Parameter estimation and Test of General Relativity using 3.5PN phasing formula

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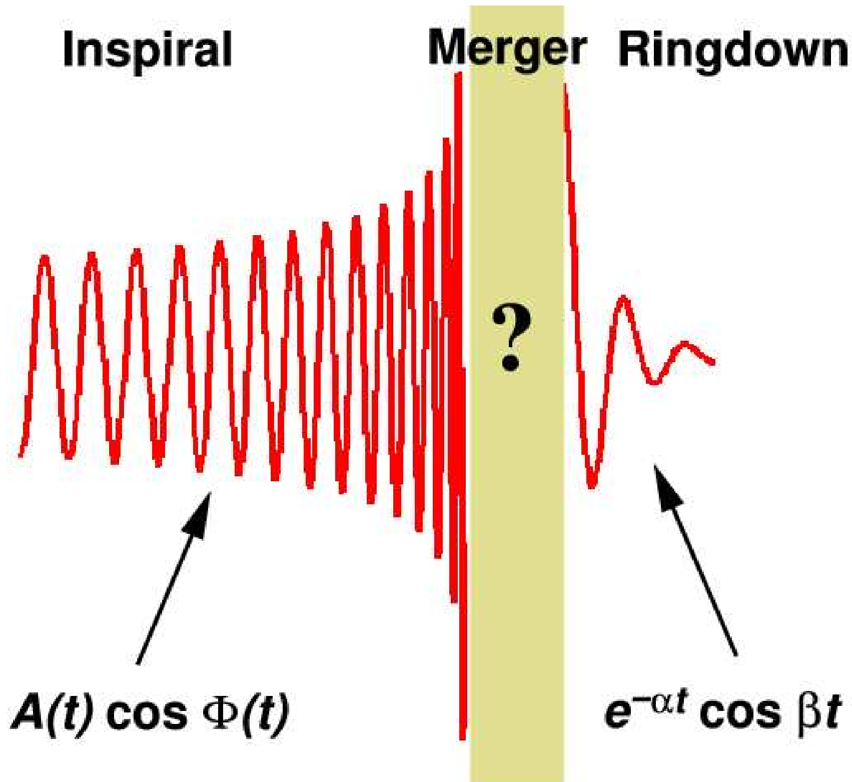
Outline

- Introduction
- Parameter estimation for ground-based detectors
- Parameter estimation problem in the *LISA* context
- Testing Post-Newtonian theory using GW phasing

What are ICBs?

- Compact binaries (NS/BH)
- Time varying quadrupole moment $\Rightarrow$ gravitational wave (GW) emission
- GW takes away energy, angular momentum (and linear momentum) which makes the orbit to decay $\Rightarrow$ inspiral
- They come closer and eventually merge to form a single object $\Rightarrow$ Merger
- The newly formed object (BH) will radiate away its asymmetries and settle to a Kerr-like geometry $\Rightarrow$ Ringdown.

The GW signal from ICBs



$$h(t) = A(t)\,\cos\Phi(t)$$

Amplitude and frequency increase with time ⇒ Chirp

How to look for a GW chirp

Matched filtering!

- One can model the GWs from the inspiral phase using general relativity (GR)
- Solving the 2-body problem in GR.
- Exact analytical solution too hard $\Rightarrow$ Analytically using approximations or fully numerically.
- We follow analytical method based on many approximation schemes.
- Analytically constructed waveforms are used for matched filtering: cross correlating the data with the theoretical waveforms (templates)
- One needs a very accurate model of the waveform for efficient matched filtering: Challenge to the theorists.

Usual waveform model

- Post-Newtonian waveform: expansion in powers of v/c, where v is the source velocity: ($\frac{v}{c} \ll 1$).
- Assumes orbit of the binary to be circular: Due to gravitational radiation reaction, eccentricity decays quite fast and most of the binaries, if not all, will have negligible eccentricity towards the late stages of inspiral(Peters & Mathews, 1963, Peters, 1964).
- Adiabatic approximation: ($\frac{\dot{\omega}}{\omega^2} \ll 1$)
- The Fourier domain waveform is obtained using stationary phase approximation:
$d \ln A/dt \ll d\phi(t)/dt$ and $|d^2\phi/dt^2| \ll (d\phi/dt)^2$.
- Neglects PN corrections to amplitude, ignores the presence of harmonics other than the dominant one and keeps phase of the wave to maximum PN accuracy $\Rightarrow$
Restricted waveform (RWF)

Parameter estimation problem for the nonspinning ICBs using 3.5PN phasing

GW phasing formula

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)},$$

$$\tilde{h}(f) = \frac{1}{\sqrt{30}\,\pi^{2/3}} \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} e^{i\psi(f)},$$

and to 3.5PN order the phase of the Fourier domain waveform is given by

$$\psi(f) \quad = \quad 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\,\eta\,v^5} \sum_{k=0}^{7} \alpha_k\, v^k,$$

Notation: G=c=1, $v^n \sim \frac{n}{2}$ PN.

The coefficients in the phasing formula

$$
\begin{aligned}
\alpha_0 &= 1, \text{Einstein , Landau and Lifshitz, 1941, Fock, 1959} \\
\alpha_1 &= 0, \\
\alpha_2 &= \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right), \text{Wagoner and Will, 1976} \\
\alpha_3 &= -16\pi, \text{Blanchet and Schaefer(1993), Poisson(1993)} \\
\alpha_4 &= 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\,\eta + \frac{617}{144}\,\eta^2\right) \text{(BDIWW, 1994)}, \\
\alpha_5 &= \pi\left(\frac{38645}{756} + \frac{38645}{252}\log\left(\frac{v}{v_{\rm lso}}\right) - \frac{65}{9}\eta\left[1 + 3\log\left(\frac{v}{v_{\rm lso}}\right)\right]\right) \text{(Blanchet, 1996)}, \\
\alpha_6 &= \left(\frac{11583231236531}{4694215680} - \frac{640\,\pi^2}{3} - \frac{6848\,\gamma}{21}\right) \\
&\quad + \eta\left(-\frac{15335597827}{3048192} + \frac{2255\,\pi^2}{12} - \frac{1760\,\theta}{3} + \frac{12320\,\lambda}{9}\right) \\
&\quad + \frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{6848}{21}\log(4\,v) \text{(BFIJ, 2002, BDEI, 2004)}, \\
\alpha_7 &= \pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\,\eta - \frac{74045}{756}\,\eta^2\right) \text{(BFIJ, 2002)}.
\end{aligned}
$$

where $v = (\pi\, m\, f)^{1/3}$, $m = \mathcal{M}\,\eta^{-3/5}$; $\eta = \frac{m_1\, m_2}{m^2}$
$\theta = -\frac{11831}{9240}$, $\lambda = -\frac{1987}{3080}$ (Damour, Jaranowski and Schäfer (2001), Blanchet, Damour and Esposito-Farése (2004)).

Parameter Estimation

- Very important for using GWs as tools of Astronomy.
- Matched filtering $\Rightarrow$ detector output is 'filtered' using pre-calculated waveforms with different signal parameters.
- The 'measured' values of the signal parameters correspond to that of the template which has maximum SNR.
- They need not be the 'actual' parameters due to the noise present.
- Parameter estimation aims at calculating the probability distribution for the measured values of a signal and to compute the interval in which the true parameters of the signal lie
- Our error estimates are obtained using covariance matrix.
- The two inputs needed are the Fourier domain gravitational waveform and the noise PSD of the detector.

Statistical Vs Systematic errors

There are two types of errors

- Statistical errors due to the noise present
- Systematic errors due to approximate waveform model we employ.

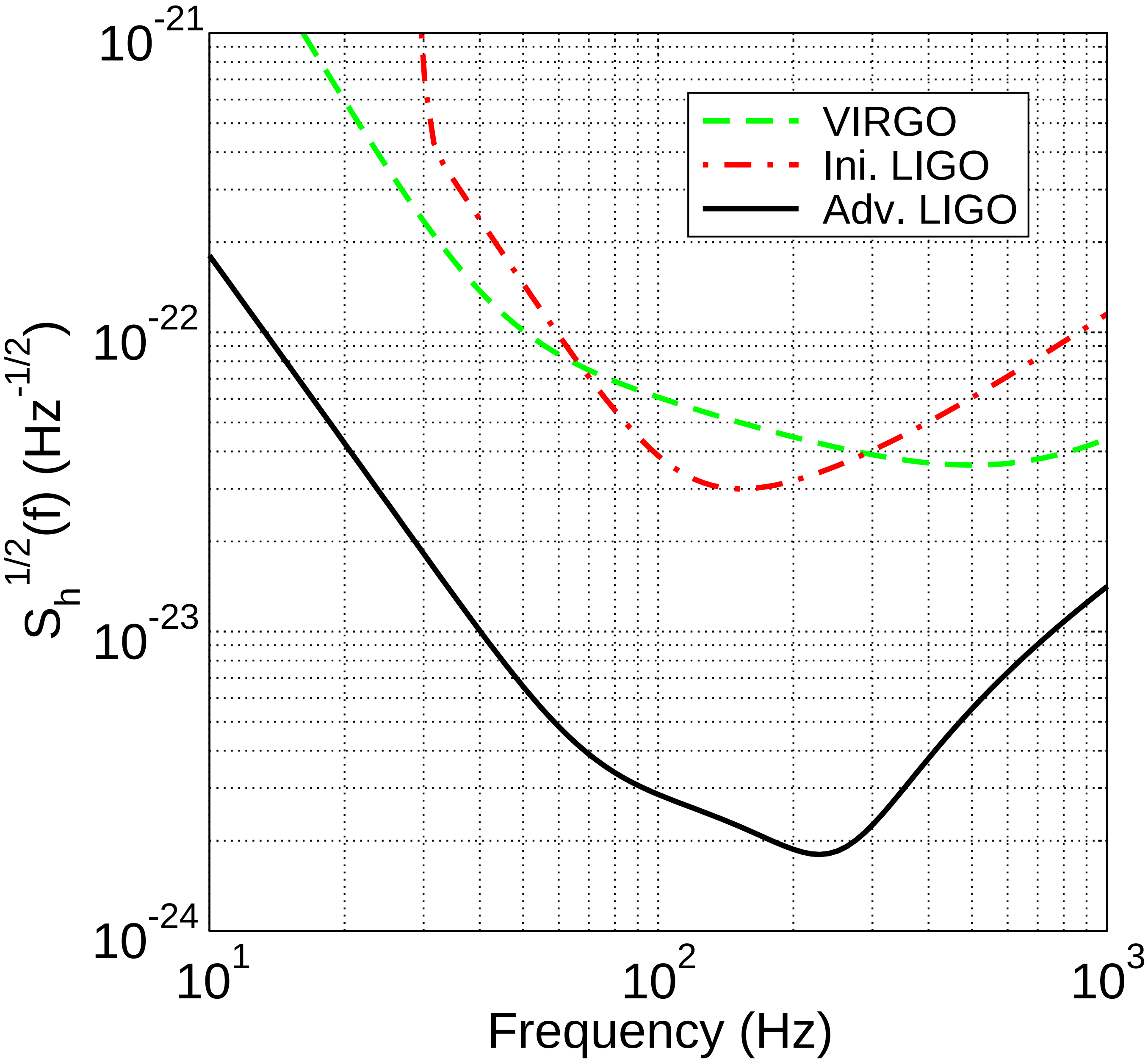
In this work we are addressing the statistical errors and variation of them with PN orders

Present Work

- Theoretical study of the parameter estimation problem using 3.5PN phasing: Ground-based detectors.
  - Implications of higher PN order modelling of the binary in the context of parameter estimation of the chirp signal.
  - Comparison (theoretical) of detector performances of initial LIGO, advanced LIGO and VIRGO interferometers.
  - Effect of Bandwidth and sensitivity on Parameter estimation.
- Implications of the 3.5PN phasing for LISA:
  - SMBH coalescences
  - Implications for Intermediate mass ratio inspirals.
  - Effect of lower cut-off of LISA on parameter estimation and PN convergence

Present analysis is for the case of nonspinning binaries.

The noise PSDs: Plots



The Scheme of Analysis

- The set of parameters describing the chirp is

$$\theta^a = \{t_c, \phi_c, \mathcal{M}, \eta\}$$

- We construct the corresponding 4×4 Fisher matrix and invert it to get the associated errors.
- Do it for all PN orders and for different detectors and systems.
- The analysis is done for fixed SNR as well as for sources at fixed distance in order to study the effect of BW and Sensitivity (respectively).
- We consider 3 prototypical systems of NS ($1.4 M_\odot$) and BH ($10 M_\odot$), viz, NS-NS, NS-BH and BH-BH.
- Limits of integration: source is assumed to last from $f_{\rm lower}$ of the detector up to $f_{\rm lso} = (6^{3/2}\,\pi\, m)^{-1}$

Errors at diff. PN orders: fixed SNR

(KGA, Iyer, Sathyaprakash, Sundararajan, 2005)

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TABLE I: Convergence of measurement errors from 2PN to 3.5PN at a SNR of 10 for the three prototypical binary systems: NS-NS, NS-BH and BH-BH using the phasing formula, in steps of 0.5PN. For each of the three detector noise curves the table presents Δt_c (in msec), $\Delta\phi_c$ (in radians), $\Delta\mathcal{M}/\mathcal{M}$ and $\Delta\eta/\eta$.

	NS-NS				NS-BH				BH-BH			
PN Order	Δt_c	$\Delta\phi_c$	$\Delta\mathcal{M}/\mathcal{M}$	$\Delta\eta/\eta$	Δt_c	$\Delta\phi_c$	$\Delta\mathcal{M}/\mathcal{M}$	$\Delta\eta/\eta$	Δt_c	$\Delta\phi_c$	$\Delta\mathcal{M}/\mathcal{M}$	$\Delta\eta/\eta$
Advanced LIGO												
2PN	0.4623	1.392	0.0143%	1.764%	0.7208	1.848	0.0773%	2.669%	1.404	2.850	0.6240%	10.79%
2.5PN	0.5090	1.354	0.0134%	1.334%	0.9000	1.213	0.0686%	1.515%	1.819	1.555	0.5300%	5.934%
3PN	0.4938	1.326	0.0135%	1.348%	0.8087	1.126	0.0698%	1.571%	1.544	1.559	0.5466%	6.347%
3.5PN	0.5198	1.273	0.0133%	1.319%	0.9980	0.9203	0.0679%	1.456%	2.086	1.137	0.5237%	5.730%
Initial LIGO												
2PN	0.4109	1.816	0.0423%	3.007%	1.148	3.597	0.2903%	6.316%	2.900	7.179	2.851%	32.82%
2.5	0.4605	1.642	0.0384%	2.129%	1.467	1.964	0.2491%	3.305%	3.836	3.070	2.351%	16.48%
3PN	0.4402	1.610	0.0389%	2.170%	1.286	1.787	0.2554%	3.474%	3.159	3.069	2.446%	17.94%
3.5PN	0.4760	1.507	0.0383%	2.098%	1.668	1.311	0.2455%	3.148%	4.531	1.851	2.313%	15.75%
VIRGO												
2PN	0.1562	0.7515	0.0098%	1.085%	0.5918	1.561	0.0611%	2.215%	1.395	2.667	0.5199%	9.625%
2.5PN	0.1743	0.7015	0.0091%	0.7957%	0.7384	1.035	0.0541%	1.263%	1.787	1.527	0.4417%	5.370%
3PN	0.1671	0.6890	0.0092%	0.8083%	0.6632	0.9625	0.0551%	1.309%	1.532	1.528	0.4552%	5.724%
3.5PN	0.1799	0.6527	0.0091%	0.7854%	0.8195	0.7914	0.0536%	1.214%	2.031	1.150	0.4366%	5.193%

Inferences from the table

- The values listed are for a fixed SNR of 10
- Compared to 2PN phasing, the 3.5PN phasing provides an improved estimation of the mass parameters $\mathcal{M}$ and η.
- Improvement is as high as 19% and 52% for $\mathcal{M}$ and η for a BH-BH binary in the initial LIGO sensitivity band.
- Errors oscillate at every half-a-PN order
- More massive systems have larger errors associated with their parameters
- Larger improvements for massive systems.

Inferences contd...

- At fixed SNR, VIRGO provides the least errors followed by Adv. LIGO and Initial LIGO configurations: VIRGO observes the signal over a large BW
- While $\mathcal{M}$, η and ϕ_c estimations improve as one goes from 2PN to 3.5PN, estimation of t_c worsens!!!!
- The improved parameter estimation cannot be correlated only to the number of GW cycles at each order.

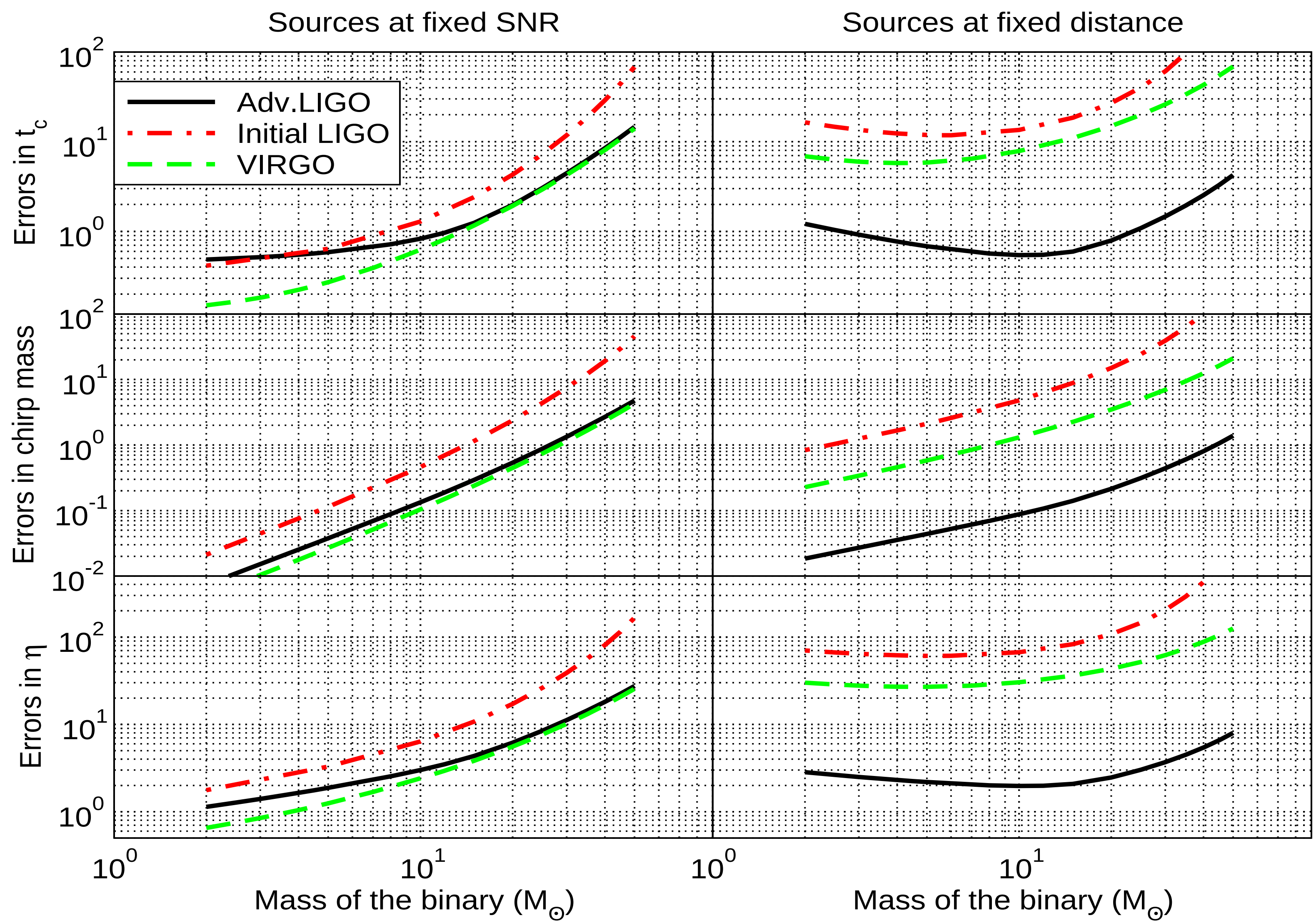
Effect of Bandwidth

For the fixed SNR case, where the Sensitivity aspect of the detector is fully suppressed, the detector with the largest BW provides the best estimate.

Errors for fixed distance

- Errors at fixed SNR cannot be used to gauge detector performance as by keeping SNR constant one ie effectively suppressing the sensitivity of a better detector. A more sensitive detector has larger SNR for a given source and hence lesser errors.
- Errors $\sigma \propto 1/\rho$ (ρ is SNR).
- We can re-tabulate the errors for sources at fixed distance by rescaling the fixed SNR results by $10/\rho_a$, where ρ_a, where ρ_a is the SNR at the fixed distance of 300 Mpc.

Results for fixed-SNR and fixed-Distance



(KGA, Iyer, Sathyaprakash, Sundararajan, 2005)

Effect of sensitivity

For sources at a fixed distance, both BW and sensitivity plays a role in the parameter estimation. Advanced LIGO gains an improvement of 30-60 times compared to initial LIGO. Of this 3-4 times is from BW and 10-15 times is due to its sensitivity over the initial LIGO config.

Conclusions

- This study emphasizes the significance of higher PN order modelling of the ICBs for the parameter estimation.
- Relative to 2PN phasing the 3.5PN phasing provides a better estimate of the mass parameters $\mathcal{M}$ and η. Errors in t_c worsens.
- At fixed SNR, VIRGO has the least errors due to its larger BW, followed by Adv. LIGO and Initial LIGO
- For sources at fixed distances, Adv. LIGO provides the most accurate estimates due to its better sensitivity. VIRGO performs better than initial LIGO
- Number of cycles alone cannot quantify the trends seen in parameter estimation across PN orders.

Parameter estimation problem for the supermassive BH binaries using *LISA*

Laser Interferometer Space Antenna

- $\mathrm{ESA + NASA = LISA}$
- Space based detector in the freq range 10^{-4} or 10^{-5} Hz to 1Hz.
- Three Space craft constellation, distance b/w any two detectors is 5 million kilometers.
- Proposed launch by 2016+
- Complements the high freq observations made by the Ground based detectors.
- Science goals include observation of supermassive BHs, Strong field tests of Gravity
- Future upgrades of LISA: BBO, DECIGO

LISA features

- Three arms at 60 degrees $\Rightarrow$ one can construct two detector outputs $\Rightarrow$ one detector and two detector configuration.
- Orbital motion $\Rightarrow$ encodes information about angular coordinates of the source $\Rightarrow$ even with single detector configuration location and orientation of the source can be measured.
- Location and orientation measurement by orbital modulation

LISA features

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2}\mathcal{A}\, f^{-7/6}\, e^{i\Psi(f)}\, \left\{\frac{5}{4}\tilde{A}_\alpha(t(f))\right\} e^{-i\left(\varphi_{p,\alpha}(t(f))+\varphi_D(t(f))\right)}\,,$$

where $\varphi_{p,\alpha}(t(f))$ and $\varphi_D(t(f)$ are the polarization phase and Doppler phase respectively. $\tilde{A}_\alpha(t(f))$ correspond to the amplitude modulations induced by the LISA's orbital motion. which depends on the pattern functions $F_+^\alpha(t)$ and $F_\times^\alpha(t)$ and hence vary with time.

- Calculation now is more involved than for the ground based detectors.
- Three cases: using a pattern averaged waveform, without pattern averaging for one detector and without pattern averaging for 2 detector network.

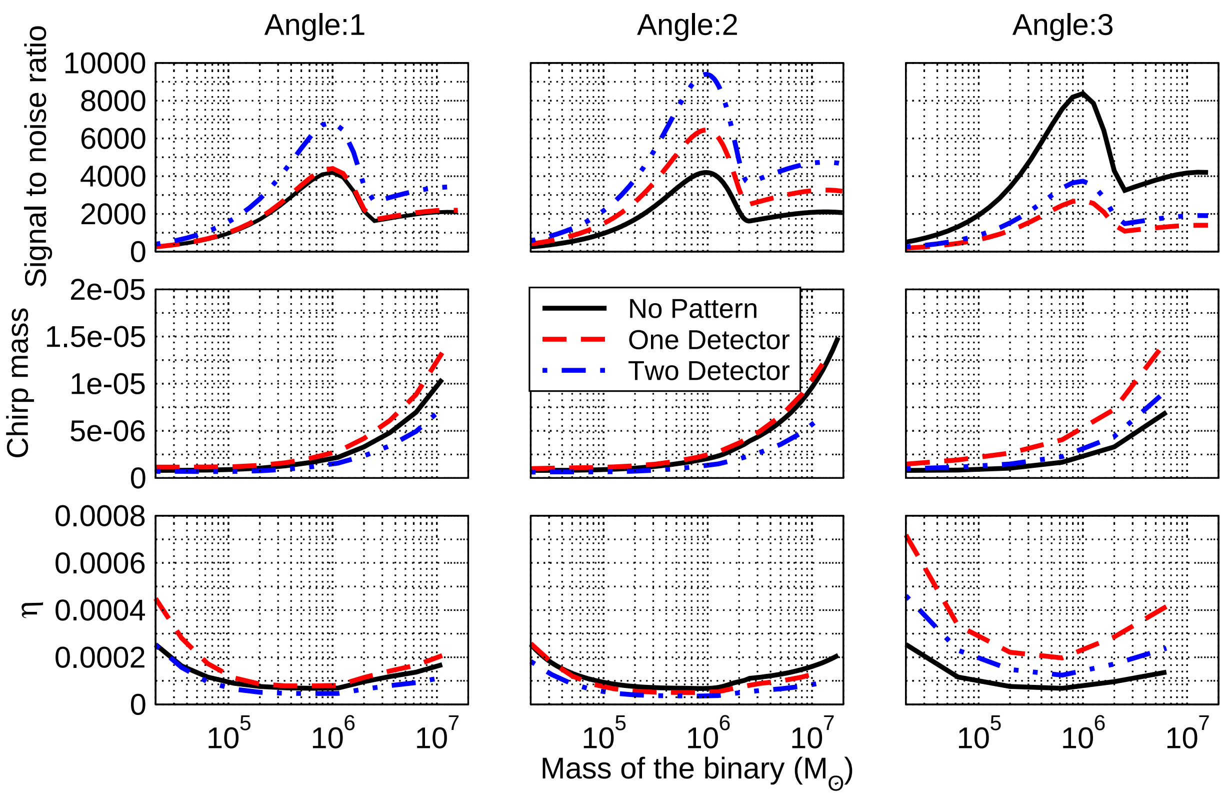
Choice of upper and lower cut-off frequencies

- The upper limit of integration is $f_{\rm fin} = {\rm Min}[f_{\rm lso}, f_{\rm end}]$, where $f_{\rm lso}$ is the frequency of the innermost stable circular orbit for the test particle case, $f_{\rm lso} = (6^{3/2}\,\pi\,m)^{-1}$ and $f_{\rm upper}$ corresponds to the upper cut-off of the LISA noise curve $f_{\rm end} = 1{\rm Hz}$.
- We have chosen the lower limit of frequency $f_{\rm start} = {\rm Max}[f_{\rm in}, f_{\rm lower}]$ where $f_{\rm in}$ is calculated by assuming the signal to last for one year in the LISA sensitivity band

Parameter estimation using non-pattern averaged WF

- Four more parameters corresponding to the luminosity distance and orientation/location are added to the parameter space: $\{t_c\,,\phi_c\,,\mathcal{M}\,,\eta\,,D_L\,,\bar{\mu}_L\,,\bar{\mu}_S\,,\bar{\phi}_S\,,\bar{\phi}_L\}$
- Dimension of the parameter space increased to 9.
- Increased dimensionality $\Rightarrow$ increased errors in estimation of the existing 4 parameters.
- But there is an increase/decrease in SNR due to the inclusion of pattern functions also. Increase/decrease depends on the orientation of the source.

Results including the pattern functions



(KGA, 2006)

Inferences

- Among the different effects, the change (increase/decrease) in SNR is the most dominant effect..
- The improvement in going from 2PN to 3.5PN waveform depends very much on the source's location and orientation.
- General conclusions cannot be drawn.
- Monte-Carlo methods may have to be used to study PN trends in this case.
- But irrespective of the location and orientation, the higher order PN terms do NOT improve the estimation of D_L and angular resolution.

PN convergence with pattern averaged waveform

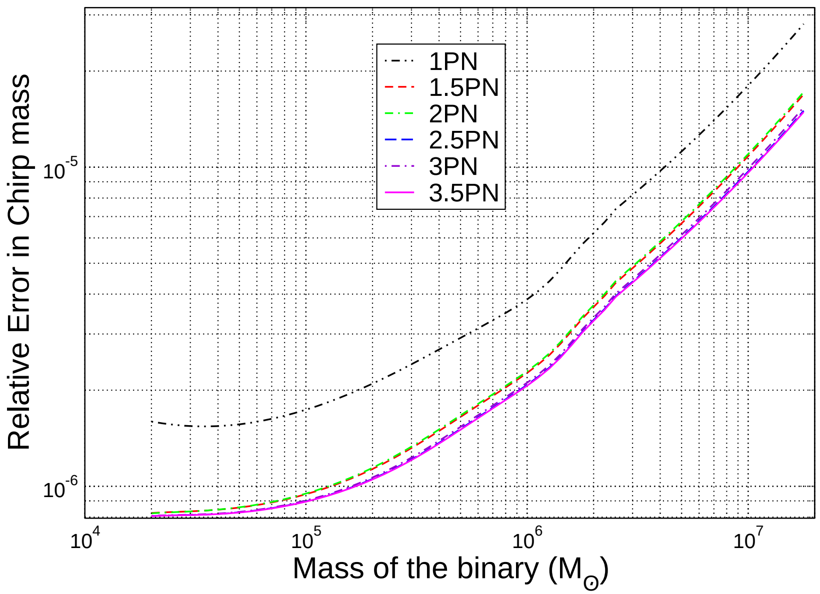
- Justification: Earlier MC simulations (Berti, Buonanno & Will, 2005) using 2PN phasing showed that the results with the two detector case without pattern averaging is very close to that of pattern averaged waveform.
- Hence we believe the trends obtained using the pattern averaged waveform will give useful insights about the problem, though it has to be supported with a MC simulation in future.
- Similar to the ground-based detector case for sources in the LISA band also, 3.5PN phasing improves estimation of mass parameters. Improvement could be as high as 13% for $\mathcal{M}$ and 45% for η (with a lower cut-off of 10^{-5} Hz).

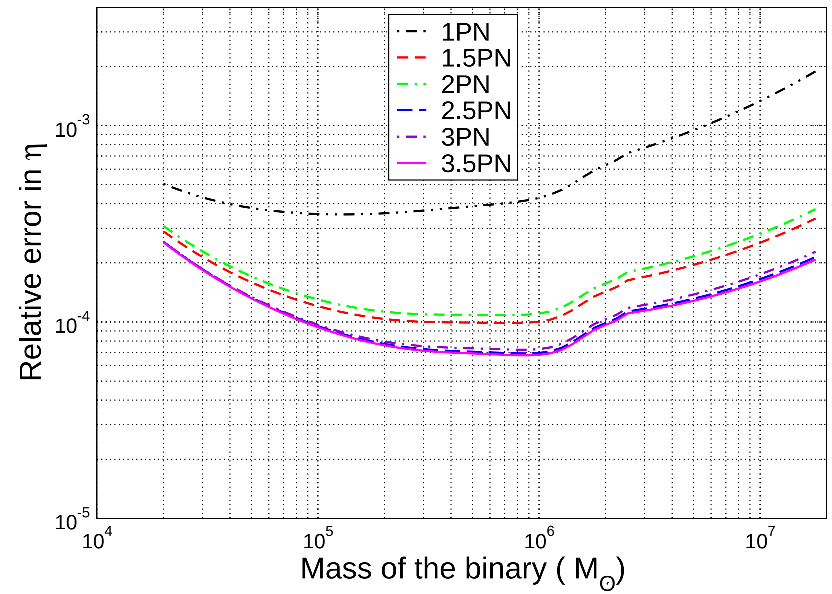
LISA errors across PN orders

PN Order	Δt_c (sec)	$\Delta\mathcal{M}/\mathcal{M}$ (10^{-6})	$\Delta\eta/\eta$ (10^{-4})	$N_{\rm cycles}$
1PN	0.2474	6.217	6.287	2414.03
1.5PN	0.3149	3.648	1.427	2310.26
2PN	0.3074	3.694	1.572	2305.52
2.5PN	0.3947	3.320	0.9882	2314.48
3PN	0.3435	3.377	1.033	2308.73
3.5PN	0.4399	3.300	0.9661	2308.13

(KGA, 2006)

Results for LISA: PN convergence



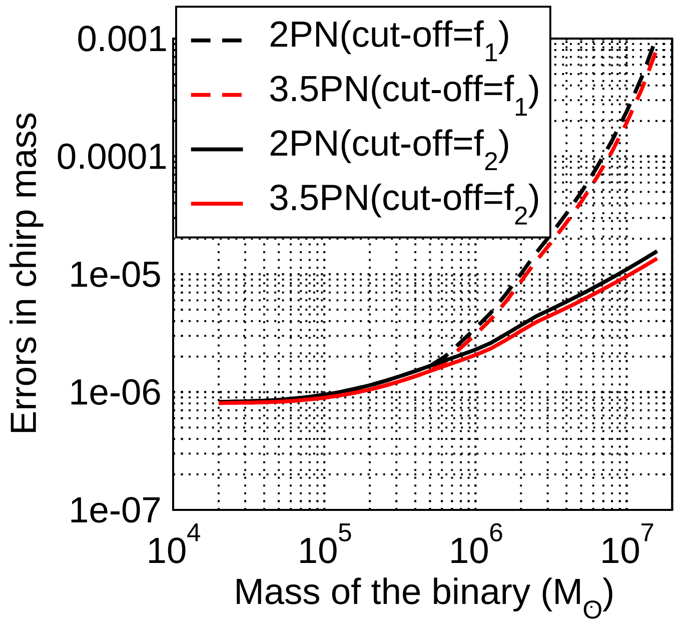


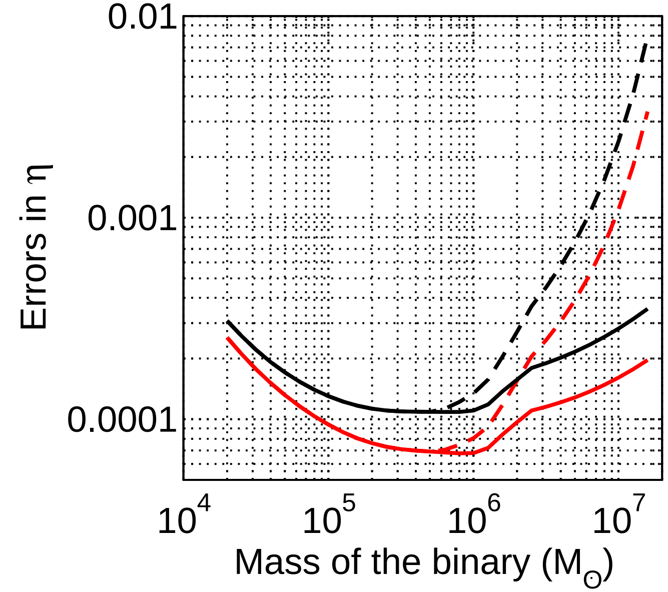
(KGA, 2006)

Errors in chirp mass and η for different PN orders

- % improvement in $\mathcal{M}$ for $2 \times 10^6 M_\odot$=11% (∼20%)
- % improvement in η for $2 \times 10^6 M_\odot$=40% (∼60%)

Effect of lower cut-off for LISA





$$f_1 = 10^{-4} \text{ Hz}, \; f_2 = 10^{-5} \text{ Hz}$$

(KGA, 2006)

Improvement is smaller with smaller lower cut-off $\Rightarrow$ convergence is better.

Errors for unequal mass coalescences

- Fisher matrix inversion for Extreme mass ratio inspirals are very much ill-conditioned.
- We consider IMBH-SMBH coalescences to understand the unequal mass case.
- For a $10^4\ M_\odot$-$10^7\ M_\odot$ system, the SNR is about a hundred and improvement with a 10^{-5} Hz cut-off is about 20% for chirp mass and 62% for symmetric mass ratio.
- Larger improvements for asymmetric systems is not special to LISA.

Conclusions

- Higher order phasing terms are very much significant for LISA as well.
- The improvement is sensitive to the lower cut-off LISA has (for massive systems).
- Compared to equal mass case, improvement is larger for inspirals with intermediate mass ratios.
- Percentage improvements are very much sensitive to the location and orientation of the sources when orbital motion of LISA is put in.

- Testing PN theory with GW phasing

Theme

- How post-Newtonian (PN)structure of general relativity (GR) can be probed by the observation of a gravitational wave (GW) inspiral signal with high signal to noise ratio (SNR).
- How accurately can this test be performed?
- Which generation of detectors can test (upto) which PN order?

GW phasing for nonspinning binaries

The gravitational waveform in the Fourier Domain reads as

$$\tilde{h}(f) = \frac{1}{\sqrt{30}\,\pi^{2/3}} \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} e^{i\psi(f)},$$

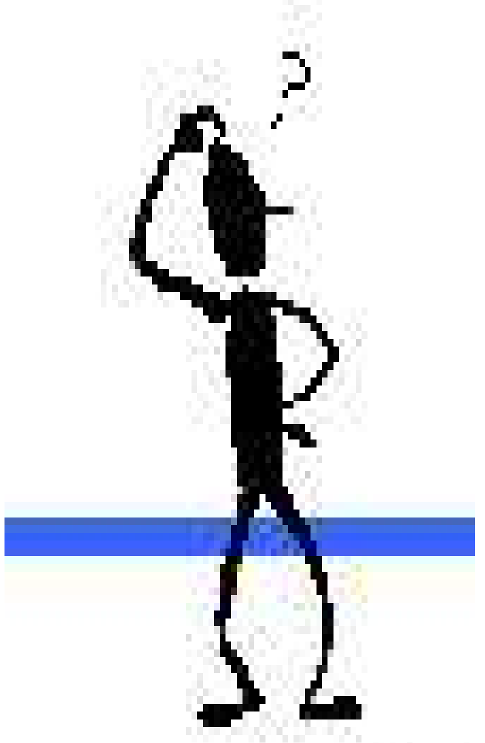
and to 3.5PN order the phase of the Fourier domain waveform is given by

$$\psi(f) \;=\; 2\pi f t_c - \phi_c - \frac{\pi}{4} + \sum_{k=0}^{7} (\psi_k + \psi_{kl} \ln f)\, f^{\frac{k-5}{3}},$$

$$\psi_k(m_1, m_2) = \frac{3}{128\left(\frac{m_1\, m_2}{m^2}\right)} (\pi\, m)^{\frac{(k-5)}{3}} \alpha_k(m_1, m_2)$$

Coefficient of each PN order ψ_k is function only of the individual masses of the binary (for the nonspinning case).

How to use the phasing coefficients to ‘test’ post-Newtonian theory?



!dea

- Lets assume a GW (inspiral) signal is detected with a high SNR meeting all the detection criteria.

Assuming that 3.5PN waveform is a good representation of the actual waveform, fit the signal by a bank of templates where all PN coefficients are treated as independent parameters.
These parameters depend only on the masses of the binary, m_1 and m_2 which when measured from each coefficient should be consistent$\Rightarrow$ Test of PN theory
(Blanchet and Sathyaprakash, (1995).)

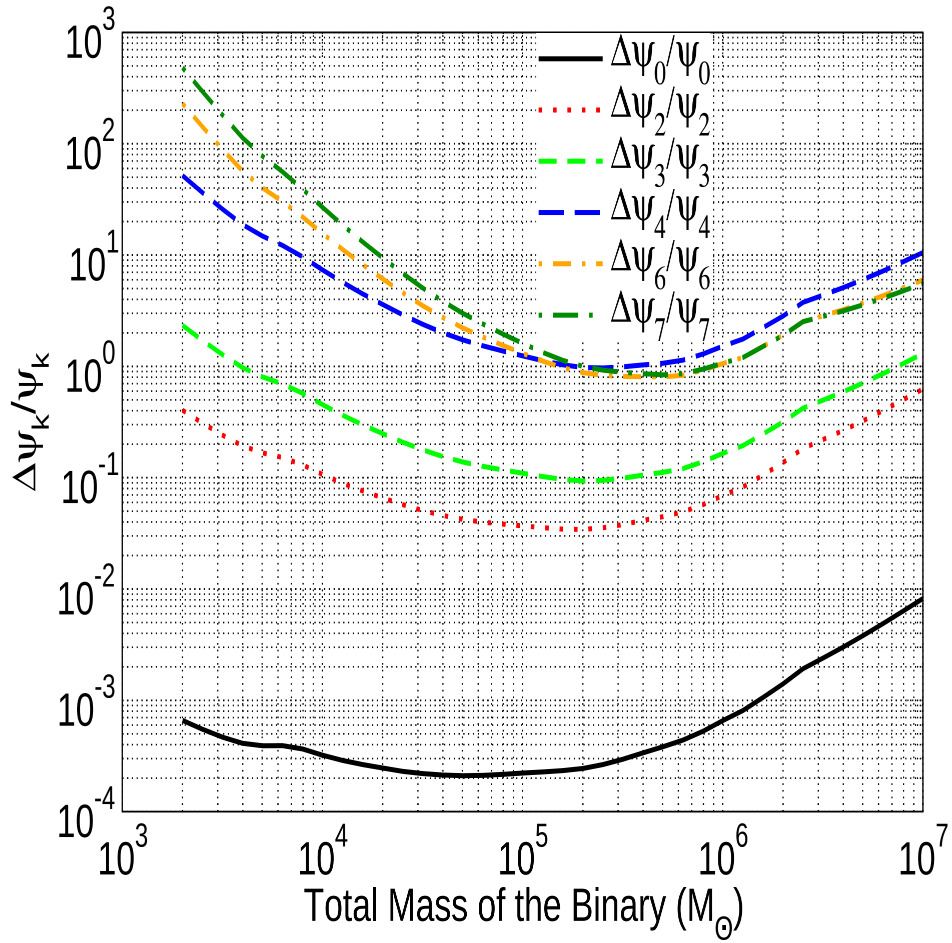
- The ‘most exhaustive test’ possible

A demo of the test

- We consider typical systems of interest for each detector, and calculate the corresponding errors using covariance matrix.

- We estimated the typical errors associated with the estimation of different ψ_k in the contexts of the ground based and space based GW experiments assuming the detection of a signal from a prototypical source.

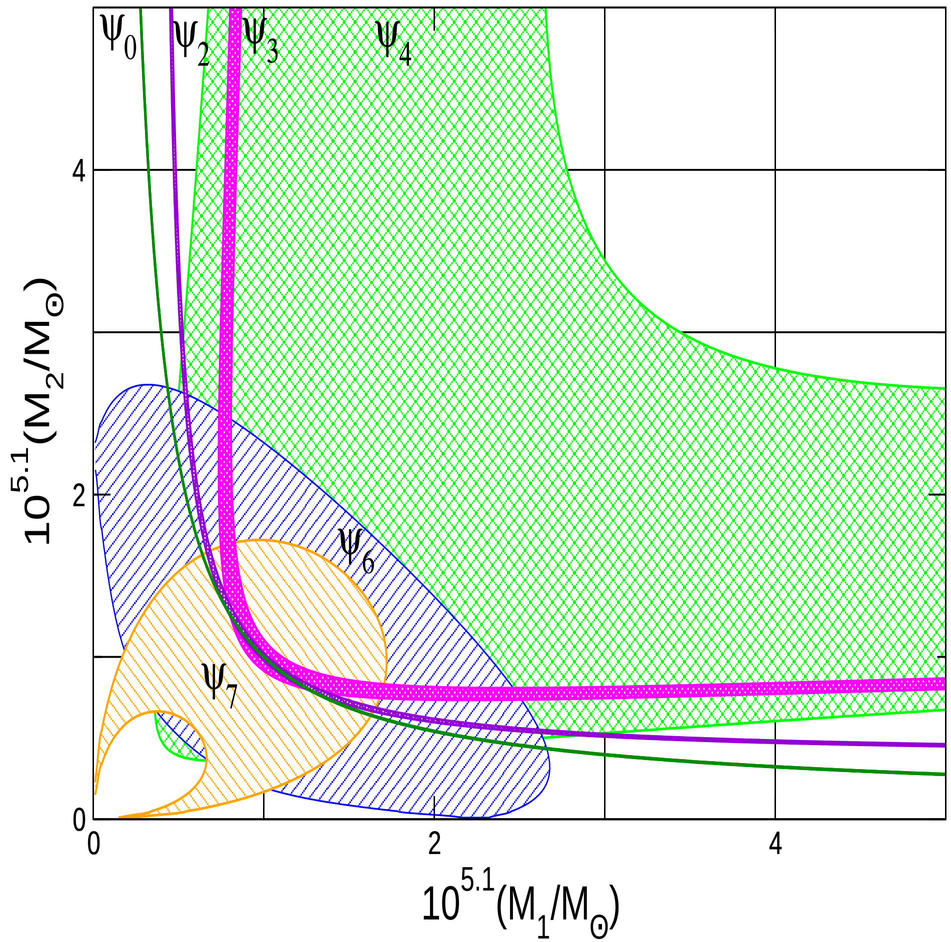
Results for LISA



(KGA, Iyer, Qusailah and Sathyaprakash. 2006)

Source assumed to a distance of 3 Gpc.

$m_1 - m_2$ plot for LISA



(KGA, Iyer, Qusailah and Sathyaprakash(2006))

Source assumed to a distance of 1 Gpc.

A $2+1$ *Approach*

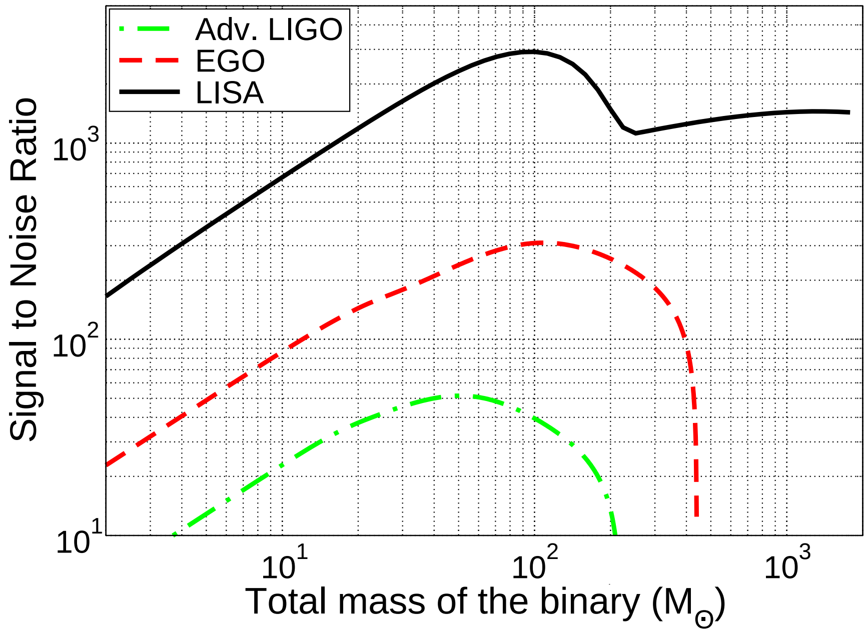
Use 3 parameters at a time, parametrize the waveform in terms of two of them and use the third parameter as test

- This way many tests are possible!
- E.g.: Use ψ_0 and ψ_2 to parametrize the waveform and use one of $\{\psi_3, \psi_4, \psi_5, \psi_{5l}, \psi_6, \psi_{6l}, \psi_7\}$ as test.
- For each ψ_k, plot the region enclosed between $\psi_k + \Delta\psi_k$ and $\psi_k - \Delta\psi_k$ in the $m_1 - m_2$ plane of the binary
- All the test parameters will have to enclose the common region of intersection if the theory is correct.

Similar to binary pulsar tests!!!

Different detectors

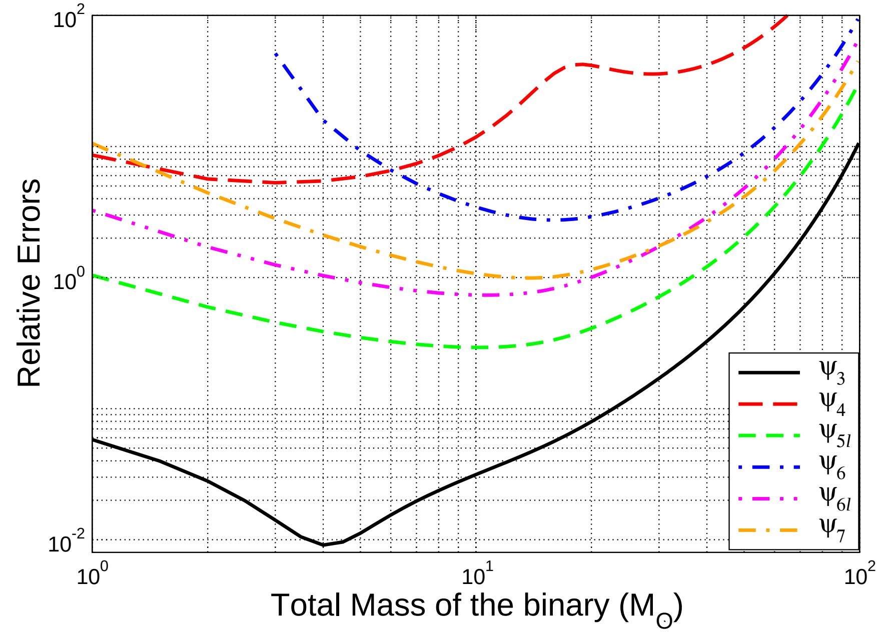
We performed this analysis with 2 ground based detector configurations (Adv LIGO and EGO) and the space based detector **LISA**.



KGA, Iyer, Qusailah and Sathyaprakash (2006)

Note: Masses for the LISA case is scaled down by 10^4 for LISA.

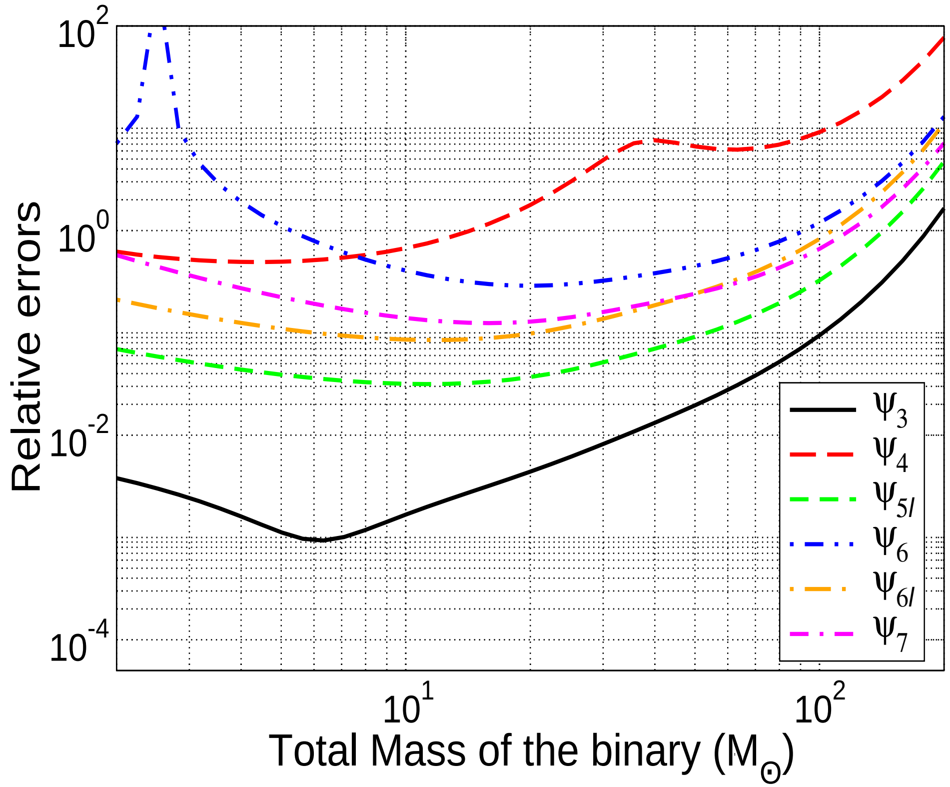
Results: Adv LIGO



(KGA, Iyer, Qusailah and Sathyaprakash (2006))

- Only 4 parameters can be determined with $\frac{\Delta\psi_k}{\psi_k} \leq 1$ accuracy for a prototypical system of a binary BH of $20 M_\odot$ at 200 Mpc (SNR $\simeq 50$)
- Log terms at 2.5PN and 3PN can be tested

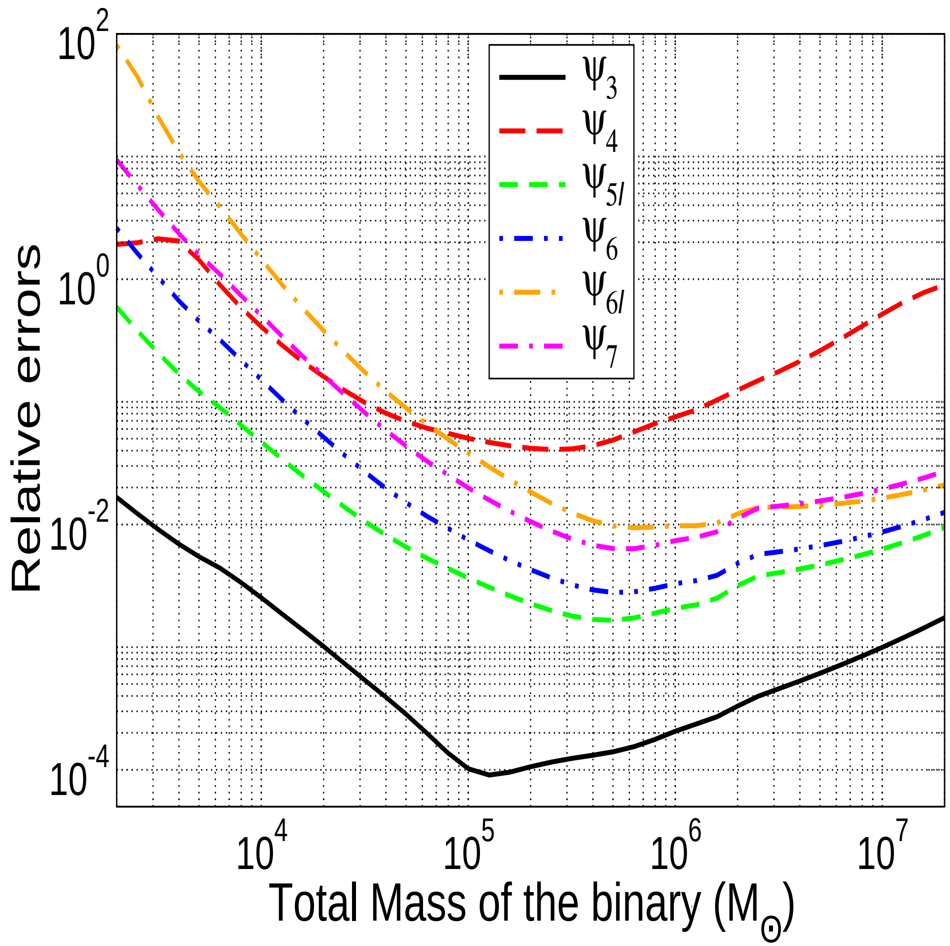
Results: EGO



(KGA, Iyer, Qusailah and Sathyaprakash (2006))

- Except ψ_4 all other parameters can be determined with $\frac{\Delta\psi_k}{\psi_k} \leq 1$ for a system of a binary BH of $20M_\odot$ (SNR$\simeq 150$)
- Better estimation compared to Adv LIGO.

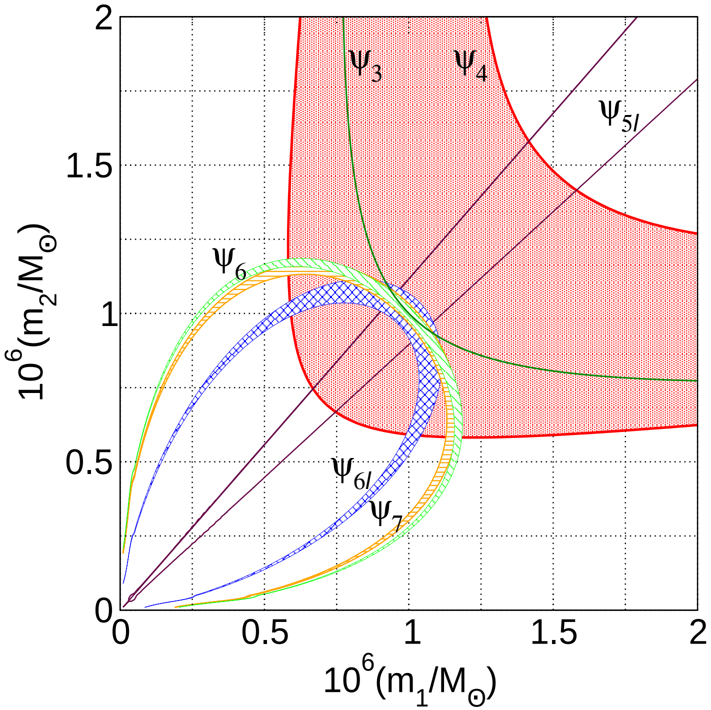
Results for LISA



(KGA, Iyer, Qusailah and Sathyaprakash, 2006)

LISA will measure ALL ψ_k computed till date with incredible precision of a few times 10^{-2} or better!!!

The $m_1 - m_2$ plane plot for LISA



(KGA, Iyer, Qusailah and Sathyaprakash, 2006)

Plot showing the regions in the m_1-m_2 plane that correspond to the uncertainties in the test parameters $\psi_T = \psi_3, \psi_4, \psi_{5l}, \psi_6, \psi_{6l}, \psi_7$ for a $(10^6,\, 10^6)M_\odot$ supermassive black hole binary at a redshift of $z = 1$ as observed for a year by LISA.

Can we test Alternate theories of gravity?

- YES! in principle.
- Any theory of gravity with similar PN structure in the phasing can be tested with the current proposals
- This test can distinguish GR from another theory of gravity, like Brans-Dicke theory or massive graviton theories, which will have different PN coefficients and hence different predictions for the masses of the binary.
- Higher order phasing terms are not available for alternate theories

Caveats and Future directions

- Include systematic effects due to neglect of spin, eccentricity
- Effect of using non-restricted waveform
- Careful Analysis for alternate theories of gravity

Summary

- Next generation GW detectors can probe the PN structure of general relativity to very high degree of accuracy.
- Tests very similar to the binary pulsar tests will be possible with the LISA with incredible precision.
- We have demonstrated the capability of LISA to test the nonlinear aspects of GR by using two of the PN coefficients to parametrize the waveform and using any one of the other 7 as test parameter, LISA can test for their mutual consistency in the $m_1 - m_2$ plane.

Thorough probes of strong field gravity possible with *LISA*

Thank You!