Regular DGP in the core of a hypermonopole

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with Christophe Ringeval

UCL

Introduction

- Do we live in $D > 4$ dimensions?

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- Maybe traces at LHC

What is gravity?

- How can gravity be 4D?

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- DGP: non-compact, infinite volume

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- Chosen profile for $M_*(X)$ [Kolanovic, Porrati, Rombouts PRD (2003)][Shaposnikov, Tinyakov, Zuleta PRD (2004)]

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- Cascading DGP: ghost appears but cured via CC
- Is there a classical FT which has DGP mechanism?

Other attempt

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- No-go theorem: Gravity was 6D

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- Positively curved, $n = 2$

Hints

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- 't Hooft-Polyakov Monopole

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Directions

- Choose ansatz

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- Find background solution

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Background geometry

- Static ansatz

$$ds^2 = e^{\sigma(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + \omega(r)^2 d\Omega^2$$

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- Gauge fields

$$\vec{C}_\theta = \frac{1-Q(r)}{q}\vec{u}_\phi \quad \vec{C}_\phi = -\frac{1-Q(r)}{q}\sin\theta\vec{u}_\theta$$

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- 3 param. family

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- Compactify infinity into a segment, $0 < \rho < 1$,

$$r = \frac{\rho}{1 - \rho}$$

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- Continuity of fields and deriv

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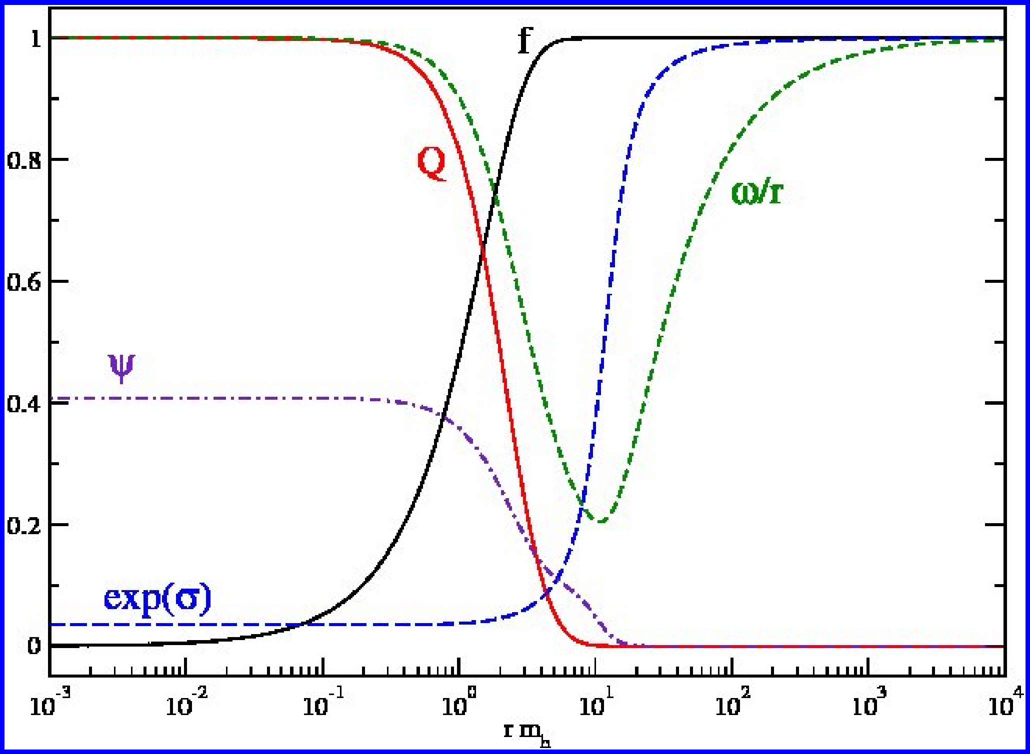
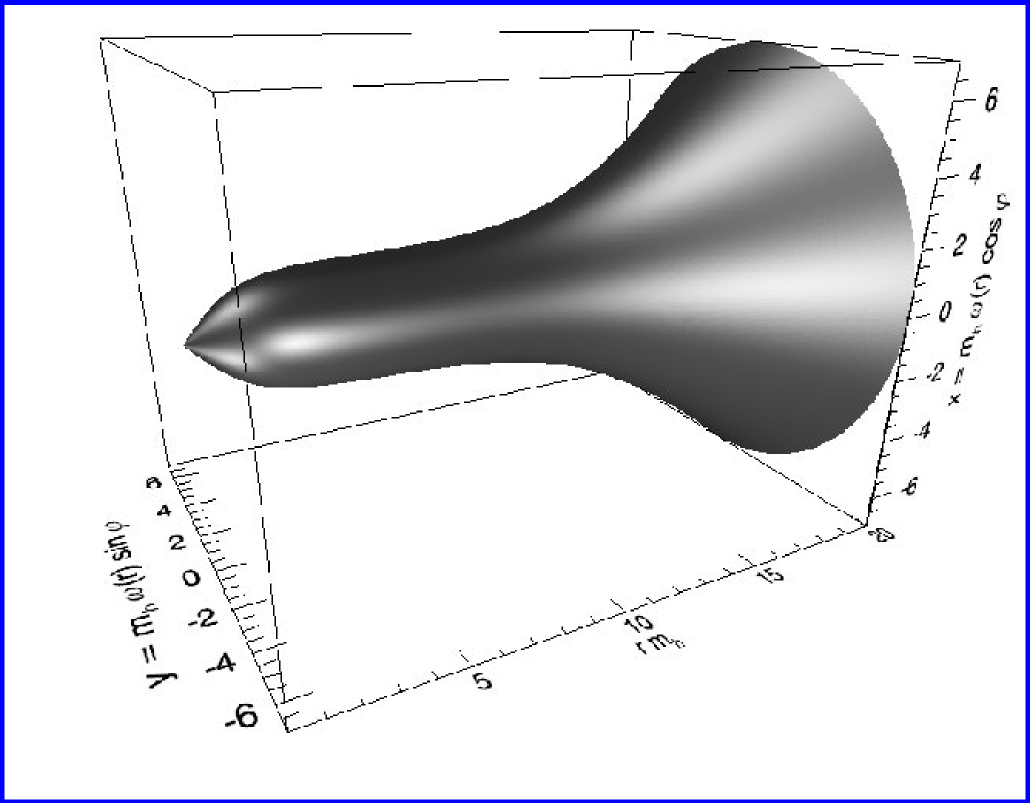


Example $\alpha = 2$, $\epsilon = 0.5$, $\beta = 1$. Typical topological configuration for the fields. Minkowski at boundary. TWPBVP: Cash and Mazzia algorithm, JCAM (2005).



Region where ω no longer grows, cylindrically shaped ED, resonant gravitons at this scale

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- Dilaton, no strong influence

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- Look for (meta-)stable modes

Tensor fluctuations

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- Linearize Einstein equations

- Decoupled linear PDE's

$$h''_{\mu\nu} + F' h'_{\mu\nu} + m_h^{-2} \Box h_{\mu\nu} - \frac{A}{\omega^2} L^2 h_{\mu\nu} = 0$$

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- 4D Fourier + Spherical Harmonics

$$h = \sum_{m,l} Y_{lm}(\theta,\phi) \int d\bar{M}\,\hat{h}_{lm}(\bar{M},z)\Delta_{\bar{M}}(x)$$

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- Δ eigenfun of $m_h^{-2}\Box$ with $\bar{M}^2$ eigenval

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- $W = \frac{3}{4}\sigma' + \omega'/\omega + \frac{1}{2}\psi'$

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- Not normalizable

SUSY-QM

- QM with central potential $V_{2,l=0} \equiv W^2 + W'$

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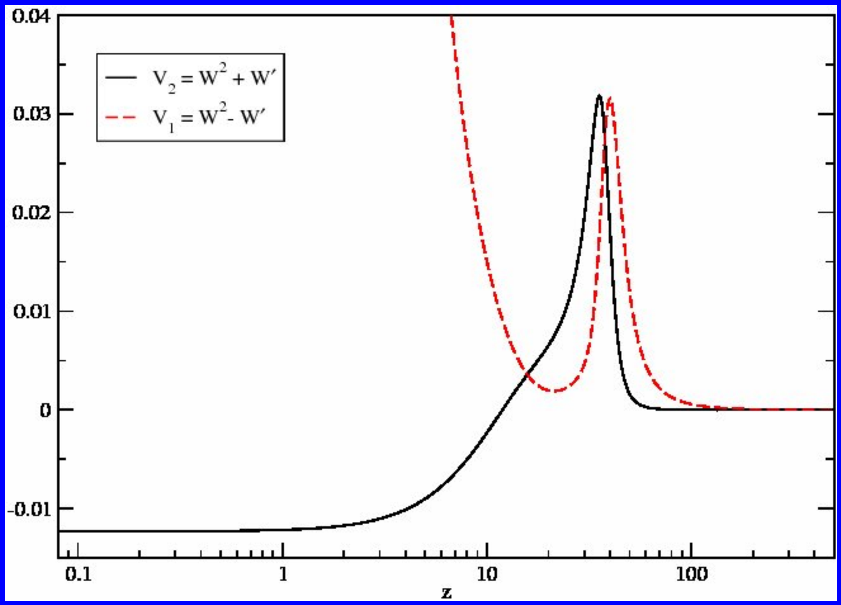

Depends on α, β, ϵ. For V_2: ground state ξ_0 not normalizable. For V_1, $1/\xi_0$ not regular in 0. SUSY is broken and spectrum $M^2 > 0$.

Solution of radial SE

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- $u_{M,l}(z)$ solution of Schr.Eq.
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- Completeness relation

$$\int_0^\infty u^*_{M,l}(z_1)u_{M,l}(z_2)dM = \delta(z_1 - z_2)$$

- Retarded Green function for TT source $S_{\mu\nu}$ [Garriga, Tanaka PRL (2000)]

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- Find

$$h_{\mu\nu}(X_1) = -\frac{2\kappa^2}{m_h^2\omega} e^{-\psi(z_1)-3\sigma/4} \times \int G_\xi(X_1; X_2) e^{-\psi(z_2)+3\sigma/4} \omega(z_2) S_{\mu\nu} d^7 X_2$$

GF expression

$$G''_\xi + (-V_2 + \Box + e^\sigma \omega^{-2} L) G_\xi = \delta^4(x^\mu - x^{\mu\prime})$$
$$\times\, \delta(\cos\theta - \cos\theta')\delta(\phi - \phi')\delta(z - z')$$

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$$G_\xi = -\int \frac{d^4p}{(2\pi)^4} e^{ip_\mu(x_1^\mu - x_2^\mu)} \sum_{l,m} Y_l^m(\theta_1, \phi_1) Y_l^{m*}(\theta_2, \phi_2)$$
$$\times \int \frac{u_{M,l}(z_1) u^*_{M,l}(z_2) dM}{M^2 + \vec{p}^2 - (p^0 + i\epsilon)^2}$$

Flat case

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- Radial solution: $u^{\flat}_{M,l} = \sqrt{Mz}J_{l+1/2}(Mz)$

- Contribution in the origin

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$$h^{\flat}_{\mu\nu} = \lim_{z\to 0} \frac{\kappa^2}{8\pi^2 m_h^2} \int d^3\vec{x}_2 s_{\mu\nu} \int dM \frac{|u^{\flat}_{M,0}(z)|^2}{z^2} \frac{e^{-M\Delta\vec{x}}}{|\Delta\vec{x}|}$$

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- In $d = 3$, $1/r$ dependence: 4D gravity
- $4\pi^3$: $d - 2$ times the surface of $d - 1$ unit sphere.

Monopole case

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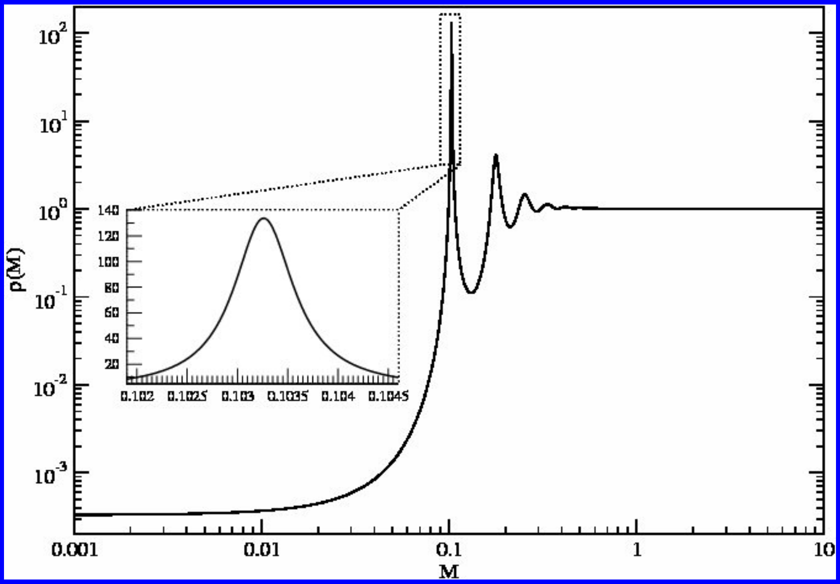
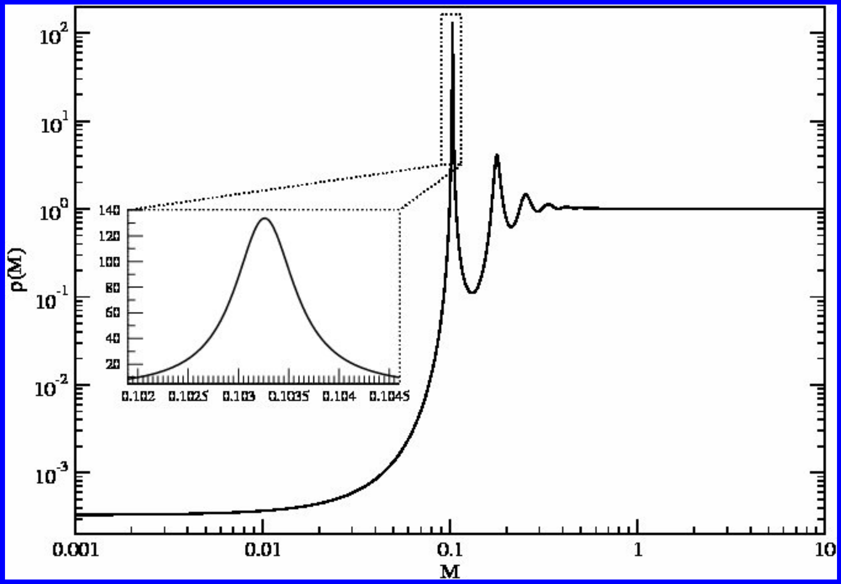
- Need spectral density $\rho(M) \equiv |u_{M,0}(0)|^2/|u^\flat_{M,0}(0)|^2$
- Therefore one finds

$$h_{\mu\nu} = \frac{2\kappa^2}{8\pi^3 m_h^2} \int d^3\vec{x}_2 \frac{\mathcal{L}\{\rho(M)M^2\}}{|\Delta\vec{x}|} s_{\mu\nu}(\vec{x}_2)$$



There are at least two trapped metastable gravitons. Mass $m_g \approx 0.103$ Breit-Wigner with $\Gamma \approx 3.5 \times 10^{-4}$ (m_h units)



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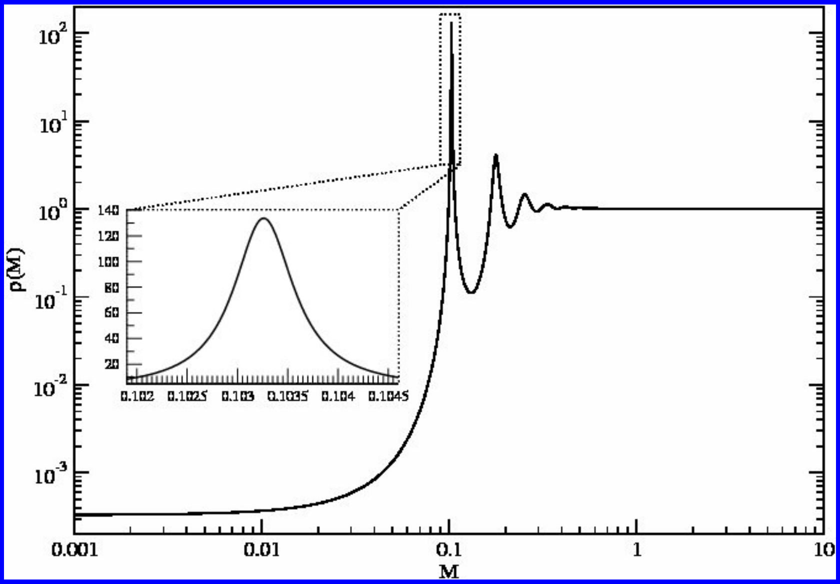

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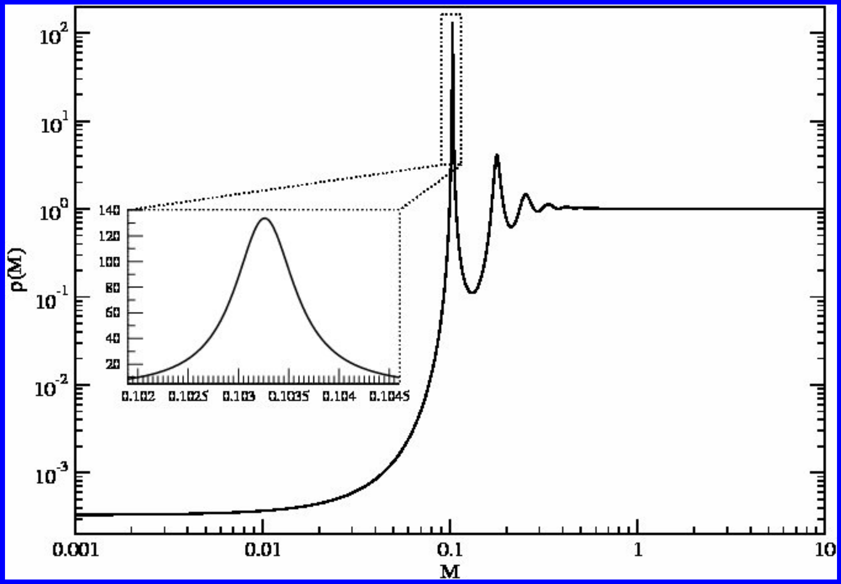
- ρ strongly peaked: resonant metastable modes



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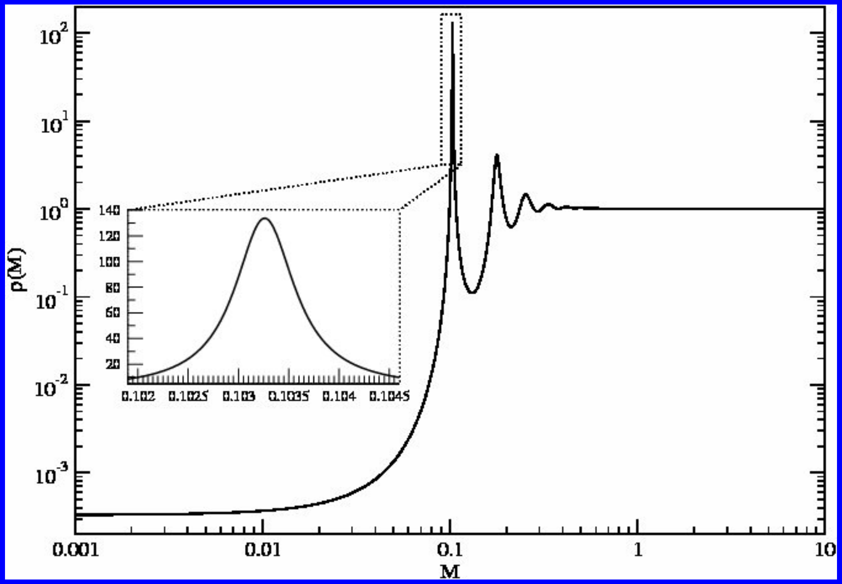

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