Exploring dark energy models with linear perturbations: Fluid vs scalar field

Masaaki Morita
(Okinawa Natl. College Tech., Japan)


Beautiful ocean view from my laboratory
in Henoko, Okinawa
Futenma Air Base will really move to this sea...?

1. Present Status of Observational Cosmology

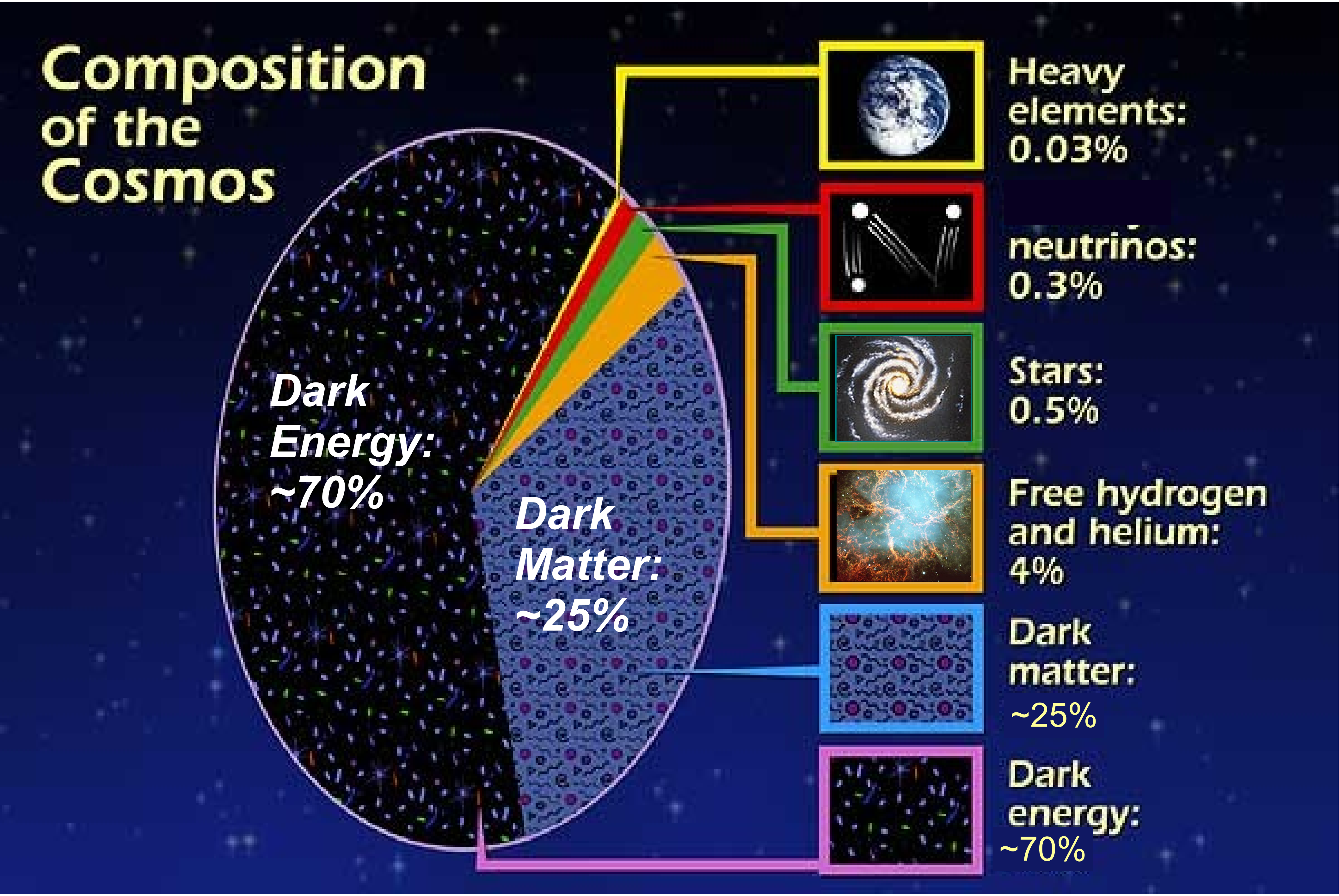
Recent cosmological observations have revealed two main features of the universe:

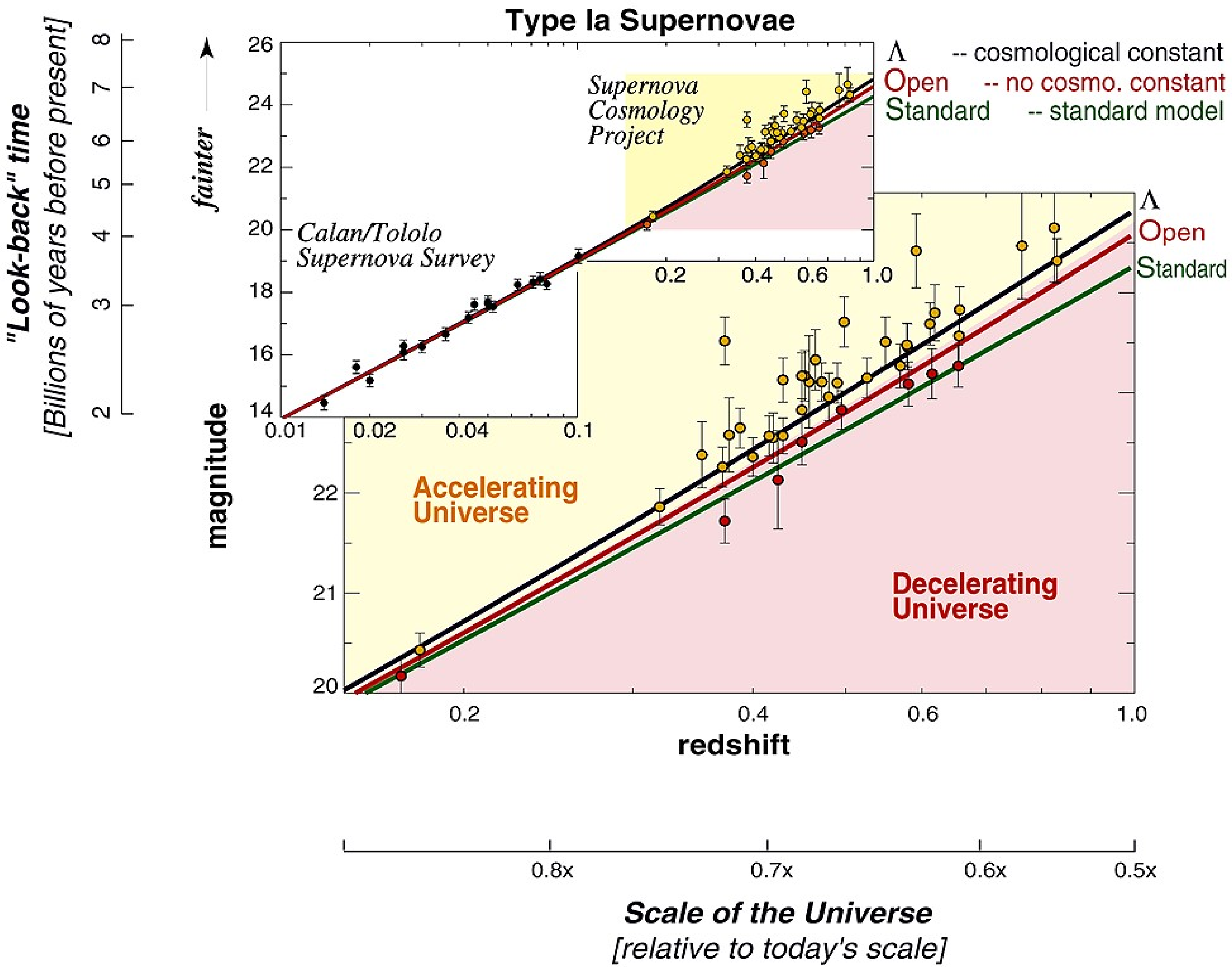
- High-z type Ia supernovae
  ⇒ late-time acceleration of cosmic expansion

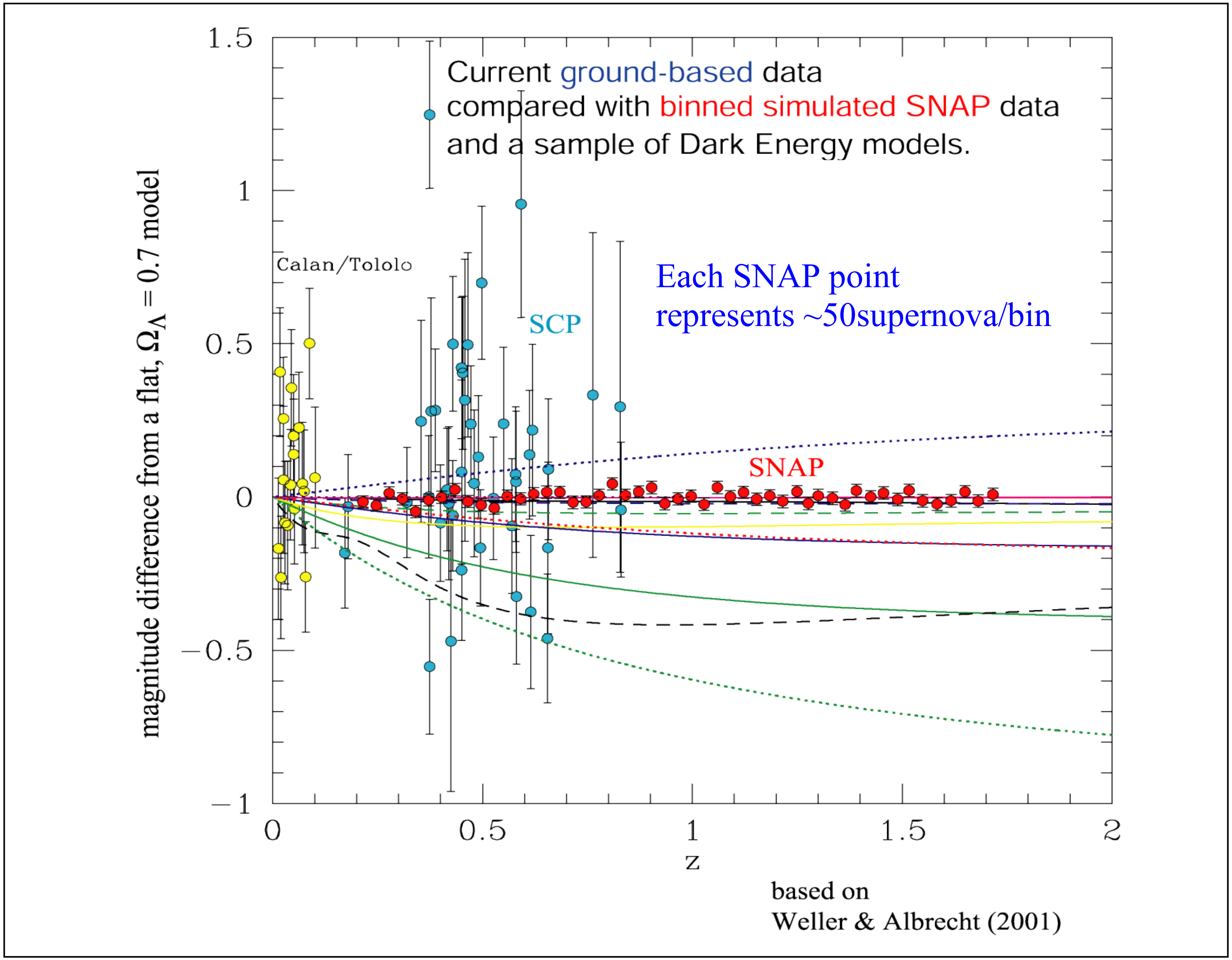
- Cosmic Microwave Background by WMAP Satellite
  ⇒ the universe is almost spatially flat

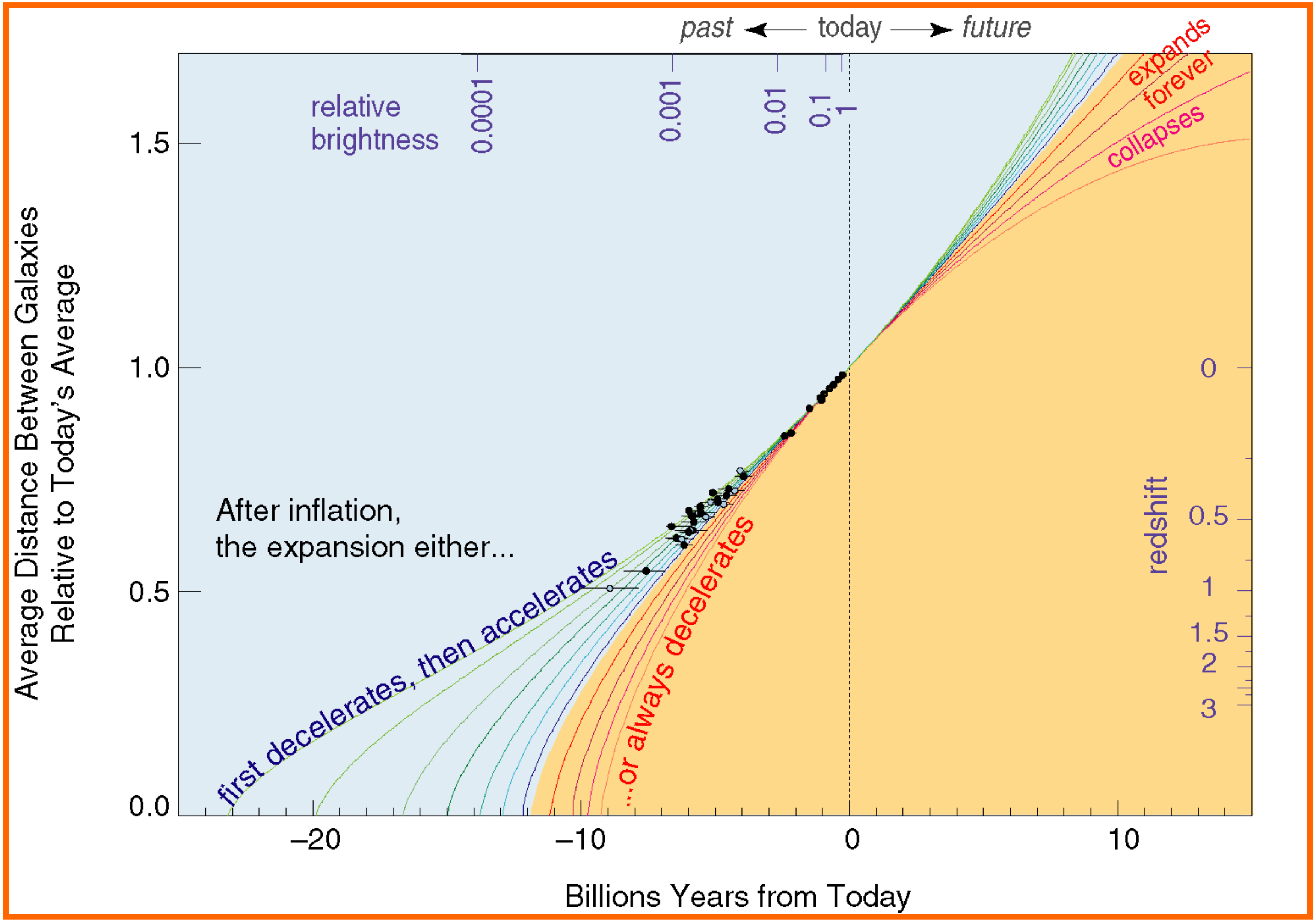
Dark Energy (DE) – 70 %
Dark Matter (DM) – 25 %

Almost all components of the universe is unknown

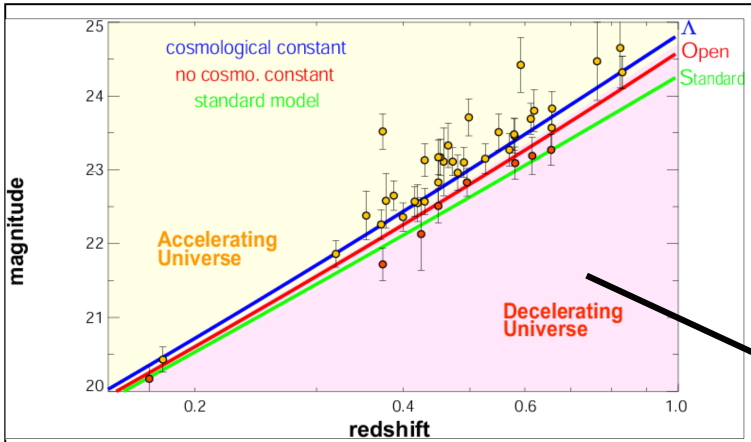

Composition of the Cosmos
Dark Energy: ~70%
Dark Matter: ~25%
Heavy elements: 0.03%
[redacted]
neutrinos: 0.3%
Stars: 0.5%
Free hydrogen and helium: 4%
Dark matter: ~25%
Dark energy: ~70%


Type Ia Supernovae
"Look-back" time
[Billions of years before present]
8
7
6
5
4
3
2
fainter
magnitude
26
24
22
20
18
16
14
0.01
0.02
0.04
0.1
0.2
0.4
0.6
1.0
Supernova Cosmology Project
Calan/Tololo Supernova Survey
Λ -- cosmological constant
Open -- no cosmo. constant
Standard -- standard model
Λ
Open
Standard
Accelerating Universe
Decelerating Universe
22
21
20
0.2
0.4
0.6
1.0
redshift
0.8x
0.7x
0.6x
0.5x
Scale of the Universe
[relative to today's scale]


Current ground-based data
compared with binned simulated SNAP data
and a sample of Dark Energy models.
Calan/Tololo
SCP
Each SNAP point
represents ~50supernova/bin
SNAP
magnitude difference from a flat, Ω_Λ = 0.7 model
1.5
1
0.5
0
−0.5
−1
0
0.5
1
1.5
2
z
based on
Weller & Albrecht (2001)

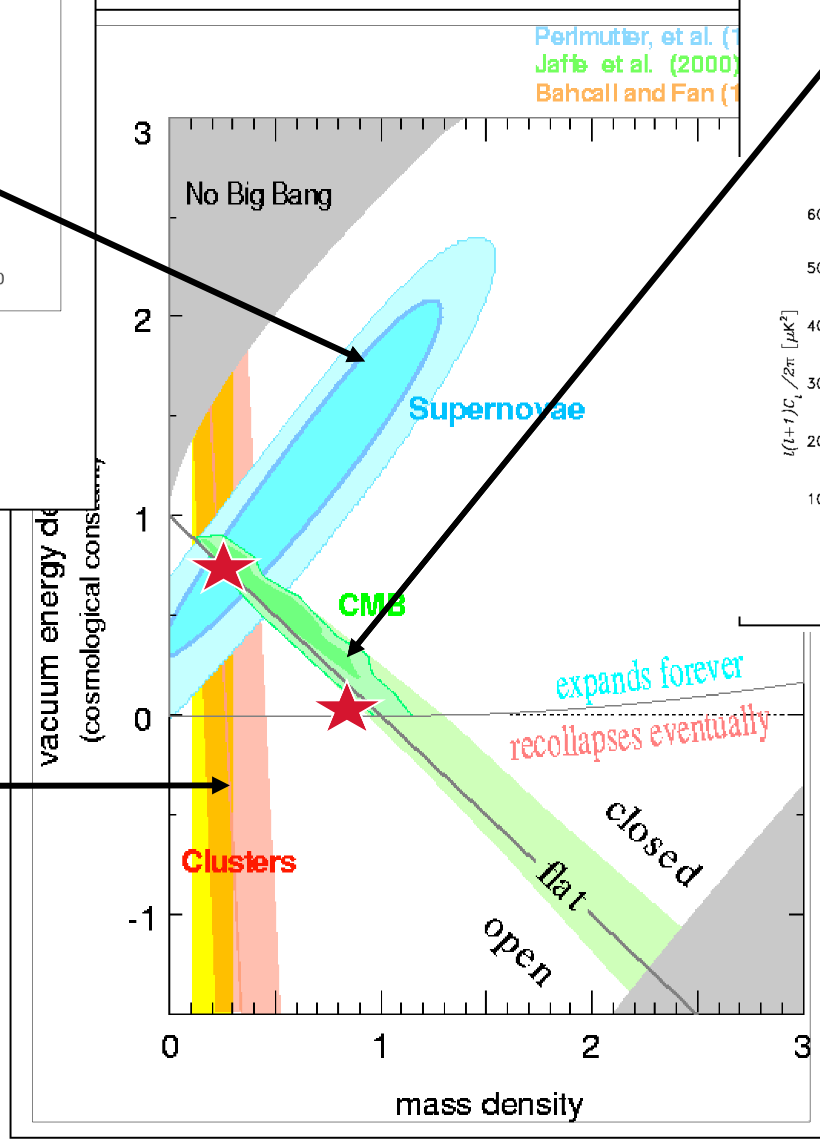

past ← today → future
relative brightness
0.0001
0.001
0.01
0.1
1
expands forever
collapses
1.5
1.0
0.5
0.0
Average Distance Between Galaxies Relative to Today's Average
0
0.5
1
1.5
2
3
redshift
After inflation,
the expansion either...
first decelerates, then accelerates
...or always decelerates
−20
−10
0
10
Billions Years from Today

A Revolution in Cosmology



Ω_{DE}= 0.7, Ω_M= 0.3

for a flat universe

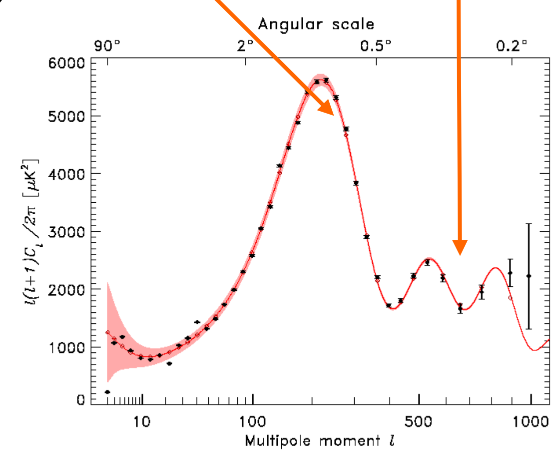


Flat universe

Ω_{total}= 1.02+/-0.02

Baryon Density

Ω_B= 0.044+/-0.004

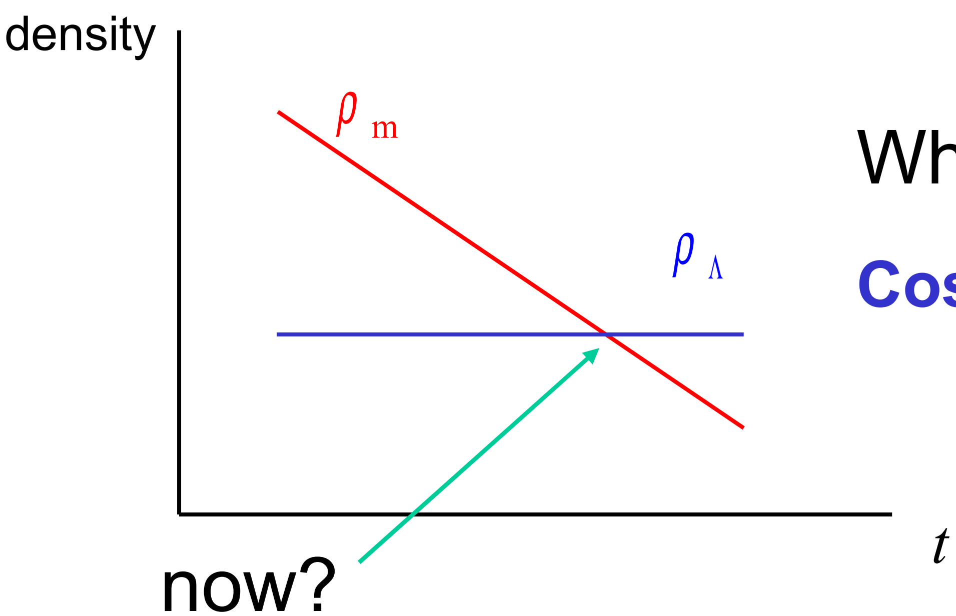


- Weak lensing mass census
- Large scale structure measurements

Ω_M= 0.3

Lambda CDM cosmology gives a best fit

Cosmological constant (DE)
+ Cold Dark Matter (CDM)



Why now?

Cosmic coincidence problem

- Physical substance of DE and DM
- Cosmic coincidence problem

2. Theoretical Basics

Einstein eq. $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$\Rightarrow \quad H^2 := \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \ , \quad \frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3P)$$

for a homogeneous and isotropic universe (spatially flat)

For acceleration of cosmic expansion, $\rho + 3P < 0$

violation of Strong Energy Condition

Eq. of motion $\nabla_\mu T^\mu{}_\nu = 0$

$\Rightarrow$ $\dot{\rho} + 3H(\rho + P) = 0$

$\Rightarrow$ $\rho \propto a^{-3(1+w)}, \quad w := P/\rho$

$\Rightarrow$ $a \propto t^{2/[3(1+w)]}$ (for const w)

For the acceleration, $w < -\frac{1}{3}$

Many possible models of Dark Energy

- Dynamical DE
  with a standard scalar field (quintessence)

- K-essence with a non-canonical scalar field

- Phantom field with a negative kinetic term

- (Generalized) Chaplygin gas $P = -A\rho^{-\alpha}$
  a fluid model unifying DE and DM

- $f(R)$-gravity (modified gravity)

...

(a) Standard scalar field model (quintessence)

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi-V(\varphi)\right]$$

$$\rho=\frac{1}{2}\dot{\varphi}^2+V(\varphi)\quad,\qquad P=\frac{1}{2}\dot{\varphi}^2-V(\varphi)$$

$$w=\frac{P}{\rho}=\frac{\dot{\varphi}^2/2-V}{\dot{\varphi}^2/2+V}$$

Similar to inflation,
but the potential has the form of inverse power law

$$V(\varphi)\propto\varphi^{-n}\quad(n>0)$$

(b) General scalar field model (k-essence) including non-canonical kinetic terms

$$S = \int d^4 x \sqrt{-g}\; p(\varphi, X) \quad ; \qquad X = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi$$

$$T^{\text{scalar}}_{\mu\nu} = p_{,X} \nabla_\mu \varphi \nabla_\nu \varphi + p\, g_{\mu\nu}$$

$\Updownarrow$

$$T^{\text{fluid}}_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

$$\rho = 2 X p_{,X} - p \quad ; \qquad u_\mu = \frac{\nabla_\mu \varphi}{(2X)^{1/2}}$$

e.g. $\quad p(\varphi, X) = K(\varphi) X + L(\varphi) X^2$

(c)(Generalized) Chaplygin Gas

Kamenshchik et al (2001), Bento et al (2002), ...

$$P = -A\rho^{-\alpha}$$

$$\dot{\rho} + 3H(\rho + P) = 0 \quad \rightarrow \quad \rho = \left(A + B a^{-3(1+\alpha)}\right)^{1/(1+\alpha)}$$

early time $P \approx 0$ late time $P \approx -\rho$

Fluid models unifying DE and DM

interesting possibility to solve the coincidence problem

But... strongly constrained from structure formaion and CMB

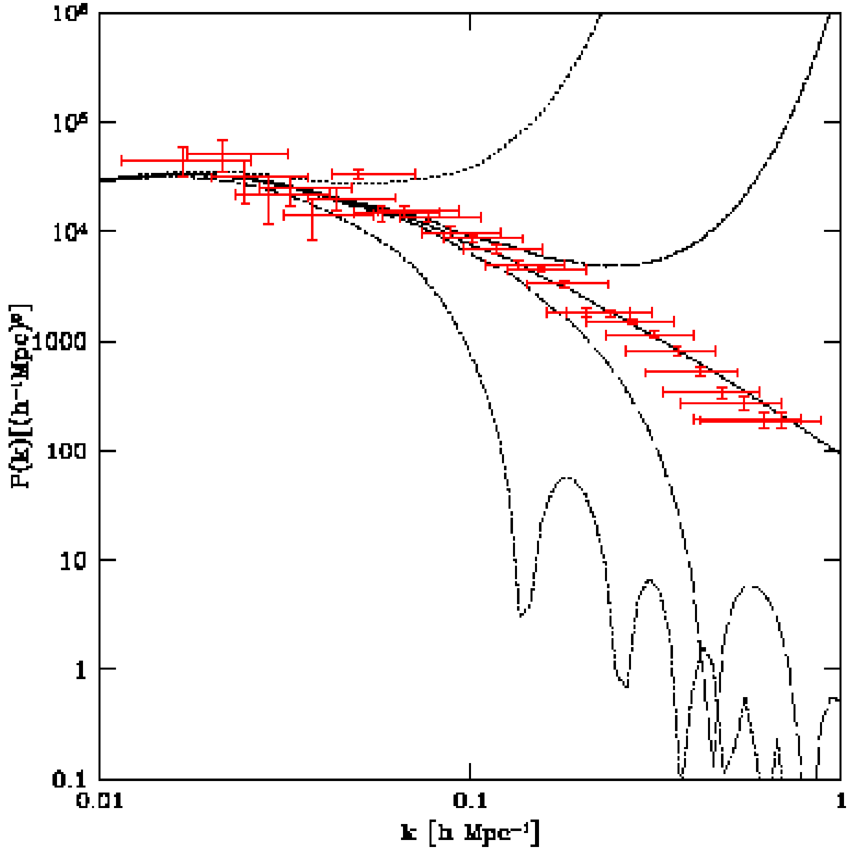


FIG. 1. UDM solution for perturbations as function of wavenumber, k. From top to bottom, the curves are GCG models with $\alpha = -10^{-4}$, -10^{-5}, 0 (ΛCDM), 10^{-5} and 10^{-4}, respectively. The data points are the power spectrum of the 2df galaxy redshift survey.

The End of Unification?

Linear power spectrum in Chaplygin gas models

Sandvik et al, PRD 69 (2004) 123524

A generalized Chaplygin gas model is *equivalent* to a type of quintessence models:

fluid $\quad\quad\quad\quad\quad\quad\quad\quad$ scalar field

$$P = -A\rho^{-\alpha} \quad \Longleftrightarrow \quad V(\phi) = (A^{1/2}/2)(\cosh\alpha\phi + (\cosh\alpha\phi)^{-1})$$

These two give the same cosmic expansion.
zeroth-order dynamics

But how is the linear **perturbation dynamics**?

The difference of the perturbation dynamics may give a clue to distinguish the physical substance of DE.

What we want to do:

Compare the linear perturbation dynamics under the same background dynamics

between fluid and scalar field DE models

3. Perturbations in dark energy cosmology

Model: Einstein gravity
+ DE (fluid or scalar field)
+ pressureless matter

$$G^{\mu}_{\ \nu} = 8\pi G\left(T^{\mu}_{(\mathrm{f})\nu} + T^{\mu}_{(\mathrm{m})\nu}\right)$$

$$T^{\mu\nu}_{(\mathrm{f})} = (\rho_{\mathrm{f}} + P_{\mathrm{f}})\, u^{\mu}_{(\mathrm{f})} u^{\nu}_{(\mathrm{f})} + P_{\mathrm{f}}\, g_{\mu\nu}$$

or $$G^{\mu}_{\ \nu} = 8\pi G\left(T^{\mu}_{(\varphi)\nu} + T^{\mu}_{(\mathrm{m})\nu}\right)$$

$$T^{(\varphi)}_{\mu\nu} = p_{,X} \nabla_{\mu}\varphi \nabla_{\nu}\varphi + p\, g_{\mu\nu}$$

Longitudinal gauge:

$$ds^2 = -(1+2\Phi)\,dt^2 + a^2(1-2\Phi)\,\gamma_{ij}\,dx^i\,dx^j$$

Perturbed Einstein tensor

$$\delta G^0_{\ 0} = 2\frac{k^2}{a^2}\Phi + 6H(H\Phi + \dot{\Phi})$$

$$\delta G^0_{\ i} = 2(H\Phi + \dot{\Phi})_{,i}$$

$$\delta G^i_{\ j} = \left[2(H\Phi + \dot{\Phi})^{\cdot} + 6H(H\Phi + \dot{\Phi}) + 2\dot{H}\Phi\right]\delta^i_{\ j}$$

Perturbed Einstein eqs for fluid DE

$$\delta G^{\mu}_{\nu} = 8\pi G\left(\delta T^{\mu}_{(\mathrm{f})\nu} + \delta T^{\mu}_{(\mathrm{m})\nu}\right)$$

$$\frac{k^2}{a^2}\Phi + 3H(H\Phi + \dot{\Phi}) = -4\pi G(\delta\rho_{\mathrm{f}} + \delta\rho_{\mathrm{m}})$$

$$H\Phi + \dot{\Phi} = -4\pi G a^2\left[(\rho_{\mathrm{f}} + P_{\mathrm{f}})v_{\mathrm{f}} + \rho_{\mathrm{m}} v_{\mathrm{m}}\right]$$

$$(H\Phi + \dot{\Phi})^{\cdot} + 3H(H\Phi + \dot{\Phi}) + \dot{H}\Phi = 4\pi G\,\delta P_{\mathrm{f}}$$

⇩

$$c_{\mathrm{f}}^2 := dP_{\mathrm{f}}/d\rho_{\mathrm{f}}$$

$$\ddot{\Phi} + (4 + 3c_{\mathrm{f}}^2)H\dot{\Phi} + \left[2\dot{H} + 3H^2(1 + c_{\mathrm{f}}^2) + \frac{c_{\mathrm{f}}^2 k^2}{a^2}\right]\Phi = -4\pi G c_{\mathrm{f}}^2\,\delta\rho_{\mathrm{m}}$$

Eq. of motion $\nabla_{\mu} T^{\mu}_{(\mathrm{m})\nu} = 0$

Assumption: no interaction between fluid DE and CDM except gravity

$$\dot{\Delta}_{\mathrm{m}} - 3\dot{\Phi} - k^2 v_{\mathrm{m}} = 0 \qquad \left(\Delta_{\mathrm{m}} := \delta\rho_{\mathrm{m}} / \rho_{\mathrm{m}}\right)$$

$$\dot{v}_{\mathrm{m}} + 2 H v_{\mathrm{m}} + \frac{1}{a^2} \Phi = 0$$

Perturbed Einstein eqs for scalar field DE

$$\delta G^{\mu}_{\nu} = 8\pi G\left(\delta T^{\mu}_{(\varphi)\nu} + \delta T^{\mu}_{(\mathrm{m})\nu}\right)$$

$$\frac{k^2}{a^2}\Phi + 3H(H\Phi + \dot{\Phi}) = -4\pi G(\delta\rho_{\varphi} + \delta\rho_{\mathrm{m}})$$

$$H\Phi + \dot{\Phi} = 4\pi G\left(p_{,X}\dot{\varphi}\,\delta\varphi - \rho_{\mathrm{m}} a^2 v_{\mathrm{m}}\right)$$

$$(H\Phi + \dot{\Phi})^{\cdot} + 3H(H\Phi + \dot{\Phi}) + \dot{H}\Phi = 4\pi G\,\delta p$$

$$\ddot{\Phi}+\left[(4+3c_\varphi^2)H+Z\right]\dot{\Phi}+\left[2\dot{H}+3H^2(1+c_\varphi^2)+Z+\frac{c_\varphi^2 k^2}{a^2}\right]\Phi$$
$$=-4\pi G\rho_\mathrm{m}(c_\varphi^2\Delta_\mathrm{m}+Z a^2 v_\mathrm{m})$$

$$c_\varphi^2 := \frac{p_{,X}}{p_{,X}+p_{,XX}\dot{\varphi}^2}=\frac{p_{,X}}{\rho_{,X}} \quad , \quad Z:=\frac{p_{,X}\,p_{,X\varphi}\dot{\varphi}^2-p_{,\varphi}\,p_{,XX}\dot{\varphi}^2-2p_{,\varphi}\,p_{,X}}{p_{,X}\dot{\varphi}(p_{,X}+p_{,XX}\dot{\varphi}^2)}$$

Remember the fluid case:

$$\ddot{\Phi}+(4+3c_\mathrm{f}^2)H\dot{\Phi}+\left[2\dot{H}+3H^2(1+c_\mathrm{f}^2)+\frac{c_\mathrm{f}^2 k^2}{a^2}\right]\Phi=-4\pi G c_\mathrm{f}^2\rho_\mathrm{m}\Delta_\mathrm{m}$$

In the case of quintessence,

$$\rho = X + V(\varphi) \ , \quad p = X - V(\varphi)$$

$$\Rightarrow \quad c_\varphi^2 = p_{,X} / \rho_{,X} = 1 \ , \qquad Z = 2 V_{,\varphi} / \dot{\varphi}$$

(After some calculations, we find)

Both coincide in the large-scale limit if CDM component is negligible

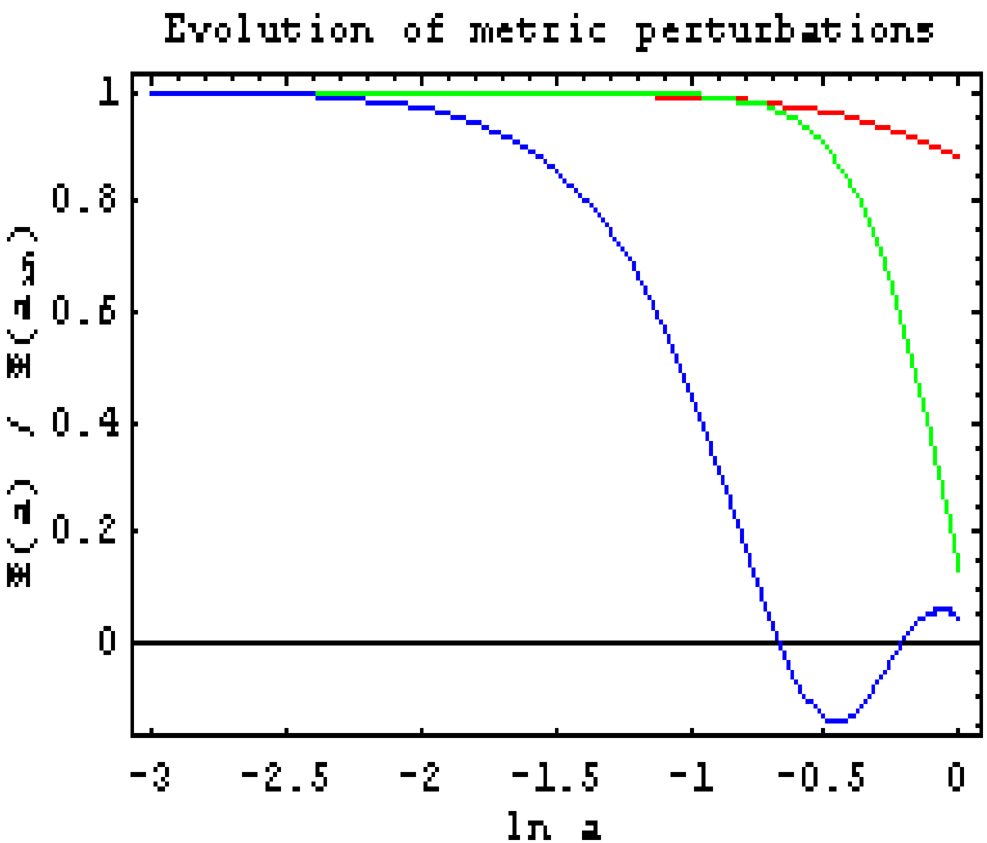
Two differences:
sound velocity and **source from CDM component**

- **Does not coincide with a fluid case generally even in the large-scale limit**

- **In purely kinetic k-essence** $p=p(X)$, **the left-hand side coincides with a fluid case because** $Z=0,\ c_X^2=c_s^2$ **but still different in the source**

- May coincide with a fluid case generally in large-scale in uniform-field gauge $\delta\varphi=0$
  If so, ``fluid analogy'' is a gauge-dependent notion in cosmological perturbations

4. Illustrations of the perturbation dynamics



LCDM

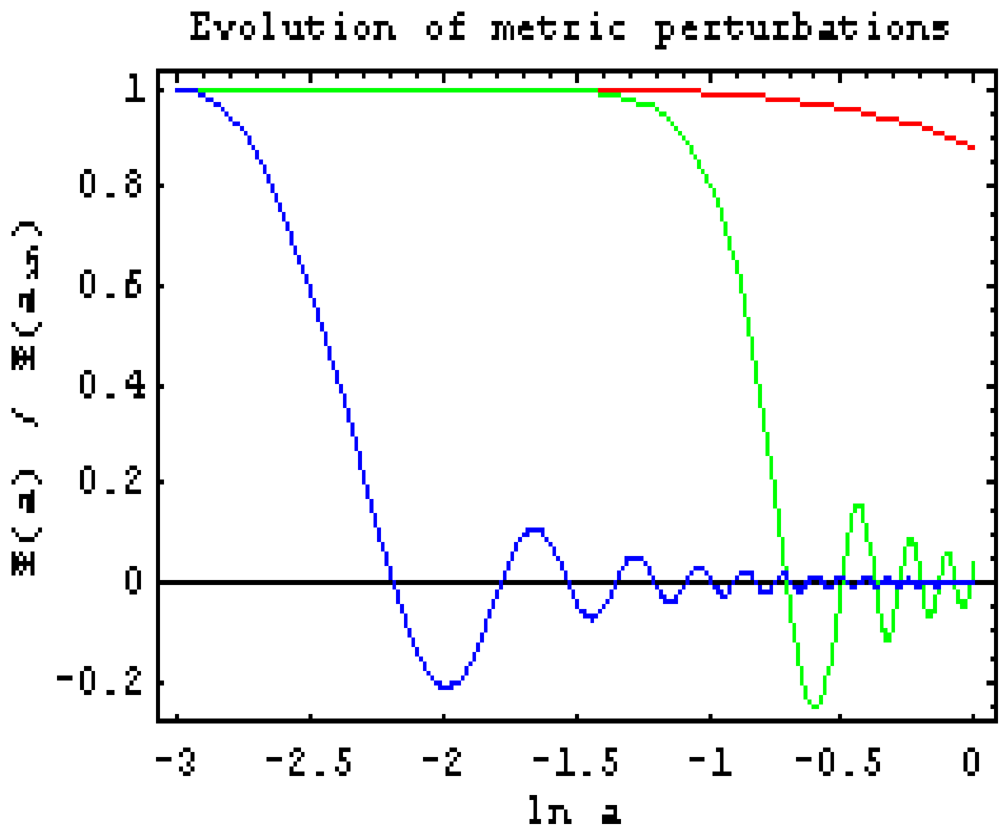
$(\Omega_m = 0.3,\ \Omega_\Lambda = 0.7)$

Chaplygin fluid

$(\alpha = 0.5,\ \xi = 10)$

Corresponding quintessence

$\xi = k / H_0$



LCDM

$(\Omega_m = 0.3,\ \Omega_\Lambda = 0.7)$

Chaplygin fluid

$(\alpha = 0.5,\ \xi = 100)$

Corresponding quintessence

5. Summary & Discussions

- Dynamics of linear perturbations can be a tool of exploring unified dark energy models

- Dynamics of linear perturbations coincides between a fluid dark energy and the corresponding scalar-field quintessence in the large-scale limit but are quite different in small scales.

- Does not coincide between a fluid model and a general scalar-field model generally even in the large-scale limit.

- Damping of metric perturbations will cause the Integrated SW effect on CMB anisotropy and is strongly constrained by observations. (Amendola et al 2003) $0 \leq \alpha < 2$

- To construct a viable model of DE-DM unification, the sound velocity should be designed to be small.