Making relativistic $\perp$ shocks with a spectral 1D PIC code UZEIN

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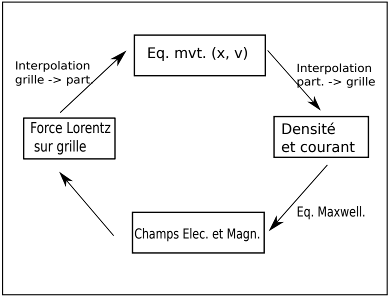
Birdsall&Langdon 1985 *"Plasma Physics via Computer Simulation"*,
Dawson 1983

- Finite size macroparticles : shape factors
- Time step : $\Delta t < 0.2\omega_{\rm pe}^{-1}$;
- Grid step : $\Delta x \sim \lambda_D$;
- CFL stability criterion : $c\Delta t < \Delta x$;
- Relativistic bias : grid-Cerenkov radiation

Scheme of 1 iteration



Normalisation :

$$
\begin{aligned}
\tilde{x} &= \frac{x}{\Delta} \\
\tilde{t} &= \omega_{\mathrm{pe}} t \\
\tilde{v} &= \frac{v}{\omega_{\mathrm{pe}}\Delta} \\
\tilde{p_\alpha} &= \frac{p_\alpha}{m_\alpha \omega_{\mathrm{pe}}\Delta} \\
\tilde{E} &= \frac{\mathrm{q}E}{m_e \omega_{\mathrm{pe}}^2 \Delta} \\
\tilde{B} &= \frac{\mathrm{q}B}{m_e \omega_{\mathrm{pe}}^2 \Delta} \\
\tilde{J} &= \frac{J}{\mathrm{q}\omega_{\mathrm{pe}}\Delta} \\
& \left(\text{and not } \frac{J}{n_0 \mathrm{q}\omega_{\mathrm{pe}}\Delta}\right)
\end{aligned}
$$

Move particles :

$$
\frac{\mathrm{d}\vec{p}_i}{\mathrm{d}t} = q_i(\vec{E} + \frac{\vec{p}_i}{\gamma_i m_i c} \times \vec{B})
$$

Solved by time-centered finite difference method.

Update fields on grid ($x \rightarrow k$ transformed) :

$$
\begin{aligned}
E_x(k_x) &= -i4\pi\rho(k_x)/k_x \\
\frac{\partial B_y}{\partial t} &= ick_x E_x(k_x) \\
\frac{\partial B_z}{\partial t} &= -ick_x E_y(k_x) \\
\frac{\partial E_y}{\partial t} &= -ick_x B_z(k_x) - 4\pi J_y(k_x) \\
\frac{\partial E_z}{\partial t} &= ick_x B_y(k_x) - 4\pi J_z(k_x)
\end{aligned}
$$

Example set of input parameters for Hada et al. 03 simulation :

Param.	Valeur	Quant.	Valeur
Δx	1	$\tilde{\rho}_{L,e}$	0.4
Δt	$0.1\ \omega_{pe}^{-1}$	$\tilde{\rho}_{L,i}$	26.74
$\tilde{c}$	$3\ \Delta x \cdot \omega_{pe}$	ω_{pe}	1
n_0	50 part/Δ/spec	ω_{pi}	0.1
J_0	-110	$\lambda_{De} = V_{th,e}$	0.2
B_0	1.5	ω_{ce}	0.5
Θ_0	90	ω_{ci}	0.006
m_i/m_e	84	V_A	0.149
T_e/T_i	1.58	β_e	0.0356

A realistic simulation ?

For an electron-proton shock in the ISM we need :

1. $m_i/m_e = 1835$
2. $c/v_A \simeq c/c_s \simeq 10^4$
3. $T_{simulation} \geq \Omega_{ci}^{-1}$.

Some numbers :

- $\omega_{pe} \simeq 10^4 \text{ s}^{-1}$
- $\Delta t \inf 10^{-4}\omega_{pe}^{-1}$
- $\rho_I = 10^5\Delta$
- $1\ \Omega_{CI} \sim 3 10^9 \Delta t$
- Box length $= 10^6 - 10^8\Delta$, and $N_p > 10^8$

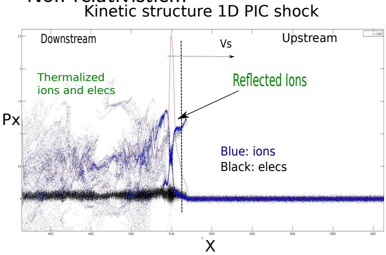
In the code fiducial setup we have :

1. $m_i/m_e \sim 100$
2. $c/v_A \simeq c/c_s \simeq 10$
3. $T_{simulation} \geq \Omega_{ci}^{-1}$.

$\rightarrow$ Need to focus on relevant physics.

- $\Delta t = 0.1\omega_{pe}^{-1}$
- $\rho_I \sim 10^2\Delta$
- $1\ \Omega_{CI} = 10^2 - 10^3\Delta t$
- Box length $= 10^4\Delta$, and $N_p < 10^7$

Non-relativistic...



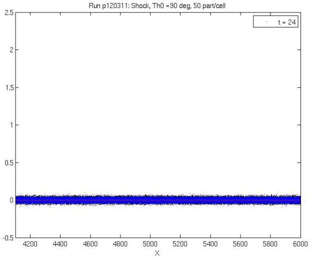
- Rankine-Hugeniot conditions $\pm$ ok.
- Shock front reformation (non-rel).
- Electron heating.

3 different methods tested

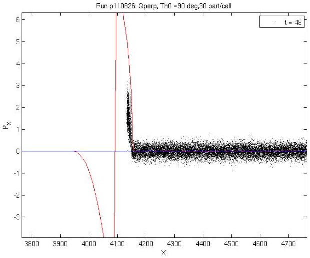
1. Magnetic Piston
2. Reflexion on wall
3. Beam injection in the plasma at rest

Piston method (1)

Electron-ion simulation (phase space X-Px)
(e.g. Hada et al. 03)



Electron-positron simulation ($V_s = 0.97c$)



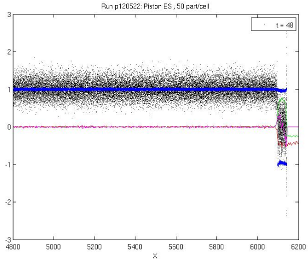
Piston method (2)

Relativistic electron-ion shock ?
Need $J_0 >> 0$, non-linear behavior $V_s = f(J_0)$. Difficult to control the shock speed and go up to the relativistic regime. Other methods tested.

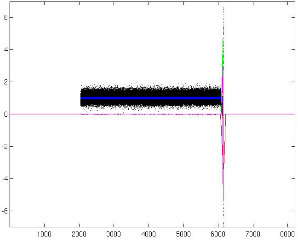
Reflexion of bulk plasma on a wall (1)

Most popular method, but not suited to the spectral code... $k = 0$ problem.
Difficult to handle $\vec{v}$ in $\vec{E_0} + \vec{v} \times \vec{B_0} = 0$ in all plasma components.

Moderate injection speed + low magnetisation :

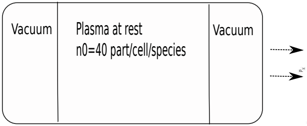


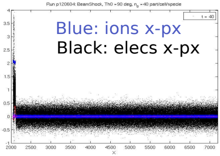
Moderate injection speed + high magnetisation :



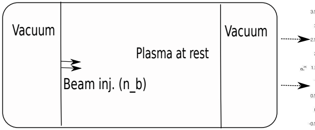
Beam injection in a plasma at rest (1)

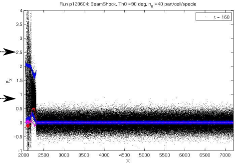
t=0



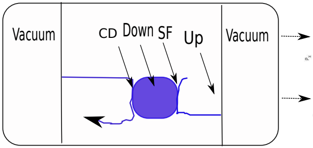


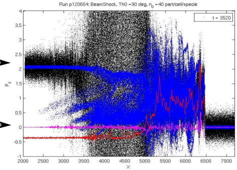
0<t<T(form)





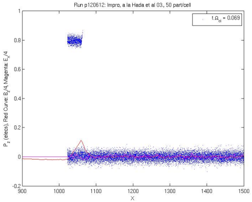
t>>T(form)



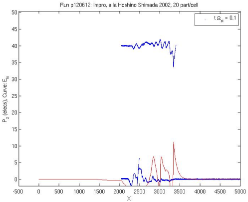


Beam injection (2) : non-relativistic "Benchmarks"

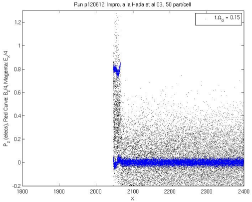
Biskamp & Welter 72 (Whistlers)



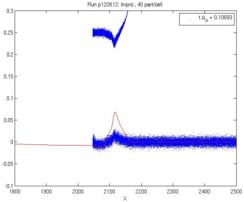
Hoshino & Shimada 2002



Hada et al. 03

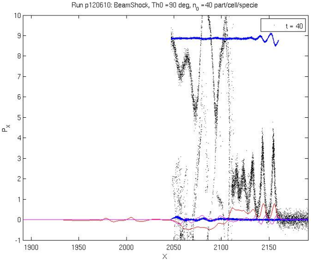


Scholer & Matsukiyo 04



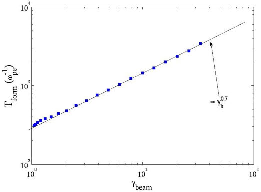
Parameters : $n_b = n_0$, $\tilde{c} = 3$, $\gamma_b \in [1, 30]$.
Example :

- $\gamma_b = 3.12$
- $V_S/c = 0.76$,$\gamma_S = 1.54$
- Buneman instability in the shock foot.
- Shock reformation at $\tau_{ref} \sim \tau_{ci}/3$.

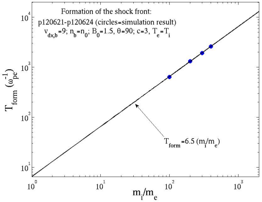


Parameters : $n_b = n_0$, $\tilde{c} = 3$, $\gamma_b \in [1, 30]$.
Shock formation time and shock speed as function of the beam speed.

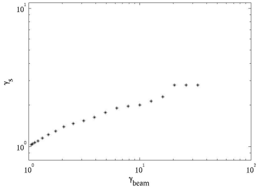
γ_b .vs. T_{form}



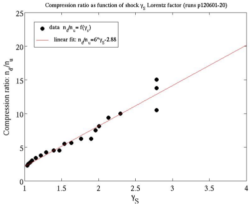
m_i/m_e .vs. T_{form}



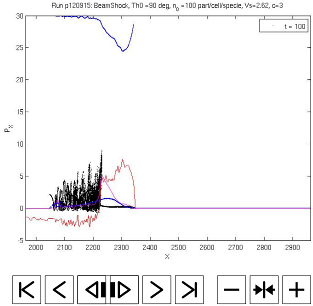
γ_b .vs. γ_S



γ_s .vs. n_d/n_0

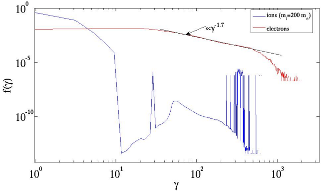


Animation : red line : $B_z/4$, magenta : $E_x/4$. Black dots $u_{x,elec}/\sqrt{m_i/m_e}$.



$V_S \simeq 0.9c$.

Distributions downstream



1. Difficult to deal with initial inhomogeneous plasma drifts in a spectral code.
2. Time formation for a relativistic shock...
3. Electron acceleration ?

Thank You !