Gauge invariant averages for the cosmological backreaction: examples and perspectives

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Introduction

- The study of the possible dynamical influence of (small) inhomogeneities on the evolution of a cosmological background has recently attracted considerable interest, from both a theoretical and a phenomenological point of view
- One needs a well defined averaging procedure for smoothing-out the perturbed (non-homogeneous) geometric parameters.
- The computation of these averages is affected in principle by a well-known ambiguity due to the possible choice of different “gauges”.

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Outline

- Gauge freedom in cosmology
- Gauge trasformation vs Gauge Invariant variables.
- Gauge (non)-invariance of space-time integrals
- Covariant averaging prescription
- Gauge (non)-invariant averaging and the present cosmological evolution
- Gauge invariant averaging for the quantum backreaction
- Growing-Curvature cosmology: a key example
- Conclusions

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Gauge freedom in a FRW universe, 1

Let us consider a cosmological background sourced by a scalar field ϕ and described by the simple four-dimensional action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

with spatially flat FLRW background geometry

$$ds^2 = -dt^2 + a(t)^2 \, \delta_{ij} \, dx^i dx^j$$

Gauge freedom in a FRW universe, 2

The background fields $\{\phi, g_{\mu\nu}\}$ can be expanded, up to second order, in the non-homogeneous perturbations as follows:

$$\phi(t,\vec{x}) = \phi^{(0)}(t) + \delta\phi^{(1)}(t,\vec{x}) + \delta\phi^{(2)}(t,\vec{x}),$$

$$g_{00} = -1 - 2\alpha^{(1)} - 2\alpha^{(2)}, \qquad g_{i0} = -\frac{a}{2}\left(\beta^{(1)}_{,i} + B^{(1)}_i\right) - \frac{a}{2}\left(\beta^{(2)}_{,i} + B^{(2)}_i\right),$$

$$g_{ij} = a^2\left[\delta_{ij}\left(1 - 2\psi^{(1)} - 2\psi^{(2)}\right) + D_{ij}(E^{(1)} + E^{(2)}) + \frac{1}{2}\left(\chi^{(1)}_{i,j} + \chi^{(1)}_{j,i} + h^{(1)}_{ij}\right) + \frac{1}{2}\left(\chi^{(2)}_{i,j} + \chi^{(2)}_{j,i} + h^{(2)}_{ij}\right)\right],$$

where $D_{ij} = \partial_i\partial_j - \delta_{ij}(\nabla^2/3)$.
One obtains 11 degrees of freedom which are in part redundant.
To obtain a set of equations (Einstein equations + equation of motion of ϕ) well defined, order by order, we have to set to zero two scalar perturbations among $\delta\phi$, α, β, ψ and E, and one vector perturbation between B_i and χ_i.

Gauge freedom in a FRW universe, 3

The choice of such variables is called a choice of gauge.

For the scalar sector (first or second order) we can have:

$\psi = 0$, $E = 0$ Uniform Curvature Gauge
$\beta = 0$, $E = 0$ Longitudinal Gauge
$\alpha = 0$, $\beta = 0$ Synchronous Gauge
$\delta\phi = 0$, β or ψ or $E = 0$ Uniform Field Gauge
etc.

We do not consider the vector sector, vector perturbations can be neglected for our purpose. In particular, as we will see, all first order vector inhomogeneities are vanishing (Mena, Mulryne and Tavakol (2007)).

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Gauge trasformation vs Gauge Invariant variables, 1

How to connect differents gauge? Gauge trasformations!

General coordinate transformations (GCT) $\Longleftrightarrow$ Gauge transformations (GT).

Consider, for example, a (typically non-homogeneous) scalar field $S(x)$. Under a GCT:

$$x \rightarrow \tilde{x} = f(x), \quad x = f^{-1}(\tilde{x}), \quad S(x) \rightarrow \tilde{S}(\tilde{x}) = S(x)$$

Under the associated GT old and new fields are evaluated at the same space-time point x and

$$S(x) \rightarrow \tilde{S}(x) = S(f^{-1}(x)).$$

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Gauge trasformation vs Gauge Invariant variables, 2

To connect different gauge we need an infinitesimal gauge transformation.
This can be parametrized by a first-order, $\epsilon^{\mu}_{(1)}$, and a second-order, $\epsilon^{\mu}_{(2)}$, vector generator, and is given by (Bruni, Matarrese, Mollerach, Sonego (1997)):

$$x^{\mu} \to \tilde{x}^{\mu} = x^{\mu} + \epsilon^{\mu}_{(1)} + \frac{1}{2}\left(\epsilon^{\mu}_{(1),\nu}\epsilon^{\nu}_{(1)} + \epsilon^{\mu}_{(2)}\right).$$

and a tensor T changes as

$$T^{(1)} \to \tilde{T}^{(1)} = T^{(1)} - \mathcal{L}_{\epsilon_{(1)}} T^{(0)},$$

$$T^{(2)} \to \tilde{T}^{(2)} = T^{(2)} - \mathcal{L}_{\epsilon_{(1)}} T^{(1)} + \frac{1}{2}\left(\mathcal{L}^{2}_{\epsilon_{(1)}} T^{(0)} - \mathcal{L}_{\epsilon_{(2)}} T^{(0)}\right)$$

Gauge trasformation vs Gauge Invariant variables, 3

Request: Physics results should not depend on the gauge chosen to describe these.

Answer: Gauge Invariant (GI) formalism (Bardeen (1980), for a review see: Mukhanov, Feldman, Brandenberger(1992)).

Physically meaningful variable $\leftrightarrow$ GI variable.

A GI variable F is defined as a function of our perturbations which takes always the same value independently of the gauge chosen

$$F(\delta\phi, \alpha, \beta, ...) \rightarrow F(\delta\tilde{\phi}, \tilde{\alpha}, \tilde{\beta}, ...) = F(\delta\phi, \alpha, \beta, ...)$$

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Power Spectrum

The scalar power spectrum associated with a model of inflation is defined using the GI curvature perturbation ξ. Such perturbation is given, to first order, by

$$\xi^{(1)} = \frac{H}{\dot{\phi}} Q^{(1)} \quad \textit{with} \quad Q^{(1)} = \delta\phi^{(1)} + \frac{\dot{\phi}}{H}\left(\psi^{(1)} + \frac{1}{6}\nabla^2 E^{(1)}\right)$$

where $Q^{(1)}$ is the first order GI Mukhanov variable (Mukhanov (1988)). So one obtains

$$P_\zeta(k) = \frac{k^3}{2\pi^2}\left(\frac{H}{\dot{\phi}}\right)^2 |Q_k|^2$$

Gauge (non)-invariance of space-time integrals, 1

Consider now the space-time integral of a scalar S over a four-dimensional region Ω defined in terms of a window function W_Ω:

$$F(S,\Omega) = \int_{\Omega(x)} d^4x \sqrt{-g(x)}\, S(x) \equiv \int_{\mathcal{M}_4} d^4x \sqrt{-g(x)}\, S(x) W_\Omega(x).$$

The integral will be gauge invariant only if under a GT

$$W_\Omega(x) \to \tilde{W}_\Omega(x) = W_\Omega(f^{-1}(x)),$$

$F(S,\Omega)$ is invariant under GT only if the region Ω itself changes as a scalar under GT!

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Gauge (non)-invariance of space-time integrals, 2

In our (cosmological) case $W_\Omega(x)$ can be represented as a step-like window function, selecting a cylinder-like region with temporal boundaries determined by the two space-like hypersurfaces on which a function $A(x)$ (with time-like gradient) takes the constant values A_1 and A_2 and by the coordinate condition $B(x) < r_0$, where $B(x)$ is a suitable function with space-like gradient . More explicitly:

$$W_\Omega(x) = \theta(A(x) - A_1)\theta(A_2 - A(x))\theta(r_0 - B(x))$$

In this case the integral will be GI only if the functions $A(x)$ and $B(x)$ are scalars.

Gauge (non)-invariance of space-time integrals, 3

For the cosmological backgrounds all fields are naturally of quasi-homogeneous type, and their gradients are typically time-like. In such a context we cannot covariantly define the spatial boundaries for lack of appropriate fields at our disposal and we have a non gauge invariant integral.

$$\tilde{F}(\tilde{S},\Omega) - F(S,\Omega) = \int_{\mathcal{M}_4} d^4x \sqrt{-g(x)}\, S(x) \Delta W_\Omega(x)$$

where

$$\Delta W_\Omega(x) = \theta(A(x) - A_1)\theta(A_2 - A(x)) \left[\theta(r_0 - B(f(x))) - \theta(r_0 - B(x))\right].$$

The breaking of gauge invariance comes from the region $r \sim r_0$ and seems to go away for large enough volumes.

Covariant averaging prescription, 1

Depending on the context in which the backreaction is considered, there are two types of averaging procedure: spatial (or *ensemble*) average of classical variables, and (vacuum) expectation values of quantized fields.

In both cases, ones has to face the problem of the gauge dependence of the results.

Is it possible to define a gauge-invariant averaging prescription?

As first step we define a covariant averaging prescription.

A particular spatial volume average can be covariantly obtained from the four-dimensional integrals discussed before simply by using the following delta-like window function:

$$W_\Omega(x) = u^\mu \nabla_\mu \theta(A(x) - A_0)\theta(r_0 - B(x))$$

with $u^\mu = \frac{\partial^\mu A}{(-\partial^\mu A \partial^\nu A g_{\mu\nu})^{1/2}}$.

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Covariant averaging prescription, 2

We define:

$$\langle S\rangle_{A_0,r_0} = \frac{F(S,\Omega)}{F(1,\Omega)} = \frac{\int d^4x\sqrt{-g}\, S\, u^\mu \nabla_\mu \theta(A(x)-A_0)\theta(r_0-B(x))}{\int d^4x\sqrt{-g}\, u^\mu \nabla_\mu \theta(A(x)-A_0)\theta(r_0-B(x))}$$

and, in the time $\bar{t}$ for which A is homogeneous (defined by $t = h(\bar{t},\vec{x})$), one obtains

$$\langle S\rangle_{A_0,r_0} = \frac{\int_{\Sigma_{A_0}} d^3x\sqrt{|\overline{\gamma}(t_0,\vec{x})|}\ \overline{S}(t_0,\vec{x})\,\theta(r_0 - B(h(t_0,\vec{x}),\vec{x})}{\int_{\Sigma_{A_0}} d^3x\sqrt{|\overline{\gamma}(t_0,\vec{x})|}\,\theta(r_0 - B(h(t_0,\vec{x}),\vec{x}))}$$

where we have called t_0 the time $\bar{t}$ when $A^{(0)}(\bar{t})$ takes the constant values A_0 and we are averaging on a section of the three-dimensional hypersurface Σ_{A_0}, hypersurface where $A(x) = A_0$.

As seen, in the cosmological context we cannot covariantly define the spatial boundaries for lack of appropriate fields at our disposal.

So we obtain gauge invariant average only in the limit of an infinite spatial volume.

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Gauge (non)-invariant averaging and the present cosmological evolution, 1

Inhomogeneities could affect in a non-trivial way the present cosmological evolution.

Recent interpretations have tried to see the dark energy as the "backreaction effect" of appropriately smoothed-out inhomogeneities.

Averaging formalism (T. Buchert (2001))

$$\langle S \rangle_D = \frac{\int_D d^3x \sqrt{|\gamma|}\, S}{\int_D d^3x \sqrt{|\gamma|}}$$

used in different gauge as, for example, longitudinal gauge (Rasanen (2004)) and synchronous gauge (Kolb, Matarrese and Riotto (2006)).

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The point that we want put in evidence is that the averaging formalism above can be seen as a particular case of the previous averaging prescription

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where D is defined by $\theta(r_0 - B(h(t_0,\vec{x}),\vec{x}))$, if we take a scalar $A(x)$ which is homogeneous in the particular gauge chosen to make the calculation.

In a realistic calculation of the BR effects of the inhomogeneities, on the present cosmological evolution, this region D can be as large as the Hubble radius, not more.

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Gauge (non)-invariant averaging and the present cosmological evolution, 3

So, also if the different calculations in the literature can be seen as covariant averaging prescriptions on different hypersurface calculated in a particular gauge, they are not strictly gauge invariant.

In perturbation theory, up to second order, this non-gauge invariance can be parametrized by the following formula

$$\langle \tilde{S} \rangle_{A_0,r_0} - \langle S \rangle_{A_0,r_0} = \frac{\int_{\Sigma_{A_0}} d^3x \, \overline{S}^{(1)} \, \theta(r_0 - B(t_0,\vec{x})) \int_{\Sigma_{A_0}} d^3x \, \frac{\partial B}{\partial x^i} \epsilon^i_{(1)} \, \delta(r_0 - B(t_0,\vec{x}))}{\int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0,\vec{x})) \int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0,\vec{x}))}$$

$$- \frac{\int_{\Sigma_{A_0}} d^3x \, \overline{S}^{(1)} \, \frac{\partial B}{\partial x^i} \epsilon^i_{(1)} \, \delta(r_0 - B(t_0,\vec{x}))}{\int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0,\vec{x}))}$$

it is easy to see that this quantity goes to zero for $r_0 \to +\infty$.

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$$\langle \tilde{S} \rangle_{A_0, r_0} - \langle S \rangle_{A_0, r_0} = \frac{\int_{\Sigma_{A_0}} d^3x \; \overline{S}^{(1)} \, \theta(r_0 - B(t_0, \vec{x}))}{\int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0, \vec{x}))} \frac{\int_{\Sigma_{A_0}} d^3x \; \frac{\partial B}{\partial x^i} \epsilon^i_{(1)} \, \delta(r_0 - B(t_0, \vec{x}))}{\int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0, \vec{x}))} - \frac{\int_{\Sigma_{A_0}} d^3x \; \overline{S}^{(1)} \frac{\partial B}{\partial x^i} \epsilon^i_{(1)} \, \delta(r_0 - B(t_0, \vec{x}))}{\int_{\Sigma_{A_0}} d^3x \, \theta(r_0 - B(t_0, \vec{x}))}$$

it is easy to see that this quantity goes to zero for $r_0 \to +\infty$.

Gauge (non)-invariant averaging and the present universe, 4

Next step: to use this formula to evaluate the impact of the problem on different approaches present in the literature to study the BR of the inhomogeneities on the present cosmological evolution $\Longrightarrow$ Work in progress!

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Gauge invariant averaging for the quantum BR, 1

The prescription used for the BR on the present cosmological evolution became gauge invariant in the limit of an infinite spatial volume.

In this limit the step-like boundary disappears, and we obtain:

$$\langle S \rangle_{A_0} = \frac{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, \vec{x})|} \; \overline{S}(t_0, \vec{x})}{\int_{\Sigma_{A_0}} d^3x \sqrt{|\overline{\gamma}(t_0, \vec{x})|}}.$$

This results can be generalized to the quantum case. Expectation values of quantum operators can be extensively interpreted (and re-written) as spatial integrals weighted by the integration volume V, according to the general prescription

$$\langle \dots \rangle \quad \rightarrow \quad V^{-1} \int_V d^3x \, (\dots) \; ,$$

where V extends to all three-dimensional space.

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In this way the above gauge invariant prescription becomes

$$\langle S \rangle_{A_0} = \frac{\langle \sqrt{|\overline{\gamma}(t_0, \vec{x})|} \; \overline{S}(t_0, \vec{x}) \rangle}{\langle \sqrt{|\overline{\gamma}(t_0, \vec{x})|} \rangle}$$

where it is important to note that the two entries of this ratio are not separately gauge invariant, but the ratio itself, equivalent to the above prescription, is indeed invariant.

Let us now present an explicit expansion (up to second order) of the generalized average $\langle S \rangle_{A_0}$ in terms of conventional averages defined in an arbitrary gauge. Expanding to second order the previous expression we obtain

$$\langle S \rangle_{A_0} = S^{(0)} + \langle \overline{S}^{(2)} \rangle + \frac{1}{(\sqrt{|\gamma|})^{(0)}} \langle \overline{S}^{(1)} (\sqrt{|\overline{\gamma}|})^{(1)} \rangle$$

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Gauge invariant averaging for the quantum BR, 3

We can now express the transformed (barred) fields in terms of the original (unbarred) fields, in a general gauge. Considering the particular infinitesimal coordinate trasformation (see previous slide) that connects t to $\bar{t}$, we can write the transformed quantities in terms of A and of the unbarred fields and get:

$$\begin{aligned}\langle S\rangle_{A_0} &= S^{(0)} + \langle\Delta^{(2)}\rangle + \frac{1}{\left(\sqrt{|\gamma|}\right)^{(0)}}\langle\Delta^{(1)}\left(\sqrt{|\gamma|}\right)^{(1)}\rangle - \frac{1}{\dot{A}^{(0)}}\langle A^{(1)}\Lambda^{(1)}\rangle \\ &- \frac{1}{\dot{A}^{(0)}}\partial_t\left(\ln\left(\sqrt{|\gamma|}\right)^{(0)}\right)\langle A^{(1)}\Delta^{(1)}\rangle + \frac{1}{2}\frac{\Lambda^{(0)}}{(\dot{A}^{(0)})^2}\langle(A^{(1)})^2\rangle\end{aligned}$$

where:

$$\Delta^{(i)} = S^{(i)} - \frac{\dot{S}^{(0)}}{\dot{A}^{(0)}}A^{(i)}, \quad i = 1, 2$$

$$\Lambda^{(0)} = \ddot{S}^{(0)} - \frac{\dot{S}^{(0)}}{\dot{A}^{(0)}}\ddot{A}^{(0)}, \quad \Lambda^{(1)} = \dot{S}^{(1)} - \frac{\dot{S}^{(0)}}{\dot{A}^{(0)}}\dot{A}^{(1)}$$

this depends on the scalar A but, for any given choice of A, is fully gauge independent: $\langle\tilde{S}\rangle_{A_0} = \langle S\rangle_{A_0}$.

Gauge invariant averaging for the quantum BR, 4

Our result can be shown to pass several consistency checks. Suppose, for instance, that S and A are related by an arbitrary function $S = S(A)$. It is easy to check that in such case our formula simply gives $\langle S \rangle_{A_0} = S(A_0)$ as it should be.

As a second check one may replace the scalar A by $f(A)$, with f an arbitrary function, and check that $\langle S \rangle_{A_0}$ does not change.

The gauge invariance of our proposal can be very useful: it allows to compute the average in a gauge that has been conveniently chosen for other purposes; it also allows to evaluate and compare the average of a scalar $S(x)$ on different hypersurfaces, defined by different $A(x)$, while solving the dynamics of the problem in a single gauge.

We should note, instead, that the result of the conventional average procedure, i.e. $\langle S \rangle = \langle S^{(0)} + S^{(1)} + S^{(2)} \rangle = S^{(0)} + \langle S^{(2)} \rangle$, is not gauge invariant, even if this expression is computed in the barred coordinates, because of the extra term proportional to $\langle \overline{S}^{(1)} (\sqrt{|\overline{\gamma}|})^{(1)} \rangle$.

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Volume Expansion Θ

A scalar quantity that could play a central role in the computation of the backreaction is the volume expansion Θ of the observes defined by the hypersurface $A(x) = A_0$.

For a scalar field $\phi(x)$ dominated geometry, we can choose to consider the set of observes which are comoving to the source of the matter. This correspond to take $A(x) = \phi(x)$ and we have

$$\Theta = \nabla_\mu u^\mu, \qquad u_\mu = \frac{\partial_\mu \phi}{\left(-g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right)^{1/2}},$$

with $\Theta = 3H$ for the unperturbed homogeneous geometry.

If we include perturbations, up to second order, and we compute the average of Θ according to the standard prescription $\langle\Theta\rangle$, the result is notoriously gauge dependent.

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Growing-Curvature Cosmology, 1

Cosmological models based on string theory are characterized by a complementary regime, with respect to the phase of standard decreasing curvature, with a growing space-time curvature.

In these cosmology models one needs a way to go from the growing space-time curvature to the standard decreasing curvature regime avoiding a curvature singularity.

Does the quantum backreaction help us to go in the direction of a so called "graceful exit"?

We consider a cosmological background sourced by a dilaton ϕ with a four dimensional action in the Einstein frame given by

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

with $V(\phi) = V_0 e^{\pm\lambda(\phi-\phi_0)}$ and where V_0, λ and ϕ_0 are constant parameters (with $\lambda > 0$).

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Growing-Curvature Cosmology, 2

The background equations can be solved by a class of exact "scaling" solution with two branches $t < 0$ and $t > 0$, and classically disconnected by a curvature singularity at $t = 0$. For the negative-time branch the solution can be written as

$$a = (-t/t_1)^p, \qquad \phi = \phi_0 \mp \sqrt{2p}\ln(-t/t_1), \qquad t < 0.$$

with $V_0 \neq 0$, $p = 2/\lambda^2$, and the constant t_1 relate to V_0 by $t_1^2 V_0 = p(3p-1)$.

For $V = 0$ we obtain $p = 1/3$, t_1 is arbitrary and we recover the Einstein frame representation of standard pre-big bang ($\dot\phi > 0$) and ekpyrotic ($\dot\phi < 0$) backgrounds.

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Growing-Curvature Cosmology, 3

Considering the Uniform Curvature Gauge ($\psi^{(1)} = \psi^{(2)} = 0$, $E^{(1)} = E^{(2)} = 0$ and $\chi_i^{(1)} = \chi_i^{(2)} = 0$) we obtain for the first order perturbation the following equation of motion ($M_{pl}^2 = (8\pi G)^{-1}$):

$$\alpha = \frac{1}{2M_{pl}^2}\frac{\dot\phi}{H}\varphi, \quad \frac{H}{a}\nabla^2\beta = \frac{1}{M_{pl}^2}\frac{\dot\phi^2}{H}\frac{d}{dt}\left(\frac{H}{\dot\phi}\varphi\right)$$

$$\ddot\varphi + 3H\dot\varphi - \frac{1}{a^2}\nabla^2\varphi + V_{\phi\phi}\varphi = \dot\alpha\dot\phi - 2\alpha V_\phi - \frac{\dot\phi}{2a}\nabla^2\beta$$

$$\nabla^2 B_i = 0, \quad \ddot h_{ij} + 3H\dot h_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 0$$

where $\phi = \phi^{(0)}$, $\varphi = \delta\phi$ and we have neglected the suffix (1).

Growing-Curvature Cosmology, 4

The canonically normalized solutions in the Fourier space are given by

$$\varphi_k = \frac{1}{a^{3/2}}\left(\frac{\pi}{4H}\frac{p}{p-1}\right)^{1/2} H_\nu^{(1)}\left(\frac{p}{p-1}\frac{k}{aH}\right)$$

$$h_k = 2\sqrt{2}\frac{1}{a^{3/2}}\left(\frac{\pi}{4H}\frac{p}{p-1}\right)^{1/2} H_\nu^{(1)}\left(\frac{p}{p-1}\frac{k}{aH}\right)$$

with h_k a polarization component in the Fourier space of h_{ij}.

By $\nabla^2 B_i = 0$ we have that all vector inhomogeneities are vanishing (Mena, Mulryne and Tavakol (2007)).

The others first order perturbation can be connected to φ_k by using the above equations.

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Growing-Curvature Cosmology, 5

In a similar way we can find the equation of motion for the second order perturbations.
For example we have

$$\ddot{\varphi}^{(2)} + 3H\dot{\varphi}^{(2)} - \frac{1}{a^2}\nabla^2\varphi^{(2)} + \left[V_{\phi\phi} + 2\frac{d}{dt}\left(3H + \frac{\dot{H}}{H}\right)\right]\varphi^{(2)} = D$$

with D a bilinear function in the first order perturbations.

Making the v.e.v. of those second order equations we can express the v.e.v. of second order perturbations ($\langle\varphi^{(2)}\rangle$, $\langle\alpha^{(2)}\rangle$,....) in function of the v.e.v. of bilinear quantities in the first order perturbations ($\langle\varphi^2\rangle$, $\langle\varphi\dot{\varphi}\rangle$,....).

The evaluation of the v.e.v. of bilinear quantities in the first order perturbations involves a sum over momenta which is plagued by ultraviolet divergencies.

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Growing-Curvature Cosmology, 6

For this $V = 0$ case we can try to evaluate the BR of the cosmological perturbations on the background Hubble factor using the v.e.v. of the expansion rate Θ.

Let us first show the gauge dependence of the naive prescription considering the $\langle\Theta\rangle$ for differents Uniform Field Gauge (UFG)-(see previous slide for definition).

Working in the UFG fixed by $\beta^{(1)} = 0 = \beta^{(2)}$ one obtains:

$$\langle\Theta\rangle_{UFG\beta} = 3H\left[1 + \frac{45}{8}\frac{\langle Q^{(1)\,2}\rangle_{\rm REN}}{M_{\rm P}^2}\right],$$

while in the UFG with $E^{(1)} = 0 = E^{(2)}$ one obtains:

$$\langle\Theta\rangle_{UFGE} = 3H\left[1 - \frac{3}{4}\frac{\langle Q^{(1)\,2}\rangle_{\rm REN}}{M_{\rm P}^2}\right].$$

where the suffix REN denotes that the v.e.v. has been renormalized through a suitable method.

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Growing-Curvature Cosmology, 7

These two results not only differ between each other, but also differ from the one obtained with the gauge invariant prescription with $A = \phi$, which gives

$$\langle \Theta \rangle_{\phi_0} = 3H \left[1 + \frac{15}{8} \frac{\langle Q^{(1)\,2} \rangle_{\mathrm{REN}}}{M_{\mathrm{P}}^2} \right]$$

in any gauge.

It follows, in particular, that we cannot try to solve the problem of the gauge dependence of the backreaction considering the UFG as a privileged gauge, as often suggested in the literature for the case of slow-roll inflation in the long-wavelength limit.

Growing-Curvature Cosmology, 7

These two results not only differ between each other, but also differ from the one obtained with the gauge invariant prescription with $A = \phi$, which gives

$$\langle \Theta \rangle_{\phi_0} = 3H \left[1 + \frac{15}{8} \frac{\langle Q^{(1)\,2} \rangle_{\mathrm{REN}}}{M_{\mathrm{P}}^2} \right]$$

in any gauge.
It follows, in particular, that we cannot try to solve the problem of the gauge dependence of the backreaction considering the UFG as a privileged gauge, as often suggested in the literature for the case of slow-roll inflation in the long-wavelength limit.

Growing-Curvature Cosmology, 8

The results of the gauge invariant prescription used seems to hinder a graceful exit from the growing curvature phase.

Is this true? Maybe not!

There are big problems related to the interpretation of the results!!
Which is the right hypersurface $A(x) = A_0$ to use?
Which is the right scalar variable to average to see the BR of the perturbations on the background?

Possible solution: evaluation of the BR on the background acceleration between two test particle which move on neighboring geodesics. $\Longrightarrow$ Work in progress!

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Conclusions

- We have proposed a general formula for the classical or quantum average of any scalar quantity, on hypersurfaces on which another given scalar quantity is homogeneous.
  Our non-trivial proposal is gauge-invariant in the quantum case and in the classical one for averaging over all the 3-space volume.
- The evaluation of the residual gauge dependence present in the average prescription used in the literature, for the backreaction of the inhomogeneities on the present cosmological evolution, is a key point to investigate.
- By using the quantum gauge invariant average proposed one can approach in a gauge invariant way the backreaction problem in the early universe.
- Growing-curvature cosmology model is a perfect example to see the problems associated with the choice of gauge for the naive v.e.v. prescription.
- To try to solve the residual interpretation problems, associated to the calculation of the backreaction, one can try to deal with objects with a solid physical meaning as the deviation between two geodesics.

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