + V''\right)\,\delta\phi_k = 0$

$m^2 = V'' < 0$: tachyonic instability

$\dot{m} \gg m^2$: non-adiabatic production





$k_*^{\rm IR} \sim \sqrt{|V''|} \sim H \ \ll\ k_*^{\rm UV} \sim M^{3/2}/\Lambda^{1/2}$

Tachyonic dominates for $\Lambda < 10^{16}$ GeV $\Rightarrow$

Peak frequency and amplitude of GW today:

$f_* \sim \frac{\Lambda}{\rm GUT}\, 10^2\,{\rm Hz} \quad , \quad h^2\,\Omega_{\rm gw}^* \sim 10^{-7}$

In this model, GW can be further redshifted by intermediate matter-dominated stage until the inflaton decays completely

Ex: Coupling to fermions $\mathcal{L}_{\rm int} = h\,\phi\,\psi\bar{\psi} \ \Rightarrow$ Decay possible iff $m_\psi < m_\phi/2$

Non-zero vev $\phi_{\rm min}$ at the minimum $\Rightarrow\ m_\psi = h\,\phi_{\rm min}$

The decay is possible only for small coupling $h \ll 1$ and is therefore delayed

Supersymmetric Flat Directions

Combinations of complex scalar fields along which the renormalizable potential is exactly flat in the limit of unbroken SUSY.

Flatness lifted by SUSY-breaking and non-renormalizable terms

$$V = m^2\,|\phi|^2 + \left(\frac{A\,m}{M_P^{n-3}}\,\phi^n + \mathrm{h.c.} \right) + \frac{|\lambda|^2}{M_P^{2n-6}}\,|\phi|^{2n-2} + \ldots \qquad (m \sim \mathrm{TeV})$$

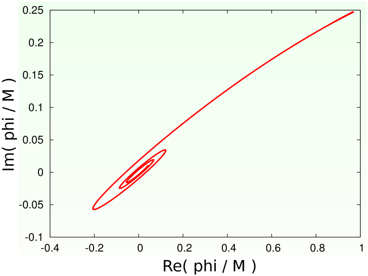
In the early universe:

[Dine, Randall, Thomas '95]

$$V = \left(m^2 - c\,H^2 \right)\,|\phi|^2 + \ldots$$

(NB: $V \supset T^2\,|\phi|^2$ for small VEVs)

ϕ can acquire a very large VEV during inflation. Subsequent evolution damped untill $H \sim m$. Then, out-of-phase oscillations of Re(ϕ) and Im(ϕ).



Ex: Affleck-Dine baryogenesis

The Non-Perturbative Decay of Flat Direction Condensates

[Olive, Peloso '06],[Basboll et al '07], [Gümrükçüoğlu et al '08], ...

Model: $V_D = g^2 \left(|\phi_1|^2 - |\phi_2|^2 \right)^2$ (D-term potential)

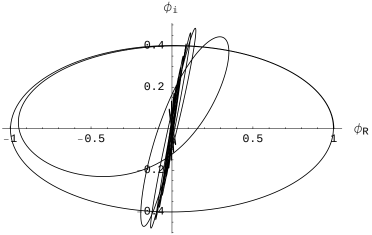
Background: $\phi_1 = \phi_2 = \Phi(t)\, e^{i\,\sigma(t)}$

Fluctuations around this background have the mass matrix:

$$\mathcal{M}^2 = g^2\,\Phi^2(t) \begin{pmatrix} 2\cos^2\sigma(t) & \sin 2\sigma(t) \\ \sin 2\sigma(t) & 2\sin^2\sigma(t) \end{pmatrix}$$

Eigenstates vary non-adiabatically with time ⇒ abundant particle production

Once the fluctations have been sufficiently amplified, they backreact on the condensate and convert most of its energy into large inhomogeneities **[JFD '09]**



GW from the Non-Perturbative Decay of Flat Directions [JFD '09]

Main differences with respect to preheating after inflation:

- $1/R_* \sim H \sim m \sim \text{TeV} \quad \Rightarrow \quad$ GW frequency in the Hz-kHz range
- When they decay, flat directions are subdominant: $\rho_{\rm flat} < \rho_{\rm tot}$
  - $\hookrightarrow \quad \Omega_{\rm gw} \propto (\rho_{\rm flat}/\rho_{\rm tot})^2$
  - $\hookrightarrow$ Equation of state and GW depend also on inflaton sector!!

Analytical estimates for the peak frequency and amplitude:

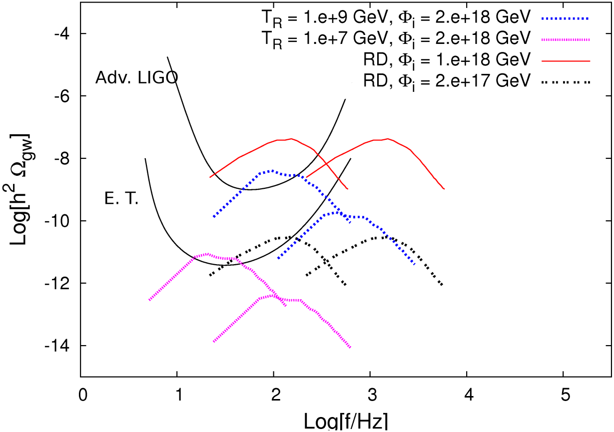
$$f_* \sim \left(\frac{a_i}{a_r}\right)^{1/4} \sqrt{\frac{m}{\rm TeV}}\, 5 \times 10^2\,{\rm Hz} \quad , \quad h^2\,\Omega^*_{\rm gw} \sim 10^{-4} \left(\frac{\Phi_i}{M_P}\right)^4 \left(\frac{a_i}{a_r}\right)$$

Key parameters: initial vev Φ_i of the condensate when it starts to oscillate, soft SUSY-breaking mass m and reheat temperature of the universe T_R

$a_i/a_r = 1 \quad \text{if} \quad T_R > 0.2\sqrt{m\,M_p}$ (radiation-domination during the decay)

$a_i/a_r \simeq \left(5\,T_R/\sqrt{m\,M_p}\right)^{4/3}$ otherwise (matter-domination during the decay)

GW spectra for $m = 100$ GeV (left) and $m = 10$ TeV (right)



Can be observable for high enough initial VEV, $\Phi_i > 10^{17}$ GeV

$\Phi_i \sim (m\, M_P^{n-3})^{1/(n-2)}$ depends on non-renormalizable terms ($W \supset \phi^n / M_p^{n-3}$)

MSSM flattest direction: $n = 9 \Rightarrow \Phi_i \sim 10^{16}$ GeV allowed by gauge invariance

But: may be forbidden by other symmetries $\Rightarrow \Phi_i \sim M_P$ possible

[Dine, Randall, Thomas '95], [Gaillard, Murayama, Olive '95]

GW from Abelian Gauge Fields at Preheating

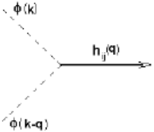
[JFD, Figueroa, Garcia-Bellido], to appear

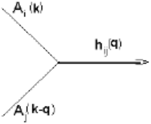
Motivation: No GW from scalar field turbulence after preheating

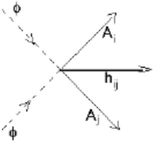
$$-\mathcal{L} = (D_\mu\phi)^*\, D^\mu\phi + \frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} + \ldots \quad \text{with} \quad D_\mu = \partial_\mu - i\, e\, A_\mu$$

Source terms: $T_{ij} \supset$ $\quad \partial_i\phi\,\partial_j\phi^* \quad \dot{A}_i\,\dot{A}_j \quad e^2\, A_i\, A_j\, |\phi|^2$

For "wave-like" fields:







Lattice simulations of gauge theories: (a nightmare...)

In the continuum: $\phi \;\to\; e^{i\alpha(x)}\,\phi \;\Rightarrow\; D_\mu\phi \;\to\; e^{i\alpha(x)}\,D_\mu\phi$

On the lattice: $\partial_i^+\phi = \frac{\phi(x+\hat{i})-\phi(x)}{dx} \;\longrightarrow\; \frac{e^{i\alpha(x+\hat{i})}\,\phi(x+\hat{i})-e^{i\alpha(x)}\,\phi(x)}{dx}$

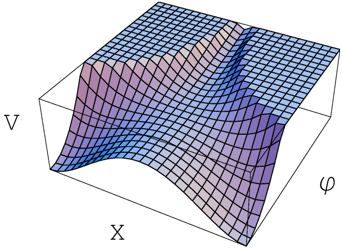
Links: $U_i(x,x+\hat{i}) \equiv e^{-ie\int_x^{x+\hat{i}} A_i dx} \approx e^{-ie\,A_i\,dx} \quad\Rightarrow\quad D_i^+\phi = \frac{U_i(x,x+\hat{i})\,\phi(x+\hat{i})-\phi(x)}{dx}$

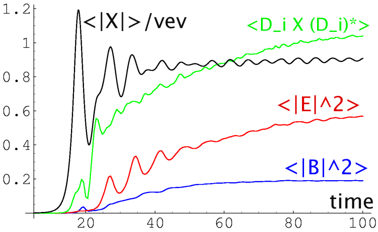
Allows to derive discrete equations of motion from a gauge-invariant lattice action. Required for constraint equation to follow from the dynamical equations that are evolved. In the gauge $A_0 = 0$, Gauss constraint: $\partial_i\dot{A}_i = e^2\, j_0$.

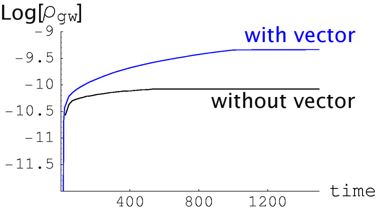
Model: Hybrid inflation with a complex "Higgs field" X coupled to a $U(1)$ gauge field A_μ

$$V = \frac{\lambda}{4}\left(|X|^2 - v^2\right)^2 + \frac{g^2}{2}\,\phi^2\,|X|^2 + V_{\rm sr}(\phi)$$

The fluctuations of X are amplified by tachyonic preheating and source the production of A_μ.



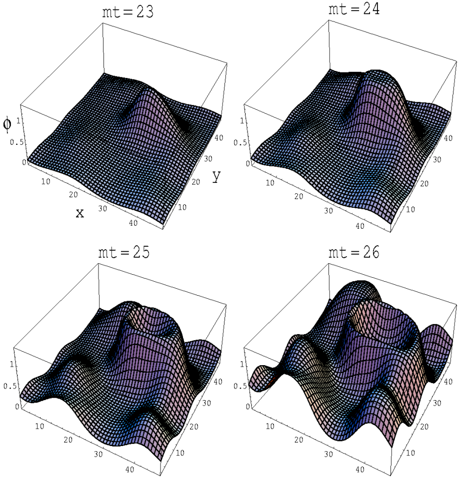


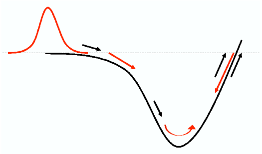


A_μ acquires a mass $m_A = e\,v$ through the Higgs mechanism. In models where X has zero VEV, its fluctuations provide a mass $m_A = e\langle X^2\rangle^{1/2}$. In $U(1)\times\ldots\times U(1)$ models, a massless A_μ is totally decoupled $\Rightarrow$ not amplified.

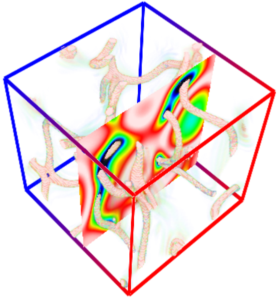
Higgs' "bubbles" and magnetic "strings" during symmetry-breaking

Spatial profile of the Higgs



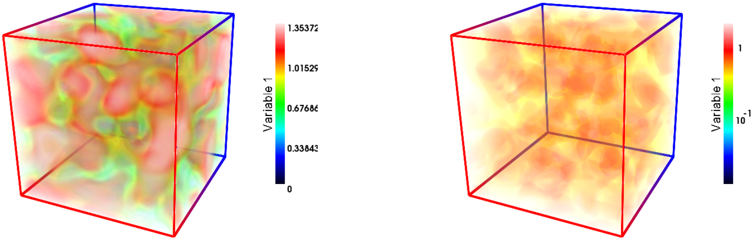


A_μ tends to be localized where $X = 0$ to minimize $\rho \supset |\partial_\mu X - ieA_\mu X|^2$

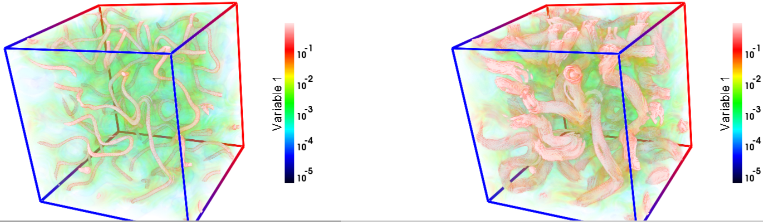


Higgs' "bubbles" and magnetic "strings" during symmetry breaking

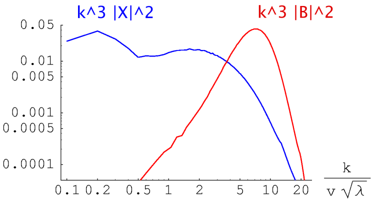
Spatial distributions of $|X|$ ("bubbles") at different times



Spatial distributions of $\vec{B}^2$ ("strings") at different times



Signatures in the GW Spectra

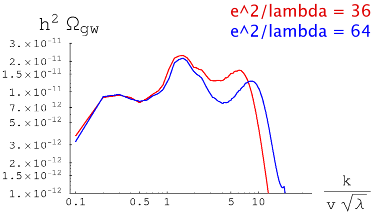


Peak of scalar spectrum: $k_* \propto \sqrt{\lambda}\, v$

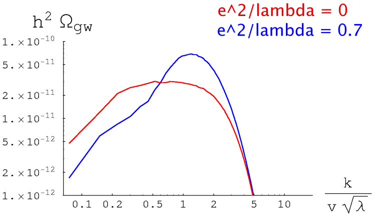
(depends on g^2, λ, initial velocity,...)

Peak of vector spectrum:

$k_* \simeq m_A = e\, v$ (string dynamics / width)



Multi-peaked GW for $e^2 \gg \lambda$



More GW with gauge fields

Conclusions and Perspectives

- **There can be several instances in the early universe where scalar field condensates decay in an explosive and highly inhomogeneous way**. This generates a stochastic background of GW that carries unique informations about these high-energy phenomena.
- **For preheating after inflation**, these GW can be observable if inflation occurs at low enough energy scales (complementary to GW from inflation itself). Depending on the model, they can cover a wide range of frequencies and amplitude.
- **For the non-perturbative decay of SUSY flat directions**, these GW fall naturally in the Hz-kHz frequency range where ground interferometers operate. They carry informations on both SUSY (scale of SUSY breaking) and inflation (reheat temperature).
- In both cases, **gauge fields can have important consequences on GW production and leave specific imprints in the GW spectra**
- Non-abelian symmetries like $SU(2) \times U(1)$ are necessary to produce **massless gauge fields** (photons) at preheating. $\Rightarrow$ GW production from long, turbulent evolution towards thermal equilibrium after preheating??
- How do **flat directions decay in realistic models like MSSM**?? Other cosmological consequences (baryogenesis, reheating, ...)??
- Applications to **other GW sources**, phase transitions, cosmic strings, ...