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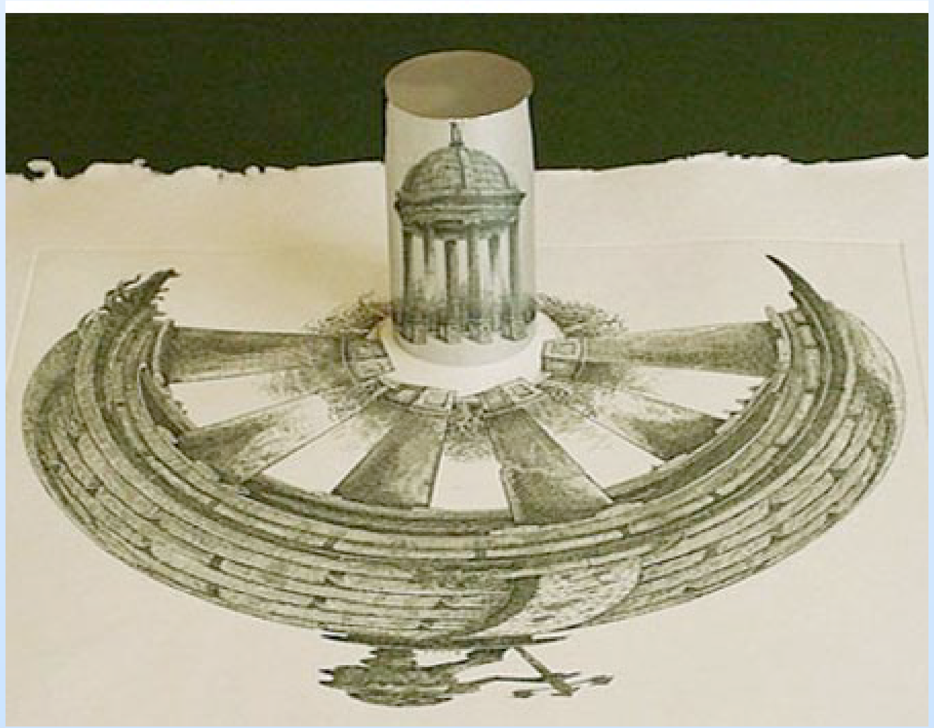
based on
S.C., J. Rocher, hep-ph/0809.4355
S.C., C. Ringeval, J. Rocher, hep-ph/0909.0402
S.C., arXiv:1006.4522

New insights in hybrid inflation



Institut d'Astrophysique de Paris (IAP), 2nd November 2010

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2. Initial conditions

2.1. Fine-tuning of IC

2.2. How to avoid fine-tuning?

2.3. Anamorphosis trajectories

2.4. Other hybrid models

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2.6. Remaining questions...

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3.1. Classical vs. Stochastic dynamics

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3.3. MCMC analysis of the parameter space

3.4 Power spectrum of adiabatic pert.

4. Conclusion and Perspectives

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- Flatness Problem
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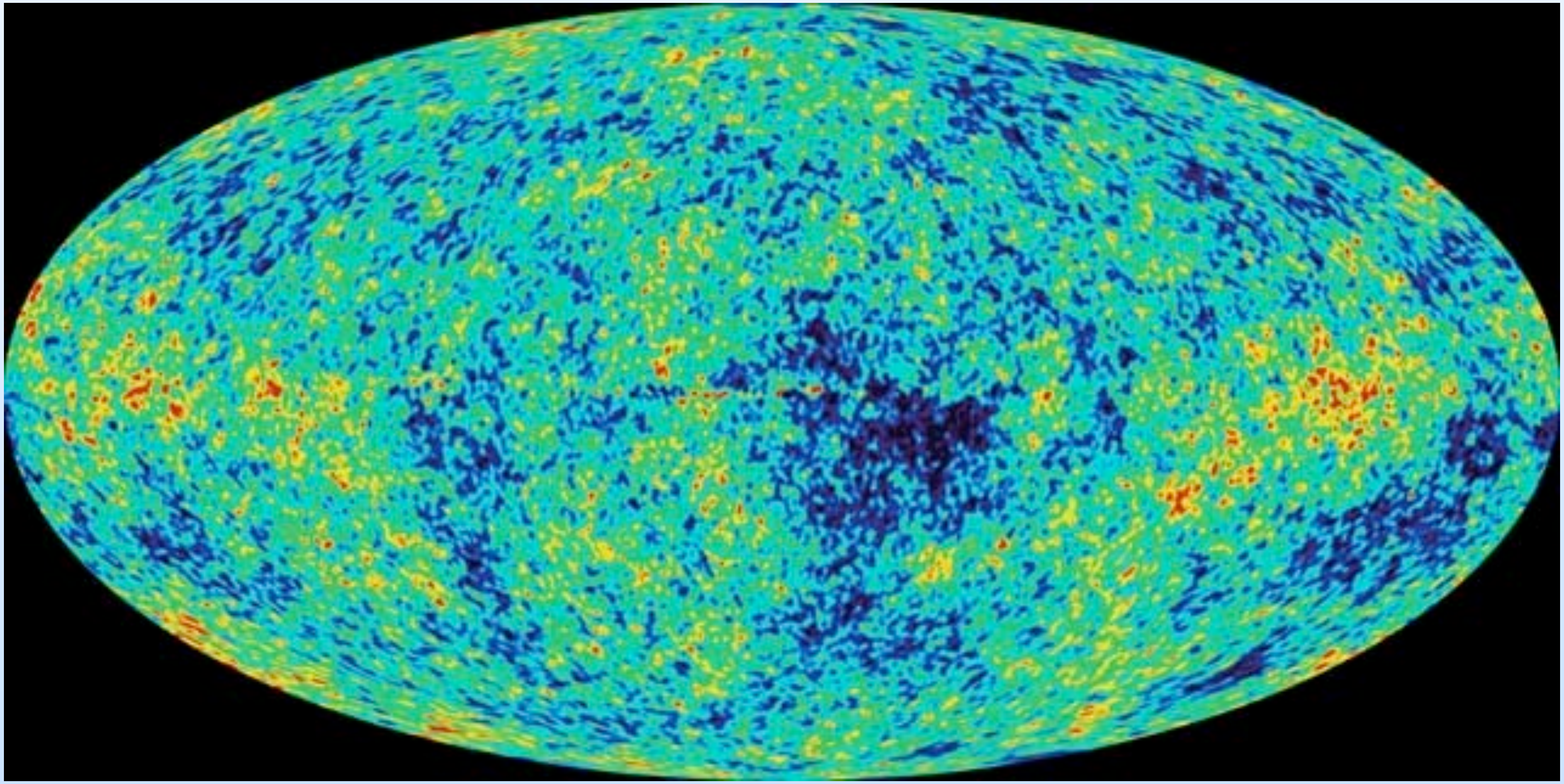

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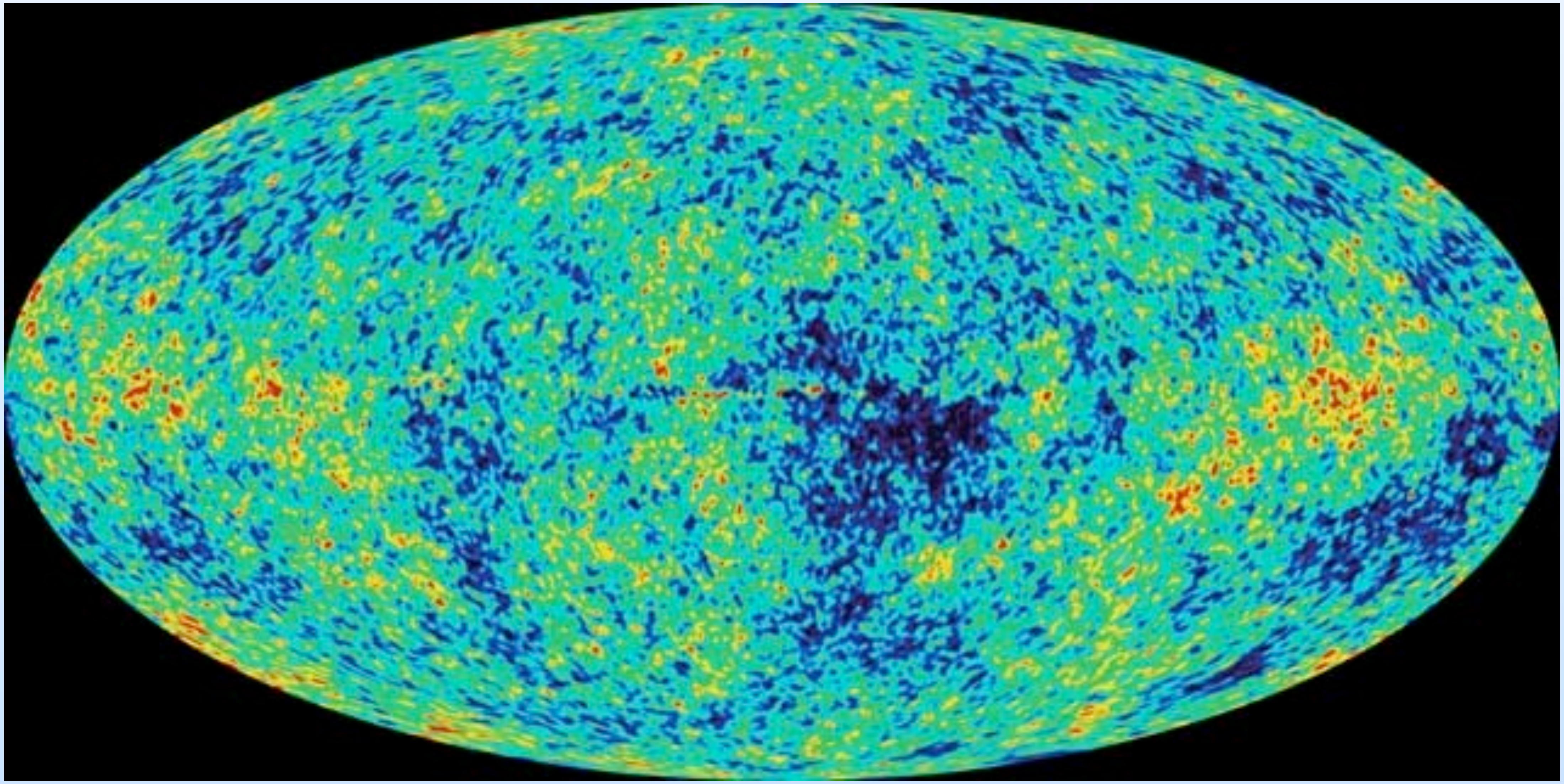

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- **Cosmological Perturbations :** $\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$
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$$\mathcal{P}_\zeta(k) = C\left(\frac{k}{k_*}\right)^{n_{\rm s}-1}$$

(nearly) scale invariance

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COBE norm. : $C = (2.43 \pm 0.11) \times 10^{-9}$

WMAP7 : $n_s = 0.963^{+0.014}_{-0.015}$

(nearly) scale invariance

Directly linked to the Potential (slow-roll)

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$$\mathcal{P}_\zeta(k) = C\left(\frac{k}{k_*}\right)^{n_{\rm s}-1}$$

$$C \sim \frac{H_*^2}{\pi\epsilon_{1*}}$$

$$n_s - 1 = -2\epsilon_{1*} - \epsilon_{2*}$$

$$\epsilon_1 = \frac{m_{\rm p}^2}{16\pi}\left(\frac{\frac{{\rm d}V}{{\rm d}\phi}}{V}\right)^2 \ll 1$$

$$\epsilon_2 = \frac{m_{\rm p}^2}{4\pi}\left[\left(\frac{V'}{V}\right)^2 - \frac{V''}{V}\right] \ll 1$$

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Other realisation : Fill the universe with TWO scalar fields

F.L. equations:

$$H^2 = \frac{8\pi}{3m_\mathrm{p}^2}\left[\frac{1}{2}\left(\dot{\phi}^2 + \dot{\psi}^2\right) + V(\phi,\psi)\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3m_\mathrm{p}^2}\left[-\dot{\phi}^2 - \dot{\psi}^2 + V(\phi,\psi)\right]$$

K.G. equations:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \qquad \ddot{\psi} + 3H\dot{\psi} + \frac{\mathrm{d}V}{\mathrm{d}\psi} = 0$$

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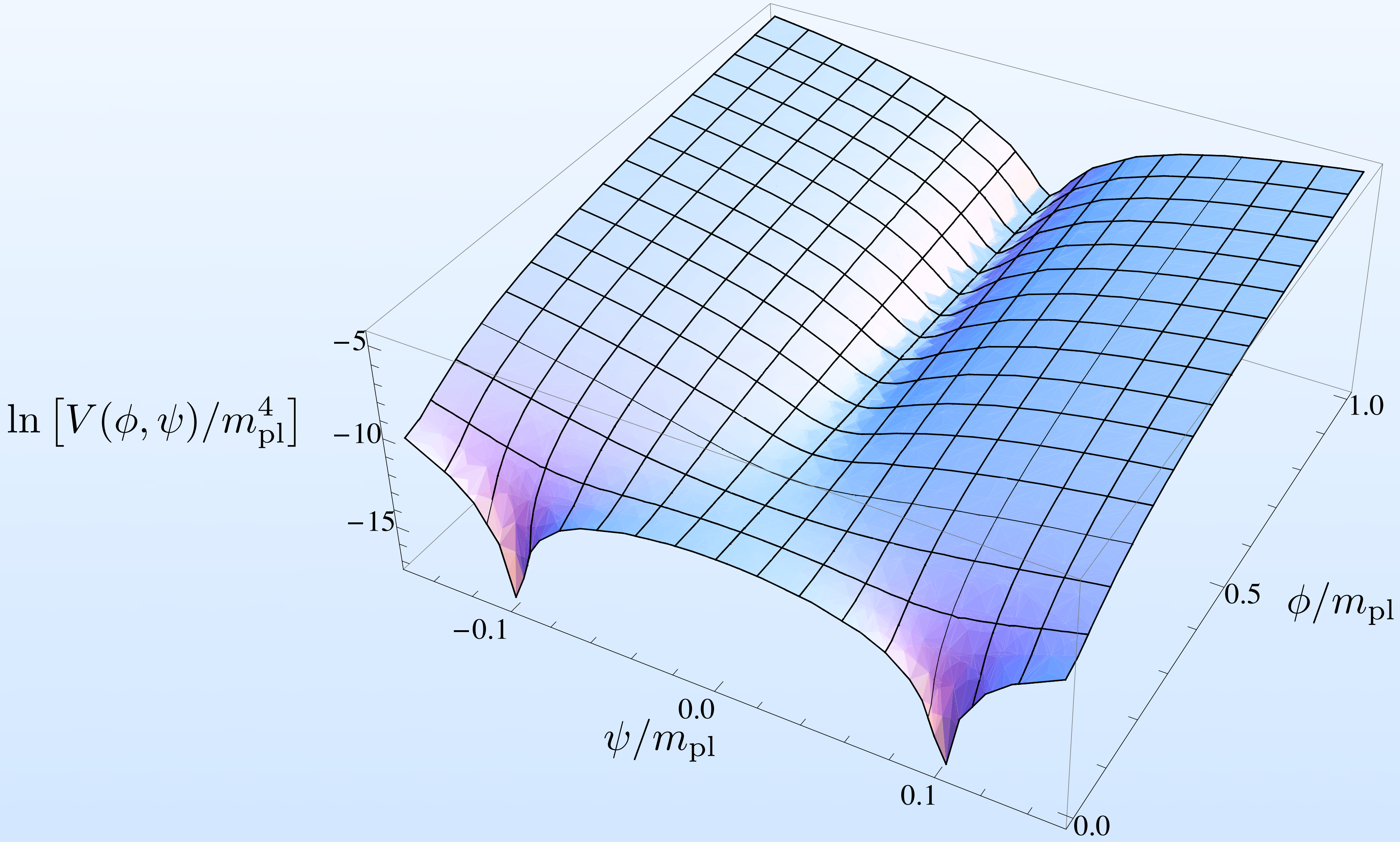
- **Higgs-type auxiliary field** ψ
- **Hybrid potential** (Linde, astro-ph/9307002) $V(\phi,\psi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2}\right) + \frac{\phi^2}{\mu^2} + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} \right]$



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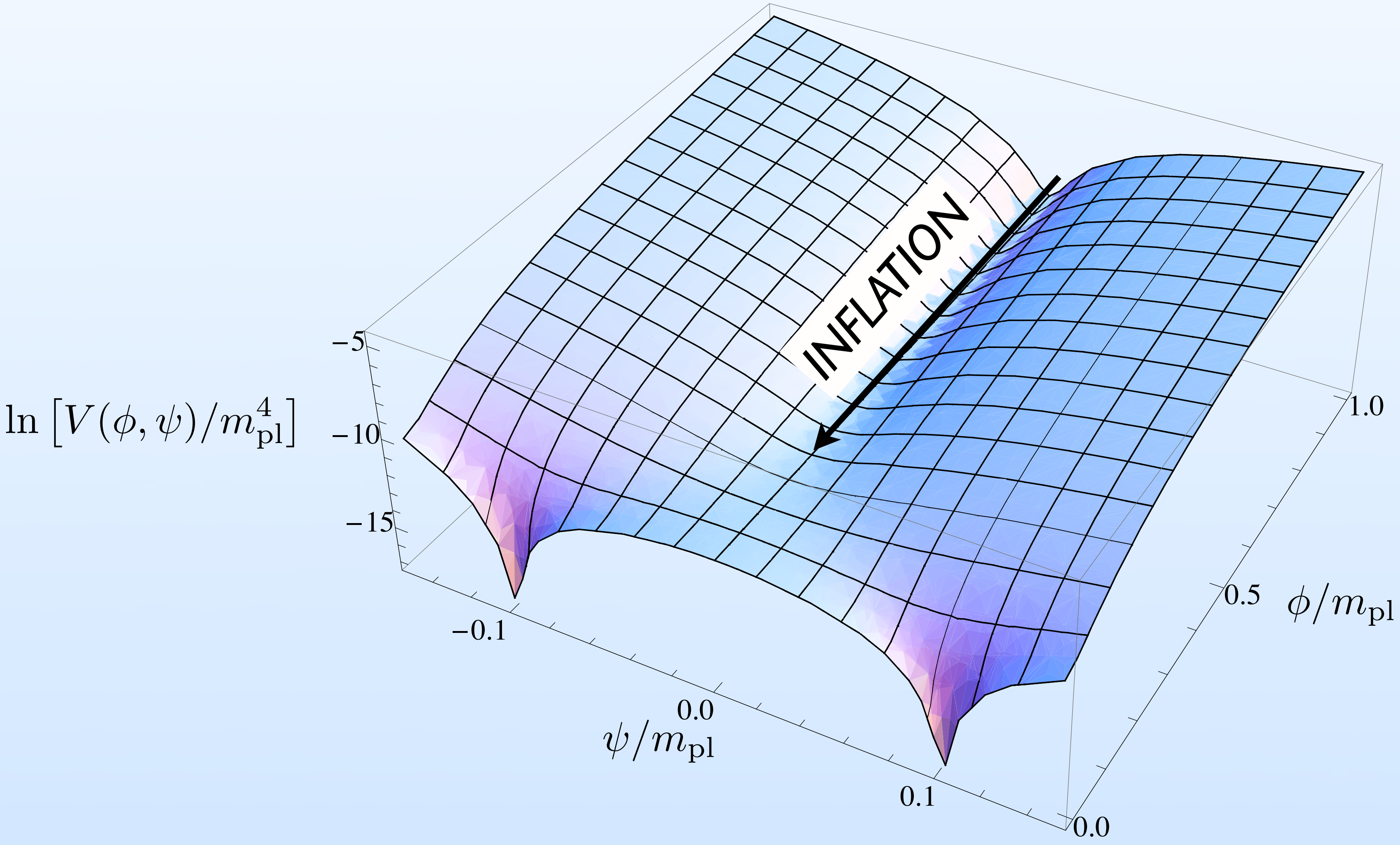

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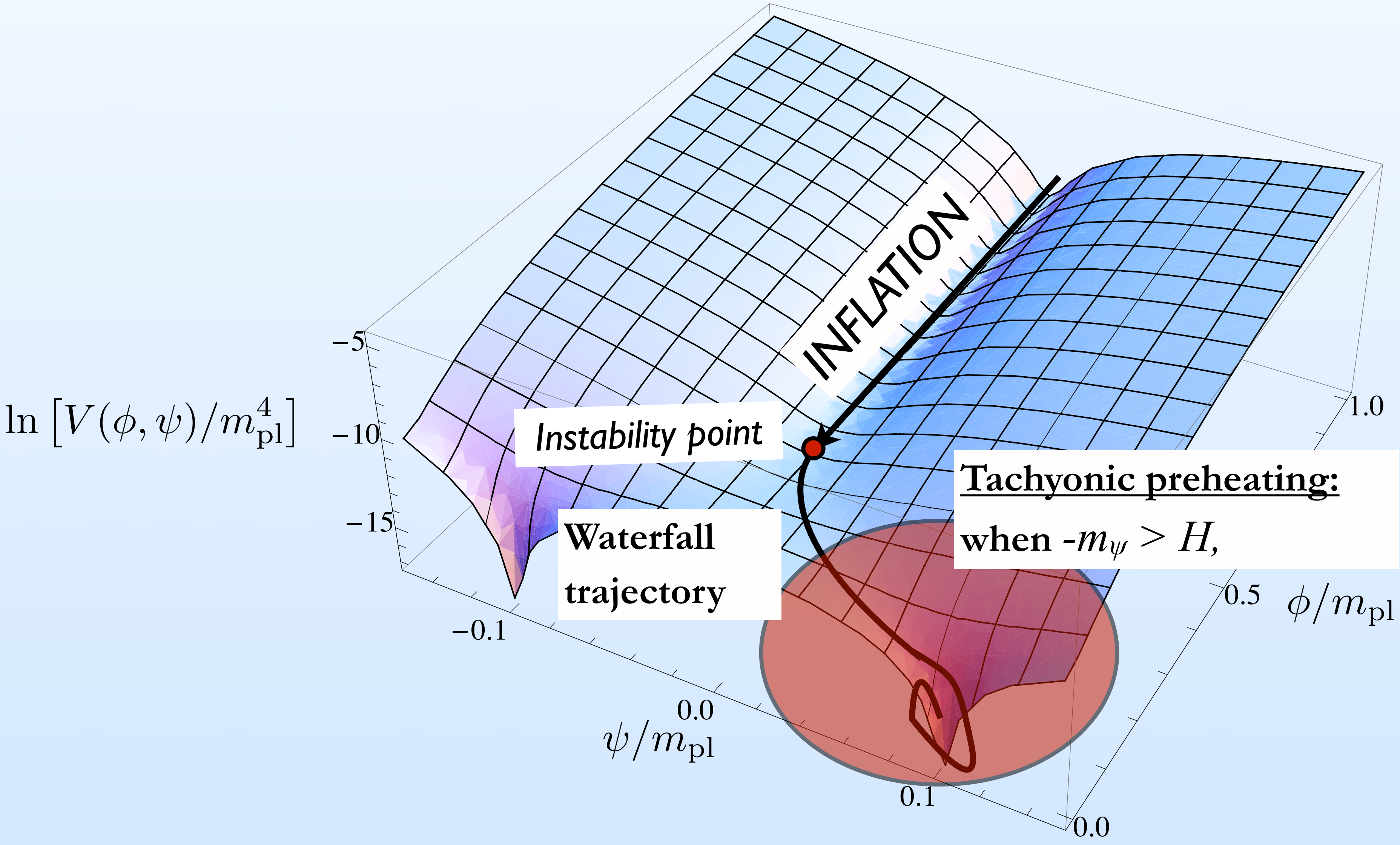

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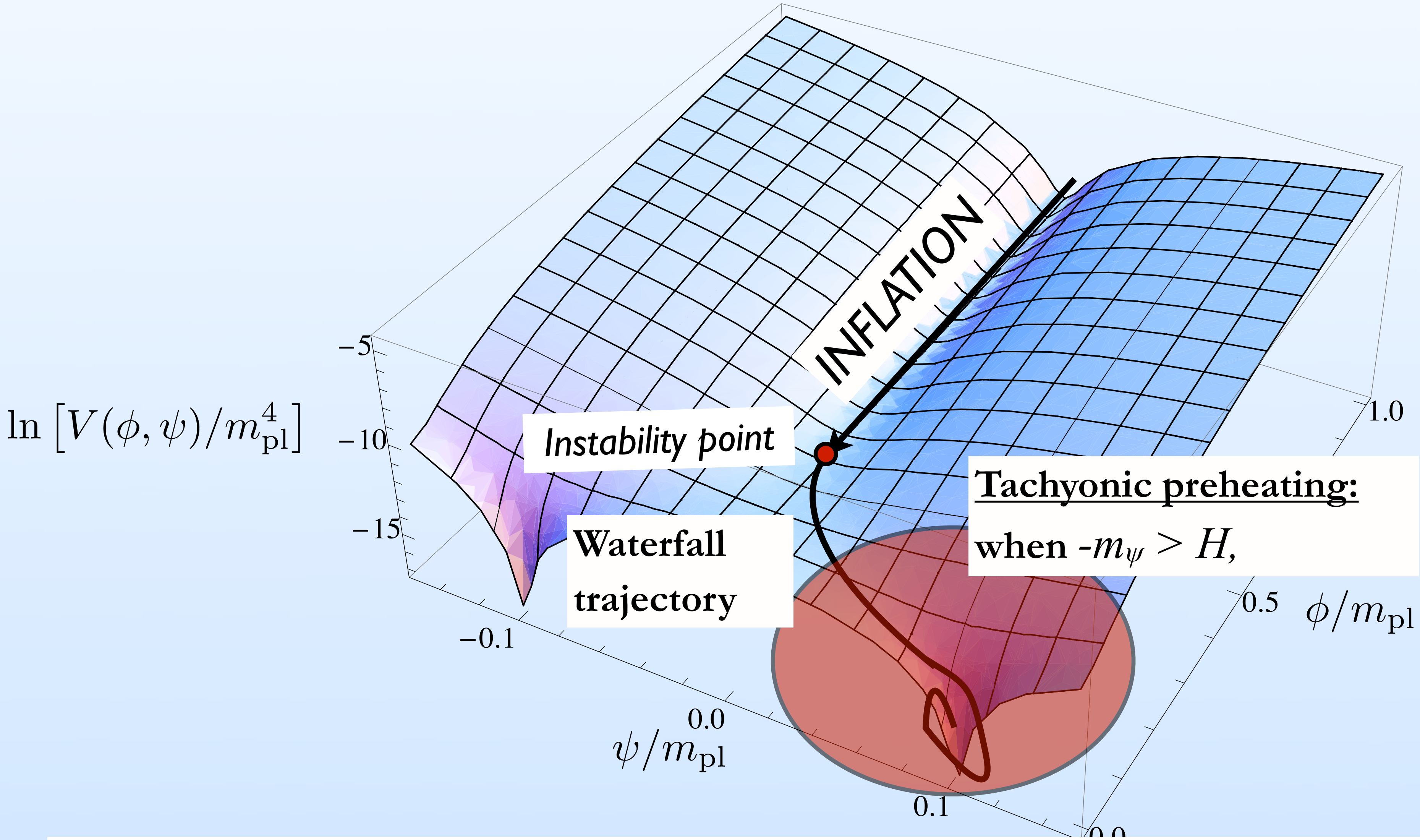

Instability point

Waterfall trajectory

Tachyonic preheating:

when $-m_\psi > H$,

Usual description

- **1-field effective potential along the valley**
- **Slow-roll approximation**
- **Quasi instantaneous end of inflation when instability is reached**
- **The primordial scalar power spectrum is slightly blue, disfavoured by CMB experiments**

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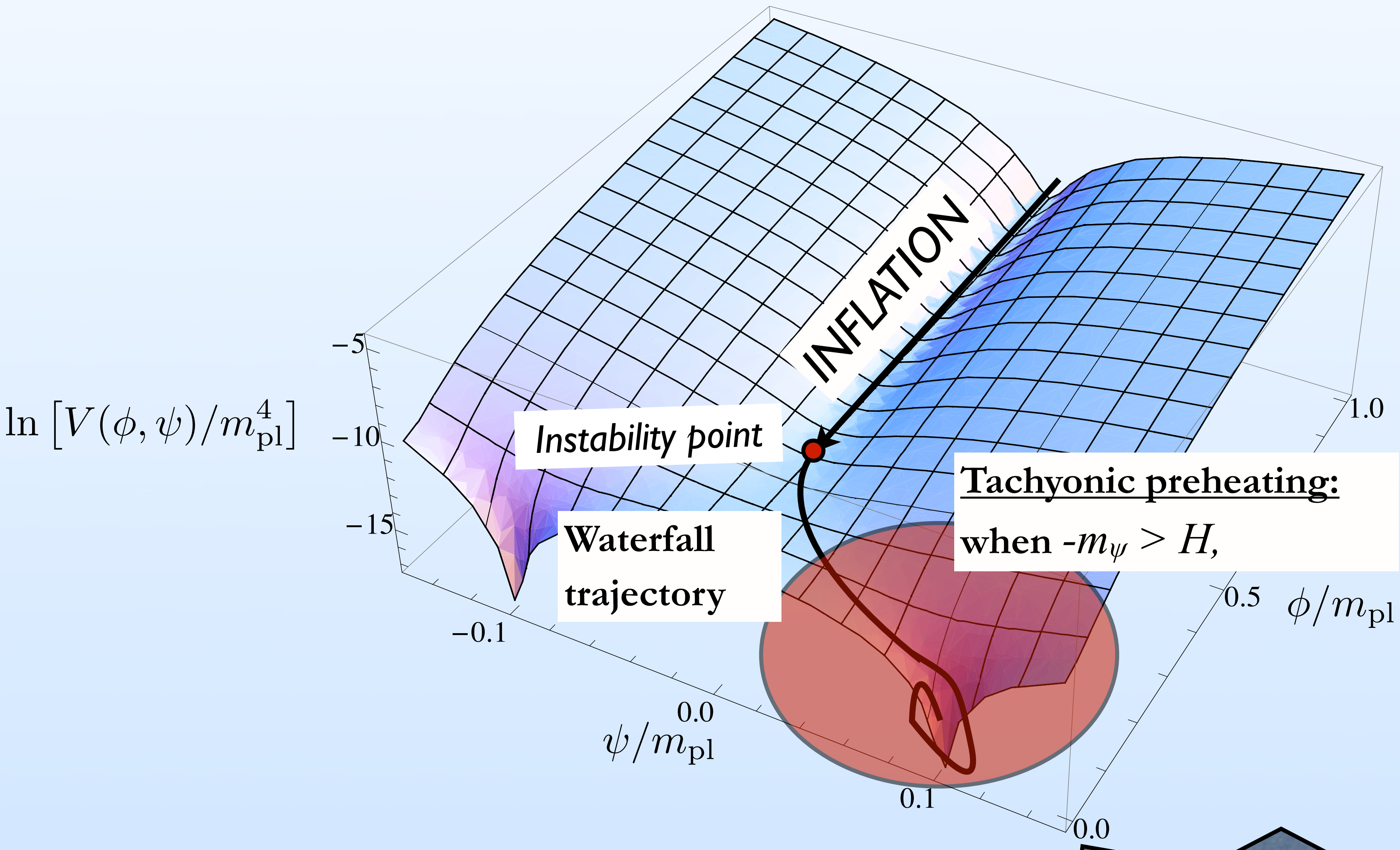

Exact 2-field dynamics

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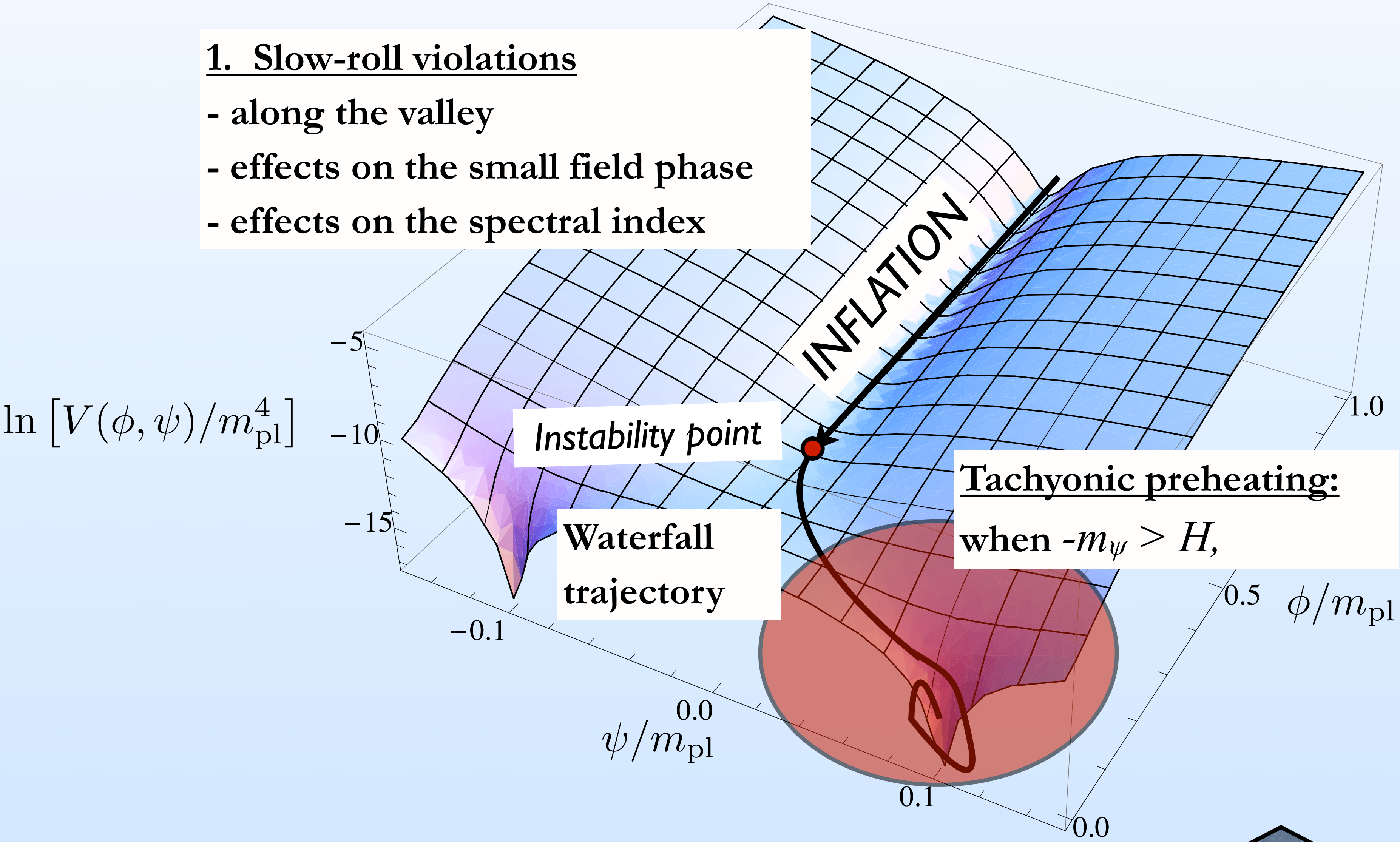

1. Slow-roll violations

- along the valley
- effects on the small field phase
- effects on the spectral index

Tachyonic preheating:

when -$m_\psi > H$,

Exact 2-field dynamics

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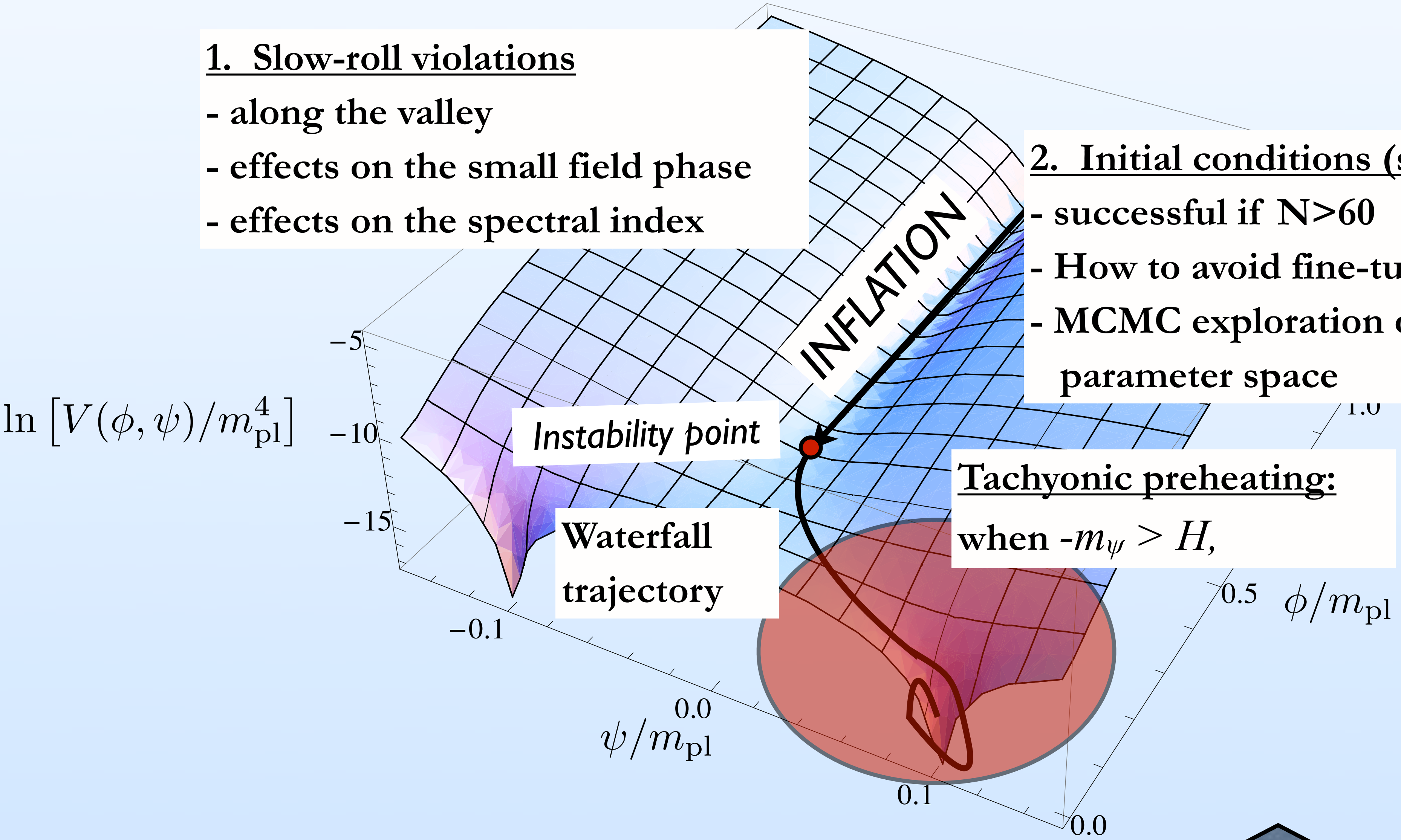

1. Slow-roll violations
- along the valley
- effects on the small field phase
- effects on the spectral index

2. Initial conditions (sub-plankian)
- successful if N>60
- How to avoid fine-tuning?
- MCMC exploration of the parameter space

Waterfall trajectory

Tachyonic preheating:
when $-m_\psi > H$,

Exact 2-field dynamics

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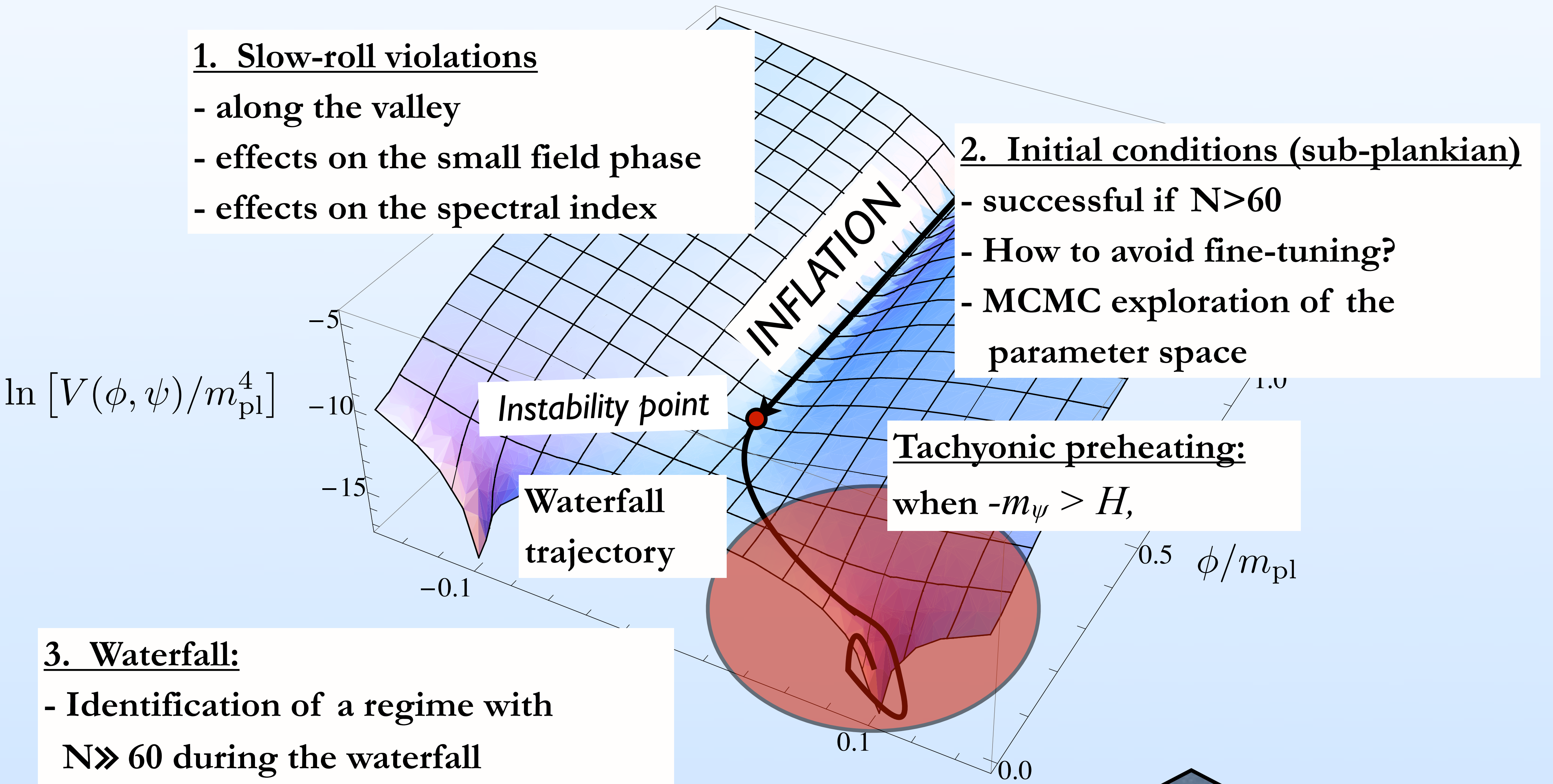

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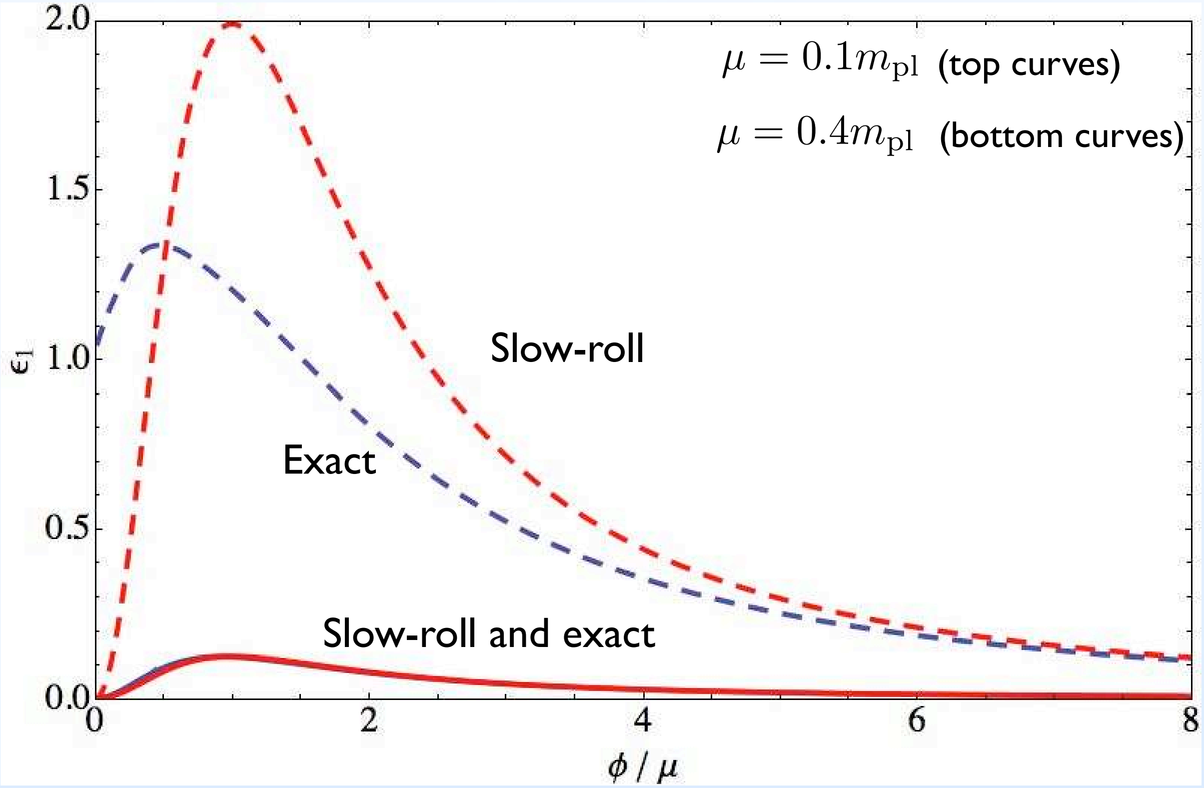
1. Slow-roll violations

- **1-field effective potential:** $V(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^2\right]^2$

- **First slow-roll parameter:** $\epsilon_1 \equiv -\frac{\dot{H}}{H^2}$

- **Slow-roll approximation:** $\epsilon_1 \ll 1$

$$\epsilon_1(\phi) = \frac{1}{4\pi}\left(\frac{m_{\rm pl}}{\mu}\right)^2 \frac{(\phi/\mu)^2}{[1+(\phi/\mu)^2]^2}$$



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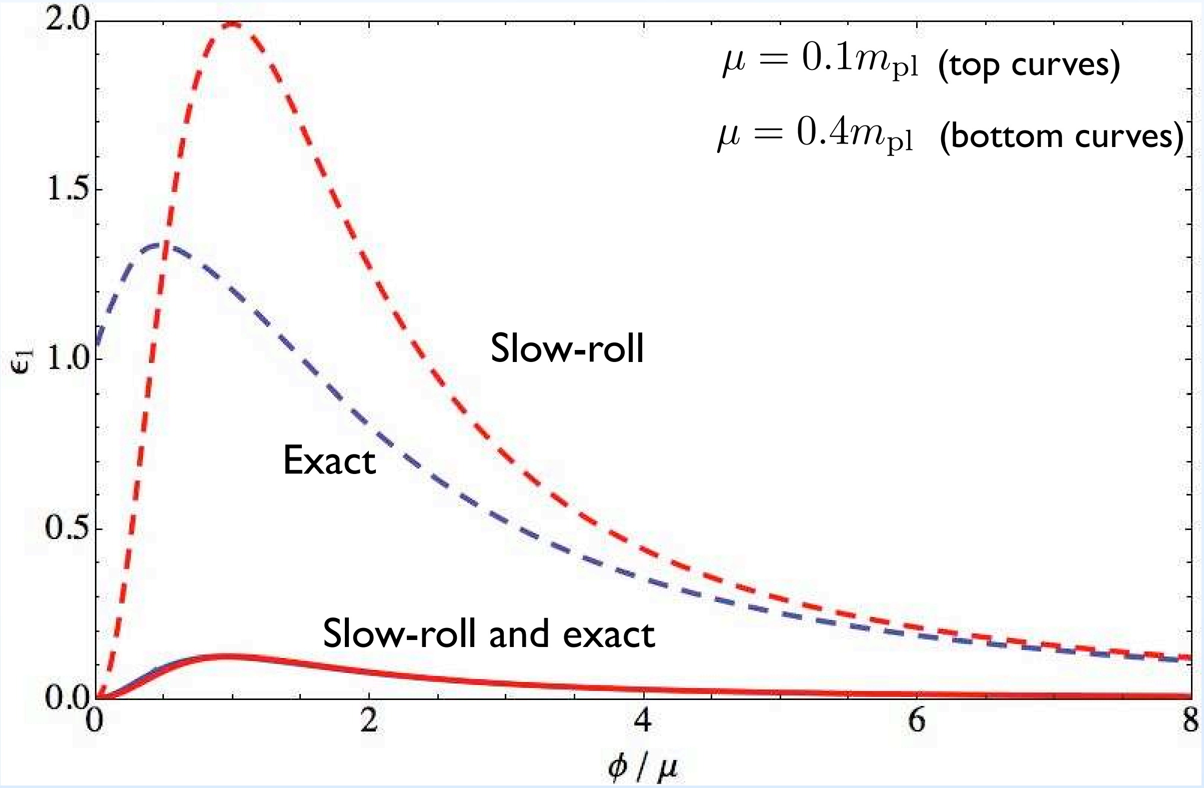

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1. if $\phi_c \gtrsim \mu$ (trivial)

2. if slow-roll is violated (non trivial, exact dynamics needed)

Implications:

- **Need of super-Planckian initial field values for N>60**
- **Red scalar power spectrum instead of blue**
- **Hybrid inflation like a chaotic model**

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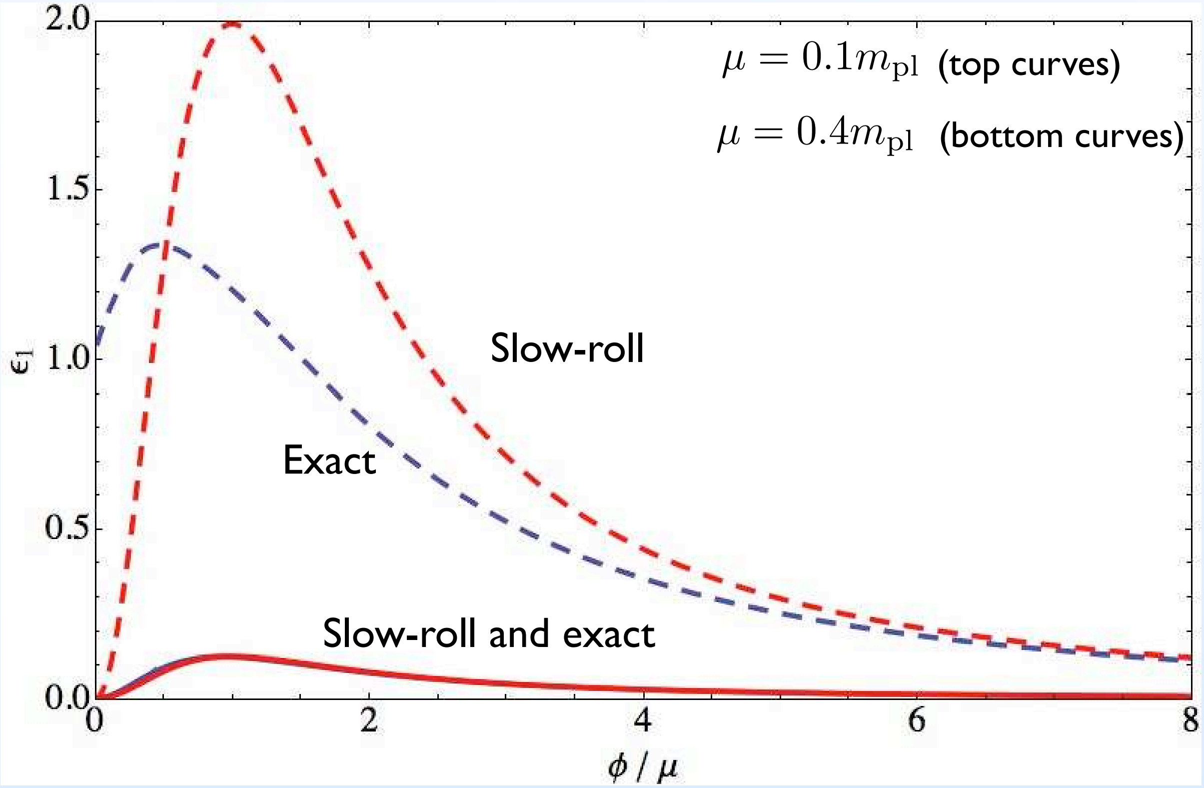

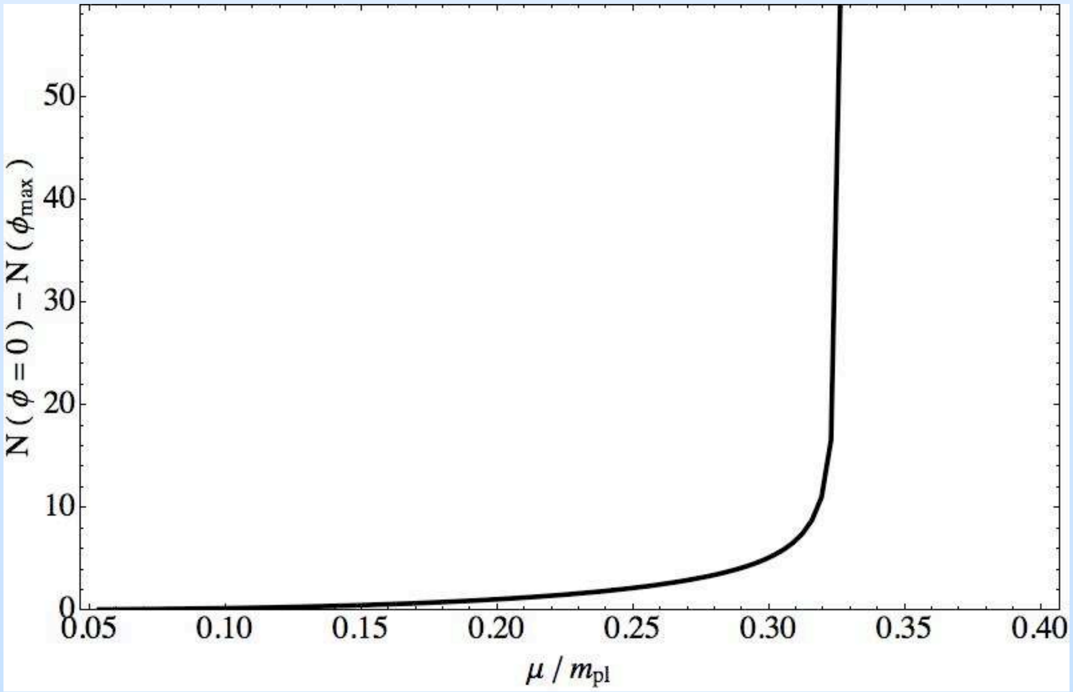

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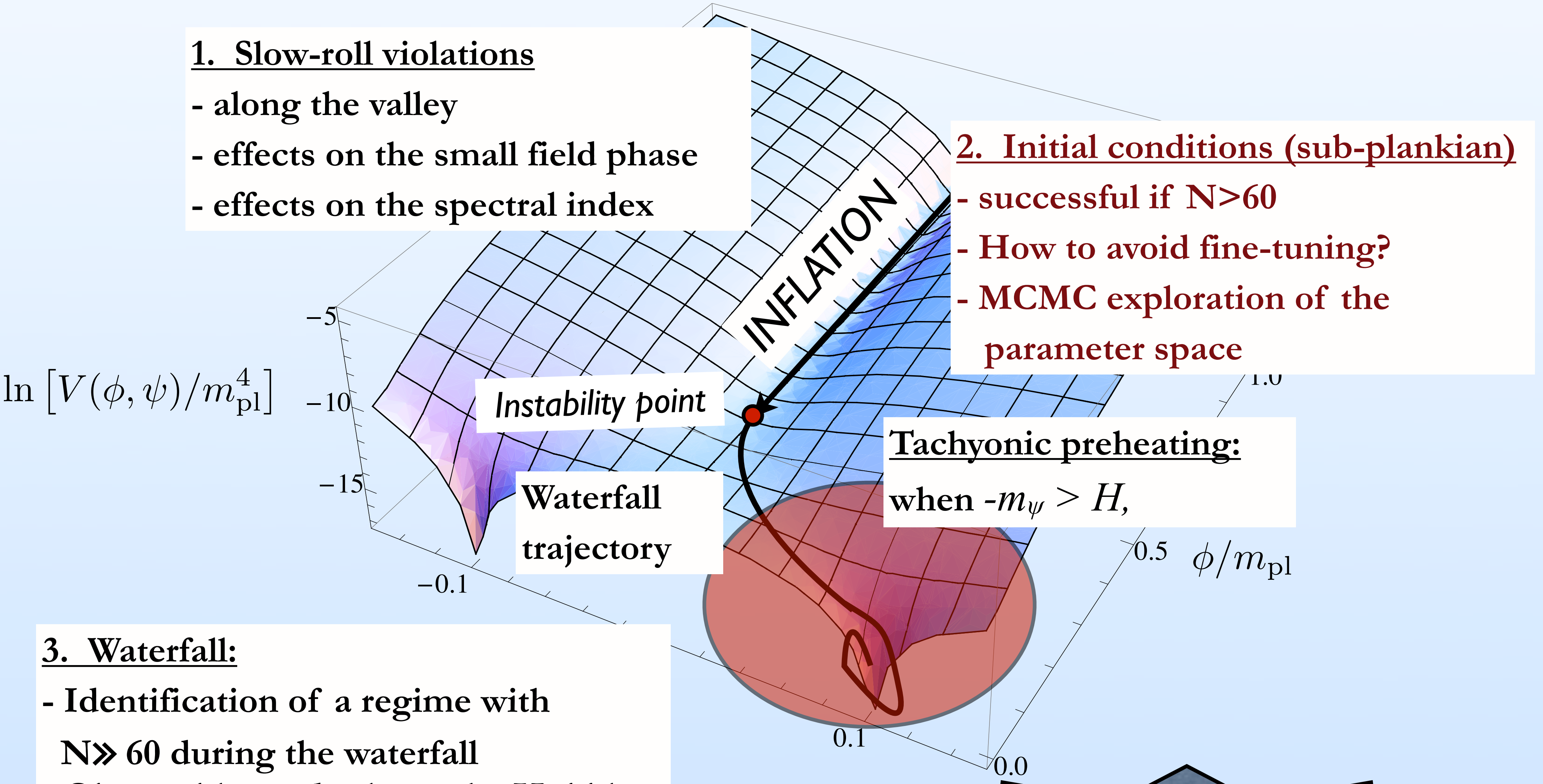

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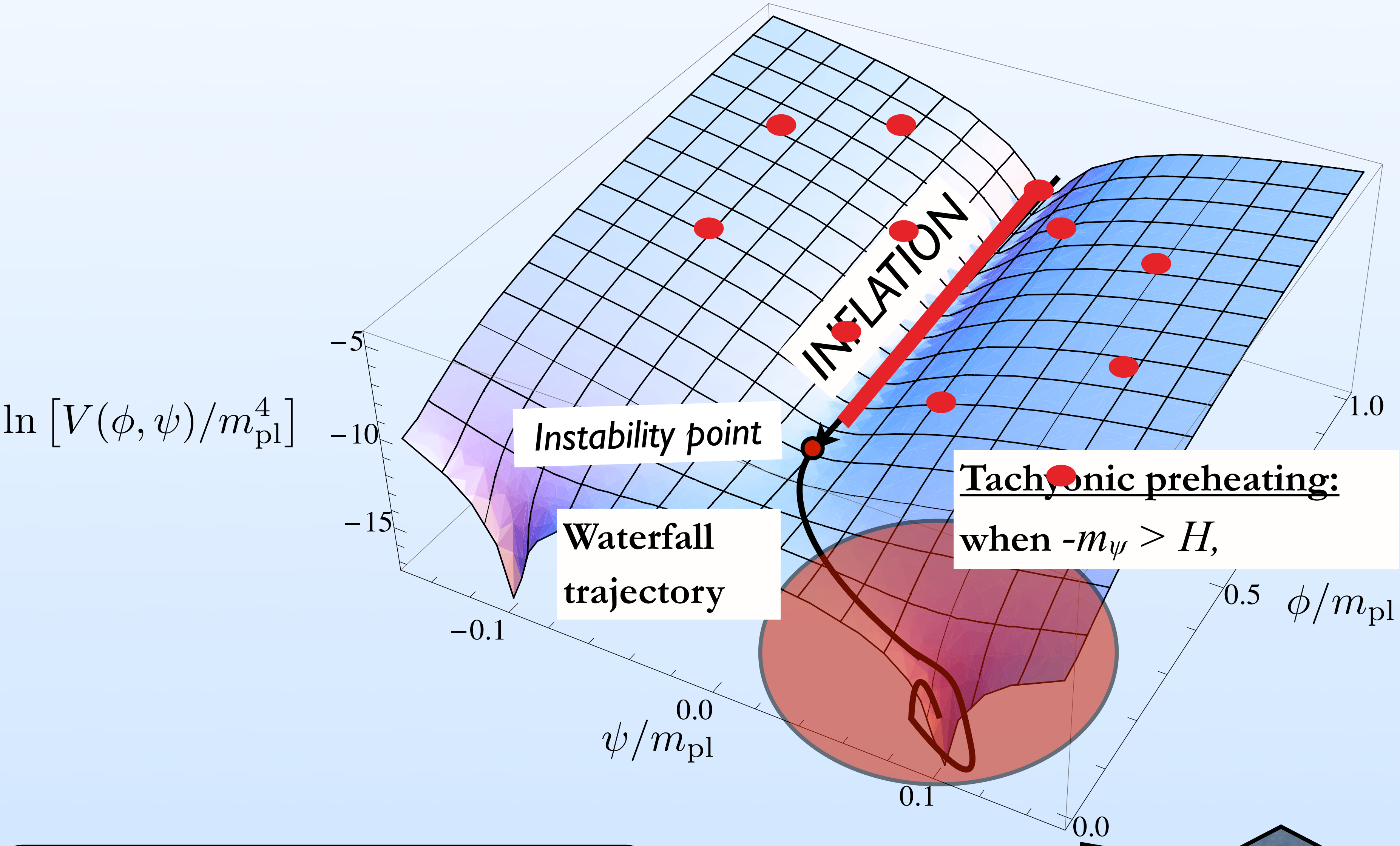

Instability point

Waterfall trajectory

Tachyonic preheating:

when $-m_\psi > H$,

Tetradis, astro-ph/9707214

Mendes, Liddle, astro-ph/0006020

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Tetradis, astro-ph/9707214

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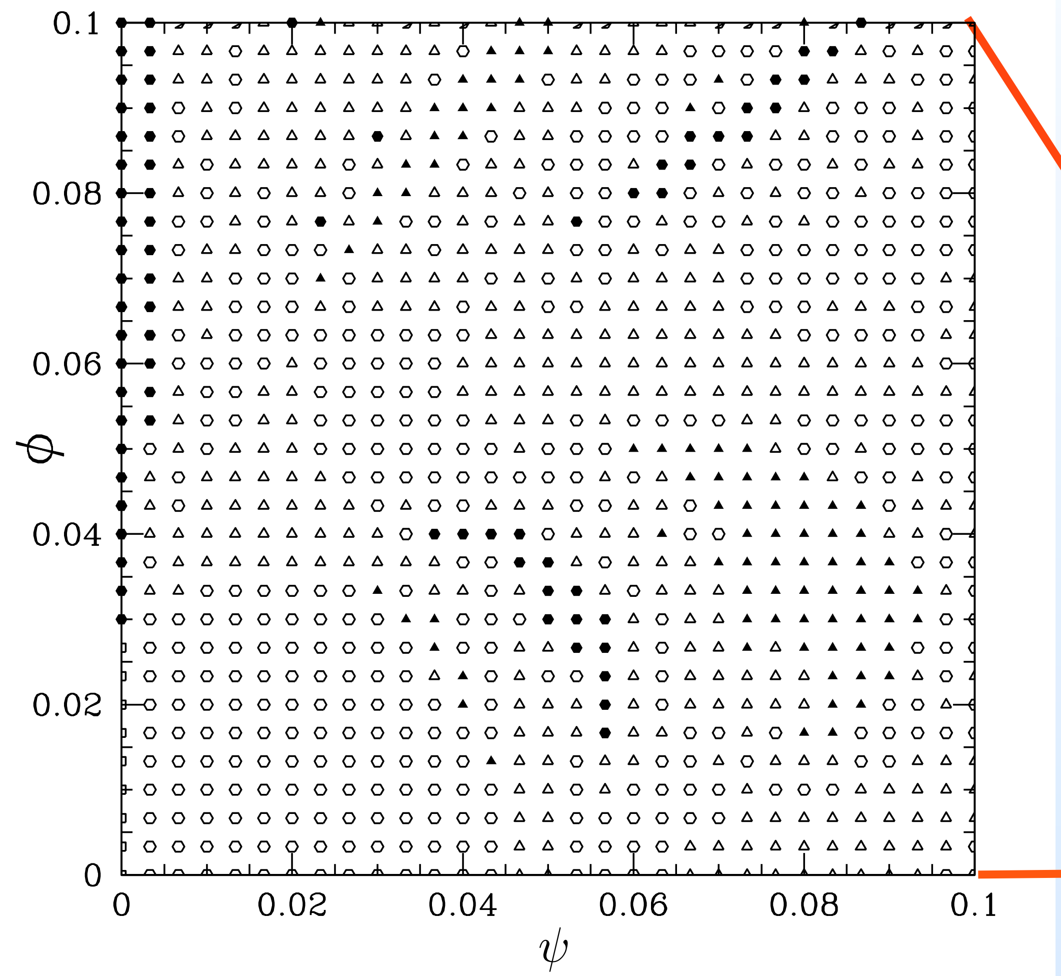
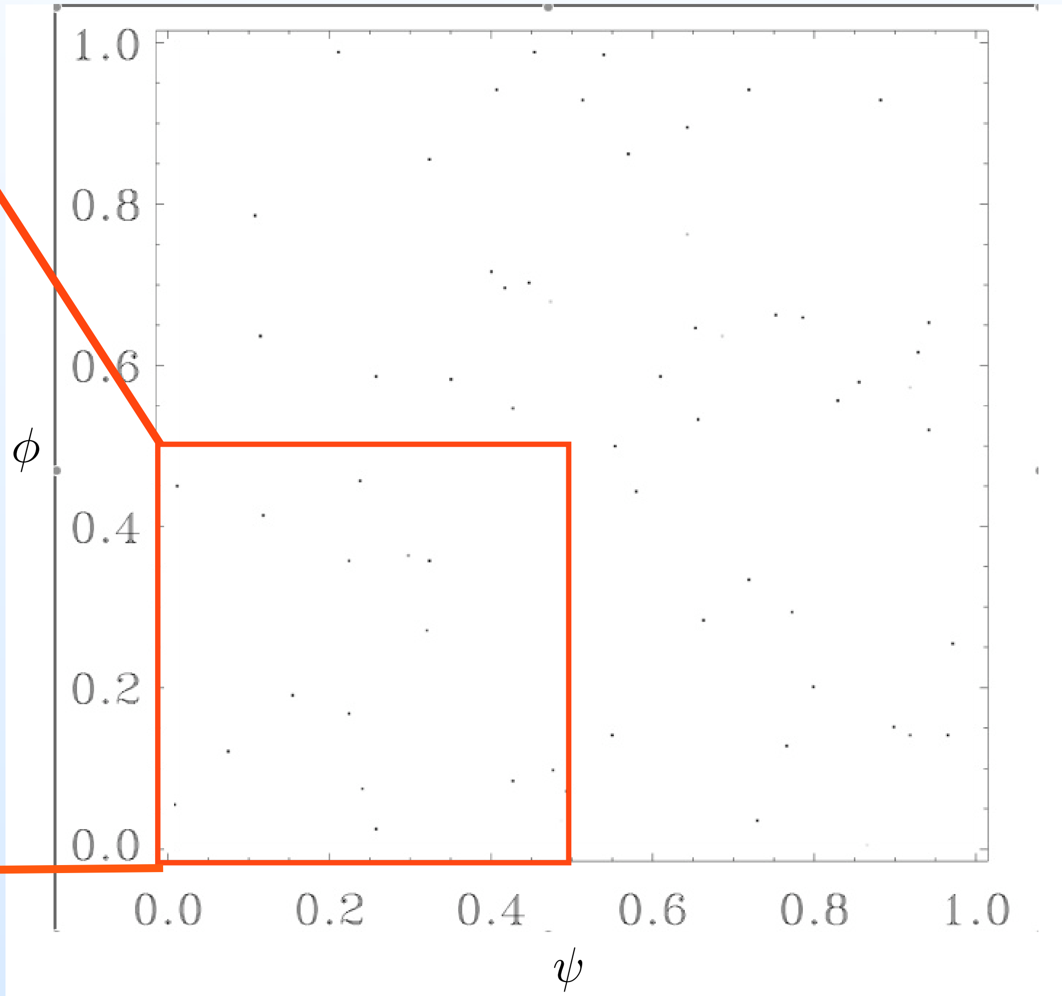


(in Planck mass)

Mendes, Liddle, astro-ph/0006020

$M = 0.004\ m_{\rm Pl},\ \phi_{\rm c} = 0.004\ m_{\rm Pl},\ \mu = 11.31\ m_{\rm Pl}$



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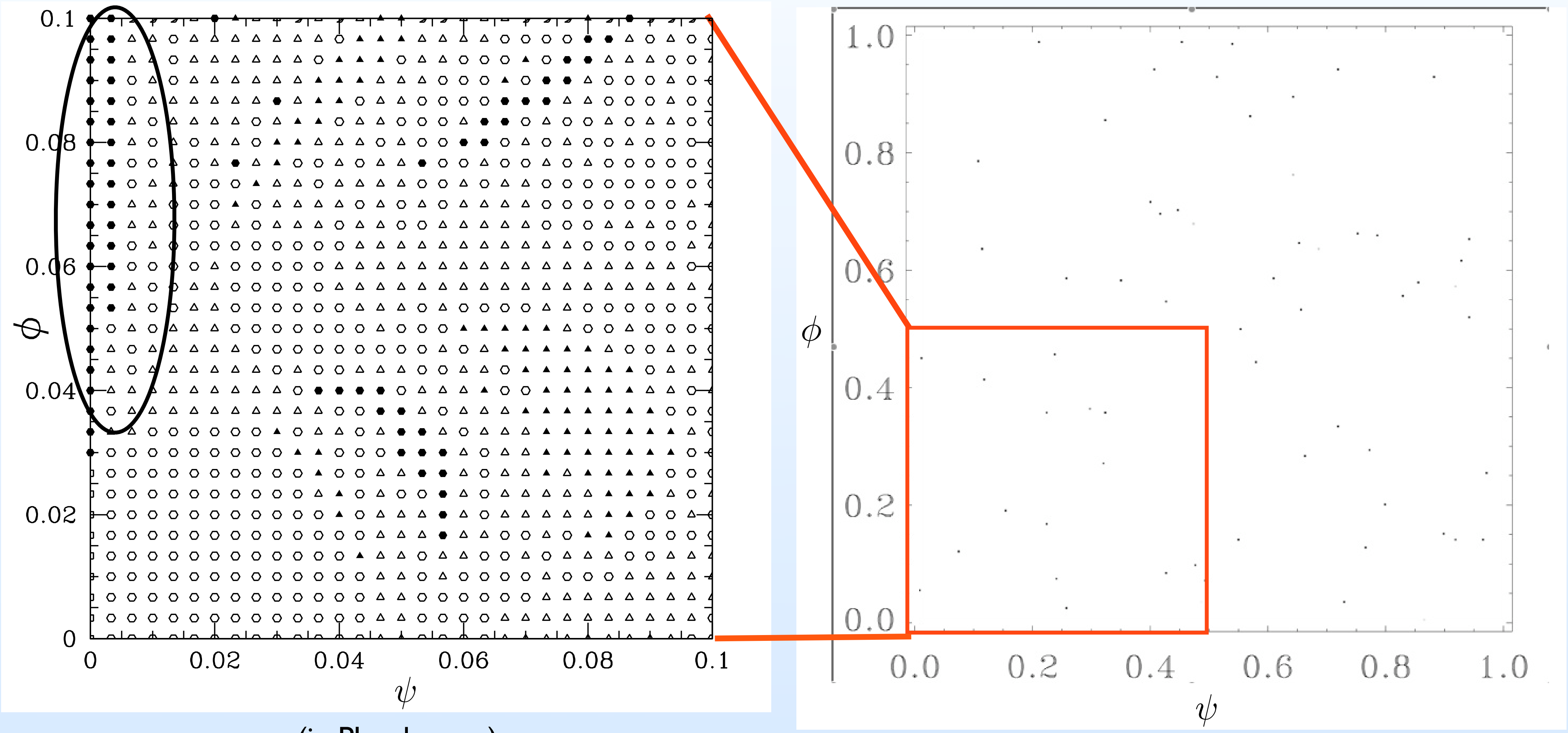

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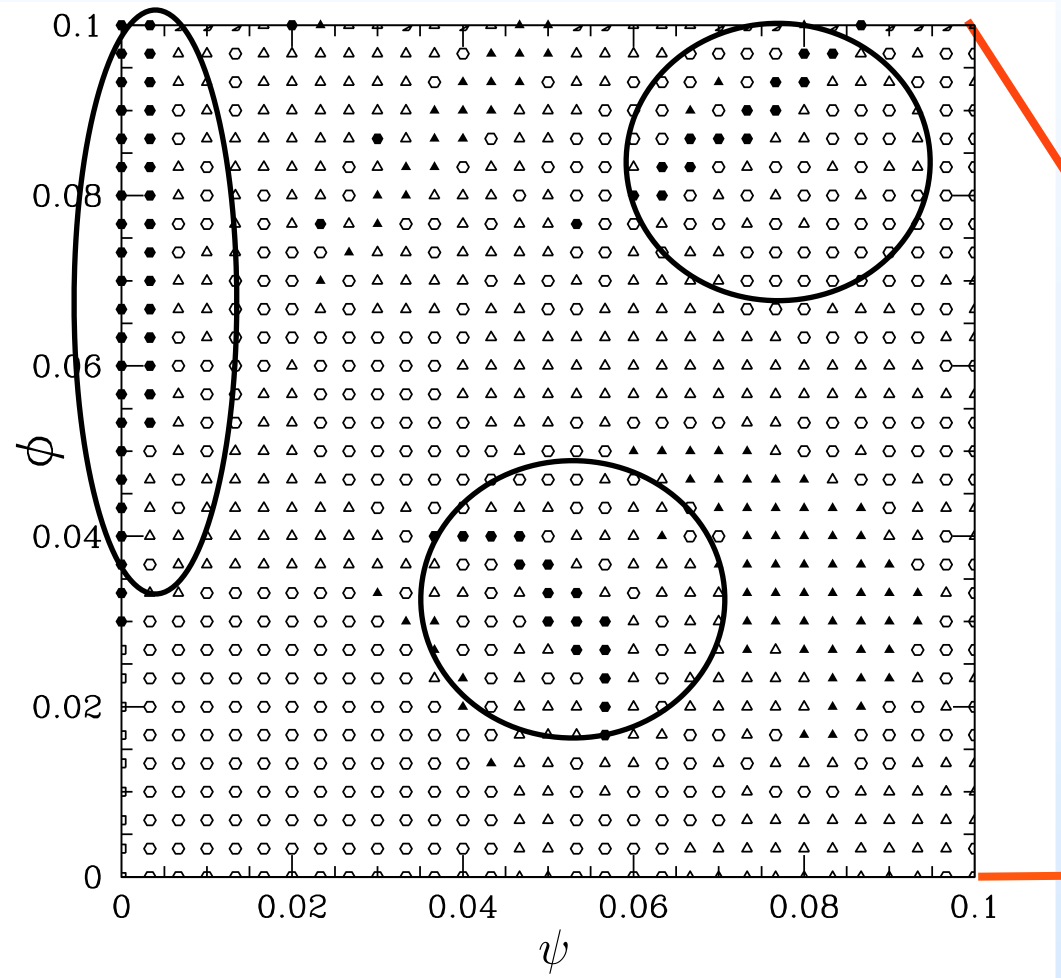

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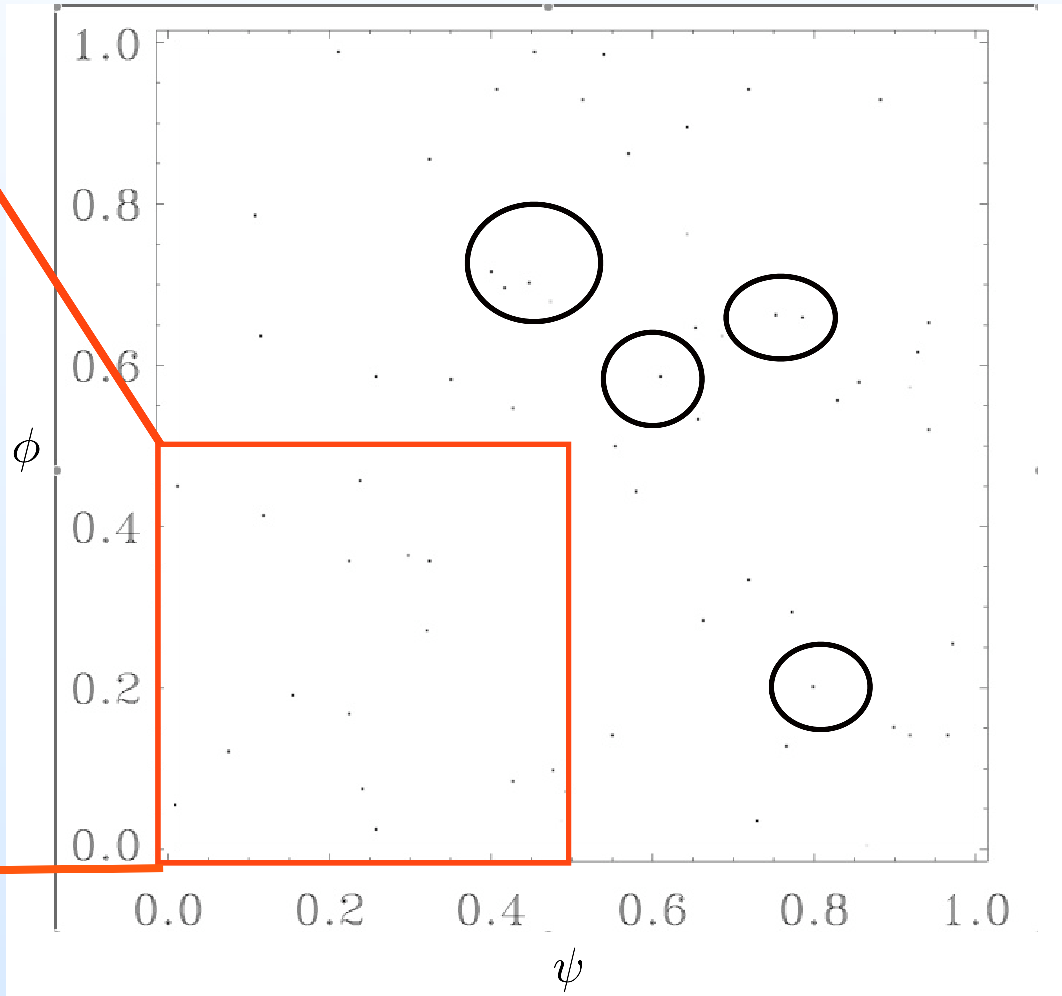

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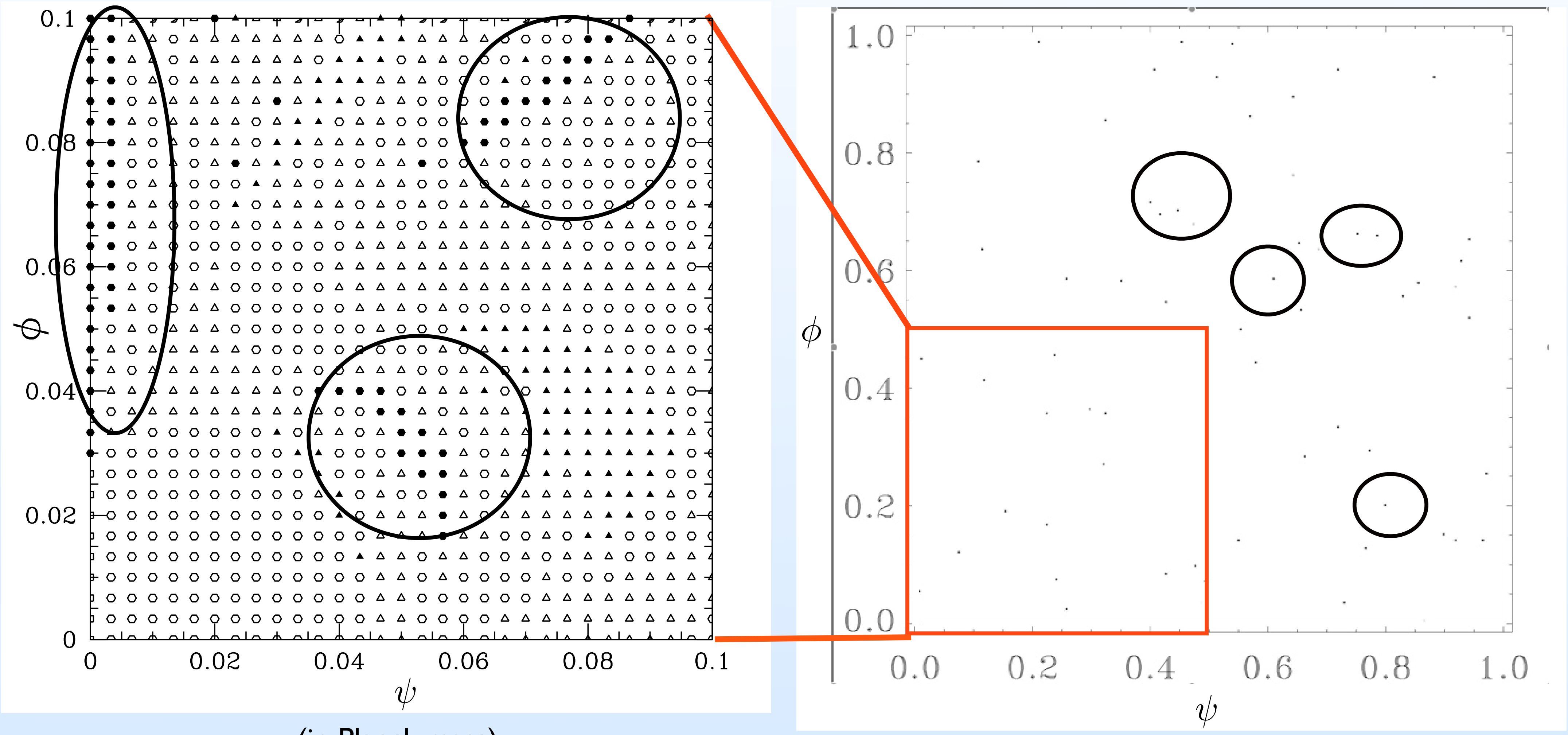

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- **Isolated points or structures ?**

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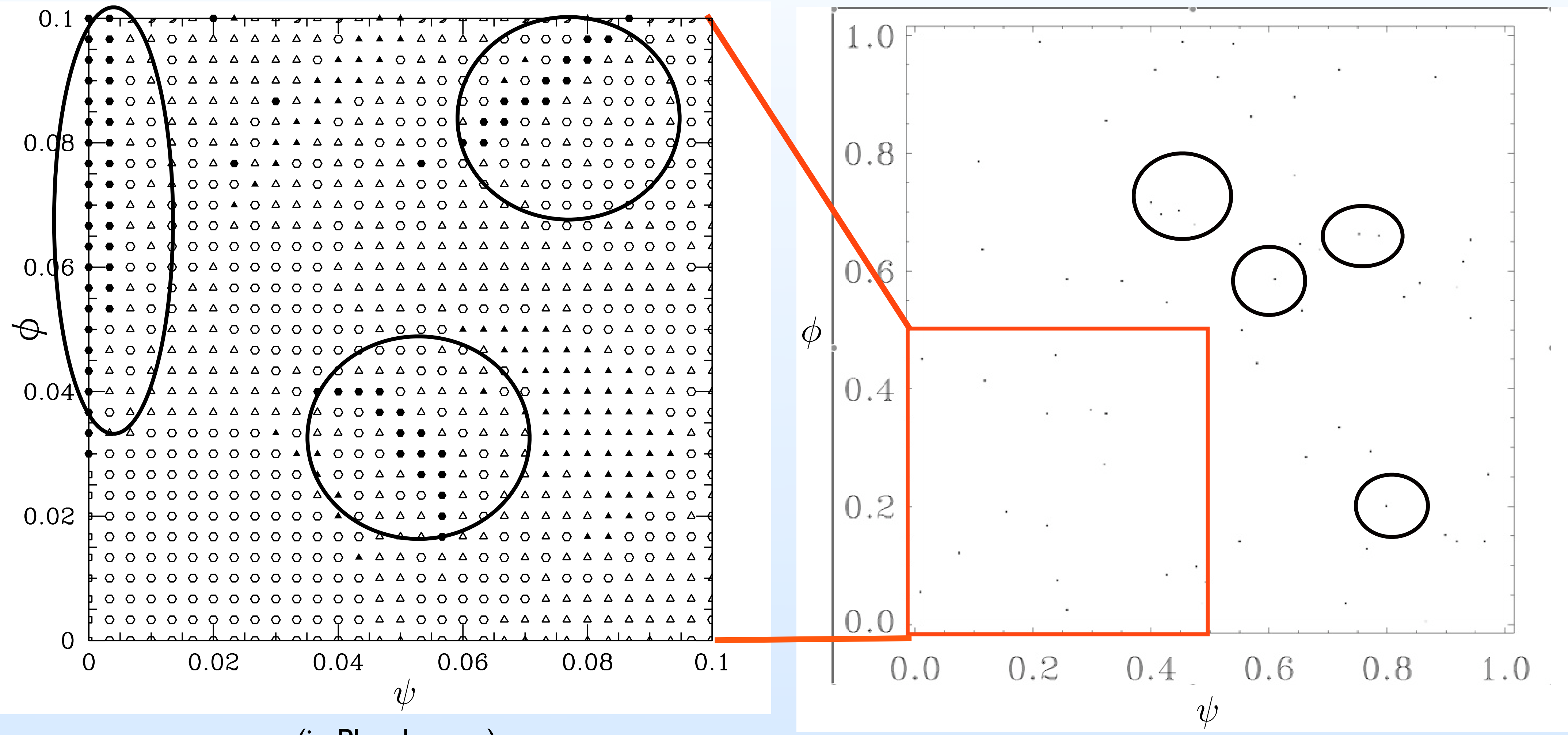

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- **Isolated points or structures ?**
- **Origin ?**

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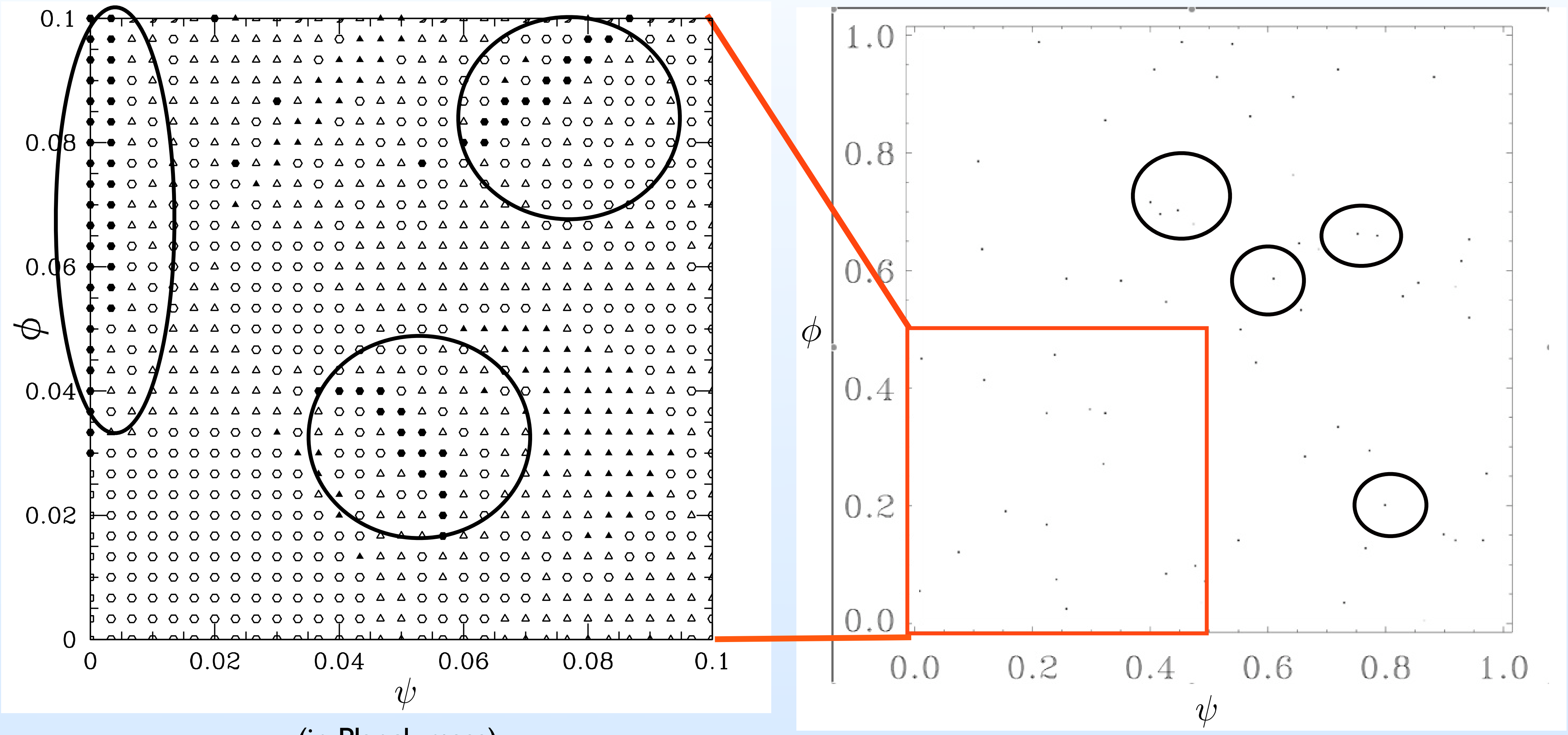

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- **Isolated points or structures ?**
- **Origin ?**
- **Quantification of successful areas ?**

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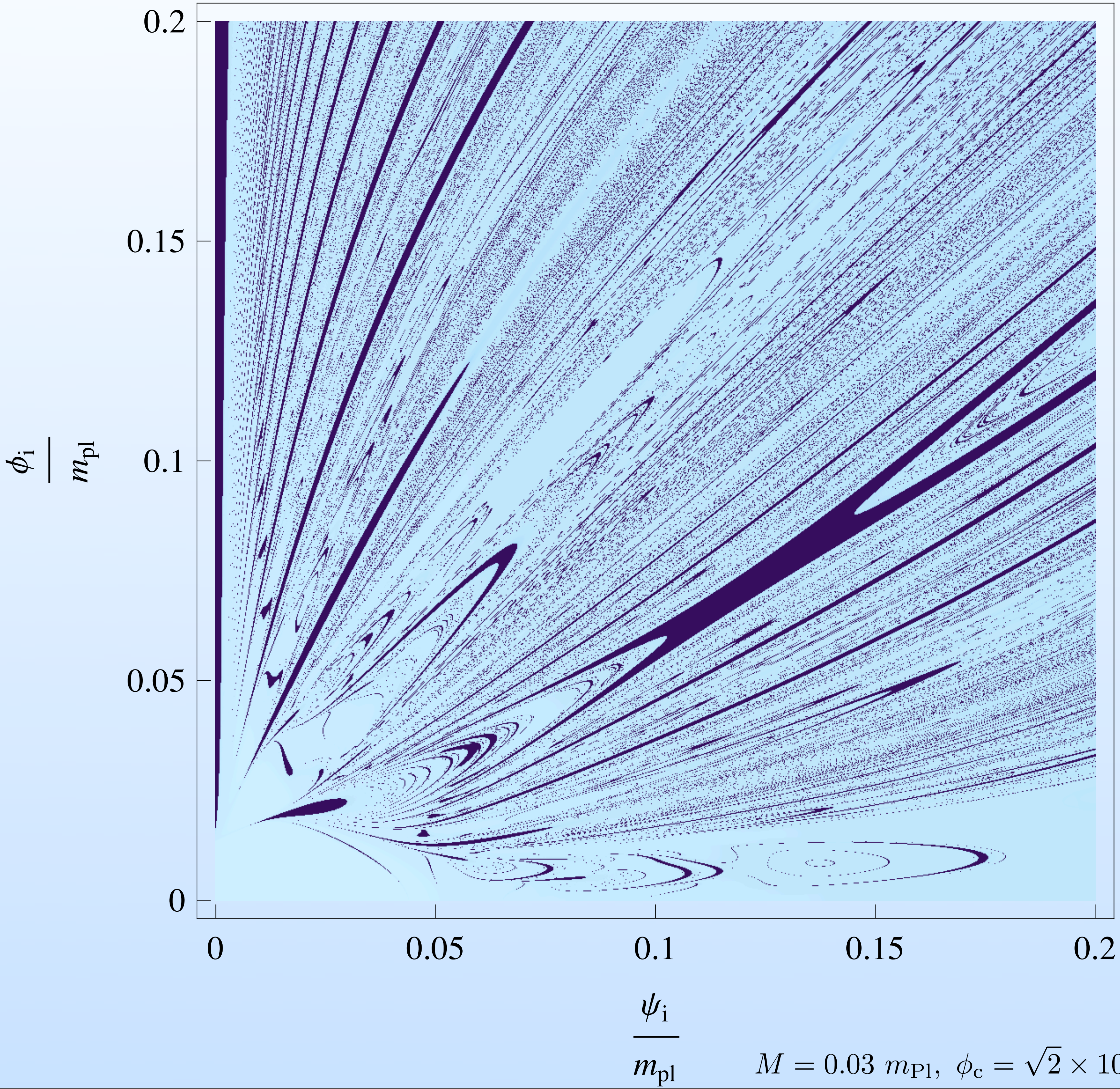
- **Set of successful initial conditions :**



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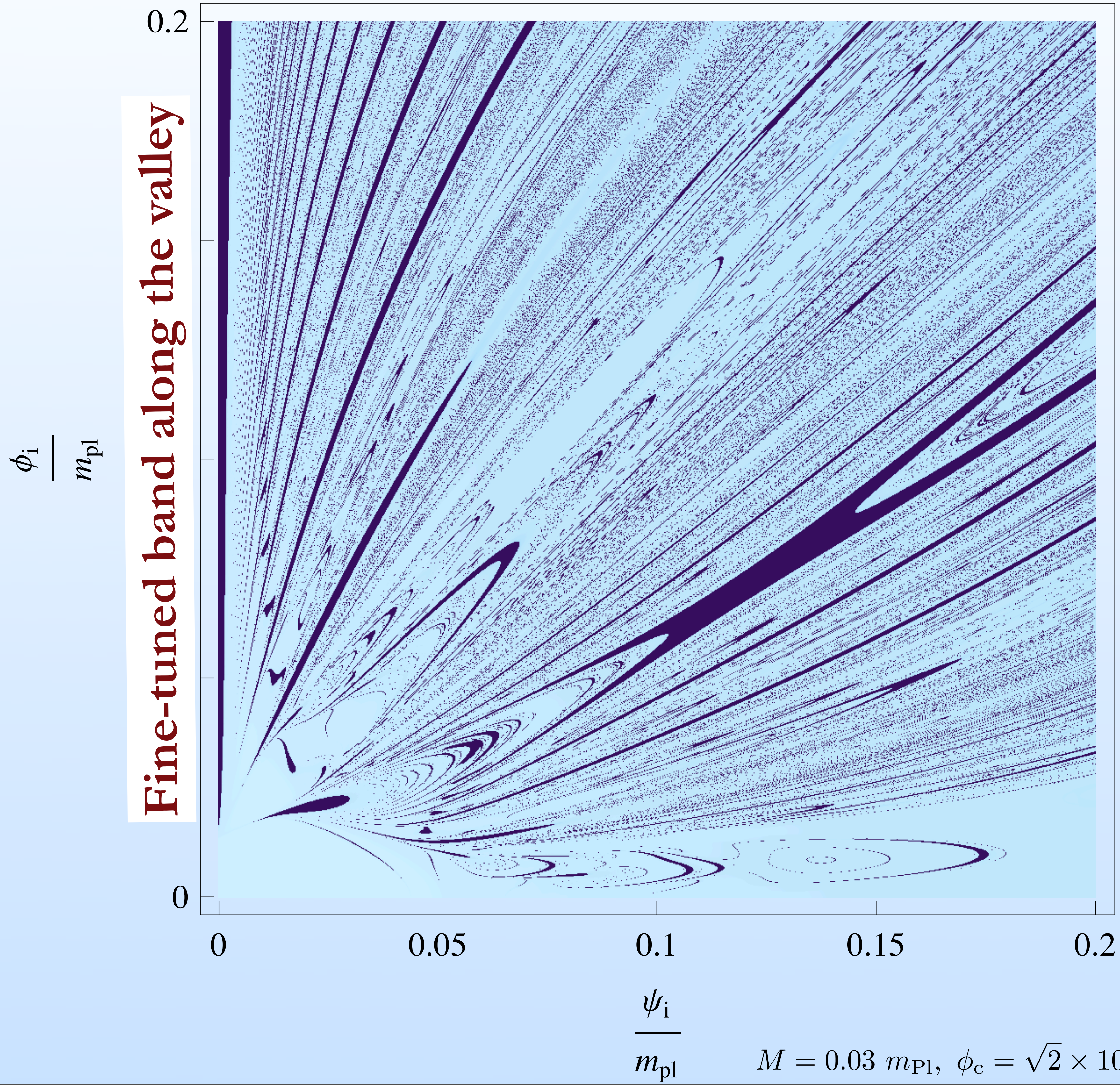

Fine-tuned band along the valley

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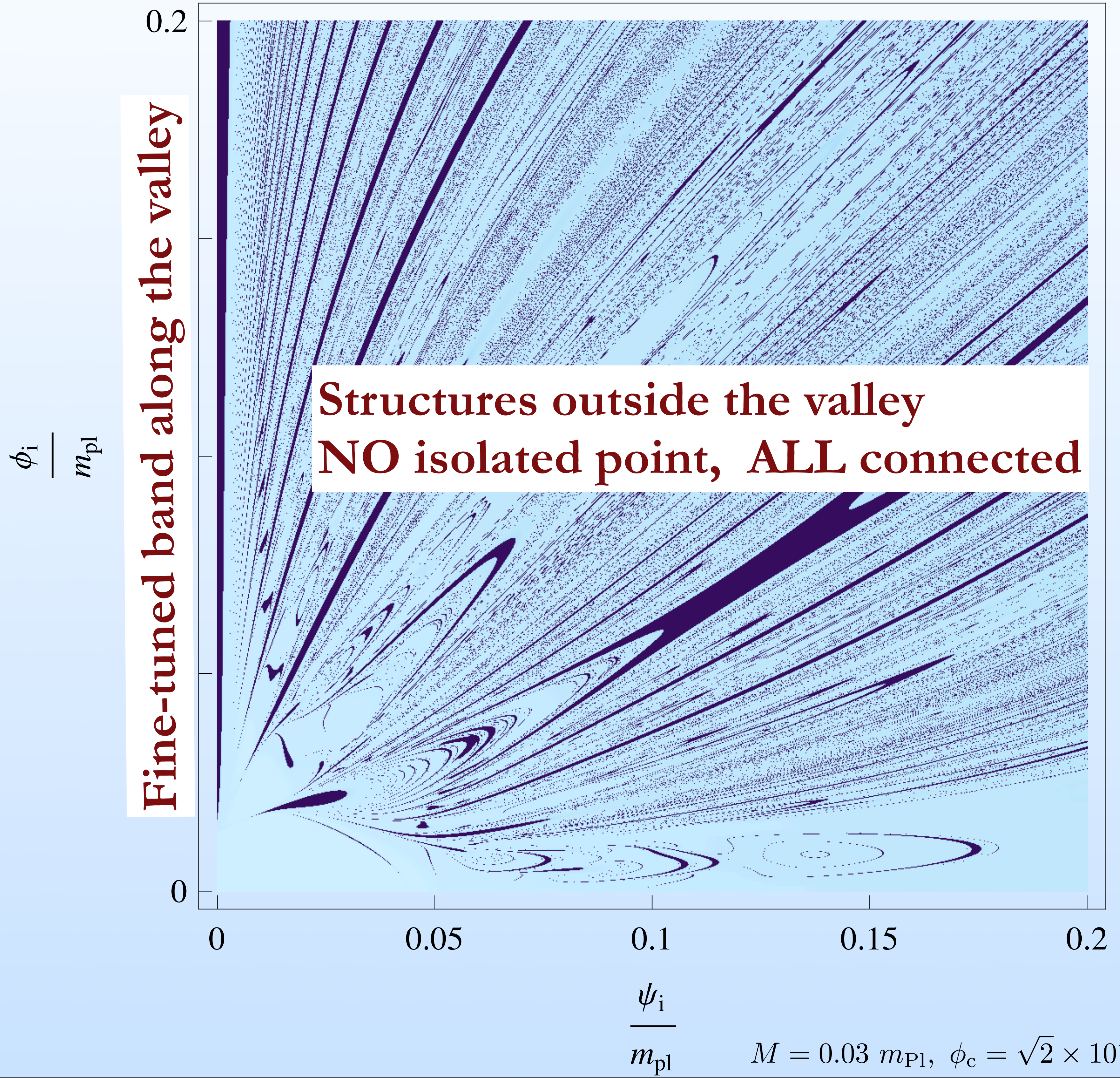

Fine-tuned band along the valley

Structures outside the valley

NO isolated point, ALL connected

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2.3 Anamorphosis trajectories

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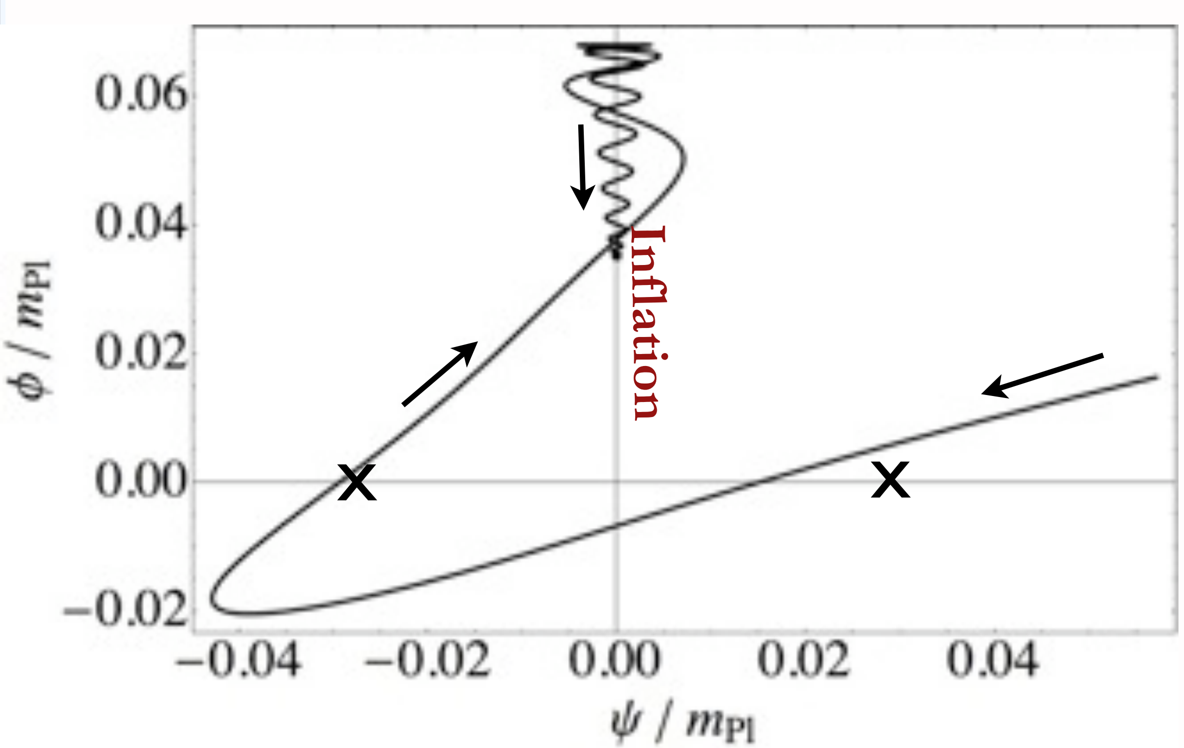

✦ Chaotic dynamical system

✦ 3 attractors

✦ Succ. points outside the valley:

- basin of attraction of the valley
- continuous map => connected set
- Fintite area with fractal boundaries (similar to Mandelbrot set)

✦ Analogy with anamorphosis

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• Set of successful initial conditions :

- ✦ Chaotic dynamical system
- ✦ 3 attractors
- ✦ Succ. points outside the valley:
- - basin of attraction of the valley
- - continuous map => connected set
- - Fintite area with fractal boundaries (similar to Mandelbrot set)
- ✦ Analogy with anamorphosis

For $\phi, \psi < 0.2 \; m_{\rm pl}$

Up to 20% of area are anamorphosis points

⇓

Anamorphosis seems to be an elegant possibility to avoid fine-tuning problem of initial conditions

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2.4 Other hybrid models...

✦ **Hybrid-type models:**

- Anamorphosis successful initial conditons :

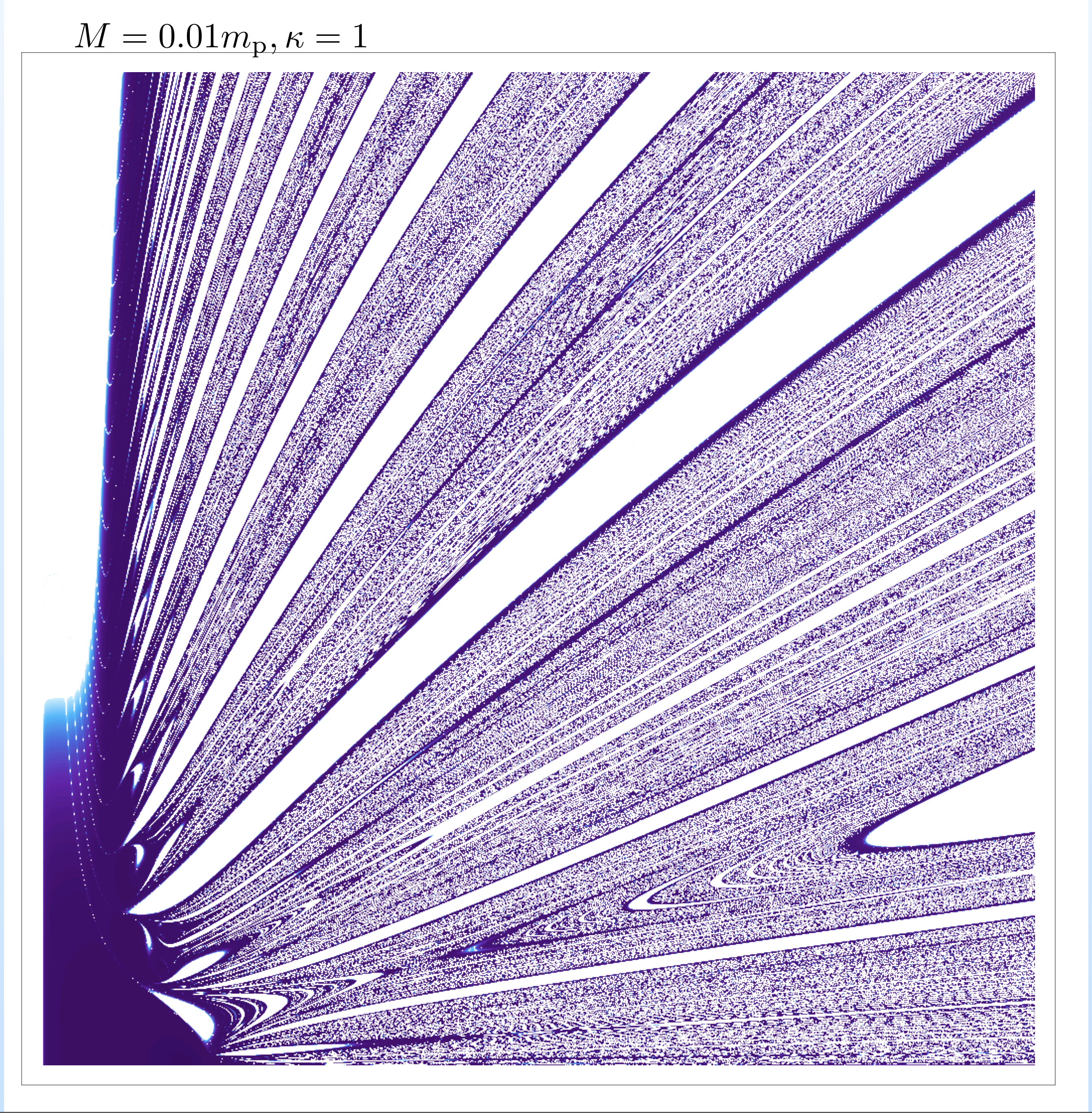
- **Smooth hybrid inflation: up to 80%**
- **in other models: Smooth+SUGRA, Shifted and Shifted+SUGRA, Radion Assisted Gauge inflation.**

2.5 Smooth inflation

- **Smooth inflation:** *(Lazarides, Panagiotakopoulos, hep-ph/9506325)*

Effective 2-field potential (SUSY): $V(\phi,\psi) = \kappa^2\left(M^2 - \frac{\psi^4}{m_{\rm Pl}^2}\right)^2 + 2\kappa^2\phi^2\frac{\psi^6}{m_{\rm Pl}^4}$

2 valleys and a flat $\psi = 0$ direction ⇒ **No topological defects**



For $\phi, \psi < 0.2\ m_{\rm pl}$

Up to 80% of area are anamorphosis points

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✦ **Questions:**

- Local in parameter space ?
- Effect of initial velocities ?

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MCMC statistical analysis of the 7D space of

initial field values

+ initial field velocities

+ potential parameters

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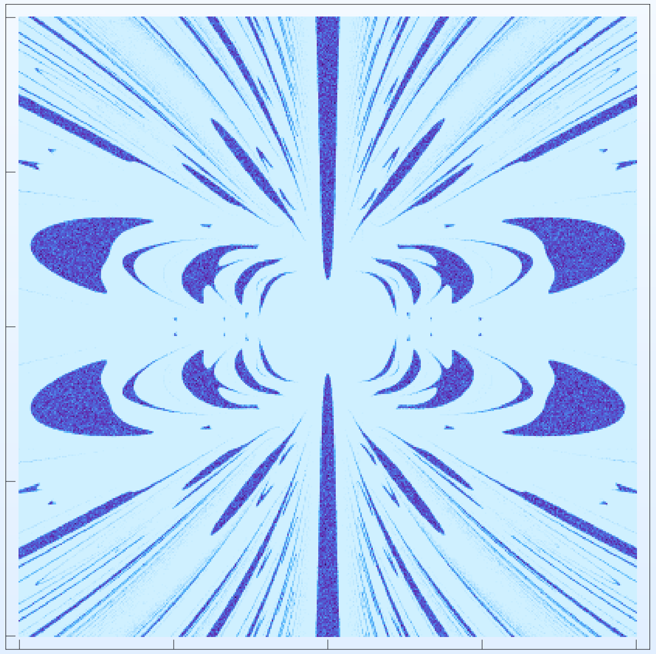
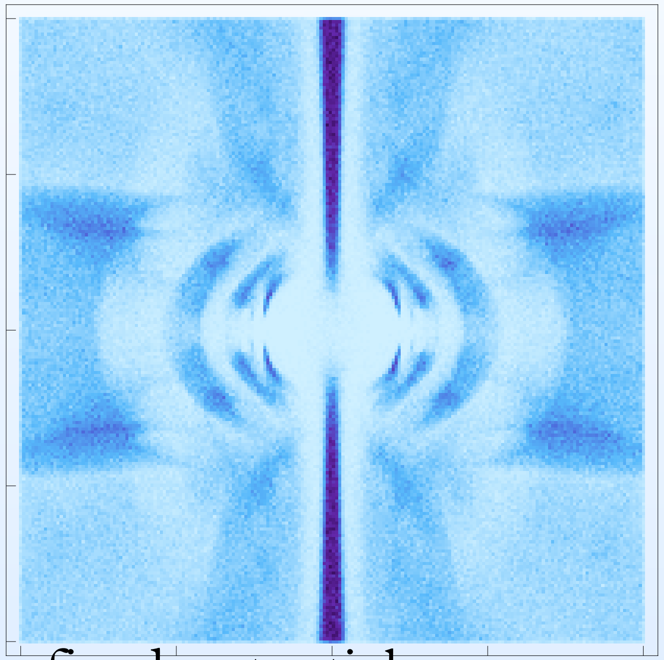
2.7 MCMC exploration - hybrid model

- **Probability density distributions of initial field values:**

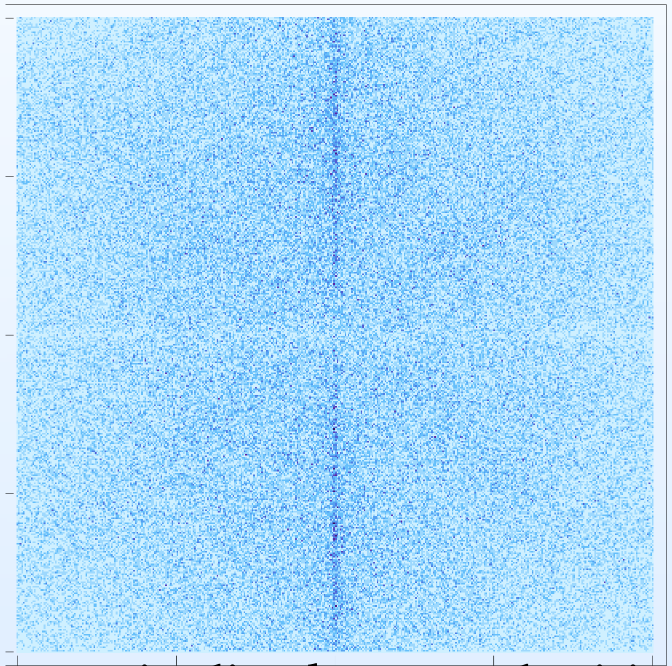
Flat Prior: $-0.2m_{\rm p} < \phi, \psi < 0.2m_{\rm p}$



Fixed potential params, vanishing initial velocities



fixed potential params, marginalised over initial velocities



marginalised over velocities and potenital params.

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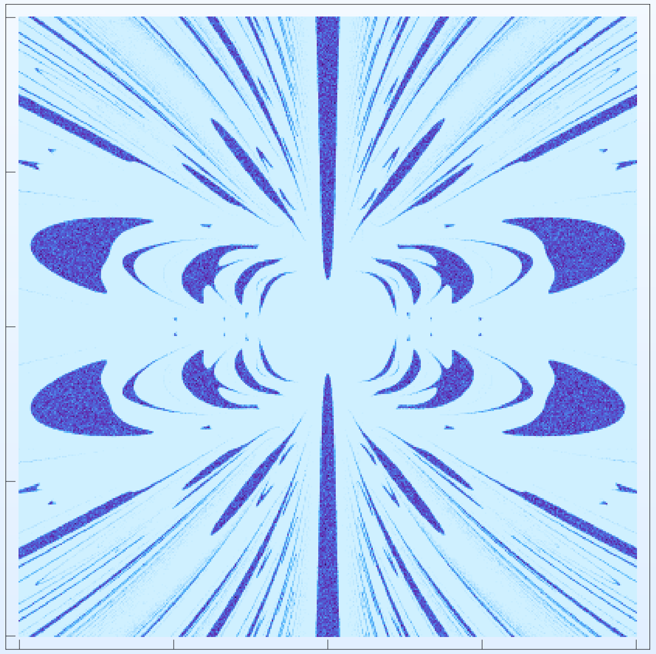

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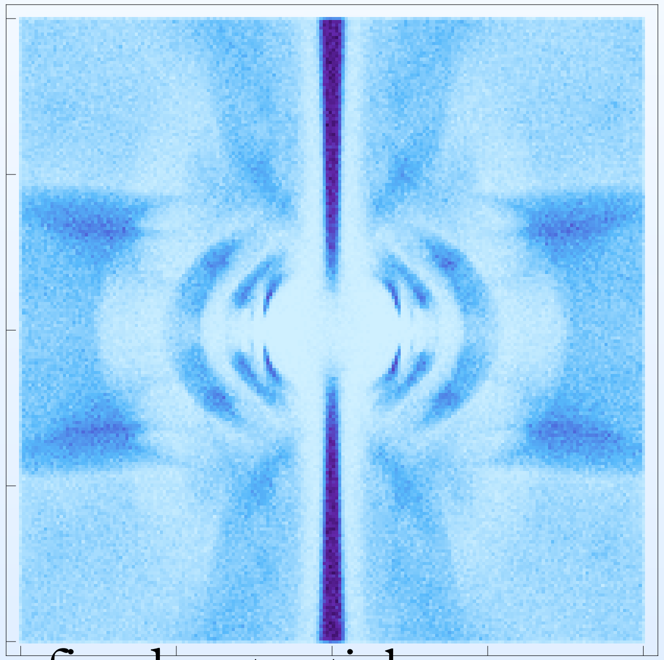

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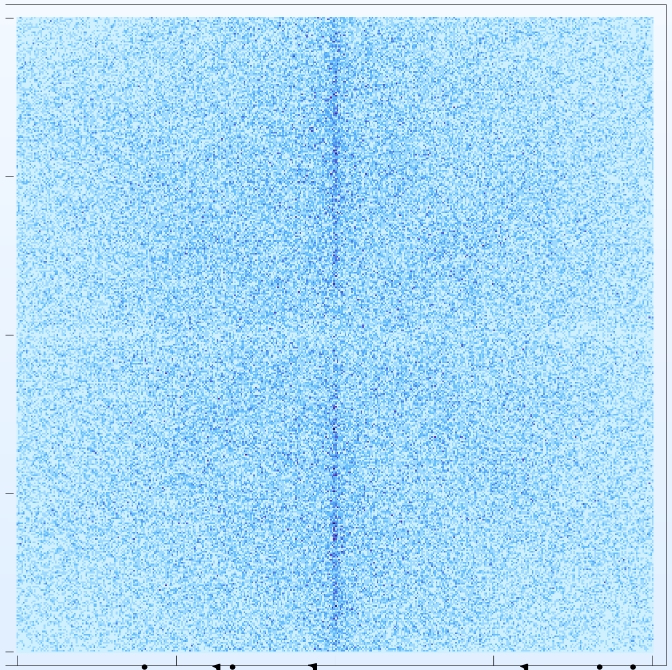

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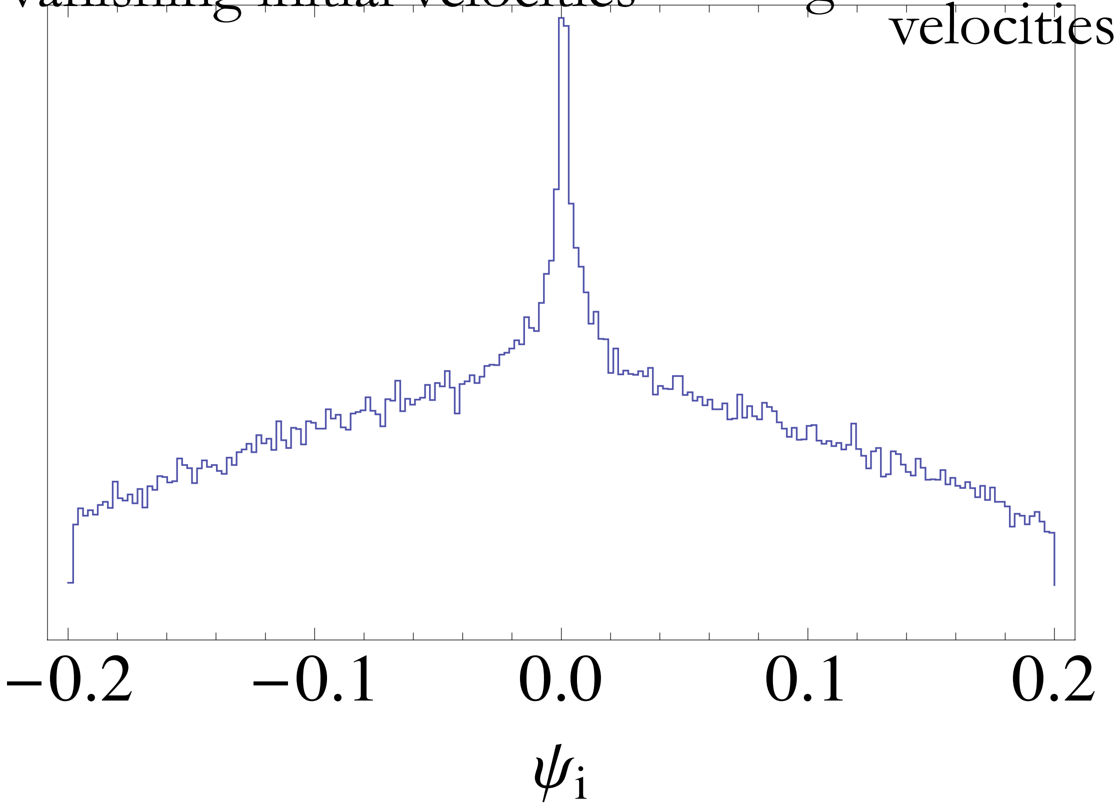

marginalised on whole 6D space

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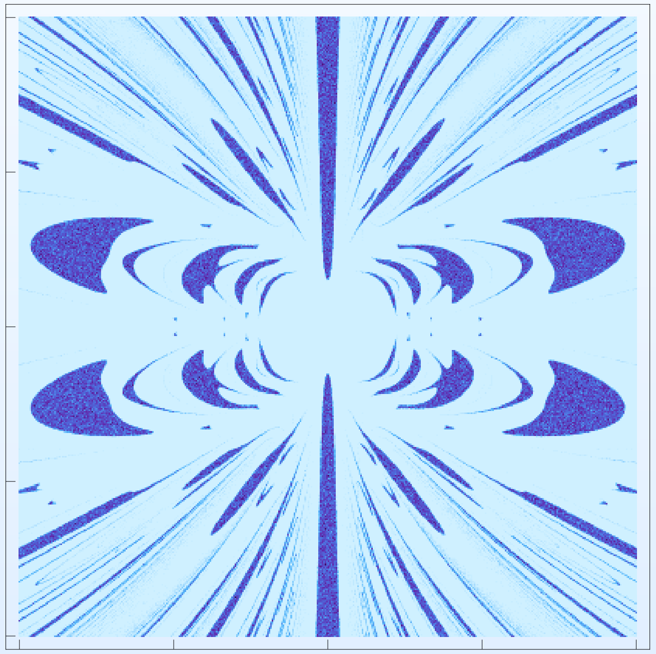

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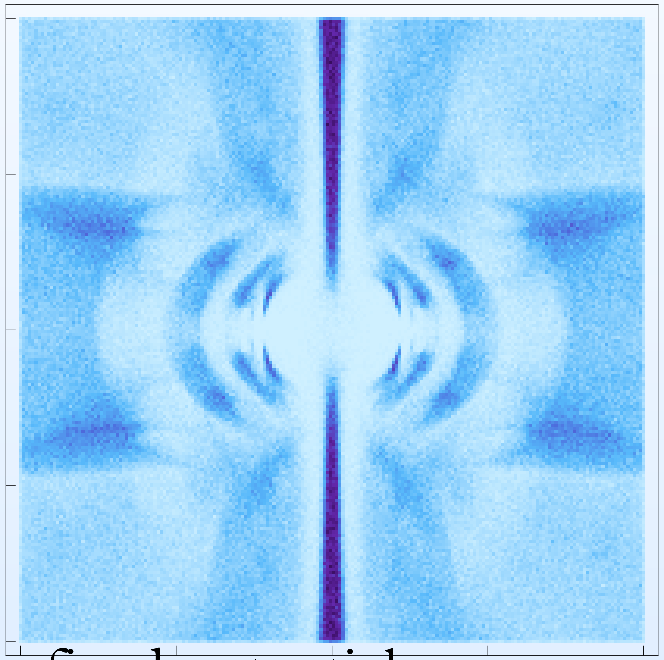
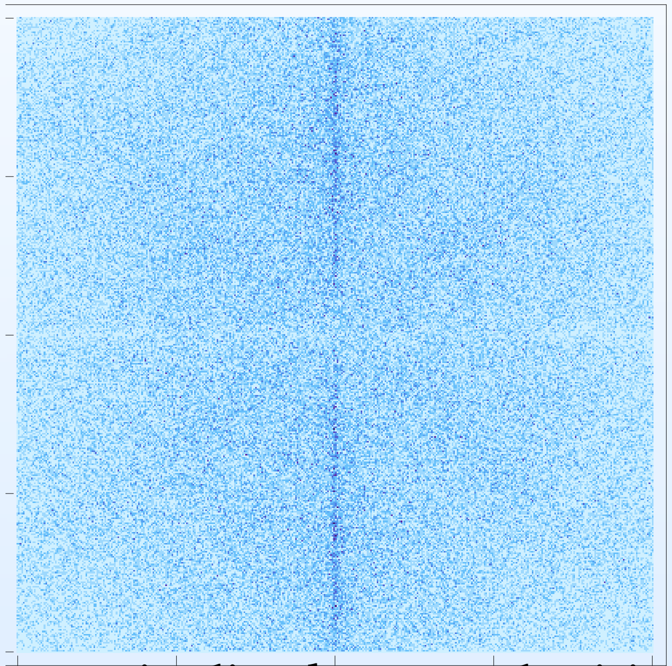


fixed potential params, marginalised over initial velocities



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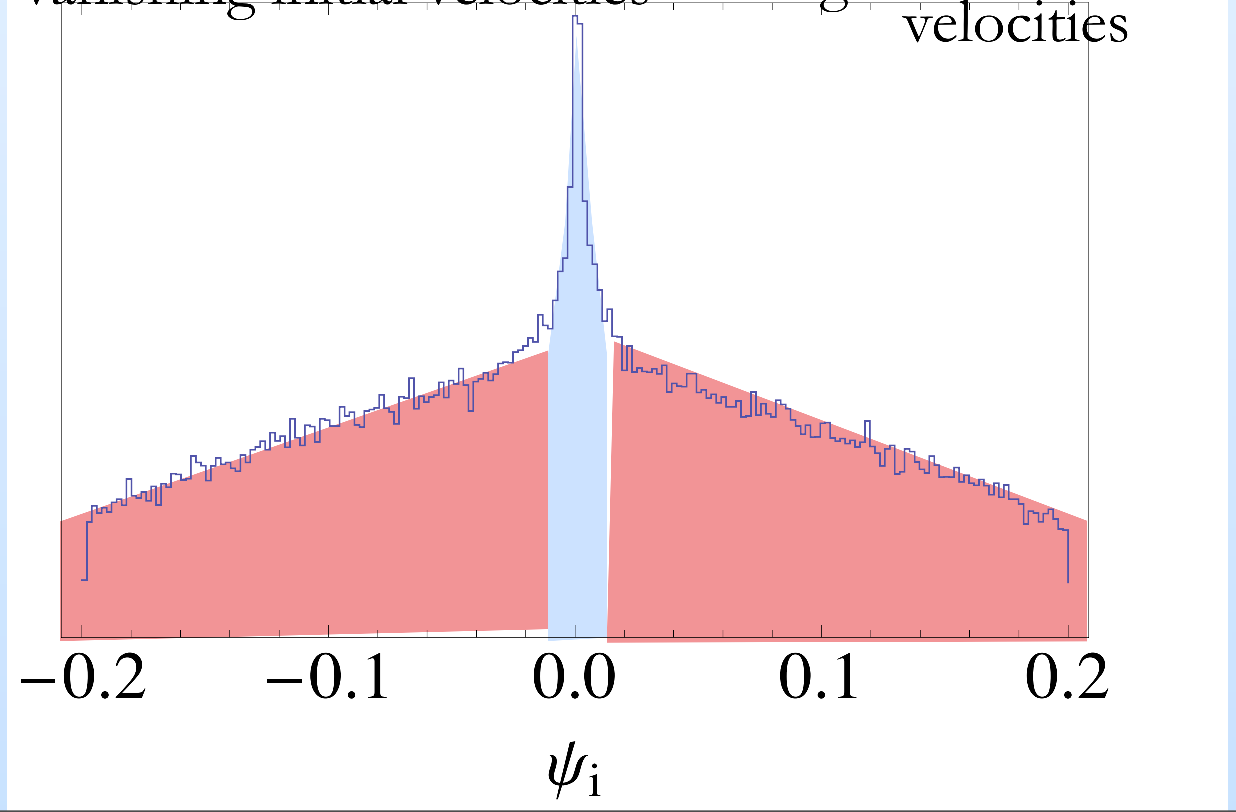

Inflation starts more probably outside the inflationary valley

marginalised on whole 6D space

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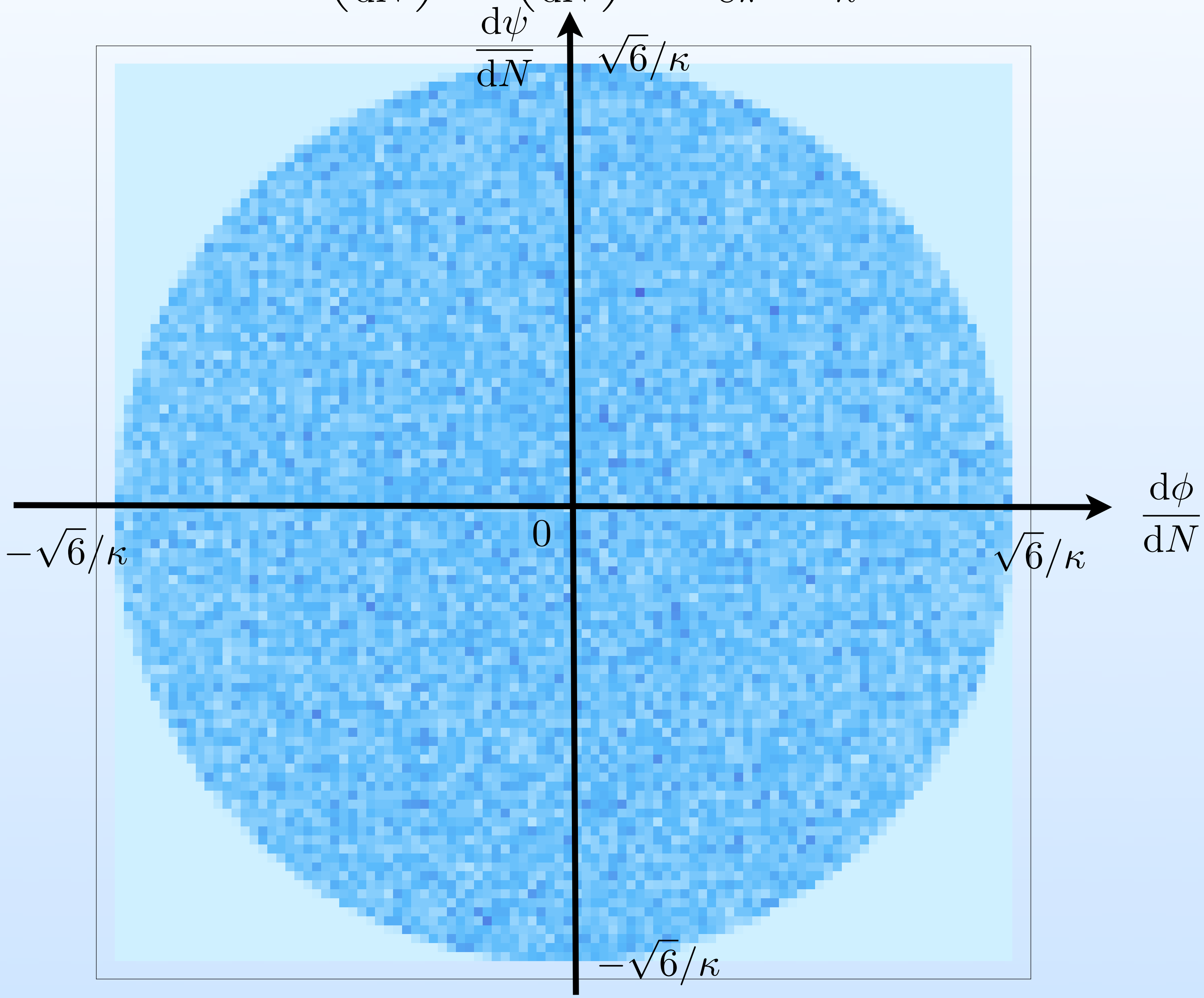
2.7 MCMC exploration - hybrid model

• Probability density distribution of initial velocities:

Flat Prior + Bound: $\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2+\left(\frac{\mathrm{d}\psi}{\mathrm{d}N}\right)^2<\frac{9m_\mathrm{p}^2}{8\pi}\equiv\frac{6}{\kappa^2}$



marginalised over initial fields and potential params.

Flat distribution for $\frac{\mathrm{d}\phi}{\mathrm{d}N}$ **and** $\frac{\mathrm{d}\psi}{\mathrm{d}N}$

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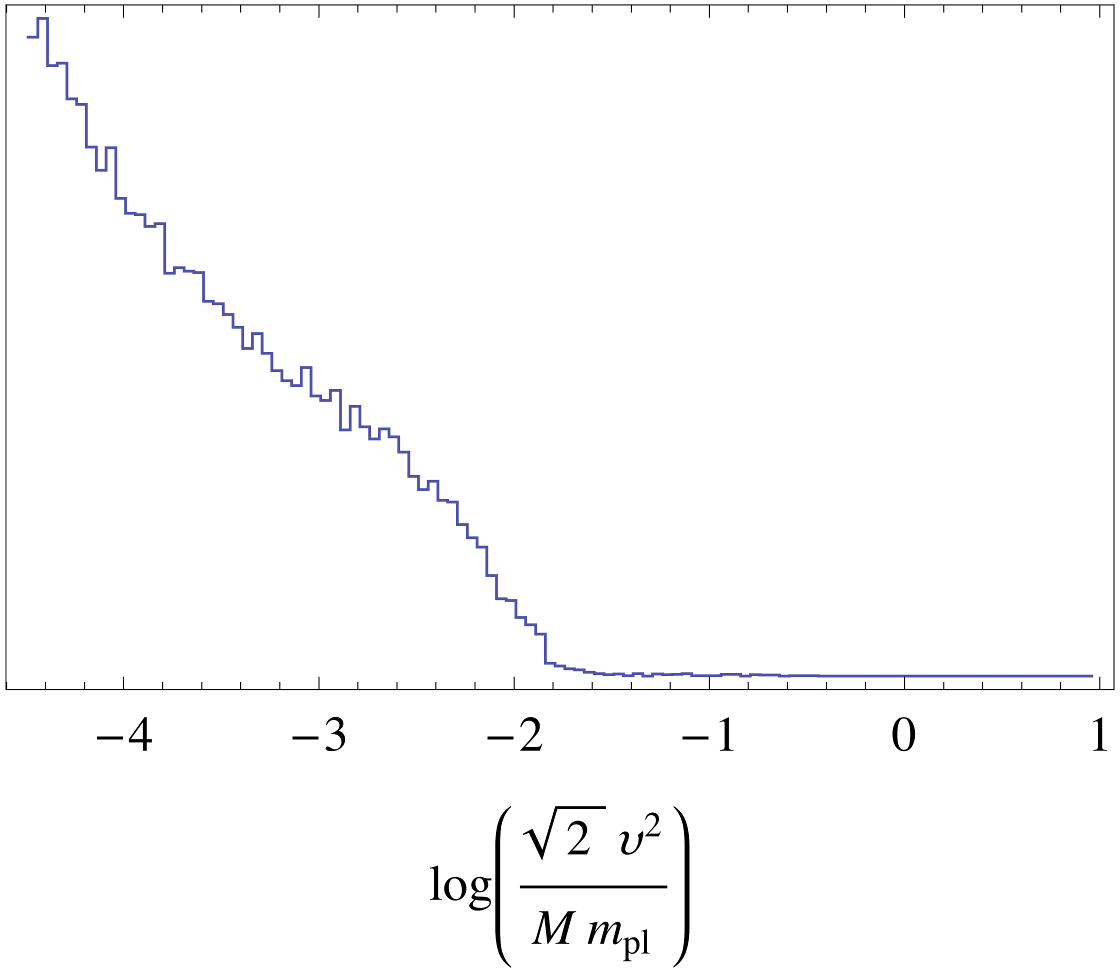
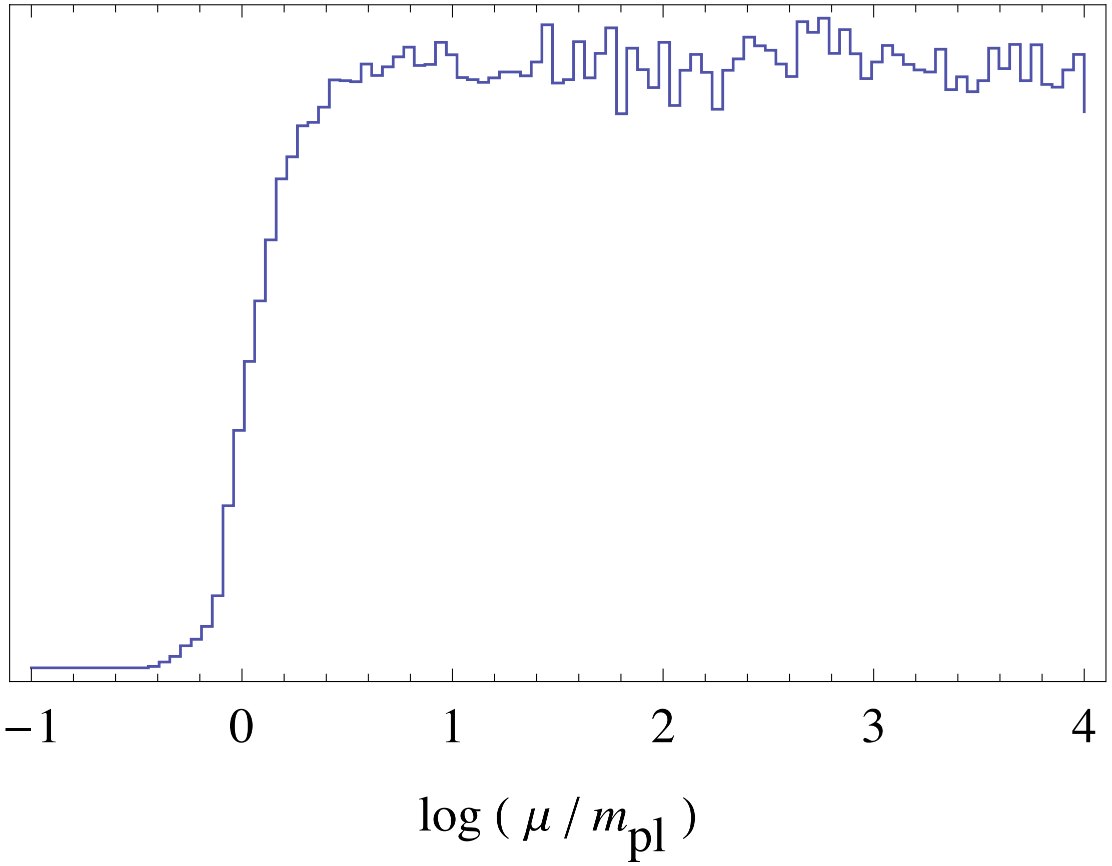
2.7 MCMC exploration - hybrid model

• Probability density distributions of parameters:

Flat prior on the logarithm

$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)+\frac{\phi^2}{\mu^2}+2\frac{\phi^2\psi^2}{\phi_{\rm c}^2M^2}\right]$$





$\phi_{\rm c} < 0.004\ m_{\rm Pl}$ 95% C.L.

$\mu > 1.7\ m_{\rm pl}$ 95% C.L.

Position

of the instability point

Due to slow-roll violations

inside the valley

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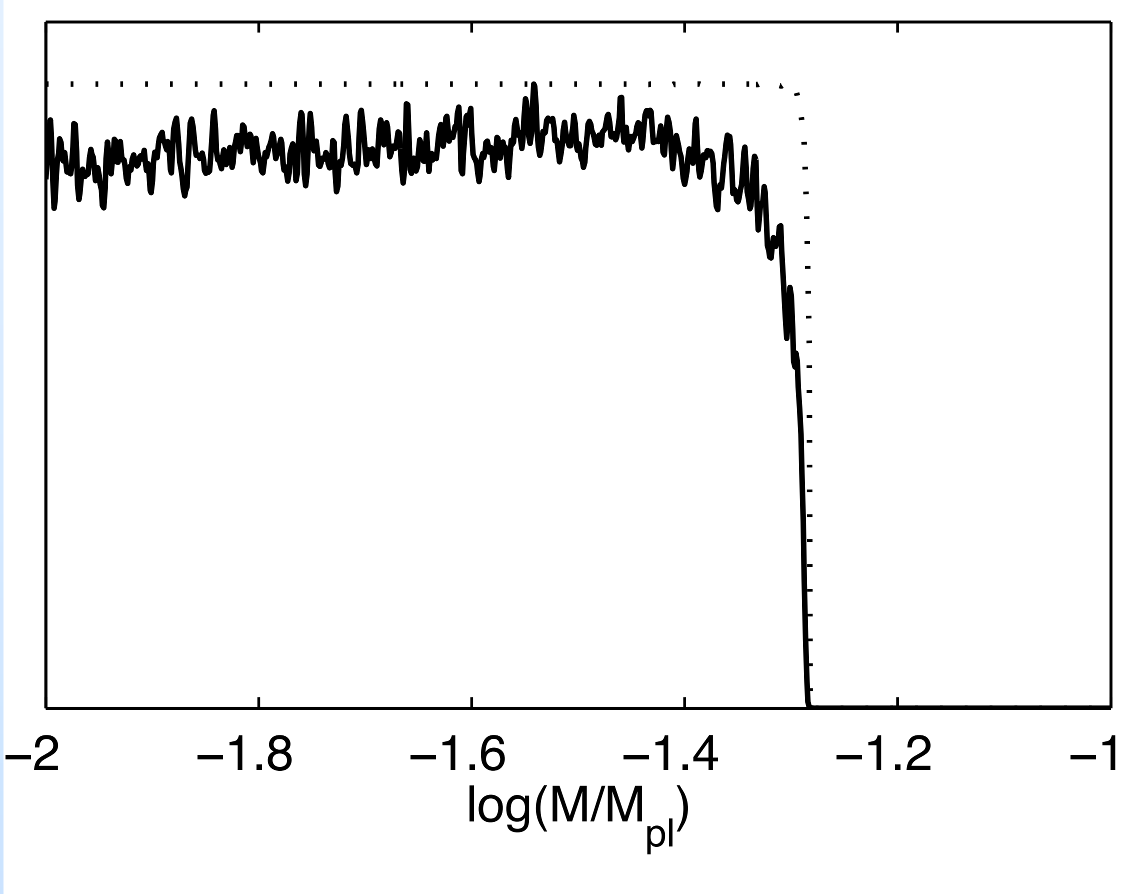
2.8 MCMC exploration - F-term SUGRA

- **Simple and realistic model in (local) SUSY framework**
- **SUGRA corrections dominate radiative correction**
- **Only one potential parameter** M

$$V_{\rm tree}^{\rm sugra}(s,\psi) = \kappa^2 \exp\left(\frac{s^2+\psi^2}{2M_{\rm pl}^2}\right) \times \left\{ \left(\frac{\psi^2}{4} - M^2\right)^2 \left(1 - \frac{s^2}{2M_{\rm pl}^2} + \frac{s^4}{4M_{\rm pl}^4}\right) + \frac{s^2\psi^2}{4}\left[1 + \frac{1}{M_{\rm pl}^2}\left(\frac{1}{4}\psi^2 - M^2\right)\right]^2 \right\} .$$

• MCMC analysis:

- **Similar results for distributions of initial fields and initial velocities**
- **Bound on the parameter:**



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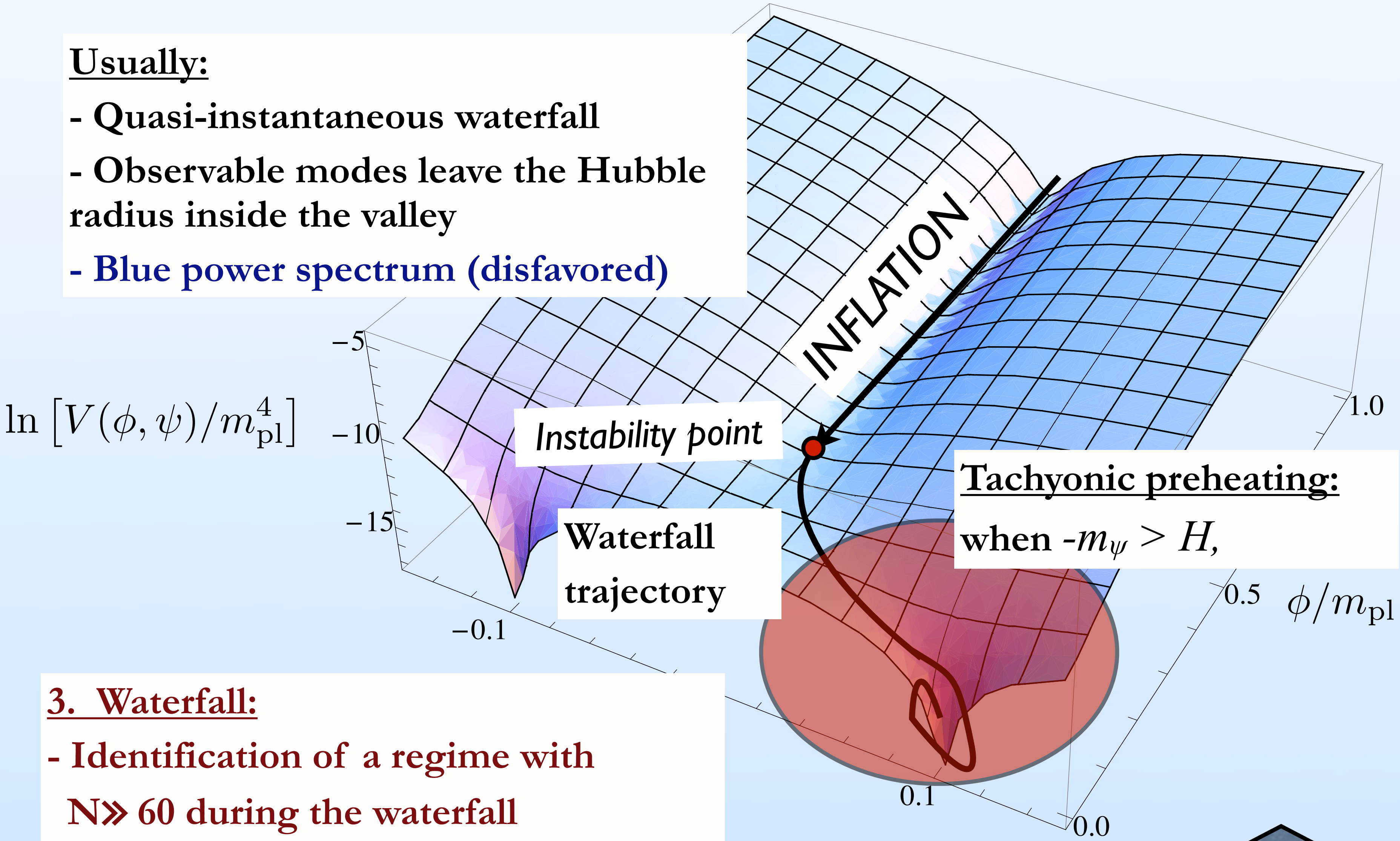
- **Classical Inflaton** ϕ **(slow-roll inflation in the valley)**
- **Higgs-type auxiliary field** ψ
- **Hybrid potential** (Linde, astro-ph/9307002)

$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)+\frac{\phi^2}{\mu^2}+2\frac{\phi^2\psi^2}{\phi_c^2M^2}\right]$$



Usually:

- **Quasi-instantaneous waterfall**
- **Observable modes leave the Hubble radius inside the valley**
- **Blue power spectrum (disfavored)**

Tachyonic preheating:

when $-m_\psi > H$,

3. Waterfall:

- Identification of a regime with N» 60 during the waterfall
- Observable modes leave the Hubble radius during the waterfall
- Initial power spectrum modified

Exact 2-field dynamics

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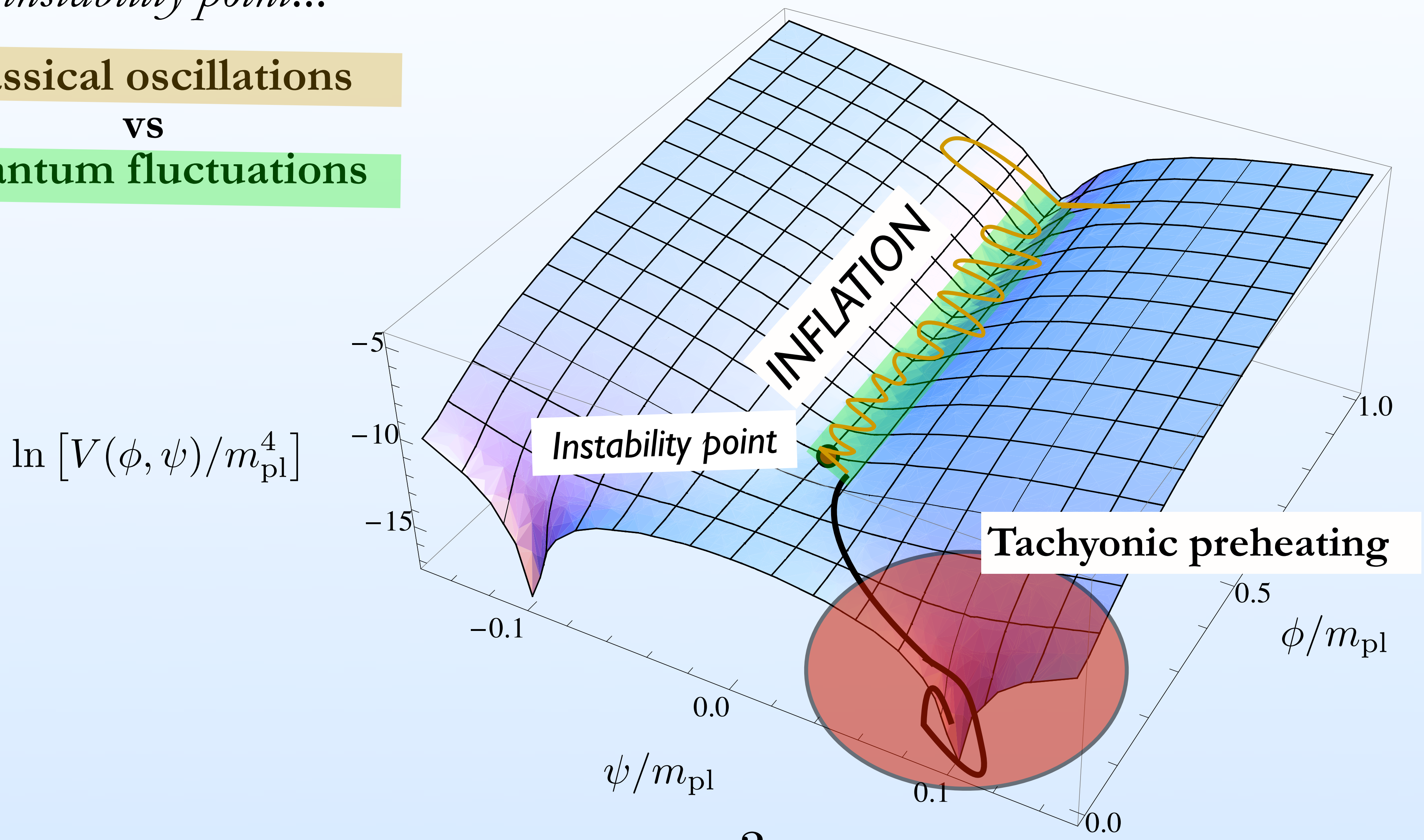
3.1 Classical vs. Stochastic dynamics

Before instability point...

Classical oscillations

vs

Quantum fluctuations



- **Classical oscillations: damped by** $\exp(-\frac{3}{2}N)$
- **Quantum fluctuations:** $\Delta\psi_{\rm qu} \simeq \frac{H}{2\pi}$ **and** $10^{-30} \lesssim \Delta\psi_{\rm qu} \lesssim 10^{-6}$

Generically, classical trajectories reach the instability point displaced from the valley line by $\Delta\psi_{\rm qu}$

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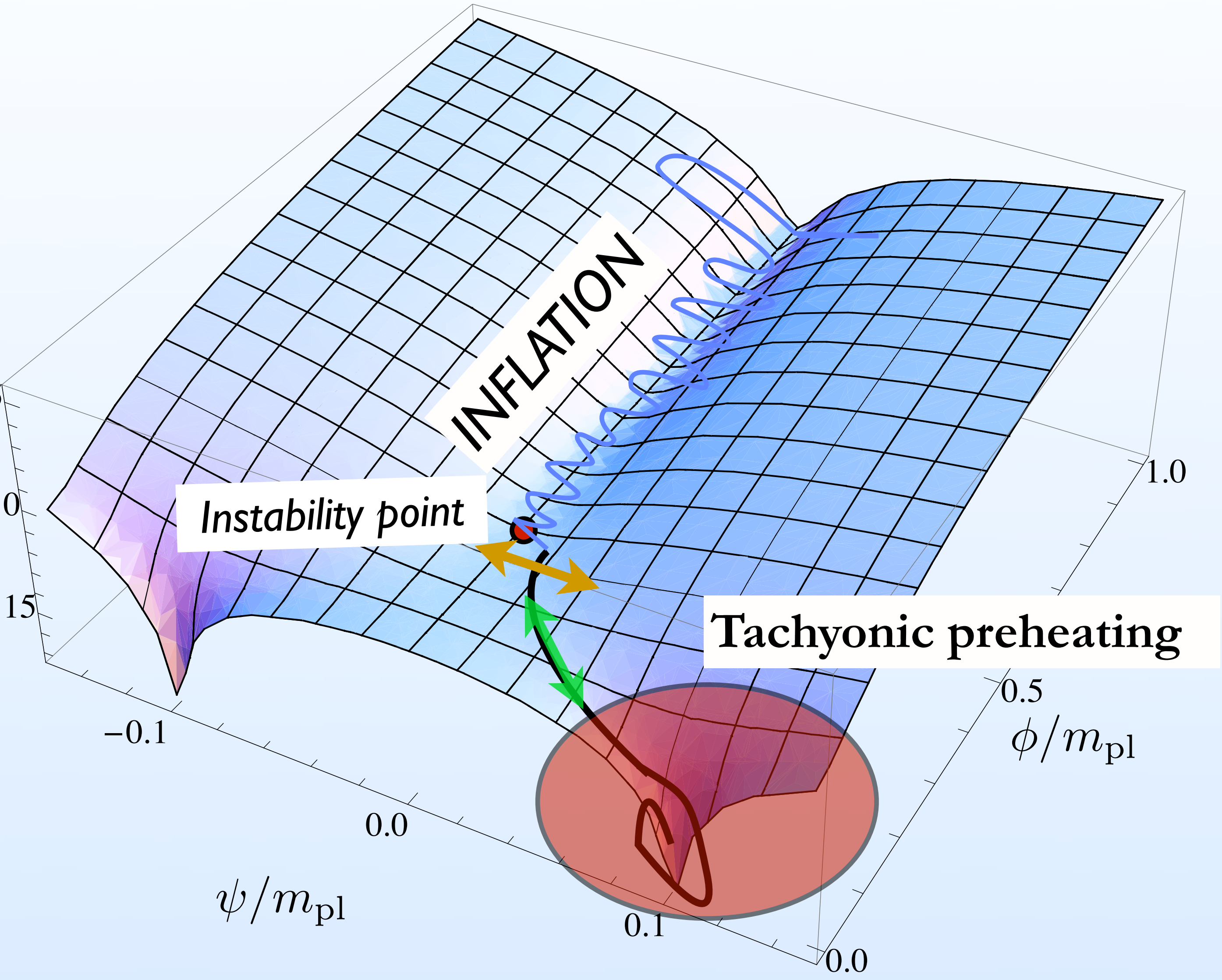
3.1 Classical vs. Stochastic dynamics

Around instability point...

Classical dynamics
(mainly ϕ evolution)
vs
Quantum backreactions
of the adiabatic field
and
Quantum backreactions
of the transverse field



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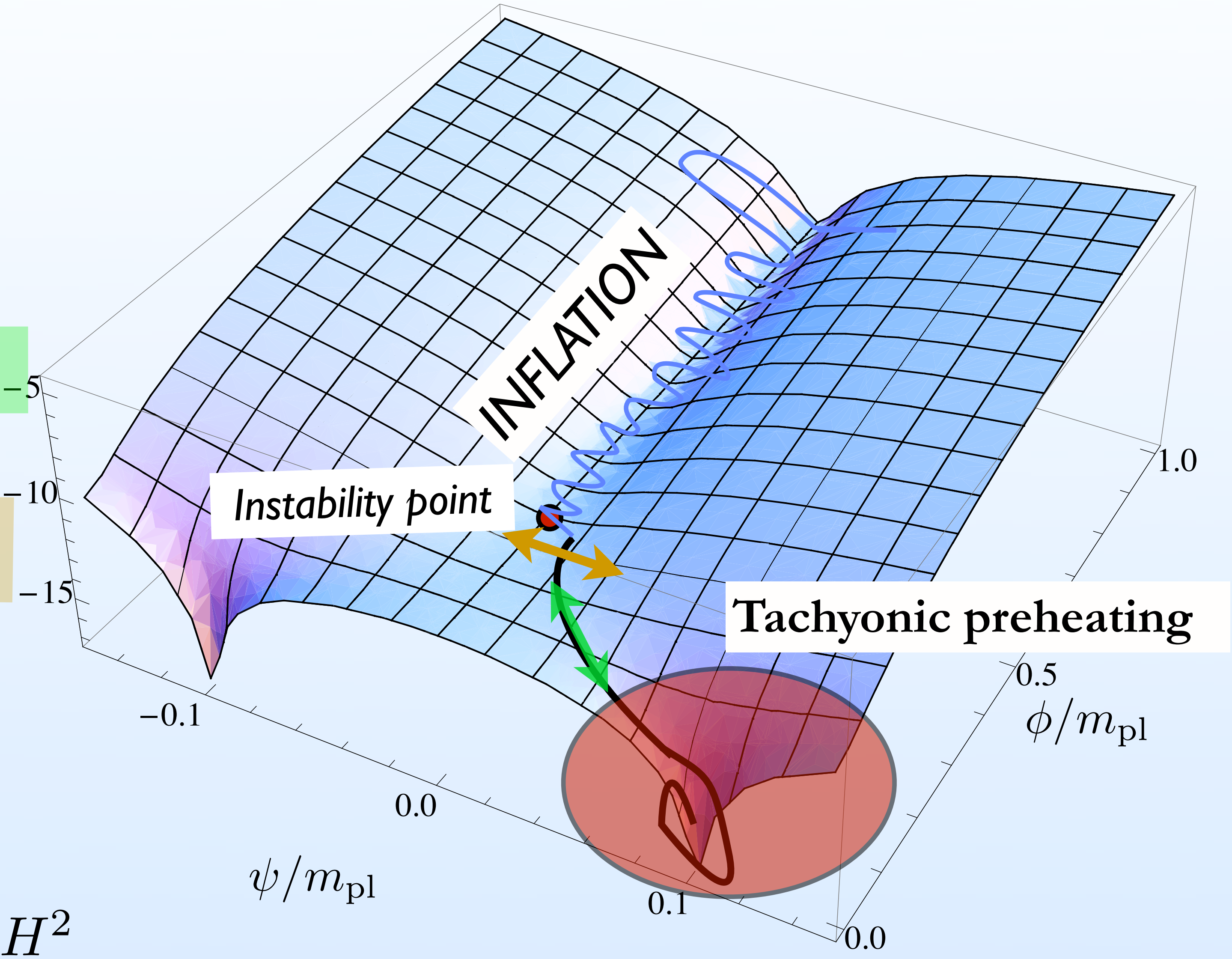

- Adiabatic field: $\epsilon_1 > \dfrac{H^2}{\pi m_{\rm pl}^2}$

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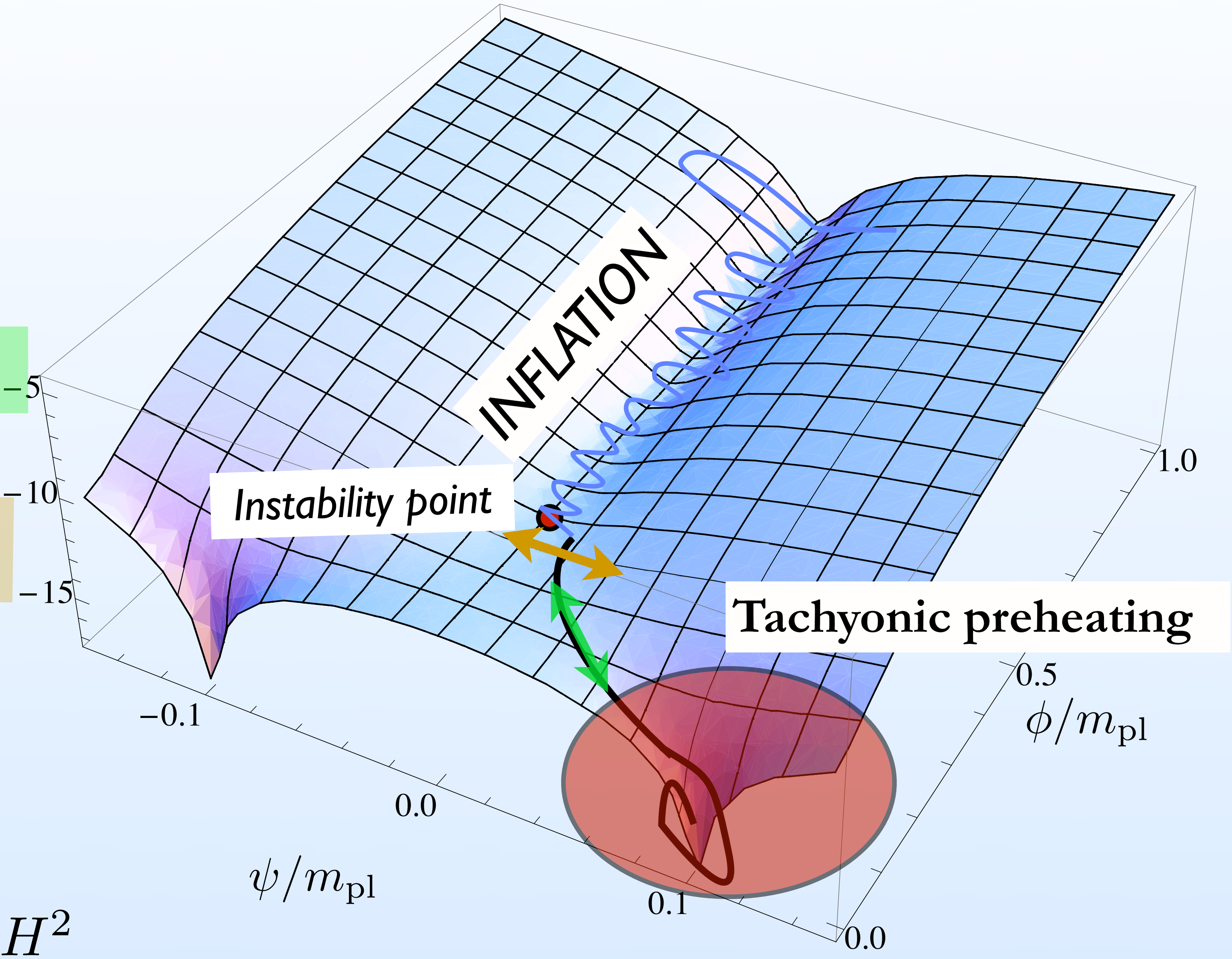

- Adiabatic field: $\epsilon_1 > \dfrac{H^2}{\pi m_{\rm pl}^2}$

- Transverse field: $\dot{\psi} + \dfrac{1}{3H}\dfrac{\mathrm{d}V}{\mathrm{d}\psi} = \dfrac{H^{3/2}}{2\pi}\xi(t)$ **(Langevin equation)**

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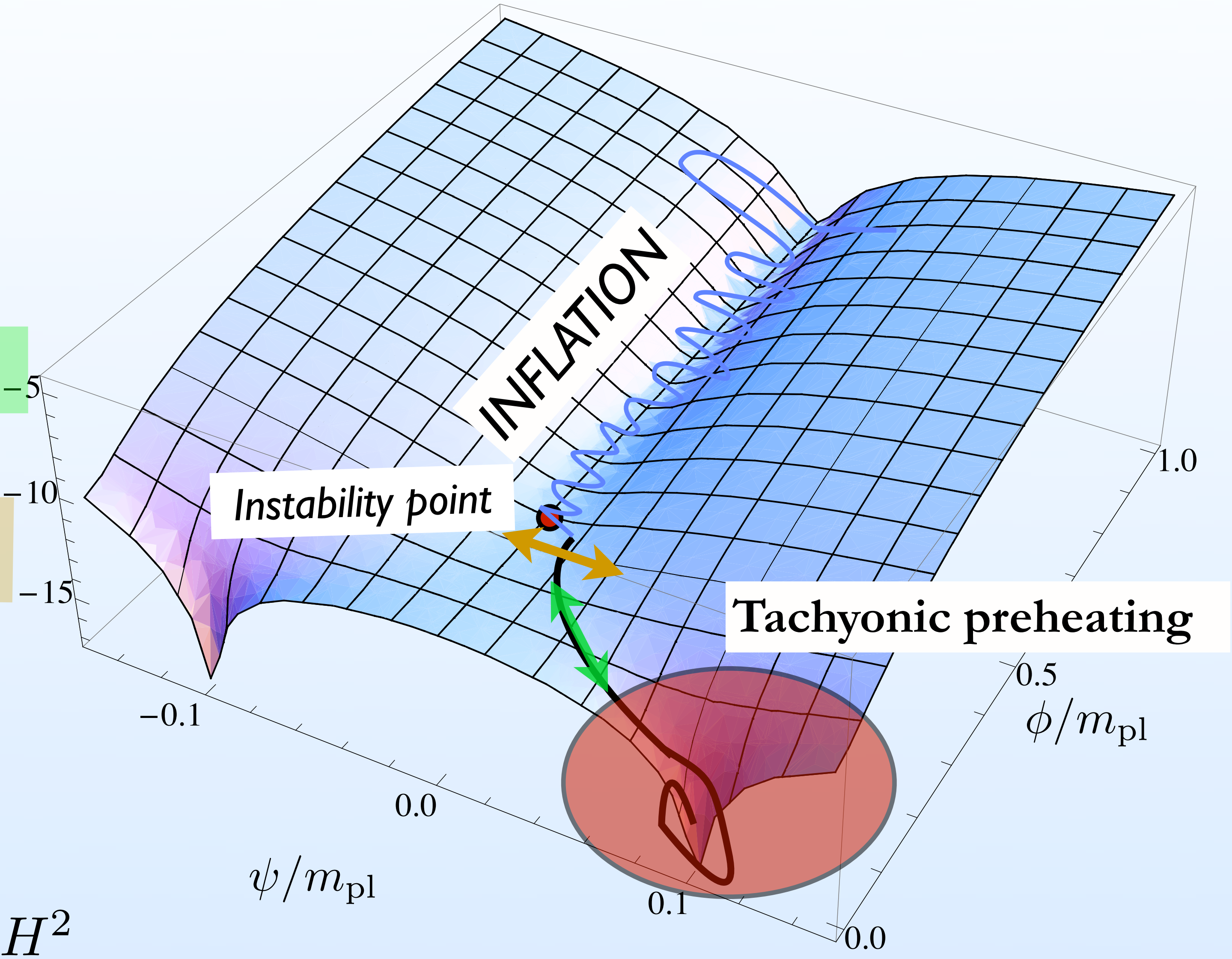

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Noise term

- **Transverse field:** $\dot{\psi} + \dfrac{1}{3H}\dfrac{\mathrm{d}V}{\mathrm{d}\psi} = \dfrac{H^{3/2}}{2\pi}\xi(t)$ **(Langevin equation)**

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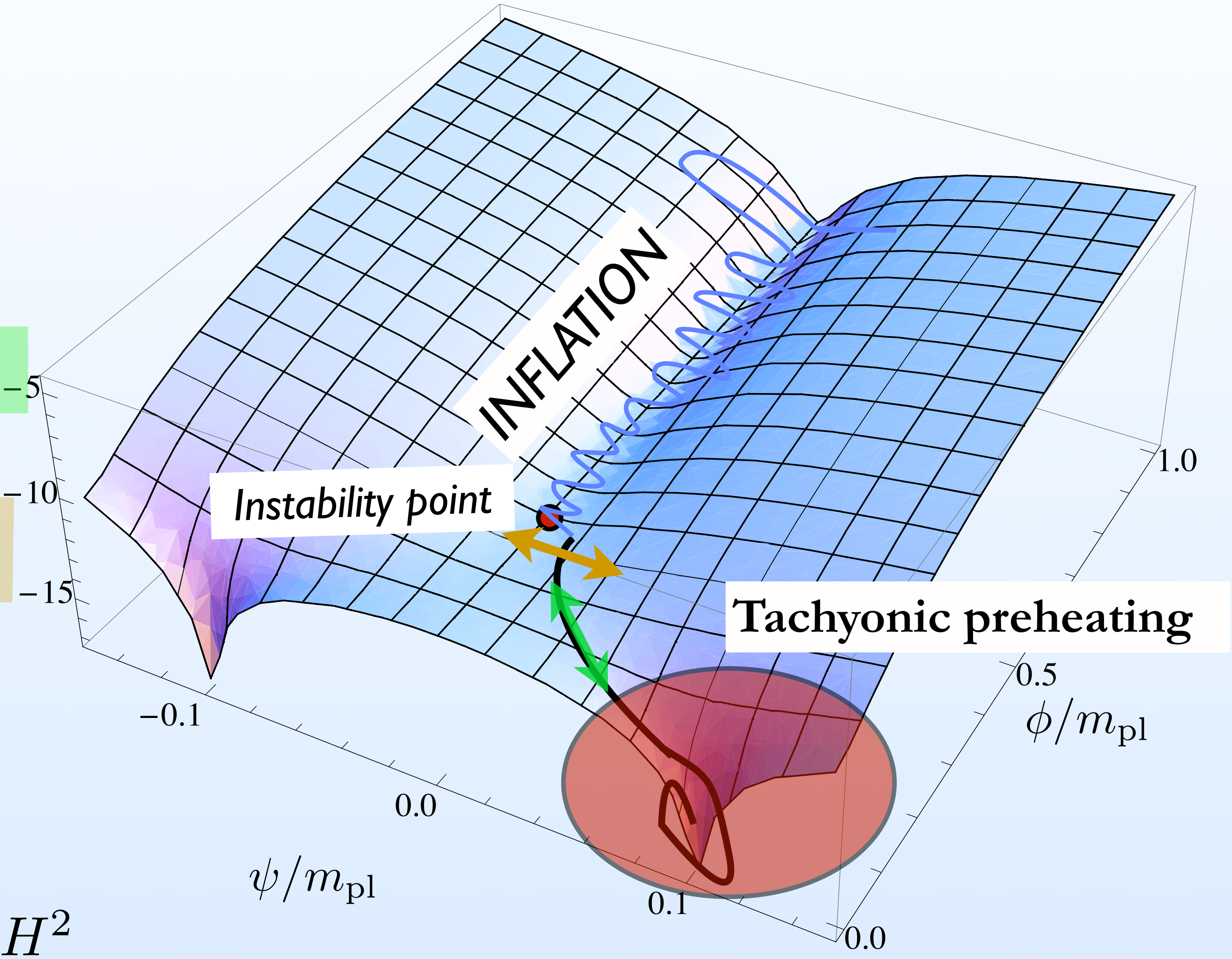

- **Adiabatic field:** $\epsilon_1 > \dfrac{H^2}{\pi m_{\rm pl}^2}$

Noise term

- **Transverse field:** $\dot{\psi} + \dfrac{1}{3H}\dfrac{{\rm d}V}{{\rm d}\psi} = \dfrac{H^{3/2}}{2\pi}\xi(t)$ **(Langevin equation)**

- **Exact solution :** $\langle\psi^2(x)\rangle = \dfrac{H^2}{8\pi^2 r}\left[\dfrac{\exp(x)}{ax}\right]^a \Gamma(a, ax)$ **fixes initial conditions at instability**

with $a \equiv 4M_{\rm pl}^2/M^2 r \qquad x \equiv \exp\left[-2r(N-N_{\rm c})\right] \qquad r \equiv \dfrac{3}{2} - \sqrt{\dfrac{9}{4} - 6\dfrac{M_{\rm pl}^2}{\mu^2}}$

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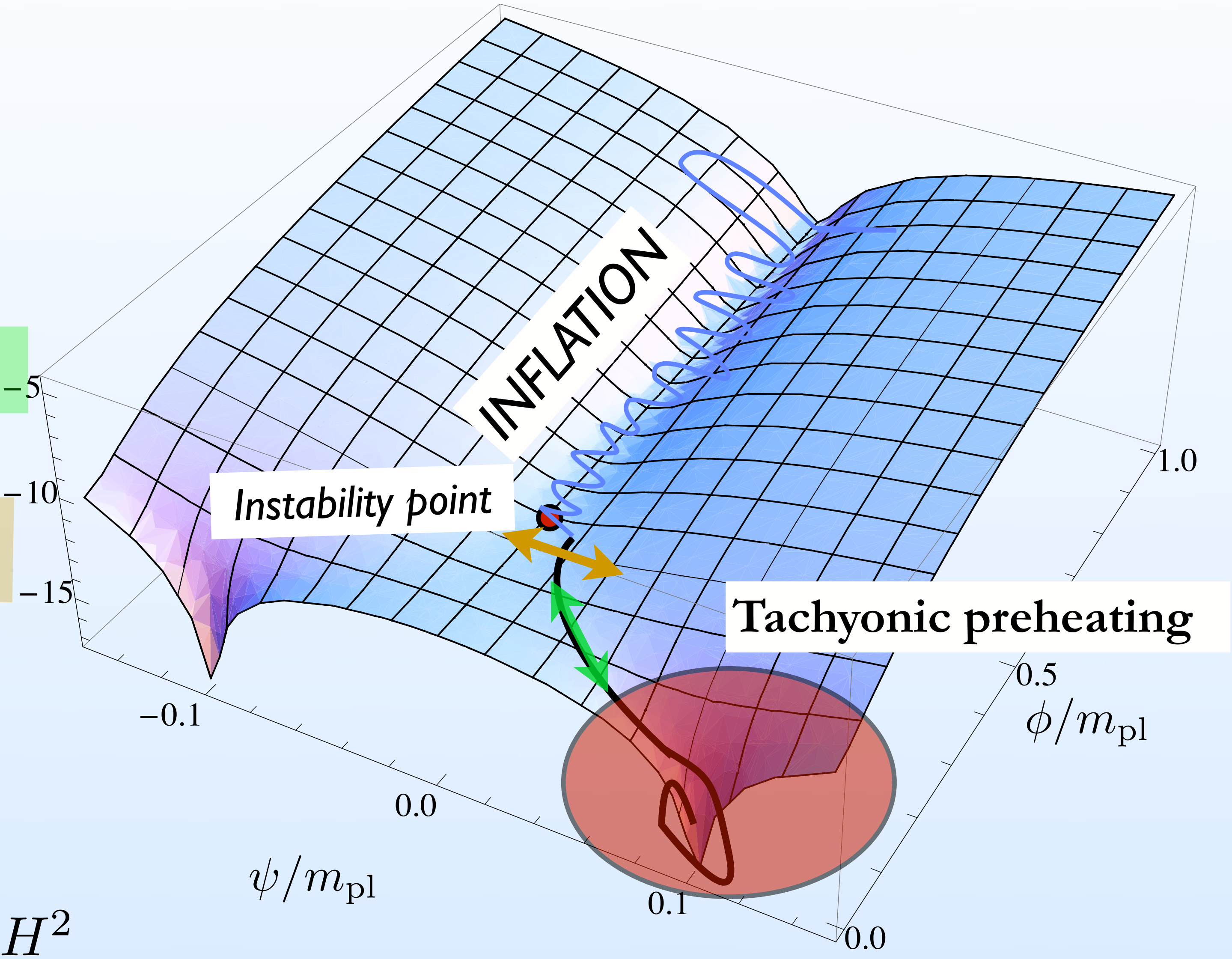
and
Quantum backreactions of the transverse field



- **Adiabatic field:** $\epsilon_1 > \dfrac{H^2}{\pi m_{\rm pl}^2}$

Noise term

- **Transverse field:** $\dot{\psi} + \dfrac{1}{3H}\dfrac{\mathrm{d}V}{\mathrm{d}\psi} = \dfrac{H^{3/2}}{2\pi}\xi(t)$ **(Langevin equation)**

- **Exact solution :** $\langle\psi^2(x)\rangle = \dfrac{H^2}{8\pi^2 r}\left[\dfrac{\exp(x)}{ax}\right]^a \Gamma(a, ax)$ **fixes initial conditions at instability**

with $a \equiv 4M_{\rm pl}^2/M^2 r$ $\quad x \equiv \exp\left[-2r(N - N_{\rm c})\right]$ $\quad r \equiv \dfrac{3}{2} - \sqrt{\dfrac{9}{4} - 6\dfrac{M_{\rm pl}^2}{\mu^2}}$

- **In the regime** $\begin{matrix}\mu \gg M_{\rm pl} \\ M < M_{\rm pl}\end{matrix}$ **, one has:** $\sqrt{\langle\psi^2\rangle} \sim H$

0. Basics on inflation
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2. Initial conditions
2.1. Fine-tuning of IC
2.2. How to avoid fine-tuning?
2.3. Anamorphosis trajectories
2.4. Other hybrid models
2.5. Smooth inflation
2.6. Remaining questions...
2.7. MCMC analysis for original hybrid model
2.8 MCMC analysis for F-term SUGRA
3. Waterfall inflation
3.1. Classical vs. Stochastic dynamics
3.2. Inflation along waterfall trajectories
3.3. MCMC analysis of the parameter space
3.4 Power spectrum of adiabatic pert.
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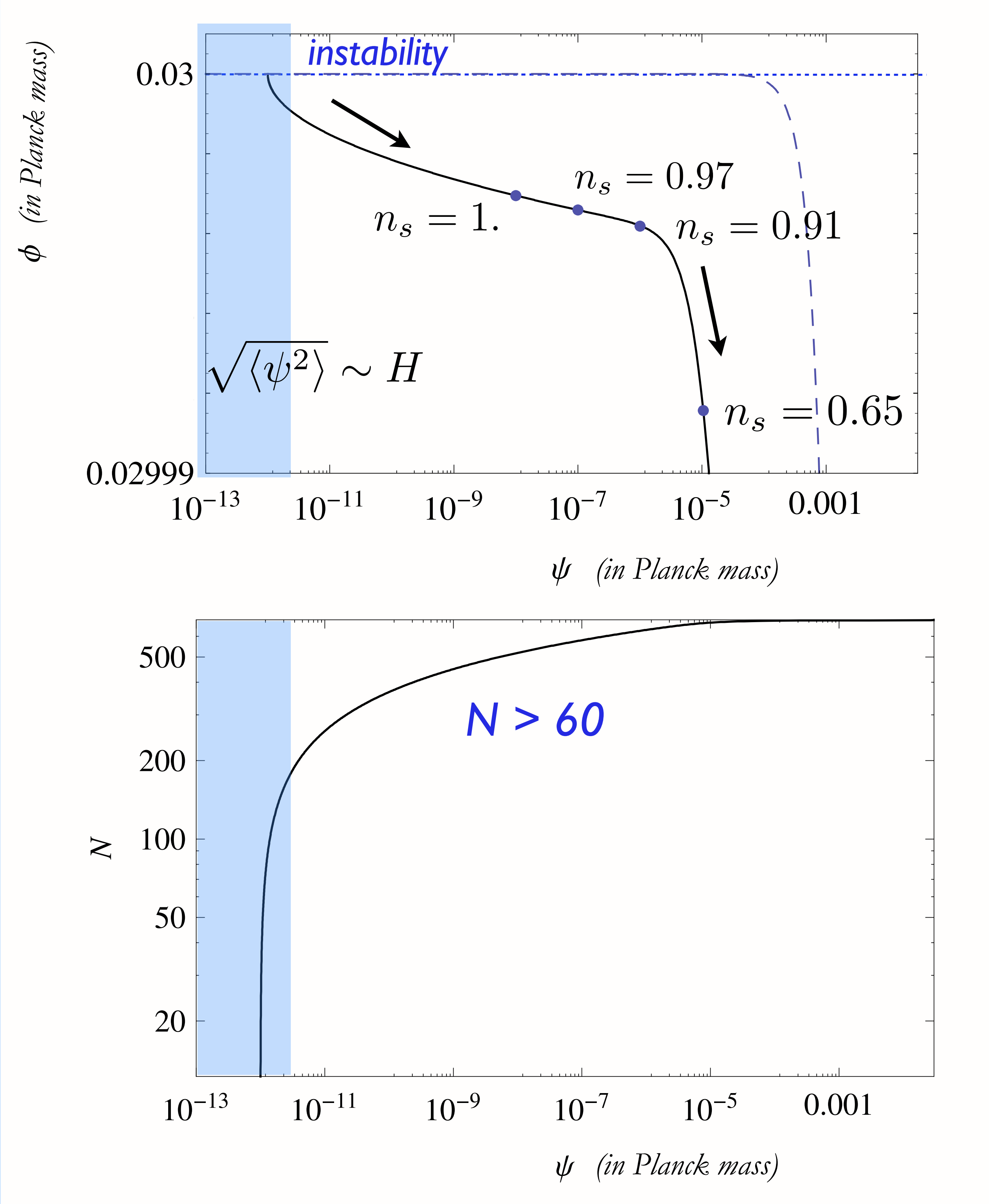
3.2 Inflation along waterfall trajectories

The exact 2-field classical dynamics of waterfall trajectories



$\phi_c = 0.03\ m_{pl}, M = 0.03\ m_{pl}, \mu = 636.4\ m_{pl}, \Lambda^4 = 10^{-24}\ m_{pl}^4$

Much more than 60 e-folds along the waterfall

Classical value of ψ quickly much larger than its standard deviation

Red power spectrum of adiabatic perturbations

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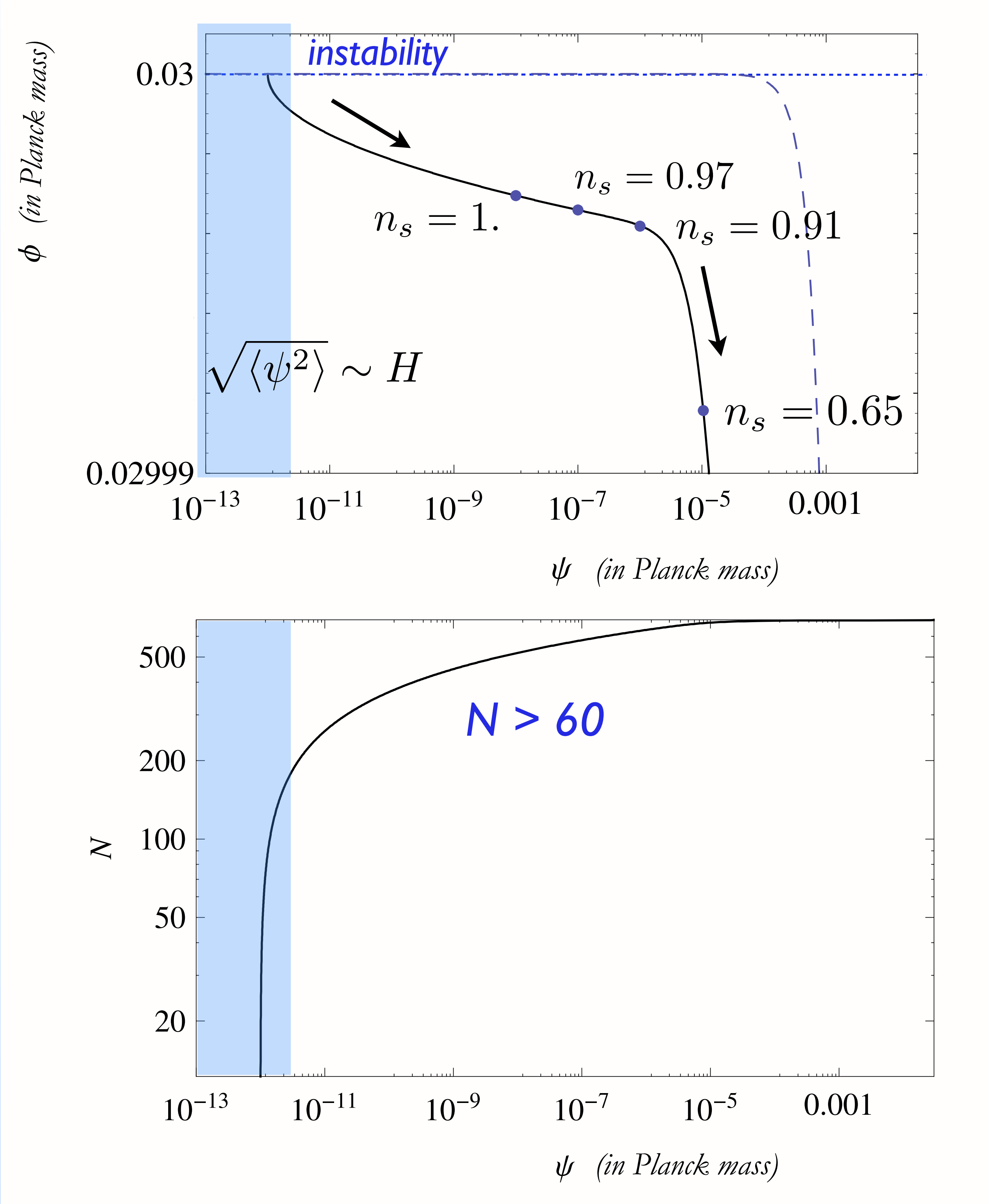

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Is it generic in the potential parameter space?

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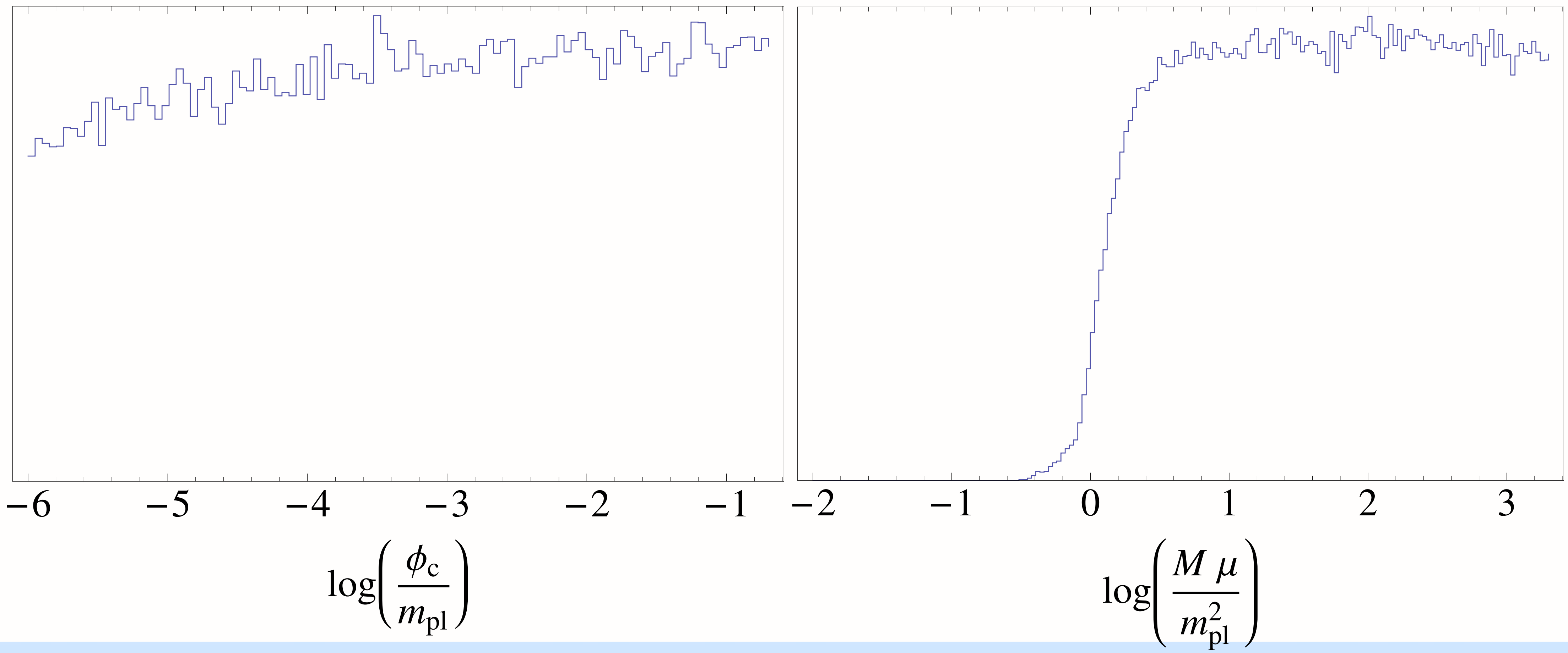
3.3 MCMC analysis of the parameter space

$$V(\phi,\psi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2}\right) + \frac{\phi^2}{\mu^2} + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} \right]$$

- **Flat priors on the log of the potential parameters**
- **Exclusion of trajectories for which quantum stochastic effects of the adiabatic field become dominant**
- **Posterior probability distributions for > 60 e-folds along waterfall :**



- **Bound on the combination** $\log\left(\frac{\mu M}{m_{pl}^2}\right) > 0.21 \quad 95\%\text{C.L.}.$

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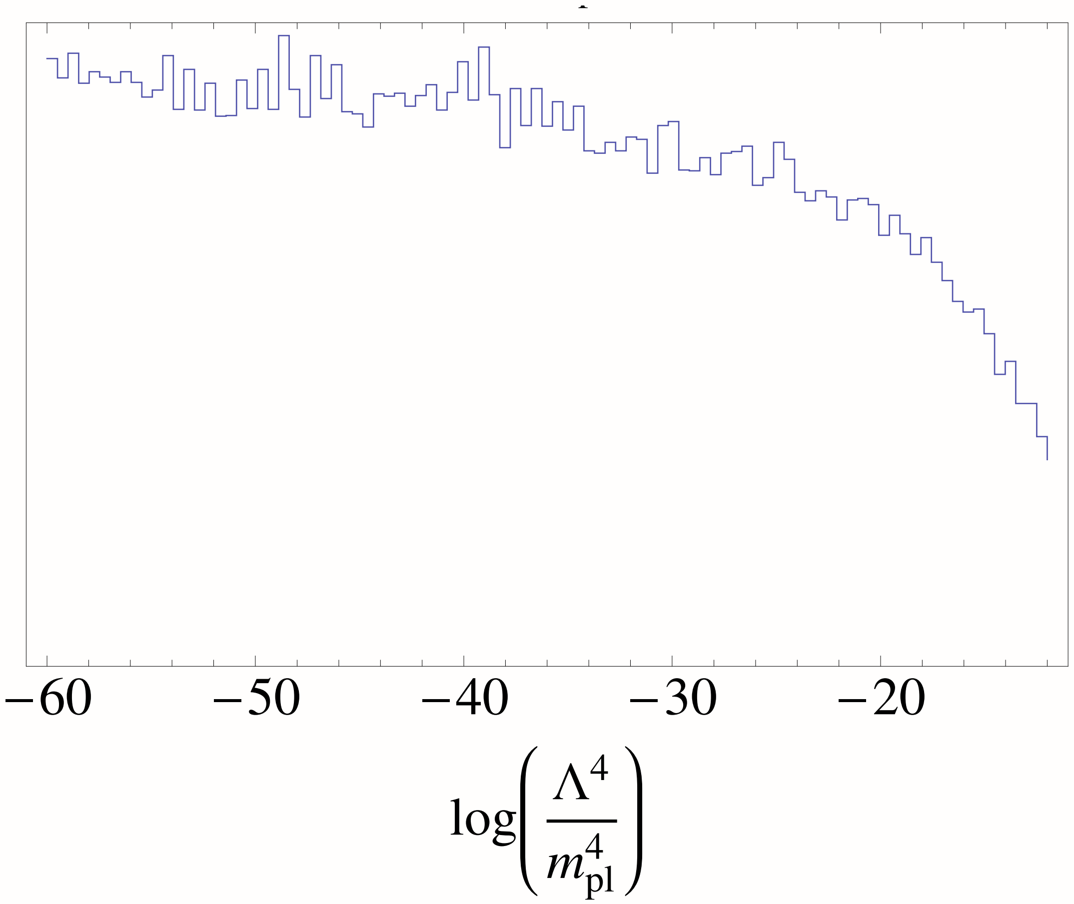
- Flat priors on the log of the potential parameters
- Exclusion of trajectories for which quantum stochastic effects of the adiabatic field become dominant
- Posterior probability distributions for > 60 e-folds along waterfall :



- High energy waterfall inflation less probable

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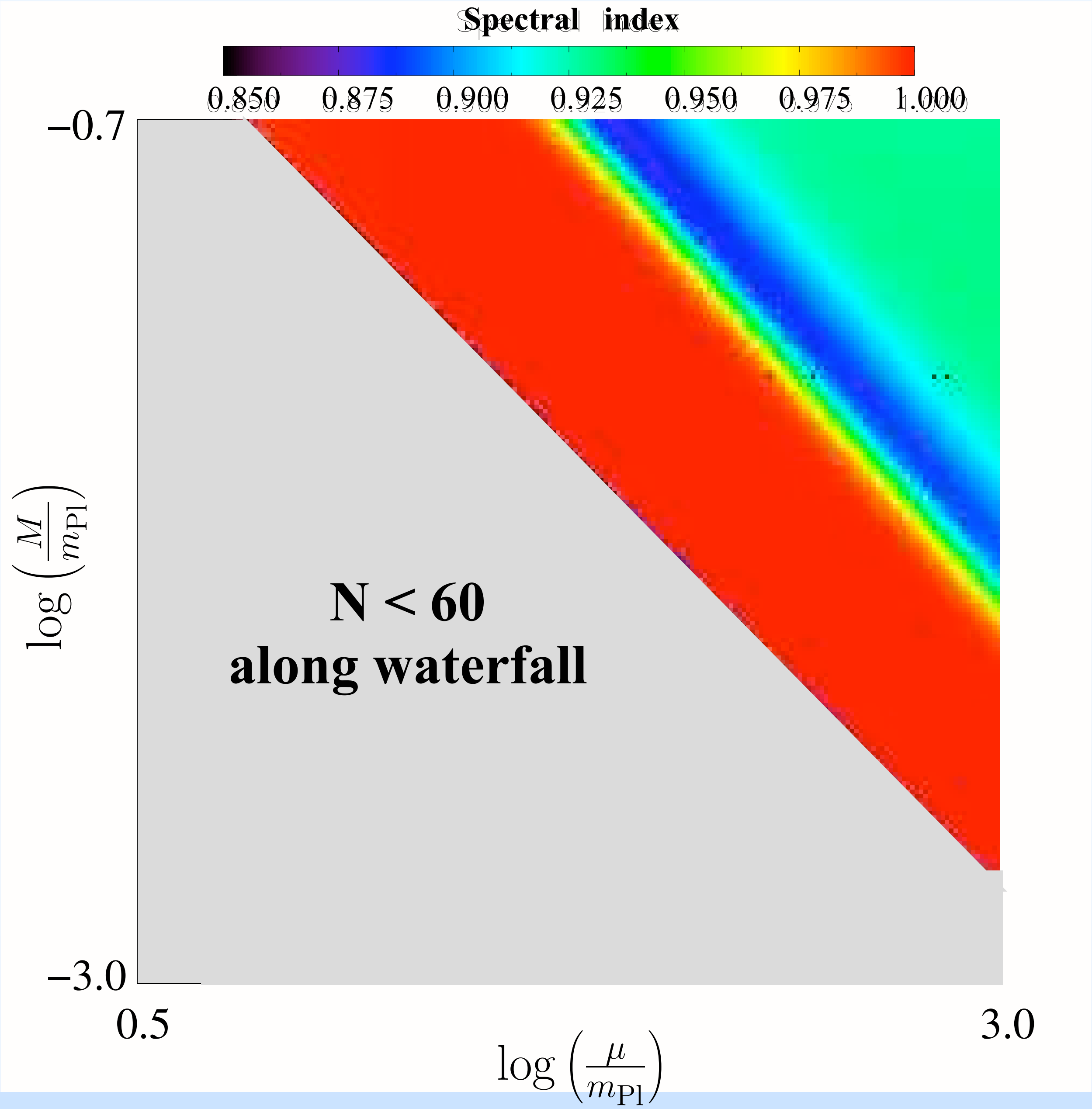
3.4 Power spectrum of adiabatic pert.

- Power spectrum (spectral index) of adiabatic perturbations (slow-roll approximation for the adiabatic field)
- Agreement with CMB constraints in a large part of the parameter space
- Iso-curvature modes contribution NOT YET included



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✦ Slow-roll violations in the valley:

- The small field phase does not take place anymore
- A critical value of μ has been established
- Super-Plankian initial field values needed
- Red power spectrum, like in chaotic inflation

✦ Initial conditions (I.C.) in hybrid inflation models:

- I.C. do not require to be fine-tuned along the valley
- $\psi_{\rm i}$ more likely to be outside the inflationary valley (anamorphosis)
- I.C. organiszd in connected structure with fractal boundaries
- successful I.C. independant of initial velocities
- Natural bounds on potential parameters from only requirement of sufficiently long inflation.
- Similar results for other hybrid models (F-term,Smooth...)

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✦ **Hybrid inflation along waterfall trajectories:**

- **Exact 2-fields dynamics**
- **Much more than 60 e-folds can be realized classically before tachyonic preheating occurs.**
- **Classical dynamics not spoiled by field quantum backreactions**
- **Bounds on potential parameters from MCMC analysis**
- **Red power spectrum of adiabatic perturbations, possibly in agreement with CMB constraints.**

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✦ Hybrid inflation along waterfall trajectories:

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✦ Perspective:

- This regime is suspected to exist for other hybrid models
  Confirmed for F-term SUSY/SUGRA hybrid model (preliminary)
- Inclusion of iso-curvature modes in the power spectrum calculation
- Schemes of symmetry breaking in GUT may be reviewed...
- What happens when the stochastic effects dominate ?

Thank you for your attention…

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Questions...

- Basics on inflation
- Slow-roll violations
- IC grids of Tetradis and Mendes, Liddle
- Super-planckian IC
- Varying parameters
- Grid with red spectrum prediction
- Shifted and Smooth models
- Radion model
- Fractal Box-Counting Dimension
- MCMC method
- Distr. init.velocitites
- 2D pdf for parmeters/initial fields
- F-term SUGRA model

More slides for questions...

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1. Basics on inflation

- Horizon Problem
- Flatness Problem
- Topological defects

Inflation : Period of acceleration of the expansion of the universe, that is $\ddot{a} > 0$, where a is the scale factor

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$$N_{\mathrm{end}} \equiv \ln \frac{a_{\mathrm{end}}}{a_{\mathrm{i}}} > 60$$

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Inflation : **Period of acceleration of the expansion of the universe, that is** $\ddot{a} > 0$ **, where** a **is the scale factor**

Simplest realisation : **Fill the universe with a scalar field** ϕ

F.L. equations:

$$H^2 = \frac{8\pi}{3m_{\rm p}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3m_{\rm p}^2}\left[-\dot{\phi}^2 + V(\phi)\right]$$

K.G. equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{{\rm d}V}{{\rm d}\phi} = 0$$

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$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

Power spectrum of scalar pert of the metric, in SR approximation

$$\mathcal{P}_\zeta(k) \simeq \frac{H_*^2}{\pi m_{\mathrm{p}}^2 \epsilon_{1*}} \sim \left(\frac{k}{k_*}\right)^{n_{\mathrm{s}}-1} \quad \textbf{with} \quad \epsilon_1 = \frac{m_{\mathrm{p}}^2}{16\pi}\left(\frac{\frac{\mathrm{d}V}{\mathrm{d}\phi}}{V}\right)^2 \ll 1$$

(nearly) scale invariance

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Scalar spectral index, in SR approximation

$$n_s - 1 = -2\epsilon_{1*} - \epsilon_{2*} \qquad \text{with} \quad \epsilon_2 = \frac{m_{\mathrm{p}}^2}{4\pi}\left[\left(\frac{V'}{V}\right)^2 - \frac{V"}{V}\right] \ll 1$$

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WMAP5 : $n_s = 0.963^{+0.014}_{-0.015}$

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Other realisation : **Fill the universe with TWO scalar fields** ϕ

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K.G. equations:

$$\ddot{\phi}+3H\dot{\phi}+\frac{\mathrm{d}V}{\mathrm{d}\phi}=0 \qquad \ddot{\psi}+3H\dot{\psi}+\frac{\mathrm{d}V}{\mathrm{d}\psi}=0$$

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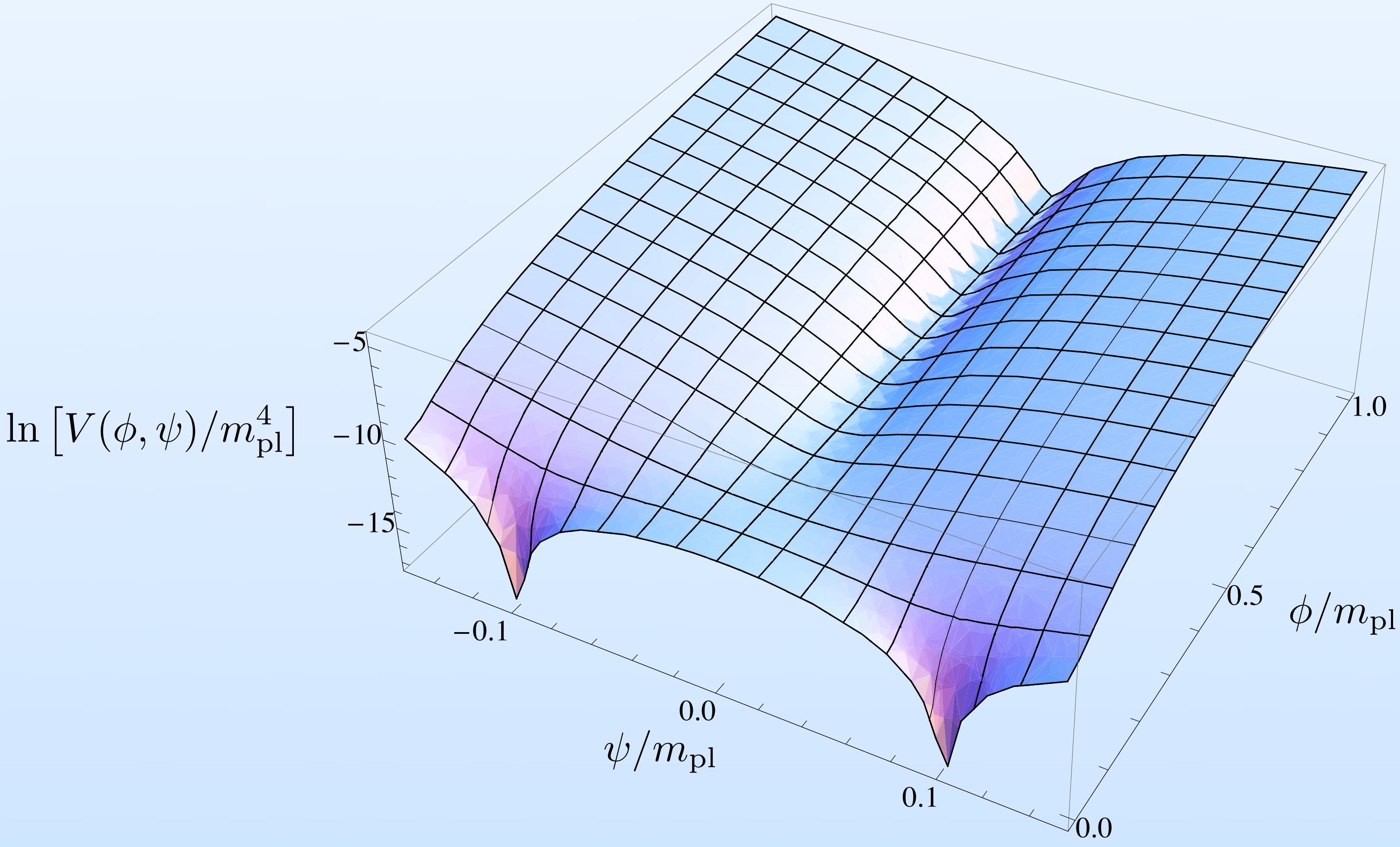
- **Inflaton** ϕ
- **Higgs-type auxiliary field** ψ
- **Hybrid potential** (Linde, astro-ph/9307002)

$$V(\phi,\psi)=\frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\left(M^2-\psi^2\right)^2+\frac{\lambda'}{2}\phi^2\psi^2$$



$\lambda=\lambda'=1, M=0.1\ m_{\rm pl},\ \ m=10^{-6}\ m_{\rm pl}$

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- **1-field effective potential** $V(\phi)=\Lambda^4\left[1+\left(\frac{\phi}{\mu}\right)^2\right]$
- **First slow-roll parameter** $\epsilon_1\equiv-\frac{\dot{H}}{H^2}$

inflation: $\epsilon_1<1$ **slow-roll approximation:** $\epsilon_1\ll 1$

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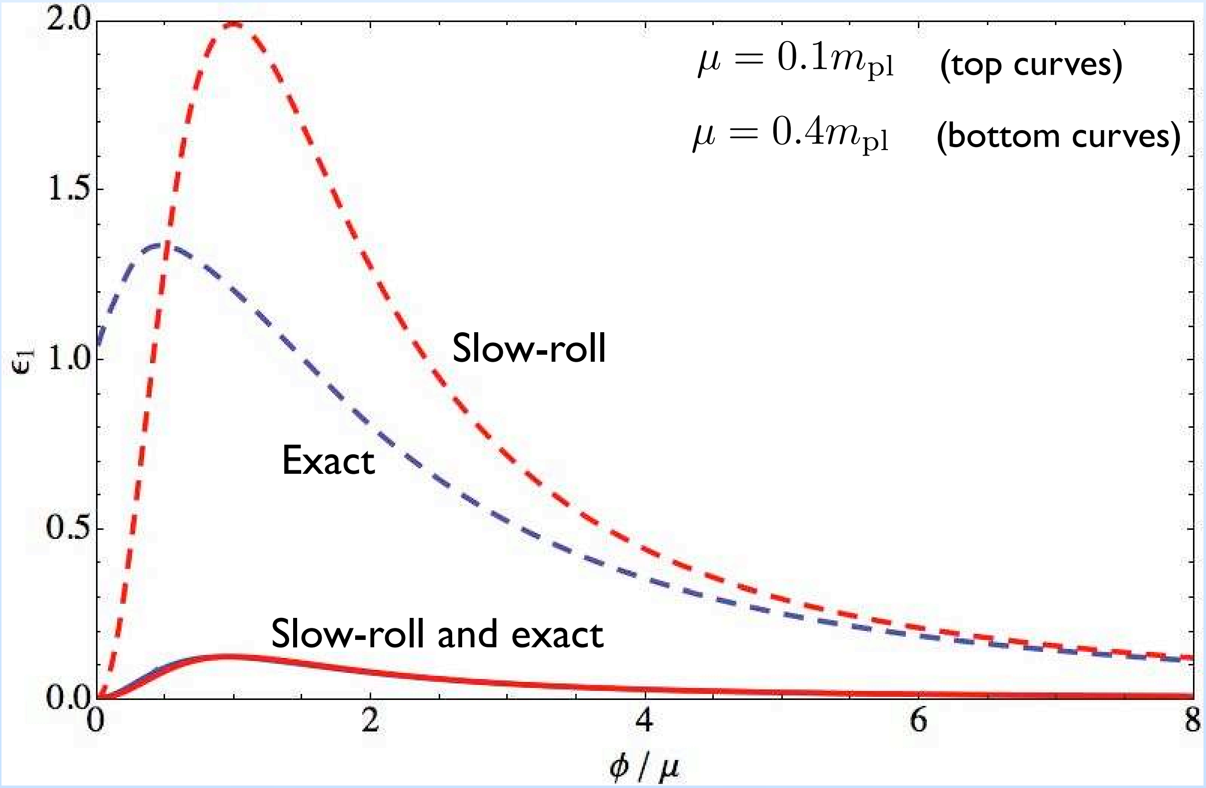

Slow-roll can be violated
⇒ Exact approach

Blue spectrum avoided

- **If critical point of instability is in the large field phase**
- **When slow-roll is violated**

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2. Fine-tuning of the initial conditions...

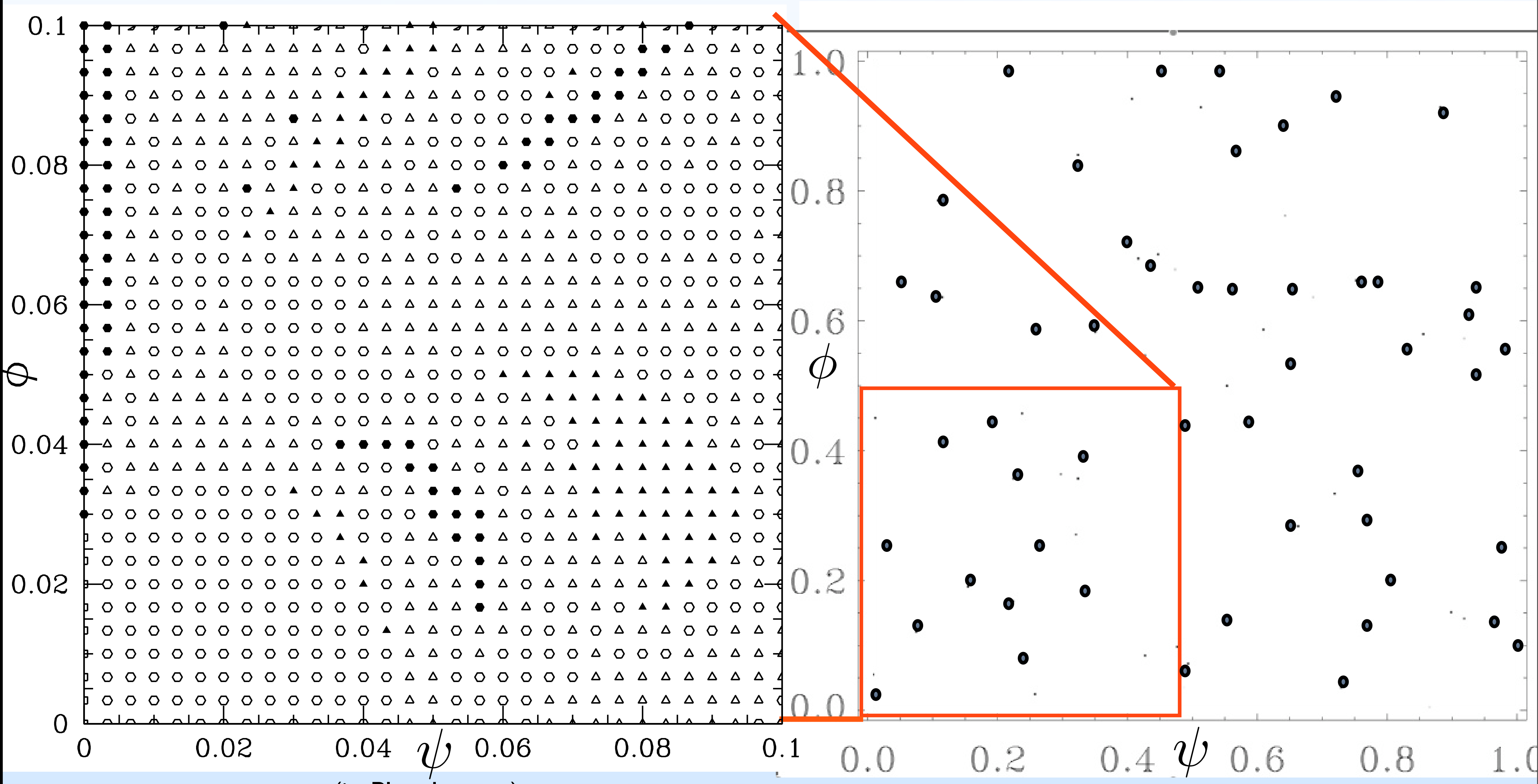
$$V(\phi,\psi)=\frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\left(M^2-\psi^2\right)^2+\frac{\lambda'}{2}\phi^2\psi^2$$

Tetradis, astro-ph/9707214

$\lambda=\lambda'=1,\ M=0.03\ m_{\rm pl}, m=10^{-6}\ m_{\rm pl}$

Mendes, Liddle, astro-ph/0006020

$\lambda=\lambda'=1,\ M=0.004\ m_{\rm pl}, m=10^{-6}\ m_{\rm pl}$



(in Planck mass)

(in reduced Planck mass)

1. Hybrid inflation
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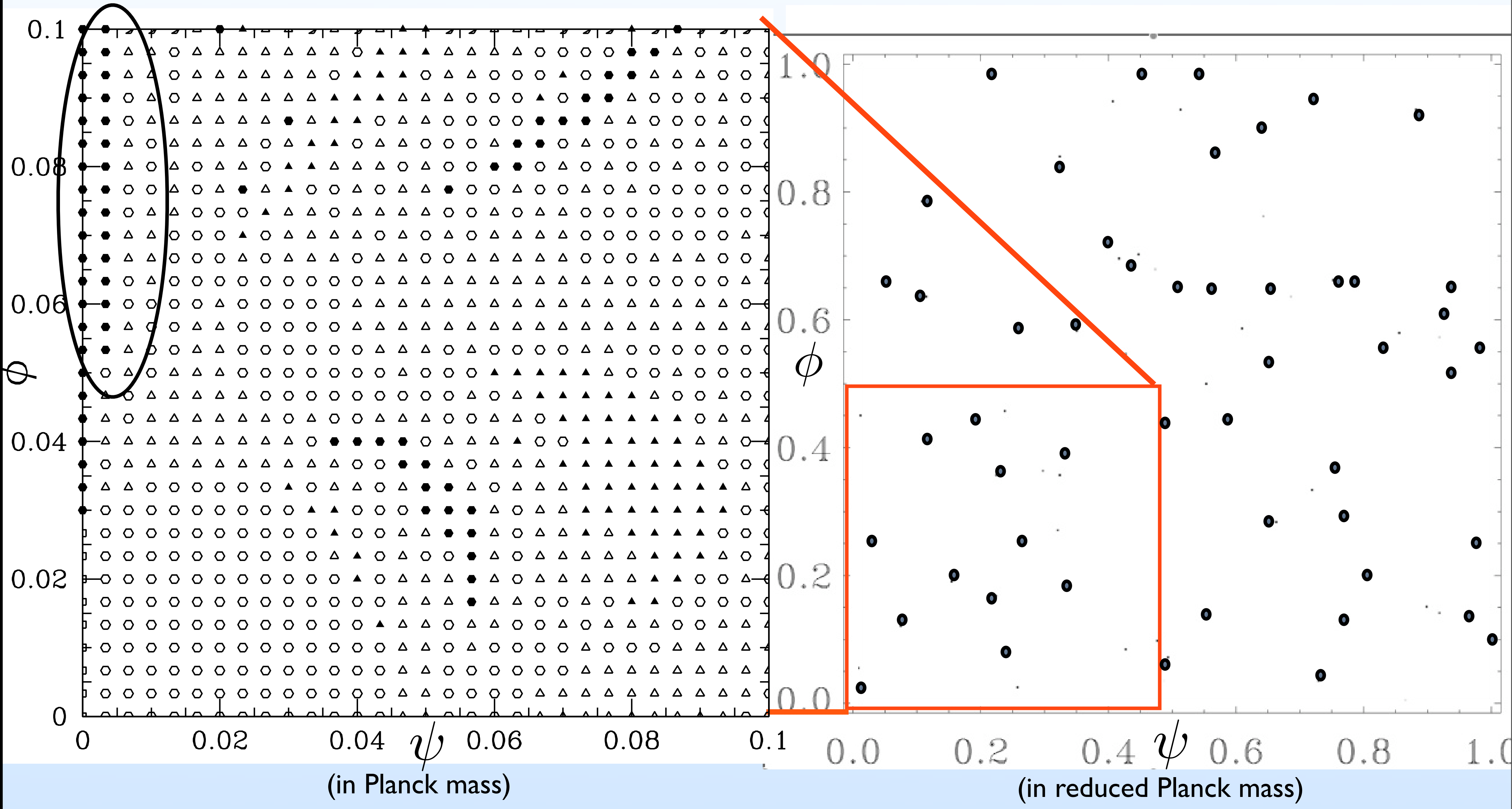
$$V(\phi,\psi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\left(M^2-\psi^2\right)^2 + \frac{\lambda'}{2}\phi^2\psi^2$$

Tetradis, astro-ph/9707214

$\lambda = \lambda' = 1, \ M = 0.03 \ m_{\rm pl}, m = 10^{-6} \ m_{\rm pl}$

Mendes, Liddle, astro-ph/0006020

$\lambda = \lambda' = 1, \ M = 0.004 \ m_{\rm pl}, m = 10^{-6} \ m_{\rm pl}$



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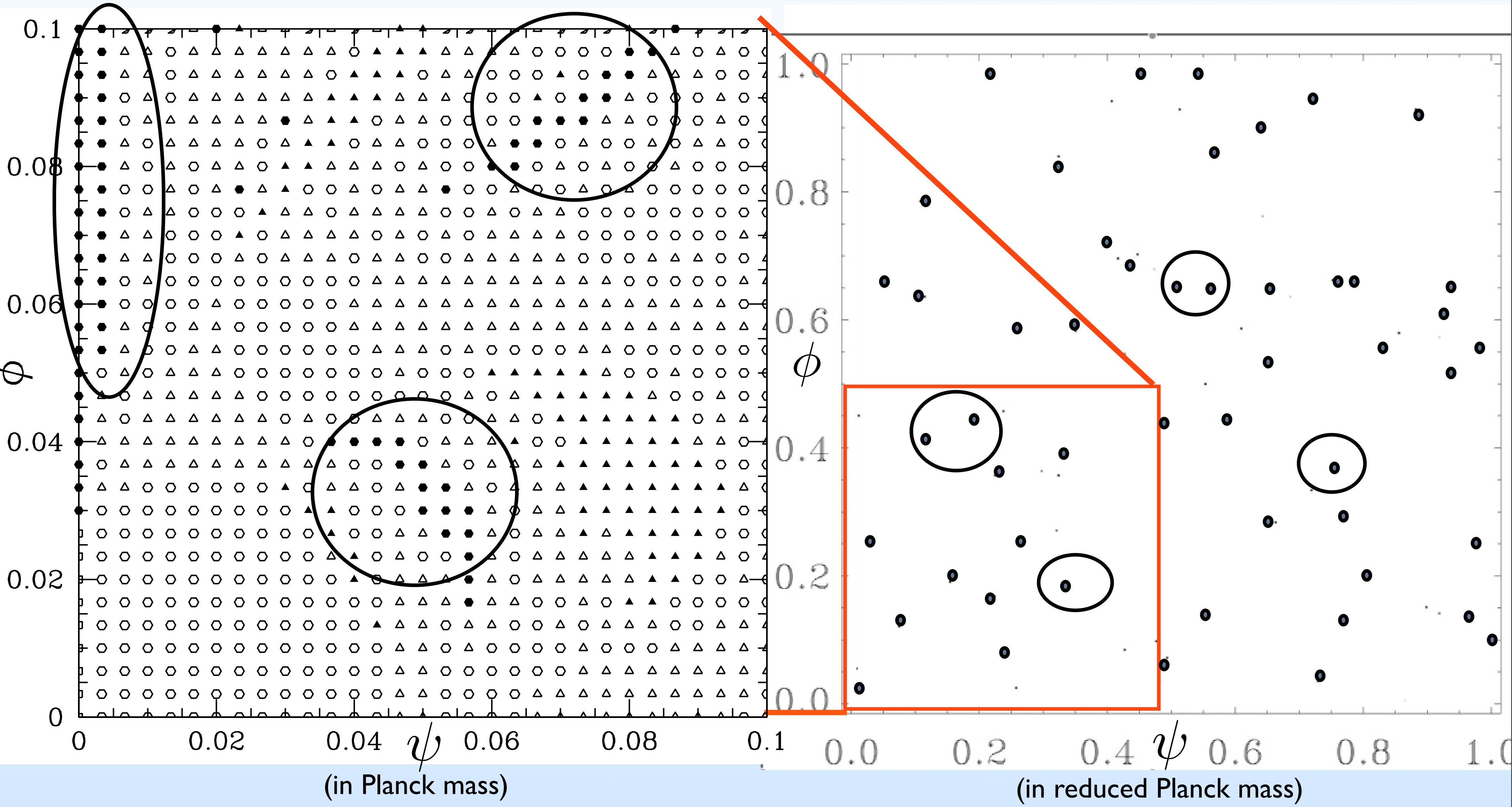

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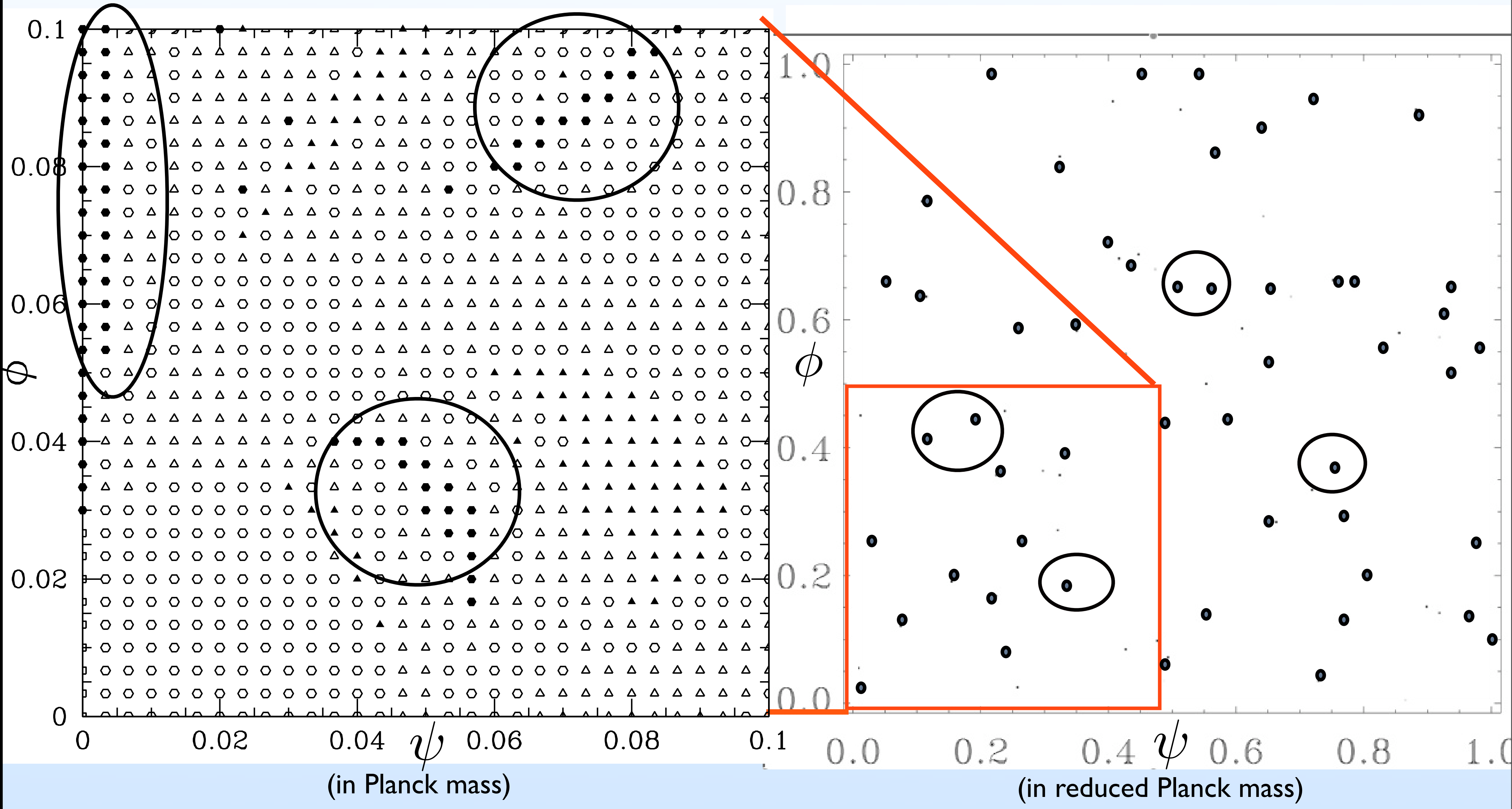

- **Isolated points or structures ?**

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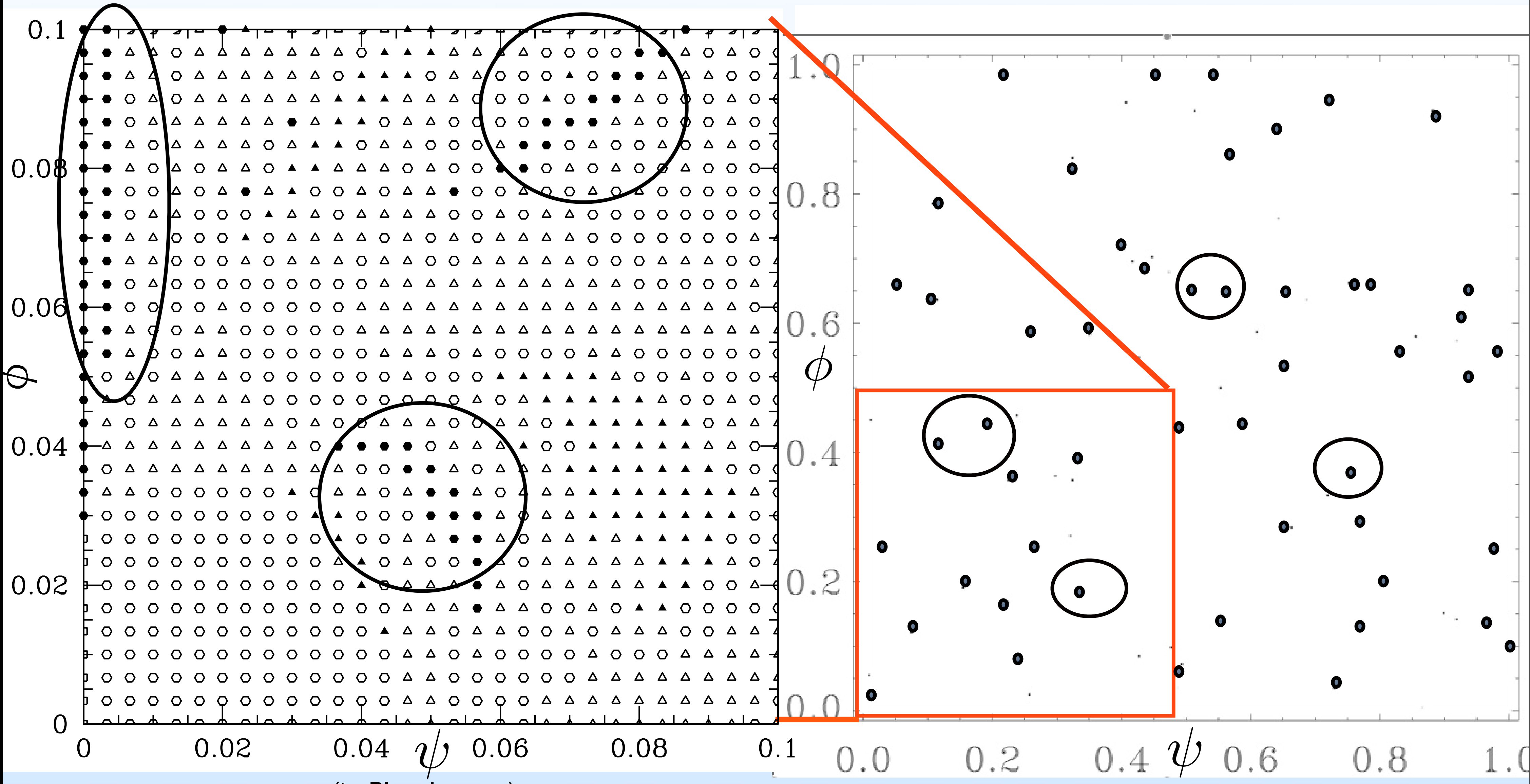

(in Planck mass)

(in reduced Planck mass)

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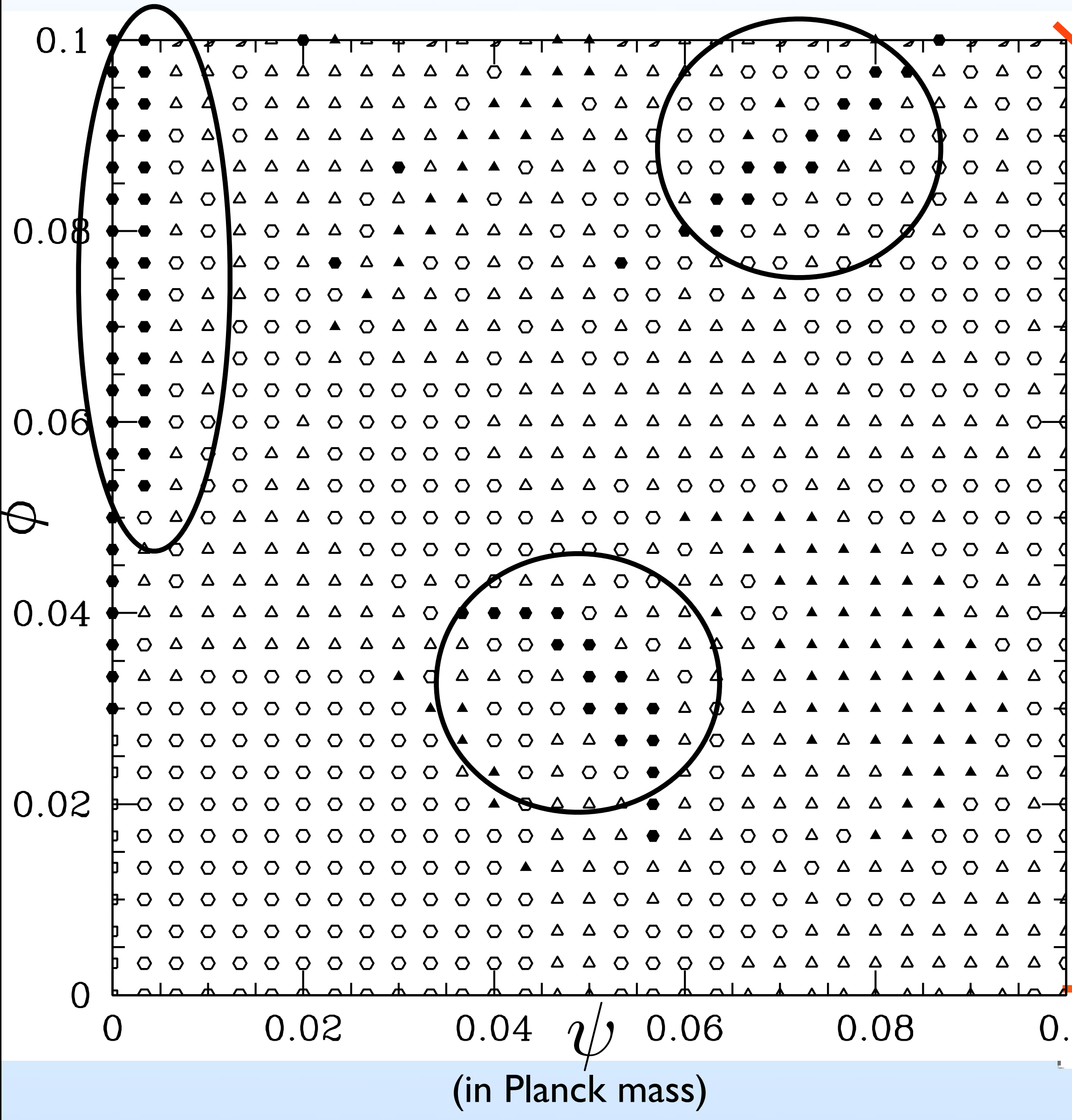

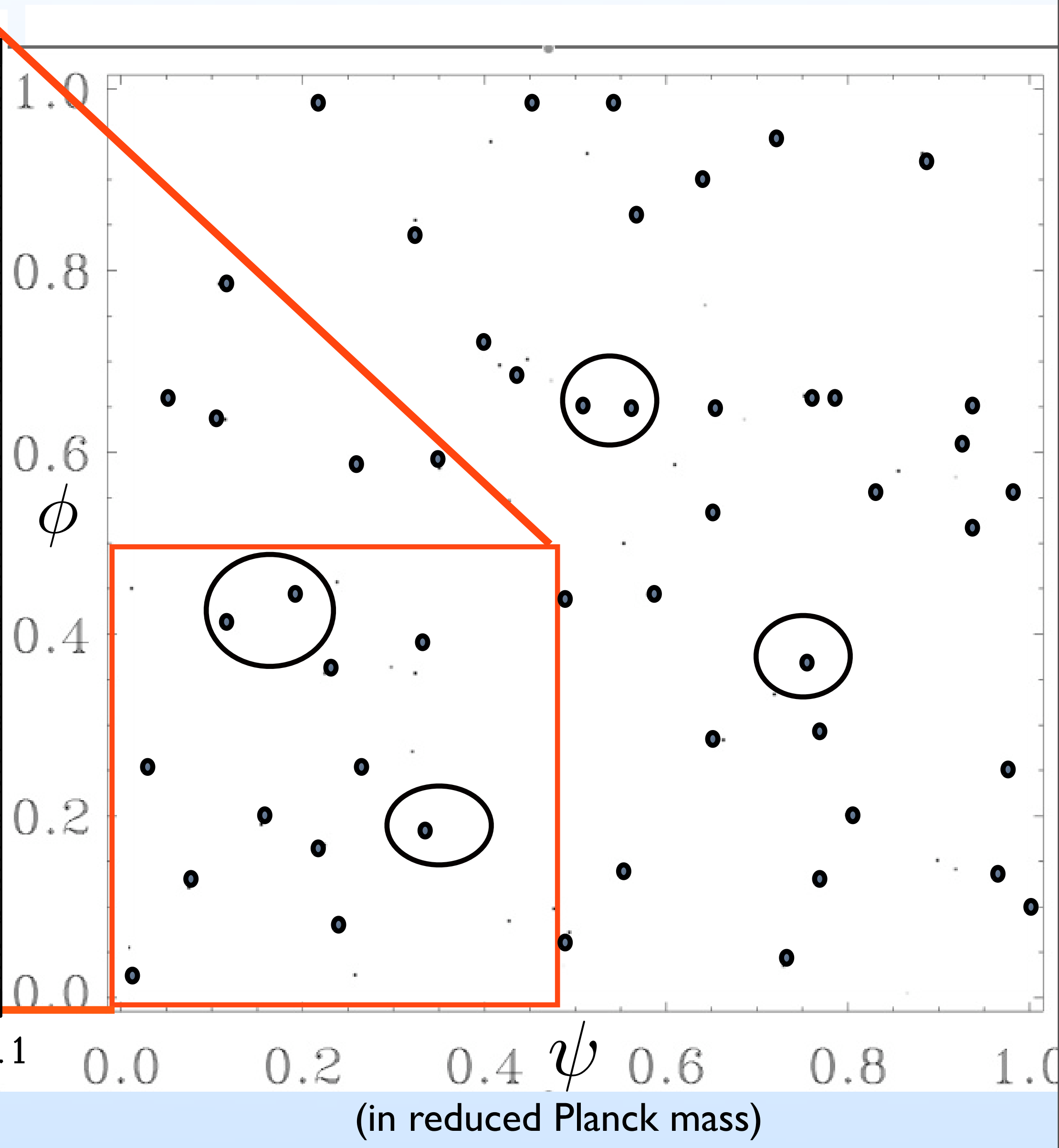

- Isolated points or structures ?
- Origin ?
- Quantification of successful areas ?

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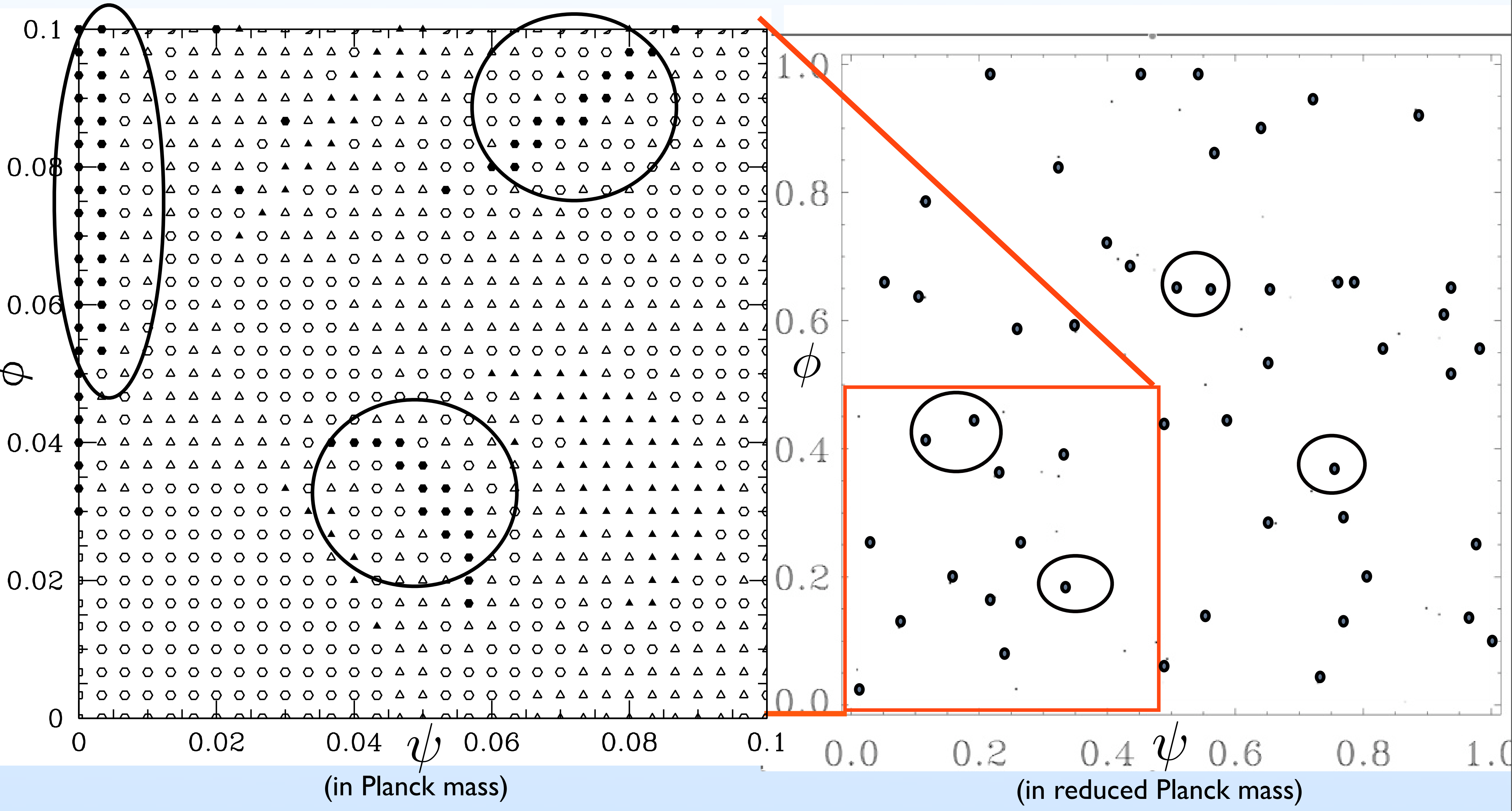

- **Isolated points or structures ?**
- **Origin ?**
- **Quantification of successful areas ?**

Numerical integration of exact 2-field dynamics to explore the space of initial conditions extended to super-planckian values

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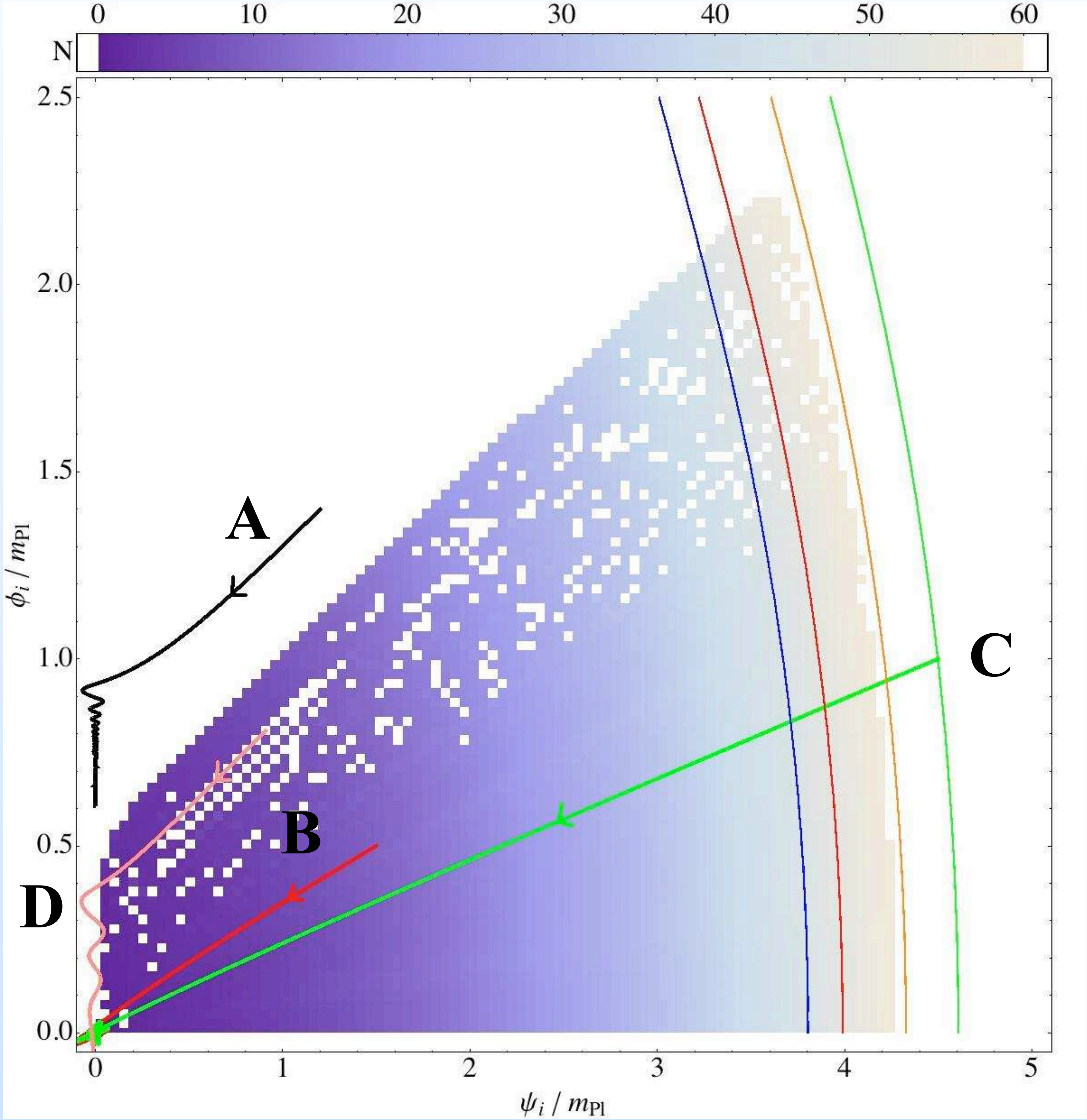
• Extended space of initial conditions



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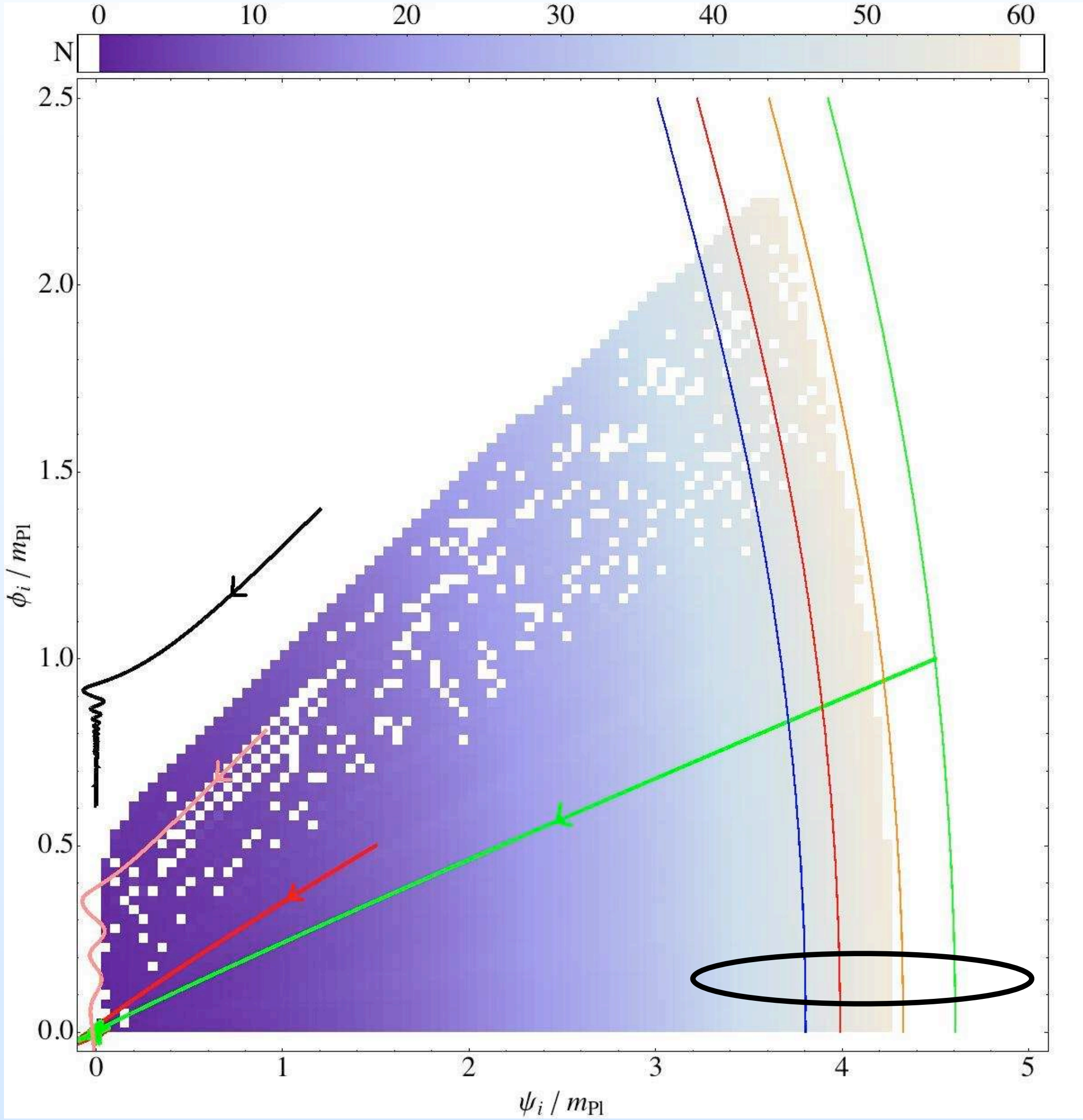
• Super-Planckian initial conditions:



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ϵ_1

$\epsilon_1 = 0.022, 0.020, 0.0167, 0.015$

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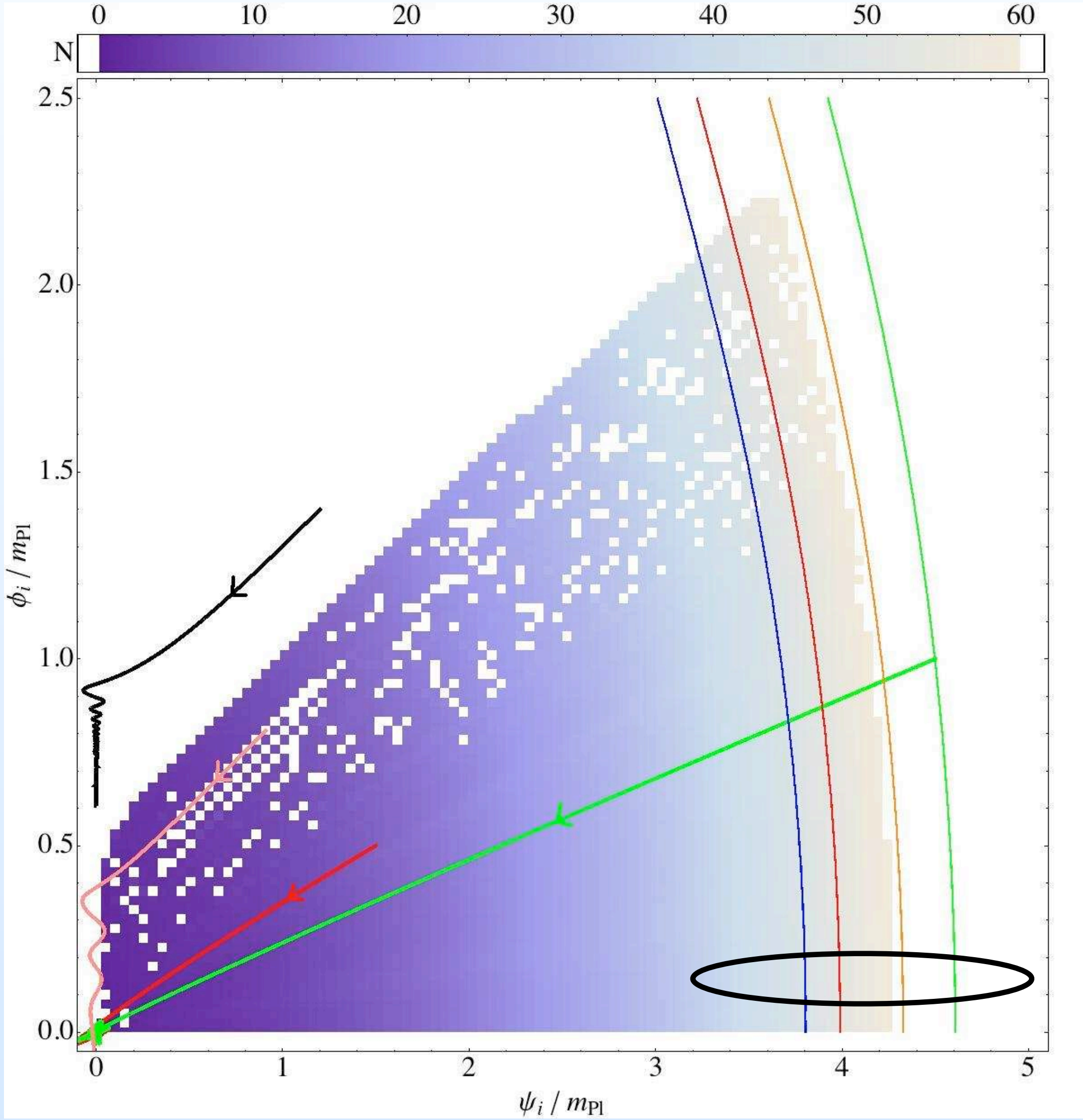

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Variation of potential parameters:

• λ' reduced
⇒ slope of the transition reduced ⇒ less "isolated" points
• M or λ increases
⇒ less "isolated" points

• m has no effect until it is small

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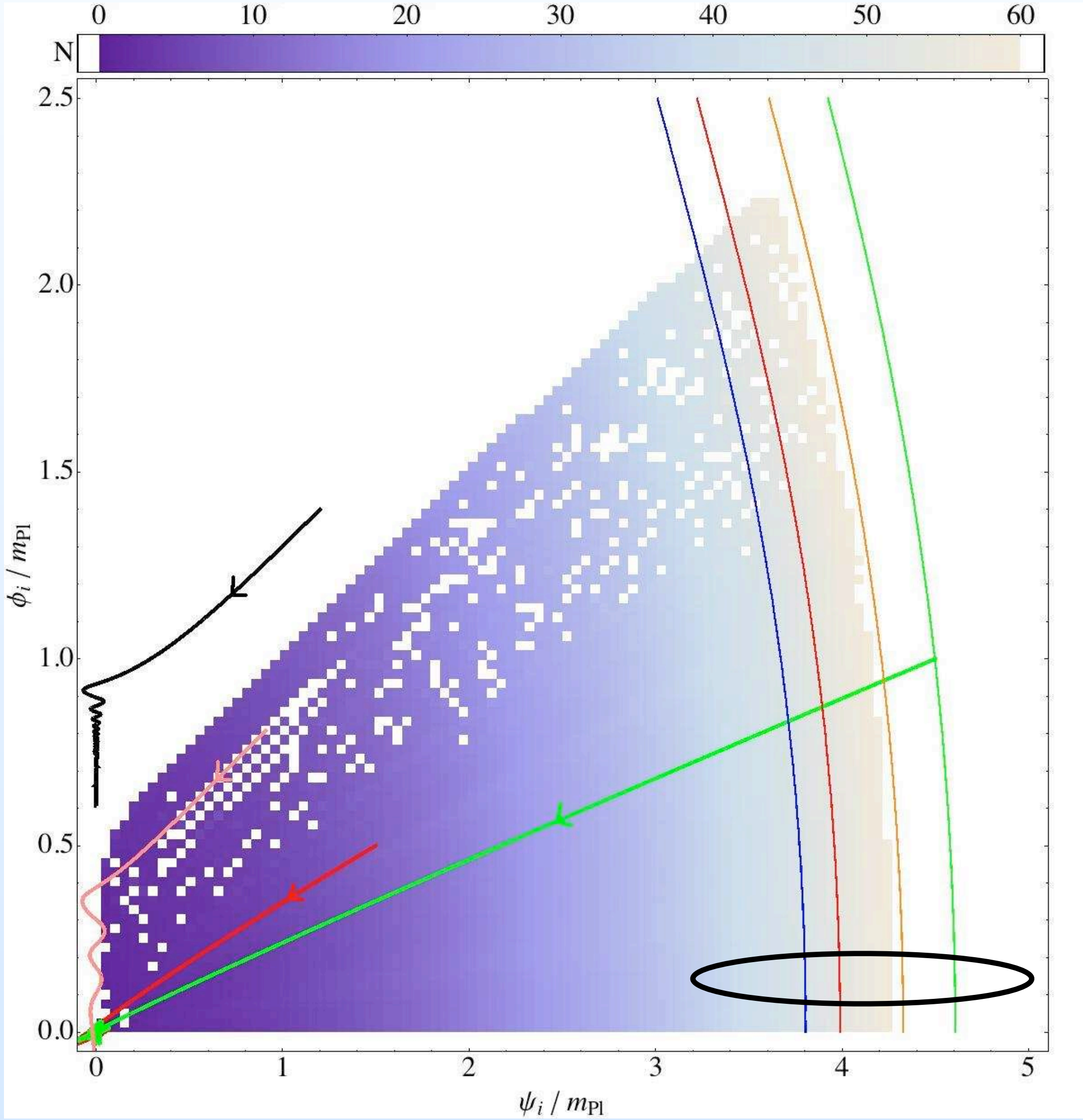

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Isocurves of ϵ_1(first slow-roll par.)

$\epsilon_1 = 0.022, 0.020, 0.0167, 0.015$

(from left to right)

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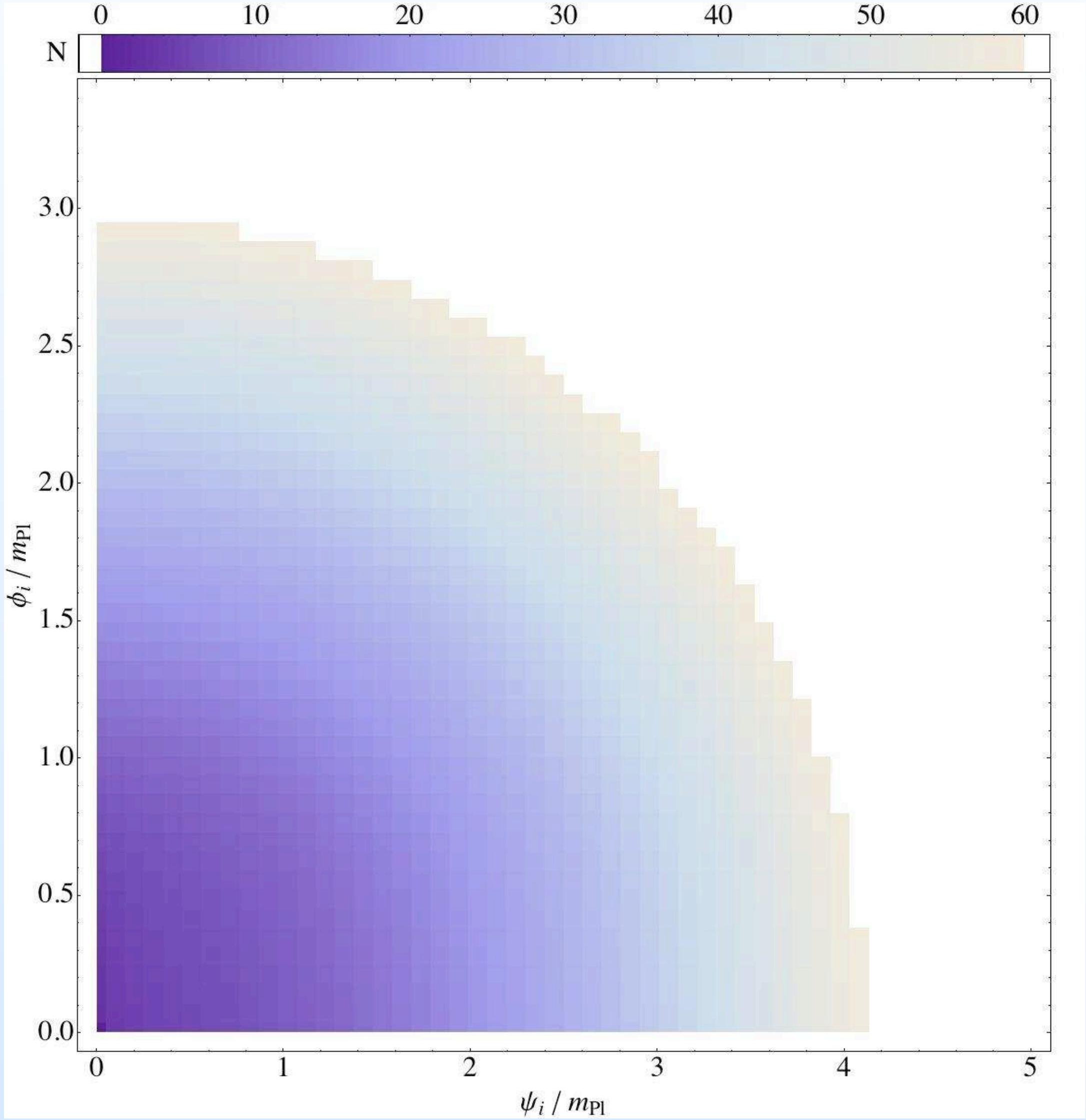

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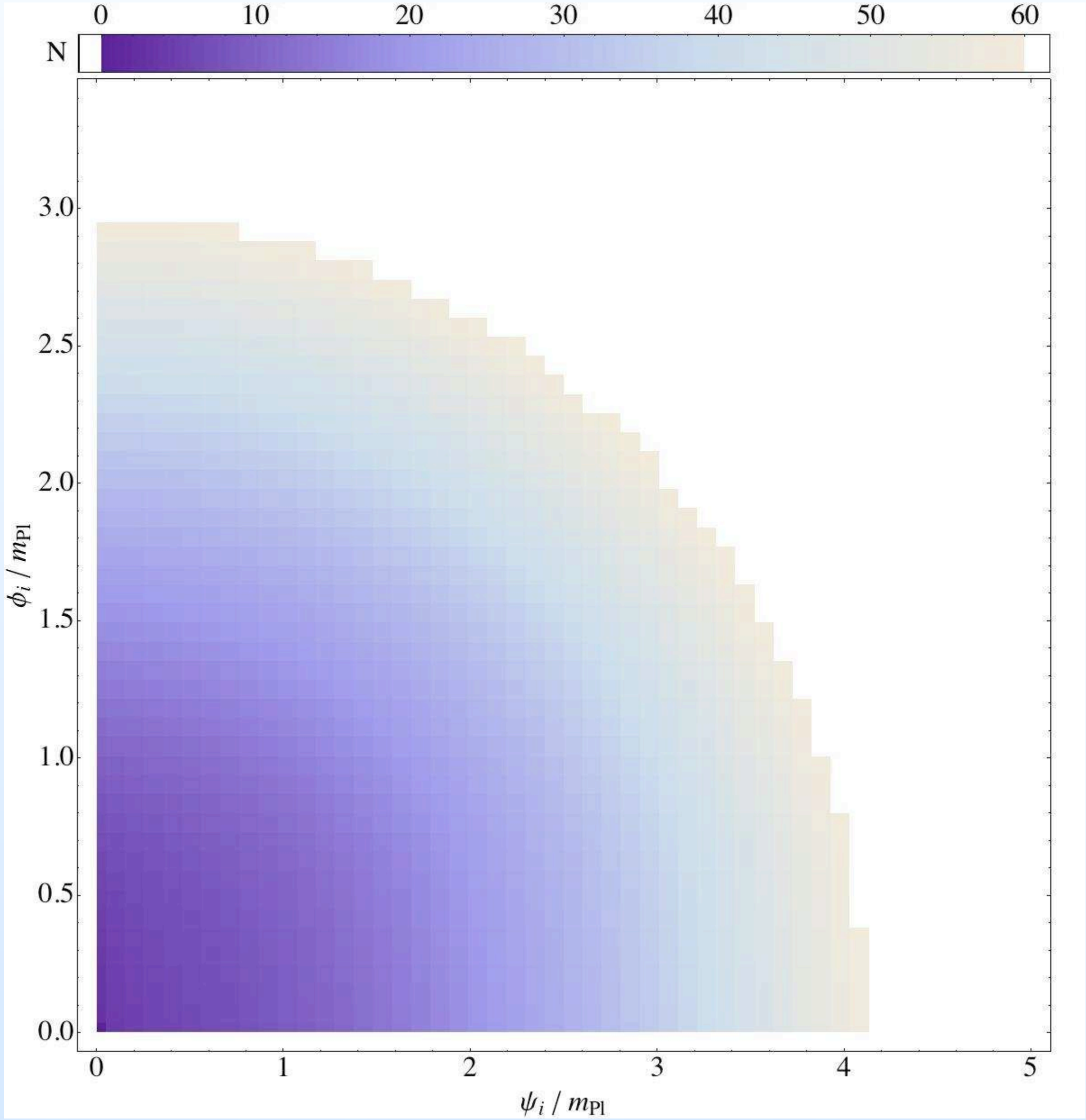

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If super-planckian values are allowed,

The fine-tuning problem is resolved!

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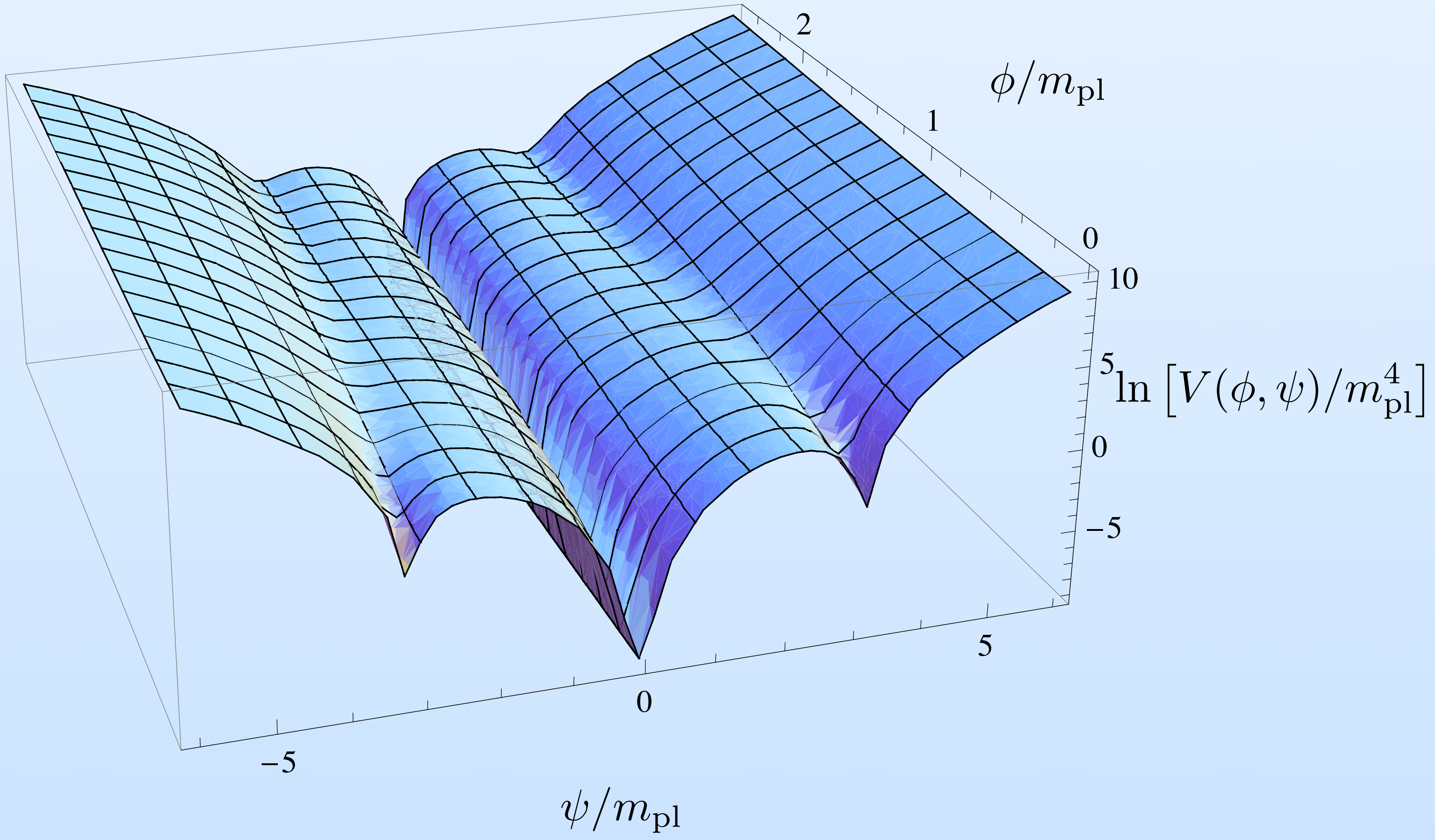
• Shifted inflation:

F-term superpotential + non-renormalizable term

Effective 2-field potential:

$$V(\phi,\psi)=\kappa^2\left(\psi^2-M^2-\frac{\beta}{\kappa}\psi^4\right)^2+2\kappa^2\phi^2\psi^2\left(1-2\frac{\beta}{\kappa}\psi\right)^2$$

1 central + 2 parallel valleys



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• Smooth inflation:

F-term superpotential + non-renormalizable term + Z_2 symmetry

Effective 2-field potential:

$$V(\phi,\psi)=\kappa^2\left(M^2-\frac{\psi^4}{m_{\mathrm{Pl}}^2}\right)^2+2\kappa^2\phi^2\frac{\psi^6}{m_{\mathrm{Pl}}^4}$$

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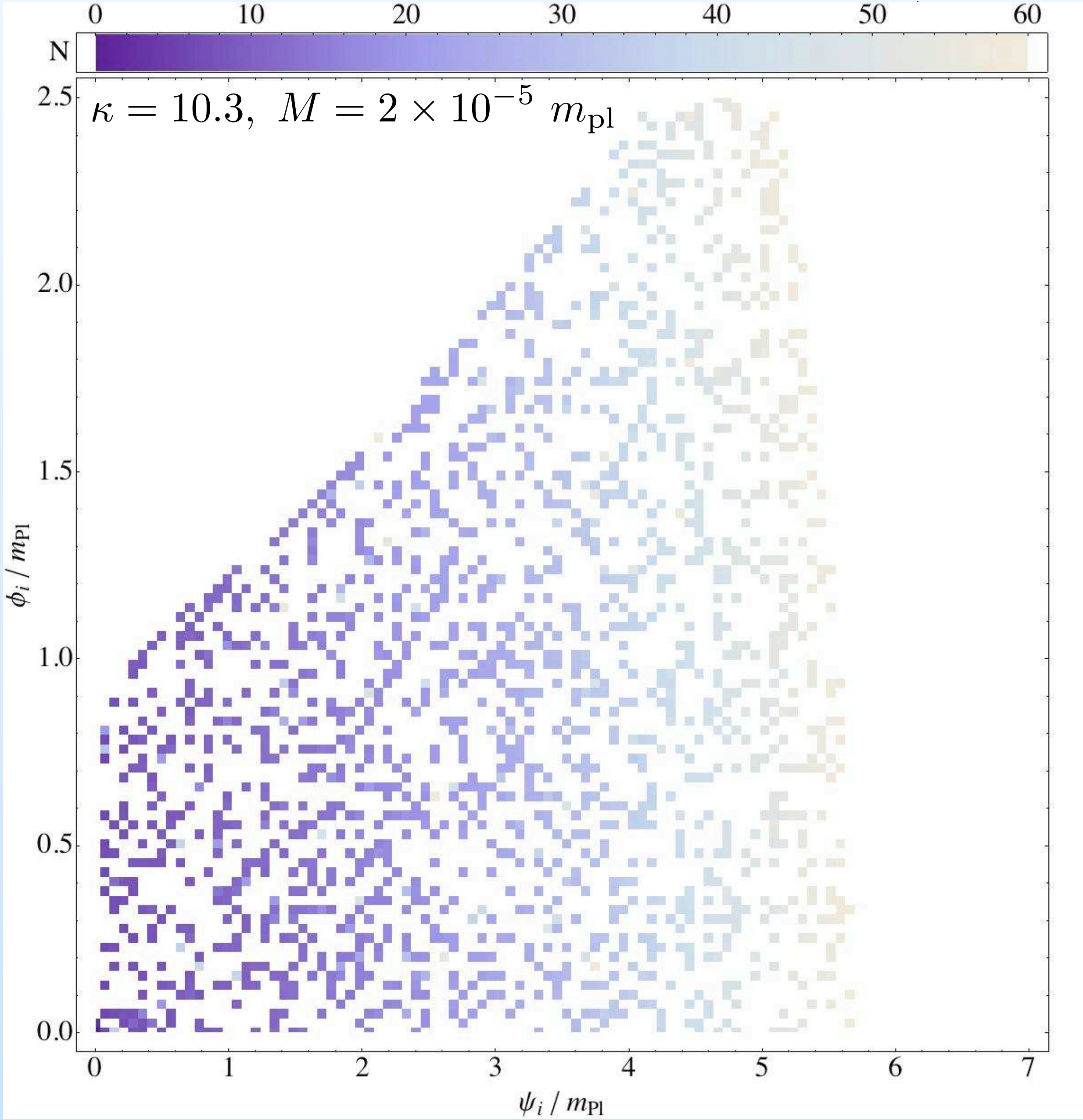
• Smooth inflation: *(Lazarides, Panagiotakopoulos, hep-ph/9506325)*

Effective 2-field potential (SUSY):

$$V(\phi,\psi)=\kappa^2\left(M^2-\frac{\psi^4}{16M_\mathrm{p}}\right)^2+\kappa^2\phi^2\frac{\psi^6}{16M_\mathrm{p}}$$

2 valleys and a flat $\psi=0$direction ⇒ **No topological defects**



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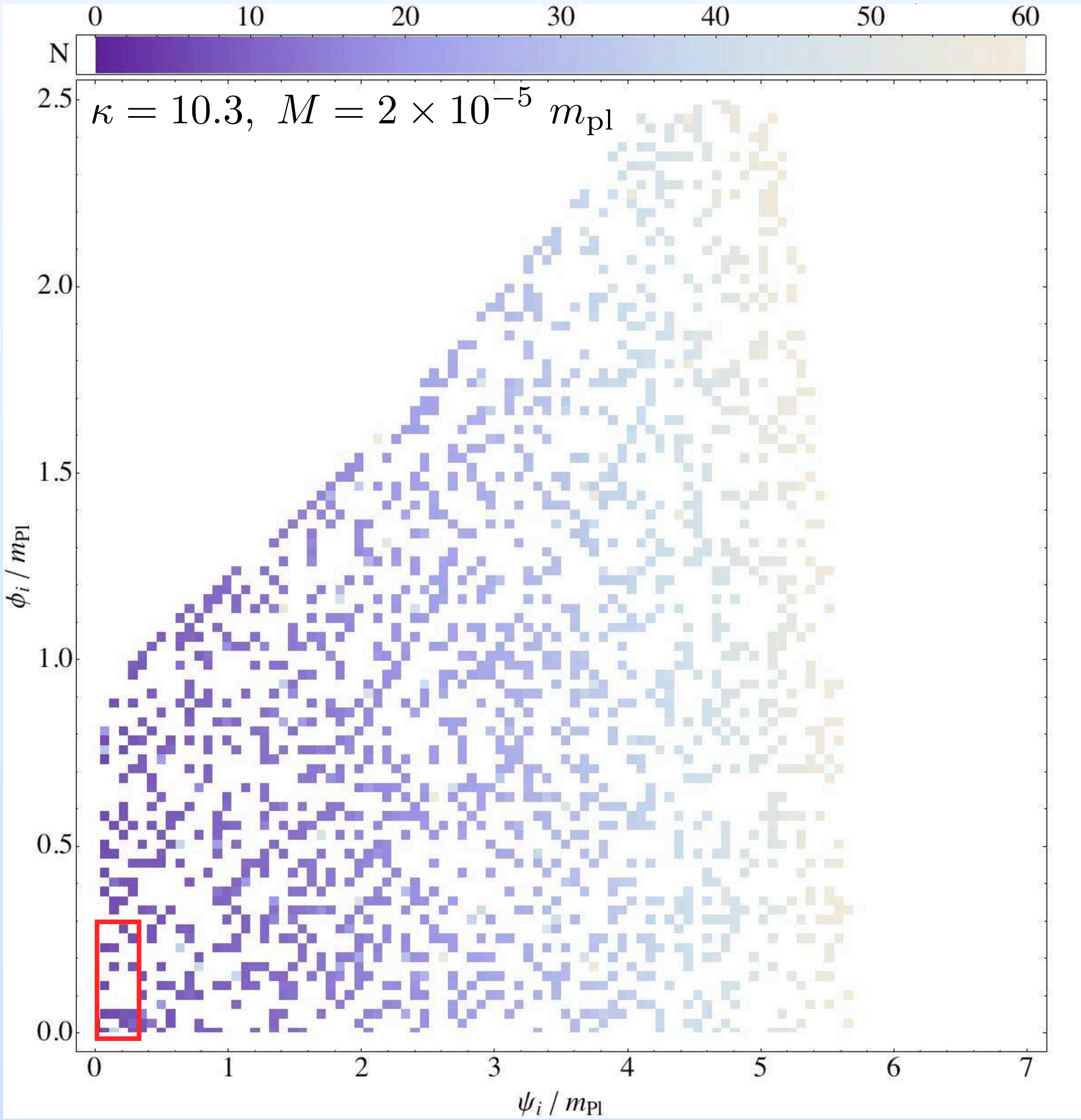

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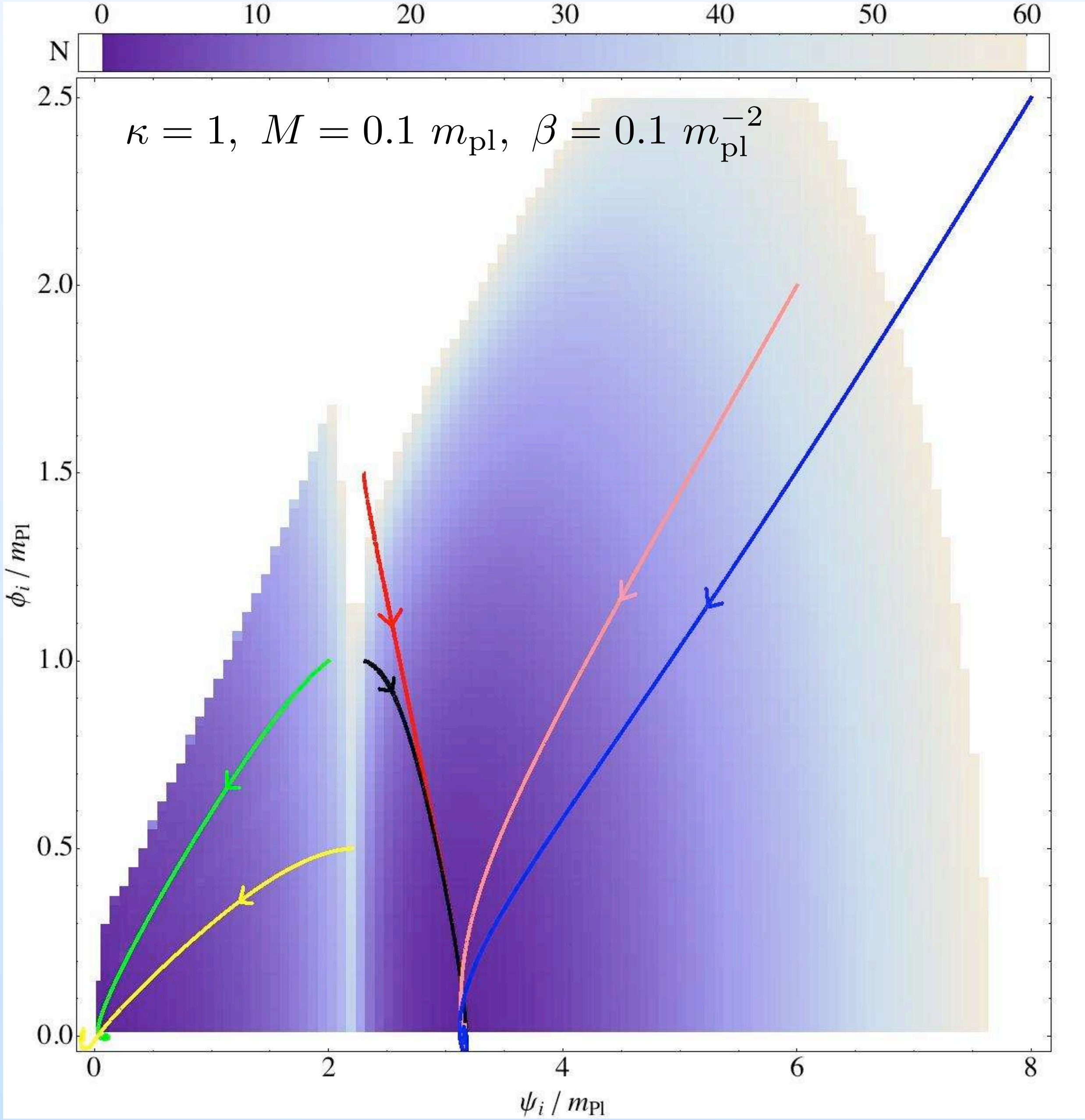

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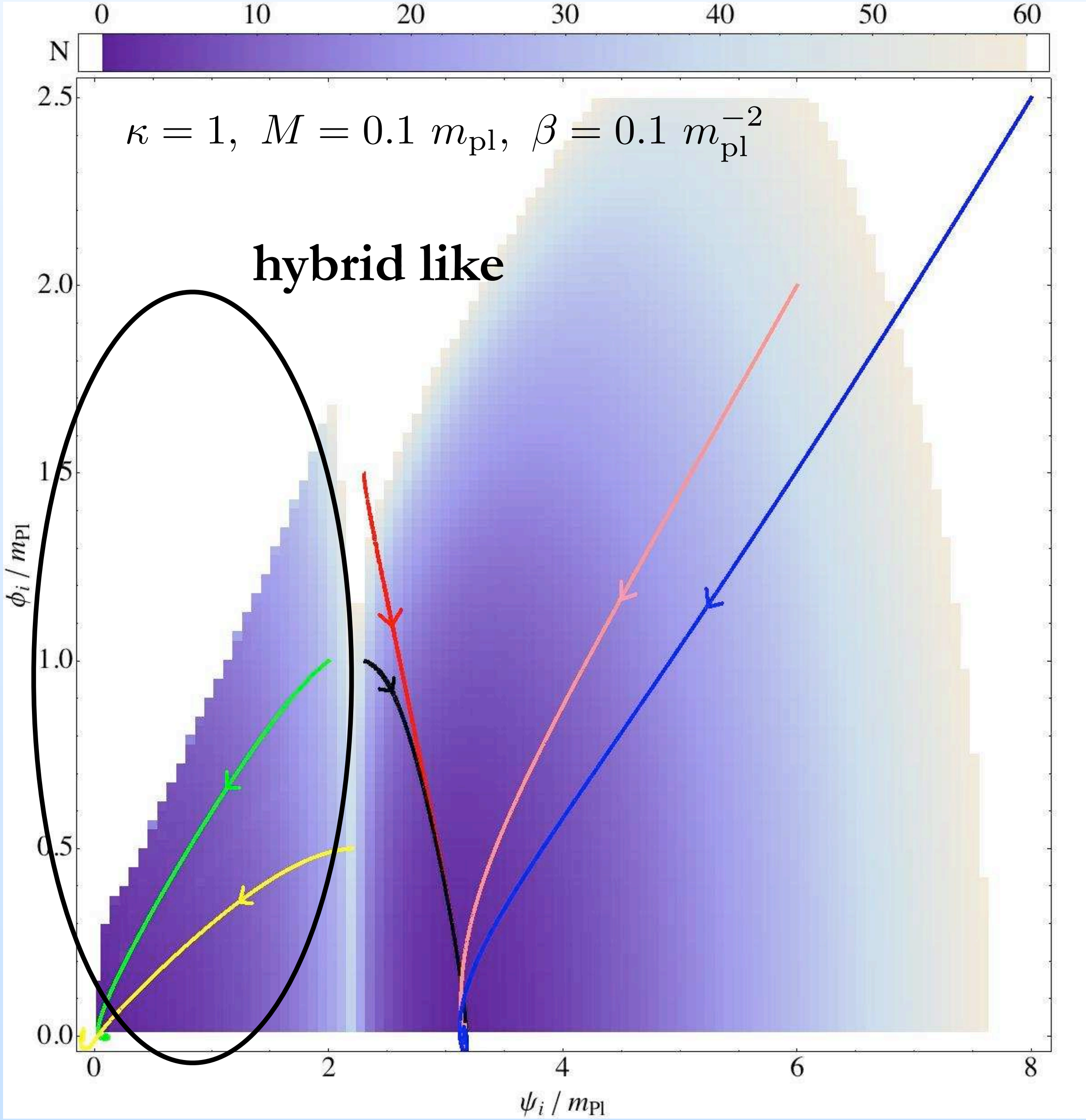

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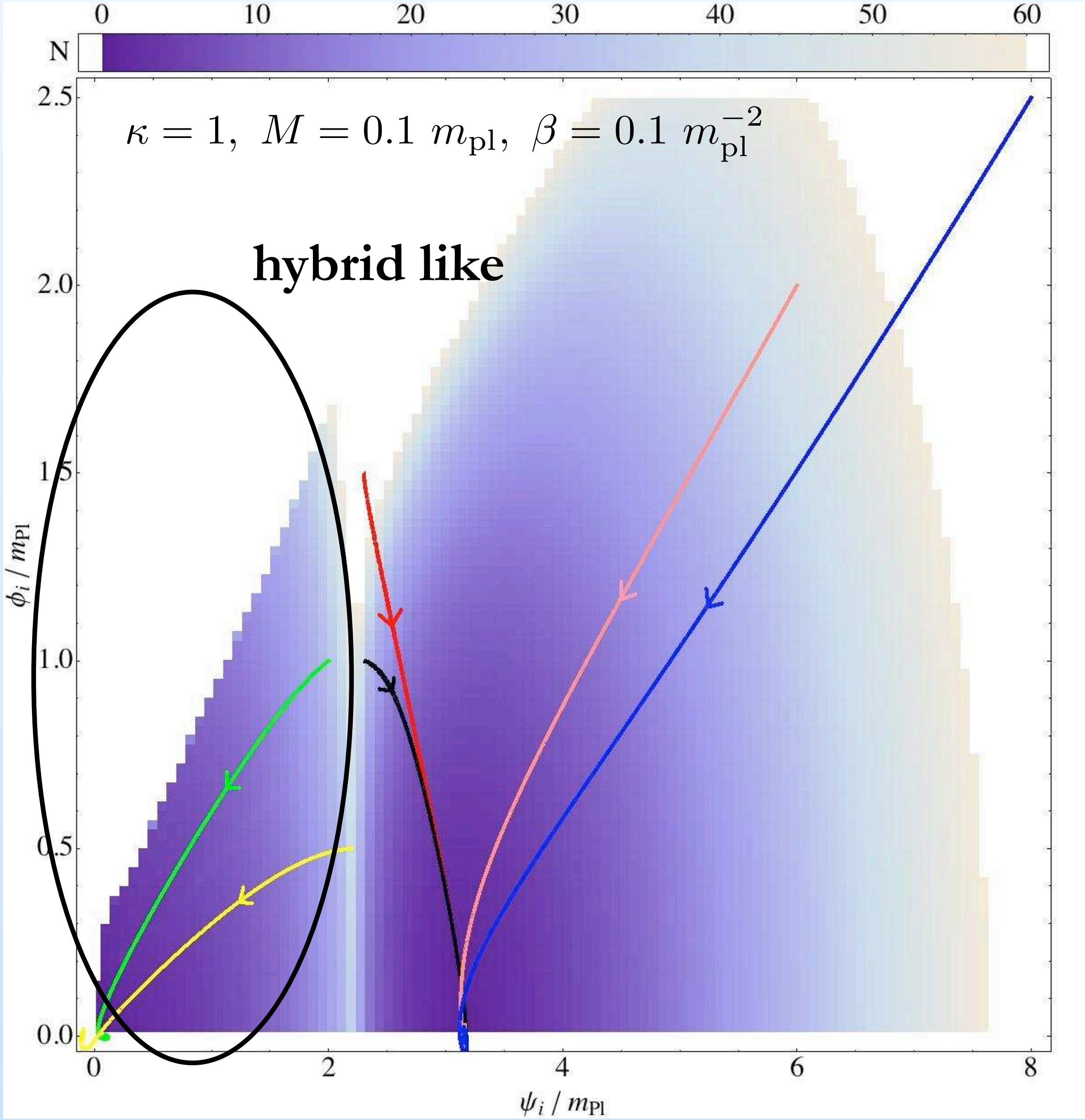

Unsuccessful region

around the parallel valley

without anamorphosis points

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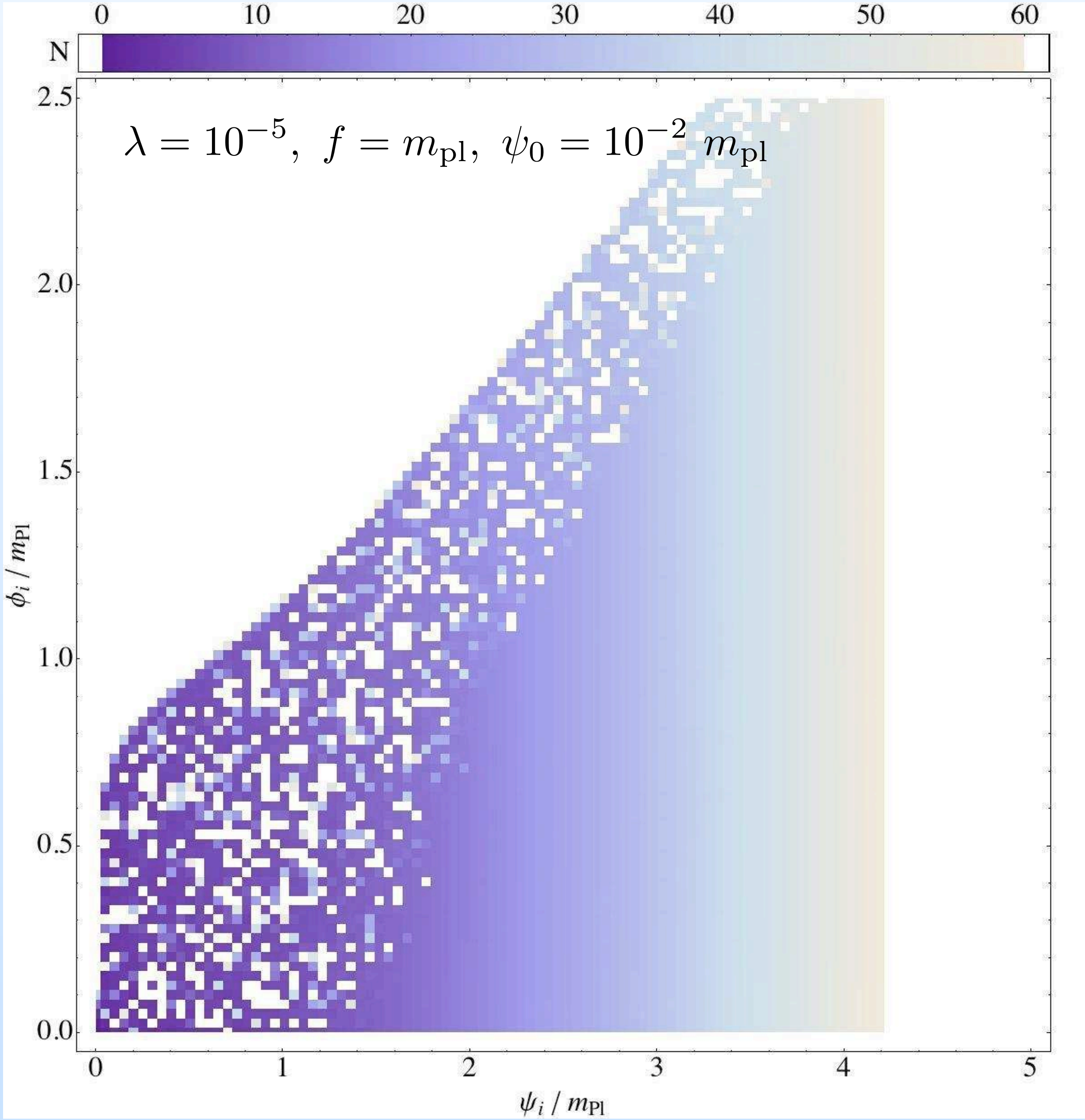
• Radion assisted gauge inflation:

(M. Fairbairn, L.Lopez-Honorez, M.Tytgat, hep-ph/0302160)

Effective 2-field potential: $$V(\phi,\psi) = \frac{1}{4}\frac{\phi^2}{f^2}\psi^4 + \frac{\lambda}{4}\left(\psi^2 - \psi_0^2\right)^2$$

Super-planckian values allowed



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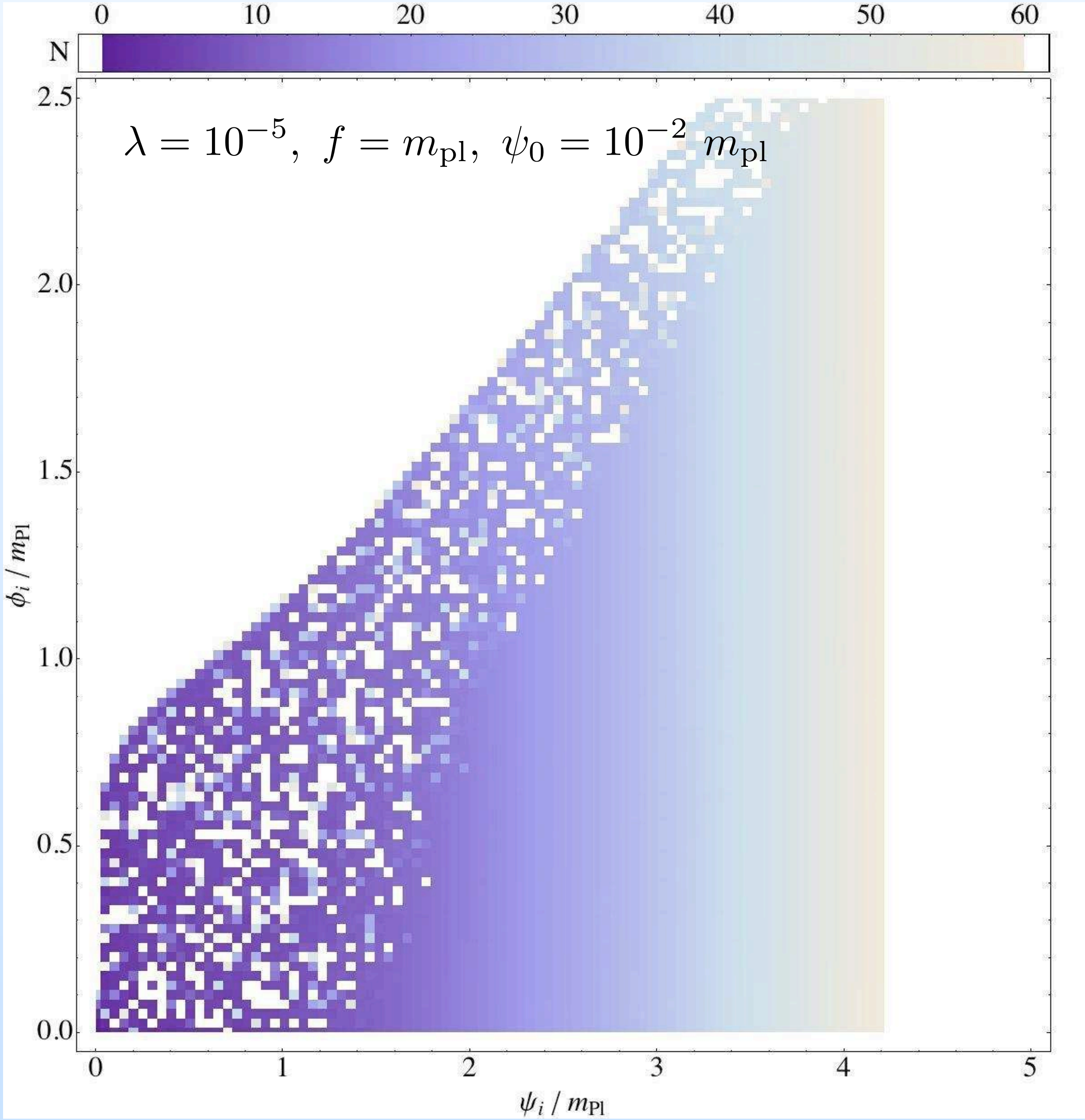

For $\phi, \psi < 0.2\ m_{\rm pl}$

Up to 25% of area are anamorphosis points

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• Radion assisted gauge inflation:

(M. Fairbairn, L.Lopez-Honorez, M.Tytgat, hep-ph/0302160)

Gauge-type inflation :

- **φ phase of a Wilson loop wrapped around a compact 5th dim.**
- **Super-planckian values allowed**
- **Varying radius *R* of the extra-dimension** $\psi \equiv (2\pi R)^{-1}$

Effective 2-field potential: $$V(\phi,\psi) = \frac{1}{4}\frac{\phi^2}{f^2}\psi^4 + \frac{\lambda}{4}\left(\psi^2 - \psi_0^2\right)^2$$

2 valleys and a flat $\psi = 0$ direction

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6. *Conclusion and Perspectives*

Questions…

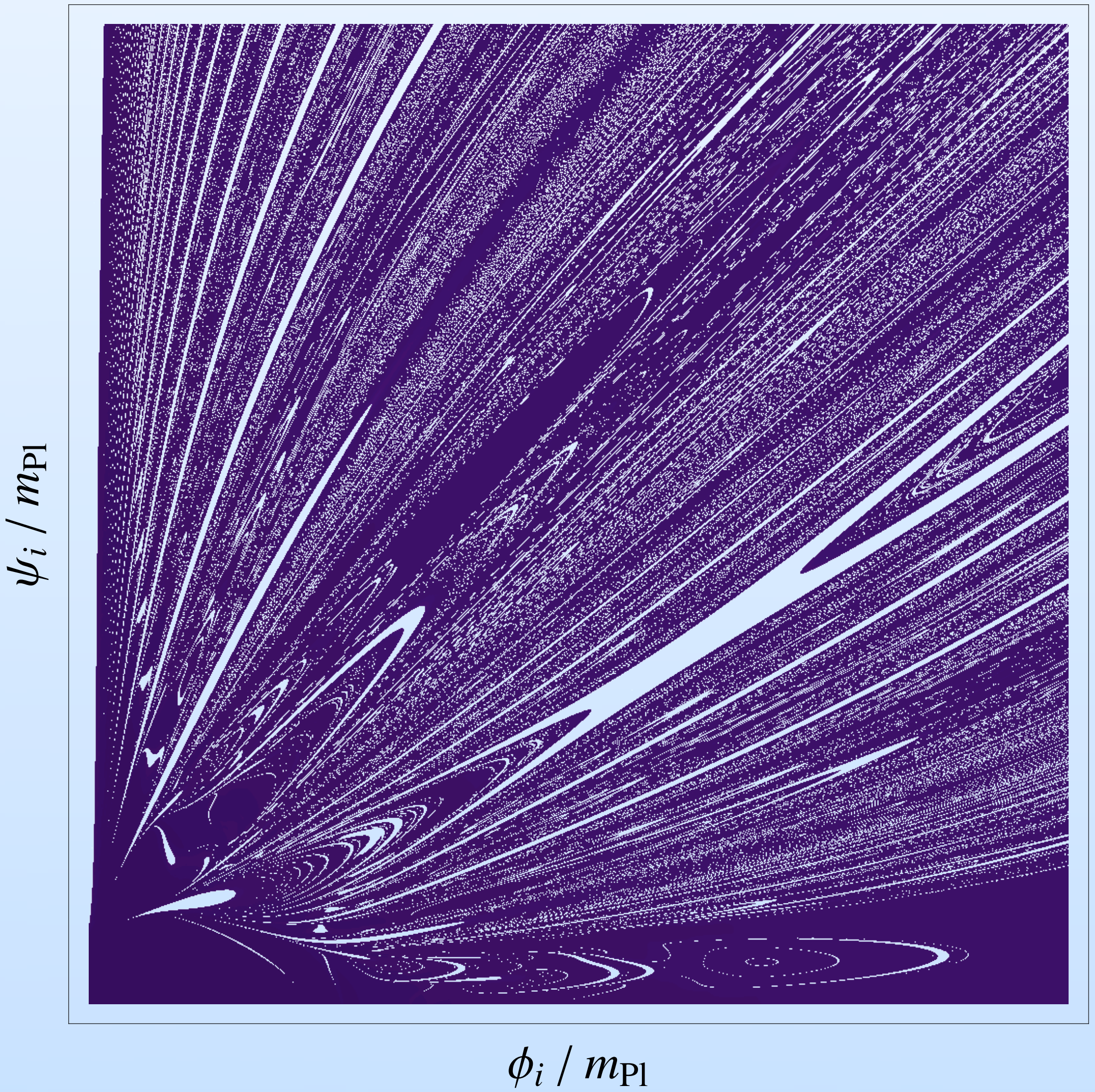
5. Fractal behaviour ?

• Fractal properties of anamorphosis points:

✦ Structure with fractal boundaries ?

✦ Fractal Surface ?

✦ Convergence of the area covered by successful points?



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✦ Structure with fractal boundaries ?	1.2
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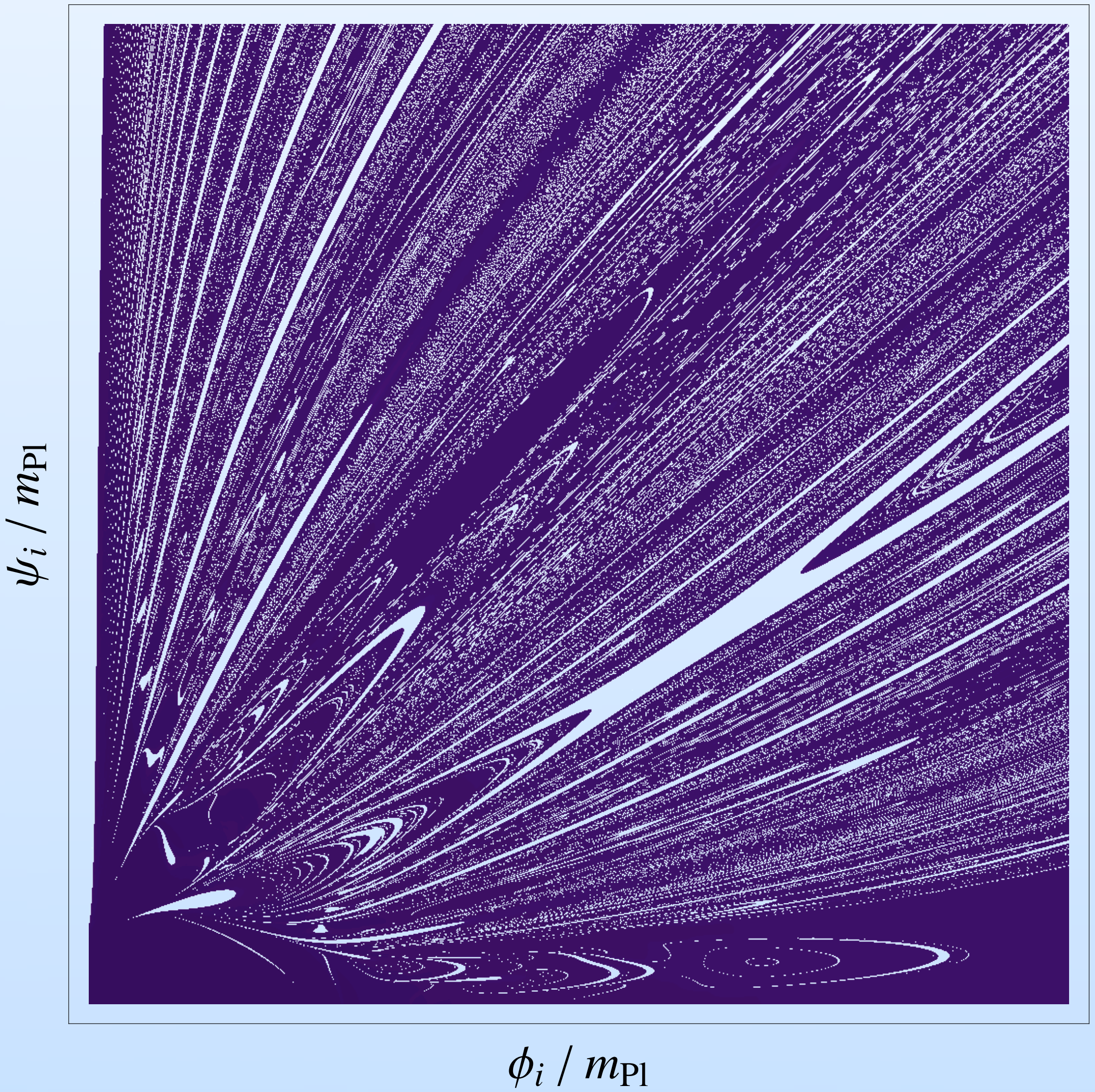

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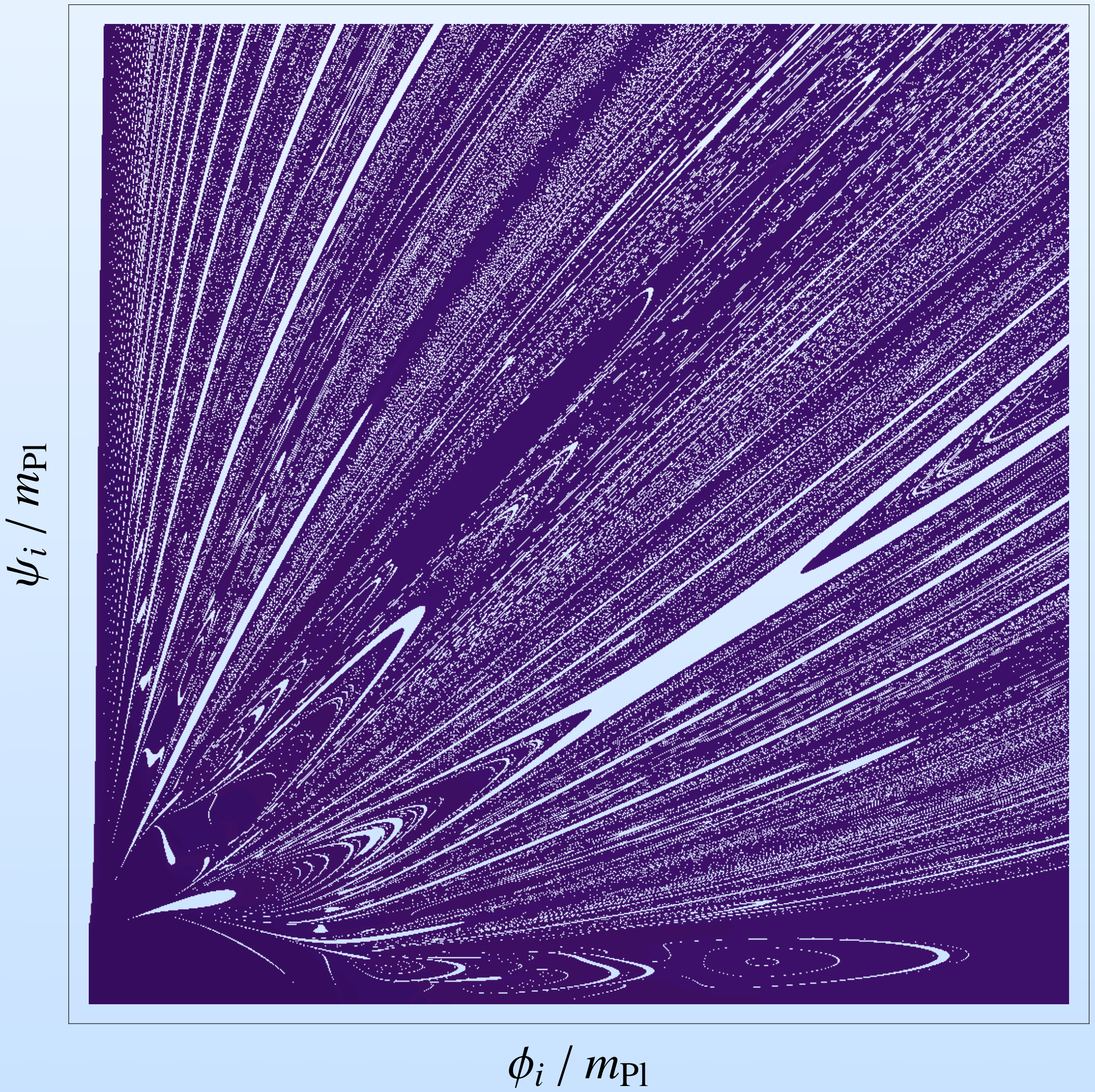

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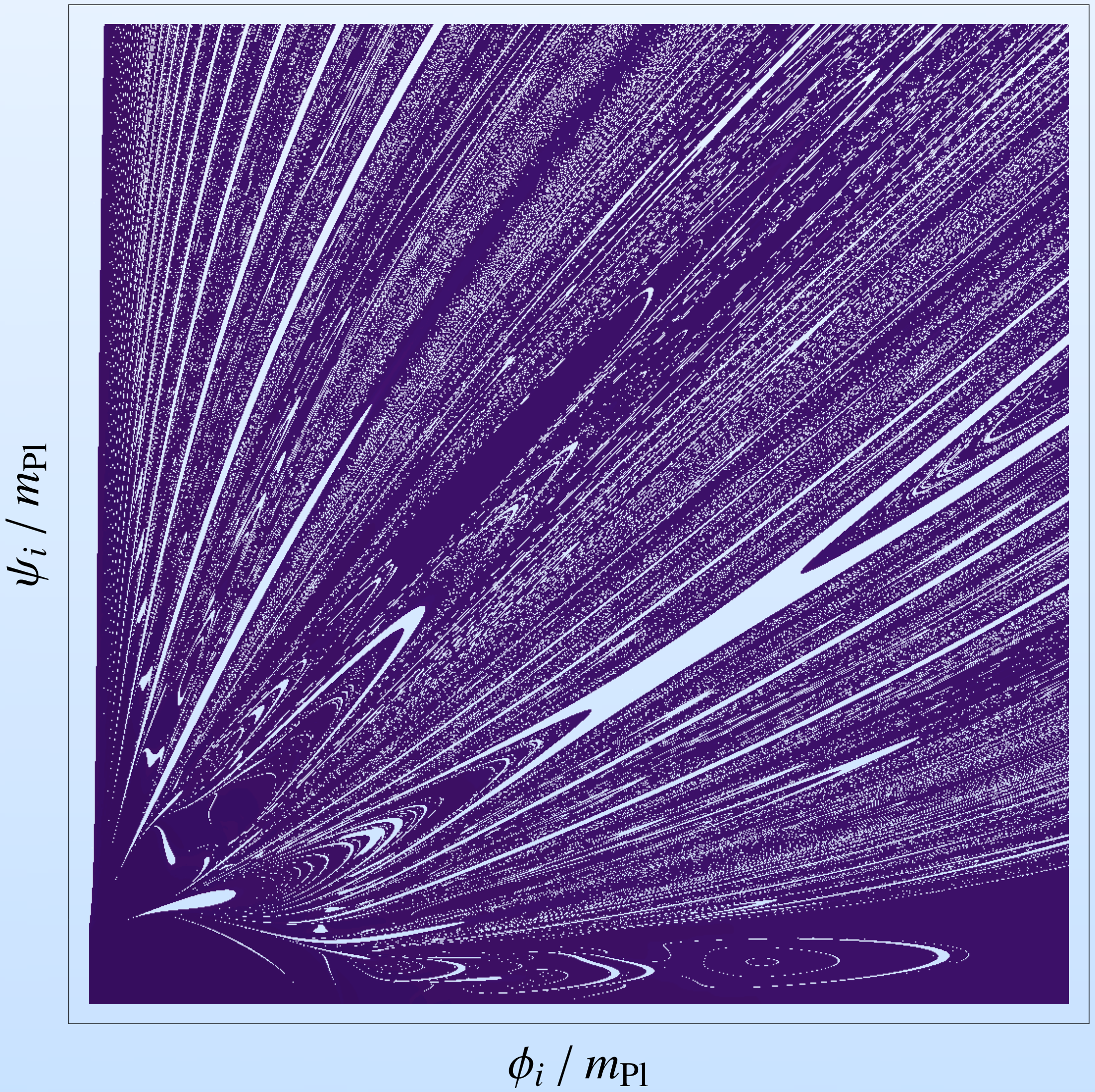
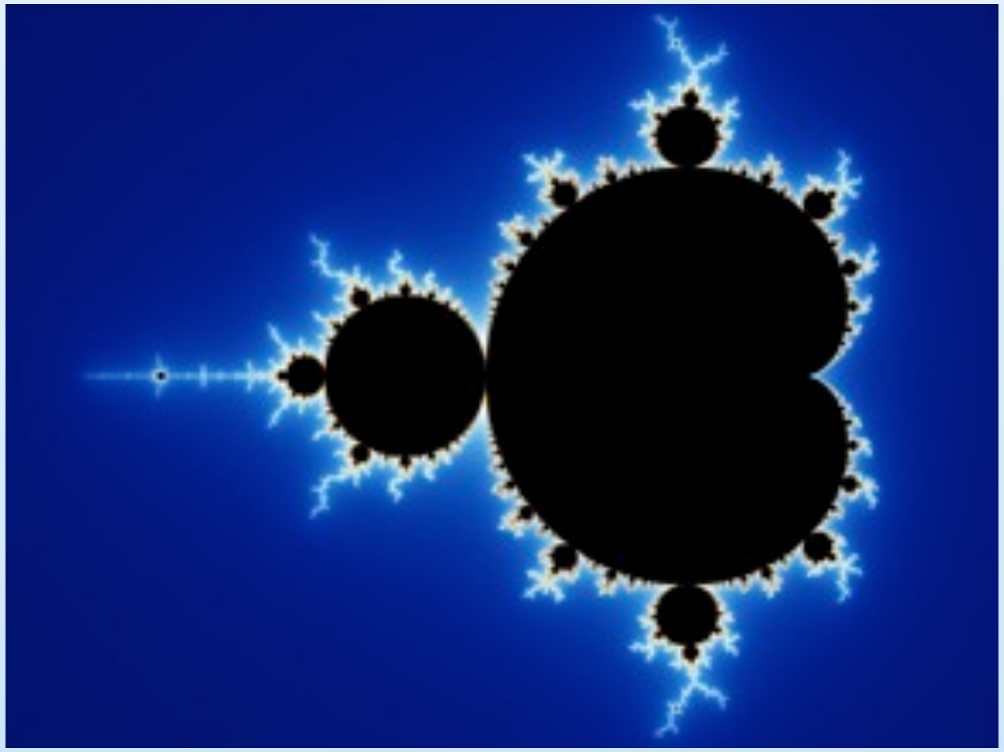


...like Mandelbrot Set



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✦**Box-Counting dimension** $D \equiv \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}$

✦ **Can be numerically evaluated:**

- **N random initial conditions** $x_n = (\phi_n, \psi_n)$
- **3 trajectories for each point:** $x_n - \epsilon,\ x_n,\ x_n + \epsilon$
- $f(\epsilon)$ **, fraction of points ending in a different attractor/all successful**

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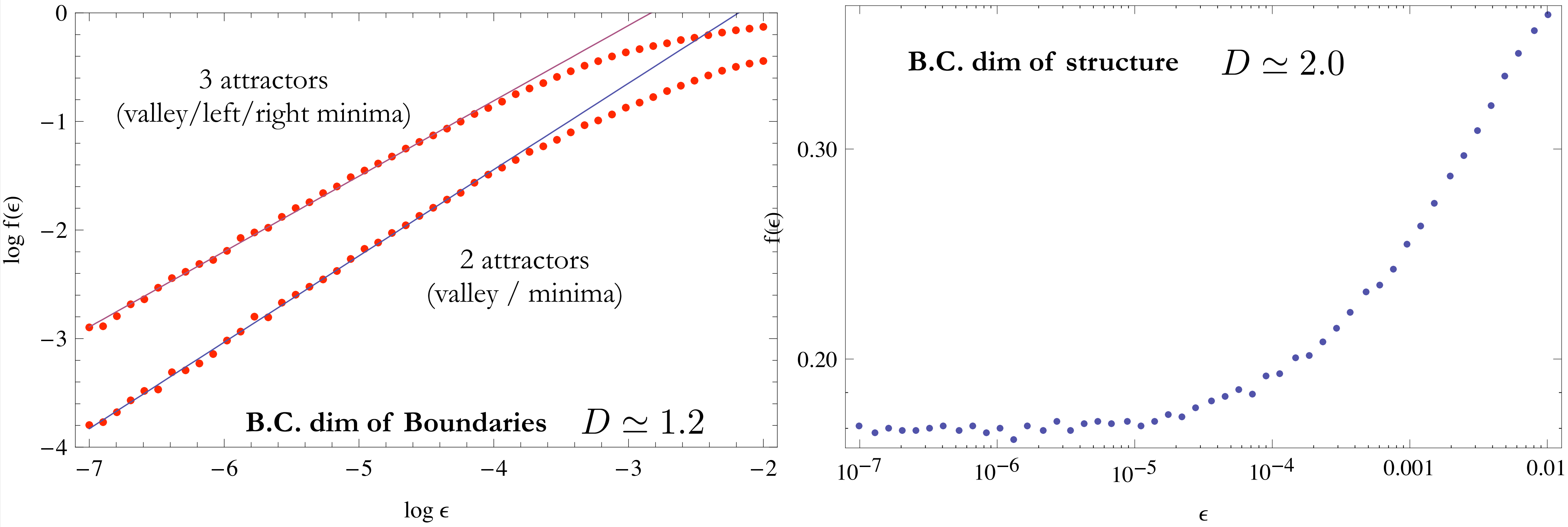
• Fractal properties of anamorphosis points:

- ✦ **Structure with fractal boundaries ?**
- ✦ **Fractal Structure ?**
- ✦ **Convergence of the area?**



3x100000 points, hybrid potential with $\lambda = \lambda' = 1, \ M = 0.03 \ m_{\rm p}, m = 10^{-6} m_{\rm p}$

7. MCMC exploration

- **The MCMC method:** Exploration of a 7D space

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• The MCMC method: Exploration of a 7D space

✦ Each element of the chain depends only of the previous one

✦ Chosen from a gaussian random distribution around the previous one, under an acceptance condition

✦ Simple acceptance condition: Trajectory leading to N>60 e-folds

✦ Flat prior for the initial fields - initial velocities - log of parameters

✦ Range $-0.2m_{\mathrm{p}} < \phi, \psi < 0.2m_{\mathrm{p}}$ and $\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 + \left(\frac{\mathrm{d}\psi}{\mathrm{N}}\right)^2 < \frac{9m_{\mathrm{p}}^2}{8\pi} \equiv \frac{6}{\kappa^2}$

$M < 0.2m_{\mathrm{p}}$

✦ Final density of points proportional to the target distribution

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$M < 0.2m_{\mathrm{p}}$

✦ Final density of points proportional to the target distribution

• The 2-fields potential:

$$V(\phi,\psi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2}\right)^2 + \frac{\phi^2}{\mu^2} + \frac{\phi^2\psi^2}{\nu^4} \right]$$

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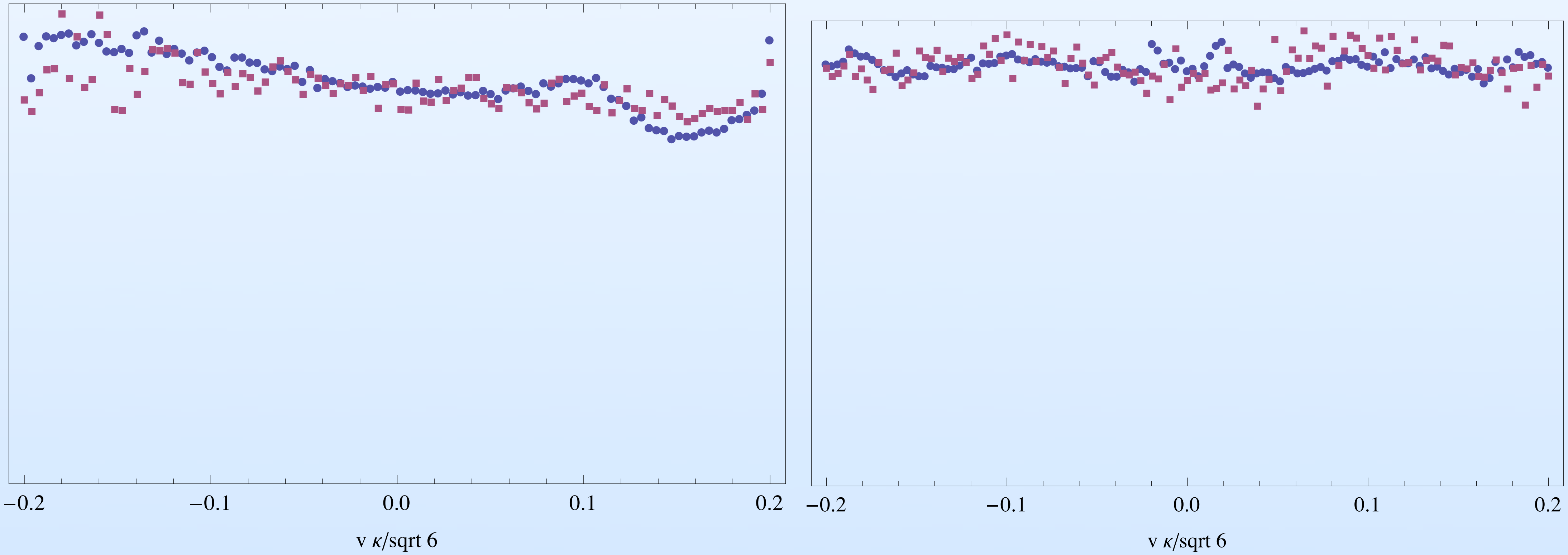
• Probability distribution of initial velocities:

Flat Prior + Bound:



Flat probability distribution for $\frac{\mathrm{d}\phi}{\mathrm{d}N}$ and $\frac{\mathrm{d}\psi}{\mathrm{d}N}$

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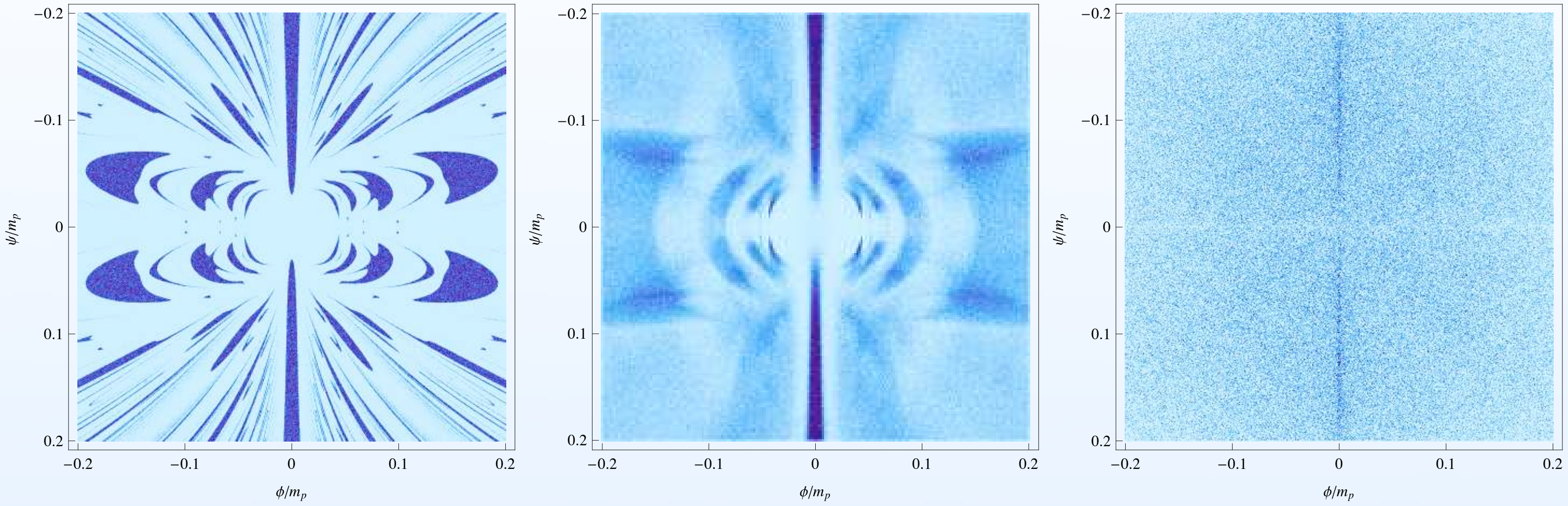

FIG. 6: 3D probability density for the fields, for MCMC on the fields only (left), including initial velocities (center), including both initial velocities and parameters (right).

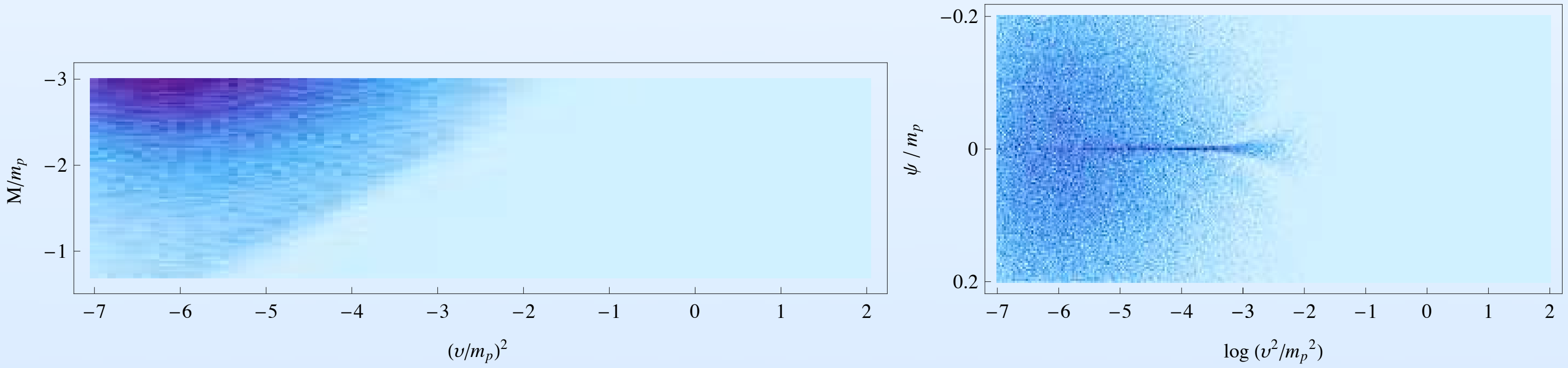

FIG. 7: Distribution of points in the plane of $\log \nu^2 - \log M$ (left) and in the plane $\log \nu^2/\psi$ (right)

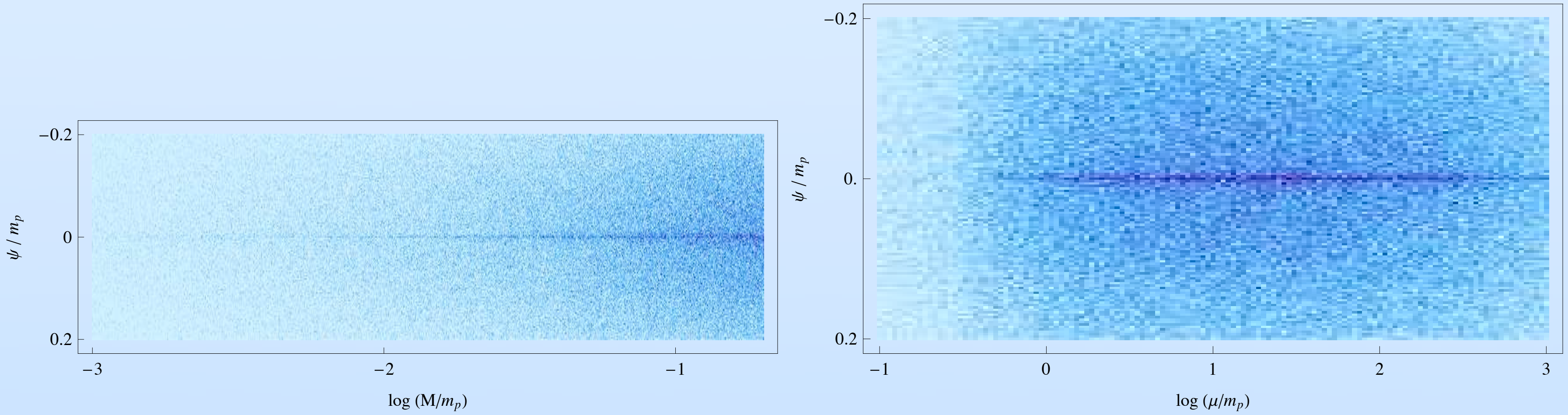

FIG. 8: Distribution of points in the plane of $\log M/\psi$ (left), and in the plane $\log \mu/\psi$

WMAP5

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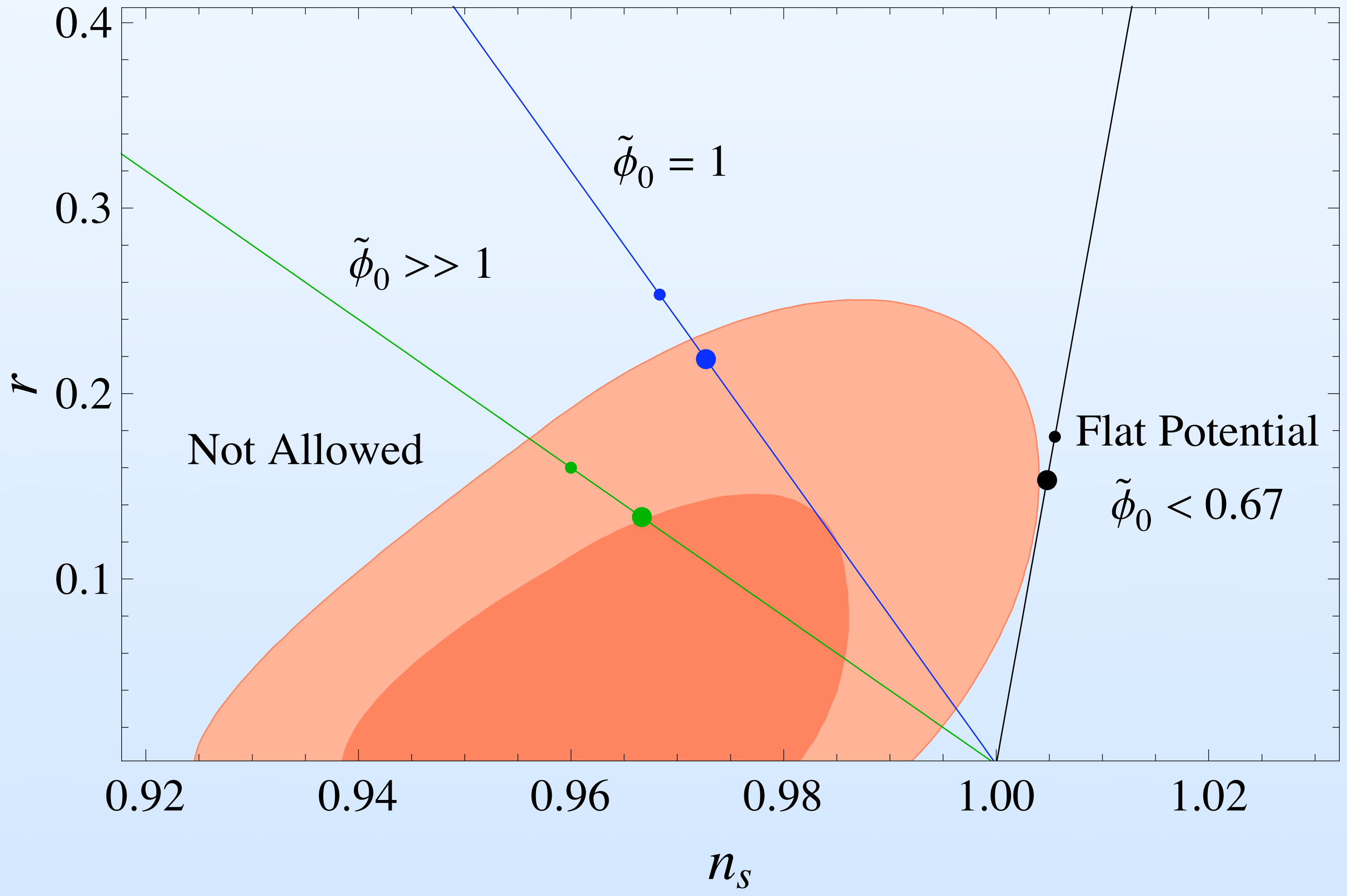

F-term SUGRA model

- Simple and realistic model in (local) SUSY framework

- SUGRA corrections dominate radiative correction

- Only one potential parameter M

- Slightly BLUE spectrum predicted:
BUT: this is the favoured case
if cosmic strings formation is taken account
BUT: it can be RED due to domination of radiative corrections
before reaching instability point in the valley

- 2 field potential:

$$V_{\rm tree}^{\rm sugra}(s,\psi) = \kappa^2 \exp\left(\frac{s^2+\psi^2}{2M_{\rm pl}^2}\right) \times \left\{ \left(\frac{\psi^2}{4} - M^2\right)^2 \left(1 - \frac{s^2}{2M_{\rm pl}^2} + \frac{s^4}{4M_{\rm pl}^4}\right) + \frac{s^2\psi^2}{4}\left[1 + \frac{1}{M_{\rm pl}^2}\left(\frac{1}{4}\psi^2 - M^2\right)\right]^2 \right\}.$$

- F-term + C.W. corrections:

$$V_{\rm 1-loop}^{\rm cw}(s) = \frac{\kappa^4 M^4 \mathcal{N}}{32\pi^2}\left[2\ln\frac{s^2\kappa^2}{\Lambda^2} + (z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1})\right],$$

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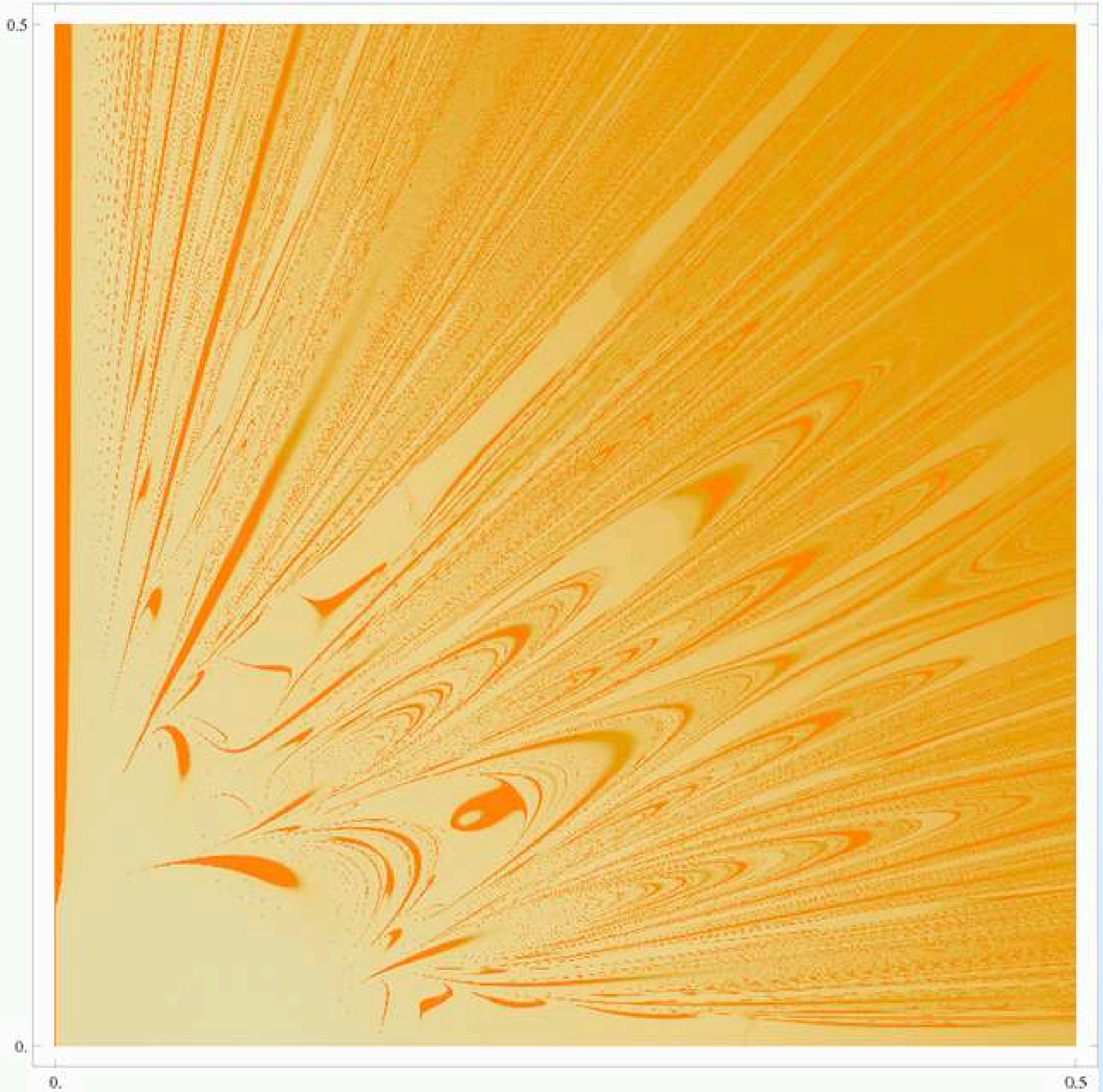

FIG. 12: Set of initial field values $(\psi_\mathrm{i}/M_\mathrm{pl}, s_\mathrm{i}/M_\mathrm{pl})$ for the SUGRA F-term model leading to more than 60 e-folds of inflation (dark red). The initial field velocities are assumed to vanish and the potential parameter is fixed at $M = 10^{-2} m_\mathrm{pl}$. As for the original hybrid model, we recover a set of dimension two with a fractal boundary.