Black Holes in String Theory

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Plan

1. A brief introduction to black holes
2. A briefer introduction to string theory
3. Black holes in string theory

Black Holes

Black holes are classical solutions of the equations of motion of general theory of relativity, possibly coupled to other fields, *e.g.* electromagnetic fields.

Their gravitational attraction is so large that even light cannot escape a black hole.

Thus they behave as perfect black bodies at zero temperature.

In quantum theory this picture of the black hole gets modified.

Hawking

A black hole is not completely black, but gives out black body radiation at a definite temperature.

$$\mathbf{T} = \frac{\hbar}{2\pi \mathbf{k_B}\, \mathbf{c}}\,\kappa$$

$\mathbf{k_B}$: Boltzmann's constant $\hbar$: Planck's constant

c: velocity of light

κ: acceleration due to gravity at the horizon of the black hole.

One also finds that, in its interaction with other objects a black hole behaves as a thermal object with definite temperature, entropy etc.

Hawking, Bekenstein 70's

In particular its entropy is given by the simple formula:

$$S_{BH} = \frac{k_B\, c^3}{4 G_N \hbar} A$$

A: Area of the event horizon

G_N: Newton's gravitational constant

For ordinary objects the entropy of a system has a microscopic interpretation.

We fix the macroscopic parameters (e.g. total electric charge, energy etc.) and count the number of quantum states – known as microstates – each of which has the same values for the macroscopic parameters.

d_{micro}: number of such microstates

$$S_{micro} = \ln d_{micro}$$

Question: Does the entropy of a black hole have a similar statistical interpretation?

Does the entropy of a black hole have a similar statistical interpretation?

Answering this question in the affirmative is essential for any consistent theory of quantum gravity.

Otherwise we can throw in a pure quantum state to form a black hole, and after the black hole evaporates completely, it comes out as a thermal (mixed) state.

– violation of the laws of quantum mechanics.

Hawking

In order to investigate the statistical origin of black hole entropy we need a quantum theory of gravity.

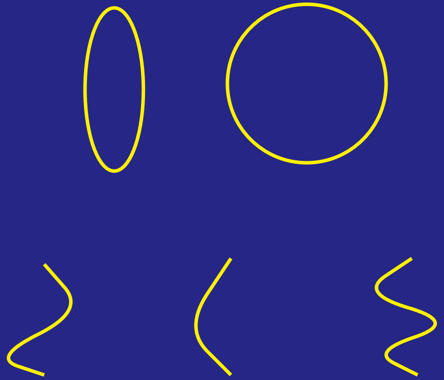
We shall carry out our investigation in string theory – the theory that attempts to give a unified description of gravity and all other interactions and also of all matter.

In all subsequent discussion we shall choose units in which

$$\hbar = 1, \quad c = 1, \quad k_B = 1$$

A brief introduction to the basic ideas of string theory

Fundamental constituents of matter are different vibrational states of a string.



Some features of string theory

1. Typical size of a string $\sim 10^{-33}$ cm.

This is much smaller than the length scale that can be probed by any present day experiment

2. String theory automatically contains a quantum theory of gravity coupled to other fields.

3. String theory is consistent only in (9+1) dimensional space-time.

6 of the extra dimensions must curl up to become small compact dimensions.

4. Even though the theory is unique, it can exist in many different stable and metastable phases.

– related to different ways of curling up the extra dimensions.

The various physical parameters, like the particle masses, their interaction etc. vary from one phase to another just as the physical properties of H_2O differ in ice, water and steam.

As a result, without knowing precisely which phase of string theory describes the part of the universe we live in, we cannot directly compare string theory to experiments.

Given this situation it would seem that until we can find the right phase of string theory which describes our world, there cannot be any further progress.

However this is not quite correct.

There are some issues which are universal, and appear in all phases of string theory.

For example, the issues involving black holes exist in all phases of string theory in which there is gravity, irrespective of whether that particular phase describes the world we live in.

Thus we can address the issues involving black hole thermodynamics in string theory without having to identify which phase of the theory we live in.

Strategy: Choose a convenient phase in which the dynamics of string theory is best understood.

Try to develop a statistical understanding of the various thermodynamic results on black holes in that phase.

Convenient phases: supersymmetric phases

– have a symmetry that relates bosonic states to fermionic states.

One advantage of such a choice of phase is that such phases are stable phases unlike most of the non-supersymmetric phases which are metastable.

Many aspects of black hole thermodynamics have been studied in such supersymmetric phases of the theory, but we shall focus our attention on one particular aspect.

– entropy of the black hole in the zero temperature limit (supersymmetric, extremal black holes).

Advantage: Such a black hole is a stable state of the theory.

Strategy:

1. Identify a supersymmetric black hole carrying a certain set of electric charges $\{Q_i\}$ and magnetic charges $\{P_i\}$ and calculate its entropy $S_{BH}(Q, P)$ using the Bekenstein-Hawking formula.

Note: since we are considering a generic phase of string theory, it may have more that one Maxwell field and hence multiple charges.

2. Identify some supersymmetric microscopic configuration in string theory carrying the same set of charges.

These will include the fundamental strings but also other objects in string theory which are required for consistency of string theory.

Calculate the number $d_{micro}(Q, P)$ of these states.

3. Compare $S_{micro} \equiv \ln d_{micro}(Q, P)$ with $S_{BH}(Q, P)$.

Initial attempts focussed on states which can be regarded as being made only of the fundamental string states.

t'Hooft; Susskind

This yielded partial success but not full success.

One could match S_{micro} and S_{BH} up to an undetermined normalization constant.

A.S.

Full success came by including more complicated configurations in string theory.

Strominger, Vafa

For a class of supersymmetric extremal black holes in string theory one indeed finds a match:

$$A/4G_N = \ln d_{micro}$$

d_{micro}: degeneracy of microstates

This formula is quite remarkable since it relates a geometric quantity in black hole space-time to a counting problem that does not make any direct reference to black holes.

The Bekenstein-Hawking formula is an approximate formula that holds in classical general theory of relativity.

While string theory gives a theory of gravity that reduces to Einstein’s theory when gravity is weak, there are corrections.

Thus the Bekenstein-Hawking formula for the entropy works well only when gravity at the horizon is weak.

Typically this requires the charges to be large.

The calculation on the microscopic side also simplifies when the charges are large.

Instead of doing exact counting of quantum states, we can use approximate methods which give the result for large charges.

For ordinary systems, thermodynamics provides an approximate description that becomes exact in the limit of large volume.

Is the situation with black holes similar, i.e. they only capture the information about the system in the limit of large size?

Or, could it be that the relation $A/4G_N = \ln d_{micro}$ is an approximation to an exact result?

Our goal will be to search for an exact formula to which the above is an approximation

In order to address this issue we have to work on two fronts.

1. Count the number of microstates to greater accuracy.

2. Calculate black hole entropy to greater accuracy.

We can then compare the two to see if they agree beyond the large charge limit.

In the rest of this talk I shall describe the progress on both fronts.

Progress in microscopic counting

In a class of phases of string theory, known as N=4 and N=8 supersymmetric string theories in four dimensions, one now has a complete understanding of the microscopic degeneracies of supersymmetric black holes.

Typically such theories have multiple Maxwell fields.

$\Rightarrow$ the black hole is characterized by multiple electric and magnetic charges, collectively denoted by (Q, P).

The degeneracy is expressed as a function $d_{micro}(Q, P)$ of the charges.

Dijkgraaf, Verlinde, Verlinde; Shih, Strominger, Yin;
David, Jatkar, A.S.; Dabholkar, Gaiotto, Nampuri; $\cdots$

In these theories $d_{micro}(Q, P)$ is expressed as Fourier expansion coefficients of some well-known functions, *e.g.* Jacobi theta functions, Igusa cusp forms etc.

$\Rightarrow$ 'experimental data' to be explained by a 'theory of black holes'.

In the large charge limit these degeneracies agree with the exponential of the Bekenstein-Hawking entropy of black holes carrying the same set of charges

Example: Degeneracies of a class of supersymmetric states in an $N = 4$ supersymmetric string theory as a function of two functions of charges D_1 and D_2

D_1	D_2	degeneracy d_{micro}	$\ln d_{micro}$	$S_{BH} = \pi\sqrt{D_1 D_2}$
2	2	50064	10.82	6.28
4	4	32861184	17.31	12.57
6	4	632078672	20.26	15.39
8	4	9337042944	22.96	17.77
10	4	113477152800	25.45	19.87

In a systematic comparison we do not compare numbers, but compare the asymptotic expansions for large charges.

On the microscopic side we have a completely systematic algorithm for finding this asymptotic expansion for this special class of theories.

In the previous example we have, for large D_1, D_2:

$$\ln d_{micro} = \pi\sqrt{D_1 D_2} - 12\log(2D_1/D_2) - 48\log\eta(i\sqrt{D_1/D_2}) + \cdots$$

η: Dedekind function

Similar formulæ are now known in many other theories.

In order to explain the difference between $\ln d_{micro}$ and the Bekenstein-Hawking entropy we need to understand corrections to the Bekenstein-Hawking formula.

This is the problem we shall now address.

In string theory there are two types of corrections to the Bekenstein-Hawking formula.

1. Stringy corrections to the classical equations of motion of general relativity.

– These originate from the fact that strings are extended objects and not point particles.

2. Quantum corrections.

We would like to look for an exact formula for the black hole entropy taking into account both types of corrections.

These are necessary if we want to compute the black hole entropy away from the large charge limit.

Stringy corrections to the Bekenstein-Hawking formula can be computed using a generalization of this formula due to Wald.

What about quantum corrections?

We now have a concrete proposal for systematically computing quantum corrections to the entropy of a zero temperature black hole. A.S.

This proposal is routed in AdS_2/CFT_1 correspondence, and gives us back Wald's formula in the classical limit.

This in principle allows us to calculate systematically all the corrections to the black hole entropy.

What is AdS_2?

Take a three dimensional space labelled by coordinates (x, y, z) and metric

$$ds^2 = dx^2 - dy^2 - dz^2$$

AdS_2 may be regarded as a two dimensional Lorentzian space embedded in this 3-dimensional space via the relation:

$$x^2 - y^2 - z^2 = -a^2$$

a: some constant giving the radius of AdS_2.

This space has an SO(2,1) isometry.

$$x^2 - y^2 - z^2 = -a^2$$

Introduce independent coordinates (η, t):

$$x = a \sinh\eta \cosh t, \quad y = a \cosh\eta, \quad z = a \sinh\eta \sinh t$$

$$dx^2 - dy^2 - dz^2 = a^2(d\eta^2 - \sinh^2\eta \, dt^2)$$

Define: $r = \cosh\eta$

$$ds^2 = a^2\left[\frac{dr^2}{r^2 - 1} - (r^2 - 1)dt^2\right], \quad r \geq 1$$

Why AdS_2?

All known black holes develop an AdS_2 factor in their near horizon geometry in the extremal limit.

– time translation symmetry gets enhanced to $SO(2,1)$ in the near horizon limit.

Reissner-Nordstrom solution in $D = 4$:

$$\begin{aligned} ds^2 &= -(1-\rho_+/\rho)(1-\rho_-/\rho)d\tau^2 \\ &\quad + \frac{d\rho^2}{(1-\rho_+/\rho)(1-\rho_-/\rho)} \\ &\quad + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned}$$

Define

$$2\lambda = \rho_+ - \rho_-, \quad t = \frac{\lambda\,\tau}{\rho_+^2}, \quad r = \frac{2\rho - \rho_+ - \rho_-}{2\lambda}$$

and take $\lambda \to 0$ limit keeping r, t fixed.

$$ds^2 = \rho_+^2\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right] + \rho_+^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$AdS_2 \quad\quad \times \quad\quad S^2$

Postulate: Any extremal black hole has an AdS_2 factor / $SO(2,1)$ isometry in the near horizon geometry.

– partially proved

Kunduri, Lucietti, Reall; Figueras, Kunduri, Lucietti, Rangamani

The full near horizon geometry takes the form $AdS_2 \times K$

K: some compact space.

Proposal:

The exact degeneracy of an extremal black hole is given by the path integral of string theory over the near horizon $AdS_2 \times K$ geometry of the black hole.

Consistency checks:

1. In the classical limit this reduces to the exponential of the Wald entropy.

2. This proposal follows naturally from the AdS_2/CFT_1 correspondence.

Many features of the microscopic formula has been reproduced by this proposed formula for the quantum black hole entropy.

More detailed tests are underway.

Summary

String theory has come a long way to explaining black hole thermodynamics and thereby resolving an outstanding problem in quantum gravity.

I have described only one aspect of the study of black holes in string theory.

We hope that eventually these studies will help us understand better not only black holes but also string theory.