Model of dark matter and dark energy based on gravitational polarization

Λ-CDM and MOND finally speaking to each other

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Based on a collaboration with L. Blanchet (astro-ph/0804.3518)

Outline

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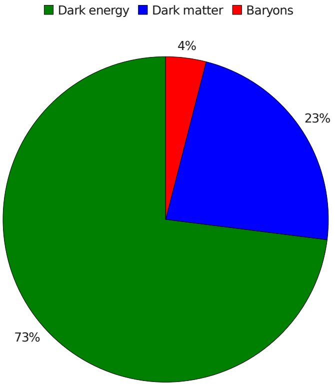
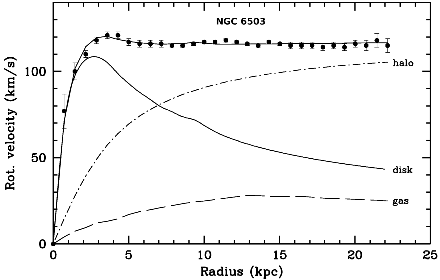
The concordance model in cosmology

- $\Omega_\Lambda = 73\%$ from Hubble diagram of supernovas
- $\Omega_\mathrm{B} = 4\%$ from Big-Bang nucleosynthesis and CMB anisotropies
- Non-baryonic dark matter accounts for the observed dynamical mass of galaxies and galaxy clusters



Galaxies are dominated by non-baryonic dark matter



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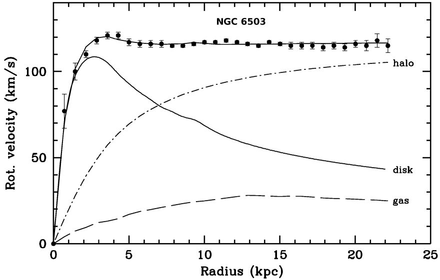

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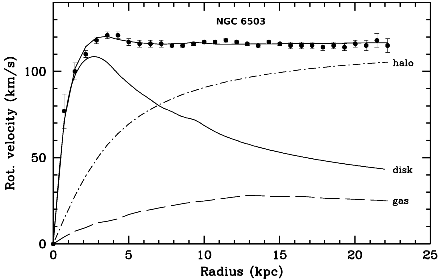
$$V_{\text{rot}}(r) = \sqrt{\frac{GM(r)}{r}}$$

- because $V_{\text{rot}} \simeq \text{cst.}$

$$M_{\text{halo}}(r) \propto r$$
$$\rho_{\text{halo}}(r) \propto \frac{1}{r^2}$$



The dark matter paradigm

- Dark matter is made of unknown non-baryonic particules, e.g.
  - Supersymmetric candidates (neutralinos, gravitinos, ...)
  - Kaluza-Klein states
  - Axions
  - ...

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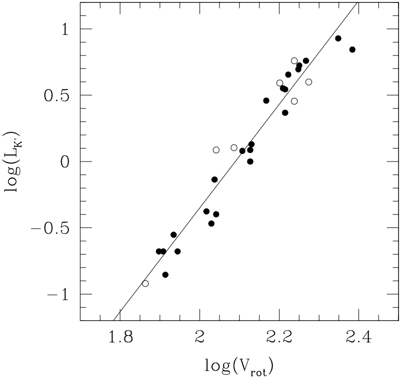
- It triggers the formation of large-scale structures by gravitational collapse and predicts the scale dependence of density fluctuations.
- It has difficulties explaining naturally the flat rotation curves of galaxies and the Tully-Fisher relation.

The Tully-Fisher empirical relation

- luminosity L
- rotation velocity $V_{\rm rot}$

$$L \propto V_{\rm rot}^4$$



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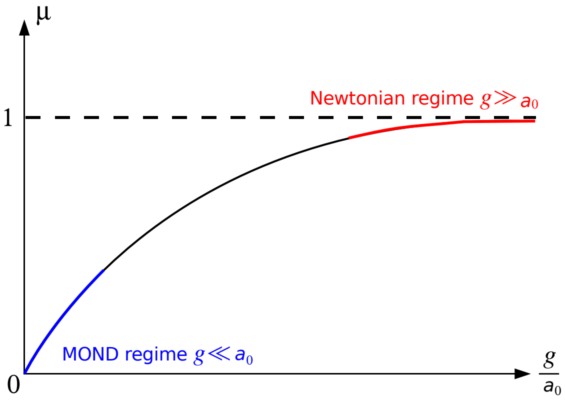
- Baryonic matter mass density ρ_b

The universal MOND function



The deep MOND regime

When $g \ll a_0$

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For a circular motion

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In order to fit the $(L, V_{\rm rot})$ diagram...

...we need $a_0 \simeq 10^{-10}$ m/s^2

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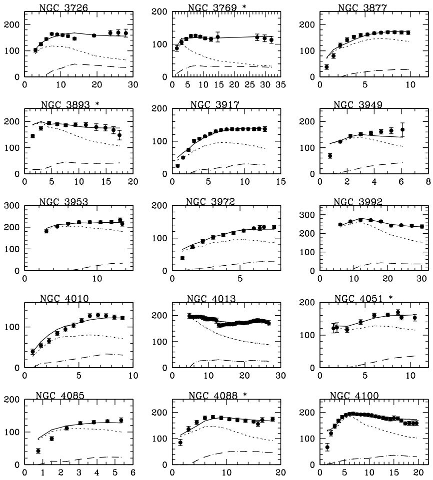

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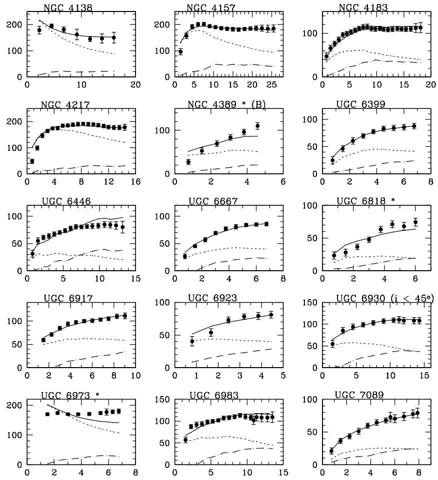

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- ✔✘ Needs a relativistic extension to be applied in cosmology, e.g. TeVeS, which is a rather complicated model.

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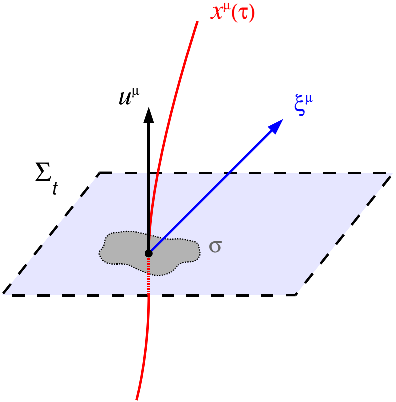
- ✔ Is based on the well-known physical mechanism of polarization.
- ✘ Is made of unknown non-baryonic particles whose fundamental structure has yet to be understood.

Action of the dipolar fluid

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, L\left[J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu}\right]$$



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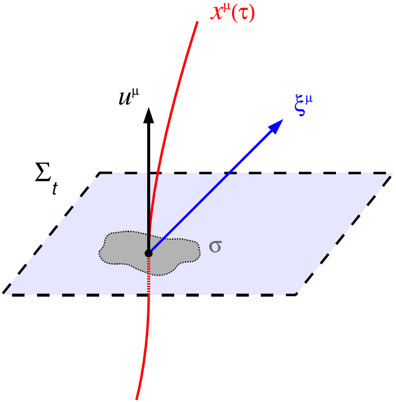

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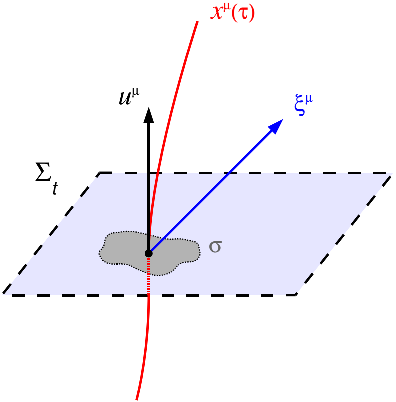

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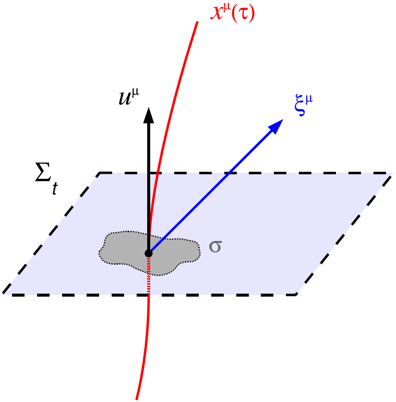

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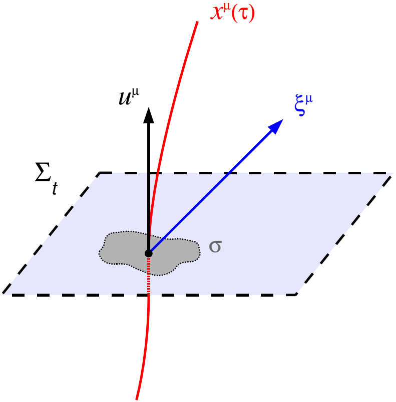
$$\dot{\xi}^\mu = u^\nu \nabla_\nu \xi^\mu$$



Lagrangian of the dipolar fluid

$$L = \sigma \left[-1 - \sqrt{\left(u_\mu - \dot{\xi}_\mu\right)\left(u^\mu - \dot{\xi}^\mu\right)} + \frac{1}{2}\dot{\xi}_\mu \dot{\xi}^\mu \right] - \mathcal{W}(\Pi_\perp)$$

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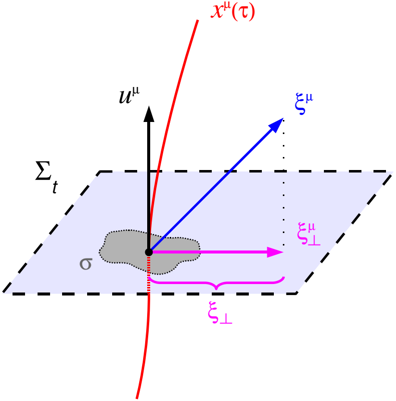
- "Fundamental" potential $\mathcal{W}$ function of the polarization $\Pi_\perp$

The polarization field $\Pi_\perp$

Project the dipole moment

$$\xi_\perp^\mu = \perp^\mu_\nu \, \xi^\nu$$



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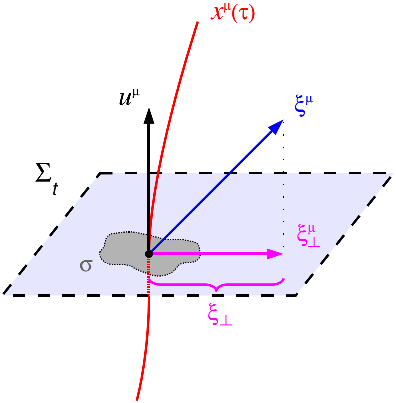
Take its norm

$$\xi_\perp = \sqrt{g_{\mu\nu} \, \xi_\perp^\mu \xi_\perp^\nu}$$



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The polarization reads

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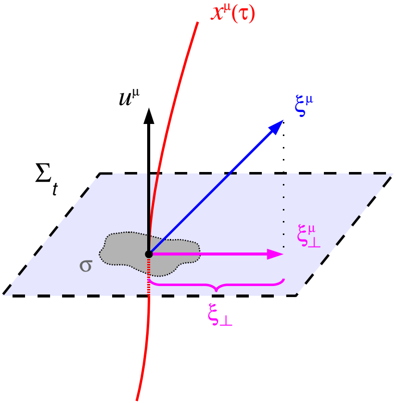

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The final equations (and stress-energy tensor) depend only on the space-like projection $\xi_\perp^\mu = \perp^\mu_\nu \xi^\nu$ of the dipole moment.

Equations of motion and evolution

Equation of motion of the dipolar fluid

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Equation of evolution of the dipole moment

$$\dot{\Omega}^\mu = -\frac{1}{\sigma} \underbrace{\nabla^\mu \left(\Pi_\perp \mathcal{W}' - \mathcal{W}\right)}_{\text{pressure term}} - \underbrace{\xi_\perp^\nu R^\mu{}_{\rho\nu\lambda} u^\rho u^\lambda}_{\text{Riemann coupling}}$$

$$\text{where} \quad \Omega^\mu = \perp^\mu_\nu \dot{\xi}_\perp^\nu + u^\mu \left(1 + \xi_\perp \mathcal{W}'\right)$$

Stress-energy tensor

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Perturbation around a FLRW background

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Standard SVT gauge-invariant formalism

$$\delta u^\mu = \frac{1}{a}\left(-A, \beta^i\right) \quad \text{where} \quad \beta^i = D^i v + v^i$$

$$\delta\xi_\perp^\mu = \left(0, \lambda^i\right) \quad \text{where} \quad \lambda^i = D^i y + y^i$$

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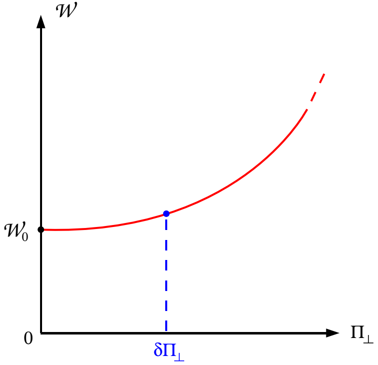
Harmonic potential

$$\mathcal{W} = \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}(3)$$



The fundamental potential

Perturbed polarization

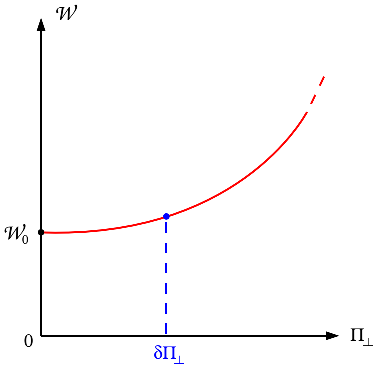
$$\Pi_\perp = \overline{\Pi}_\perp + \delta\Pi_\perp$$

Harmonic potential

$$\mathcal{W} = \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \mathcal{O}(3)$$

Internal force

$$\mathcal{F}^\mu = \hat{\Pi}_\perp^\mu \mathcal{W}' = \mathcal{W}_2\,\delta\Pi_\perp^\mu + \mathcal{O}(2)$$



Equations of motion and evolution

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Equation of motion $\dot{u}^\mu = -\mathcal{F}^\mu$

$$
\begin{aligned}
V' + \mathcal{H} V_i + \Phi &= -\mathcal{W}_2\, a^2 \overline{\sigma}\, y \\
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Equation of evolution $\ddot{\xi}_\perp^\mu = \ldots$

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- The motion is non-geodesic because of the internal force.
- The dipole moment evolution decouples!
- The scalar and vector modes satisfy the same equation!

Growth of the dipole moment

Growth of the dipole moment

The dipole moment $\delta\xi_\perp^\mu = (0, \lambda^i)$ satisfies

$$\lambda_i'' + \mathcal{H}\,\lambda_i' = \mathcal{W}_2\, a^2 \overline{\sigma}\,\lambda_i$$

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- At linear order, $D_i\lambda^i = \Delta y$ contributes in initiating the growth of structures similarly to the density contrast $\delta\rho/\overline{\rho}$ of a dark matter perfect fluid.
- Nice interplay between cosmology at large scales and galactic physics via MOND.

Dipolar fluid stress-energy tensor

At first perturbation order

$$T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$$

Dipolar fluid stress-energy tensor

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where we explicitely have

$$\begin{aligned} T^{\mu\nu}_{\rm de} &= -\mathcal{W}_0\, g^{\mu\nu} = -\frac{\Lambda}{8\pi}\, g^{\mu\nu} \\ T^{\mu\nu}_{\rm dm} &= \underbrace{\left(\sigma - D_i \Pi^i_\perp\right)}_{\text{energy density } \rho} u^\mu u^\nu + \underbrace{2Q^{(\mu} u^{\nu)}}_{\text{heat flow term}} \end{aligned}$$

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with $\delta\widetilde{u}^\mu = \delta u^\mu + \frac{Q^\mu}{\overline{\sigma}}$

This is the stress-energy tensor of a perfect fluid with vanishing pressure and a four-velocity $\widetilde{u}^\mu = \overline{u}^\nu + \delta\widetilde{u}^\mu$!

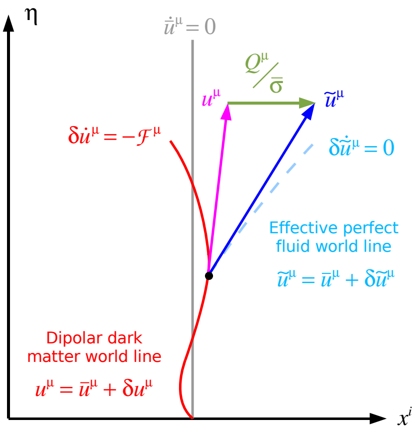
The effective four-velocity

$$\delta\widetilde{u}^\mu = \delta u^\mu + \frac{Q^\mu}{\overline{\sigma}}$$

$$\Downarrow$$

$$\widetilde{V} = V + y'$$

$$\widetilde{V}_i = V_i + y'_i$$



Dipolar dark matter as an effective perfect fluid

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Effective perfect fluid

$$\widetilde{V}' + \mathcal{H} \widetilde{V} + \Phi = 0$$
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- The dipolar DM fluid is undistinguishable from standard DM at the level of first-order cosmological perturbations.

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- The dipolar DM fluid is undistinguishable from standard DM at the level of first-order cosmological perturbations.
- Adjusting $\overline{\sigma}$ so that $\Omega_{\mathrm{dm}} \simeq 0.23$ the model is consistent with observations of CMB fluctuations.

The relativistic equations...

- Equation of motion

$$\frac{\mathrm{D}u^\mu}{\mathrm{d}\tau} = -\mathcal{F}^\mu = -\hat{\Pi}^\mu_\perp \mathcal{W}'$$

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- Einstein equations

$$G^{\mu\nu} = 8\pi \sum_{\mathrm{f}} T^{\mu\nu}_{\mathrm{f}}$$

...and their non-relativistic limit

- Equation of motion

$$\frac{\mathrm{d}v^i}{\mathrm{d}t} - g^i = -\mathcal{F}^i = -\hat{\Pi}^i_\perp \mathcal{W}'$$

- Equation of evolution

$$\sigma \frac{\mathrm{d}^2\xi^i_\perp}{\mathrm{d}t^2} - \hat{\Pi}^i_\perp \mathcal{W}' = -\partial_i \left(\Pi_\perp \mathcal{W}' - \mathcal{W}\right) + \Pi^j_\perp \partial_j g^i$$

- Continuity equation

$$\partial_t \sigma + \partial_i \left(\sigma v^i\right) = 0$$

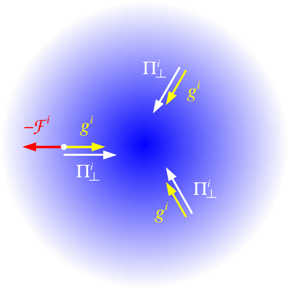
- Poisson equation

$$\partial_i g^i = -4\pi G \left(\rho_\mathrm{b} + \rho_\mathrm{dm}\right)$$

A particular solution

If baryonic matter is modeled by a mass distribution $\rho_b(r)$, there is a solution where the dipolar dark matter distribution is...

- Spherical : $\sigma = \sigma_0(r)$
- At rest : $v^i = 0$
- In equilibrium : $g^i = \mathcal{F}^i$
- Stationary : $\Pi^i_\perp = \text{const.}$
- Polarized : $\Pi^i_\perp \parallel g^i$



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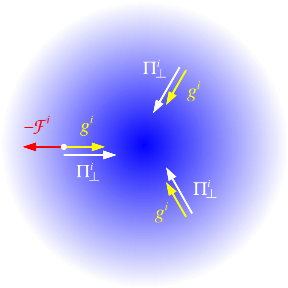
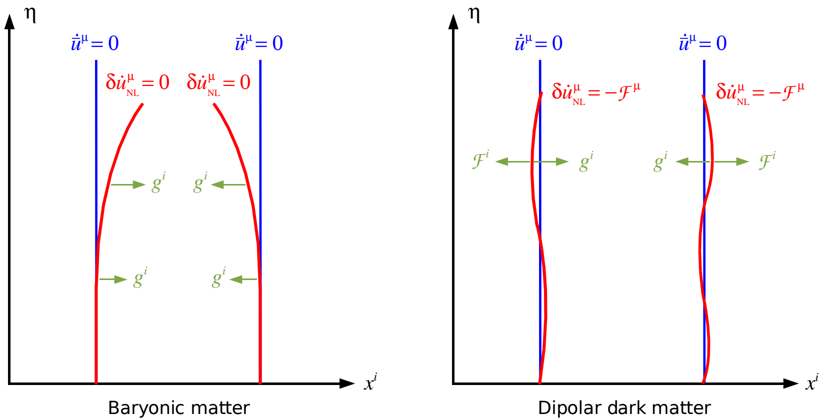


This motivates the "weak clustering hypothesis" : $\sigma \simeq \overline{\sigma} \ll \rho_b$

The weak clustering hypothesis



From the Poisson equation to the MOND equation

The Poisson equation (with $\rho_{\rm dm} = \sigma - \partial_i \Pi^i_\perp$) reads

$$\partial_i g^i = -4\pi G \left(\rho_{\rm b} + \sigma - \partial_i \Pi^i_\perp\right)$$

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But according to the equation of motion (with $v^i = 0$)

$$g^i = \mathcal{F}^i = \hat{\Pi}^i_\perp \mathcal{W}'(\Pi_\perp)$$

From the Poisson equation to the MOND equation

Inverting this relation yields

$$\Pi^i_\perp = -\frac{\chi(g)}{4\pi G}\, g^i \qquad \text{where } \chi(g) \text{ is related to } \mathcal{W}(\Pi_\perp)$$

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We finally recover the MOND equation

$$\partial_i\left(\mu\, g^i\right) = -4\pi G\, \rho_\text{b} \qquad \text{with} \qquad \mu = 1 + \chi$$

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- The dipolar dark matter benefits from the various successes of the phenomenology of MOND.
- It provides a simple explanation for this phenomenology through the physical mechanism of polarization.

The fundamental potential

In the MOND regime ($g \ll a_0 \iff \Pi_\perp \ll \Sigma \equiv a_0/2\pi G$)

$$\mathcal{W}(\Pi_\perp) = \mathcal{W}_0 + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \frac{1}{6}\mathcal{W}_3\,\Pi_\perp^3 + \mathcal{O}(4)$$

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- Dark energy as a “vacuum polarization”

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In the MOND regime ($g \ll a_0 \iff \Pi_\perp \ll \Sigma \equiv a_0/2\pi G$)

$$\mathcal{W}(\Pi_\perp) = \frac{\Lambda c^4}{8\pi G} + \frac{1}{2}\mathcal{W}_2\,\Pi_\perp^2 + \frac{1}{6}\mathcal{W}_3\,\Pi_\perp^3 + \mathcal{O}(4)$$

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Is $\mathcal{W}$ a simple function of the dimensionless variable $x \equiv \Pi_\perp/\Sigma$?

The fundamental potential

In terms of the variable $x \equiv \Pi_\perp/\Sigma$

$$\mathcal{W}(x) = 6\pi G\,\Sigma^2 \left\{\alpha^2\pi^2 + \frac{1}{3}x^2 + \frac{4}{9}x^3 + \mathcal{O}(4)\right\}$$

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- The potential $\mathcal{W}$ can be expressed in terms of the dimensionless variable $\Pi_\perp/\Sigma$ and the constants G and a_0 only.
- If one does so, the numerical coincidence $\Lambda \sim a_0^2$ noticed long ago finds a natural explanation.

Conclusion

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- ✘ Is not (yet) related to any fundamental quantum theory.

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- To clarify how large-scale structures emerge from initial perturbations in the non-linear regime.