Center for Astroparticle Physics
GENEVA

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Redshift space distortions: beyond the Kaiser formula

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IAP, June 6, 2014

Outline

- *Galaxy clustering: peaks and biasing*
- *Redshift space distortions: measuring the growth rate*
- *Halo velocity bias: time evolution*
- *Tests with N-body simulations*

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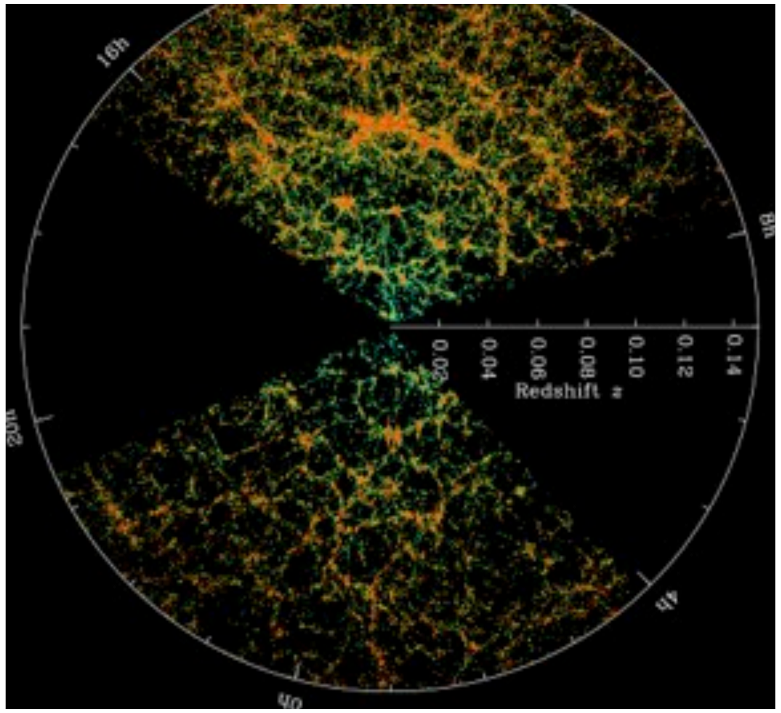
Cosmology with redshift surveys

Weak gravitational lensing
galaxy shapes



Galaxy clustering
galaxy positions

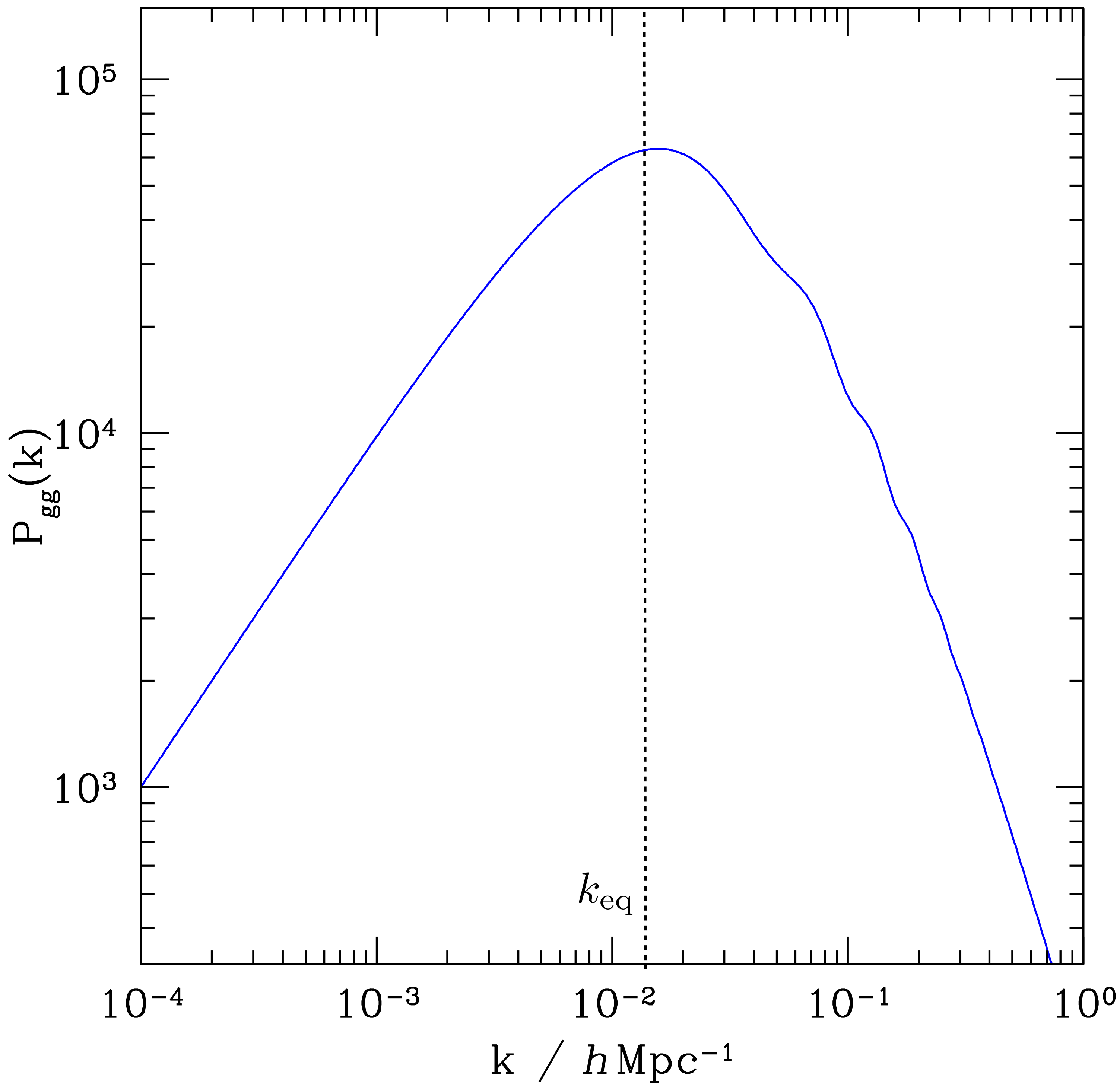
Statistical tools

The most common statistics is the 2-point correlation or power spectrum:

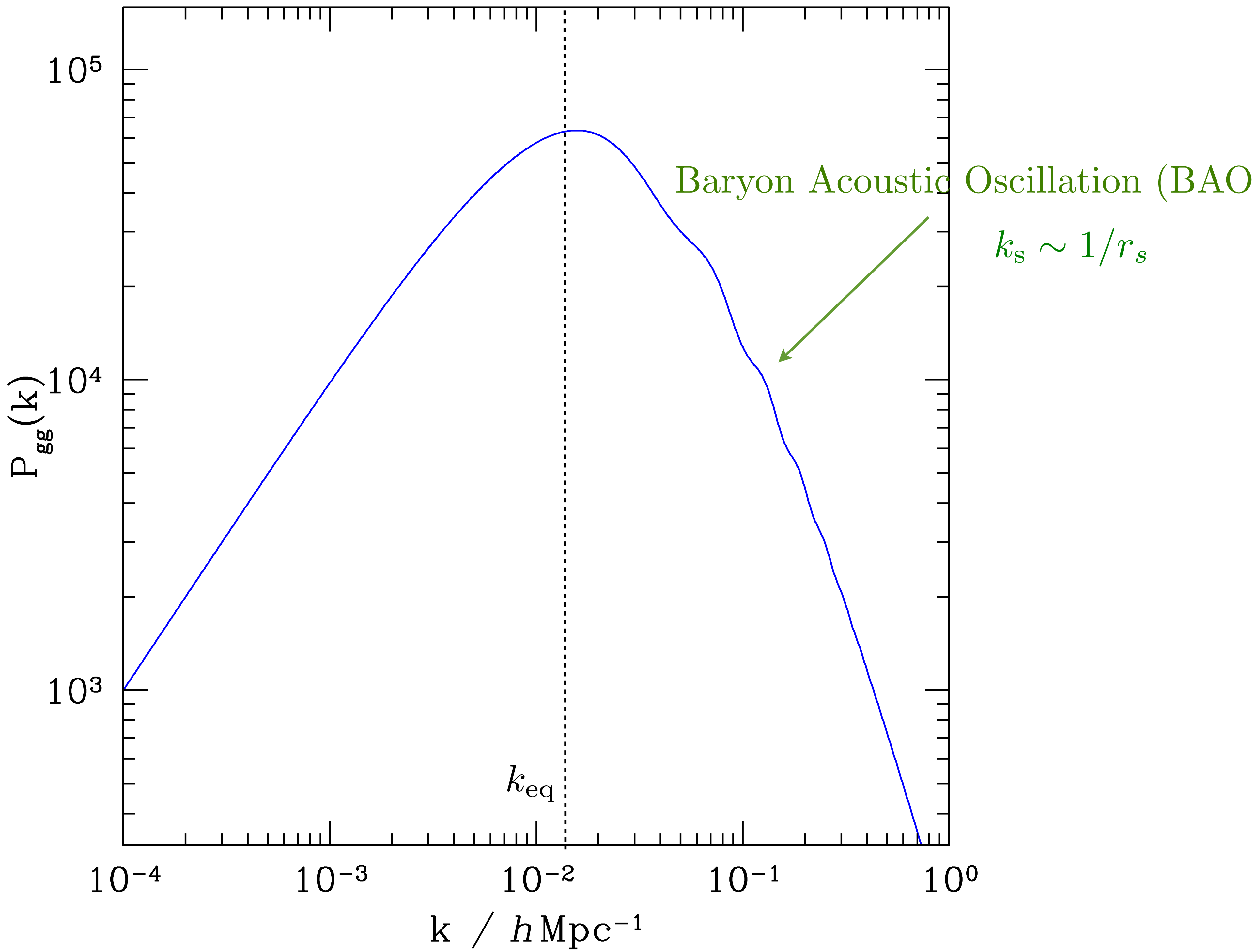
$\xi_{gg}(r)$ = *number of galaxy pairs in excess of random separated by a distance r*

$P_{gg}(k)$ = *power in fluctuations of wavenumber k*

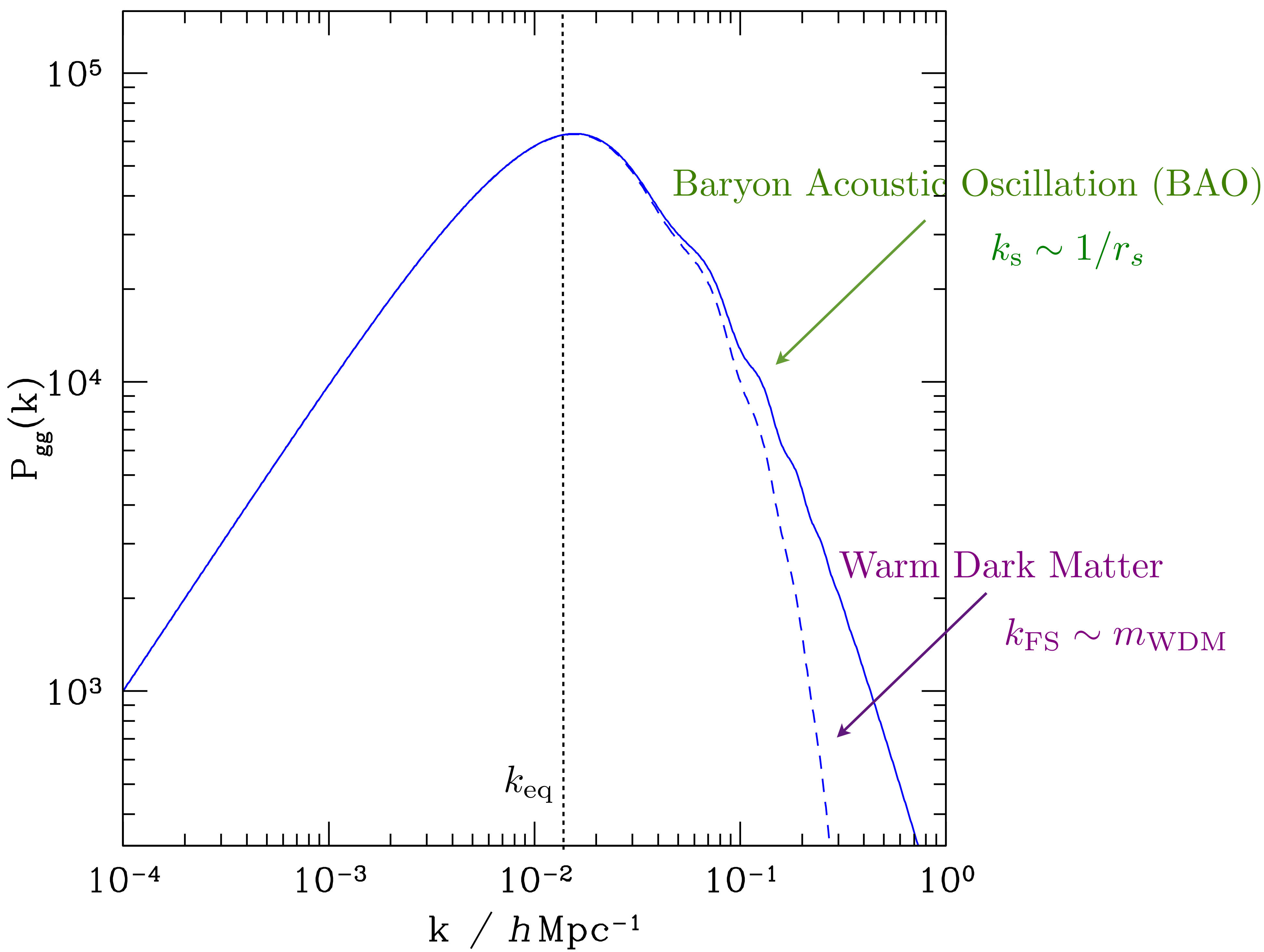
Scales



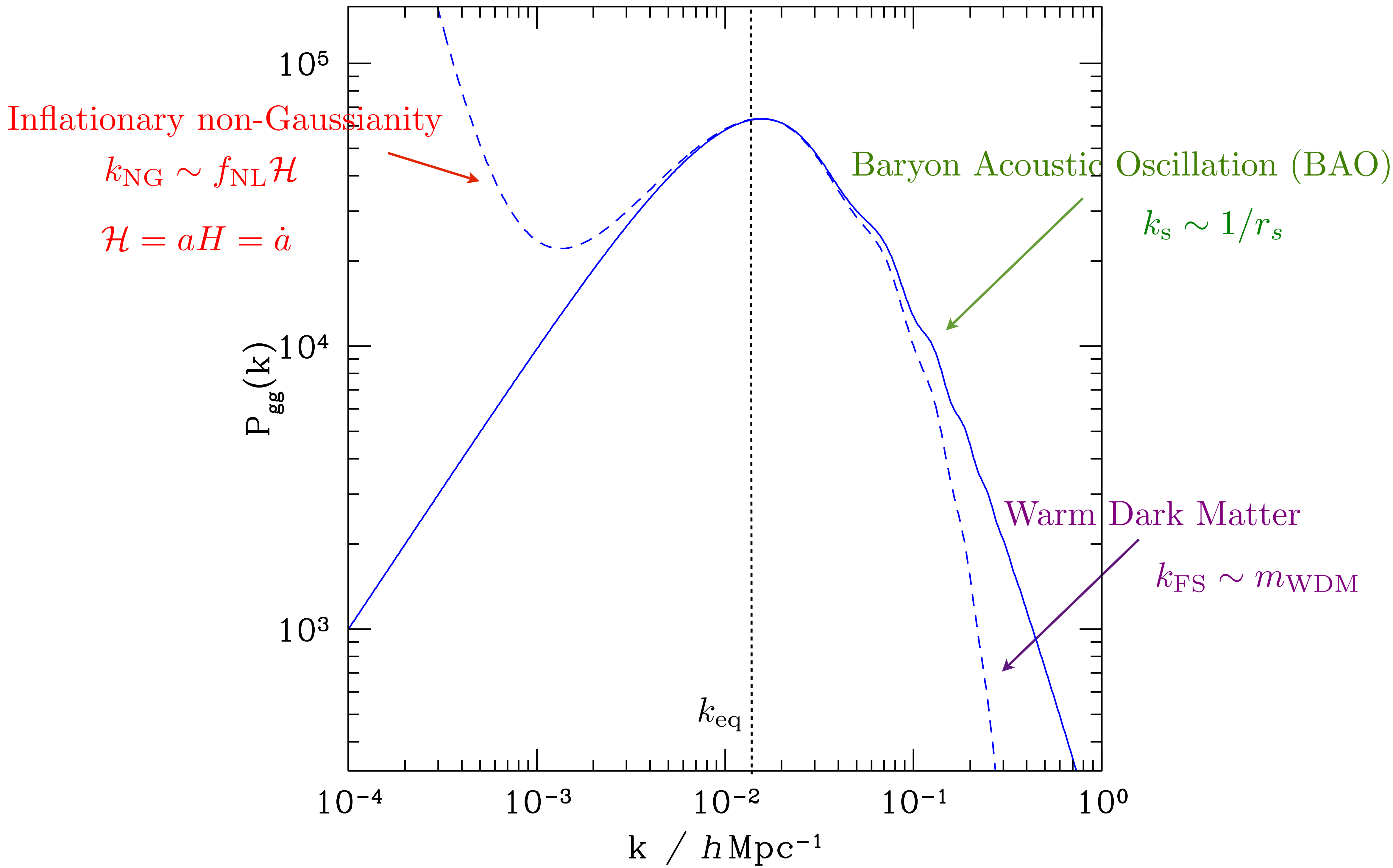
Characteristic scales



Characteristic scales



Characteristic scales



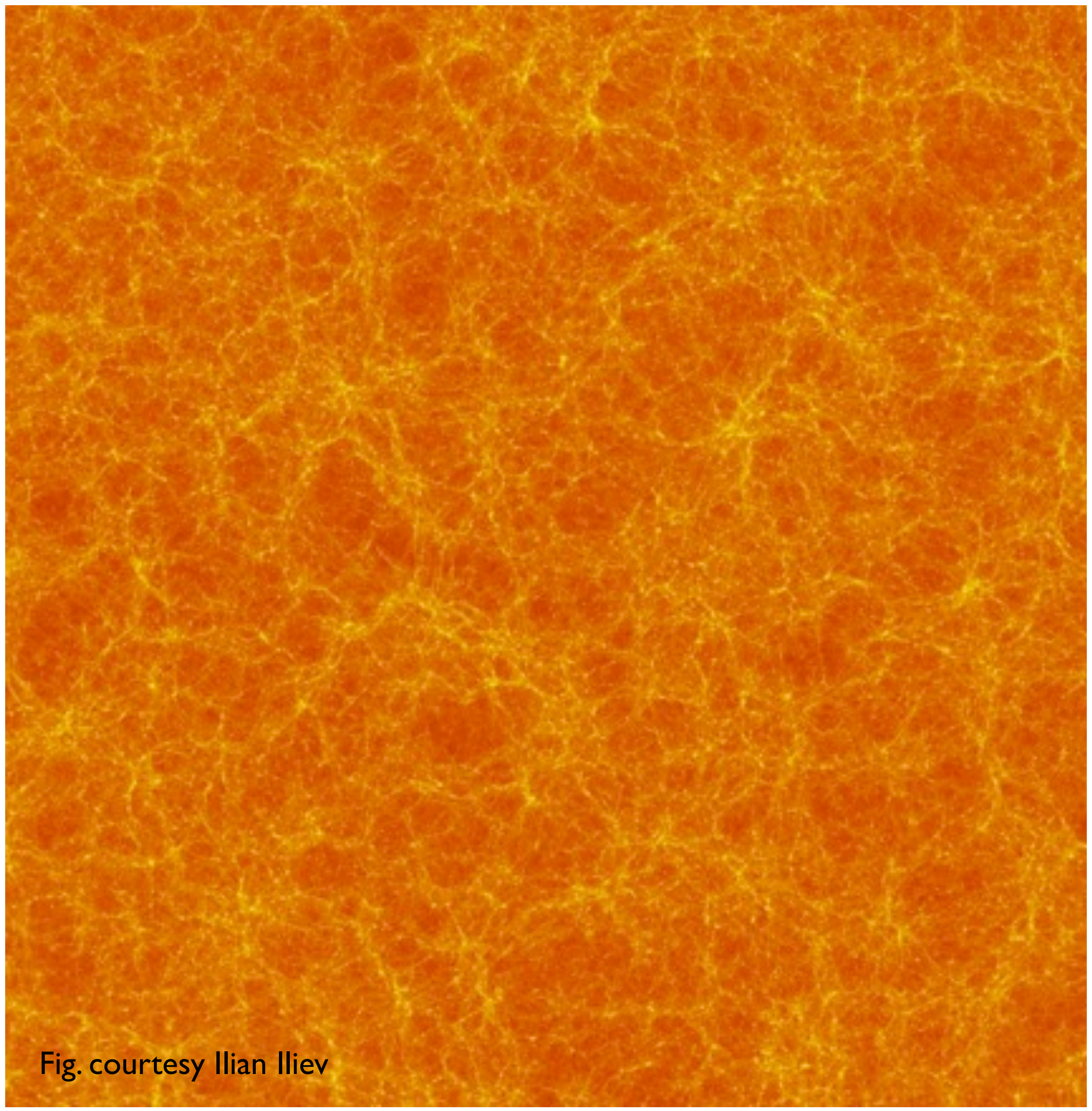
Parameter space

Parameter	Prior range	Baseline	Definition
$_{\rm b}$ $_{\rm b}h^2$	[0.005, 0.1]	. . .	Baryon density today
$_{\rm c}$ $_{\rm c}h^2$	[0.001, 0.99]	. . .	Cold dark matter density today
100 $_{\rm MC}$	[0.5, 10.0]	. . .	100 × approximation to r / $D_{\rm A}$ (CosmoMC)
. . . .	[0.01, 0.8]	. . .	Thomson scattering optical depth due to reionization
$_K$	[−0.3, 0.3]	0	Curvature parameter today with $_{\rm tot} = 1 -$ $_K$
m	[0, 5]	0.06	The sum of neutrino masses in eV
$m^{\rm eff}_{\ ,\,\rm sterile}$	[0, 3]	0	Effective mass of sterile neutrino in eV
w_0	[−3.0, −0.3]	−1	Dark energy equation of state[a], $w(a) = w_0 + (1 - a)w_a$
w_a	[−2, 2]	0	As above (perturbations modelled using PPF)
$N_{\rm eff}$	[0.05, 10.0]	3.046	Effective number of neutrino-like relativistic degrees of freedom (see text)
$Y_{\rm P}$	[0.1, 0.5]	BBN	Fraction of baryonic mass in helium
$A_{\rm L}$	[0, 10]	1	Amplitude of the lensing power relative to the physical value
$n_{\rm s}$	[0.9, 1.1]	. . .	Scalar spectrum power-law index ($k_0 = 0.05\mathrm{Mpc}^{-1}$)
$n_{\rm t}$	$n_{\rm t} = -r_{0.05}/8$	Inflation	Tensor spectrum power-law index ($k_0 = 0.05\mathrm{Mpc}^{-1}$)
$dn_{\rm s}/d\ln k$	[−1, 1]	0	Running of the spectral index
$\ln(10^{10}A_{\rm s})$	[2.7, 4.0]	. . .	Log power of the primordial curvature perturbations ($k_0 = 0.05\,\mathrm{Mpc}^{-1}$)
$r_{0.05}$	[0, 2]	0	Ratio of tensor primordial power to curvature power at $k_0 = 0.05\,\mathrm{Mpc}^{-1}$

(Planck collaboration)

+ modified gravity, primordial non-Gaussianity etc.

Simulations + analytics



noise field $\tilde{n}_g \mathcal{W}(z)\epsilon(\mathbf{x})$, and the spherical multipole function $\mathcal{M}_l(k', k)$ is defined as

$$\mathcal{M}_l(\tilde{k}, k) \equiv k \sqrt{\frac{2}{\pi}} \int_0^\infty dr\, r^2\, \mathcal{W}(r)\, j_l(\tilde{k}r)\, j_l(kr)\, \mathcal{T}_g(\tilde{k}, r)\;, \tag{26}$$

where its dimension is $[\mathcal{M}_l(\tilde{k}, k)] = L^2$. After some simplification, the spherical power spectrum in Eq. (17) eventually reads

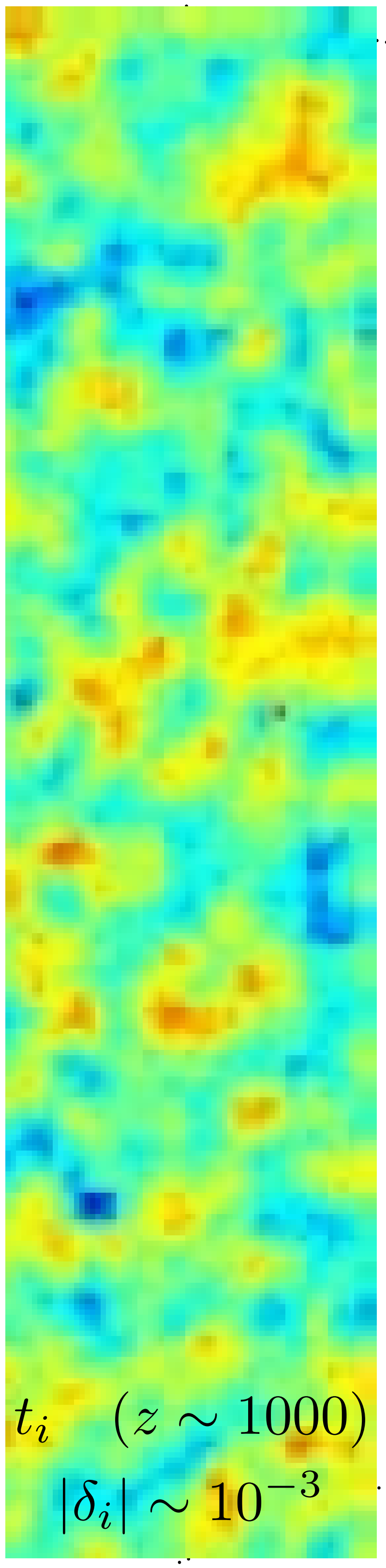
$$\begin{aligned} \mathcal{S}_l(k, k') &= 4\pi\tilde{n}_g^2 \int d\ln\tilde{k}\; \Delta^2_{\varphi_v}(\tilde{k})\mathcal{M}_l(\tilde{k}, k)\mathcal{M}_l(\tilde{k}, k') \\ &\quad + \frac{2kk'}{\pi}\tilde{n}_g \int_0^\infty dr\, r^2 \mathcal{W}(r) j_l(kr) j_l(k'r)\;. \end{aligned} \tag{27}$$

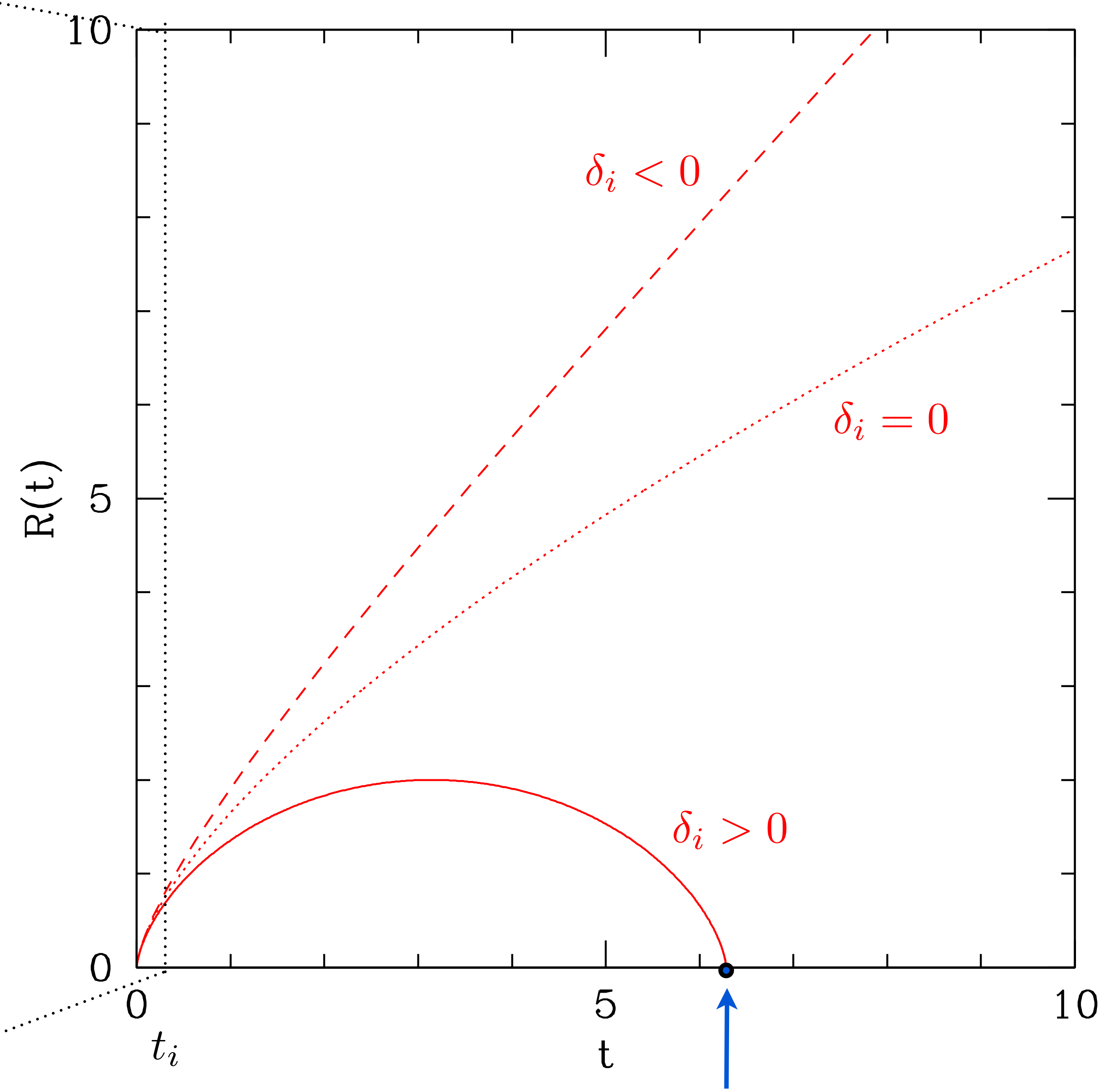
The second-term in the right-hand side is the shot-noise contribution. Using the Limber approximation (see Sec. III C), the shot-noise power spectrum can be rewritten as

$$\begin{aligned} \mathcal{N}_l(k, k') &\equiv \frac{2kk'}{\pi}\tilde{n}_g \int_0^\infty dr\, r^2 \mathcal{W}(r) j_l(kr) j_l(k'r) \\ &\approx \tilde{n}_g \mathcal{W}(\nu/k)\delta^D(k - k')\;, \end{aligned} \tag{28}$$

where $\nu = l + 1/2$. For a homogeneous and isotropic galaxy population with constant comoving number density $\bar{n}_g = \tilde{n}_g$ and power spectrum $\langle \delta_g(\mathbf{k})\delta_g^*(\mathbf{k}')\rangle = (2\pi)^3\delta^D(\mathbf{k}-\mathbf{k}')P_g(k)$, the spherical power spectrum Eq. (27) yields the well-known

Spherical collapse





Linear density @ collapse: $\delta = 1.68 \equiv \delta_c$

Nonlinear density @ collapse: $\Delta \sim 200$

(Gunn & Gott '72)

The halo model: origin

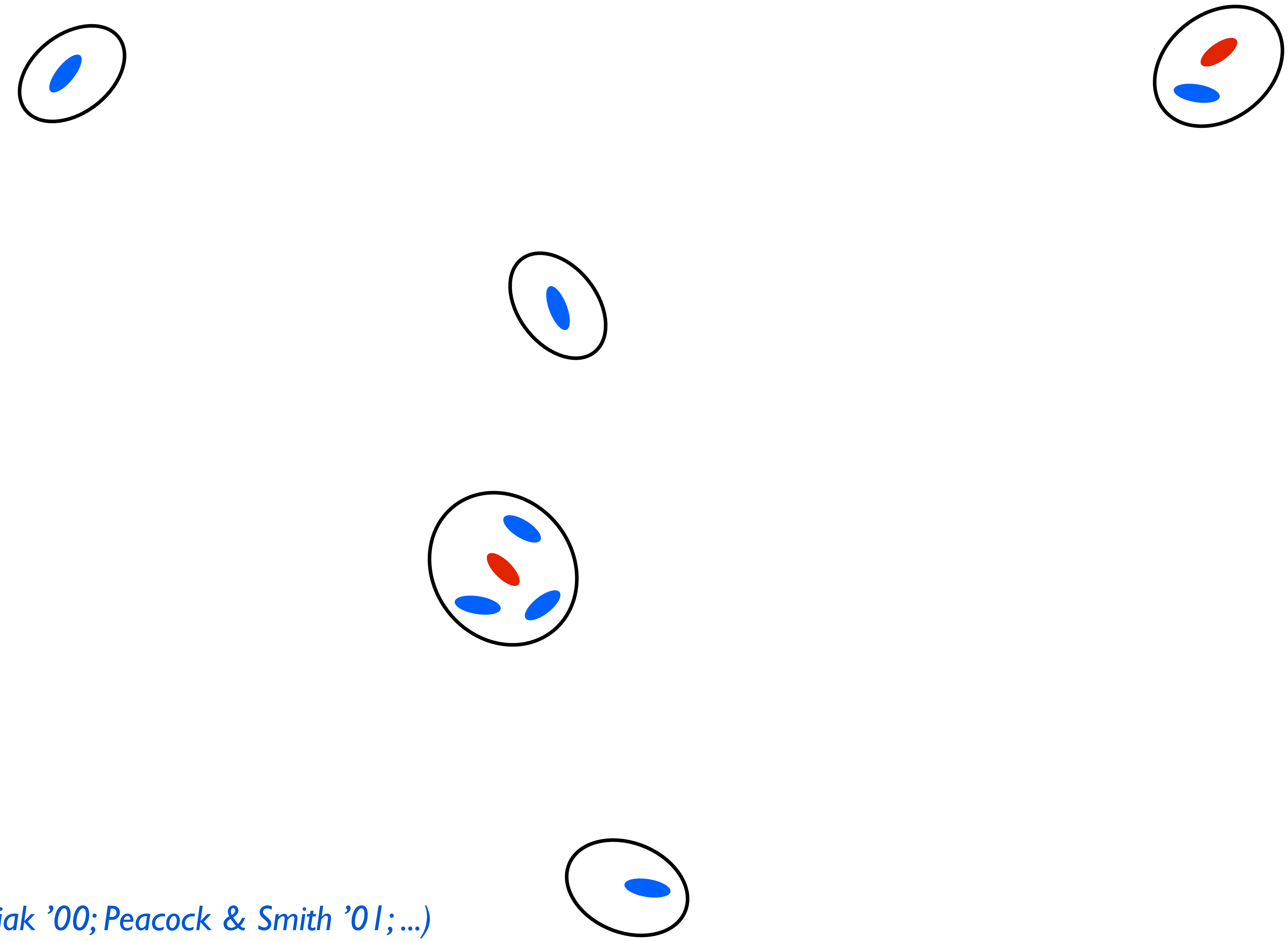
$<10^{13} h^{-1} M_\odot$. We suggest that this material constitutes the so-called 'missing mass' in clusters and the extensive halos of isolated galaxies; we further suggest that *all* the luminous matter seen in galaxies formed from residual gas that settled within the potential wells provided by the dark material at each stage of the clustering process and then collapsed to form stars. We must therefore next consider how the gravitational field of the clusters dark

(White & Rees '78)

(Binney '77; Silk '77; Rees & Ostriker '77; White & Rees '78; ...)

The halo model: clustering

$$\xi_{gg}(r) = \xi_{\rm 1halo}(r) + \xi_{\rm 2halo}(r)$$



(Ma & Fry '00; Seljak '00; Peacock & Smith '01; ...)

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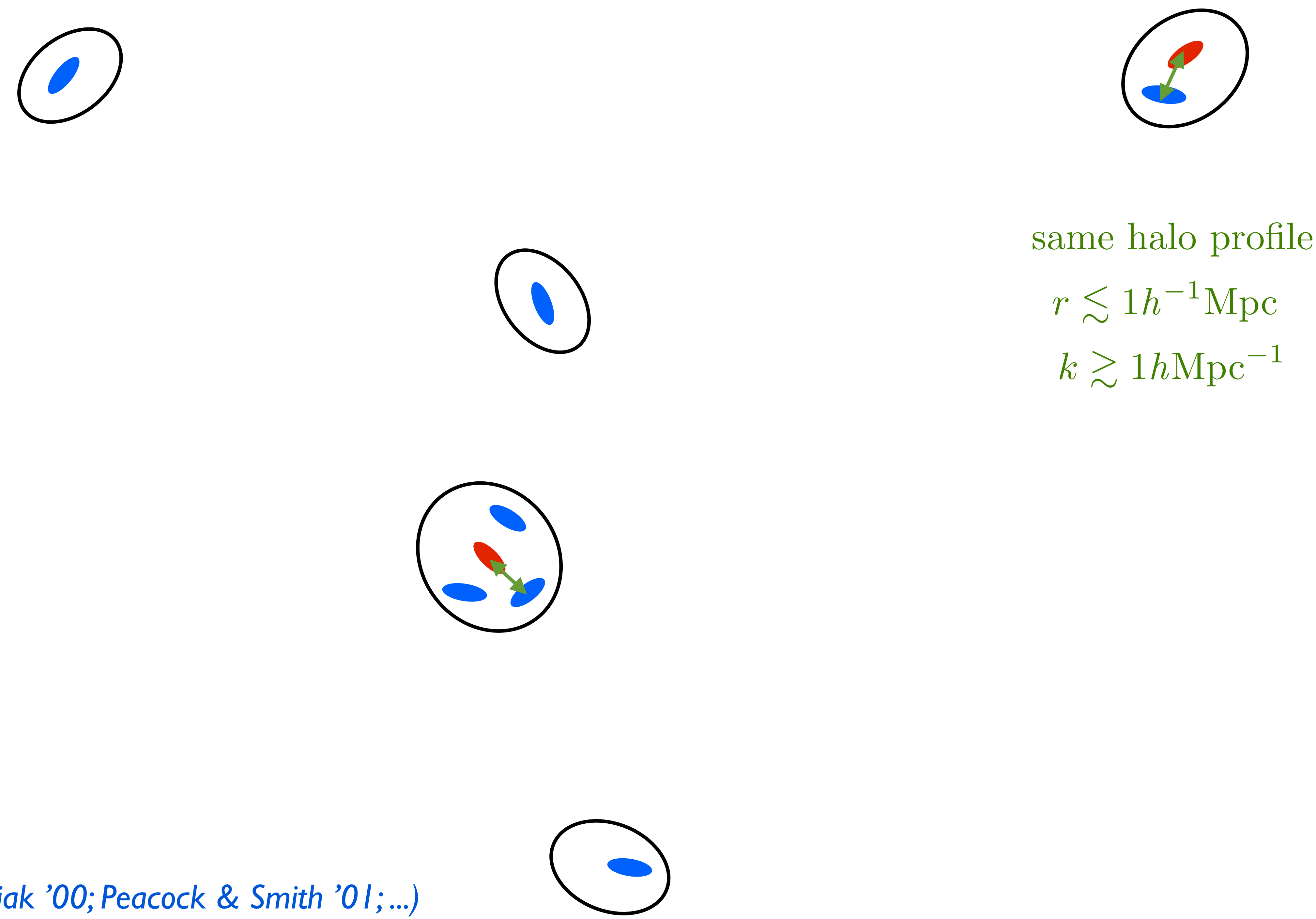

(Ma & Fry ’00; Seljak ’00; Peacock & Smith ’01; ...)

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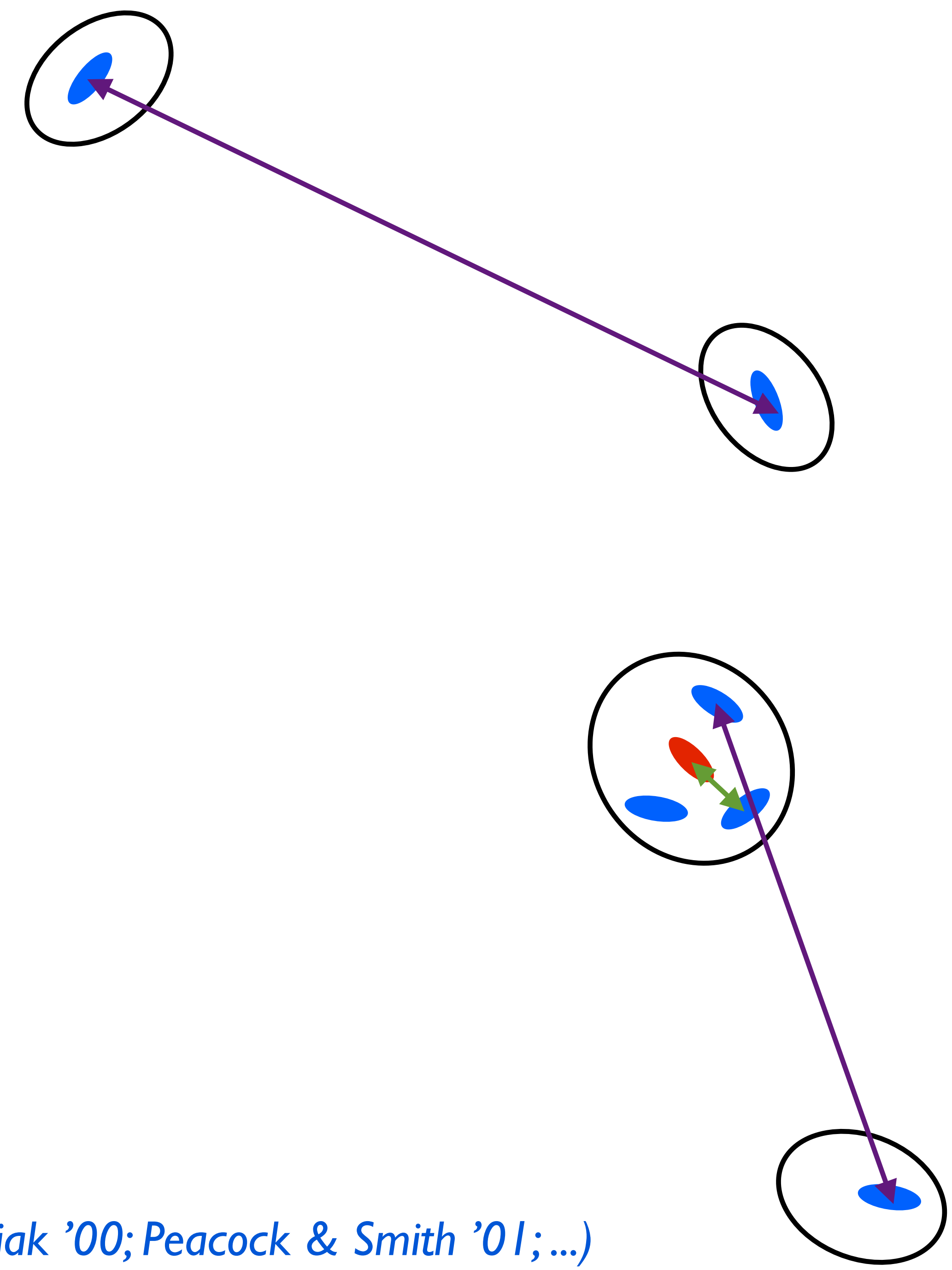
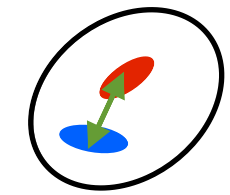




same halo profile

$r \lesssim 1 h^{-1}\mathrm{Mpc}$

$k \gtrsim 1 h\mathrm{Mpc}^{-1}$

distinct halo centres

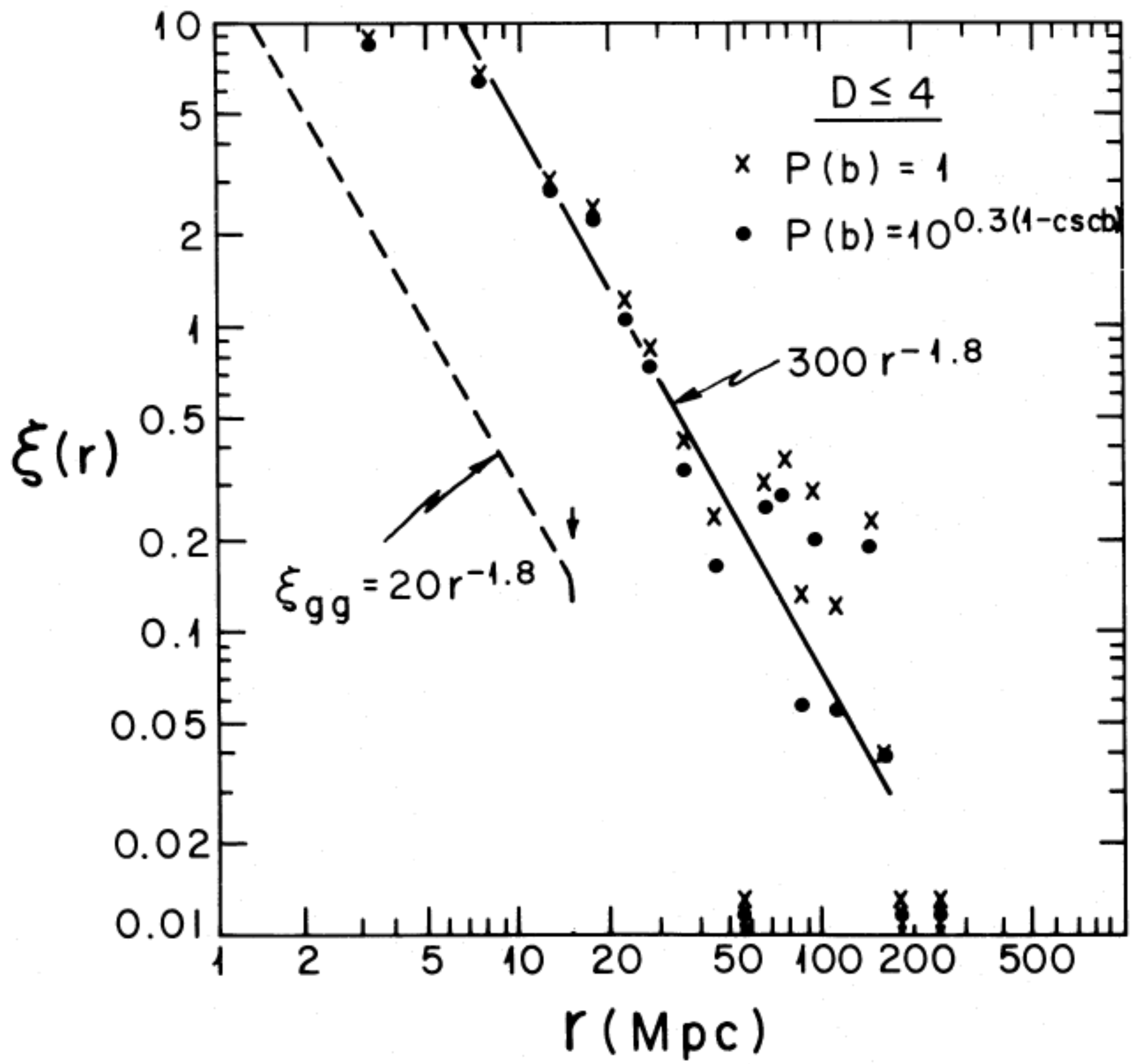
$r \gtrsim 1 h^{-1}\mathrm{Mpc}$

$k \lesssim 1 h\mathrm{Mpc}^{-1}$

(Ma & Fry ’00; Seljak ’00; Peacock & Smith ’01; ...)

Biasing

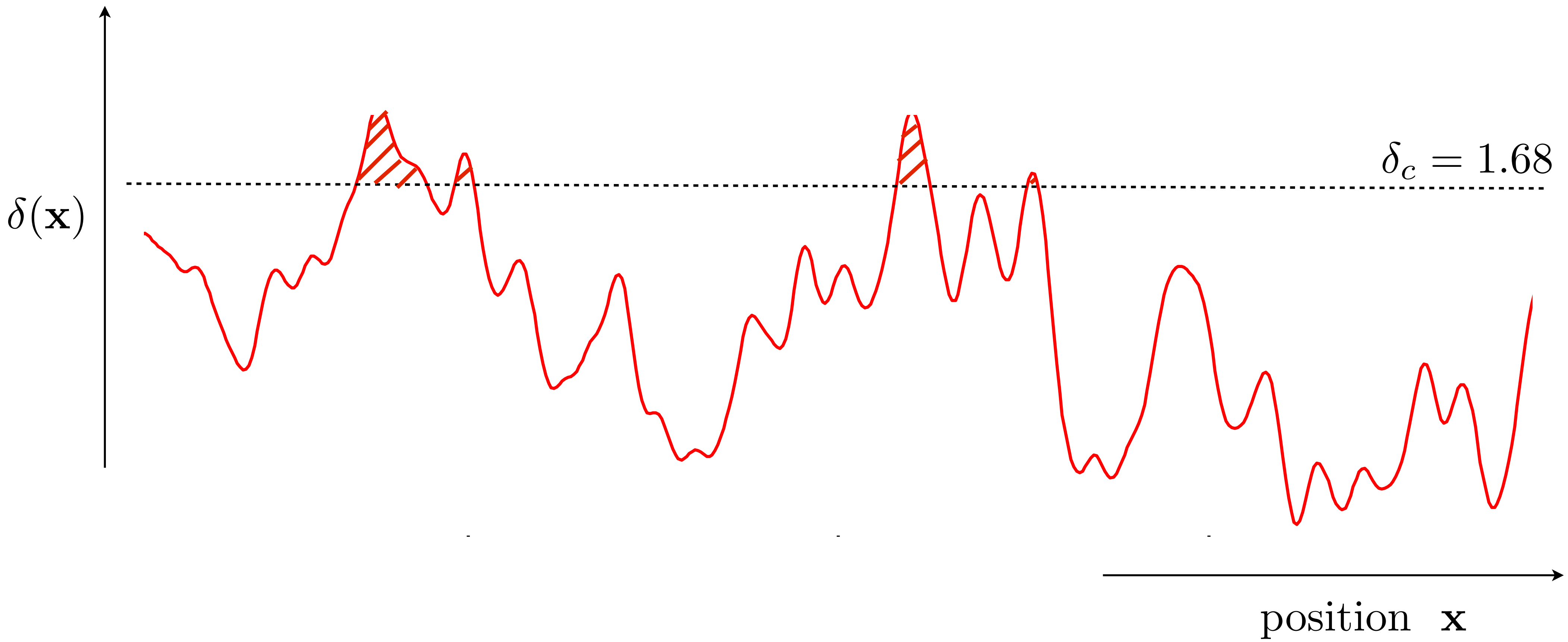
~ 100 clusters of galaxies with $z \lesssim 0.1$



(Bahcall & Soneira ’83)

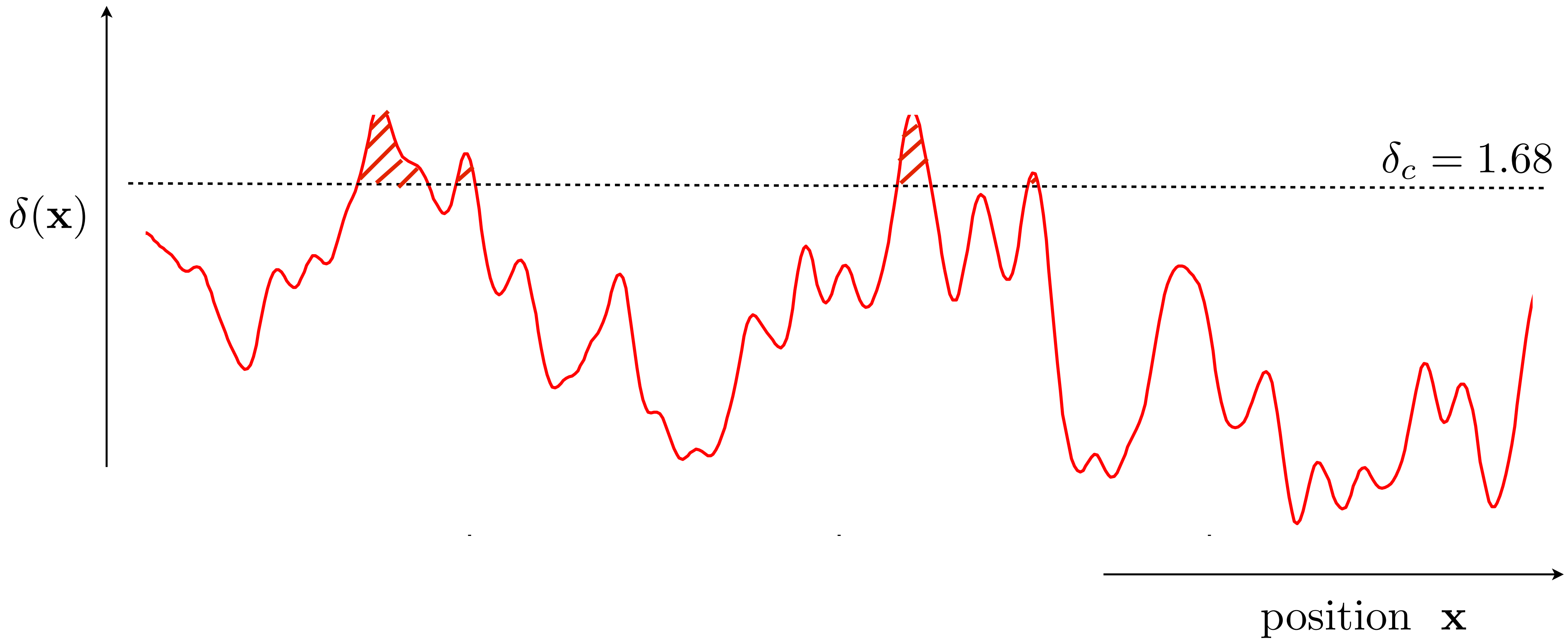
Biasing

DM halos trace linear densities above critical threshold for collapse



(Kaiser '84)

DM halos trace linear densities above critical threshold for collapse



$$\delta_g(\mathbf{x}) = b_1 \delta(\mathbf{x}) + ...$$

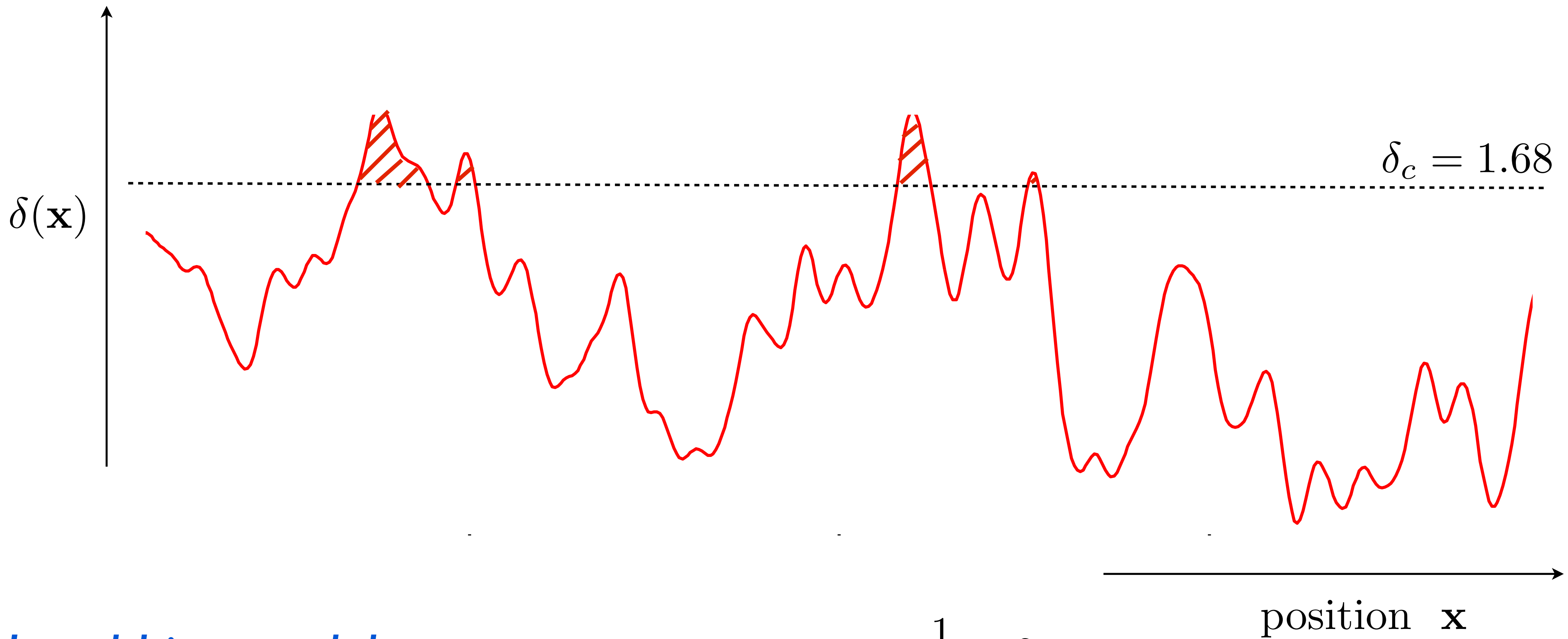
$$\xi_{gg}(r) = b_1^2 \, \xi(r) + ...$$

$$b_1 \sim \frac{\nu}{\sigma} = \frac{\delta_c}{\sigma^2}$$

(Kaiser '84)

Biasing

DM halos trace linear densities above critical threshold for collapse



Local bias model:

$$\delta_g(\mathbf{x}) = b_1\delta(\mathbf{x}) + \frac{1}{2}b_2\delta^2(\mathbf{x}) + \ldots$$

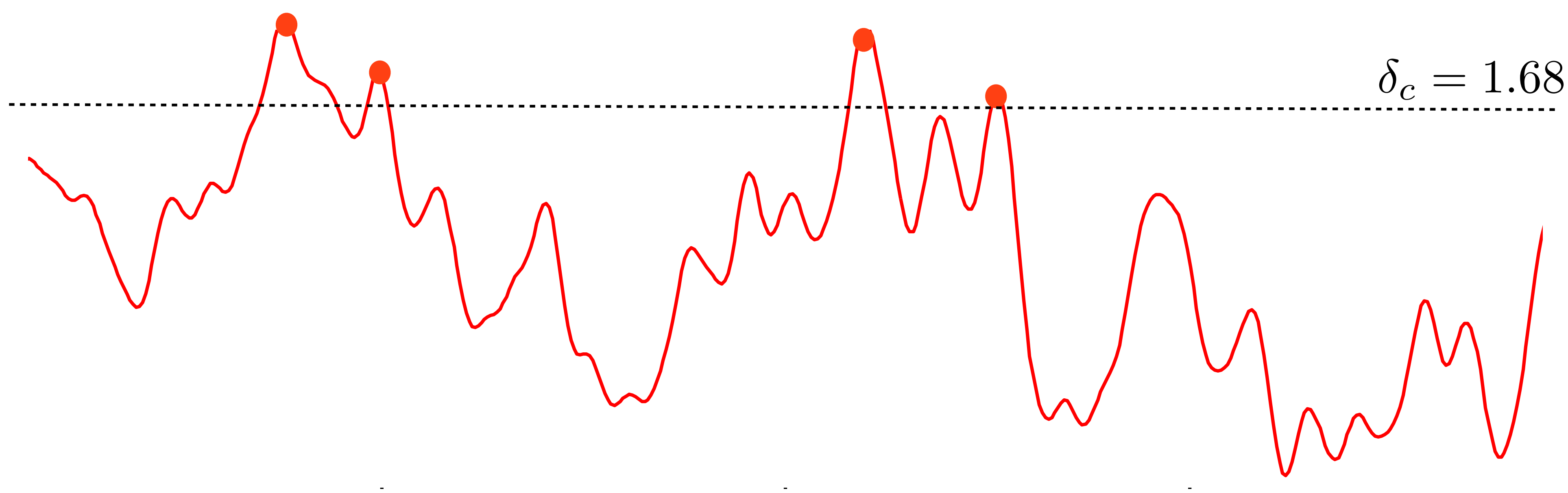
$$\xi_{gg}(r) = b_1^2\,\xi(r) + \frac{1}{2}b_2^2\,\xi^2(r) + \ldots$$

$$b_N \sim \left(\frac{\nu}{\sigma}\right)^N$$

(Kaiser '84; Szalay '88' Fry & Gaztanaga 93; ...)

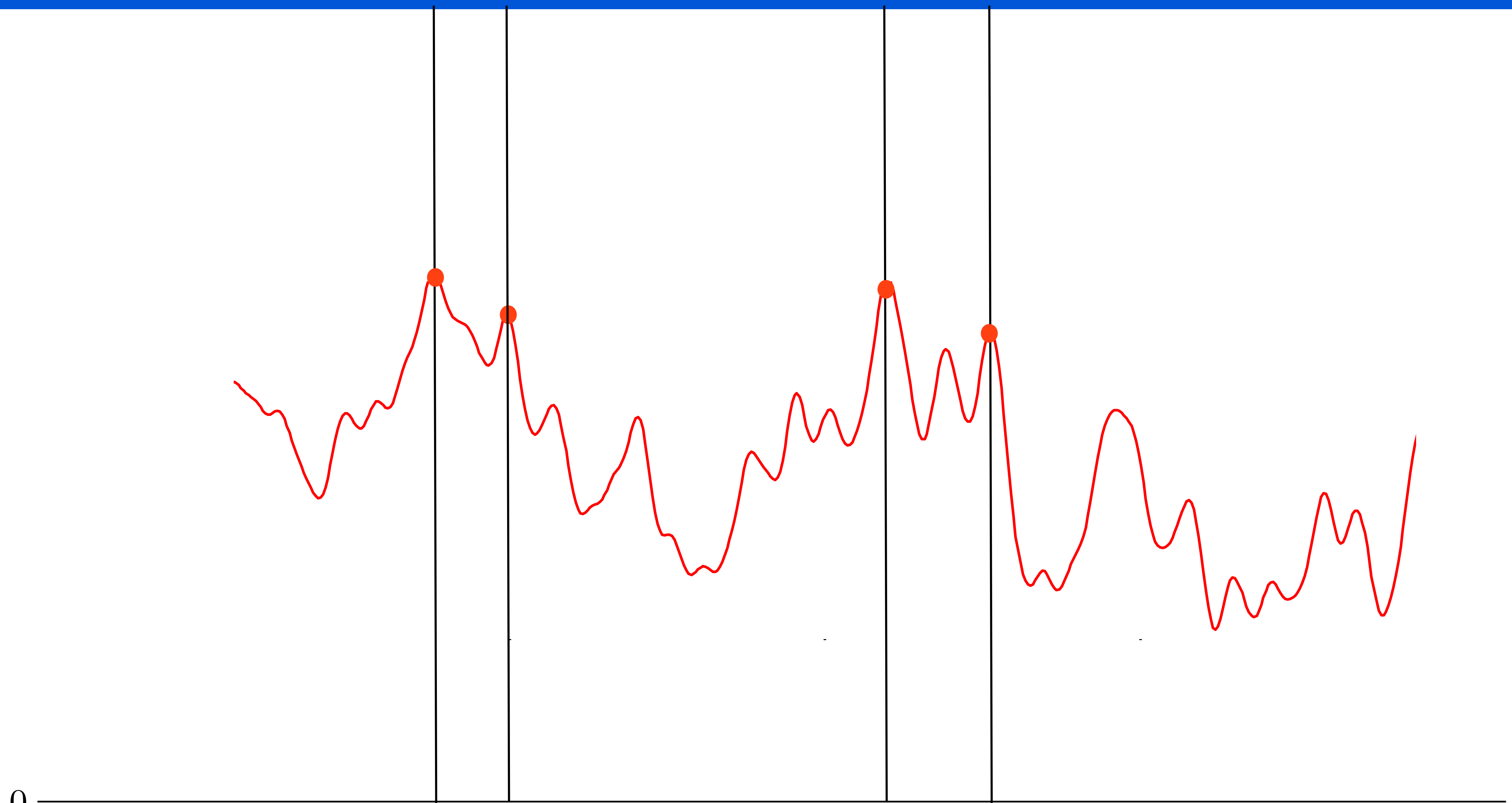
The peak model

Consider initial density maxima. Countable set, like DM halos



(Bardeen et al. '86 = BBKS)

Peak number density



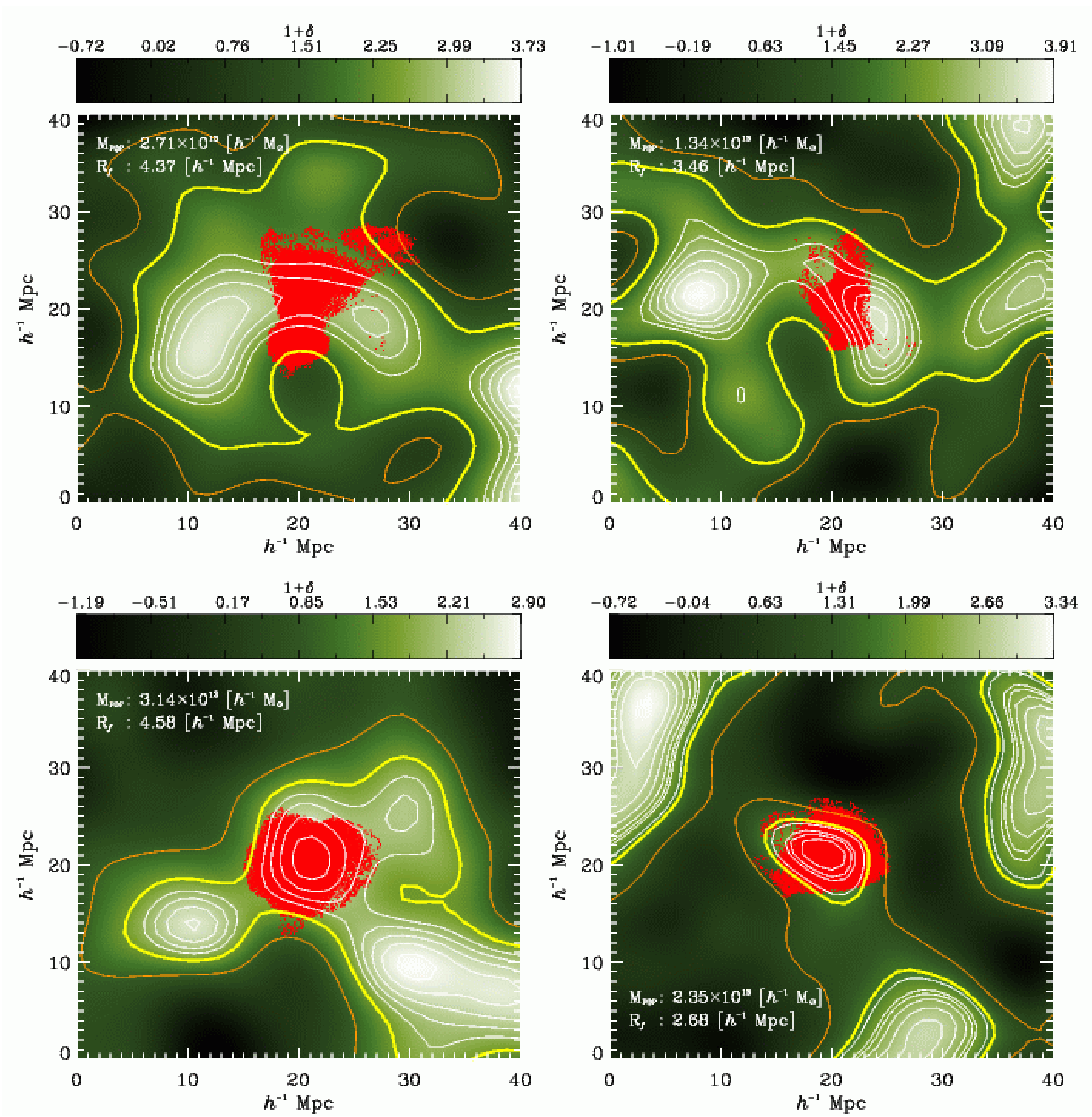
$$n_{\rm pk}(\mathbf{x}) = \sum_p \delta_{\rm D}(\mathbf{x} - \mathbf{x}_p) =?$$

Answer in Kac '43; Rice '51; BBKS

Connection initial peaks - halos

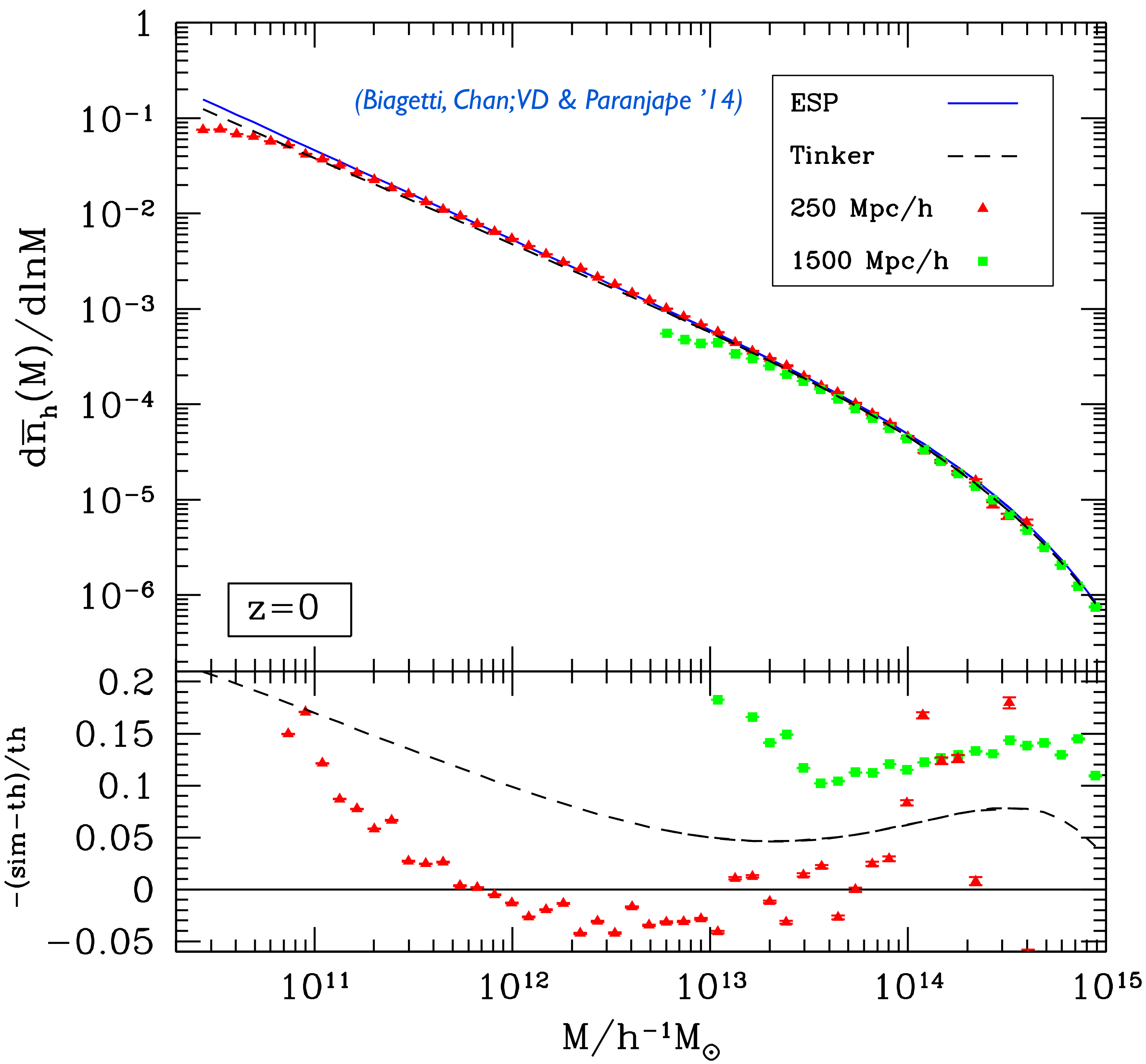
$\lesssim 10\%$

$\gtrsim 90\%$



(Ludlow & Porciani '11)

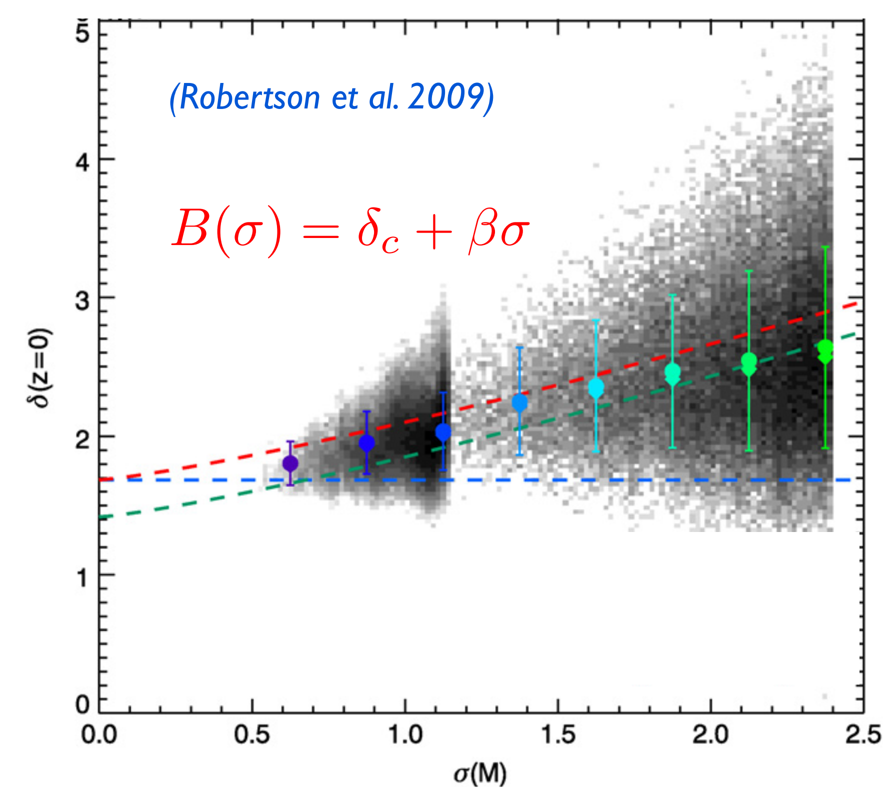
The halo mass function



(Paranjape, Sheth & VD '13)

3 ingredients in addition to BBKS:

- *Collapse barrier depends on halo mass*
- *Collapse barrier is stochastic*
- *First-crossing (no peaks in peaks)*



Linear peak bias

- *Gaussian initial conditions: density correlate with the curvature*

$$\delta_s(\mathbf{x}), \quad \nabla^2\delta_s(\mathbf{x})$$

$$P\big(\delta_s(\mathbf{x})\big|\nabla^2\delta_s(\mathbf{x})\big) \neq P\big(\delta_s(\mathbf{x})\big)$$

- *The linear bias of initial density peaks is*

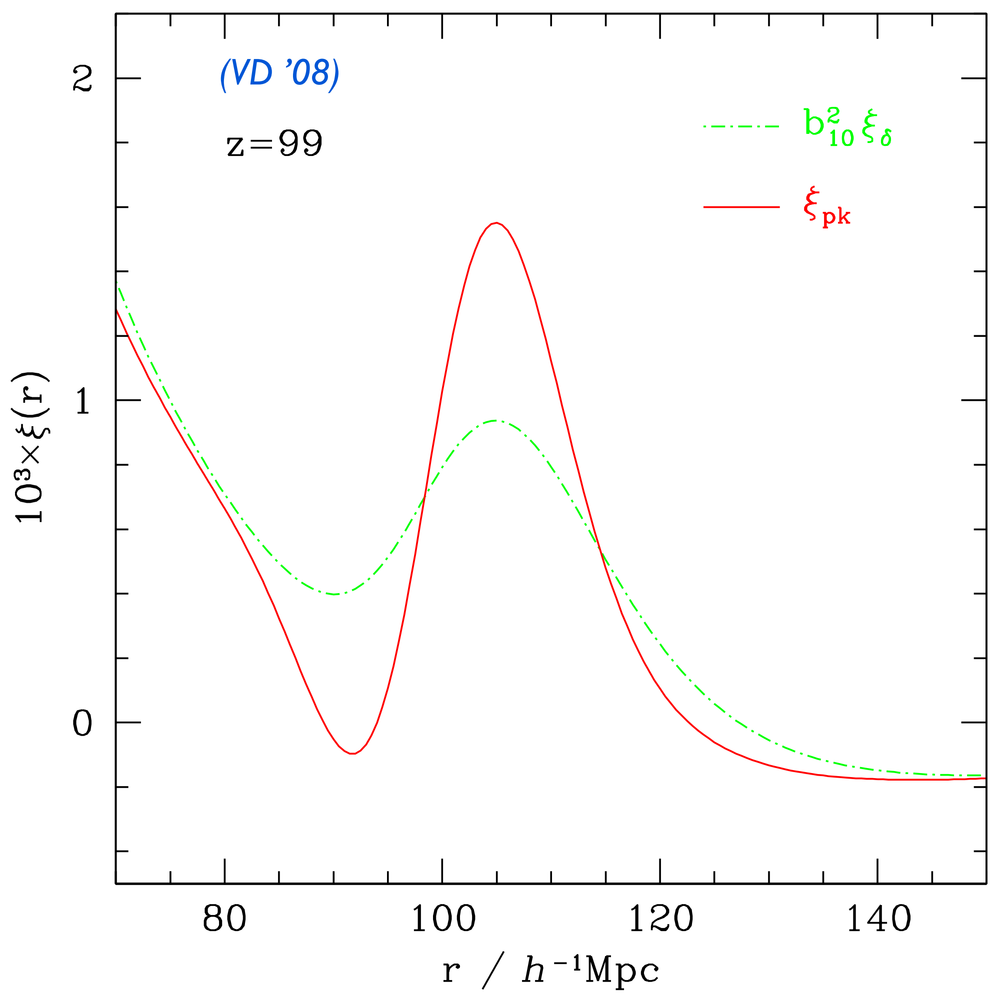
$$\delta_{\rm pk}(\mathbf{k}) = c_1(k)\delta(\mathbf{k})$$

$$c_1(k) = \left(b_{100} + b_{010}k^2 - b_{001}\frac{\partial \ln W}{\partial R_s}\right)W(kR_s)$$

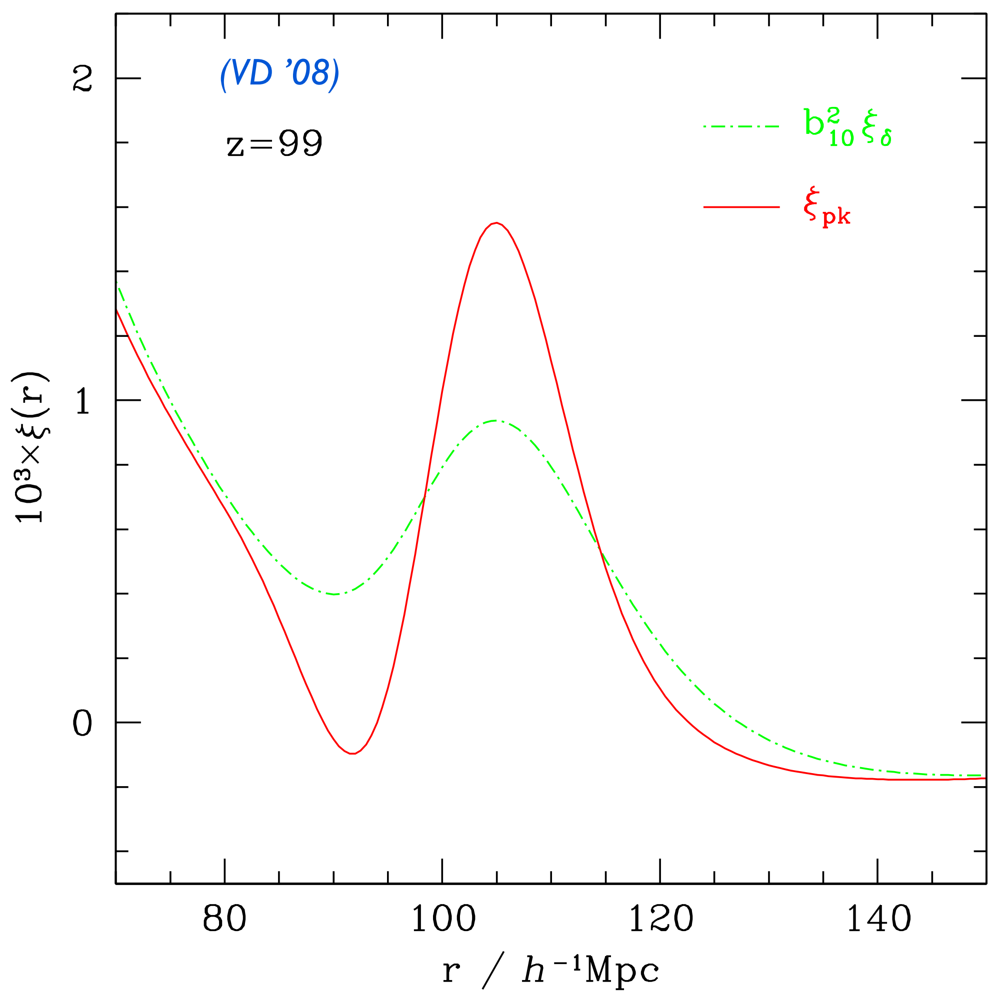
$$\approx \left(b_{10} + b_{01}k^2\right)W(kR_s)$$

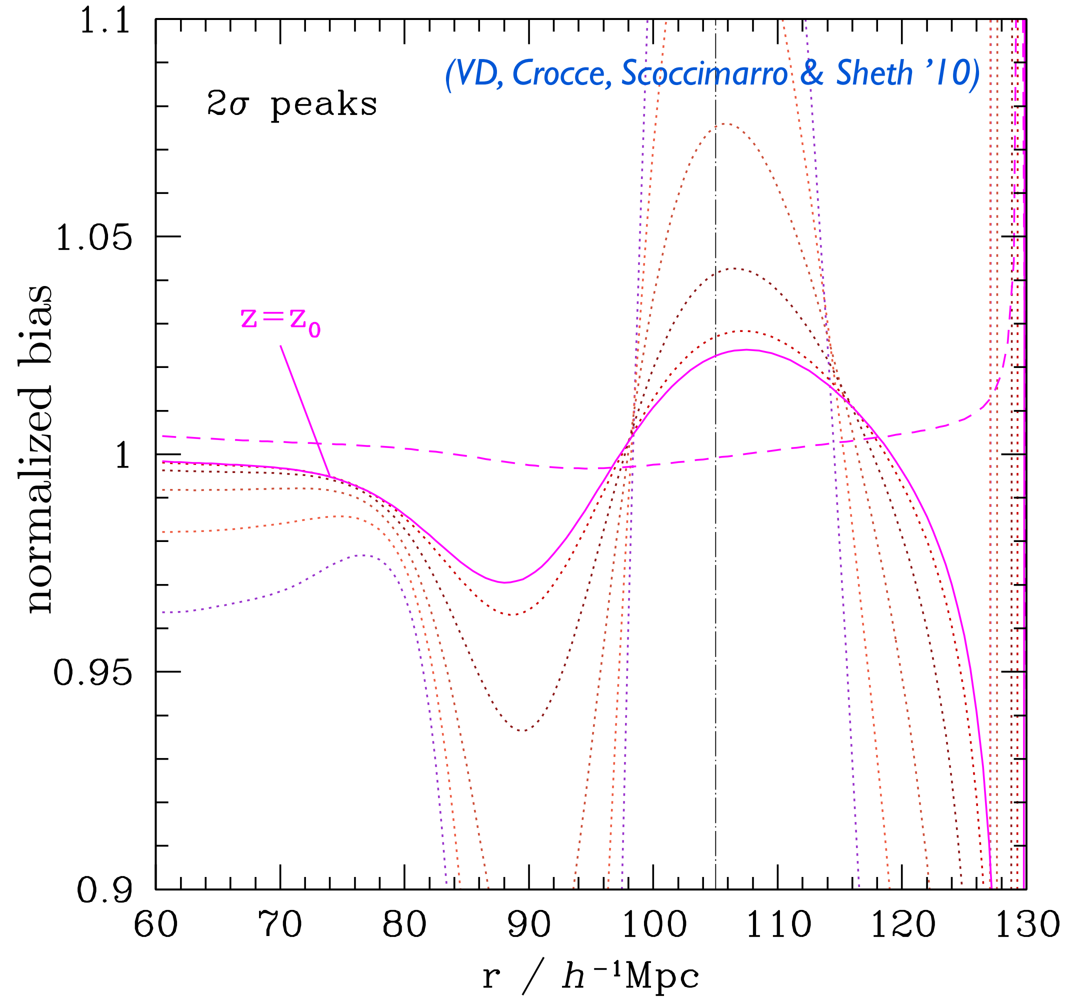
(VD '08; VD, Gong & Riotto '13)

Effect on the BAO

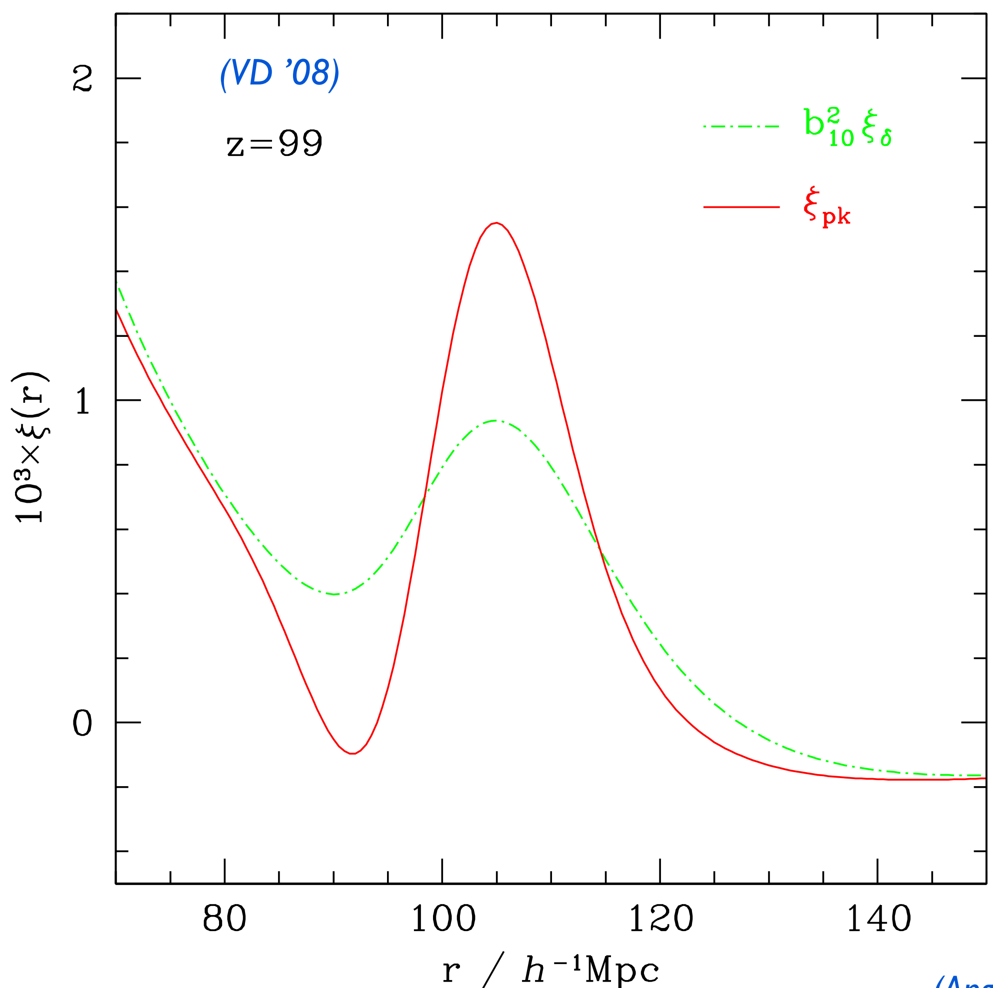


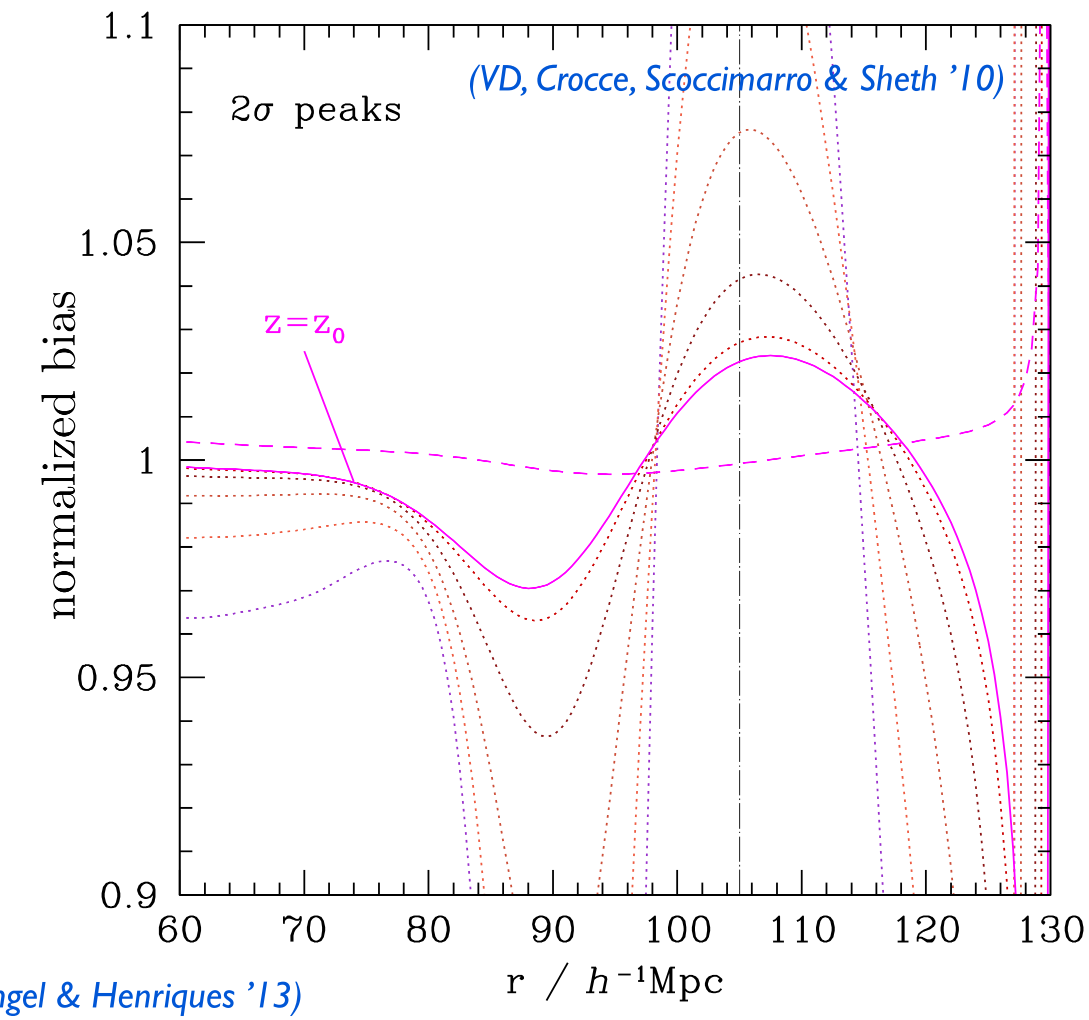
Effect on the BAO

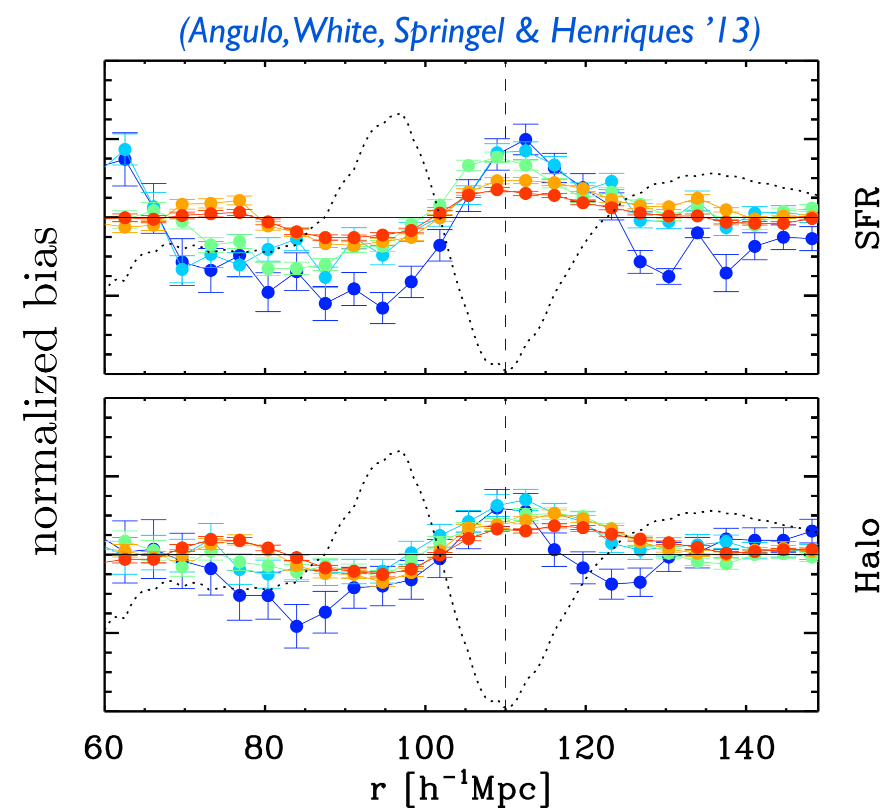




Effect on the BAO







Peak velocity dispersion

- *For Gaussian initial conditions, velocities correlate with the gradient of the density*

$$\mathbf{u}(\mathbf{x}) \sim \nabla^{-1}\delta_s(\mathbf{x}), \quad \boldsymbol{\eta}(\mathbf{x}) = \nabla\delta_s(\mathbf{x})$$

$$P\big(\mathbf{u}(\mathbf{x})\big|\boldsymbol{\eta}(\mathbf{x})\big) \neq P\big(\mathbf{u}(\mathbf{x})\big)$$

- *The velocity dispersion of initial density peaks is*

$$\sigma^2_{\rm vpk} = \sigma^2_{\rm v}\Big(1-\gamma_0^2\Big)$$

where

$$\gamma_0 = \frac{\sigma_0^2}{\sigma_{-1}\sigma_1}, \quad 0 < \gamma_0 < 1$$

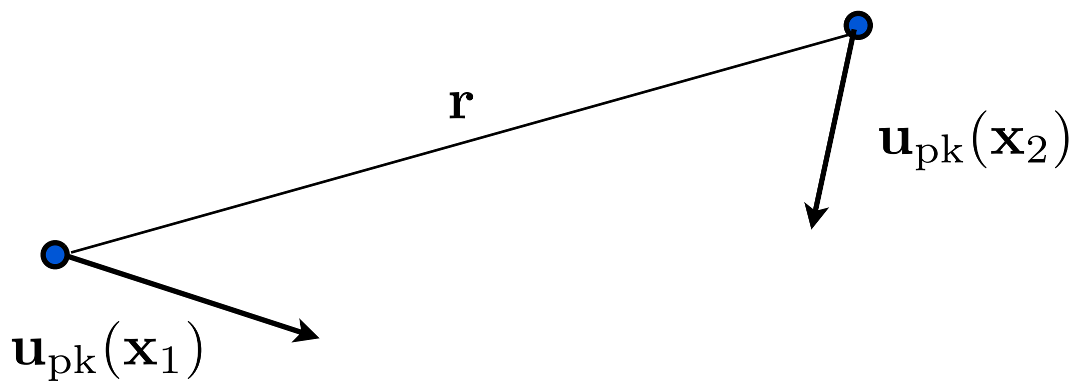
$$\sigma_n^2 = \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} W^2(kR_s)P(k)$$

(BBKS)

Peak velocity bias

- *Mean and dispersion of peak pairwise velocity*

$$\left\langle \left[\Big(\mathbf{u}_{\rm pk}(\mathbf{x}_2) - \mathbf{u}_{\rm pk}(\mathbf{x}_1) \Big) \cdot \hat{\mathbf{r}} \right]^n \right\rangle$$



- *The results can be thought as arising from*

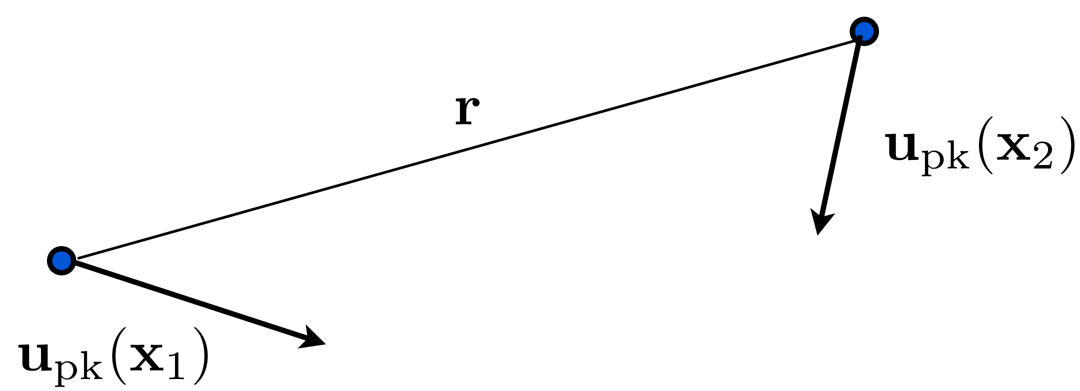
$$\mathbf{u}_{\rm pk}(\mathbf{k}) = \left(1 - \frac{\sigma_0^2}{\sigma_1^2} k^2 \right) W(kR_s)\, \mathbf{u}(\mathbf{k})$$

(VD & Sheth '10)

Peak velocity bias

- *Mean and dispersion of peak pairwise velocity*

$$\left\langle \left[\left(\mathbf{u}_{\rm pk}(\mathbf{x}_2) - \mathbf{u}_{\rm pk}(\mathbf{x}_1) \right) \cdot \hat{\mathbf{r}} \right]^n \right\rangle$$

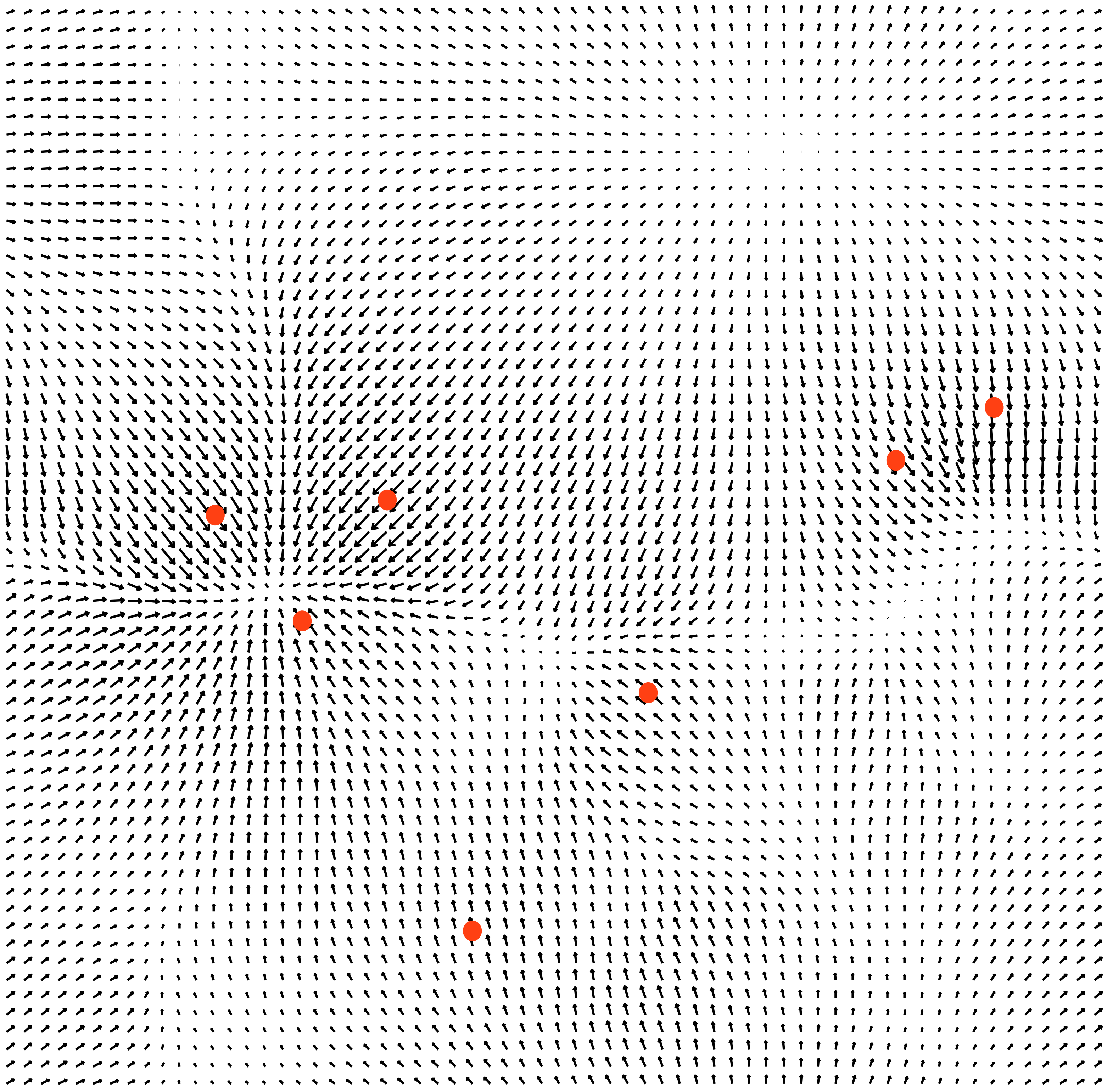


- *The results can be thought as arising from*

$$\mathbf{u}_{\rm pk}(\mathbf{k}) = \underbrace{\left(1 - \frac{\sigma_0^2}{\sigma_1^2} k^2\right) W(kR_s)}_{\equiv\, b_{\rm vpk}(k)} \mathbf{u}(\mathbf{k})$$

(VD & Sheth '10)

Peak velocity bias = statistical



Peak velocity bias = statistical

- *This explains why*

$$\sigma_{\rm vpk}^2 = \frac{1}{2\pi^2}\int_0^\infty dk\, b_{\rm vpk}^2(k)\, W^2(kR_s)P(k)$$

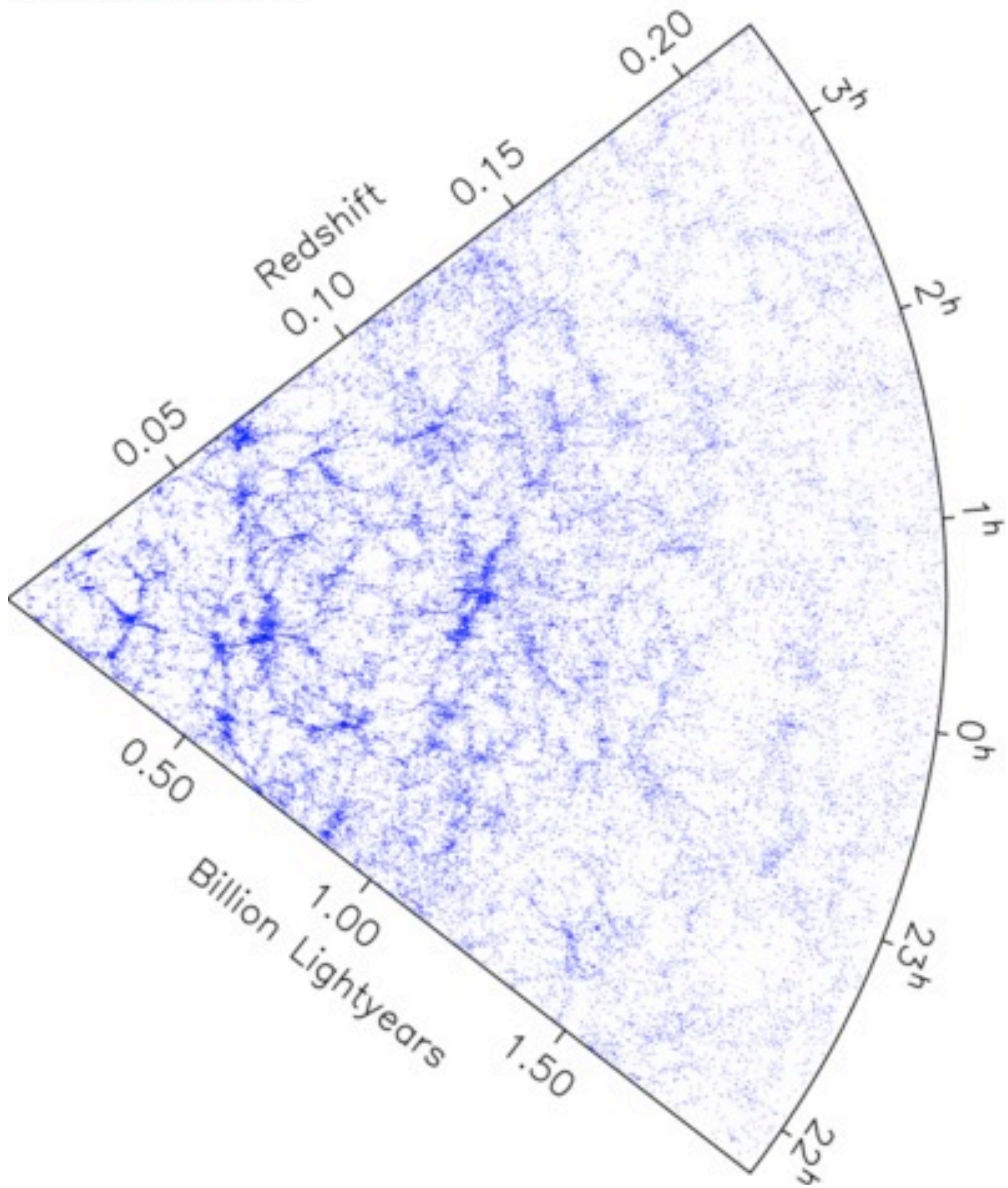
and also

$$\sigma_{\rm vpk}^2 = \frac{1}{2\pi^2}\int_0^\infty dk\, b_{\rm vpk}(k)\, W^2(kR_s)P(k)$$

- *Strongly constrains the k-dependence of* $b_{\rm vpk}(k)$

(VD & Sheth '10)

Redshift space distortions



2dF galaxy redshift survey

From real to redshift space

- *The redshift space, 3-dimensional comoving coordinate is*

$$\mathbf{s} = \mathbf{x} + \frac{\mathbf{v}(\mathbf{x}) \cdot \hat{\mathbf{x}}}{aH}$$
$$= \mathbf{x} + f\left[\mathbf{u}(\mathbf{x}) \cdot \hat{\mathbf{x}}\right], \qquad \mathbf{u} \equiv \frac{\mathbf{v}}{aHf}$$

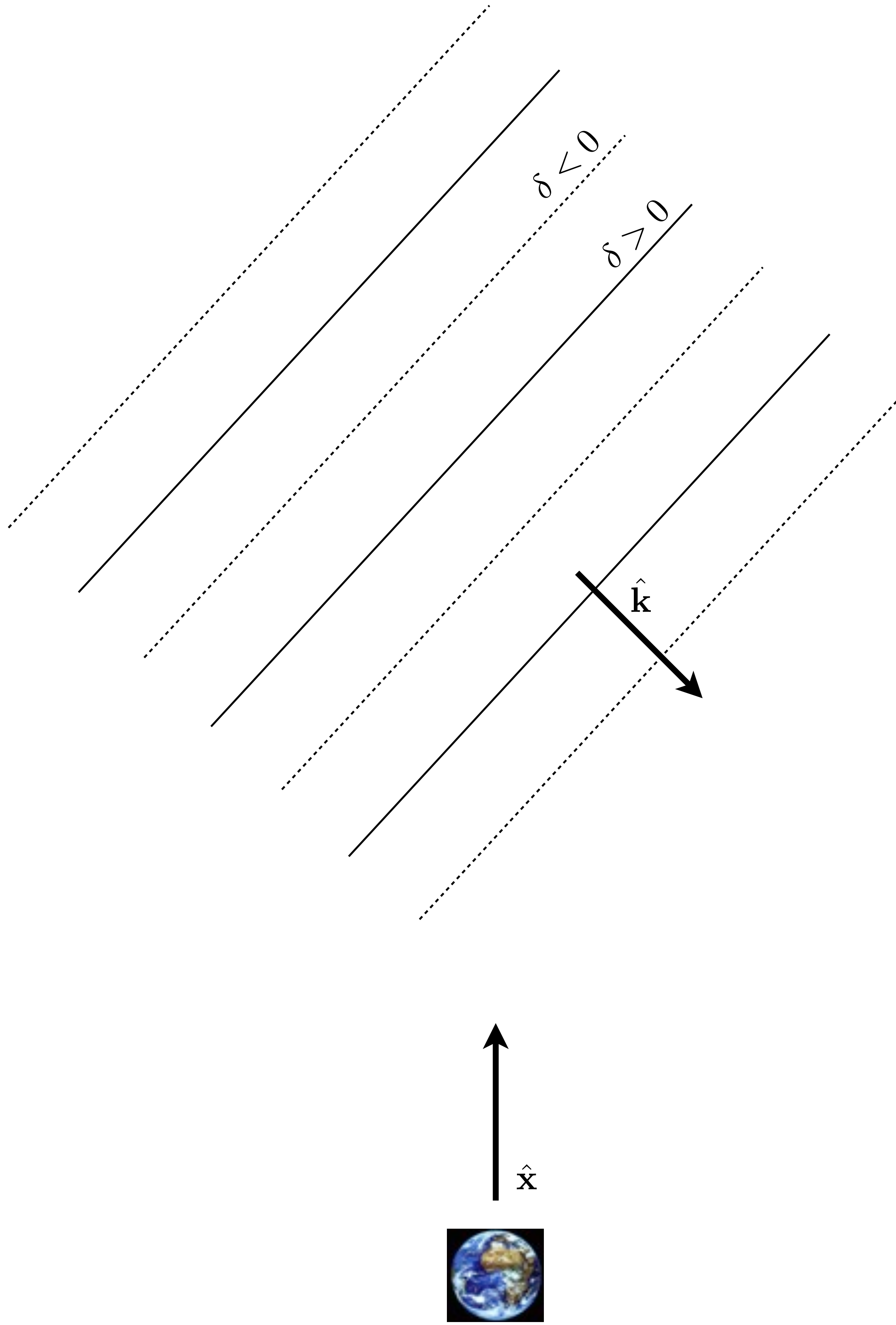
- *Logarithmic growth rate:*

$$f(a) = \frac{d\ln D}{d\ln a}$$

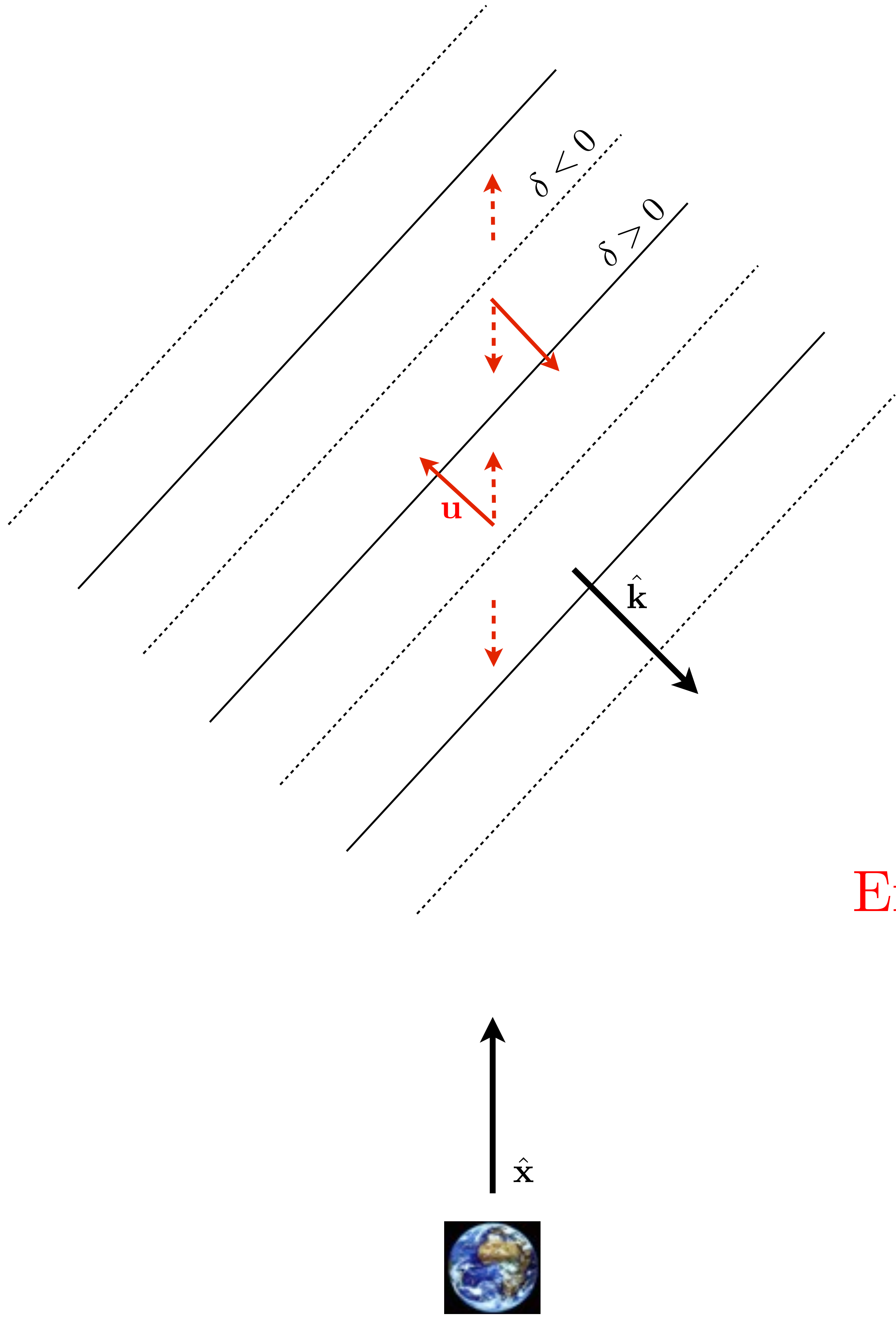
- *In linear theory, velocities are related to densities through*

$$\mathbf{u}(\mathbf{k}, a) = i\frac{\mathbf{k}}{k^2}\,\delta(\mathbf{k}, a)$$

From real to redshift space



From real to redshift space



$$\text{Effect} \propto (\hat{\mathbf{k}} \cdot \hat{\mathbf{x}})^2$$

The Kaiser formula

- *Mass conservation + plane-parallel approximation:*

$$\delta^s(\mathbf{k}) = \left(1 + f\mu^2\right)\delta(\mathbf{k}), \quad \mu \equiv \hat{\mathbf{k}} \cdot \hat{\mathbf{z}}$$

- *Redshift space power spectrum*

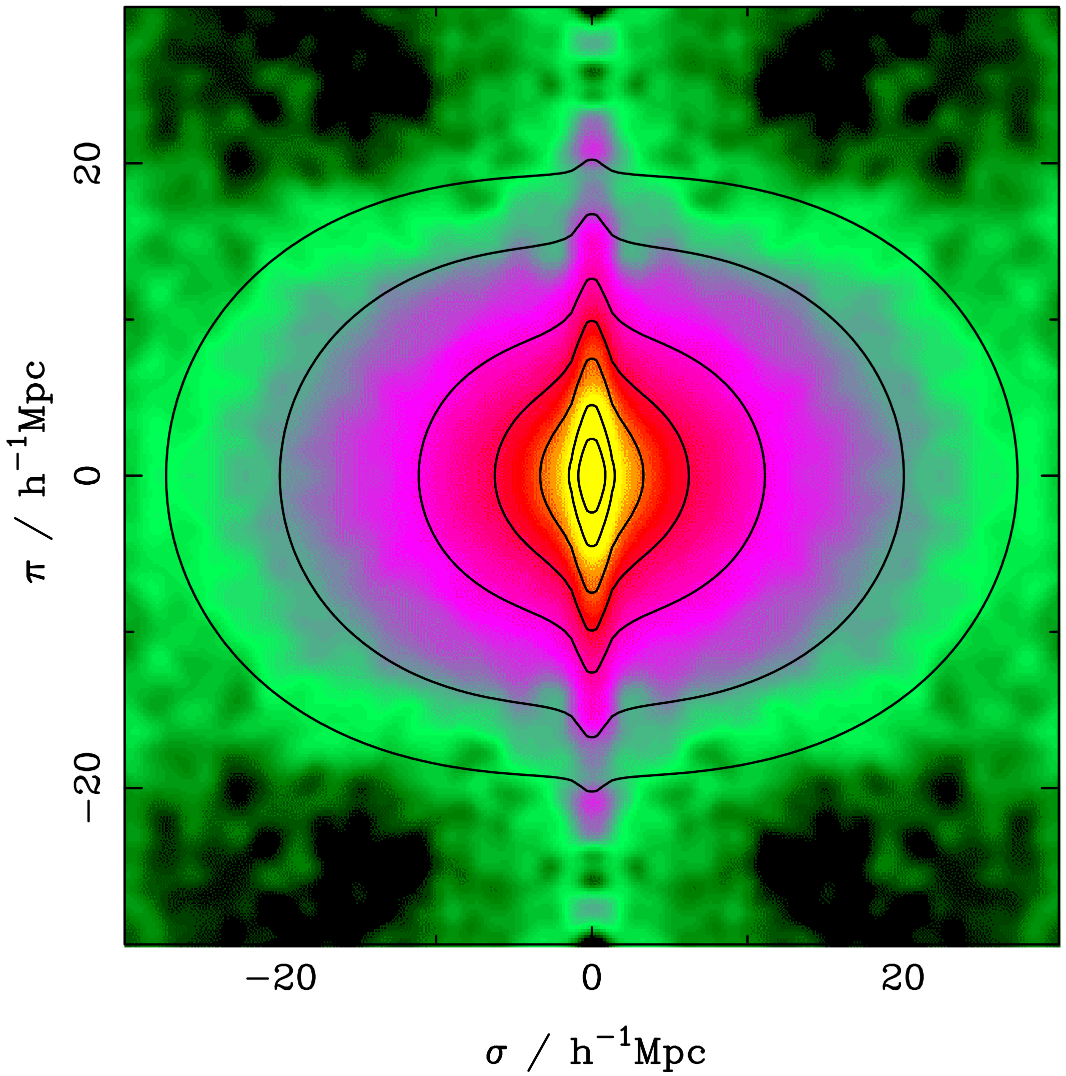
$$P^s(k,\mu) = \left(1 + f\mu^2\right)^2 P(k)$$

- *For biased tracers like galaxies:*

$$P^s_{gg}(k,\mu) = \left(b_1 + f\mu^2\right)^2 P(k)$$

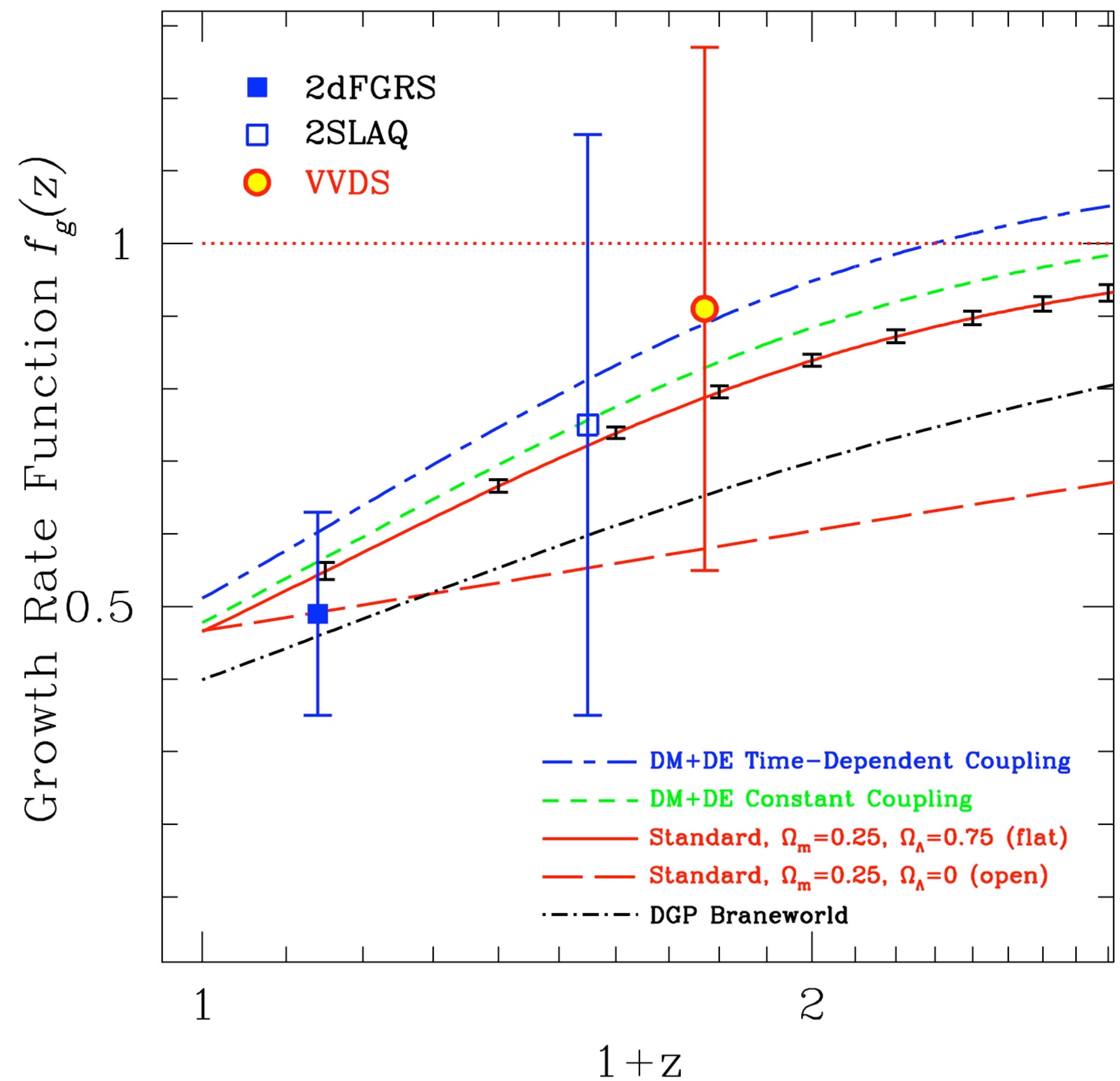
(Kaiser '87)

Measuring the growth rate



(Peacock et al. '01)

$$\text{measure} \quad \beta = \frac{f}{b_1} \quad \text{or} \quad f\sigma_8$$



(Guzzo et al. '07)

Kaiser formula for peaks

$$P^s_{\rm pk}(k,\mu) = \Big(c_1(k) + f b_{\rm vpk}(k)\mu^2\Big)^2 P_{\delta\delta}(k)$$

(VD & Sheth '10)

Kaiser formula for peaks

$$P^s_{\rm pk}(k,\mu) = \Big(c_1(k) + f b_{\rm vpk}(k)\mu^2\Big)^2 P_{\delta\delta}(k)$$

$b_{\rm vpk}(k)$ *could mimic the signature of some modified gravity, dark energy theory, massive neutrinos etc. if it is not accounted for.*

$$\beta_{\rm eff}(k) = \left(\frac{f}{b_{10}}\right) \times \frac{\left(1 - \frac{\sigma_0^2}{\sigma_1^2}k^2\right)}{\left(1 + \frac{b_{01}}{b_{10}}k^2\right)}$$

$$f_{\rm eff}(k) = f \times \left(1 - \frac{\sigma_0^2}{\sigma_1^2}k^2\right)$$

(VD & Sheth '10)

Time evolution of spatial bias

From

- *Continuity argument* *(Fry '96; Peebles & Tegmark '98)*
- *Spherical collapse* *(Mo & White '96)*

one finds

$$b_1(a) = 1 + \frac{D(a_i)}{D(a)}\big(b_1(a_i) - 1\big), \qquad a \; > \; a_i$$

Time evolution of velocity bias

Matter:

$$\frac{\partial \delta}{\partial \eta} + \theta = \mathrm{m.c.}$$

$$\frac{\partial \theta}{\partial \eta} + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta = \mathrm{m.c.}$$

Galaxies:

$$\frac{\partial \delta_{\mathrm{g}}}{\partial \eta} + \theta_{\mathrm{g}} = \mathrm{m.c.}$$

$$\frac{\partial \theta_{\mathrm{g}}}{\partial \eta} + \mathcal{H}\theta_{\mathrm{g}} + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta = \mathrm{m.c.}$$

where $\eta \equiv \int \frac{dt}{a}, \quad \delta \equiv \delta(\mathbf{k},\eta), \quad \theta \equiv \big(\boldsymbol{\nabla}\cdot\mathbf{v}\big)(\mathbf{k},\eta)$

(Chan, Scoccimarro & Sheth 2012, Baldauf et al. 2012)

Time evolution of velocity bias

Matter:

$$\frac{\partial \delta}{\partial \eta} + \theta = \mathrm{m.c.}$$

$$\frac{\partial \theta}{\partial \eta} + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta = \mathrm{m.c.}$$

Galaxies:

$$\frac{\partial \delta_{\mathrm{g}}}{\partial \eta} + \theta_{\mathrm{g}} = \mathrm{m.c.}$$

$$\frac{\partial \theta_{\mathrm{g}}}{\partial \eta} + \mathcal{H}\theta_{\mathrm{g}} + \frac{3}{2}\mathcal{H}^2\Omega_{\mathrm{m}}\delta = \mathrm{m.c.}$$

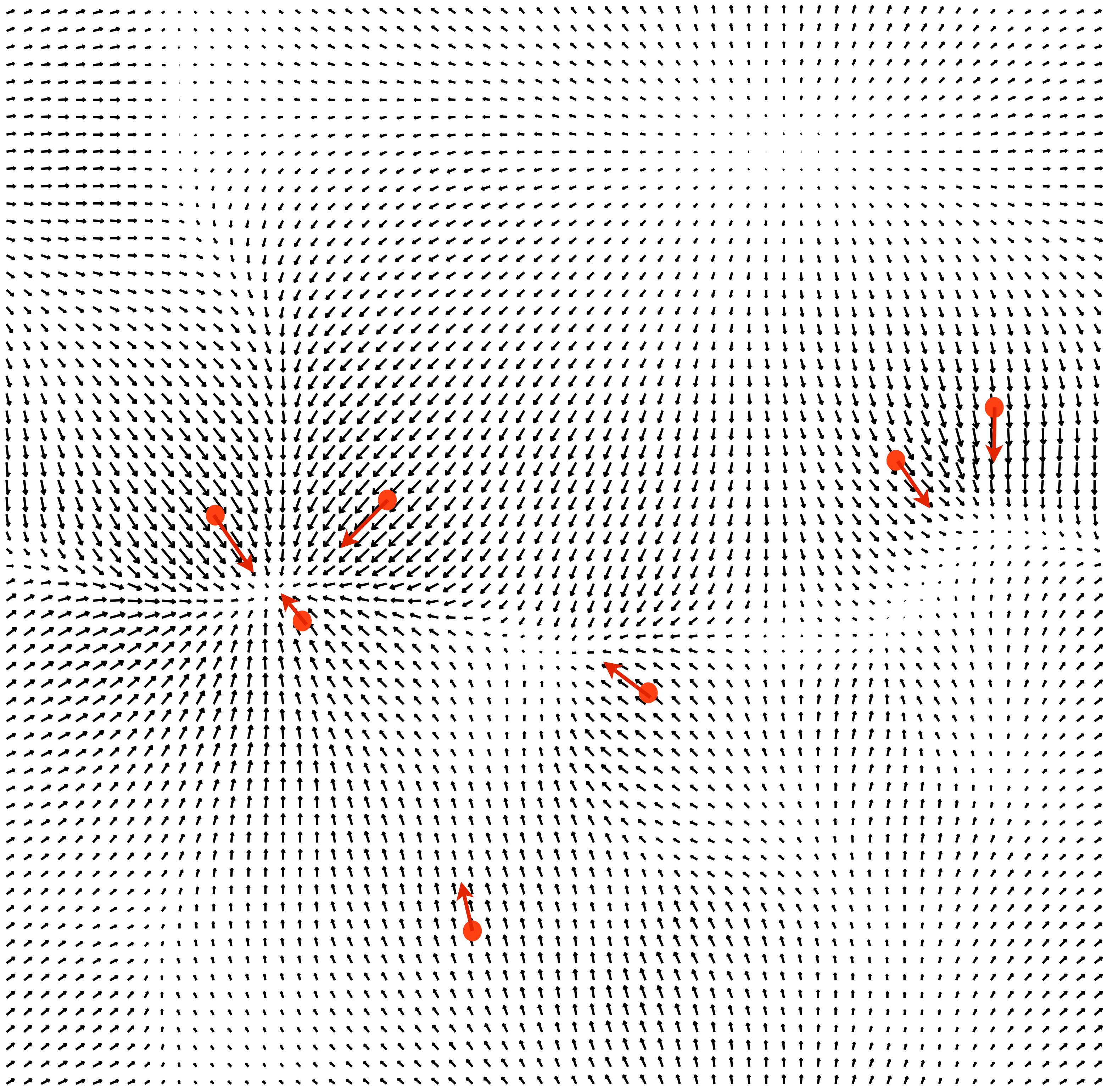
where $\eta \equiv \int \frac{dt}{a}, \quad \delta \equiv \delta(\mathbf{k}, \eta), \quad \theta \equiv \big(\boldsymbol{\nabla} \cdot \mathbf{v}\big)(\mathbf{k}, \eta)$

$$b_v(a) = 1 + \frac{D(a_i)}{D(a)}\big(b_v(a_i) - 1\big), \qquad a \ > \ a_i$$

(Chan, Scoccimarro & Sheth 2012, Baldauf et al. 2012)

Evolution of bias in the peak approach

In the Zel'dovich ('70) approximation:



Evolution of bias in the peak approach

- *At linear order, the peak bias factors evolve according to*

$$c_1(k,a) = b_{\rm vpk}(k,a_i) + \frac{D(a_i)}{D(a)} c_1(k,a_i)$$

$$b_{\rm vpk}(k,a) = b_{\rm vpk}(k,a_i)$$

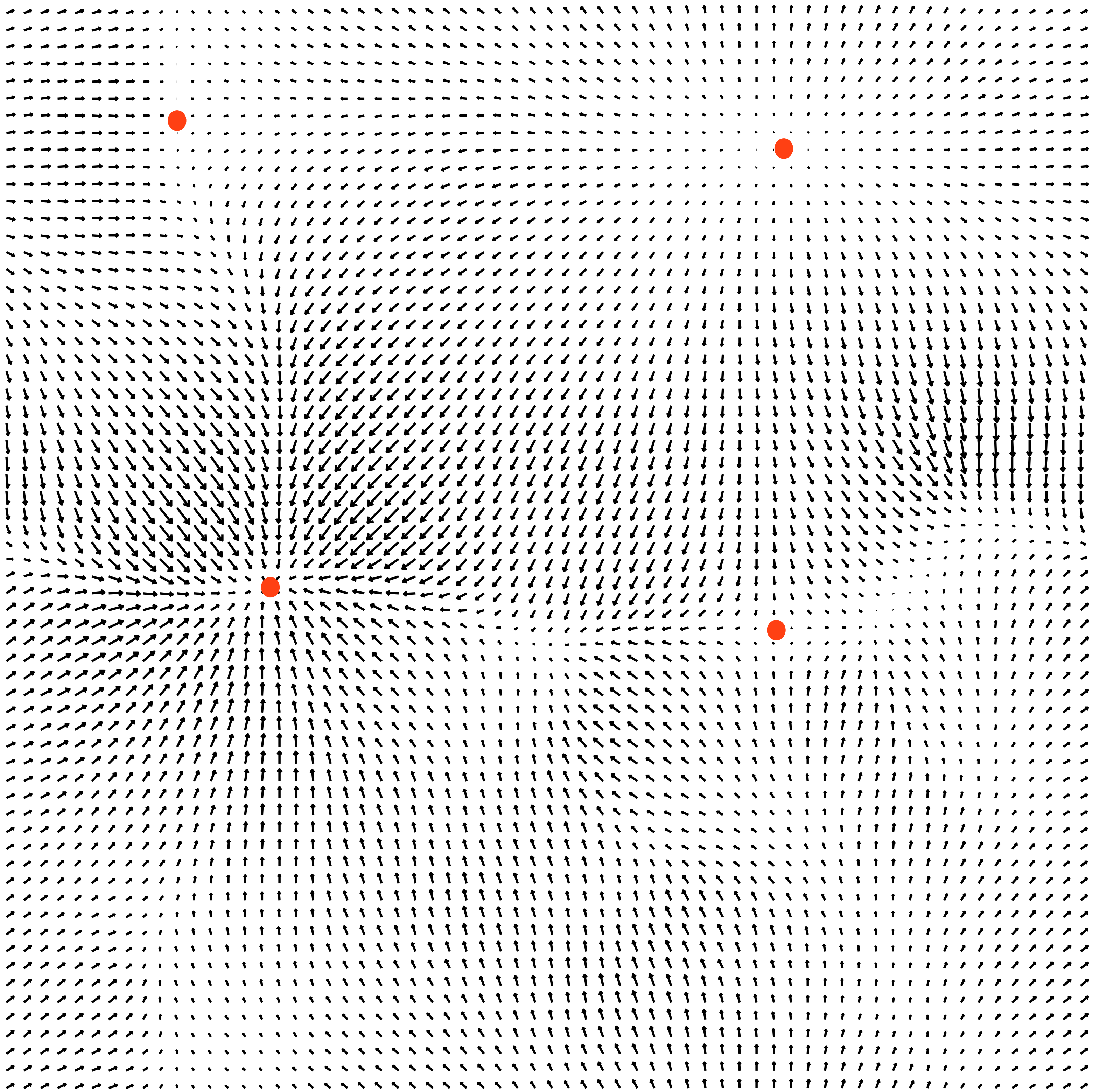
Scale-independent piece: $$b_{10}(a) = 1 + \frac{D(a_i)}{D(a)}\, b_{10}(a_i)$$

Scale-dependent piece: $$b_{01}(a) = -\frac{\sigma_0^2}{\sigma_1^2} + \frac{D(a_i)}{D(a)}\, b_{01}(a_i)$$

(VD, Crocce, Scoccimarro & Sheth '10)

Interpretation: gravity is biased

Consider points with zero initial velocity:



Interpretation: gravity is biased

Matter:

$$\frac{\partial\delta}{\partial\eta}+\theta=\mathrm{m.c.}$$

$$\frac{\partial\theta}{\partial\eta}+\mathcal{H}\theta+\frac{3}{2}\mathcal{H}^2\Omega_\mathrm{m}\delta=\mathrm{m.c.}$$

Galaxies:

$$\frac{\partial\delta_\mathrm{g}}{\partial\eta}+\theta_\mathrm{g}=\mathrm{m.c.}$$

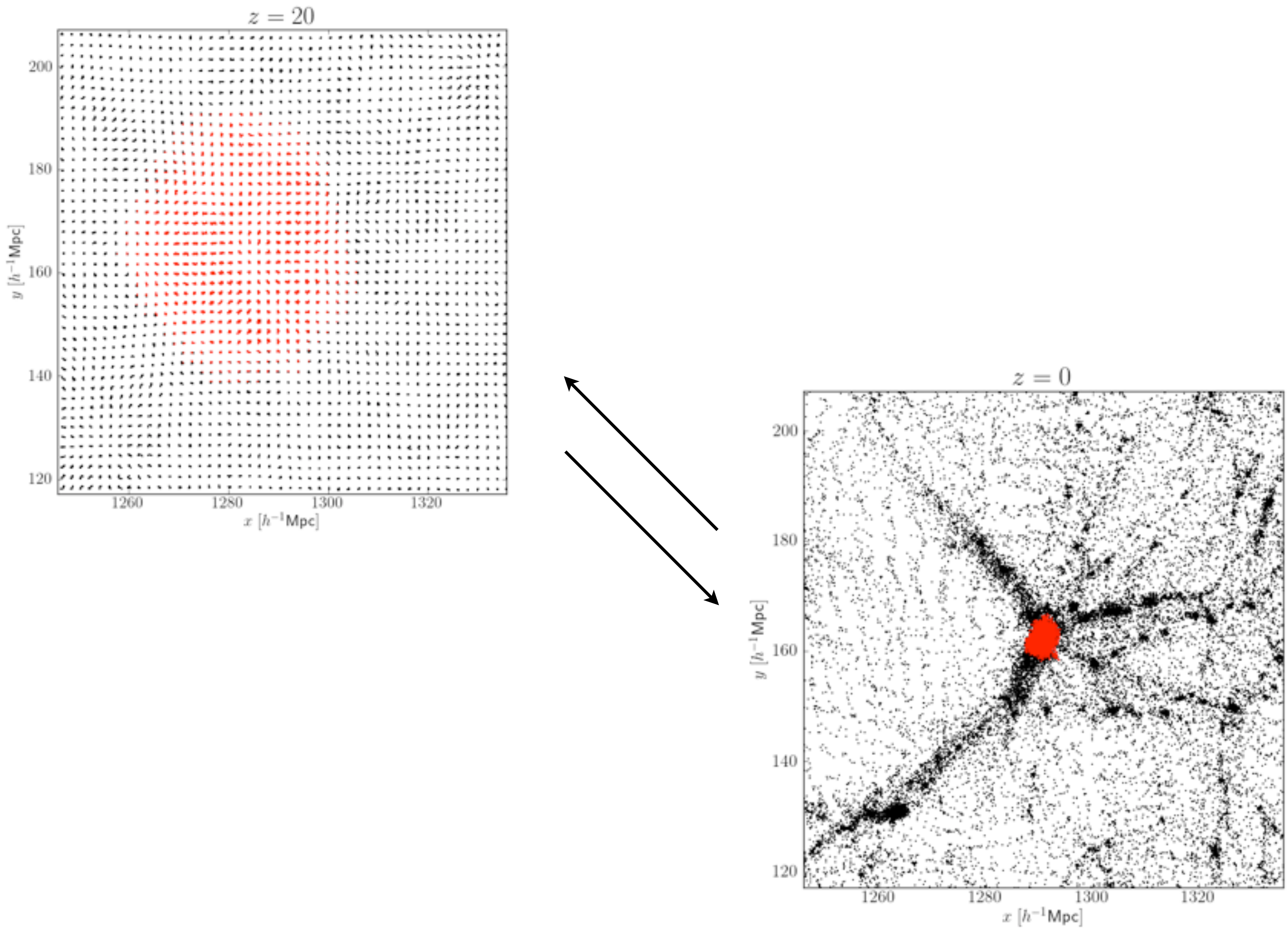
$$\frac{\partial\theta_\mathrm{g}}{\partial\eta}+\mathcal{H}\theta_\mathrm{g}+\frac{3}{2}\,b_\mathrm{vpk}\,\mathcal{H}^2\Omega_\mathrm{m}\delta=m.c.$$

We obtain:

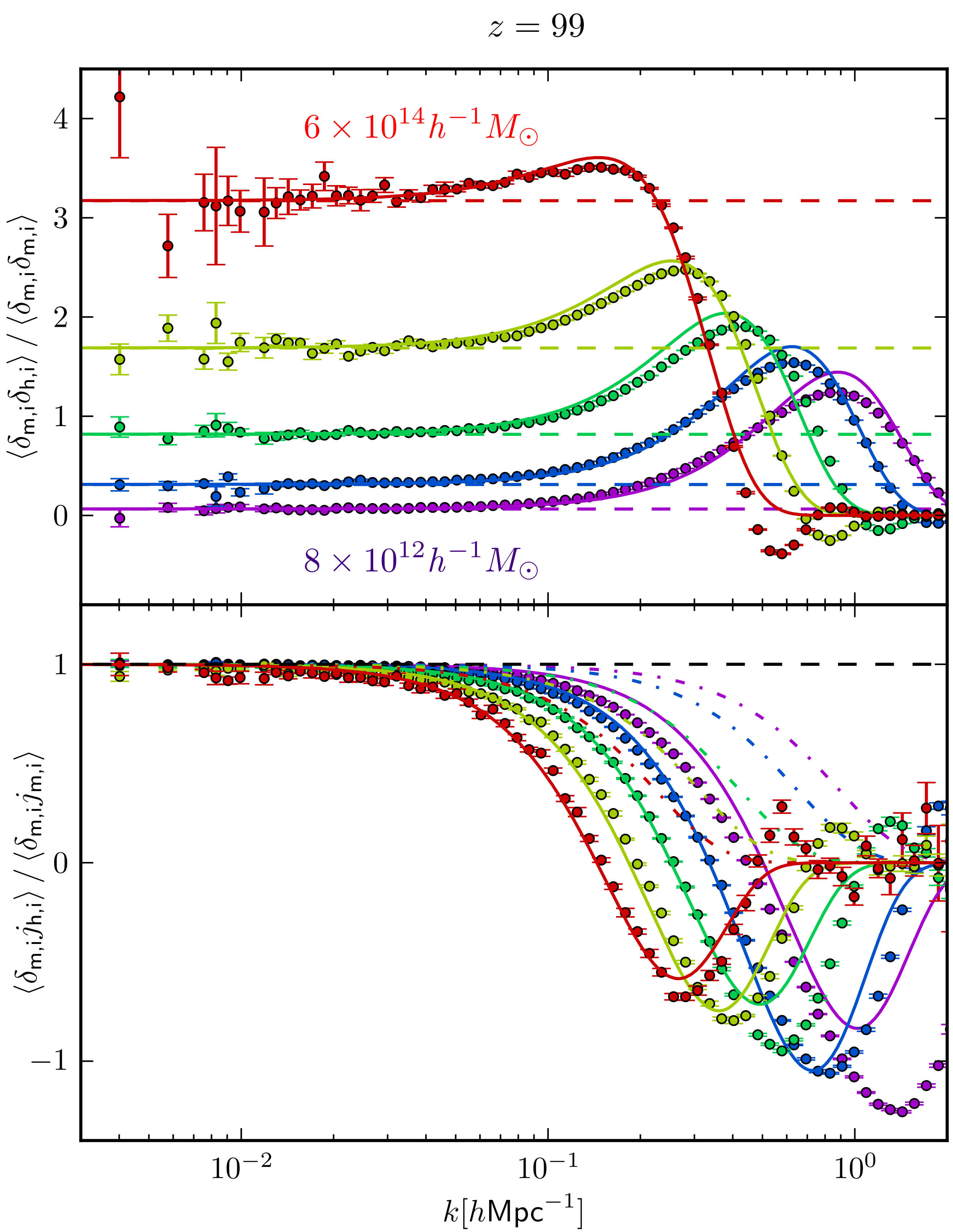
$$b_v(k,a)=b_\mathrm{vpk}(k)+\frac{D(a_i)}{D(a)}\big(b_v(k,a_i)-1\big)$$ *!!*

(Baldauf, VD & Seljak '14)

Test with numerical simulations

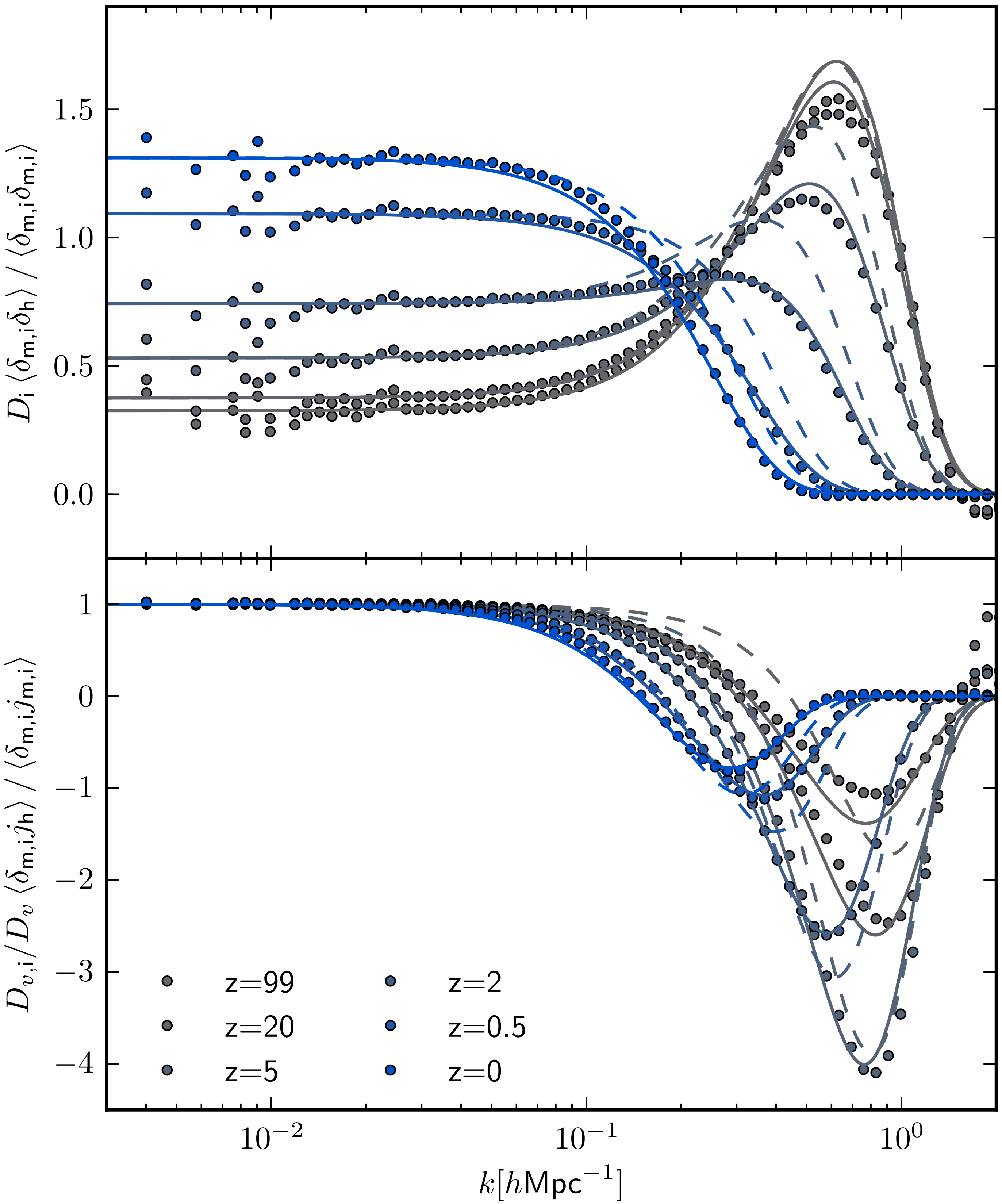


Lagrangian bias of halos



(Baldauf, VD & Seljak '14)

Evolution of halo bias



We assume that peaks move according to their initial velocity as in Zeldovich approximation. We calculate the resulting correlators by writing the evolved peak positions as $1+\delta_{\rm h}(\boldsymbol{x}) = \bar{n}_{\rm h}^{-1}\sum_{\rm h}\delta^{(\rm D)}(\boldsymbol{x}-\boldsymbol{x}_{\rm h}) = \bar{n}_{\rm h}^{-1}\int {\rm d}^3q\,\delta^{(\rm D)}(\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{\Psi}(\boldsymbol{q}))\sum_{\rm pk}\delta^{(\rm D)}(\boldsymbol{q}-\boldsymbol{q}_{\rm pk})$ following the steps laid out in [6], where $\boldsymbol{\Psi}(\boldsymbol{q})$ is the displacement field at Lagrangian position q. Since the initial matter fluctuations are Gaussian, we only select the linear terms in the bias relation. We finally obtain

$$\left\langle\delta_{\rm m}^{(1)}(\boldsymbol{k})\delta_{\rm h}(-\boldsymbol{k})\right\rangle = D_+^2\,c_1(k,a)G_{\rm pk}(k)P(k)W_R(k)\,, \quad (10)$$

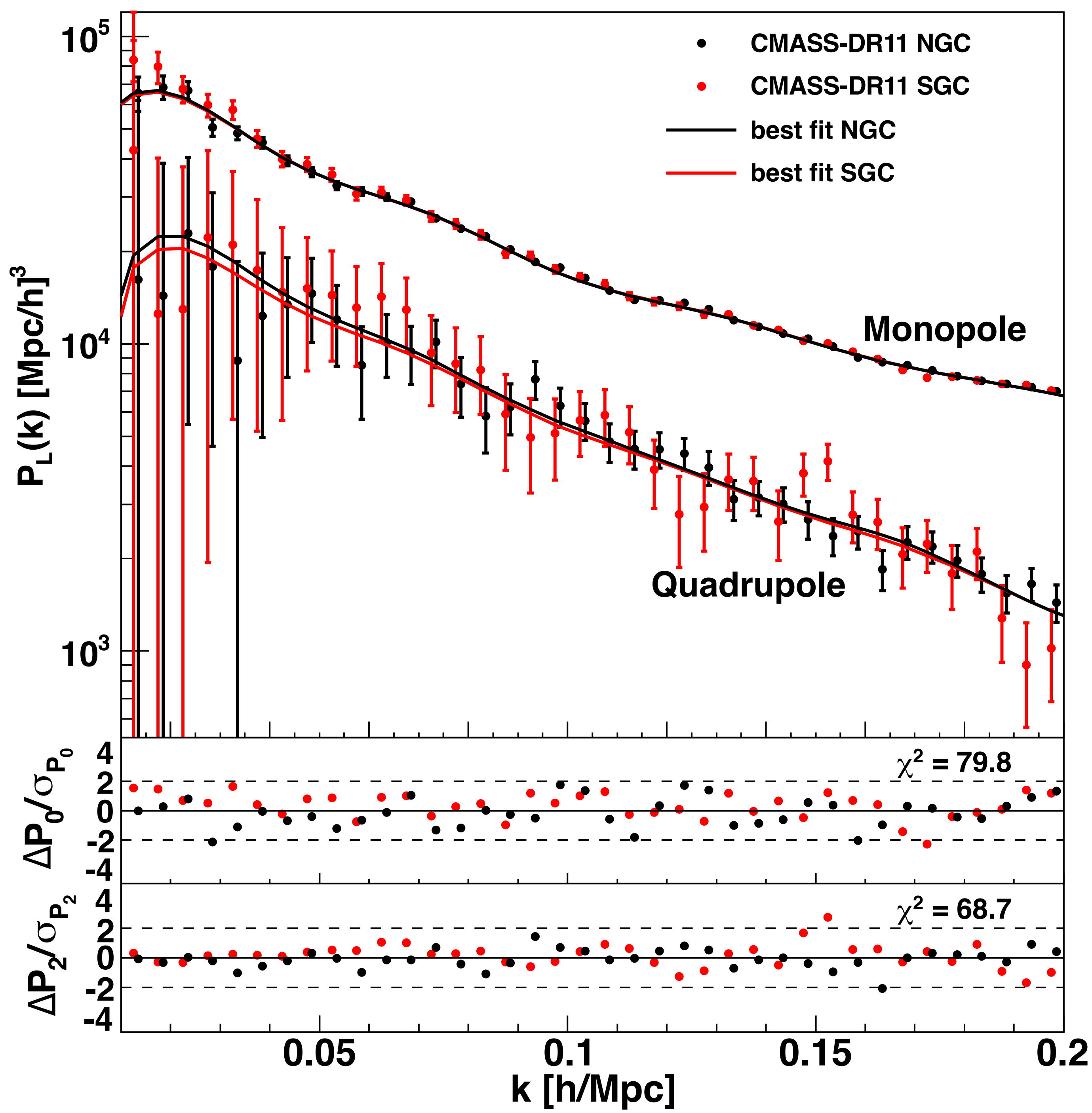
$$\left\langle\delta_{\rm m}^{(1)}(\boldsymbol{k})j_{\rm h}^z(-\boldsymbol{k})\right\rangle = \left(b_v(k) - D_+^2\sigma_{\rm d,pk}^2\,\bar{c_1}(k,a)\,k^2\right) \quad (11)$$

$$\times\,\mathcal{H}f_+D_+^2\left(i\frac{\boldsymbol{k}\cdot\hat{\mathbf{z}}}{k^2}\right)G_{\rm pk}(k)P(k)W_R(k)\,,$$

where $G_{\rm pk}(k) = e^{-\frac{1}{3}\sigma_{\rm d,pk}^2k^2D_+^2(a)}$ is the peak propagator and $\sigma_{\rm d,pk}^2$ is the peak displacement dispersion (extrapolated to the collapse epoch), given by $\sigma_{\rm d,pk}^2 = \sigma_{-1}^2 - \sigma_0^4/\sigma_1^2$

(Baldauf, VD & Seljak '14)

Implications



$10\% - 20\%$

(Beutler et al '14)

Summary

The peak approach is a great toy model to understand the nonlinearity, scale-dependence and stochasticity of bias

- *Scale-dependent corrections to the halo bias factors even at the linear level.*
- *Dark matter halos exhibit a statistical velocity bias which propagates into redshift space statistics such as the power spectrum.*
- *No new free parameters. All the bias factors are fully determined once the halo mass function is known*