Differentially rotating neutron stars: A perturbative study

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Outline

Astrophysical motivation
Outline of perturbative method
Results
Discussion-Conclusions

Astrophysical motivation

Neutron stars are born with differential rotation but as they cool differential rotation (DR) is smoothed out to uniform

Window with DR that can be astrophysically interesting

Newtonian studies

- Hansen et.al, ApJ, **217**, 151, 1977, effect of DR on mode splitting
- Karino & Eriguchi, ApJ, **578**, 413, 2002, GR reaction & f-mode

Full GR studies

- Komatsu et.al, MNRAS, **239**, 153, 1989, Construct DR models, F = f(M,J,K,n) parameter to classify equilibrium models.
- Stergioulas et.al MNRAS, **352**, 1089, 2004, splitting of f-mode
- Dimmelmeier et.al. MNRAS, **368**, 1609, 2006, Axisymmetric modes CFC

Perturbative GR studies missing….

Low T/W instability

Newtonian simulations have shown that stars with high degree of differential rotation, show a dynamical instability at low values of T/W~0.08 to 0.14.

- J. Centrella et.al, ApJ, **550**, L196, 2001→ N=3.33 polytrope, T/W ~ 0.14
- Shibata et.el MNRAS, **334**, L27, 2002 → N=1 polytrope T/W ~ 0.01
- A. Watts et.al ApJ, **618**, L37, 2005 → due to f-mode entering into corotation band
- Saio & Yoshida, MNRAS, **368**, 1429, 2006 → diagnosis with canonical angular momentum.
- Ou & Tohline, ApJ, **651**, 1068, 2006 → not necessarily rapidly rotating
- New & Shapiro, ApJ, **548**, 439, 2001 → for supermassive neutron stars

GW emitted could be detected by LISA in super massive NS

Perturbative method

Assumptions

✓ Slow rotation (star is spherical), $0<\varepsilon_e = \Omega_e / \Omega_K<1$

✓J-constant differential rotation law

$$\Omega(r,\theta) = \frac{A^2\Omega_c + e^{-2\nu}\omega(r,\theta)r^2\sin^2\theta}{A^2 + e^{-2\nu}r^2\sin^2\theta}$$

✓Relativistic polytrope, $p=K\rho^\Gamma$, $\varepsilon = \rho + p / (\Gamma-1)$

✓ Cowling approximation (neglect spacetime perturbations)

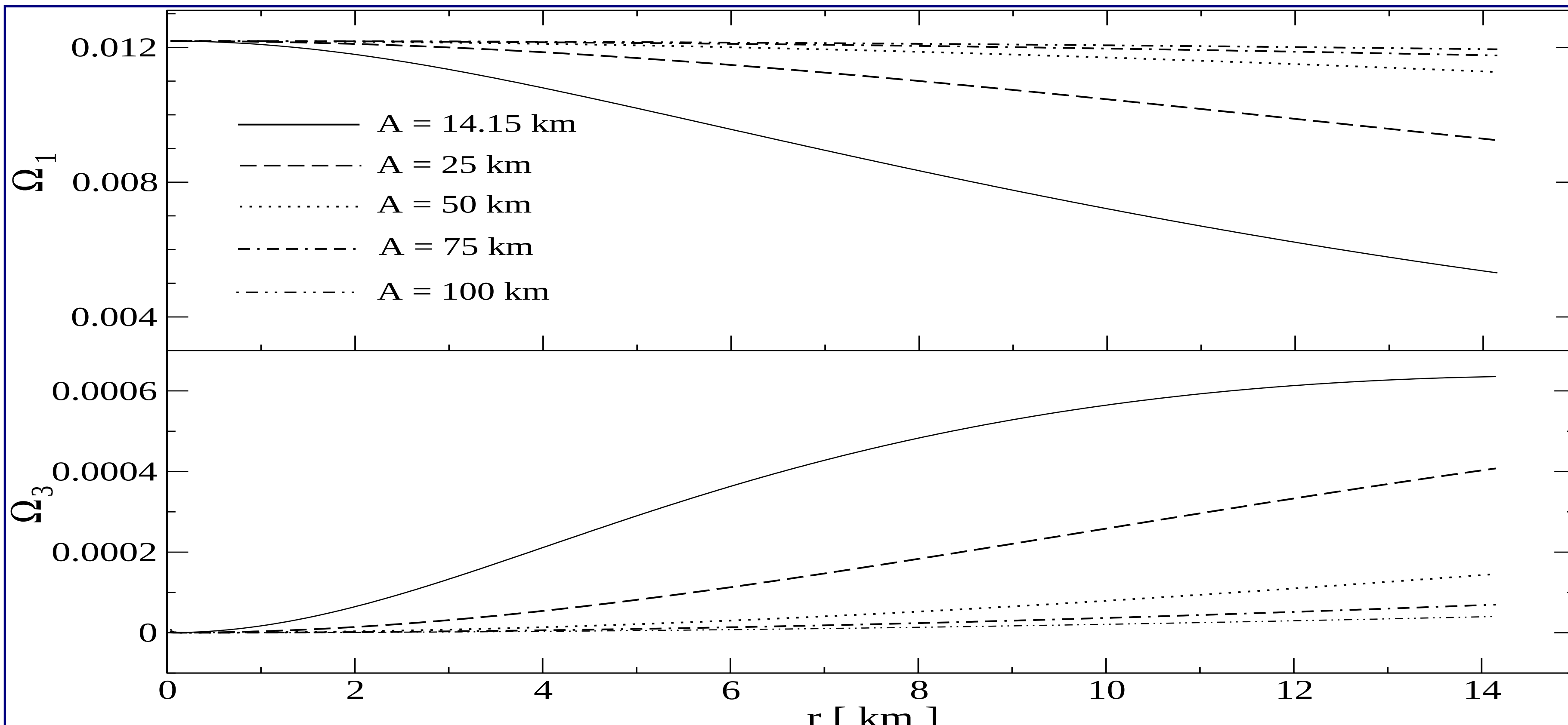
✓ Non barotropic perturbations $\Gamma \neq \Gamma_1$

Differential rotation background

TOV equations + equation for frame dragging

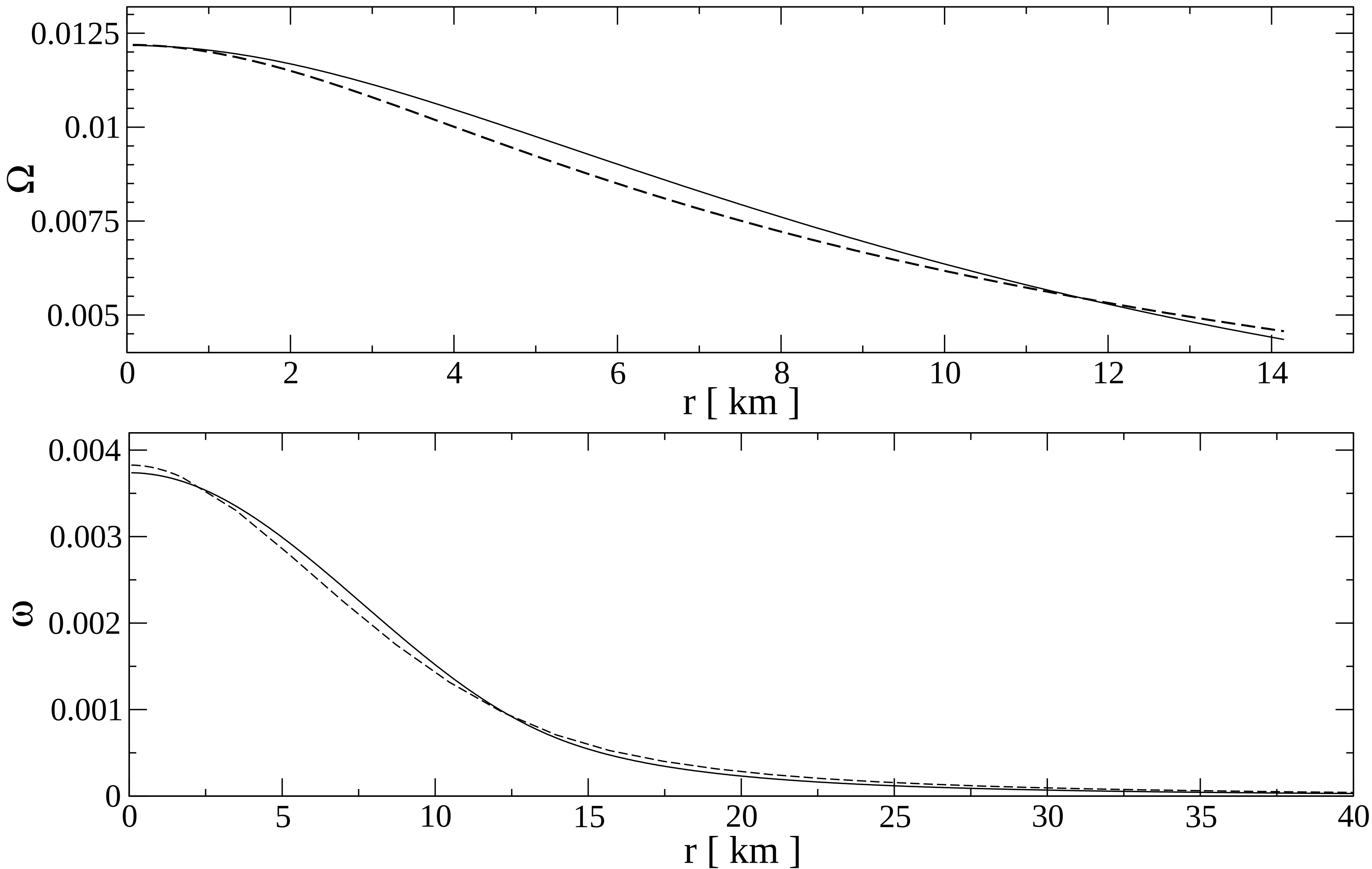
$$\omega'' - [4\pi(\varepsilon + p) r e^{2\lambda} - \frac{4}{r}]\omega' - [16\pi(\varepsilon + p) + \frac{\Lambda - 2}{r^2}] e^{2\lambda}\omega = -16\pi(\varepsilon + p) e^{2\lambda}\Omega$$

Expand terms ω, Ω in spherical harmonics, ω_1, ω_3, Ω_1, Ω_3



Comparison with non-linear backgrounds

RNS code by Stergioulas



Perturbative Equations & Method

Perturbed conservation of energy-momentum

$$\delta\,(\,T_{\mu\nu}{}^{;\,\mu}\,) = 0$$

Expand all variables in spherical harmonics

$$A(t,r,\theta,\phi) = R(r,t)\;Y_{lm}\,(\theta,\phi)$$

Integrate over solid angle to get PDEs of (r,t)

5 independent variables

4 polar (f,p,g modes)

1 axial (r,inertial modes,CS)

Infinitely coupled hyperbolic system of PDEs

$$P_{lm} + l\,m\,(\,P_{lm} + A_{l\pm1} + P_{l\pm2}\,) + A_{l\pm1} + A_{l\pm3} = 0$$

$$A_{lm} + l\,m\,(\,A_{lm} + P_{l\pm1} + A_{l\pm2}\,) + P_{l\pm1} + P_{l\pm3} = 0$$

To solve them we have to truncate for l_{max}

Results

Effects of rotation to stellar modes

- *Mixes the character of the modes*

  polar-led, polar in non rotating limit

  axial-led, axial in non rotating limit

- *Splitting of modes (like Zeeman effect in atomic physics)*

$$\sigma^{lm} = \sigma_0^{lm} \pm \alpha(l,m,A)\, \varepsilon_e$$

Study the effects with respect to the three parameters

- *Maximum number of couplings l_{max}*
- *Azimuthal index m*
- *Degree of differential rotation A*

Effect of l_{max} to quasi radial frequencies

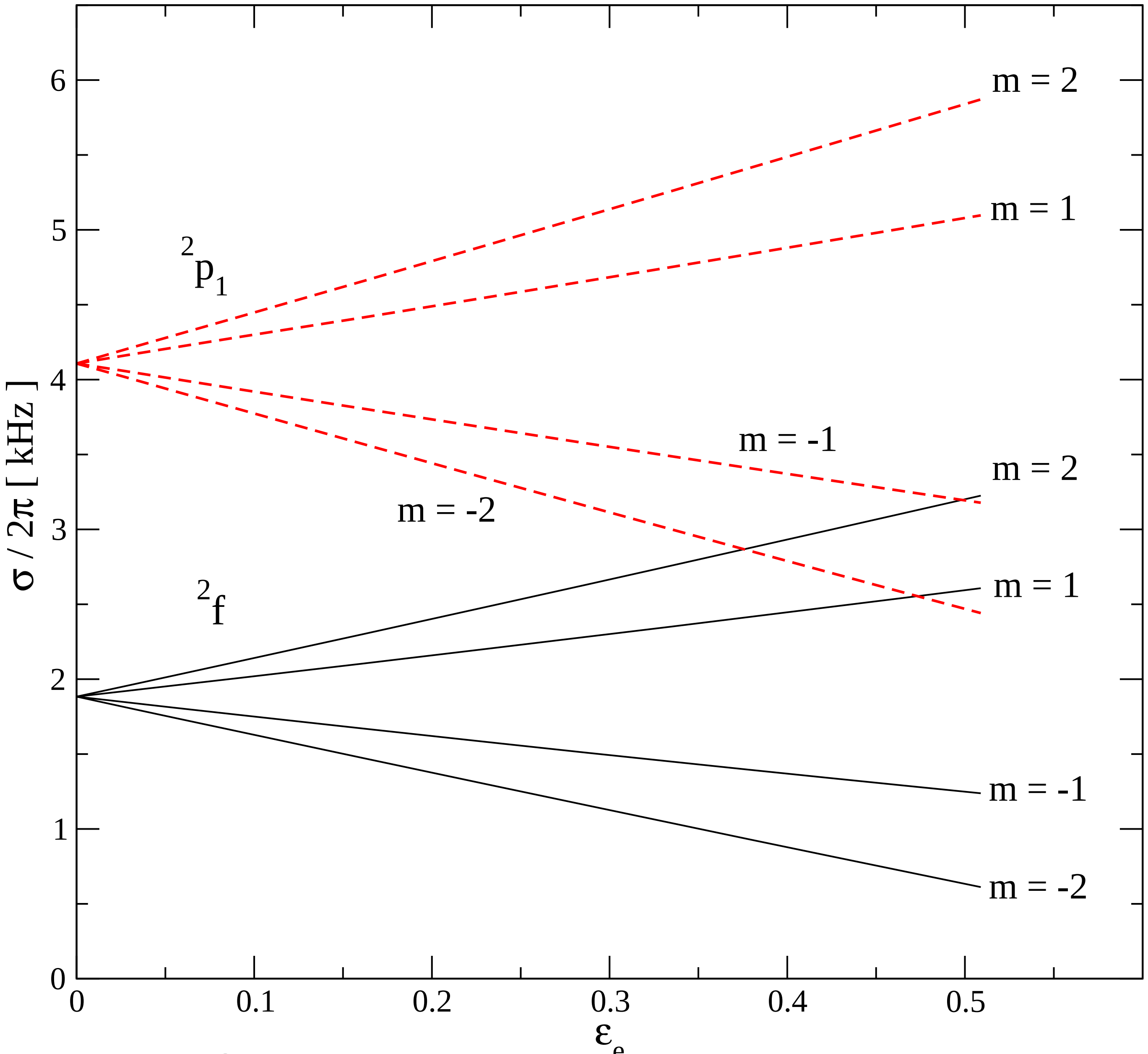
l_{max}	F (kHz)	H_1 (kHz)	
0	2.687	4.551	Radial
1	2.710	4.571	Dipole
2	2.712	4.575	Non-radial l=2
3	2.712	4.575	Non-radial l=3

Frequencies have converged for $l_{max}=2$

Axis-symmetric perturbations Comparison with non-linear results

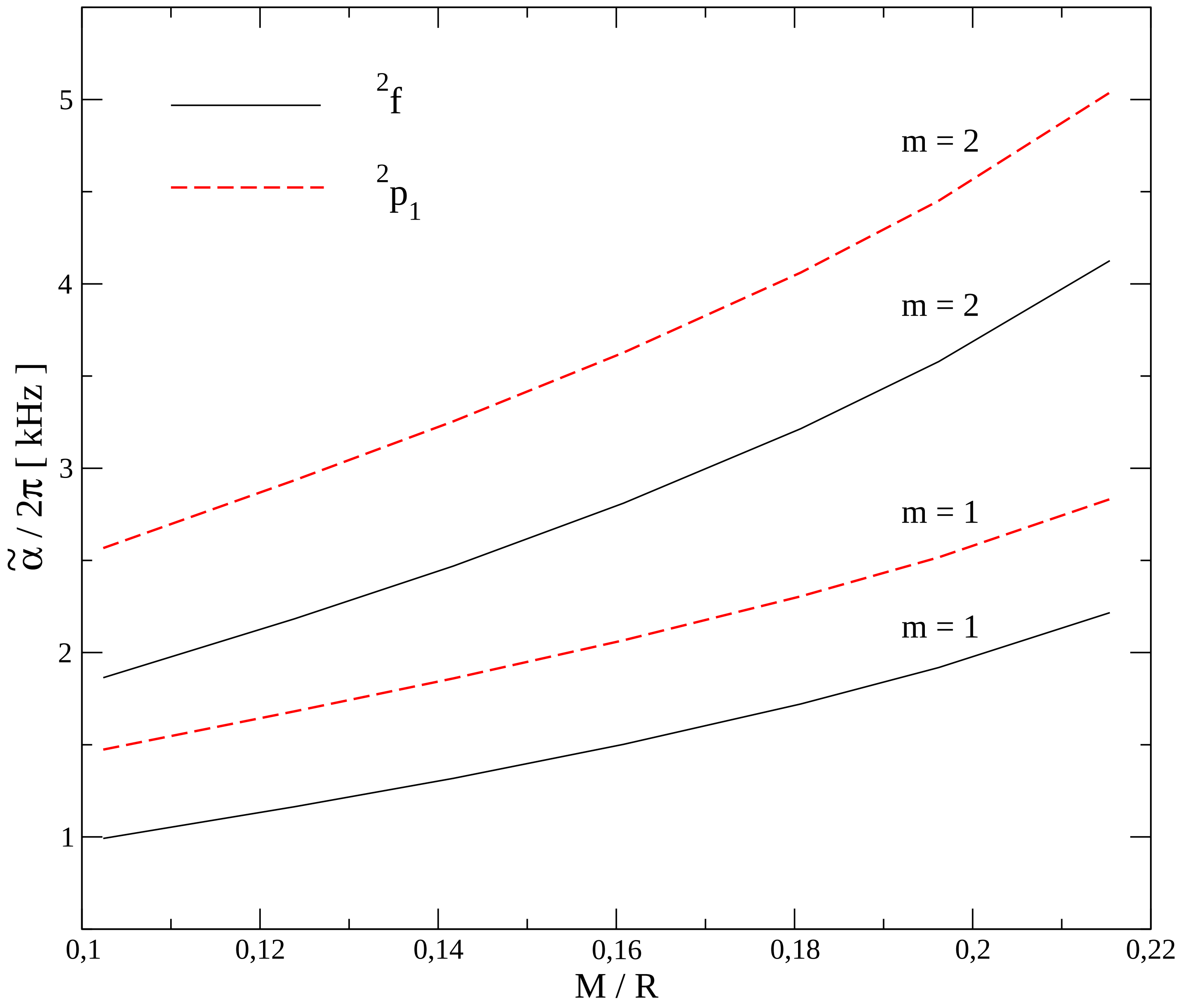
Model	F (kHz)	H_1 (kHz)	f_2 (kHz)	p_2 (kHz)
B0	2.706	4.547	1.846	4.100
B1	2.702 (2%)	4.555 (2%)	1.895 (1%)	4.117 (1%)
B3	2.735 (4%)	4.578 (4%)	1.915 (1%)	4.124 (2%)
B6	2.797 (6%)	4.624 (4%)	1.944 (1%)	4.134 (7%)
B9	2.885 (8%)	4.686 (6%)	1.974 (8%)	4.147 (14%)

Modes splitting



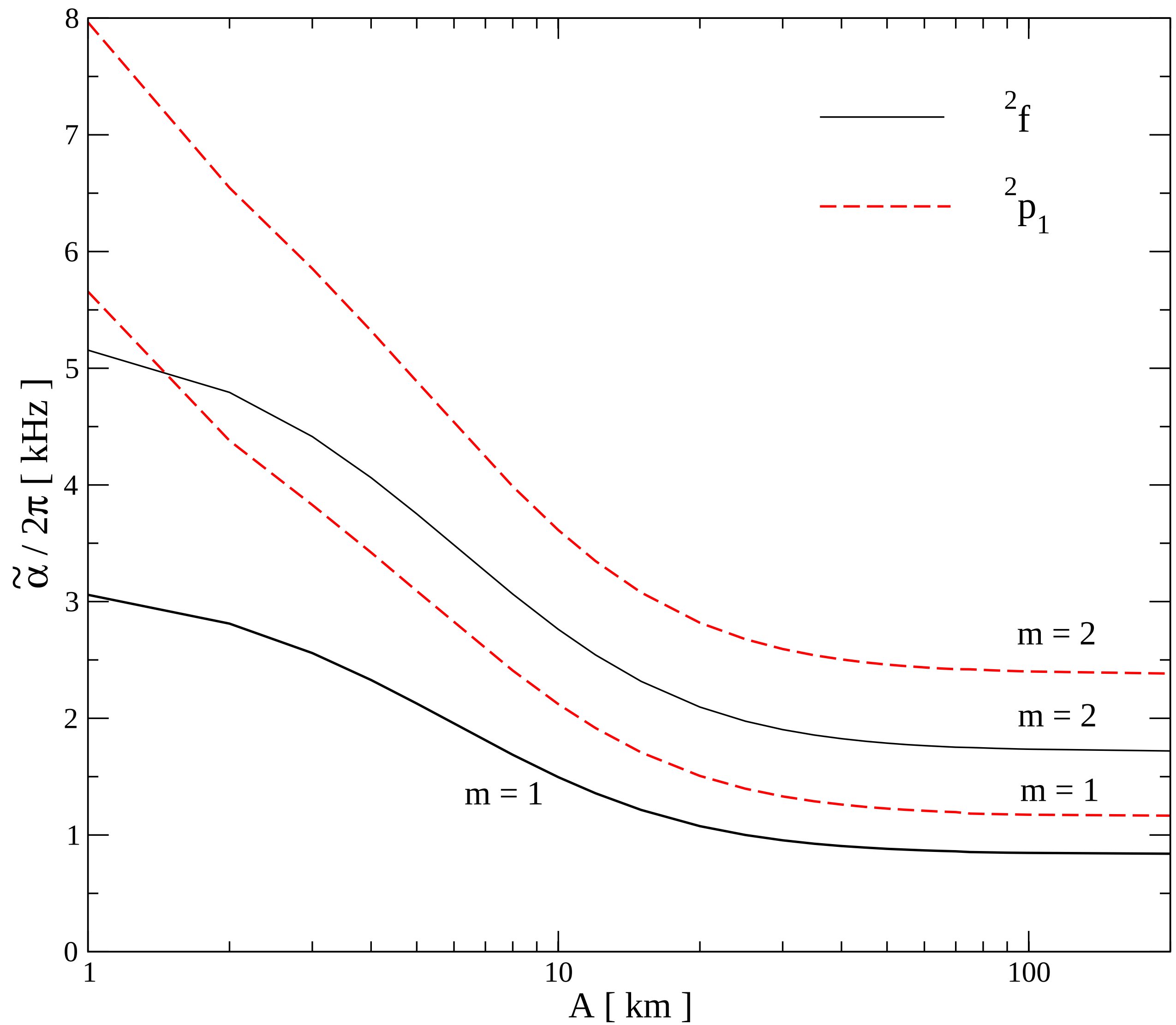
A.S, Passamonti, Kokkotas, PRD **75**, 064019, 2007.

Dependence of splitting to compactness



Passamonti, A.S, Kokkotas, PRD accepted

Dependence of splitting to degree of differential rotation



Passamonti, A.S, Kokkotas, PRD accepted

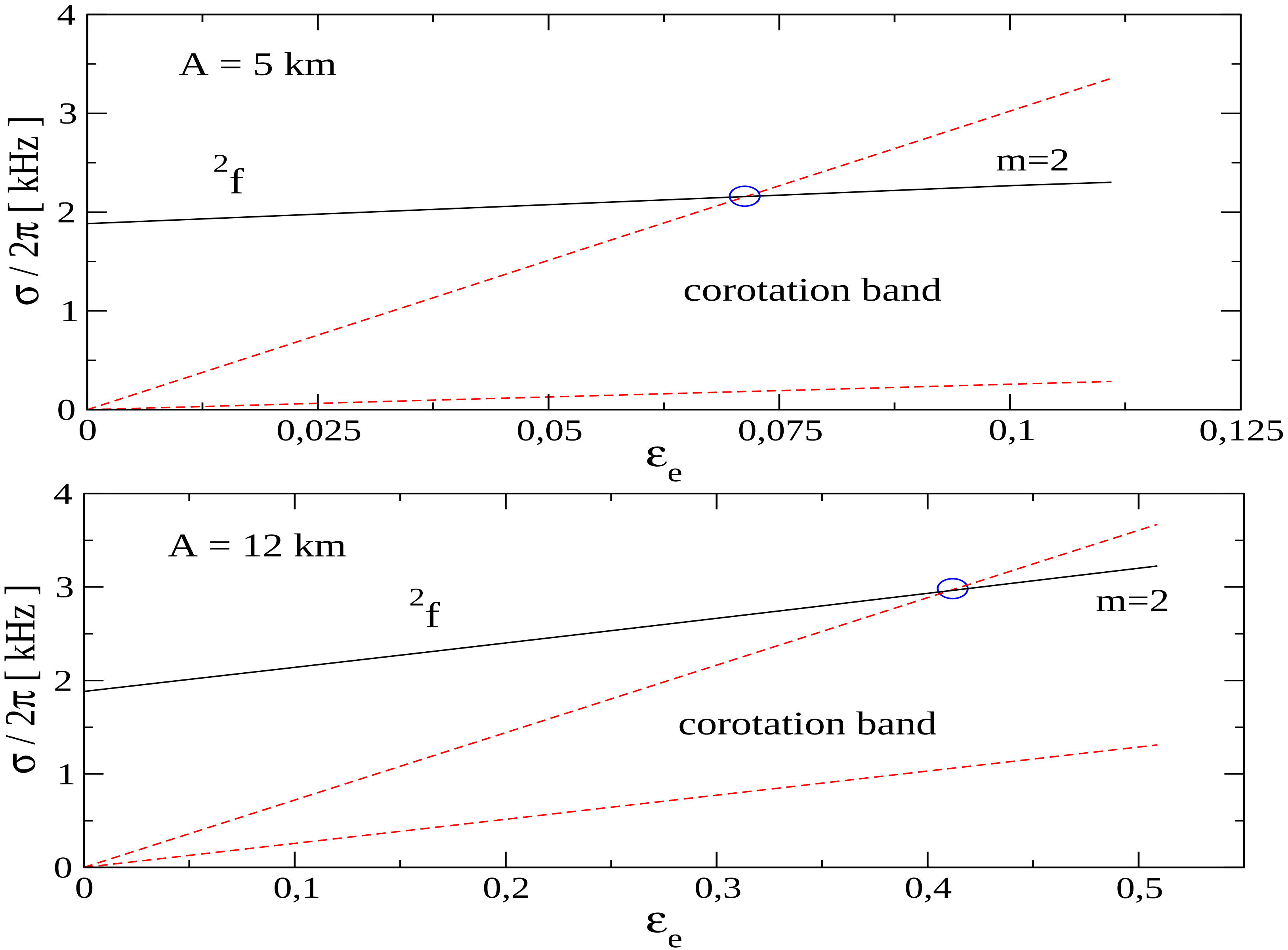
Modes and corotation

Pattern speed of the mode : σ/m

Corotation band : $\Omega_e < \Omega < \Omega_s$

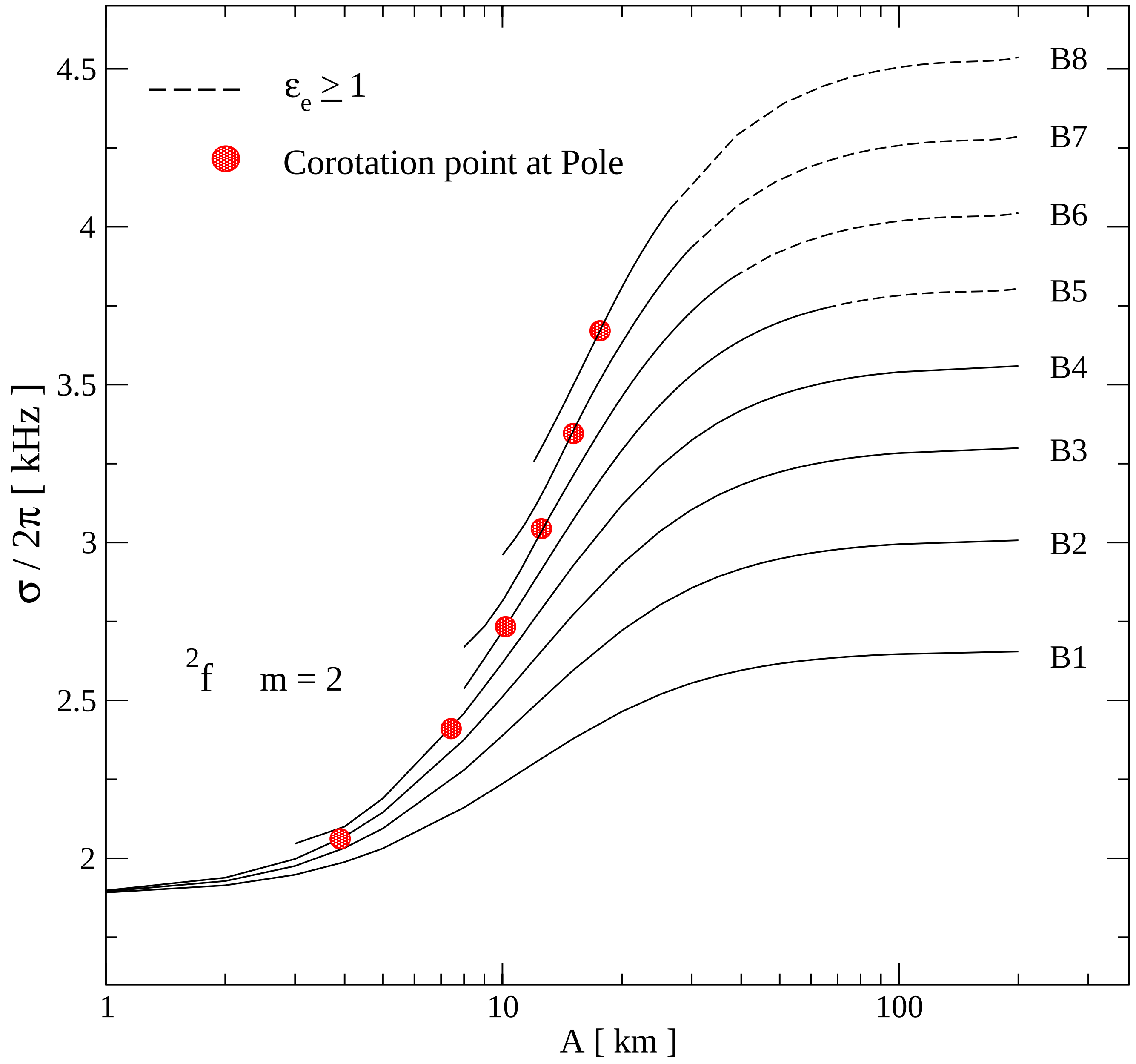
If the pattern speed of the mode is equal to the local angular velocity of the star we have a corotation mode

f - mode and Corotation band



Passamonti, A.S, Kokkotas, PRD accepted

Corotation points for different models



Epilogue Discussion

Drawbacks of our approximation

- ❑ Definition of rotational parameter $\varepsilon = \Omega_e / \Omega_K$ with respect to T/ W
- ❑ Cold polytropic EoS
- ❑ Real code, cannot see damping/growth rate of modes

On going work and extensions :

- ❑ Use hot EoS for nascent neutron stars to study g-modes
- ❑Use relativistic canonical energy (??) to study the T/W instability.

Long term goal : GW asteroseismology, estimate stellar parameter from the detected GW signal.

Based on papers:

❑A.Stavridis, A. Passamonti, K.D. Kokkotas, PRD, 75, 064019, 2007
❑A. Passamonti, A.Stavridis, K.D. Kokkotas, PRD, accepted..