Averaged inhomogeneous cosmologies and accelerated expansion: the morphon field.

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GRECO seminar

03/19/2007

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Class. Quantum Grav.23 (2006) 6379-6408 (gr-qc/0606020)

astro-ph/0609315

astro-ph/0612774

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Observational facts

In the late time Universe, matter distribution appears:

- Homogeneous on average at scales larger than ~ 100 Mpc (Yadav & al, MNRAS 2005)
- Highly structured at smaller scales (galaxies, clusters, filaments, walls, voids)

But, in General Relativity, matter and geometry of space-time are tighly coupled. How can we describe our Universe on large scales:

- dynamics?
- geometry (measure of distances)?

The cosmological principle is the guide of cosmologists.

The cosmological principles (I)

Friedmann Universe: the strong cosmological principle

- Ignore the inhomogeneities: the Universe is locally homogeneous and isotropic.
- Maximally symmetric, homogeneous and isotropic space: $ds^2 = -dt^2 + a^2(t)dl^2$; spatial section of constant curvature k.
- Dynamics: second order differential equations for $a(t)$; the so-called Friedmann equations.
- Distances are computed with the line element $ds^2 = -dt^2 + a^2(t)dl^2$.
- Structure formation explained separetely with a perturbative expansion around that background.

- Great success for the early Universe that is highly homogeneous on all scales (CMB).
- Is it still relevant in the highly structured late time Universe?

The cosmological principles (II)

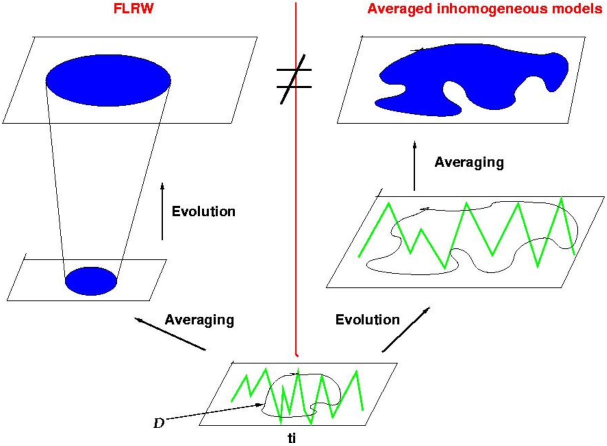
The weak cosmological principle

- We retain the averaged homogeneity on large scales: the large scale observables can be deduced from purely time-dependent functions.
- But, no assumption on the local structure of space-time and matter distribution.
- Local dynamics obeys the general Einstein equations.

- Defining the homogeneous model: averaging procedure?
- Dynamical equations?

Evolving the average or averaging the evolution?

$$[\partial_t \cdot, \langle \cdot \rangle_{\mathcal{D}}] \neq 0$$



Homogeneous Universes: Hypotheses

Framework

- 3+1 foliation of space-time:
  $ds^2 = -N(t,\vec{x})dt^2 + g_{ij}(t,\vec{x})(dx^i + N^i(t,\vec{x})dt)(dx^j + N^j(t,\vec{x})dt)$
- Late time Universe: irrotational perfect fluid of dust matter: $\rho(t,\vec{x})$.
- Observers comoving with the fluid.
- Then: $N(t,\vec{x}) = 1$ and $N^i(t,\vec{x}) = 0 \Rightarrow ds^2 = -dt^2 + g_{ij}(t,\vec{x})dx^i dx^j$

Useful quantities:

- Extrinsic curvature, K_{ij} such that: $K_{ij} = -\frac{1}{2}\partial_t g_{ij}$.
- $-K_{ij} = \frac{1}{3}\theta g_{ij} + \sigma_{ij}$ with:
  - $\sigma_{ij}(t,\vec{x})$: shear tensor. $\sigma^i_i = 0$
  - $\theta(t,\vec{x})$: local expansion rate
- Intrinsic 3-curvature: R_{ij} and the associated 3-Ricci scalar R.

Defining the averaged dynamics

Einstein equations in 3+1 splitting:

- Constraints equations: $R + K^{i\,2}_i - K^i_j K^j_i = 16\pi G\rho$; $\nabla_i K^i_j - \partial_j K^i_i = 0$
- Evolution equations: $\partial_t g_{ij} = -2K_{ij}$; $\partial_t K^i_j = K^k_k K^i_j + R^i_j - 4\pi G\rho\delta^i_j$
- Conservation of energy-momentum: $\partial_t \rho + \theta\rho = 0$

Averaging procedure (Buchert, GRG, 2000 and 2001)

- Choose a spatial domain $\mathcal{D}$
- Volume of $\mathcal{D}$: $V_{\mathcal{D}}(t) = \int_{\mathcal{D}} J d^3x$, with $J = \sqrt{\det(g_{ij})}$
- Effective volume scale factor: $a_{\mathcal{D}}(t) = \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_\mathrm{i}}}\right)^{1/3}$
- Averaging operator for scalars: $\langle \Upsilon \rangle_{\mathcal{D}} = \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \Upsilon J d^3x$

We apply this averaging procedure to the scalar part of the equations.

Effective averaged equations

$$
\begin{aligned}
H_{\mathcal{D}}^2 &= \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle R\rangle_{\mathcal{D}}) \\
3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -4\pi G\langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} \\
\langle\rho\rangle_{\mathcal{D}} &= \rho_{\mathcal{D}_\mathrm{i}} a_{\mathcal{D}}^{-3} \\
\partial_t \mathcal{Q}_{\mathcal{D}} + 6H_{\mathcal{D}} &= -\partial_t\langle R\rangle_{\mathcal{D}} - 2H_{\mathcal{D}}\langle R\rangle_{\mathcal{D}}
\end{aligned}
$$

where:

- $H_{\mathcal{D}} = \frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}$: effective Hubble parameter
- $\langle R\rangle_{\mathcal{D}}$: averaged 3-curvature
- $\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left\langle(\theta - \langle\theta\rangle_{\mathcal{D}})^2\right\rangle_{\mathcal{D}} - \left\langle\sigma^{ij}\sigma_{ij}\right\rangle_{\mathcal{D}}$: fluctuation term called kinematical backreaction.

The averaged system is not closed:
the cosmological model requires a closure condition.

A remark on curvature

One can define a constant curvature $k_{\mathcal{D}_\mathrm{i}}$ à la Friedmann for the averaged system:

$$\boxed{\frac{k_{\mathcal{D}_\mathrm{i}}}{a_{\mathcal{D}}^2} = \frac{\langle\mathcal{R}\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}}{6} + 2\frac{1}{3a_{\mathcal{D}}^2}\int_1^{a_{\mathcal{D}}} a\mathcal{Q}_{\mathcal{D}}(a)da}$$

A priori, the averaged 3-curvature is not a Friedmannian constant curvature: General Relativistic effect: coupling between averaged 3-curvature and backreaction.

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Setting up the correspondence

Formal identification of $\mathcal{Q}_{\mathcal{D}}$ and $\langle \mathcal{R} \rangle_{\mathcal{D}}$ with a scalar field:

$$-\frac{1}{8\pi G}\mathcal{Q}_{\mathcal{D}} = \epsilon\dot{\Phi}^2_{\mathcal{D}} - U(\Phi_{\mathcal{D}}) \; , \; -\frac{1}{8\pi G}\langle \mathcal{R} \rangle_{\mathcal{D}} = 3U(\Phi_{\mathcal{D}}) \qquad (1)$$

Then, the averaged equations become (Buchert, Larena & Alimi, CQG, 2006):

$$\begin{aligned}
\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 &= \frac{8\pi G}{3}\left(\langle \rho \rangle_{\mathcal{D}} + \frac{\epsilon}{2}\dot{\Phi}^2_{\mathcal{D}} + U(\Phi_{\mathcal{D}})\right) \\
\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= -\frac{4\pi G}{3}\left(\langle \rho \rangle_{\mathcal{D}} + 2\epsilon\dot{\Phi}^2_{\mathcal{D}} - 2U(\Phi_{\mathcal{D}})\right) \\
\ddot{\Phi}_{\mathcal{D}} + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\dot{\Phi}_{\mathcal{D}} + \epsilon\frac{\partial U(\Phi_{\mathcal{D}})}{\partial \Phi_{\mathcal{D}}} &= 0
\end{aligned}$$

A homogeneous model on large scales for the Universe naturally leads to an additional scalar field source for the evolution of the cosmological scale factor.
Any information on a 'Friedmanian' scalar field can provide information on the fluctuations and there coupling to the averaged 3-curvature.

A thermodynamical analogy

The scalar field description allows to identify:

$$E^{\mathcal{D}}_{kin} = \frac{\epsilon}{2}\dot{\Phi}_{\mathcal{D}} V_{\mathcal{D}}$$
$$E^{\mathcal{D}}_{pot} = -U_{\mathcal{D}} V_{\mathcal{D}}$$

$$\boxed{\frac{E^{\mathcal{D}}_{kin}}{E^{\mathcal{D}}_{pot}} = -\frac{3}{2}\left(\frac{\mathcal{Q}_{\mathcal{D}}}{\langle R\rangle_{\mathcal{D}}} + \frac{1}{3}\right)}$$

When there is no fluctuation ($\langle R\rangle_{\mathcal{D}} \propto a_{\mathcal{D}}^{-2}$), we then have a 'virial' condition: $2E^{\mathcal{D}}_{kin} + E^{\mathcal{D}}_{pot} = 0$

$\Rightarrow$ Backreaction causes deviations from 'equilibrium'.

Late time accelerated expansion: the morphon as dark energy

- $a_{\mathcal{D}}$ accelerates iff $\mathcal{Q}_{\mathcal{D}} > 4\pi G \langle \rho \rangle_{\mathcal{D}}$.
- $\mathcal{Q}_{\mathcal{D}} V_{\mathcal{D}}$ grows with the fluctuations in the expansion rate.
- Late time Universe:
  - Highly underdense and overdense regions.
  - The fluctuations in the expansion rate are important and growing.
- We can expect that $\mathcal{Q}_{\mathcal{D}}$ becomes important.
- It can provide a solution to the coincidence problem.
- A two zones toy-model implies that the significant parameter is the proportion of voids in $V_{\mathcal{D}}$ (Rasanen, JCAP, 2006).

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The scaling solutions

(Buchert, Larena & Alimi, CQG, 2006)
$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^{n}$ and $\langle \mathcal{R} \rangle_{\mathcal{D}} = \mathcal{R}_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^{p}$.

$$a_{\mathcal{D}}^{-6} \partial_t \left(a_{\mathcal{D}}^{6} \mathcal{Q}_{\mathcal{D}} \right) = -a_{\mathcal{D}}^{-2} \partial_t \left(a_{\mathcal{D}}^{2} \langle \mathcal{R} \rangle_{\mathcal{D}} \right)$$

Two types of solutions:

2 types of solutions:

- $n \neq p$:

$$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^{-6} \; ; \; \langle \mathcal{R} \rangle_{\mathcal{D}} = \mathcal{R}_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^{-2}$$

Backreaction and averaged curvature are decoupled.
Quasi-Friedmannian Universe: $\Omega_{\mathcal{R}}^{\mathcal{D}} + \Omega_{\mathcal{Q}}^{\mathcal{D}} \sim \Omega_{k}^{\mathcal{D}}$ when $a_{\mathcal{D}} \to +\infty$.

- $n = p$:

$$\mathcal{Q}_{\mathcal{D}} = r \langle \mathcal{R} \rangle_{\mathcal{D}} = r \mathcal{R}_{\mathcal{D}_{\mathbf{i}}} a_{\mathcal{D}}^{n} \; , \; n = -2(1+3r)/(1+r), \; r \neq -1$$

$r = \frac{1}{3} \frac{1+3w_{\mathcal{D}}}{1-w_{\mathcal{D}}} =$ cst is the conversion rate between kinematical backreaction and averaged curvature.
Purely General Relativistic effect.

Reconstruction of the morphon field potential

From the correspondence:

$$\begin{aligned}\dot{\Phi}_{\mathcal{D}}^2 &= -\epsilon\frac{\mathcal{R}_{\mathcal{D}_{\mathrm{i}}}}{8\pi G}\left(r+\frac{1}{3}\right)a_{\mathcal{D}}^n \\ U(\Phi_{\mathcal{D}}) &= -\frac{\mathcal{R}_{\mathcal{D}_{\mathrm{i}}}}{24\pi G}a_{\mathcal{D}}^n\end{aligned}$$

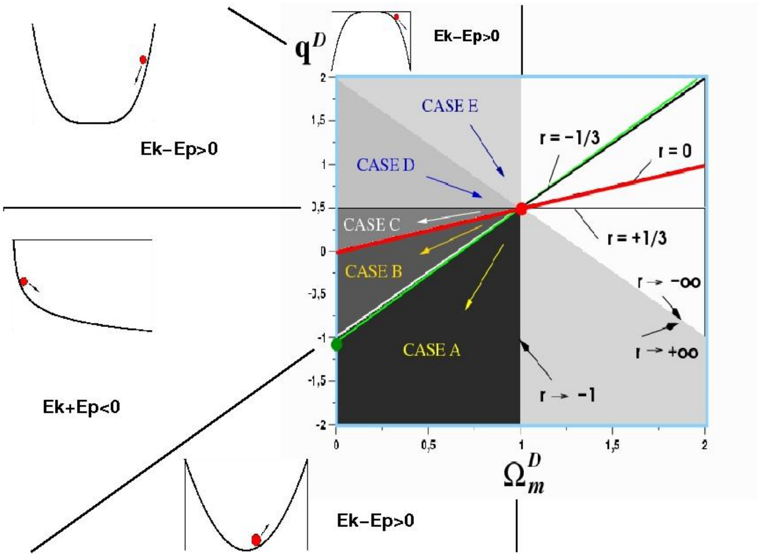
So,

$$\boxed{U(\Phi_{\mathcal{D}}) = \alpha\left(r,\mathcal{R}_{\mathcal{D}_{\mathrm{i}}},\langle\varrho\rangle_{\mathcal{D}_{\mathrm{i}}}\right)\sinh^{-4\frac{1+3r}{1-3r}}\left(\beta\left(r,\mathcal{R}_{\mathcal{D}_{\mathrm{i}}},\langle\varrho\rangle_{\mathcal{D}_{\mathrm{i}}}\right)\frac{\Phi_{\mathcal{D}}}{G}\right)}$$

(Sahni & al, JETP Lett., 2003; Sahni & al, Int. J. Mod. Phys., 2000, Copeland & al, hep-th/0603057)

- The scalar field parameters are fixed by the initial averaged quantities.
- Solution to the coincidence problem?
- Constant equation of state: $w_{\Phi}^{\mathcal{D}} = -\frac{1}{3}\frac{1-3r}{1+r}$.

The space of solutions



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Second order solution in the comoving synchronous gauge

We choose $g_{ij} = a^2(t)\left((1-2\psi(t,\vec{(x)}))\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\Delta)\chi)\right)$
Then, to second order in perturbations (Li & Schwarz astro-ph/0702043), one finds:

$$\mathcal{Q}_\mathcal{D} = \frac{\alpha_{\partial\mathcal{D}}}{a_\mathcal{D}} + \frac{\beta_{\partial\mathcal{D}}}{a_\mathcal{D}^{7/2}} + \frac{\gamma_{\partial\mathcal{D}}}{a_\mathcal{D}^{6}}$$

$$\langle R\rangle_\mathcal{D} = \frac{\delta_\mathcal{D}}{a_\mathcal{D}^2} - \frac{7\alpha_{\partial\mathcal{D}}}{3a_\mathcal{D}} - \frac{11\beta_{\partial\mathcal{D}}}{19a_\mathcal{D}^{7/2}}$$

- One mode is quintessence-like: $\frac{\alpha_{\partial\mathcal{D}}}{a_\mathcal{D}}$
- But, it does not dominate today because $\alpha_{\partial\mathcal{D}}$ is of second order.
- If initial conditions at CMB and $\Omega_m^{\mathcal{D}_0} \sim 0.3$, $\Omega_R^{\mathcal{D}_0} + \Omega_\mathcal{Q}^{\mathcal{D}_0} \sim 3\times 10^5(\Omega_R^{\mathcal{D}_i} + \Omega_\mathcal{Q}^{\mathcal{D}_i})$.
- Backreaction cannot be ignored for high precision cosmology!
- $\mathcal{Q}_\mathcal{D}$ and $\langle R\rangle_\mathcal{D}$ are gauge-invariant (they vanish on the background)
- Remark: The initial values of backreaction modes are surface terms (link with holographic cosmology?)

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Conclusion

- Averaged curvature is not, in general, a constant curvature.
- Friedmann cosmology: a very particular homogeneous model, that is saddle point in the space of homogeneous models with scaling backreaction.
- A classical physical origin to cosmological scalar fields: they naturally appear when one considers an averaged inhomogeneous model.
- A perturbative treatment leads to a quintessence-like backreaction mode.

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Perspectives

- How can we measure distances?
  - We only analysed dynamics.
  - But backreaction also modifies the way we measure distances
  - The averaged model is nor flat, neither a constant curvature space.
  - One needs a 'mock' metric that is not Friedmannian, on large scales to compute distances.
  - This metric does not obey Einstein equations. It is just 'geometric'.
  - $ds^2 = -dt^2 + a_{\mathcal{D}}(t)\left(\frac{dr^2}{1-K(t)r^2} + d\Omega\right)$?
- Extension of the morphon field to cosmology with a relativistic fluid
  - Non minimally coupled scalar field?
  - Inhomogeneous inflation
- Numerical estimate of backreaction
- Geometrical treatment of backreaction: the Ricci-Hamilton flow