"Tessellating the phase space of dark matter: A novel approach to visualizing, modeling and understanding the cosmic web."

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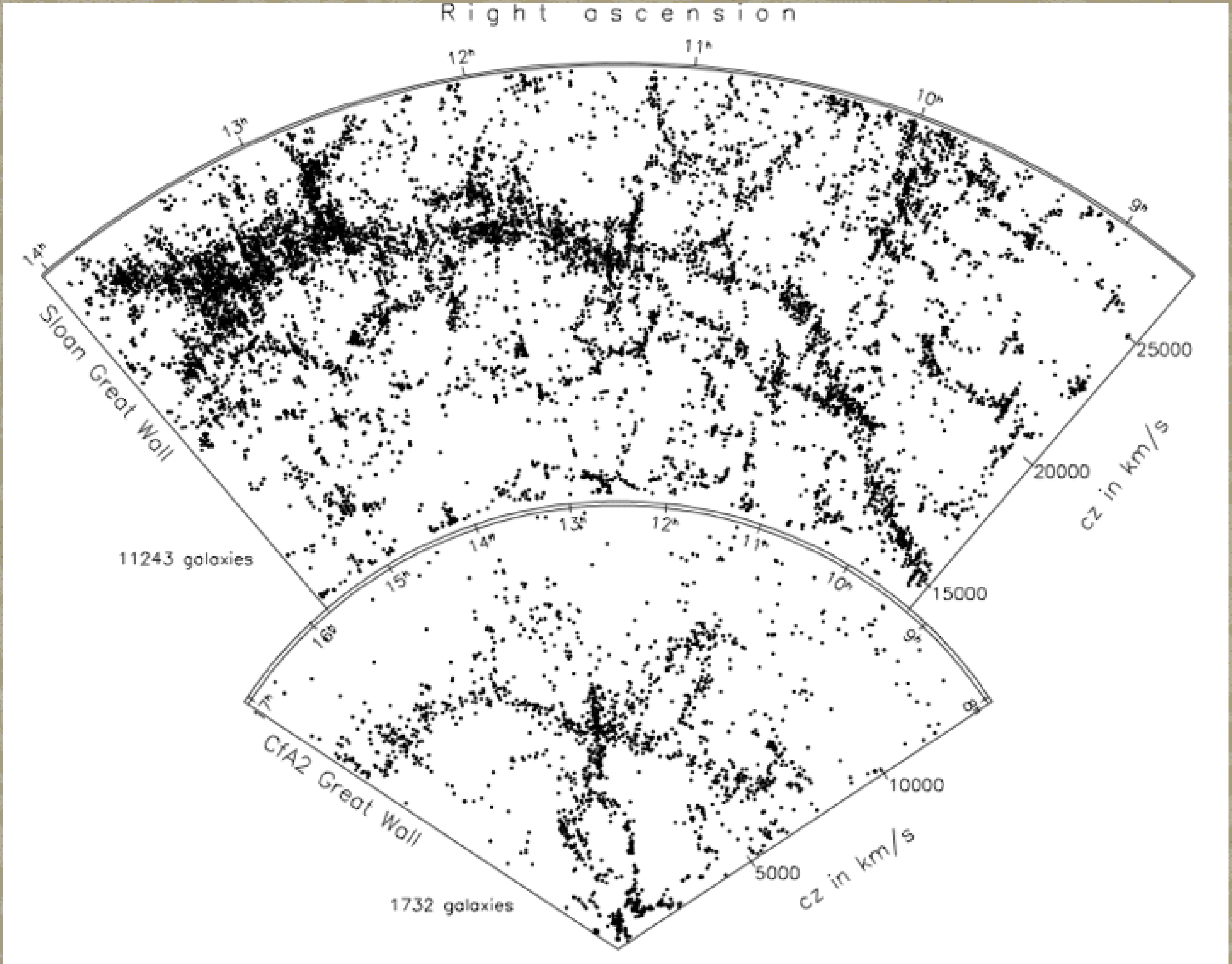
Outline

- Introduction
- Cosmological N-body simulations
- Analytical guidelines
- Multi-stream field, Caustics
- Rendering cosmic web by Voronoi, Delaunay tessellations and SPH
- Rendering cosmic web by tessellating Phase Space Sheet
- Physical voids
- Parity, Origami, Caustics in 3D
- Summary

Large-Scale Structure in Redshift Surveys

The SDSS Great Wall (Gott, et al. 2005)

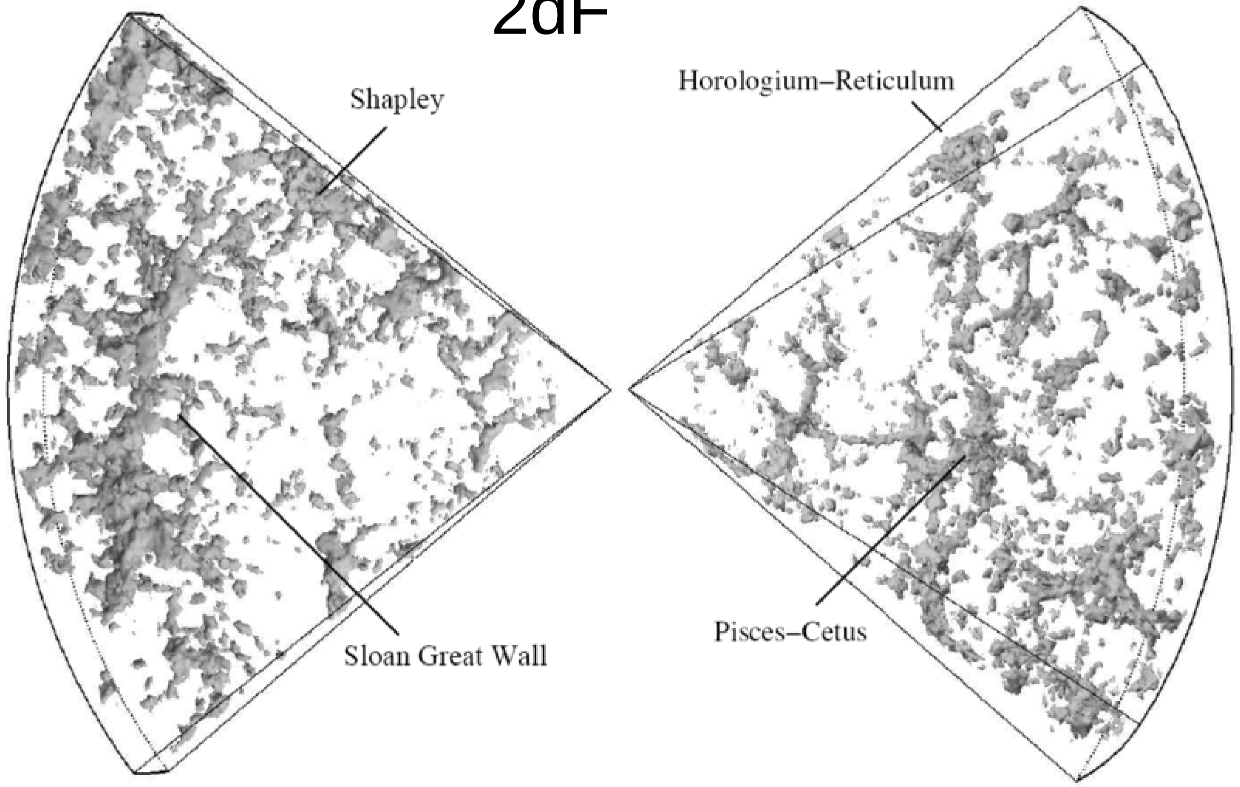


“Prominent in the map is a Sloan Great Wall of galaxies

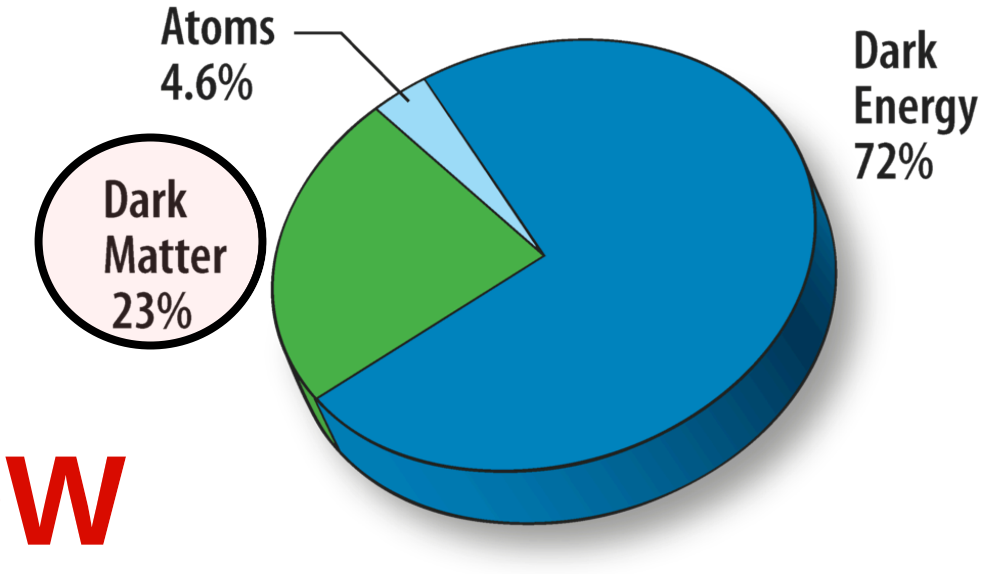
1.37 billion light years long,

80% longer than the Great Wall discovered by Geller and Huchra and therefore the largest observed structure in the universe.”

2dF



NOW

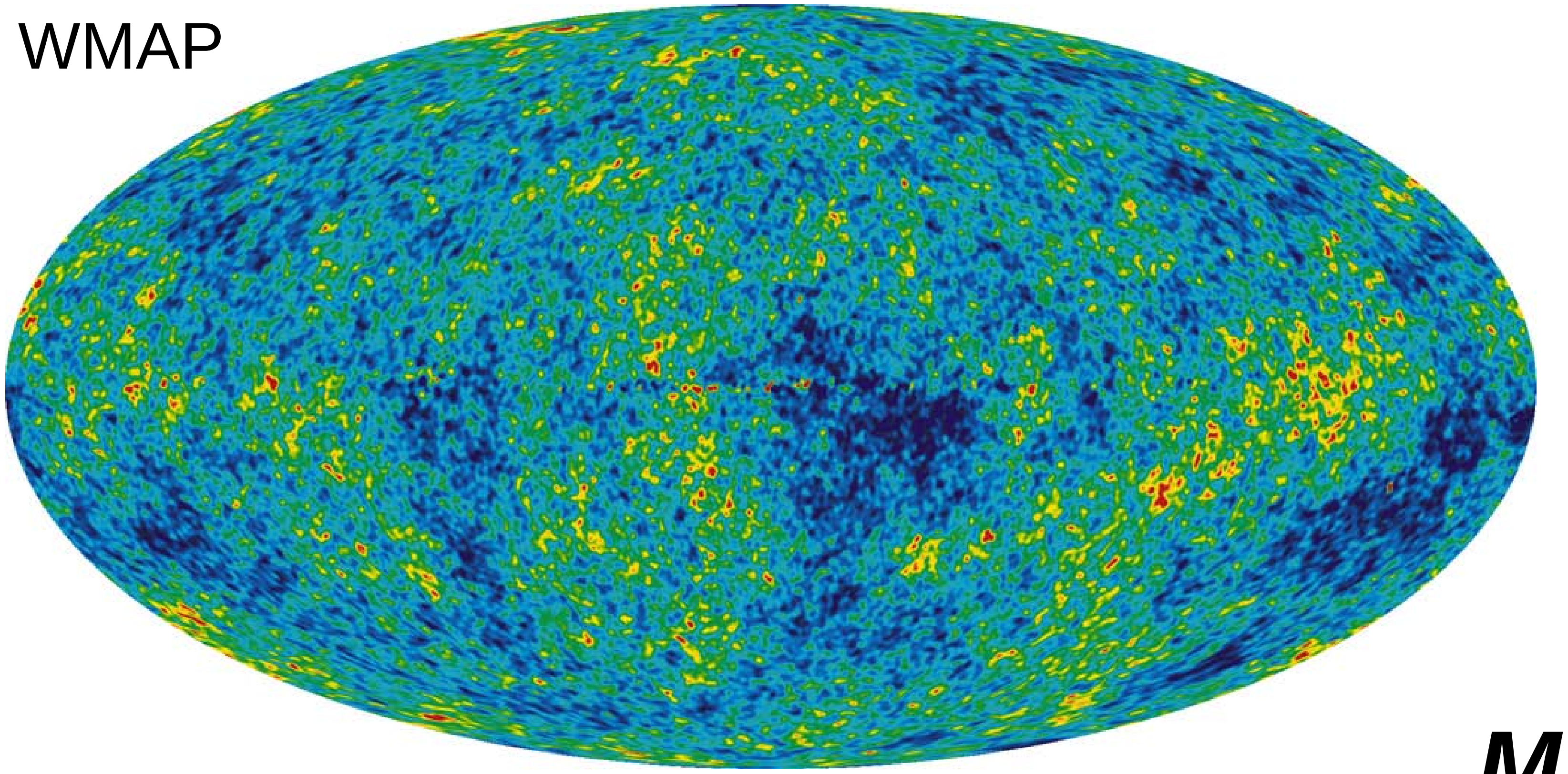


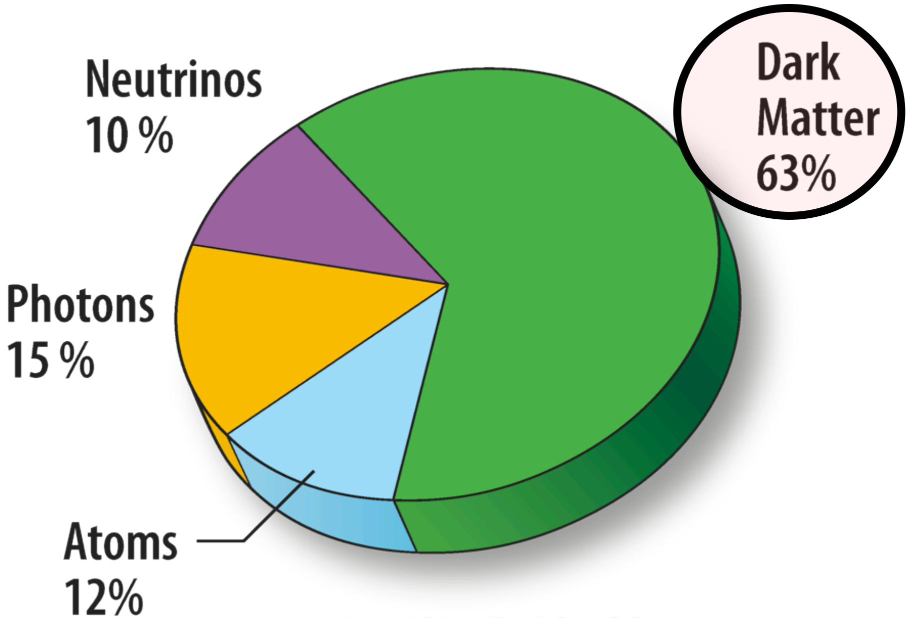
TODAY

13.7 billion years ago

THEN

WMAP



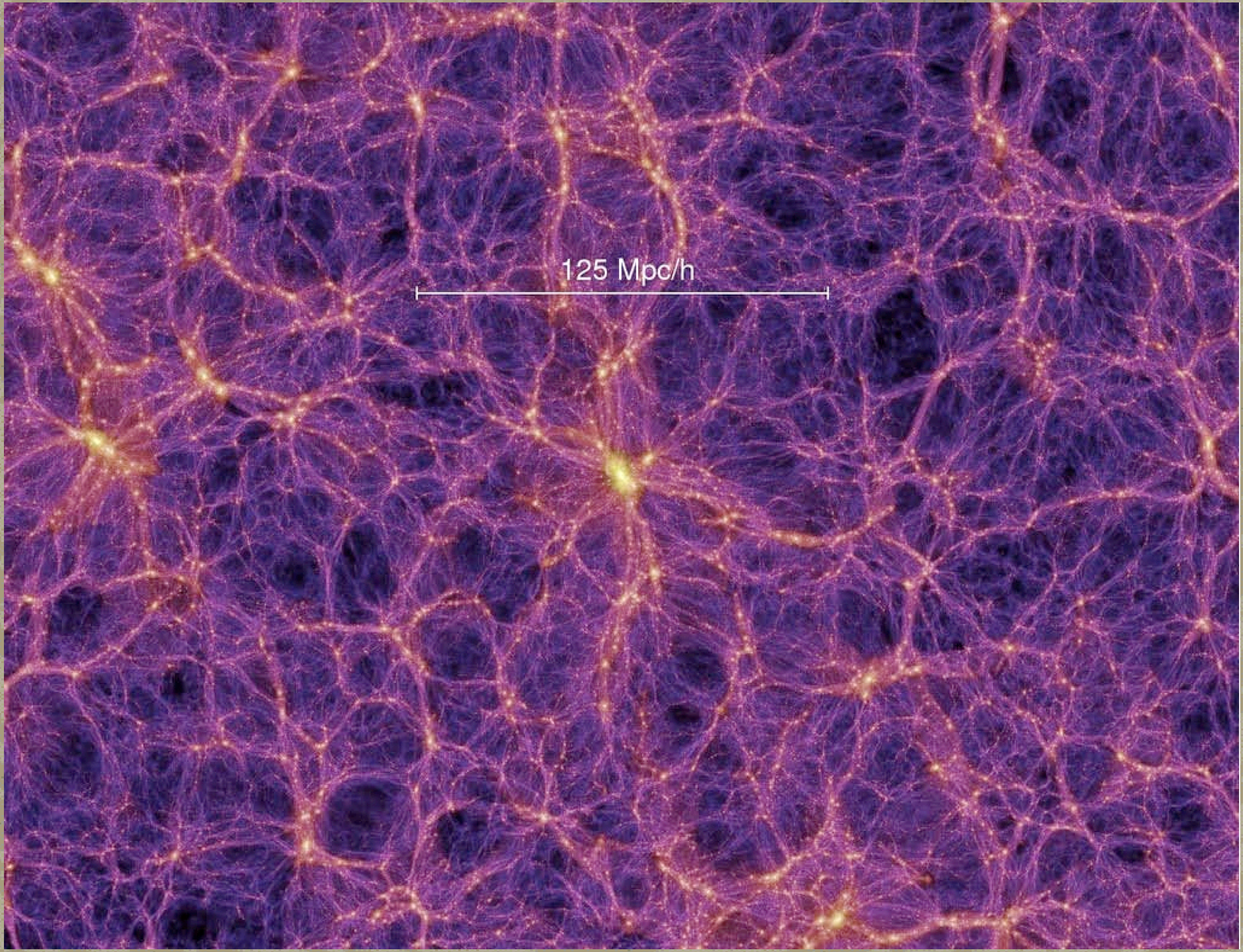


13.7 BILLION YEARS AGO
(Universe 380,000 years old)

Mass in the Universe

Structure in the Universe

Large-Scale Structure in N-body Simulations (Millennium simulation)



Physical foundation of Modern Cosmological N-body Simulations

Evolution of Dark Matter can be modeled by “particles”, which are tracers of both mass and flow

Masses, sizes, shapes of “particles” are constant.

Particles are treated as gravitationally interacting independent items

It is worth remembering!

Particles in N-B simulations have no physical meaning!

A billion (10^9) of 100 GeV WIMPs at mean density occupy a cube 5 km on a side.

There would be 10^67 particles in the Milky Way alone!

Most accurate physical model of CDM would be collisionless fluid

Brief and incomplete (hi)story of modern N-body simulations

Before: direct summation of forces, INIT.COND.: Poisson distribution

1981 Efstathiou & Eastwood P3M, INIT.COND.: Poisson distribution

1980 Doroshkevich et al. PM in 2D, INIT. COND.: regular grid perturbed by Zeldovich approximation
1983 Klypin & Shandarin PM in 3D, INIT. COND.: regular grid perturbed by Zeldovich approximation

1985 Efstathiou et al. P3M; later INIT. COND. glass

1986 Barnes & Hut	Tree–algorithm
1991 Couchman	AP3M (mesh refined particle-particle particle-mesh)
1995 Xu	Tree approach for short-range and the PM approach for long-range forces.

New: piecewise linear approximation of phase space sheet

2011 arXiv	9 Nov	Shandarin, Habib, Heitmann	2012 Phys. Rev. D, 85, 083005
2011 arXiv	16 Nov	Abel, Hahn, Kaehler	
2012 arXiv	24 Oct	Hahn, Abel, Kaehler	
2012 arXiv	5 Apr	Neyrinck	
2012 arXiv	18 Jul	Neyrink, Shandarin	

1971 Peebles A&A 11, 377

Rotation of Galaxies and the Gravitational Instability Picture

Method: Direct Summation

N particles: 90

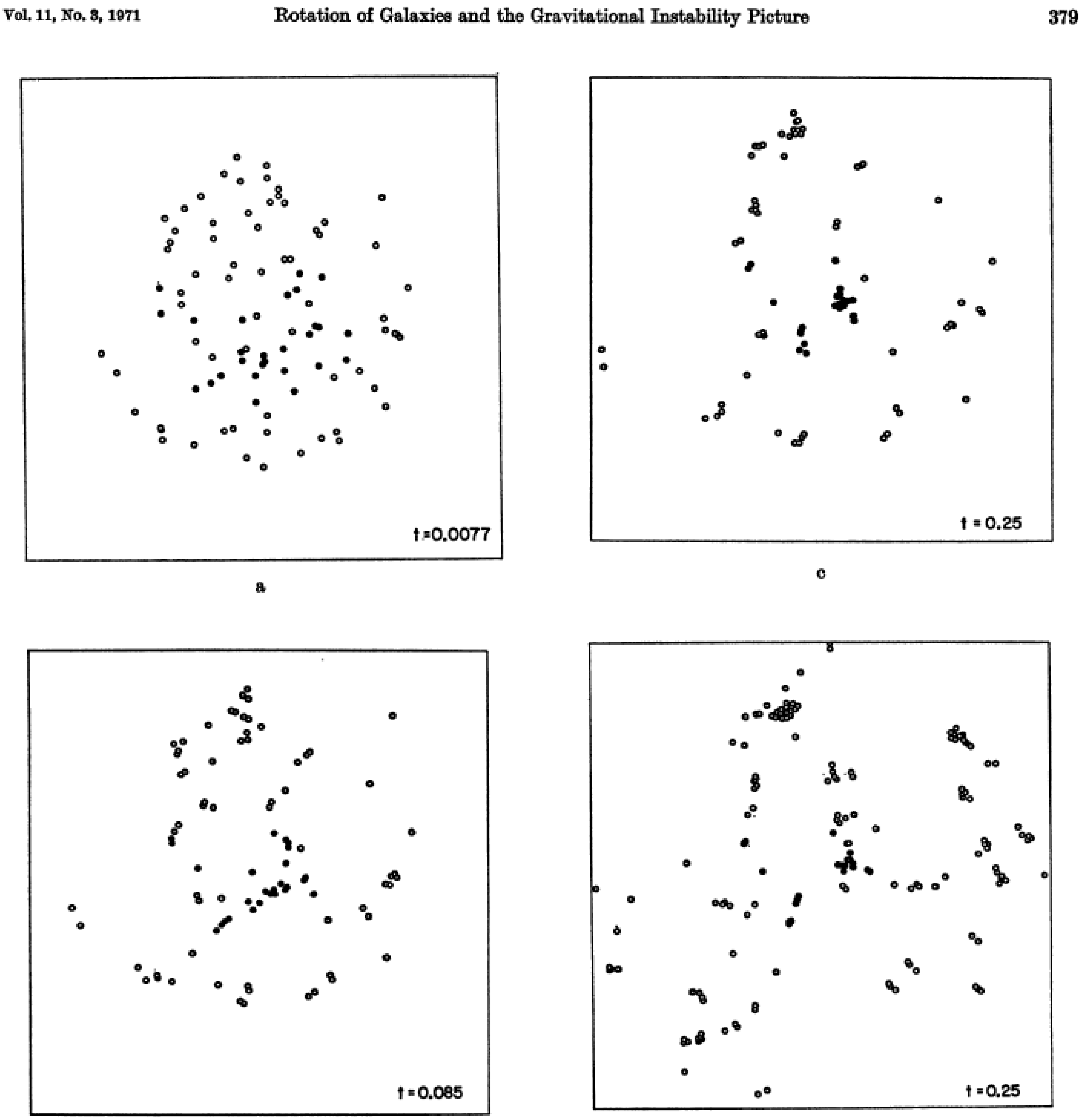
Initial conditions

coordinates: Poisson

velocities: v=Hr(1-0.05) 30 internal

v=Hr(1+0.025) 60 external

Boundary cond: No particles at R>R_0



Vol. 11, No. 3, 1971 Rotation of Galaxies and the Gravitational Instability Picture 379

1978 Peebles A&A 68, 345

Stability of a Hierarchical Clustering in the Distribution Of Galaxies

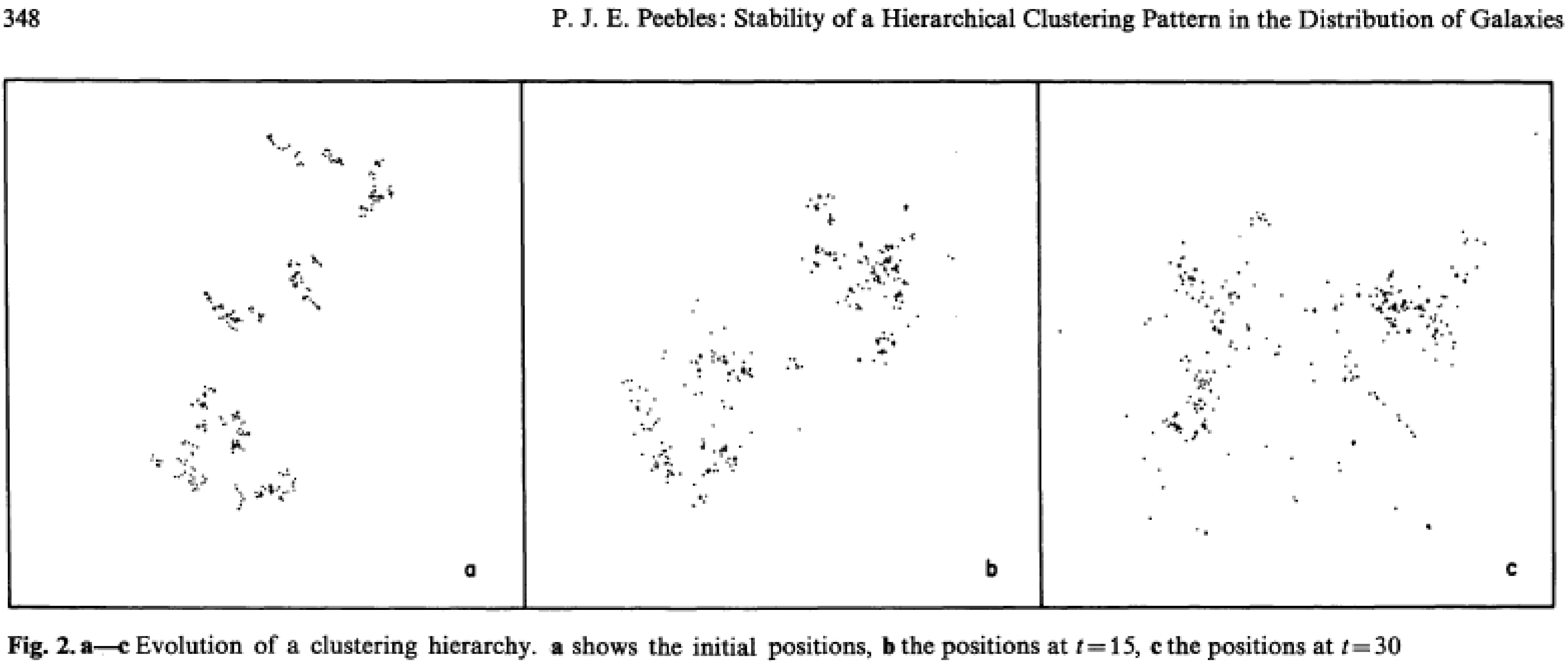
Method: Direct Summation

N particles: 256

Initial conditions
coordinates: Soneira, Peebles
velocities: virial for each subclump

Boundary cond: Empty space

(*) Two types of particles (m=1, m=0)



348 P. J. E. Peebles: Stability of a Hierarchical Clustering Pattern in the Distribution of Galaxies

Fig. 2. a—c Evolution of a clustering hierarchy. **a** shows the initial positions, **b** the positions at $t=15$, **c** the positions at $t=30$

1979 Aarseth, Gott III, Ed Turner
ApJ, 228, 664

N-body Simulations of Galaxy Clustering. I. Initial Conditions and Galaxy Collapse Time

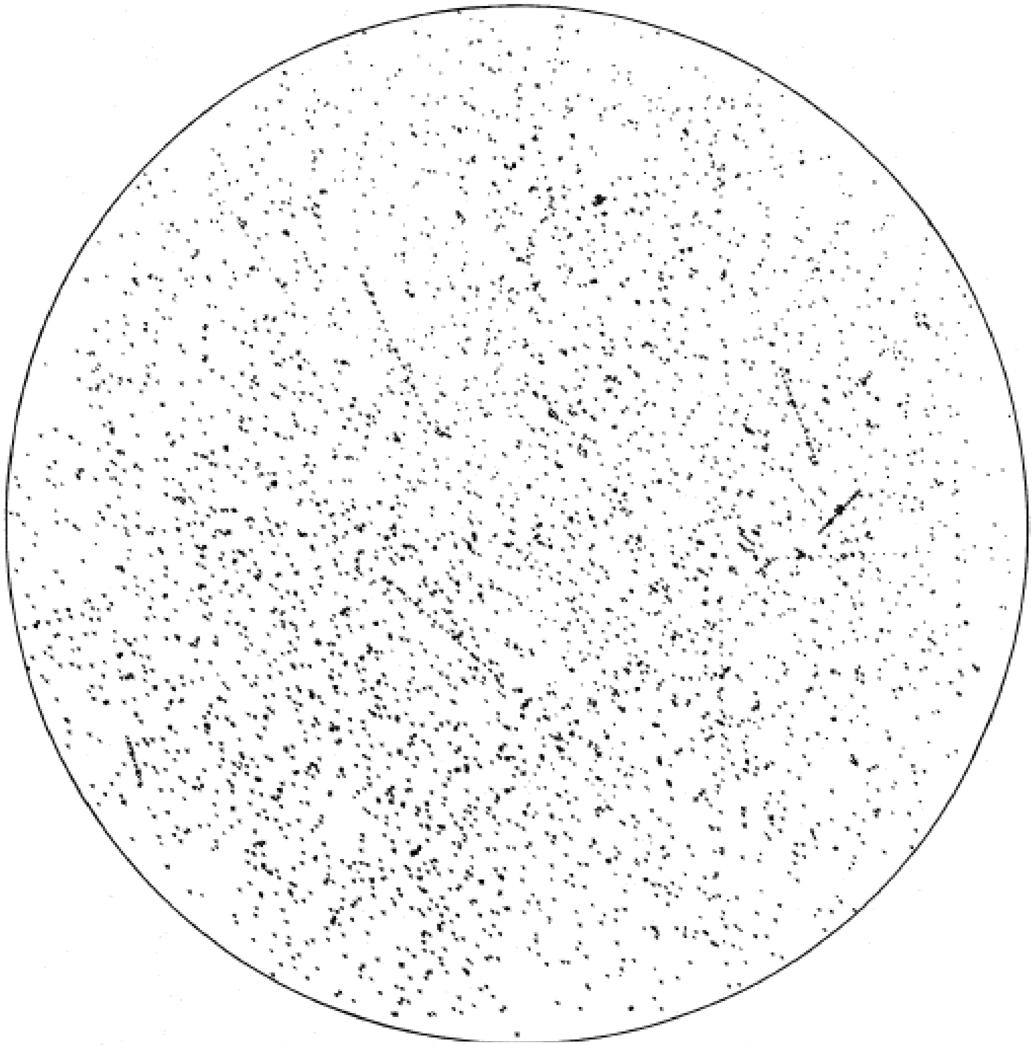
Method: **Direct Summation**
(Aarseth' code)

N particles: **4000**

Initial conditions
coordinates: **On average 8 particles are randomly placed on random 125 rods**
This mimics P = k^(-1) spectrum

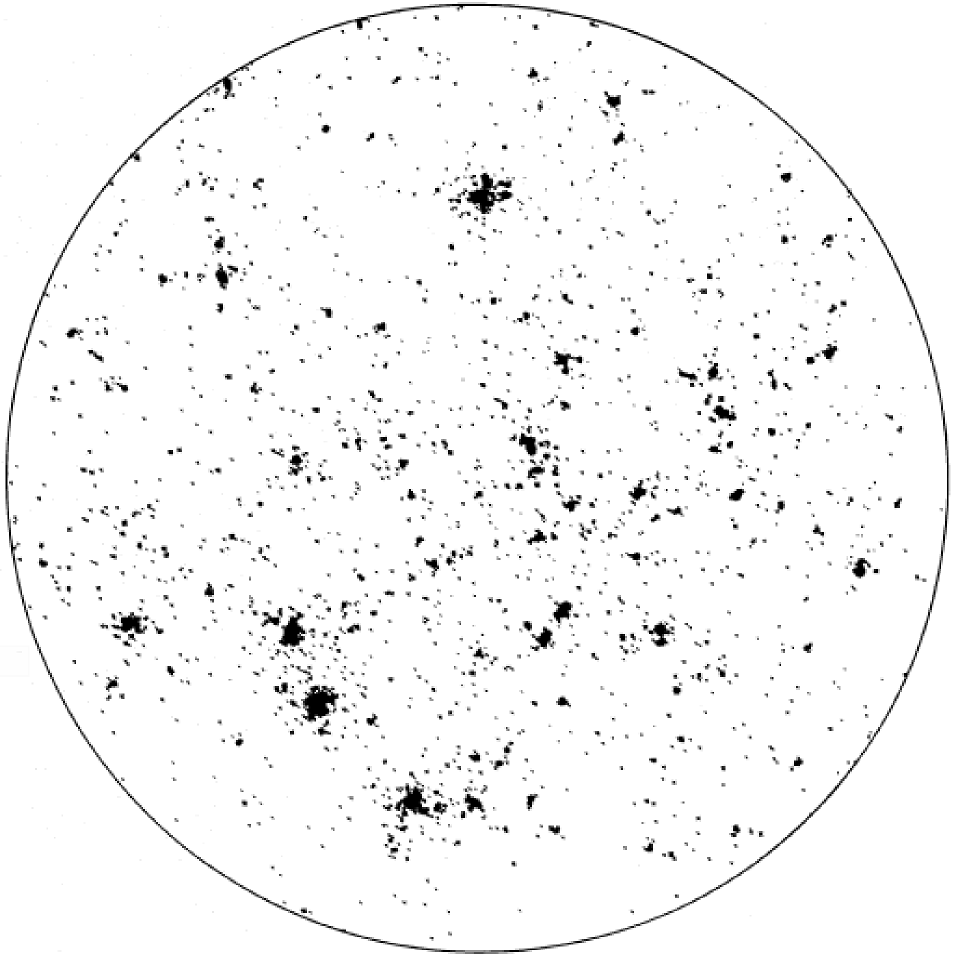
velocities: **v=Hr**

Boundary cond: **reflection on the sphere**



Z=14.2

Fig. 1a



Z=0

Fig. 1f

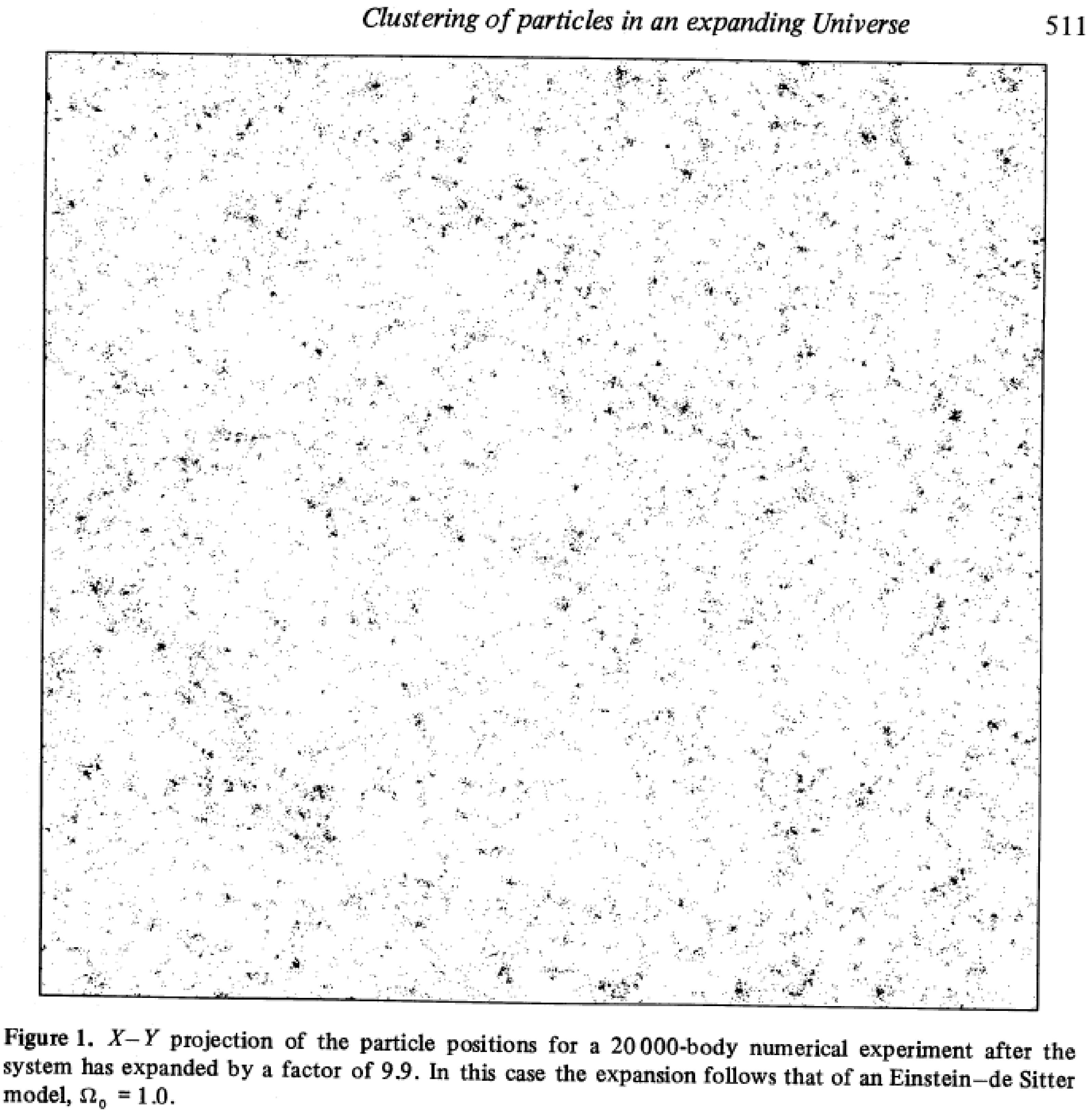
1981 Efstathiou, Eastwood MNRAS, 194, 503
On the Clustering of Particles in an Expanding Universe

Method: P^3M

N grid: 32^3
N particles: 20000 or less

Initial conditions
(i) Poisson (Om=1, 0.15)
(ii) cells distribution (Om=1)

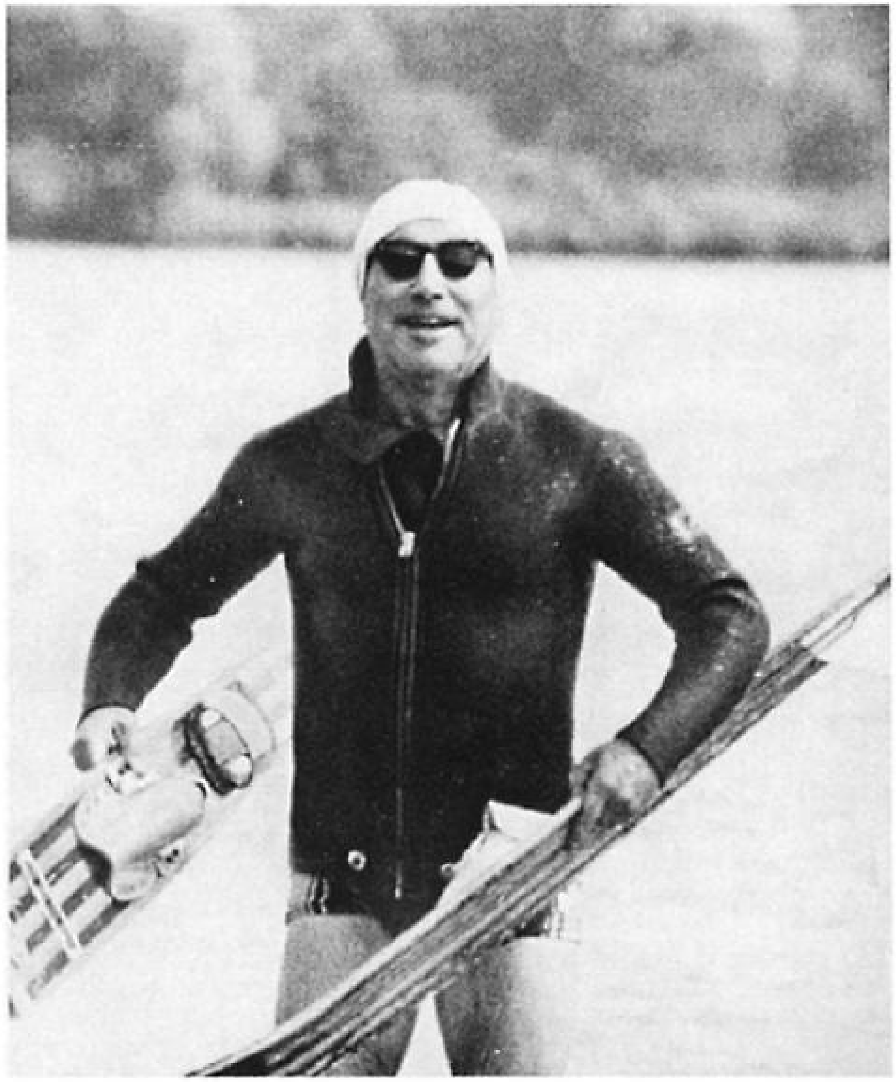
Boundary cond: Periodic



Clustering of particles in an expanding Universe 511

Figure 1. *X–Y* projection of the particle positions for a 20 000-body numerical experiment after the system has expanded by a factor of 9.9. In this case the expansion follows that of an Einstein–de Sitter model, $\Omega_0 = 1.0$.

Yakov Borisovich
Zel’dovich
1914 - 1987



3 times Hero of Socialist Labor

Theoretical Guidelines from Zel’dovich approximation (1970)

Comoving coordinates: r_i,

Zel’dovich approximation is a map: $r_i(\mathbf{q},t) = q_i + D(t)s_i(\mathbf{q})$

If $\Phi(\mathbf{q})$ is the linear perturbation of grav. potential then $s_i(\mathbf{q}) = -\partial\Phi/\partial q_i$

Density can be found from the conservation of mass

$$\rho(\mathbf{q},t) = \bar{\rho}(t)\left|\frac{\partial r_i}{\partial q_k}\right|^{-1} = \bar{\rho}\left|[(1-D(t)\alpha(\mathbf{q})]^{-1}\,[(1-D(t)\beta(\mathbf{q})]^{-1}\,[(1-D(t)\gamma(\mathbf{q})]^{-1}\right.$$

$\alpha(\mathbf{q}) \geq \beta(\mathbf{q})$ and $\beta(\mathbf{q}) \geq \gamma(\mathbf{q})$ are the eigen values of the deformation tensor

$$d_{ik}(\mathbf{q}) = \frac{\partial s_i}{\partial q_k} = -\frac{\partial^2\Phi}{\partial q_i \partial q_k}$$

Linear density fluctuations: $\delta\rho/\rho = D(t)(\alpha+\beta+\gamma)$.

The Zel’dovich approximation describes anisotropic collapse and motion.

Theoretical Guidelines from Zel'dovich approximation (1970)

x-q space is equivalent to phase space x-v

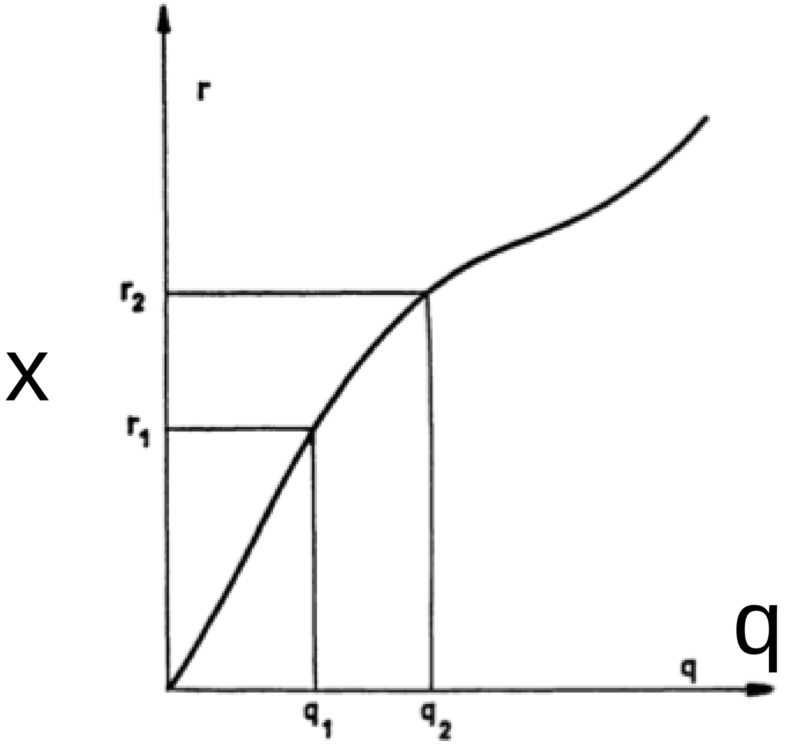
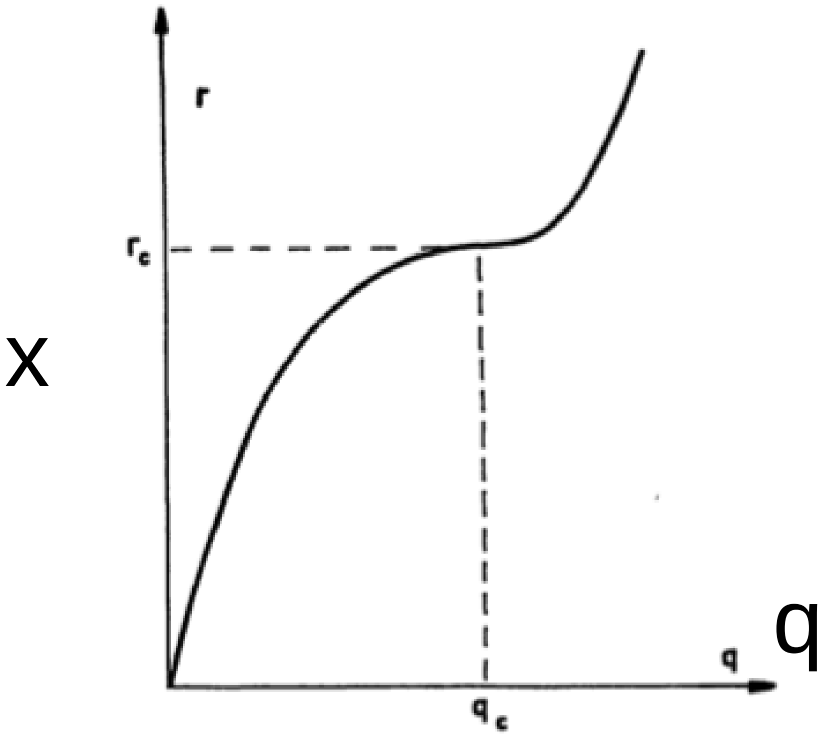
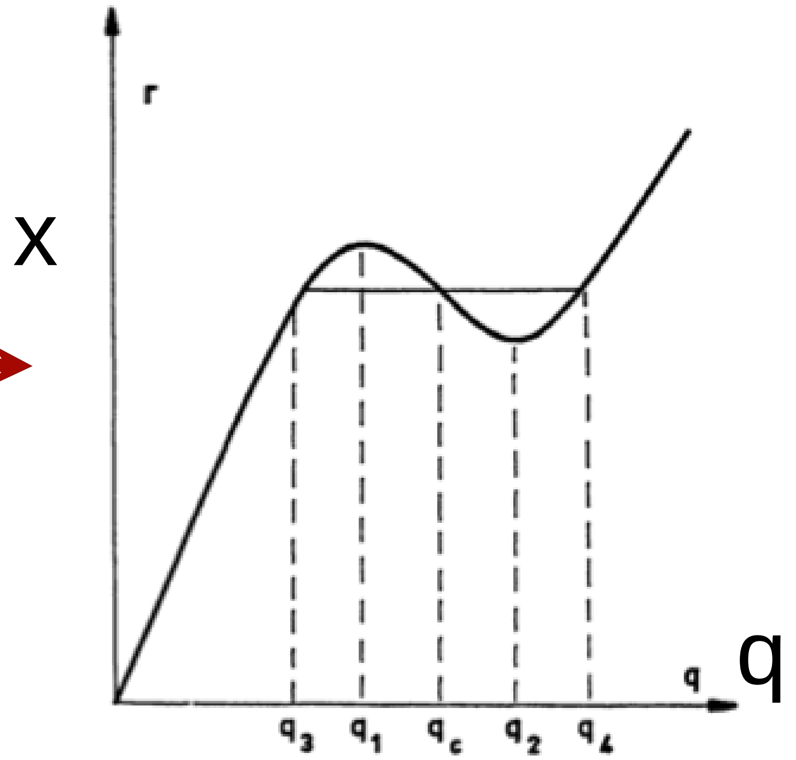


Fig. 1

Evolution

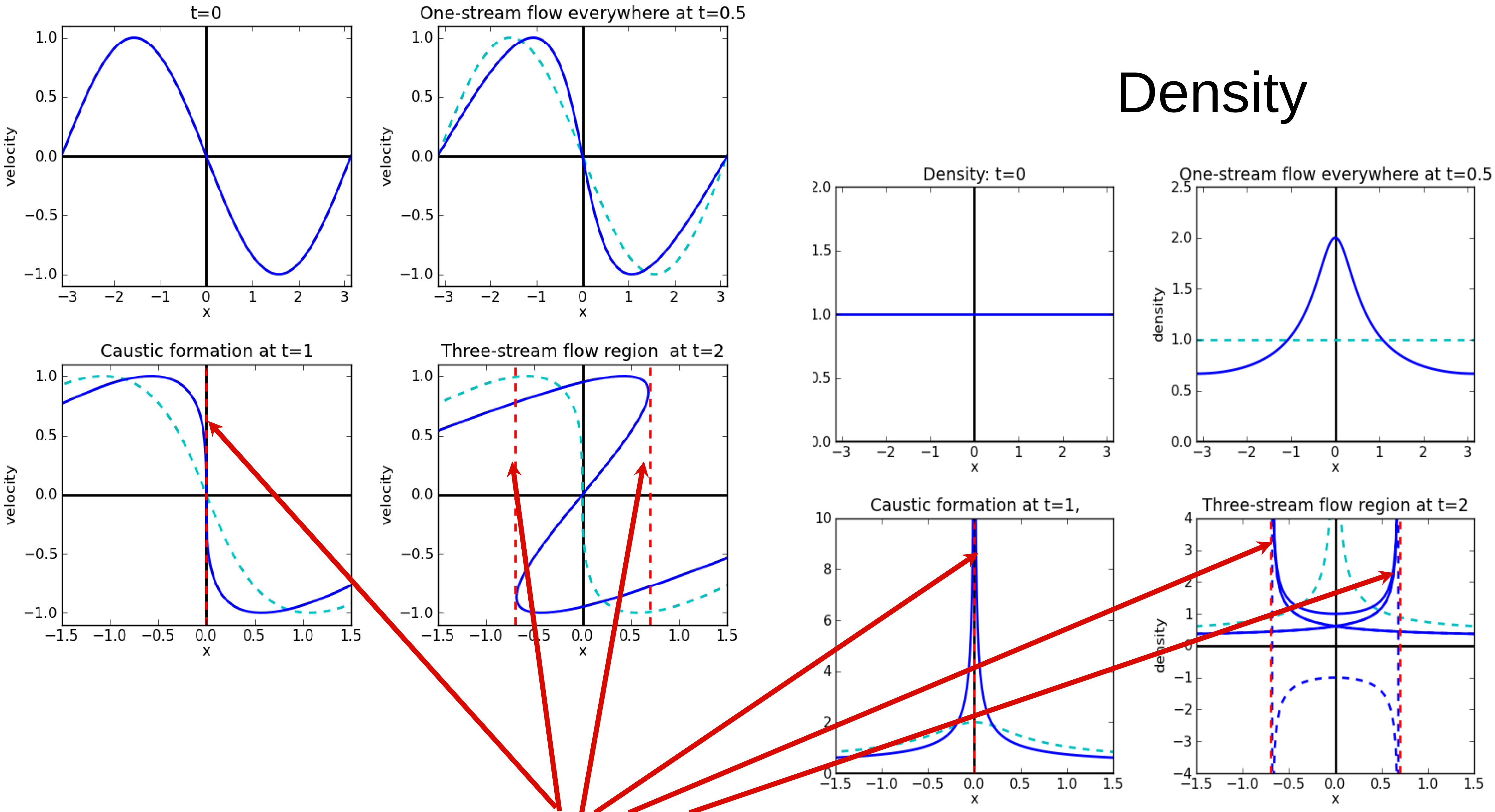




Multi-stream flows and caustics in collisionless Dark Matter
(one dimensional example)

Phase space

Density



caustics

Doroshkevich
Zeldovich
Shandarin ~1974

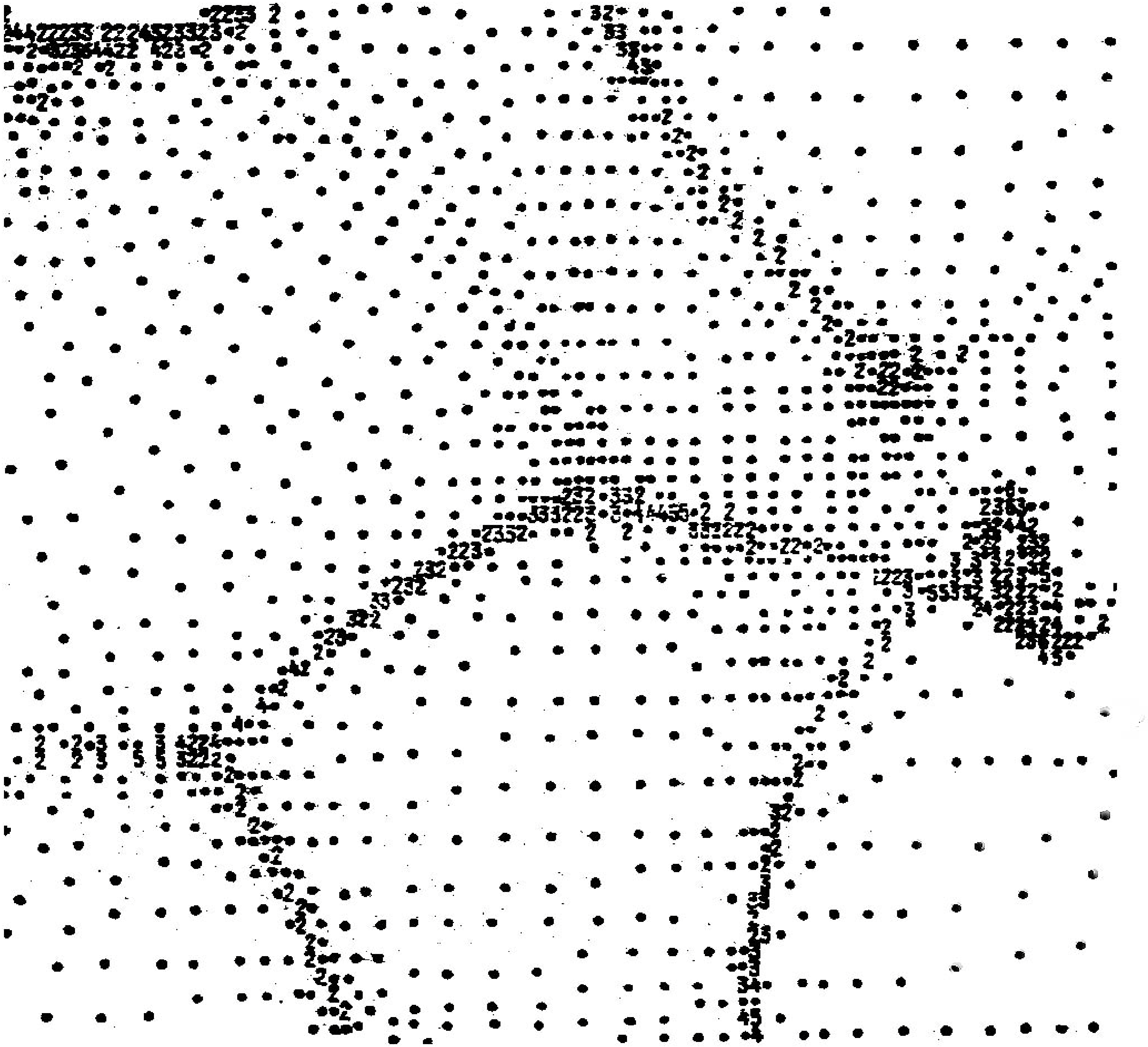
Preprint IPM

The first numerical simulation of structure in 2D by using ZA

Shandarin 1975

published in review by Doroshkevich Zeldovich Sunyaev 1975 (in Russian)

Later in Dorshkevich, Shandarin 1978



Printed on alphanumeric printer

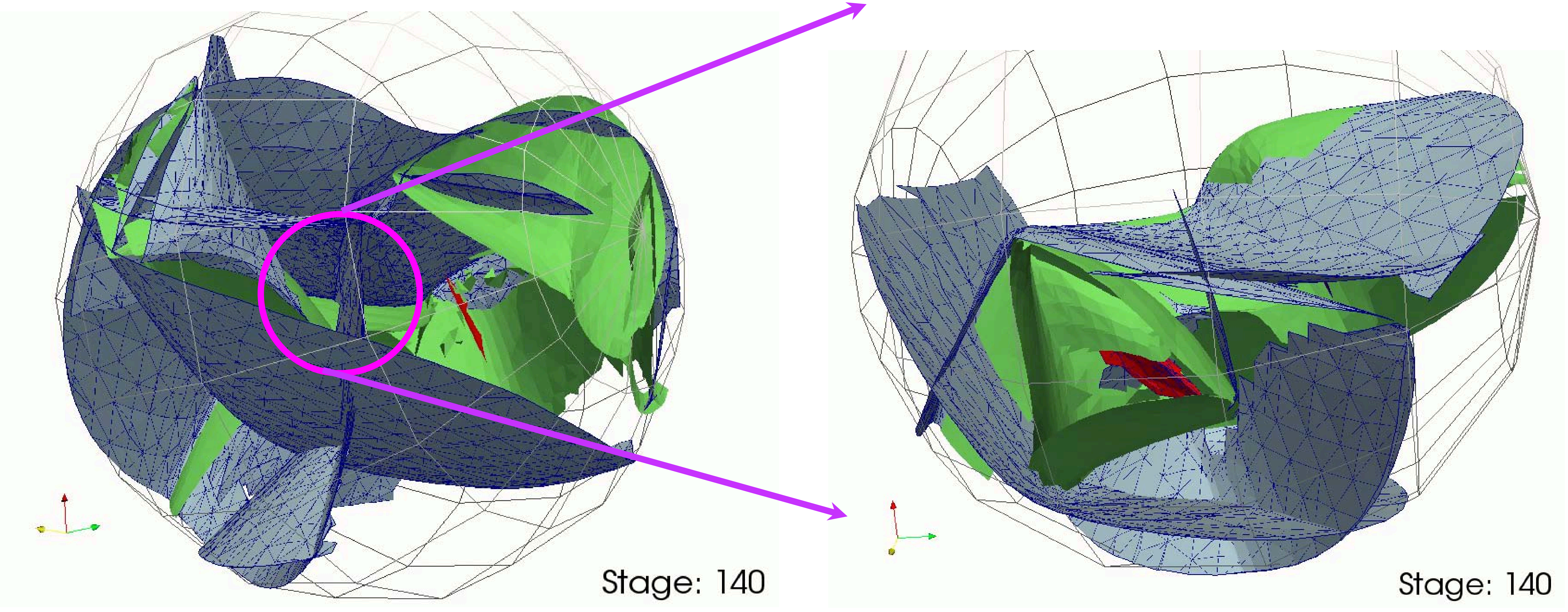
“Sketch of the formation of micropancakes, showing the topology of the singularities in configuration space”

Hogan 2001

~~Doroshkevich~~
~~Zeldovich~~
~~Shandarin ~1974~~

~~Preprint IPM~~

Caustics in 3D constructed by tessellation of phase space



Three different families of caustic surfaces
in matter distribution (blue,green,red)

Shandarin 2012

1980 Doroshkevich et al MNRAS, 192, 321

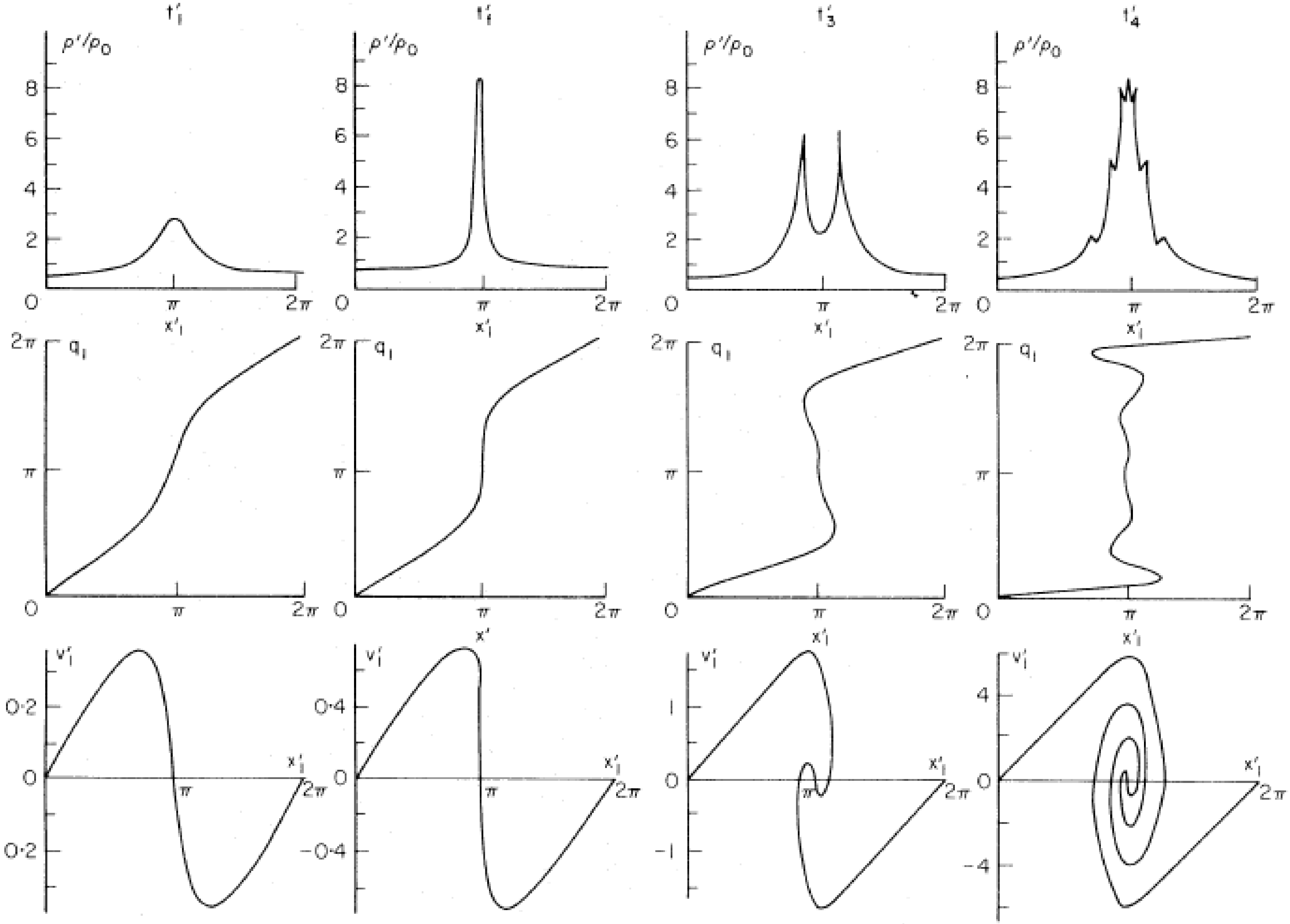
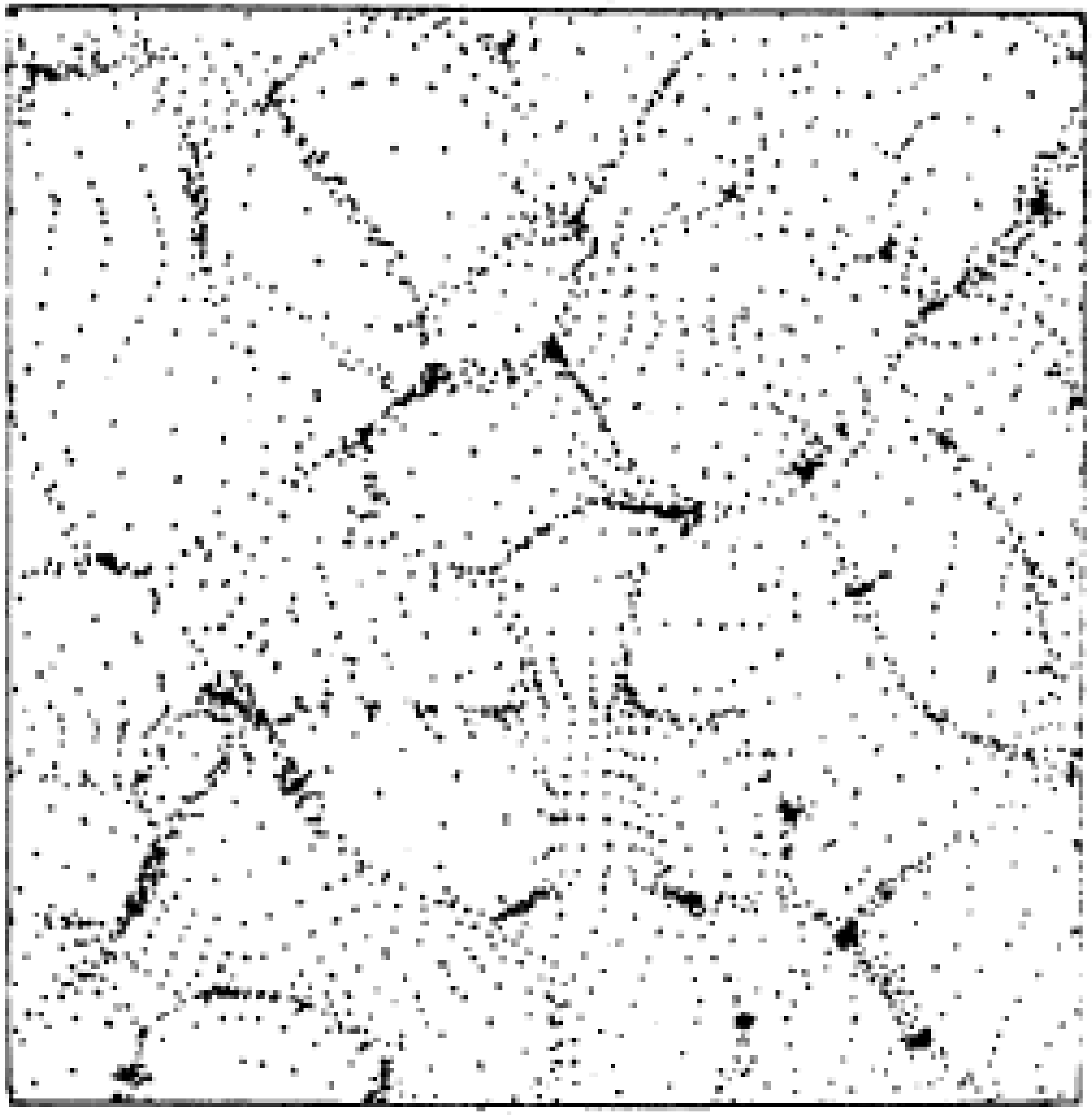


Figure 2. The system state on the phase plane $v_1' - x_1'$, on the plane $q_1 - x_1'$ and the dependence $\rho'(x_1')$ at four moments of time $t_1' < t_f' < t_3' < t_4'$. It is seen the transition of the one-flow motion with $t' = t_1'$ into the three-flow one with $t' = t_3'$ and then into the multi-flow motion with $t' = t_4'$. With $t' = t_4'$ the seven-flow distribution is observed.

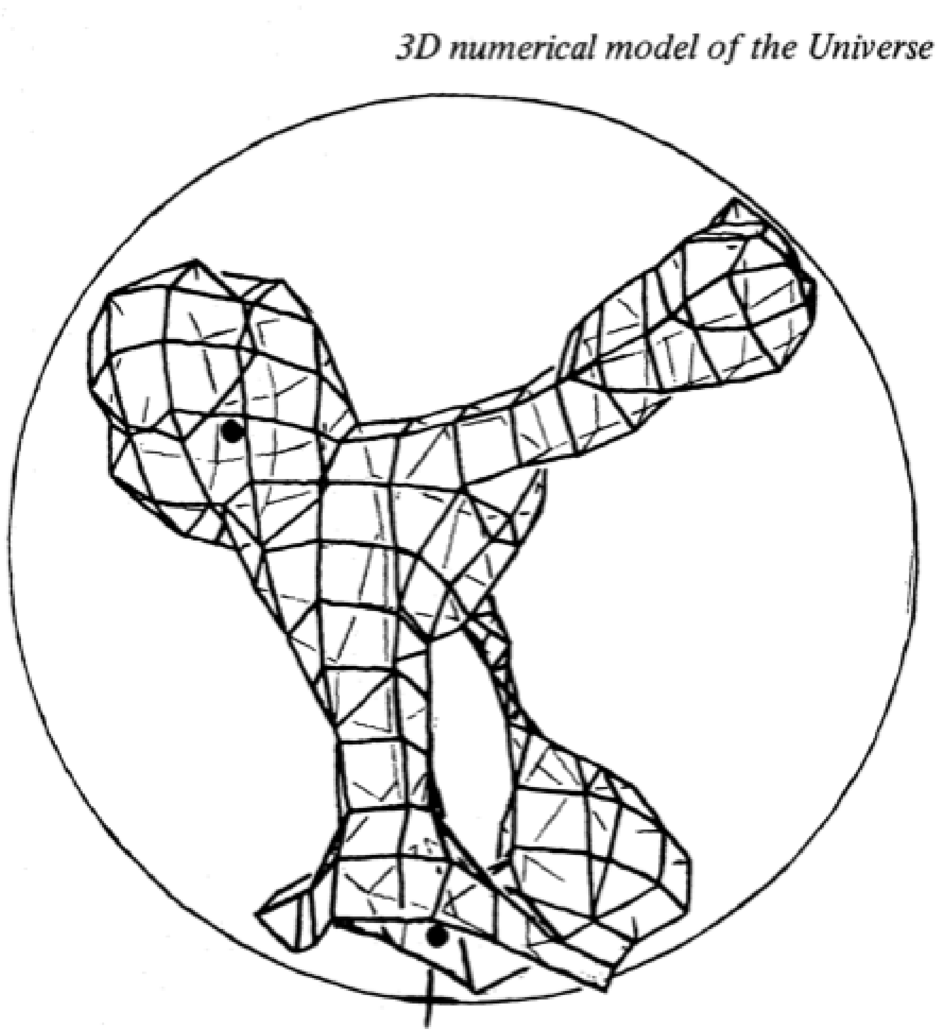
1980 Doroshkevich, Kotok, Novikov, Polyudov, Shandarin, Sigov
MNRAS, 192, 321

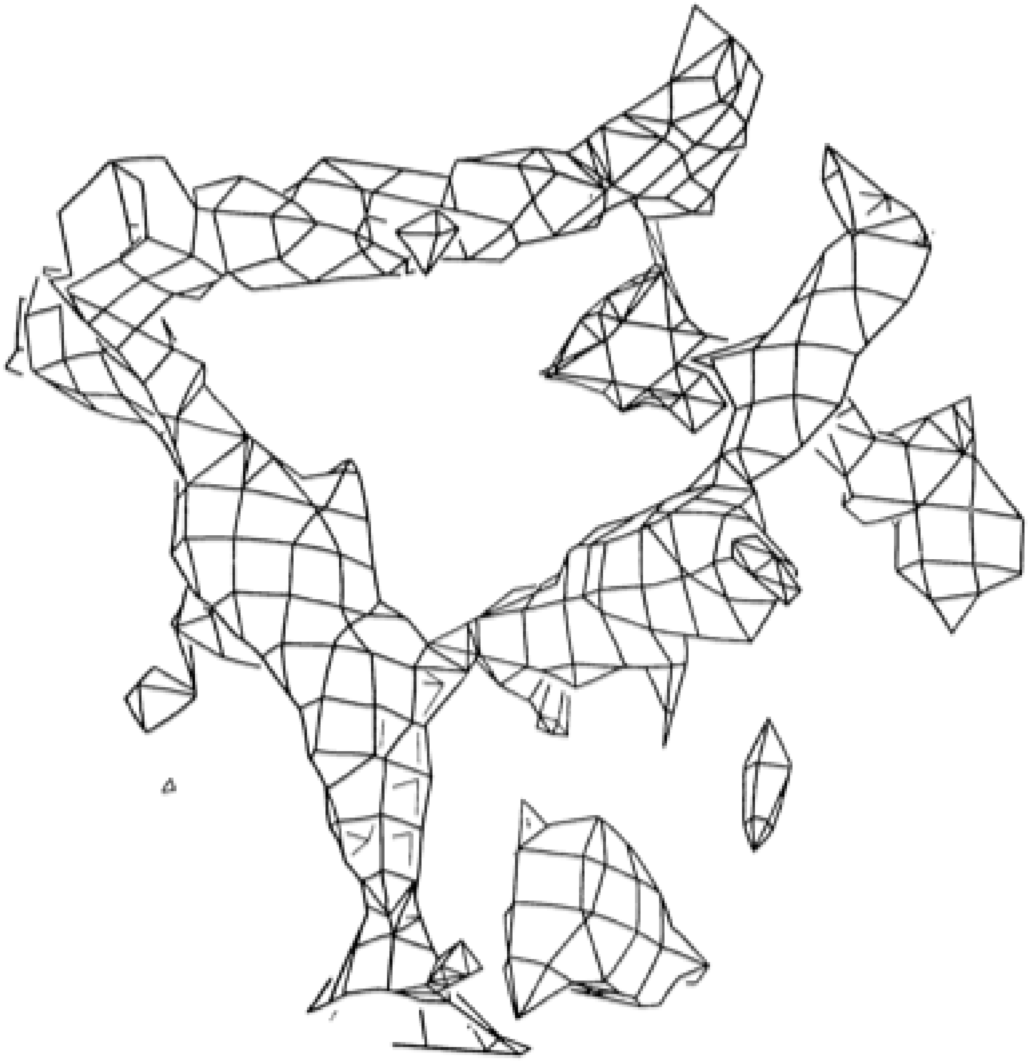
Two-dimensional Simulations of the Gravitaional System Dynamics and Formation of the Large-Scale Structure of the Universe



(b)

Filaments in N-body Simulations





Cosmological “chicken”

Klypin & Shandarin 1983,1984: First PM simulation of the collisionless hot dark matter

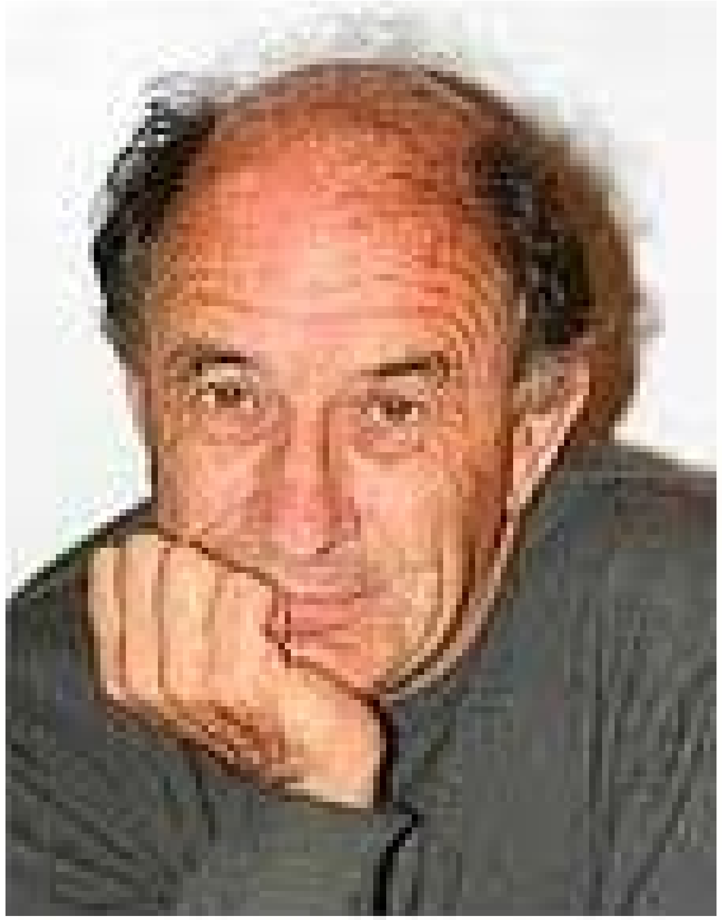
Where are Zeldovich’s pancakes?

Founding fathers of studies of caustic in cosmology

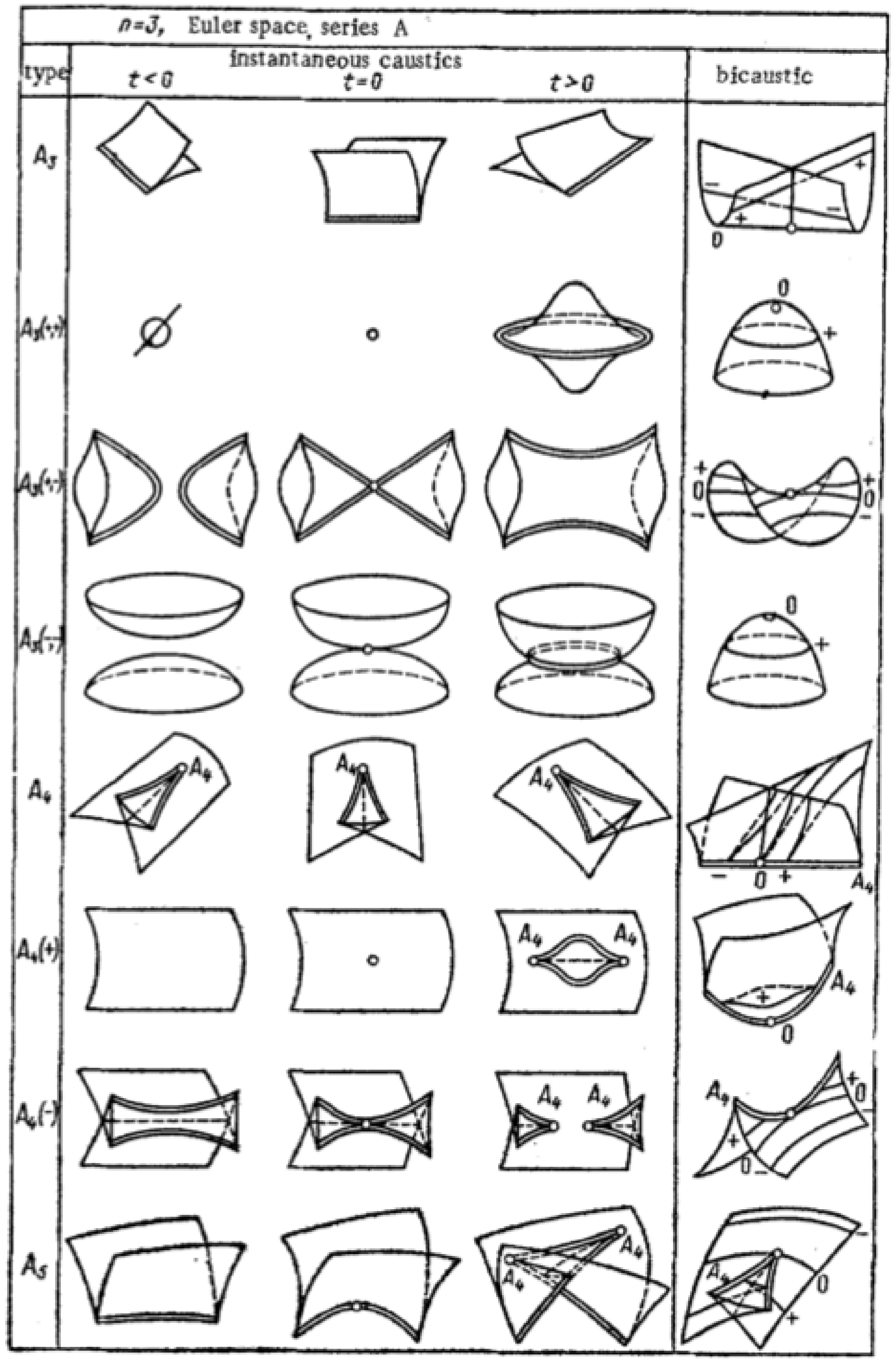
Yakov Borisovich
Zel’dovich
1914 - 1987

Singularity
Theory
I

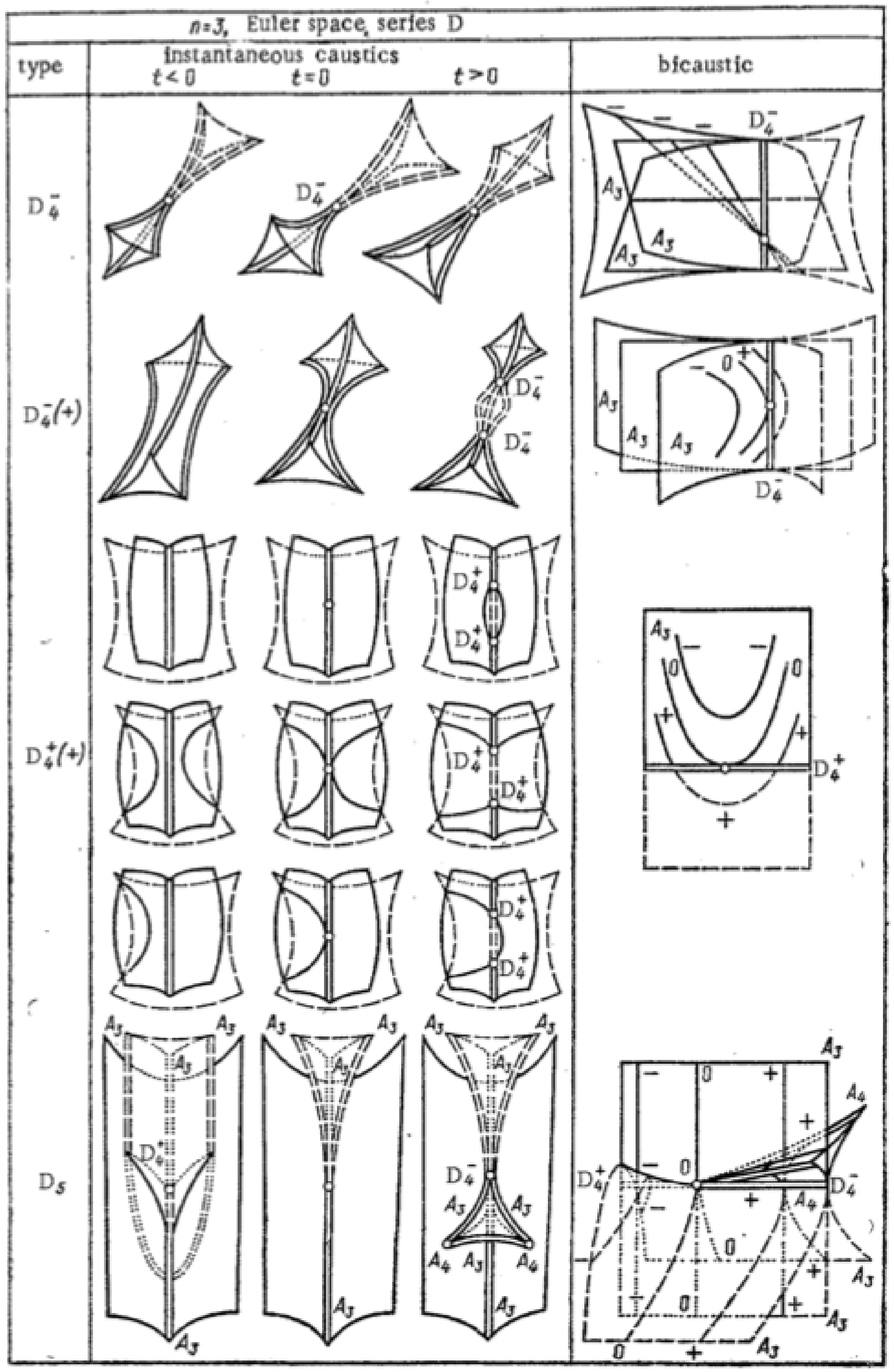
Vladimir Igorevich
Arnold
1937 - 2010



Arnold: Instantaneous caustics in 3D (normal forms)



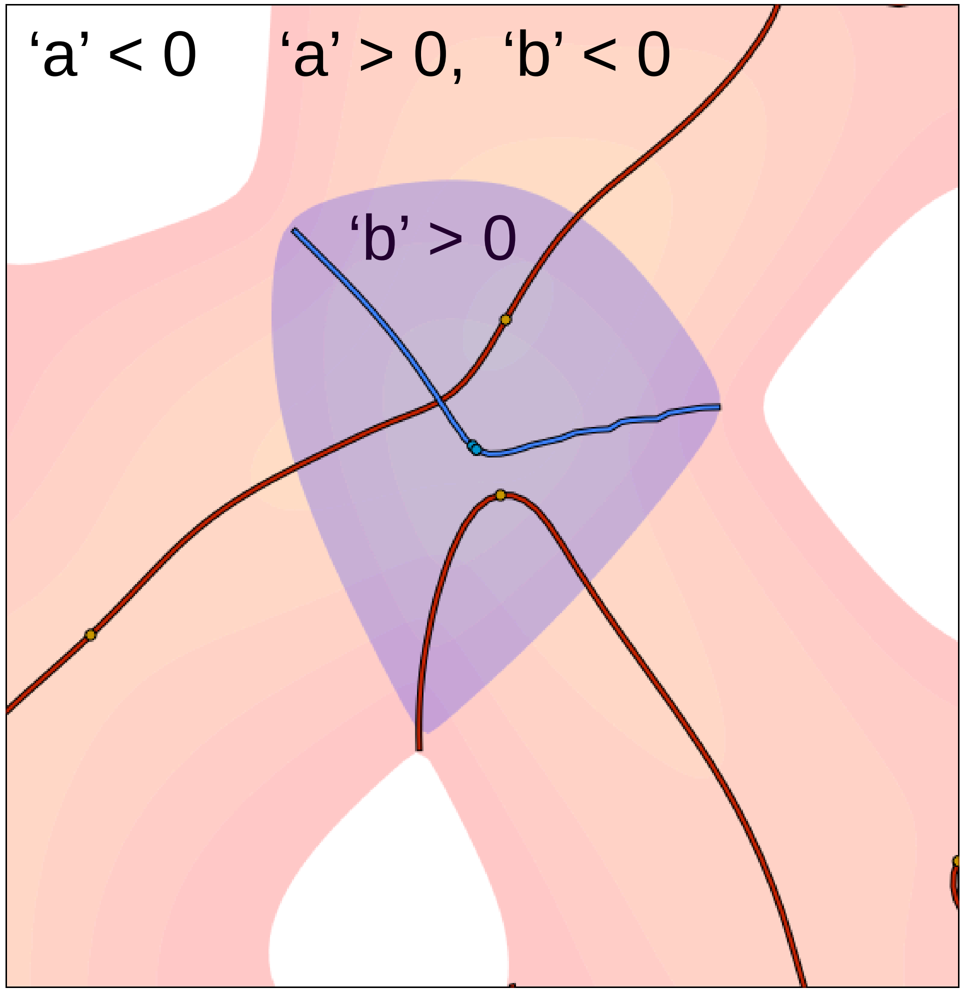
Series A



Series D

Mapping L to E in 2D

Eigen value fields ‘a’ > ’b’ control number of streams and caustics

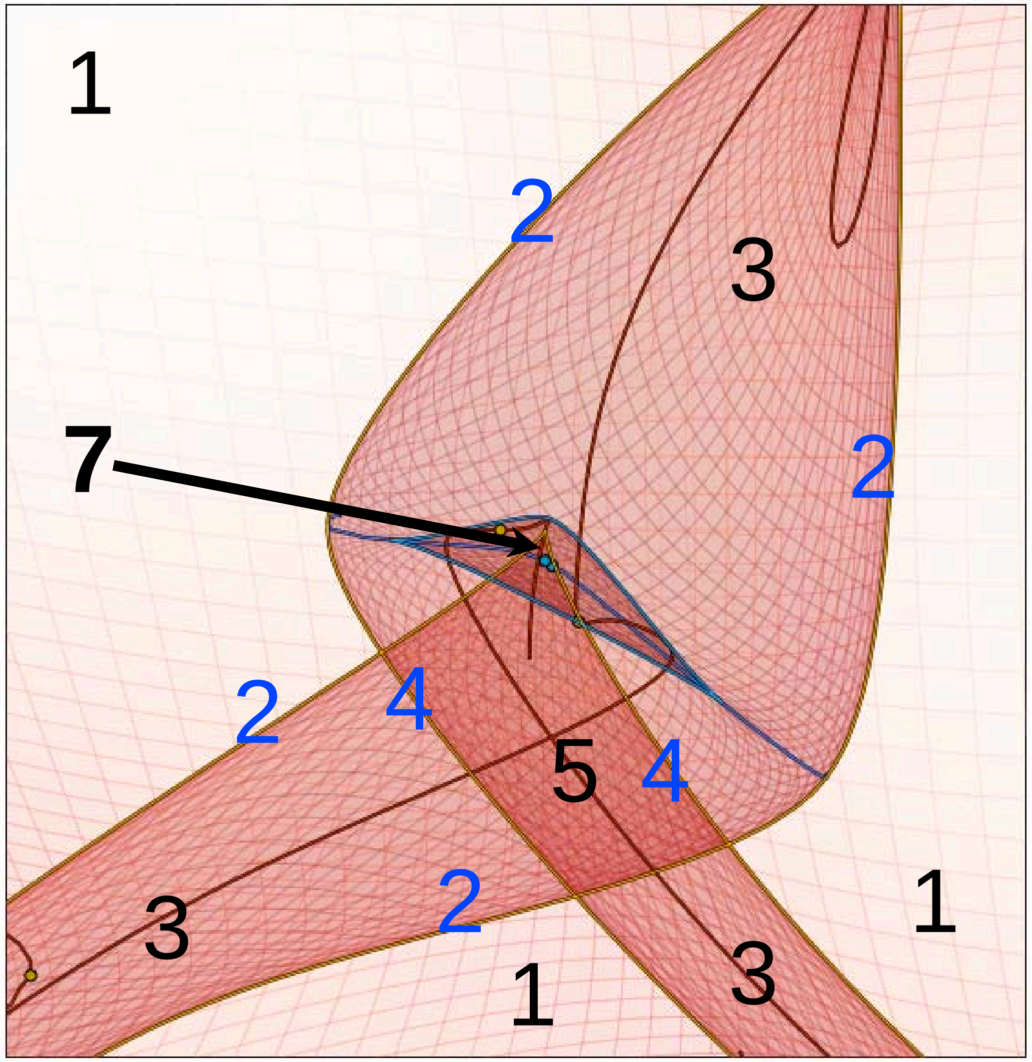


Lagrangian space

Hidding,
Shandarin,
van de Weygaert

work in progress

Number of streams in
Eulerian space

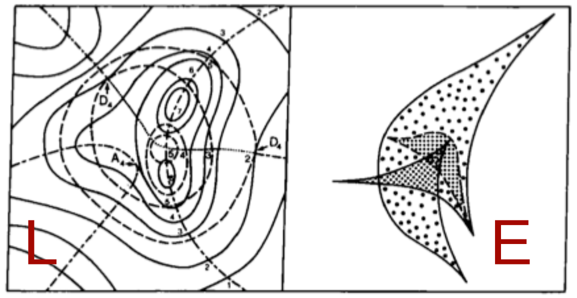


Zel'dovich Approximation (1970)   (Shand. Zeld 1989)

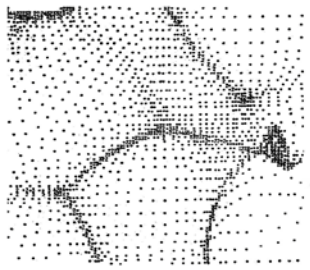
☑ Generation of the initial conditions for cosmological N-body simulations (first time in Moscow in 1973, first time in US in 1983)

Key features of cosmic web predicted by ZA

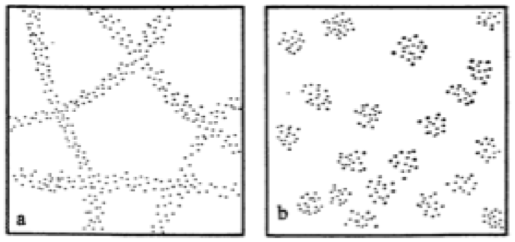
- Anisotropic collapse and anisotropic expansion:
  pancakes/walls (1970), filaments (1982),
  along with compact clumps and voids



- Full Set of Caustics (1982)

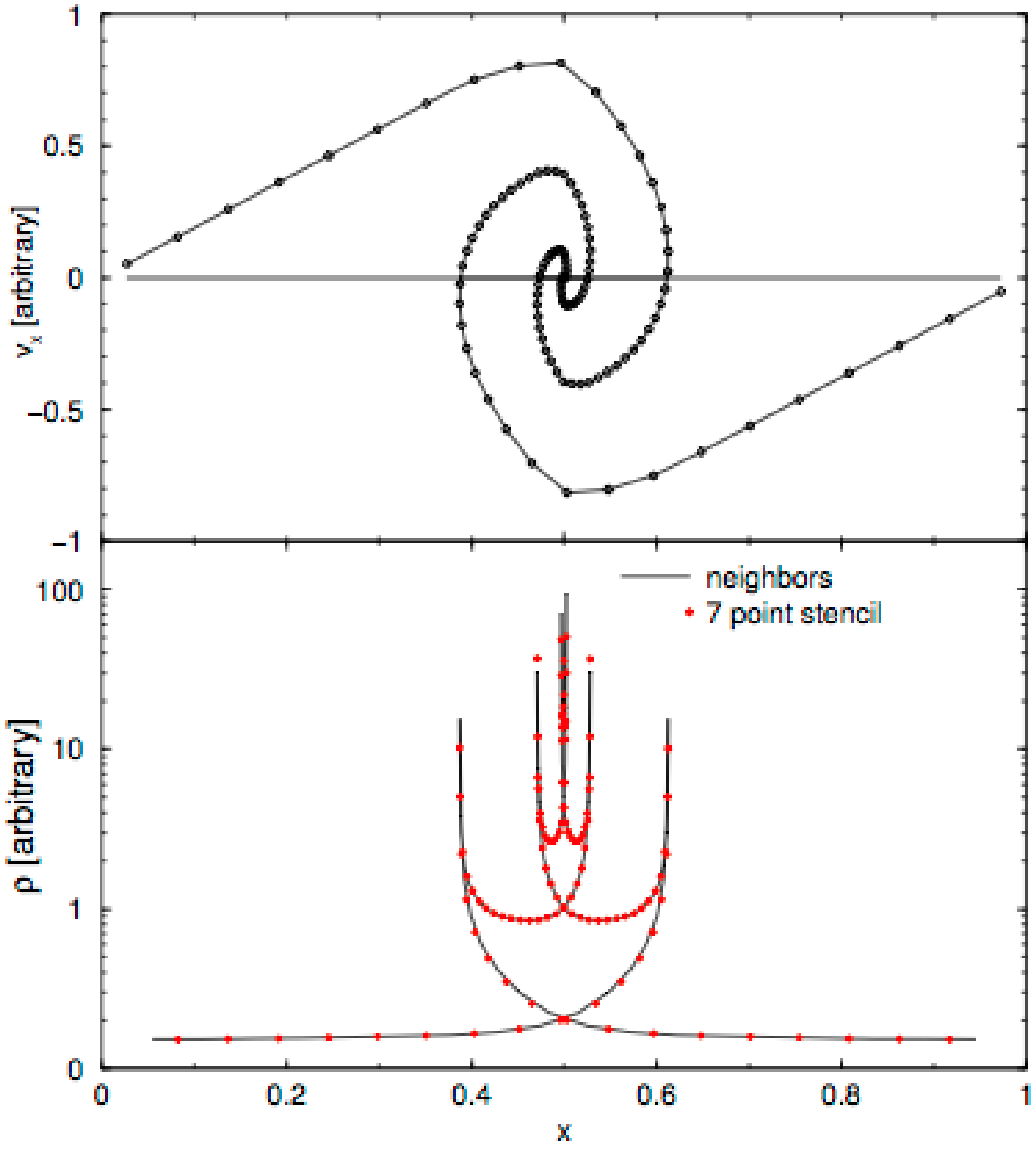


- Connectivity of the Large-Scale structure (1975)

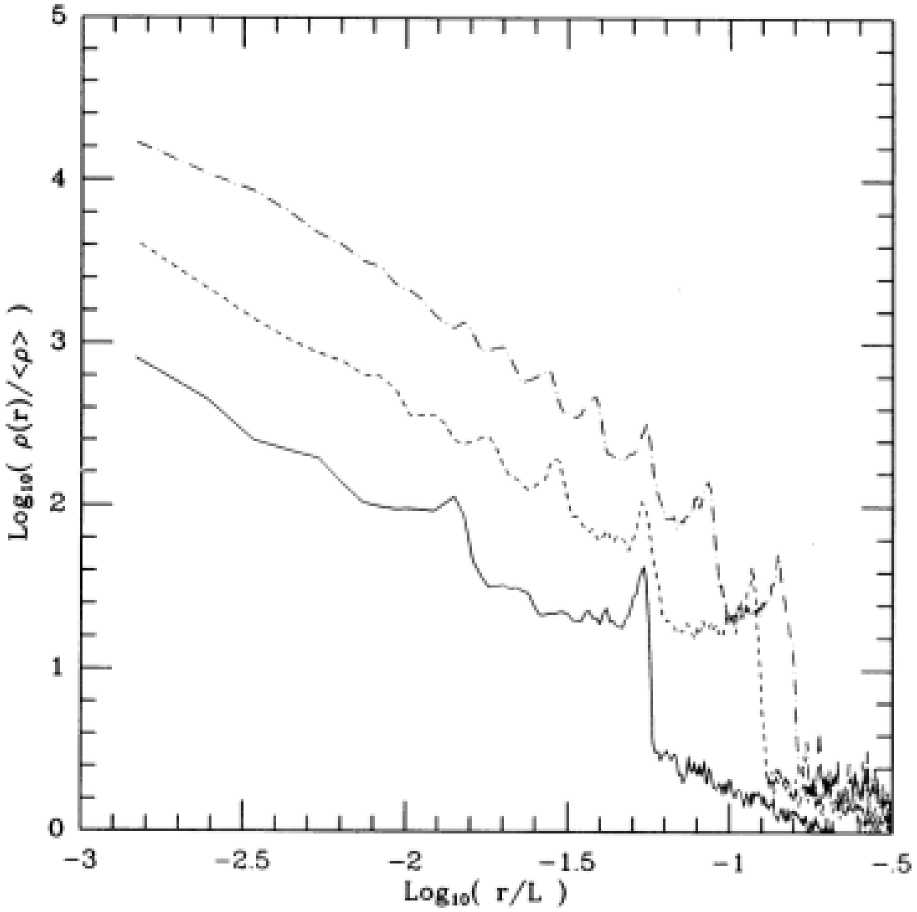


- Topology of LSS (1983)
- Multi-stream flows (1970)
- Anisotropic accretion of mass on clumps from filaments (1989)
  https://www.astro.rug.nl/~hidding/go/go.html

Phase-Space of Cold Dark Matter

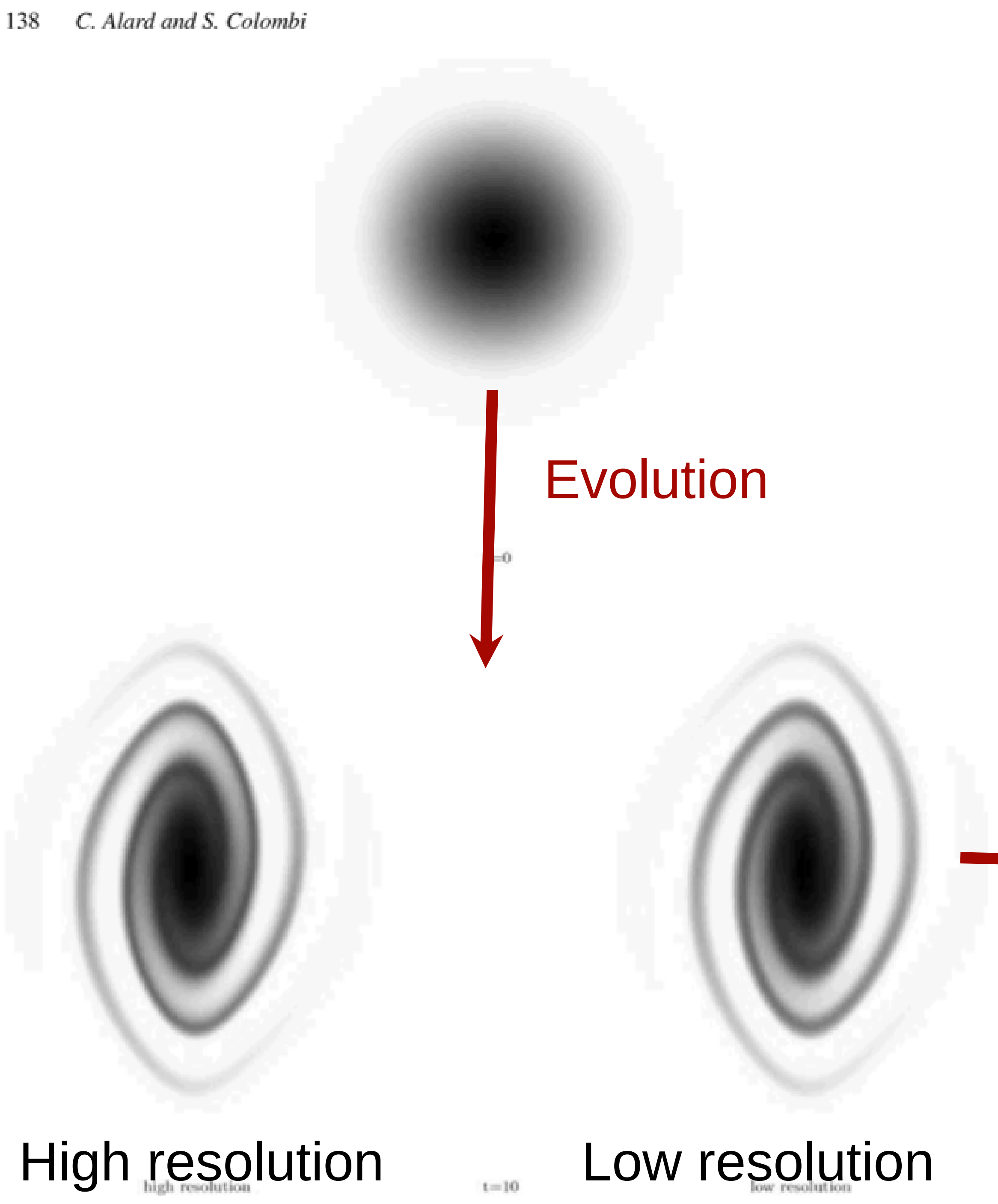


Abel, Hahn, Kaehler, 2012



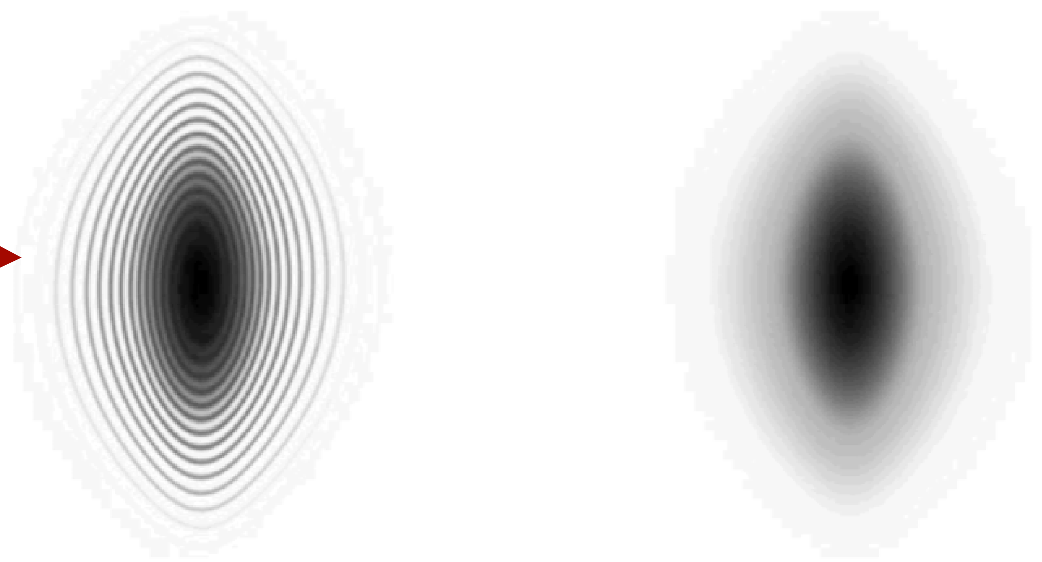
Alimi et al 1990
2D N-body simulation
of a filament

Phase-Space of Hot Dark Matter



138 *C. Alard and S. Colombi*

Evolution



Alard & Colombi 2005

High resolution

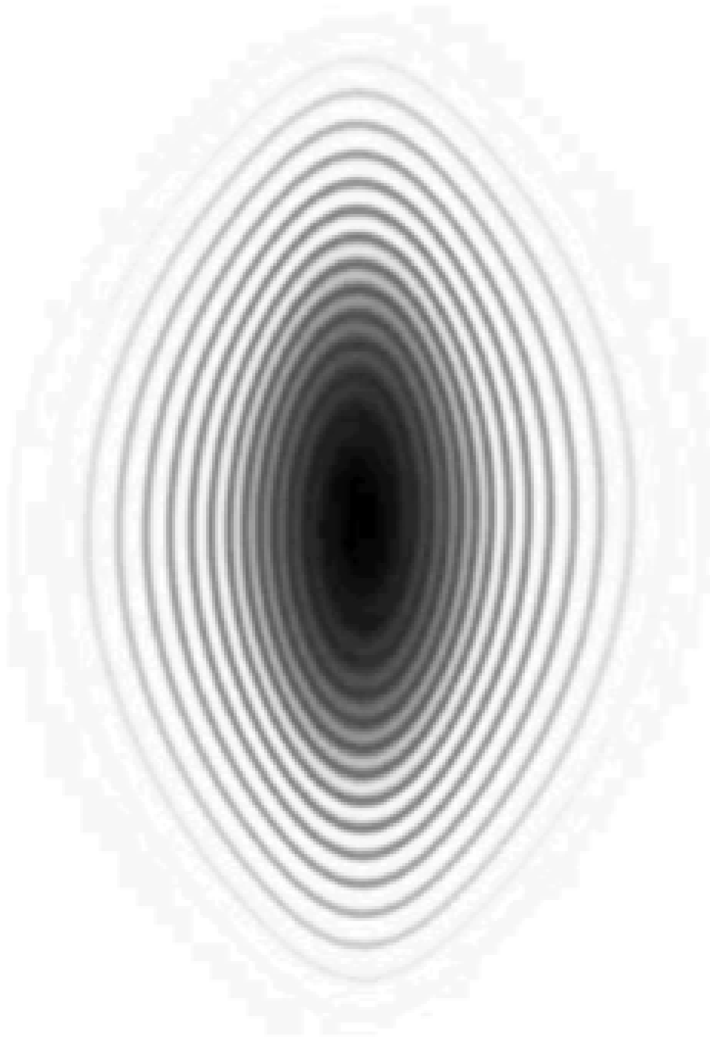
Low resolution

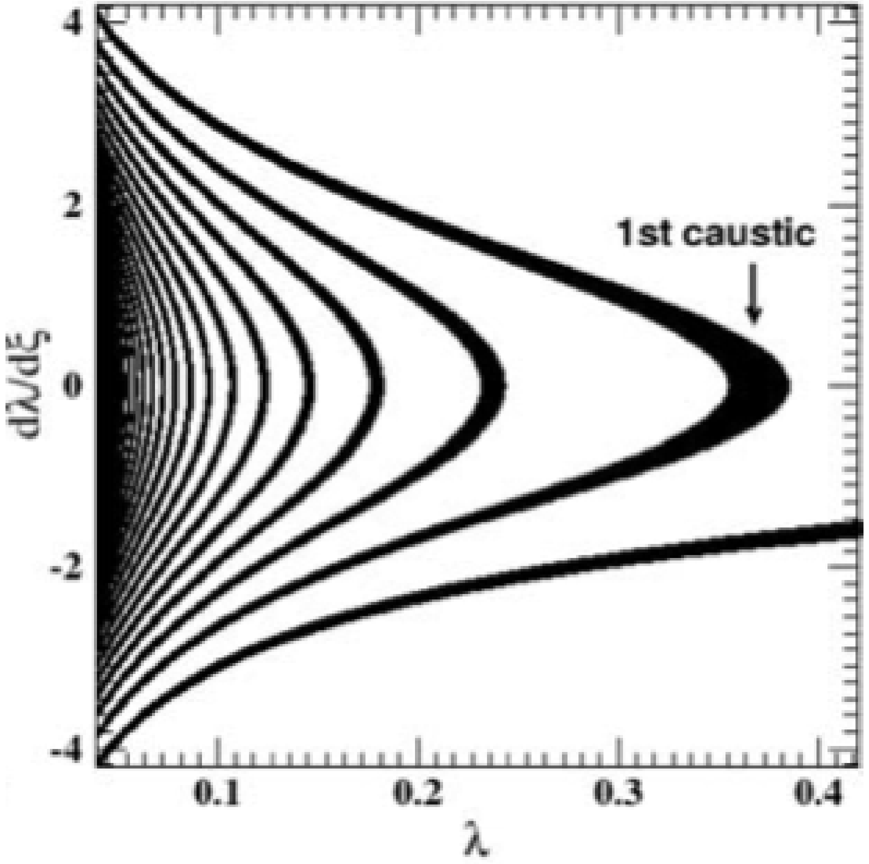
High resolution

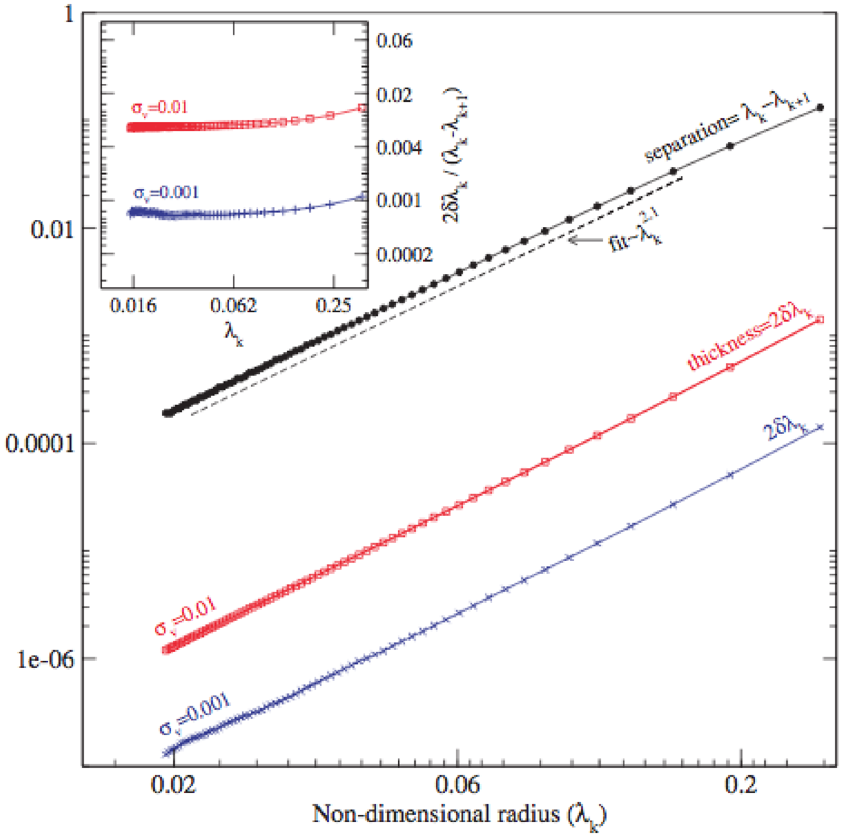
Low resolution

Phase-Space of HDM vs CDM

Mohayaee & Shandarin 2006
Self-similar spherical halo





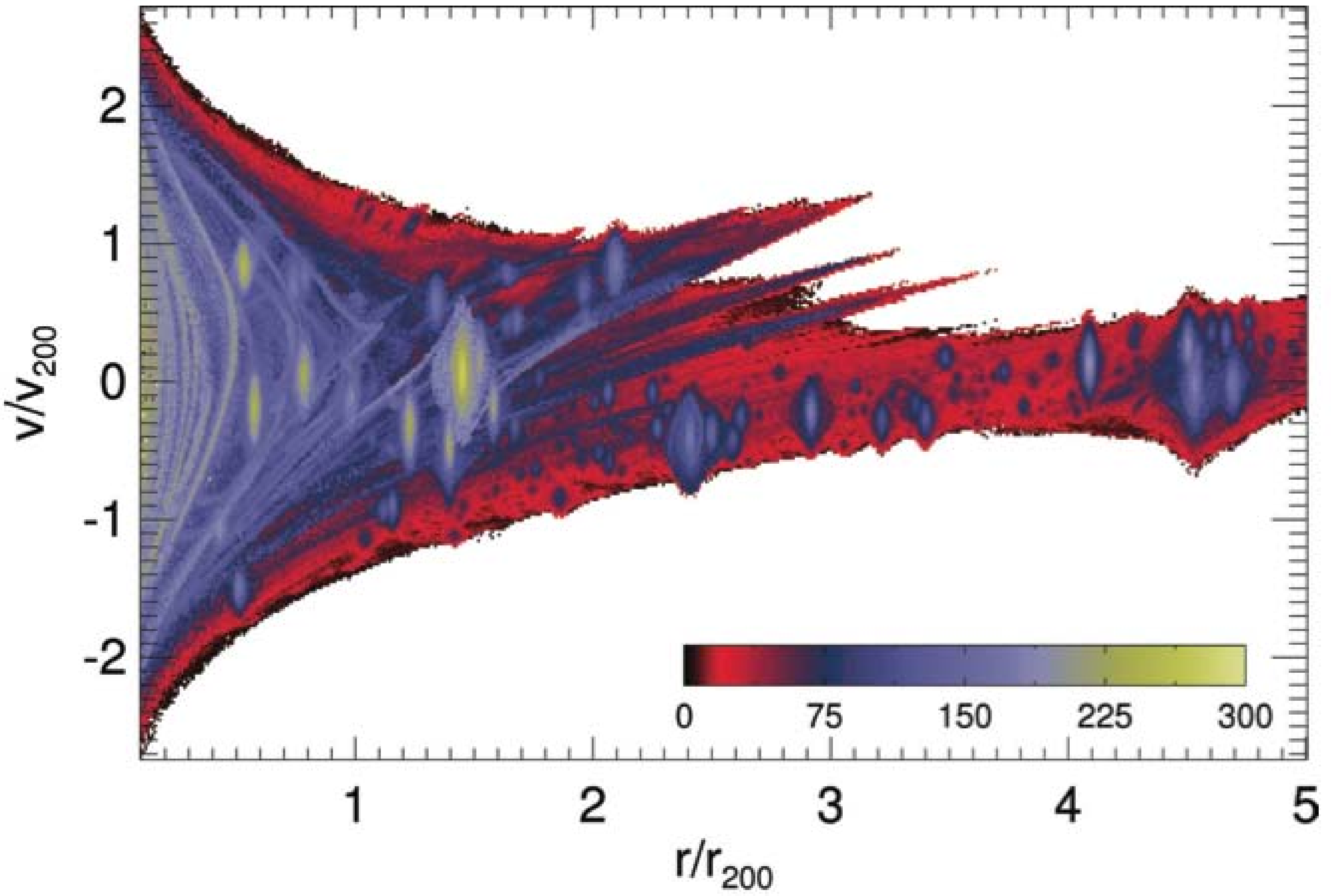


Alard & Colombi 2005

Vogelsberger & White 2011

‘Streams and caustics:

the fine-grained structure of cold dark matter haloes’

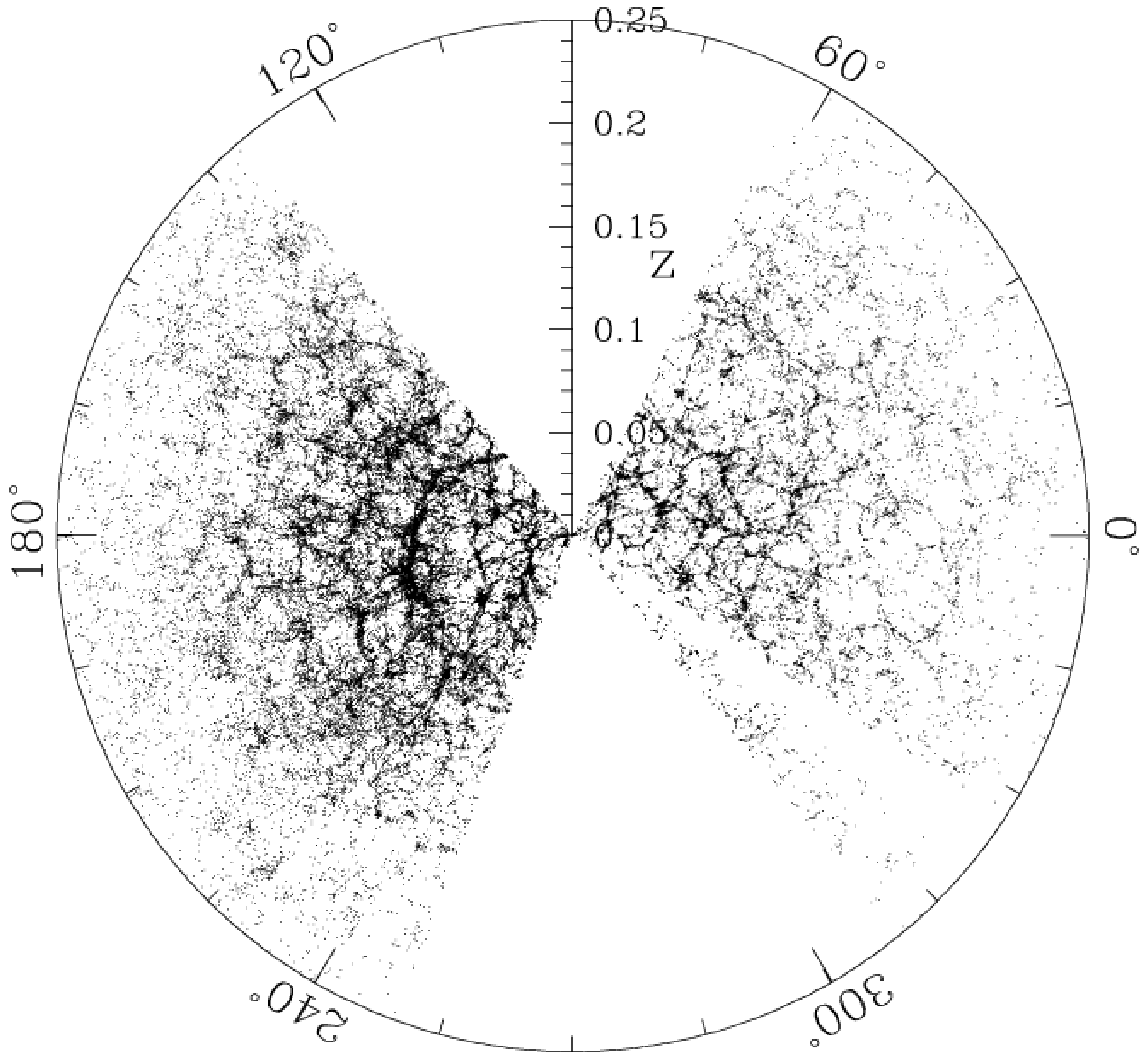


At 8 kpc from the halo centre, a typical point intersects about 10^14 streams with a very broad range of individual densities;
the ~10^6 most- massive streams contribute about half of the local dark matter density.

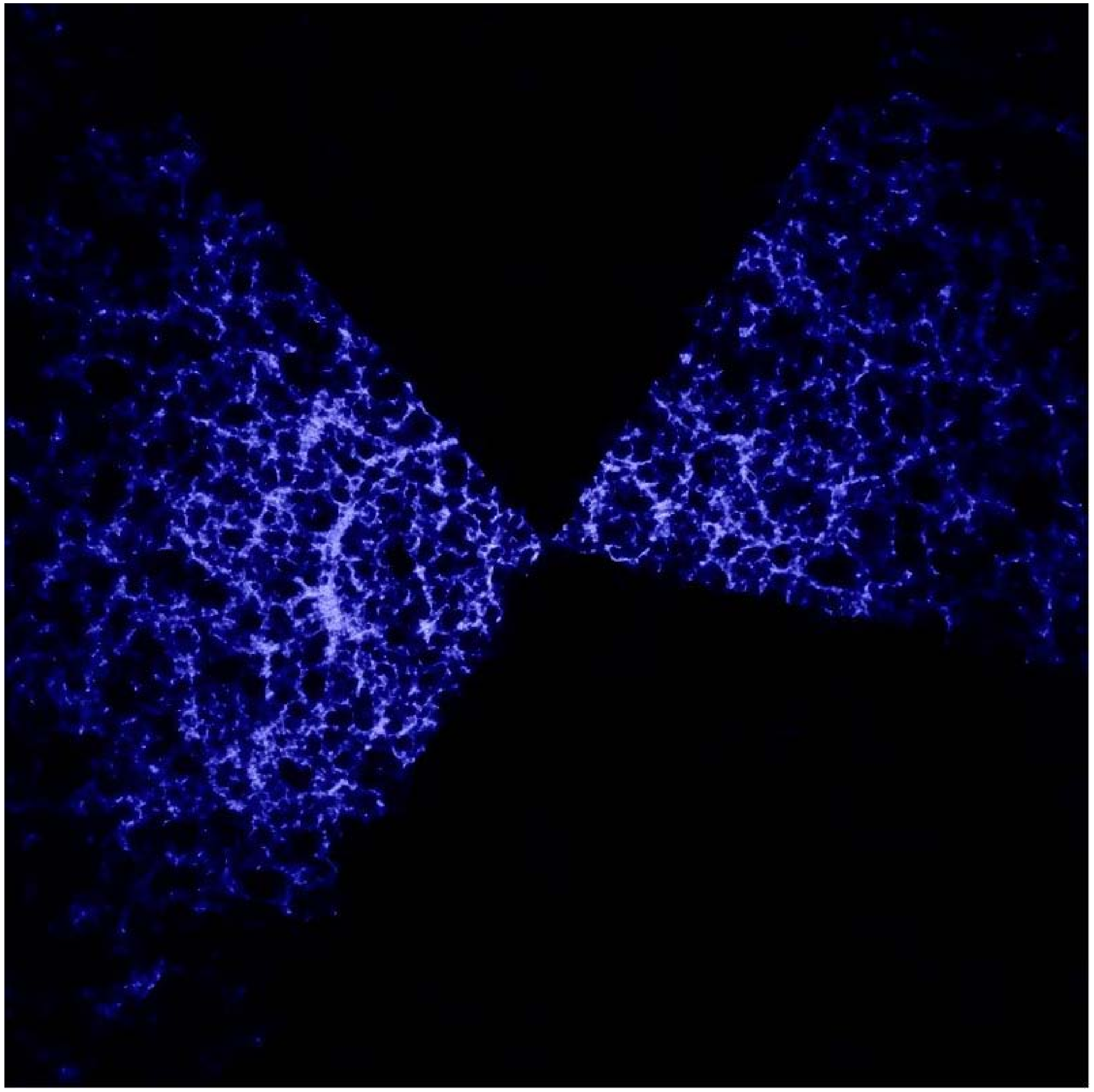
Sloan Digital Sky Survey

Dots OR Tessellation?

Blanton et al. (2003) (astro-ph/0210215)



Large−Scale Structure sample10



DelaunayTessellation Field Estimator

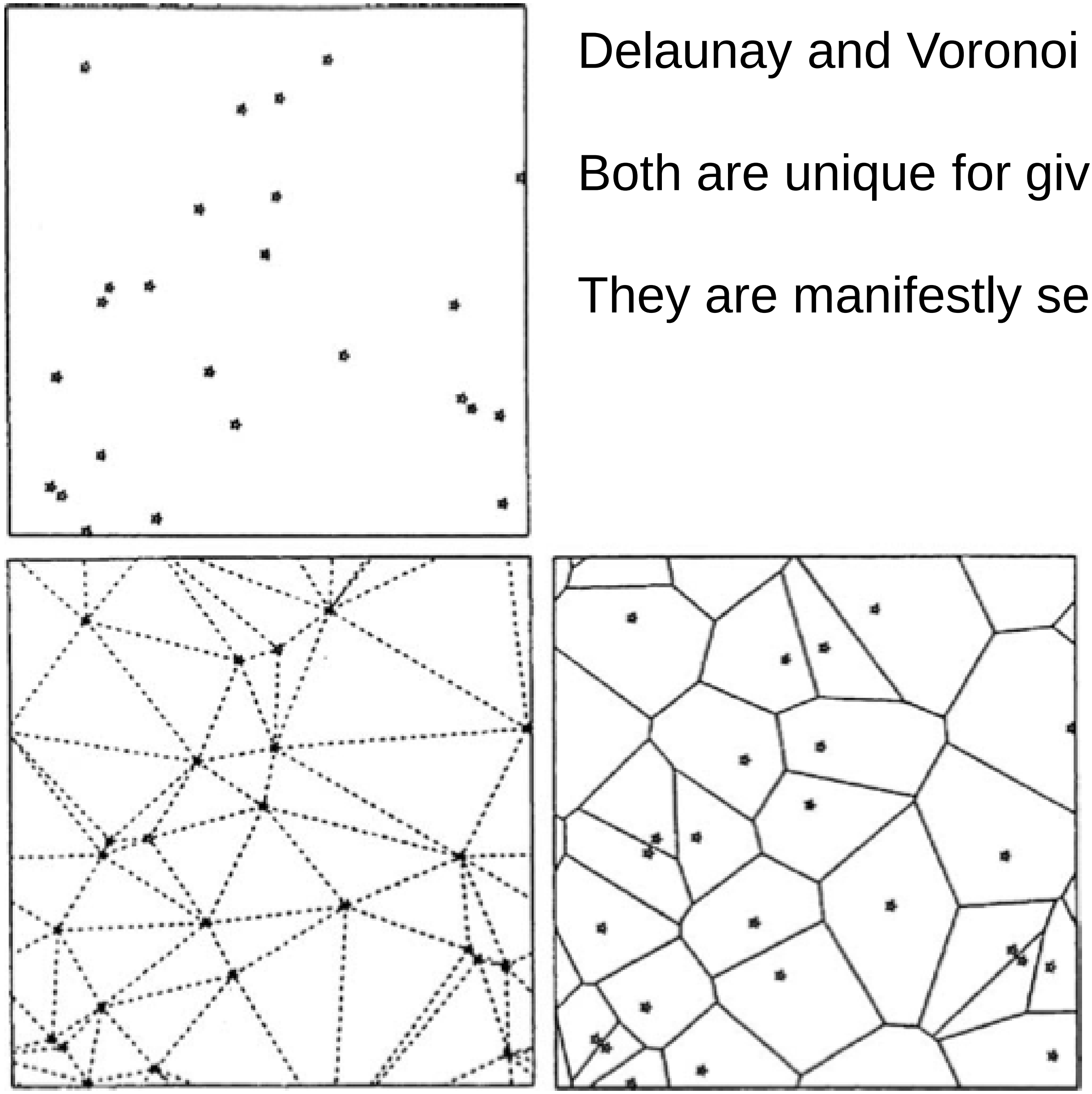
[www.astro.rug.nl/~weygaert/**dtfesdss**.html](www.astro.rug.nl/~weygaert/dtfesdss.html)

Delaunay and Voronoi Tessellations

Delaunay and Voronoi are dual tessellations

Both are unique for given set of points

They are manifestly selfadaptive

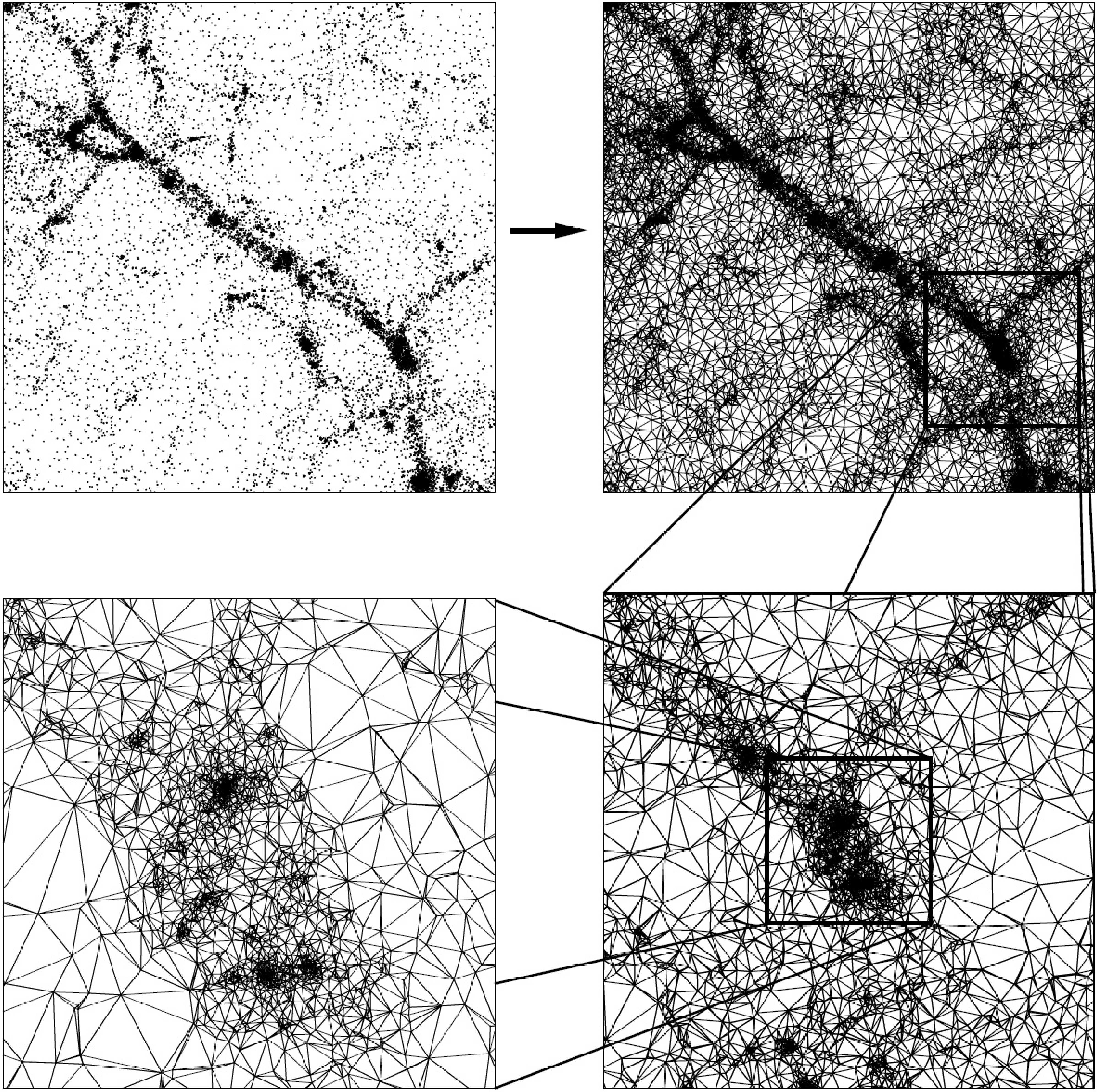


Delaunay

Voronoi

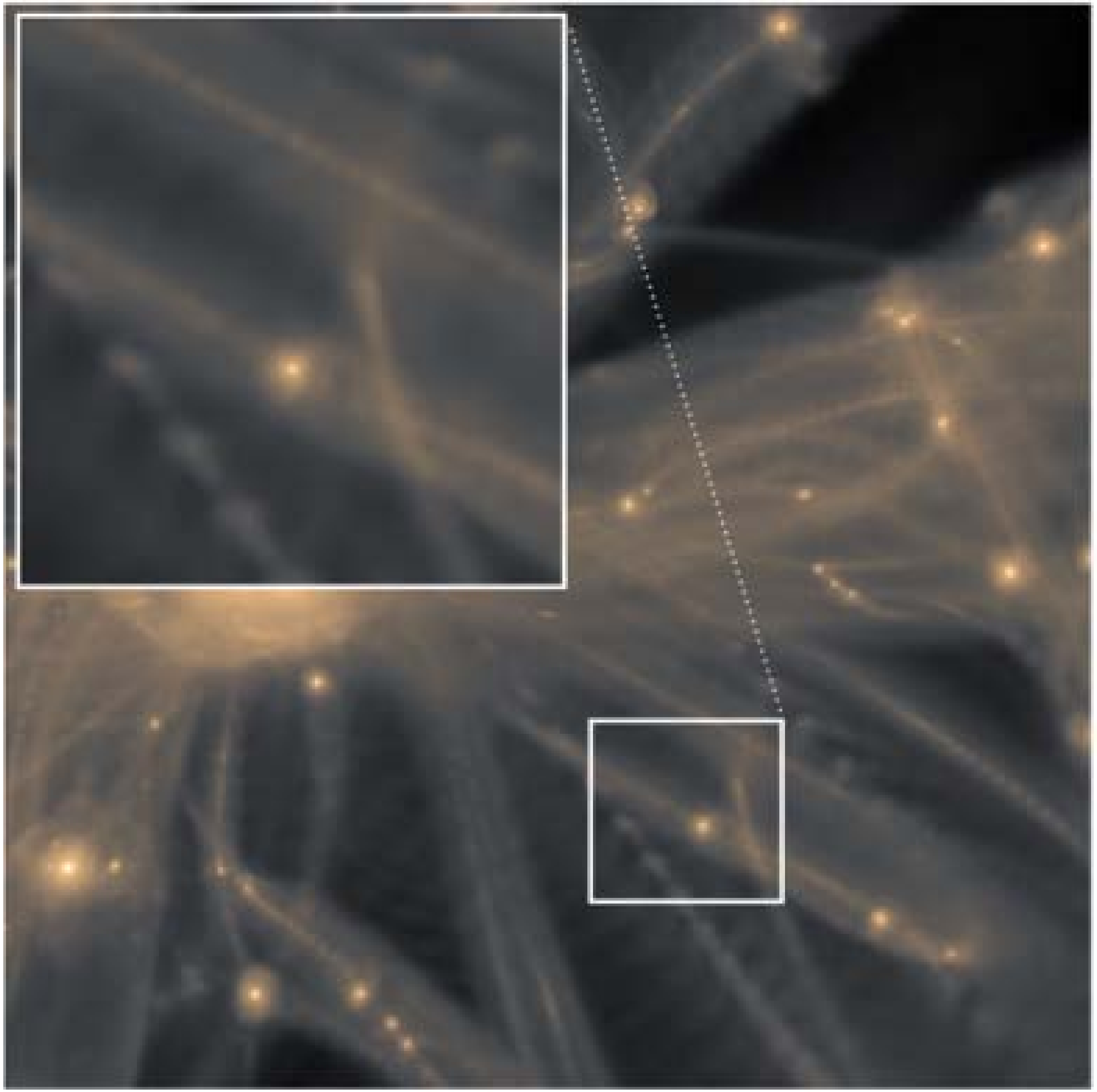
van de Weygaert & Schaap 2009

http://www.cgal.org/



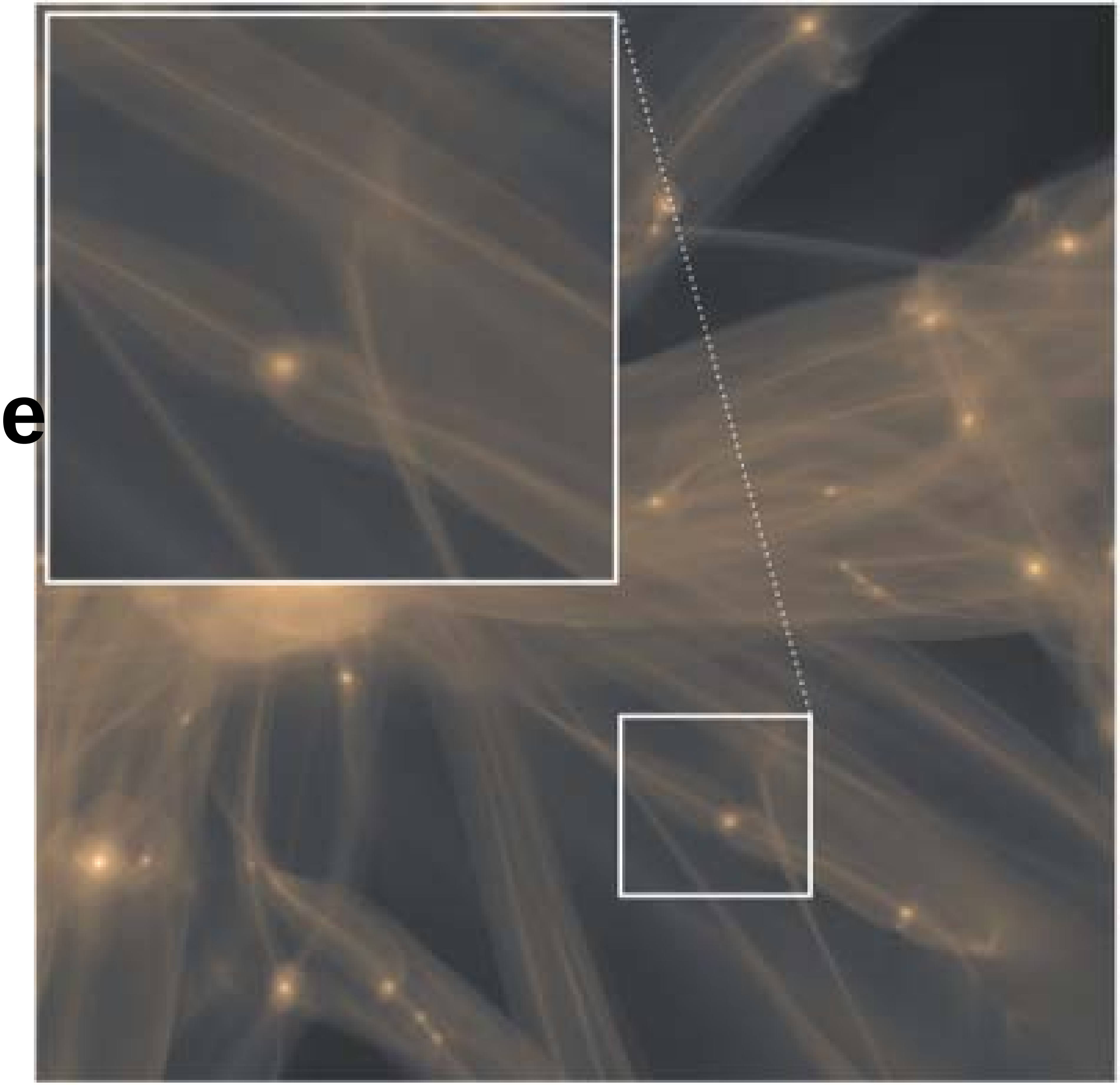
Smooth Particle Hydro

(SPH)



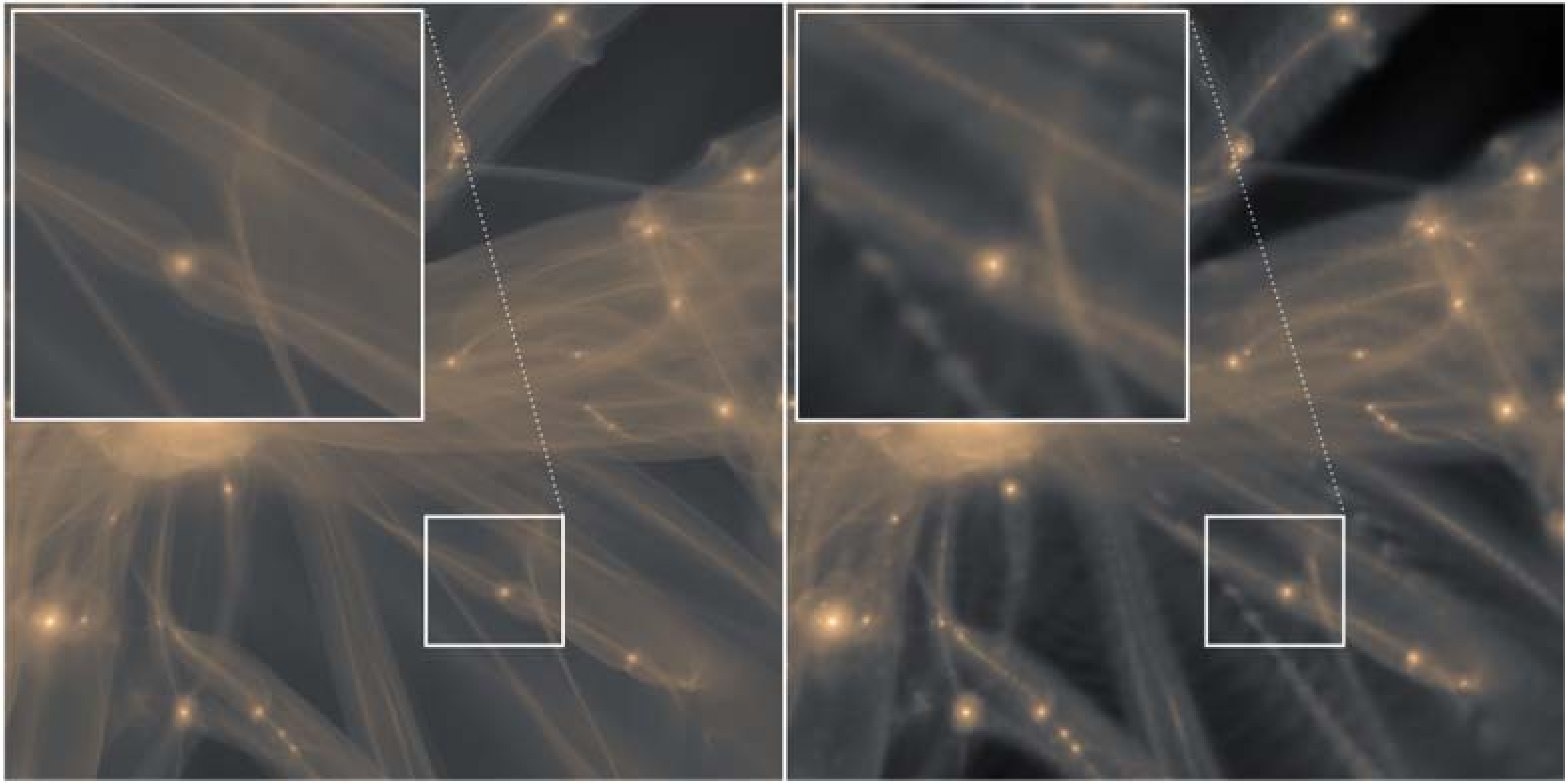
Kaehler etal 2012
(simulation of HDM)

Projection of Phase Space Sheet



Kaehler etal 2012
(simulation of HDM)

Projection of PSS vs. Smooth Particle Hydro (SPH)



Kaehler etal 2012 (simulation of HDM)

Tessellating Phase-Space Sheet (PSS)

(Shandarin, Habib, Heitmann-2011,2012; Abel, Hahn, Koehler-2011-2012)

Dynamics of the particles is same as in standard N-body code.

At initial stage 3D PSS is tessellated by 3D simplices in 6D phase space

Particles are considered as massless tracers of flow.

Mass is uniformly distributed within each tetrahedron.

Established connectivity remains in tact throughout the evolution

The set of simplices maintains continuity of 3D manifold

This is neither Delaunay nor Voronoi tessellation

Tessellating Phase-Space Sheet (PSS)

(Shandarin, Habib, Heitmann-2011,2012; Abel, Hahn, Koehler-2011-2012)

All information is stored in tessellation

It can be projected on a meshes in configuration space that have different resolutions (LOW or HIGH)

Theoretical Guidelines from Zel’dovich approximation (1970)

x-q space is equivalent to phase space x-v

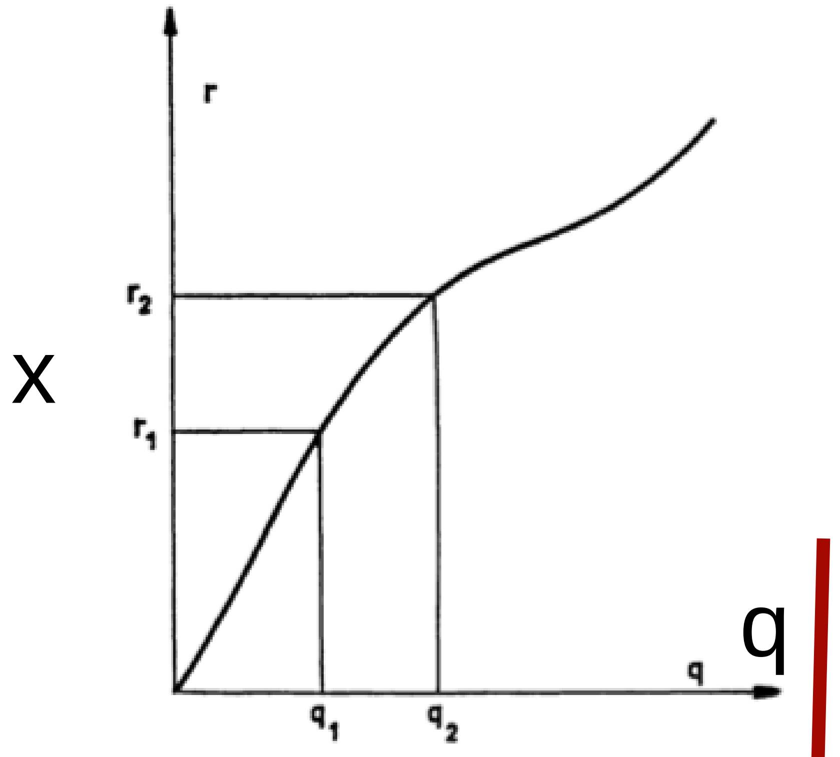
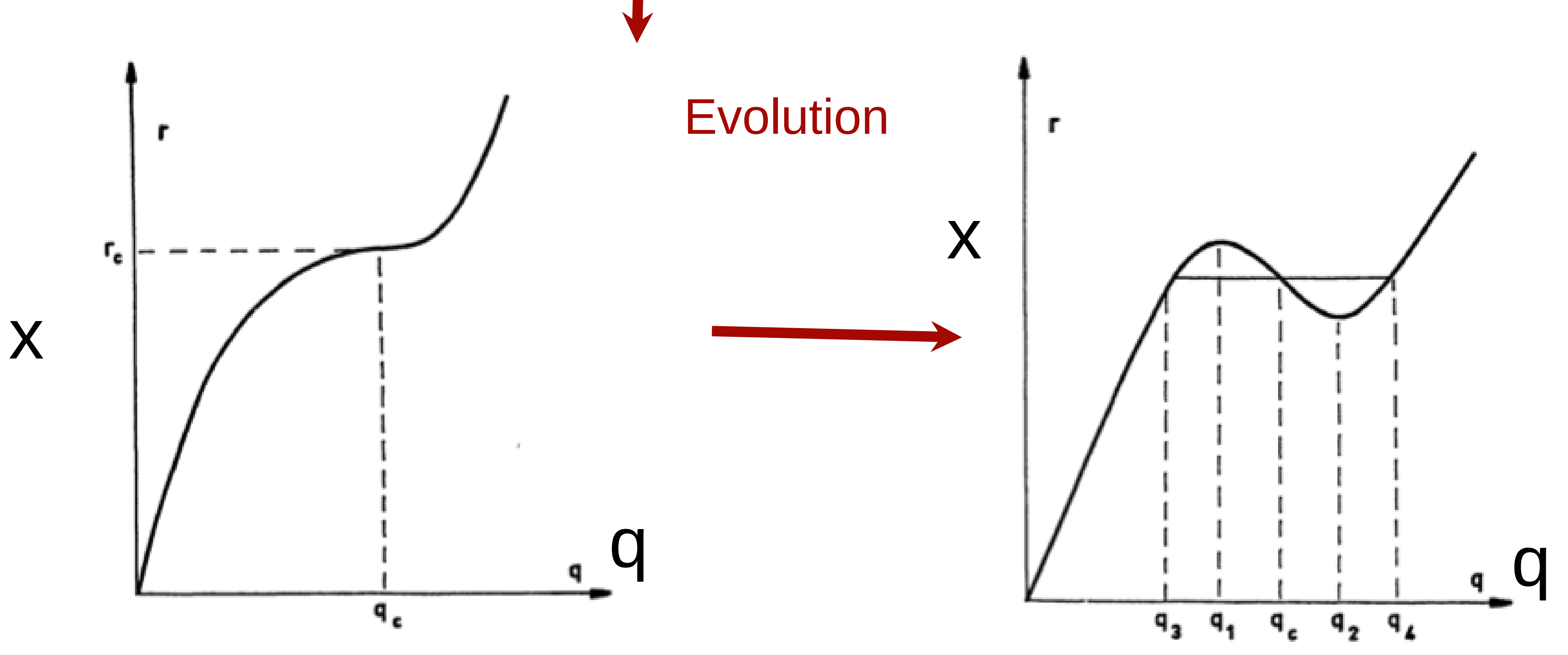
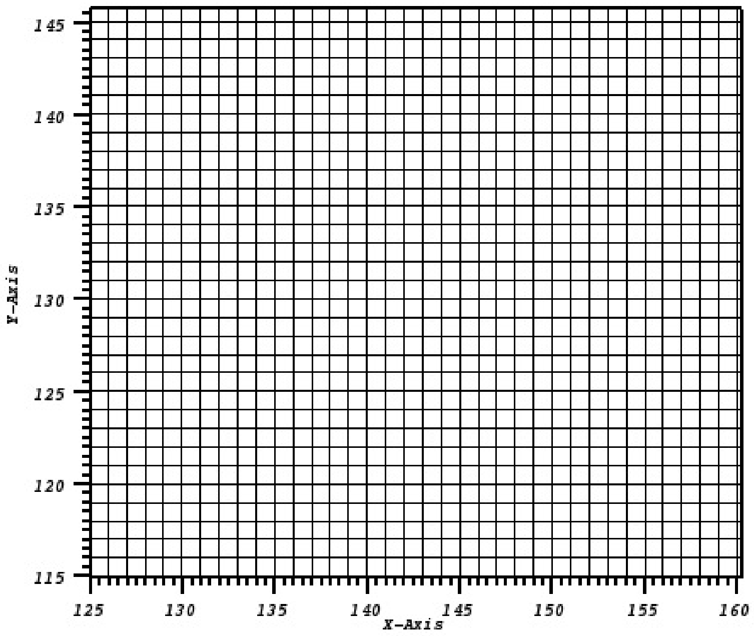
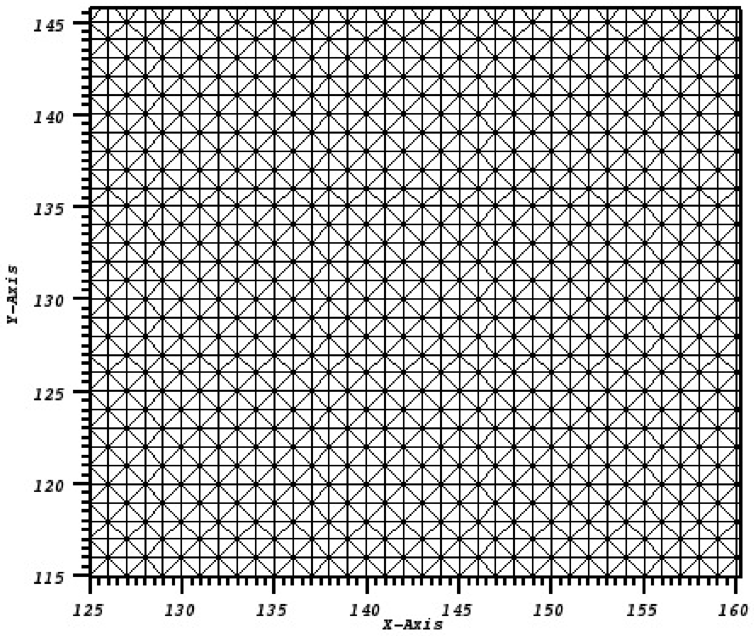


Fig. 1



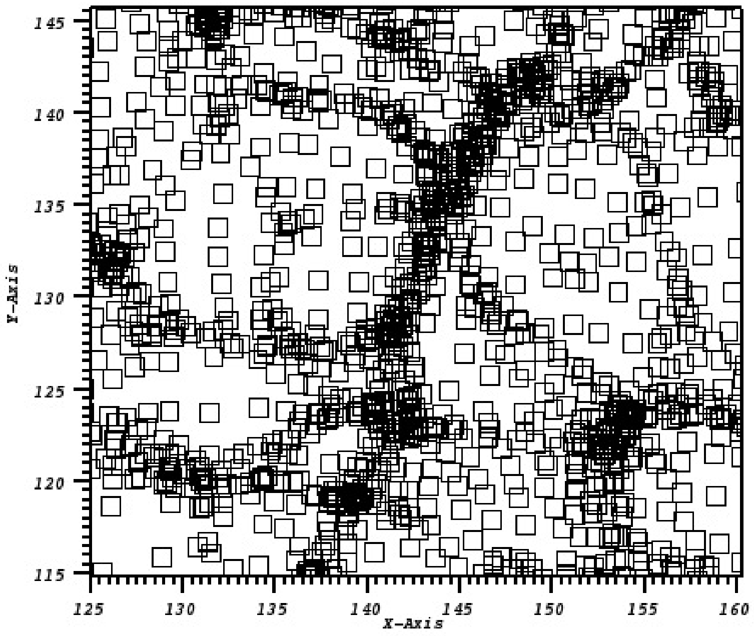
Tessellation VS. Particles (2D)

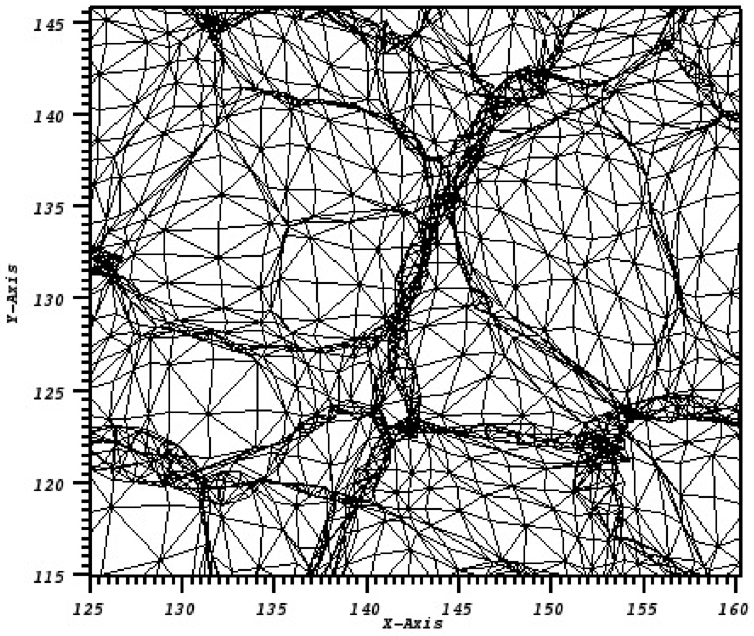




Each square voxel of Lagrangian mesh is decomposed in two triangles

den =1/area(trngl)





If triangles overlap, then the sum of den in all overlapping triangles

3D: Decomposition of a cube into 5 tetrahedra

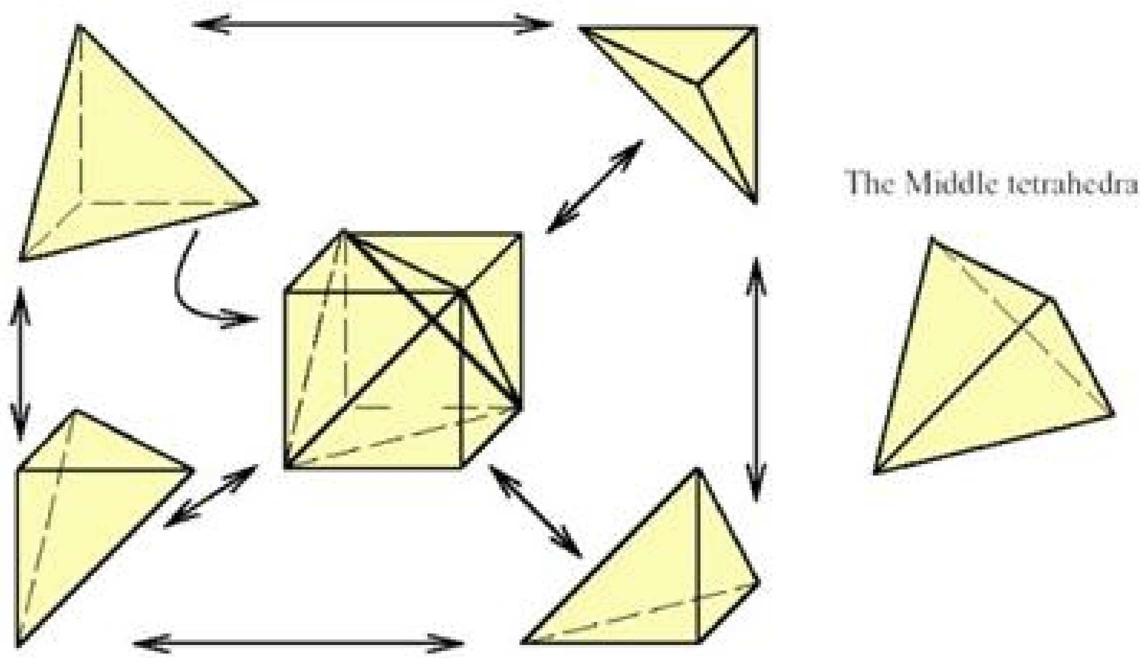
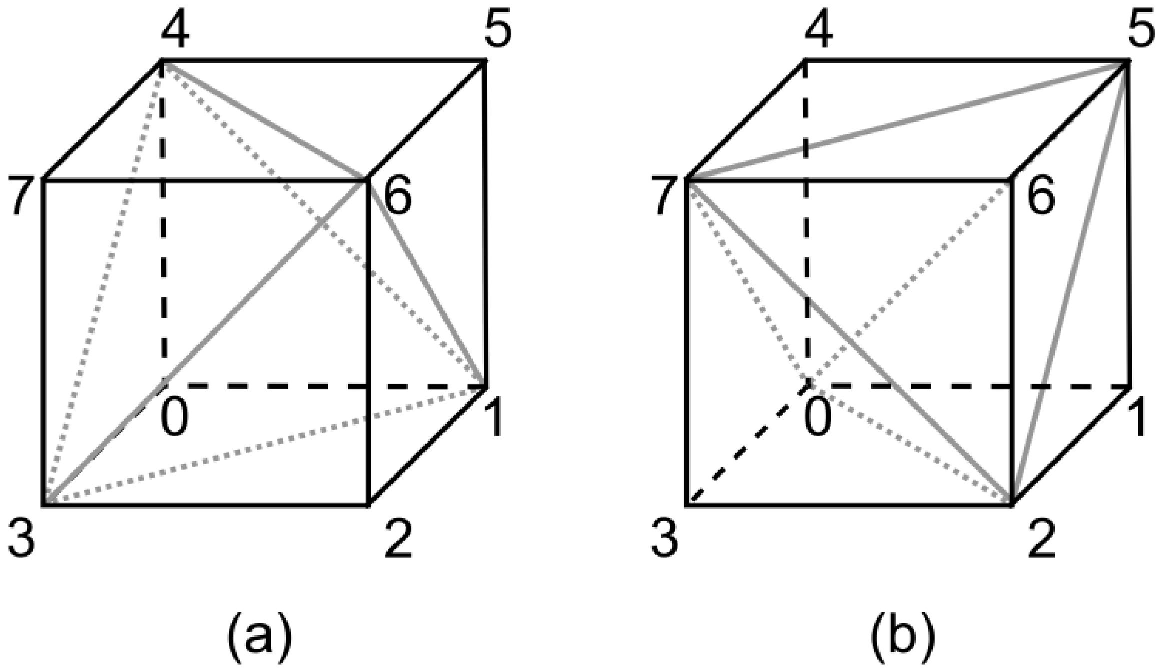


Figure 1.9: The Tetrahedra orientation within a cube



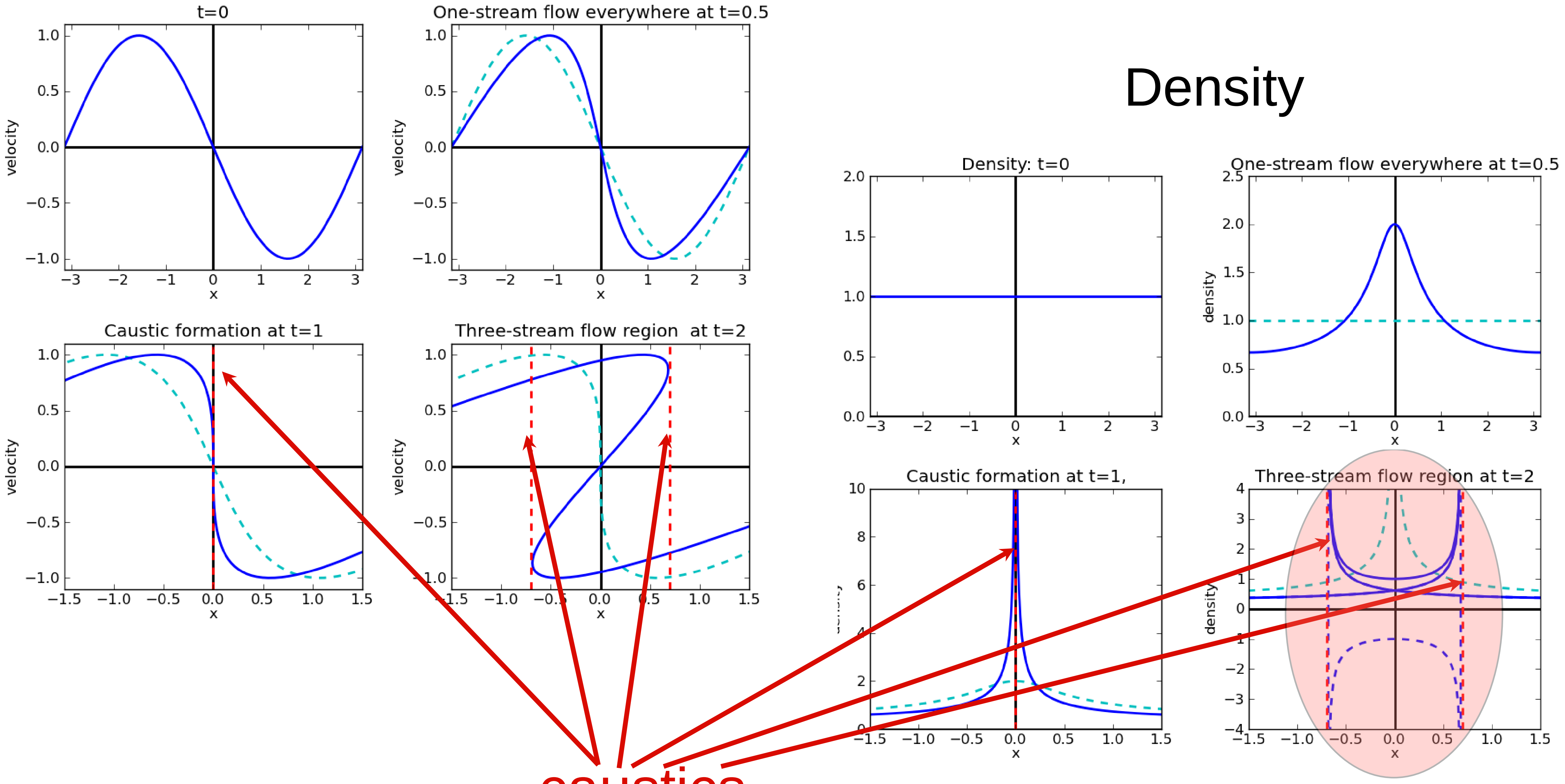
(a) (b)

Shandarin,Habib,Heitmann 2012, Phys.Rev.D

Multi-stream flows and caustics in collisionless Dark Matter (one dimensional example)

Phase space

Density

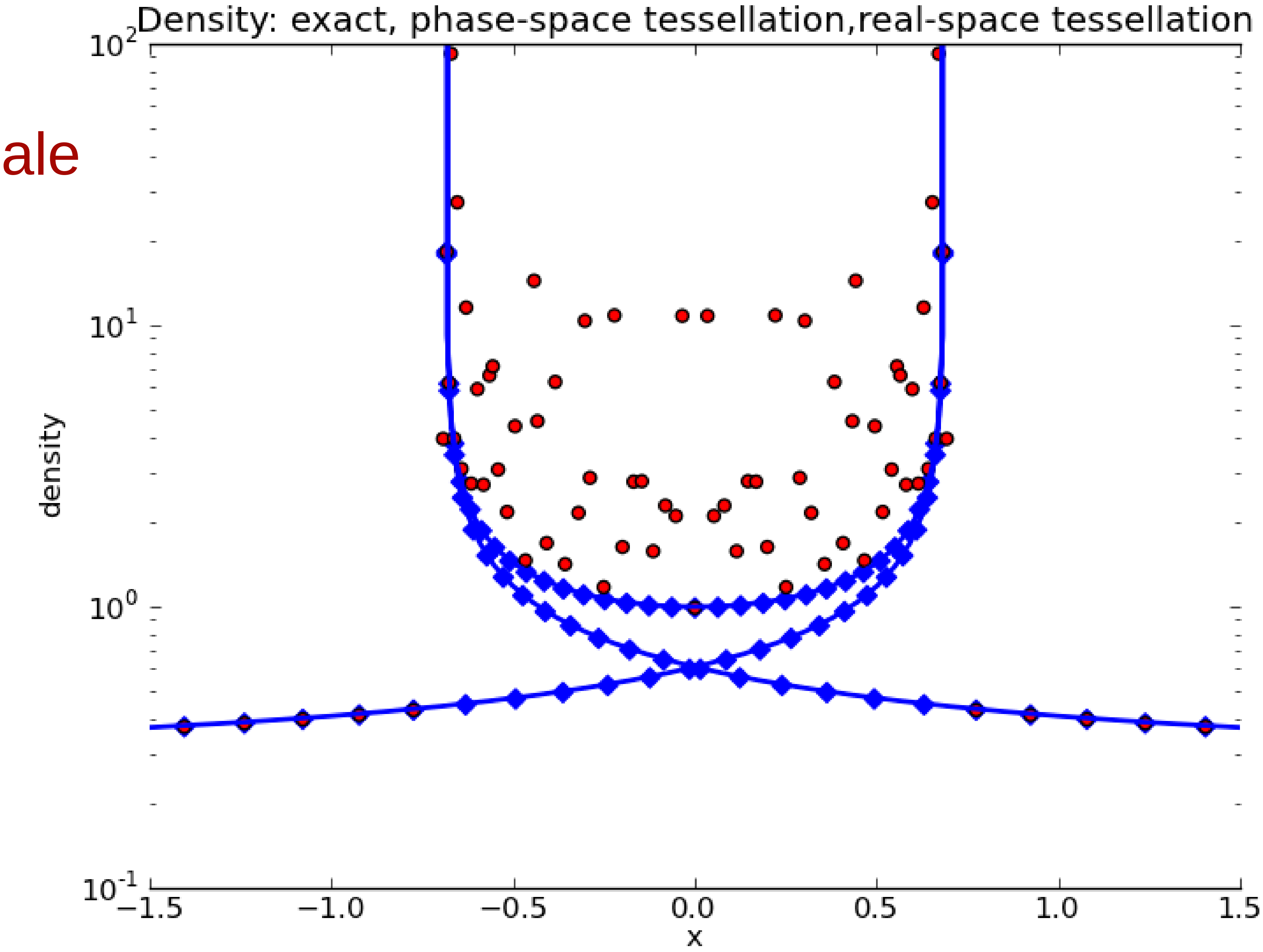


caustics

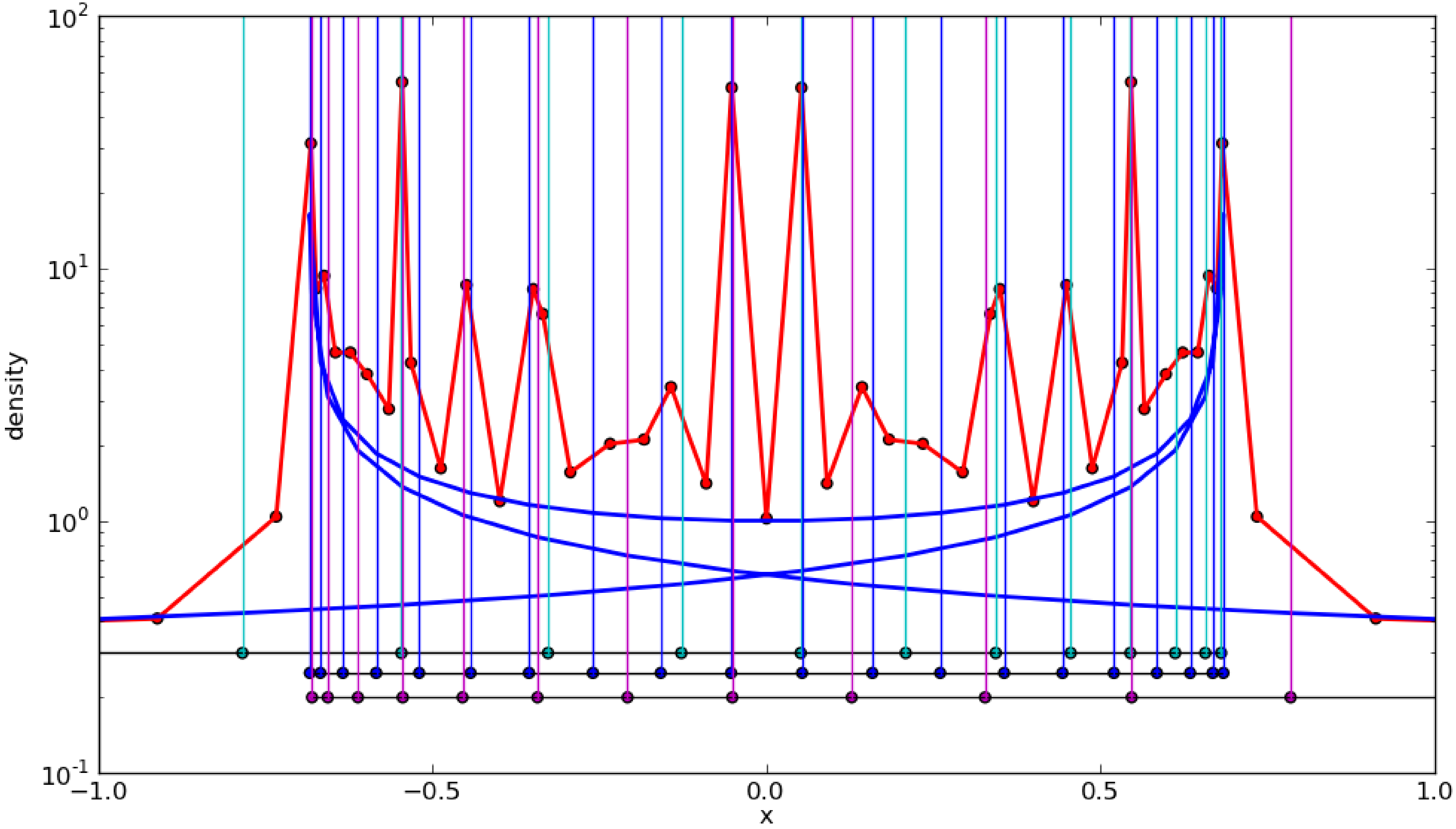
Evaluation of Density: den = 1/(x[i+1] - x[i])

(red dots: configuration space tessellation (1D Delaunay))

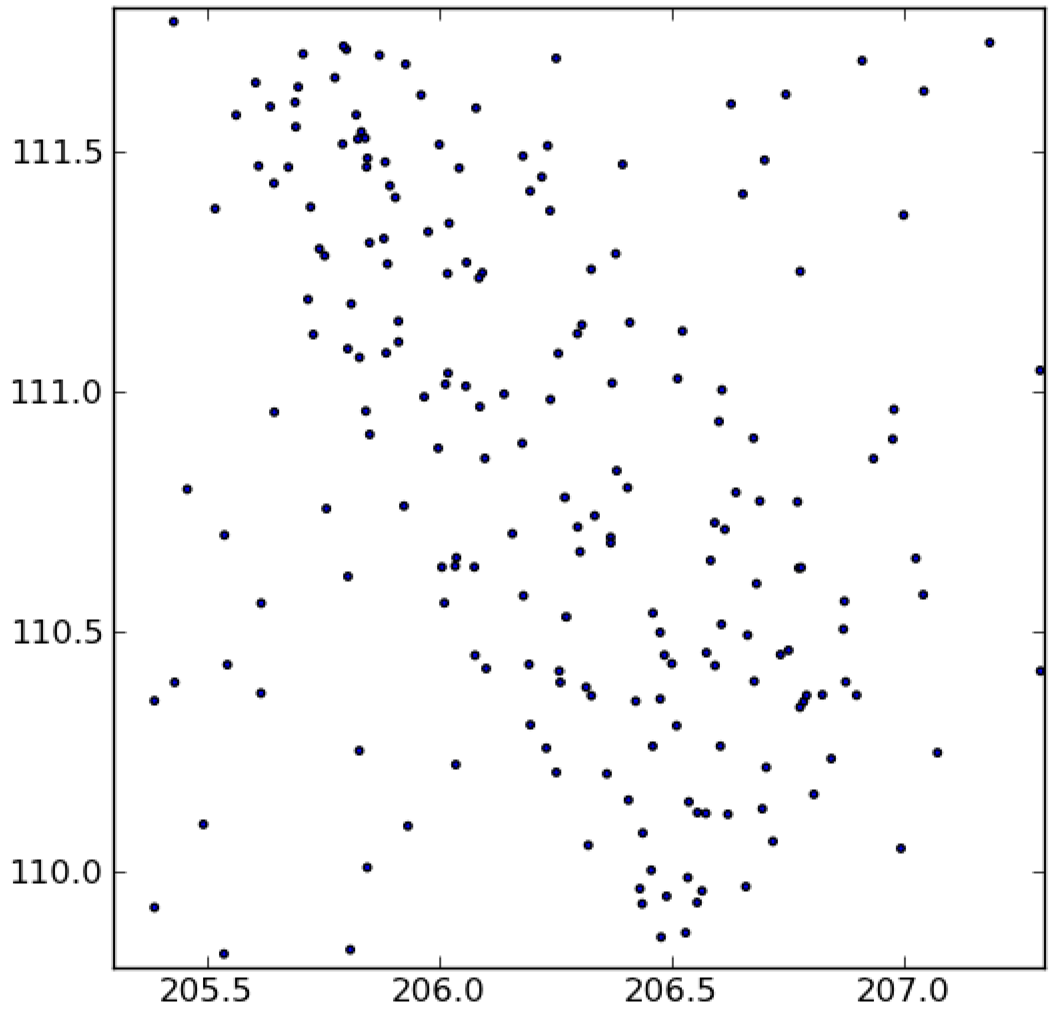
log scale

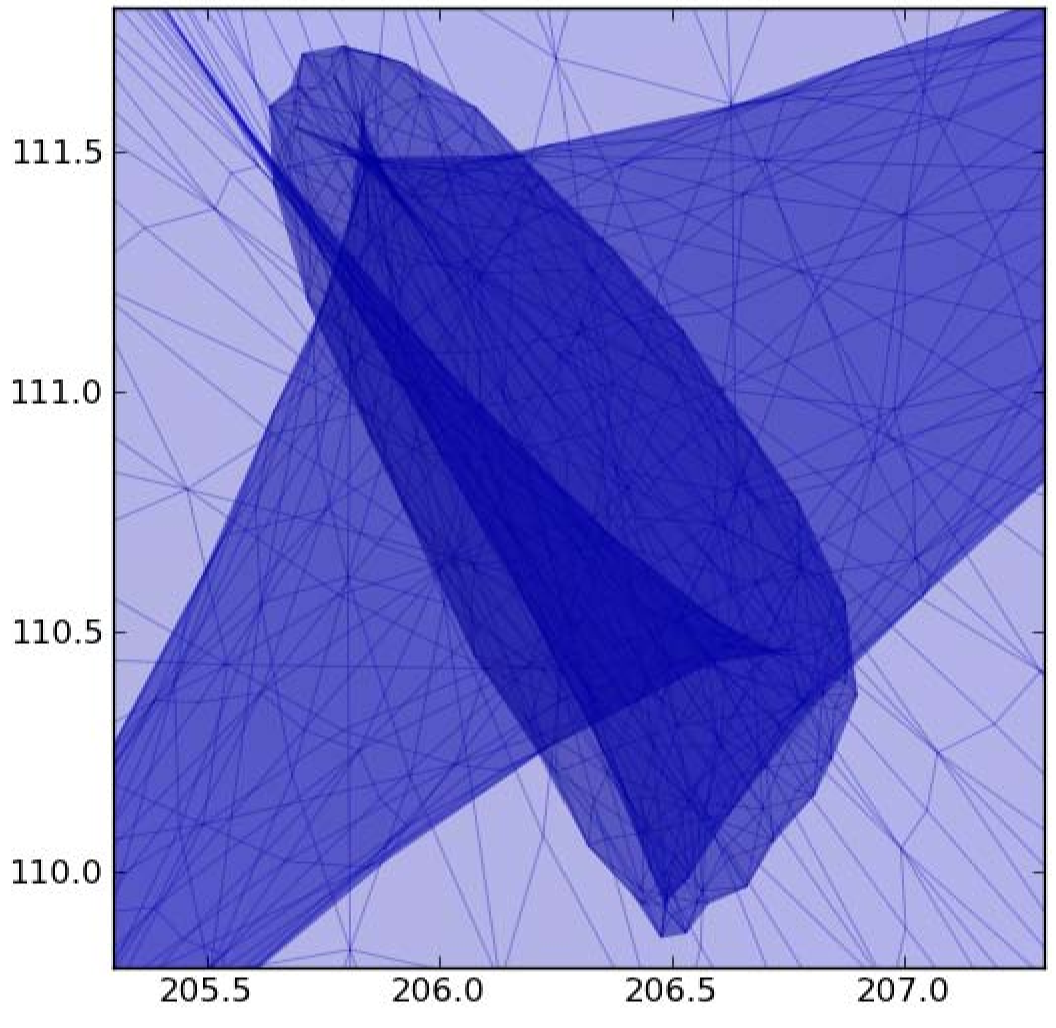


The Evaluation of Density: density = 1/(x[i+1] - x[i])



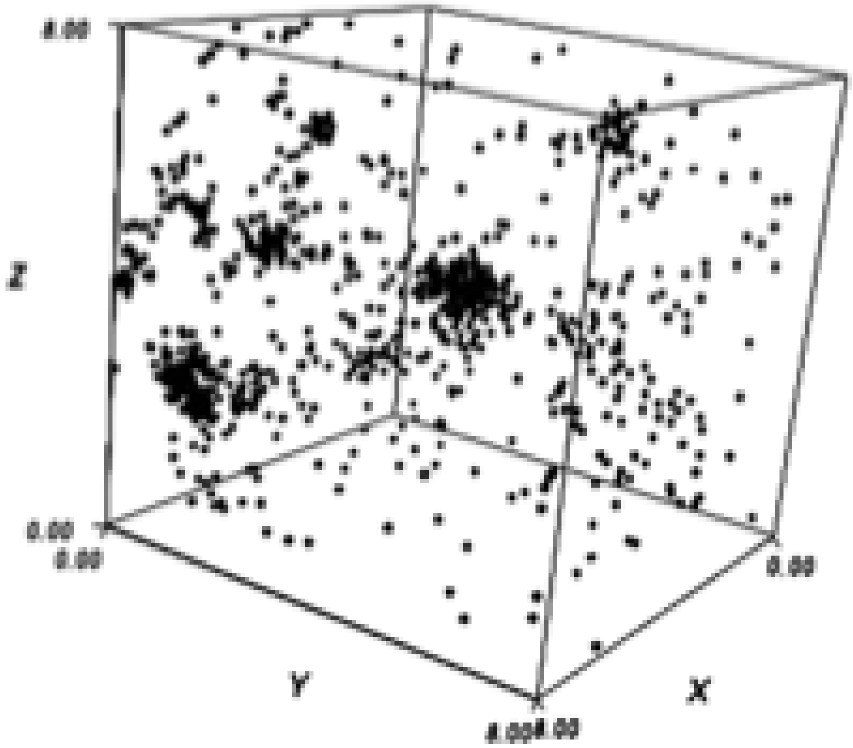
Particles VS. Tessellation (2D example)





Structure: particle representation

small box
8/h Mpc



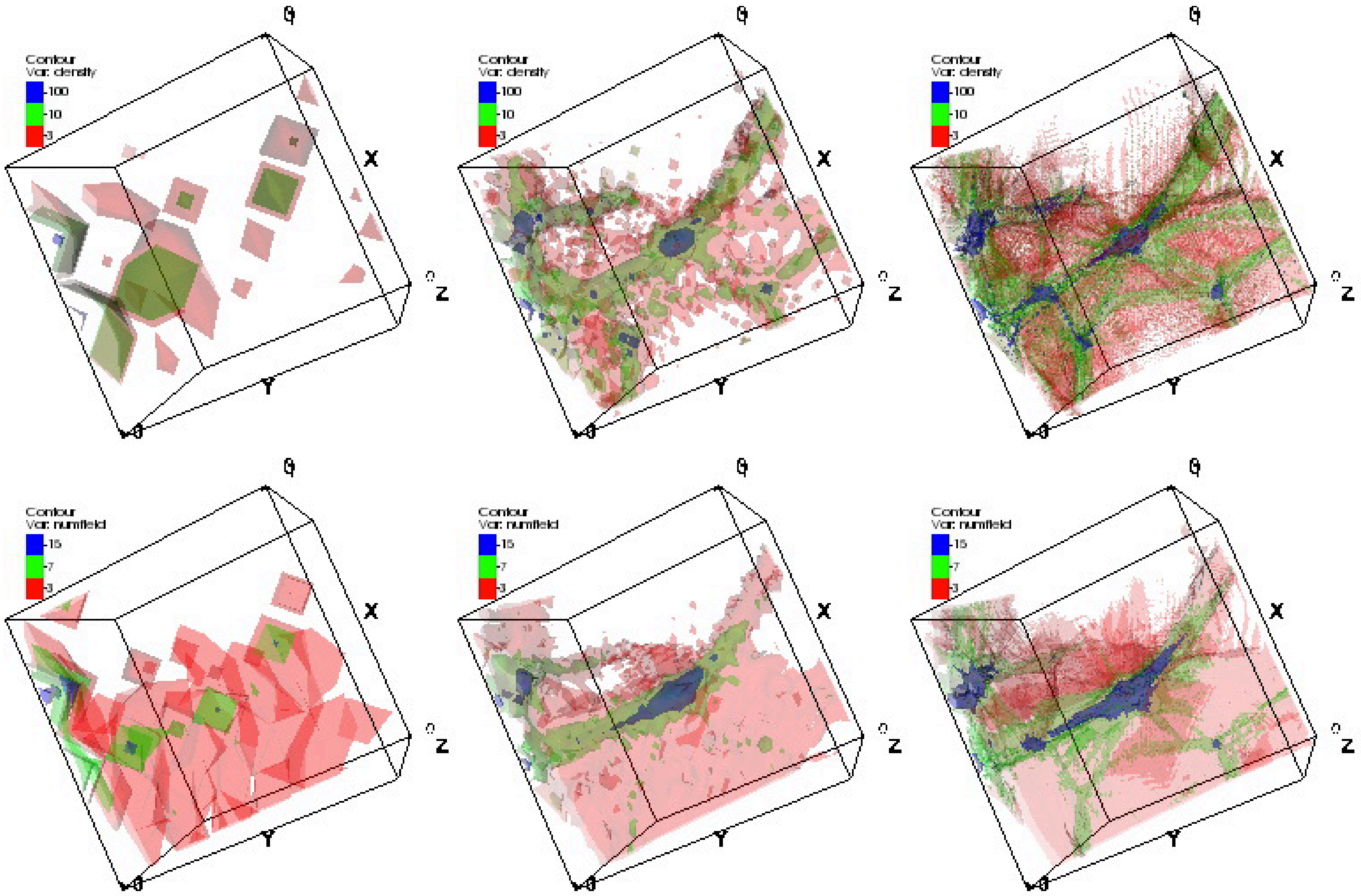
N-BODY SIMULATION (PM) - ‘Standard’ LCDM model:
h = 0.72, Omega_t = 0.25, Omega_b=0.043, n=0.97, sig_8=0.8

Full box: 512/h Mpc, N_p=512^3, Force solver 1024^3

Rendering density and multi-stream fields with increasing resolution

DENSITY FIELD

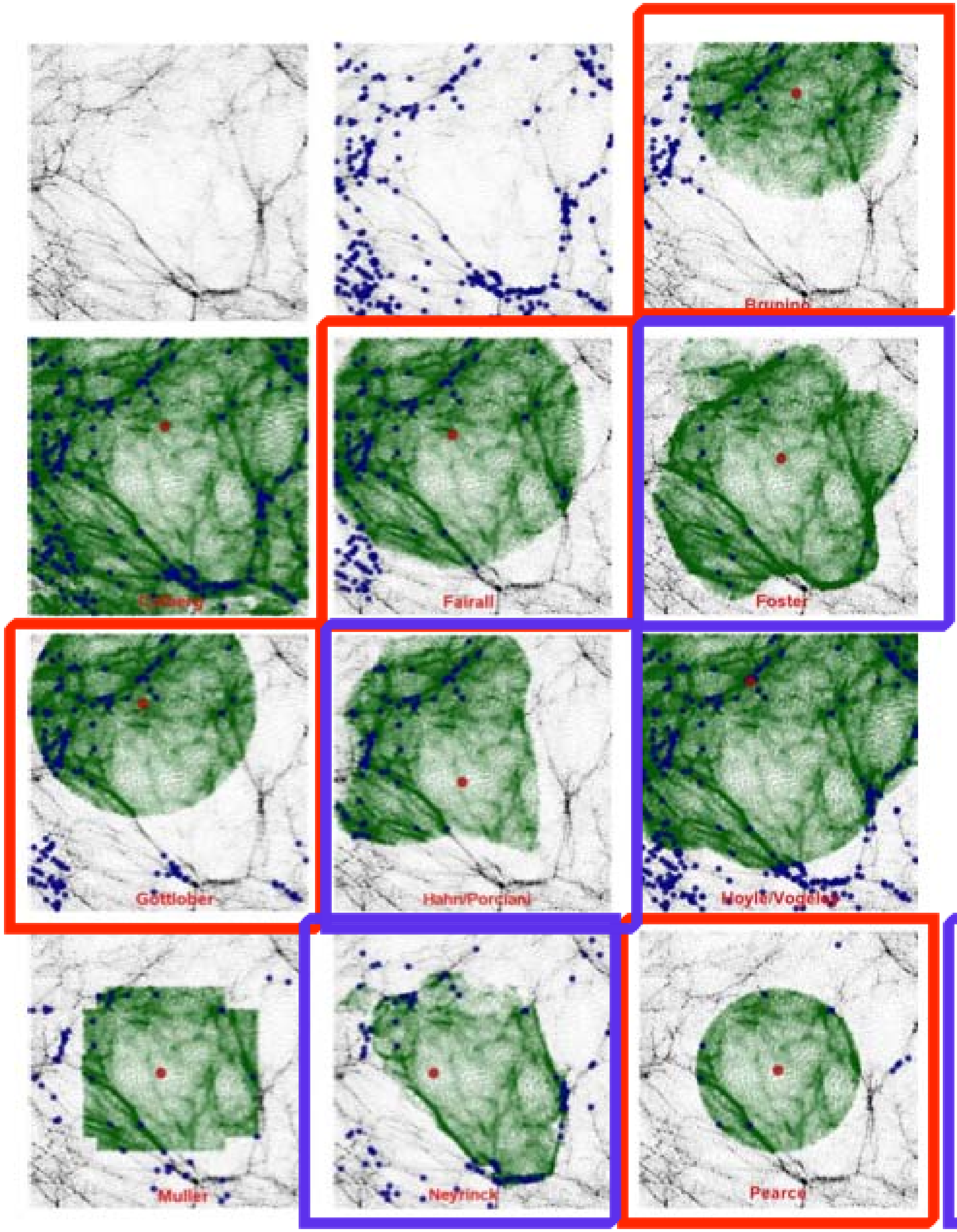
MULTI-STREAM FIELD

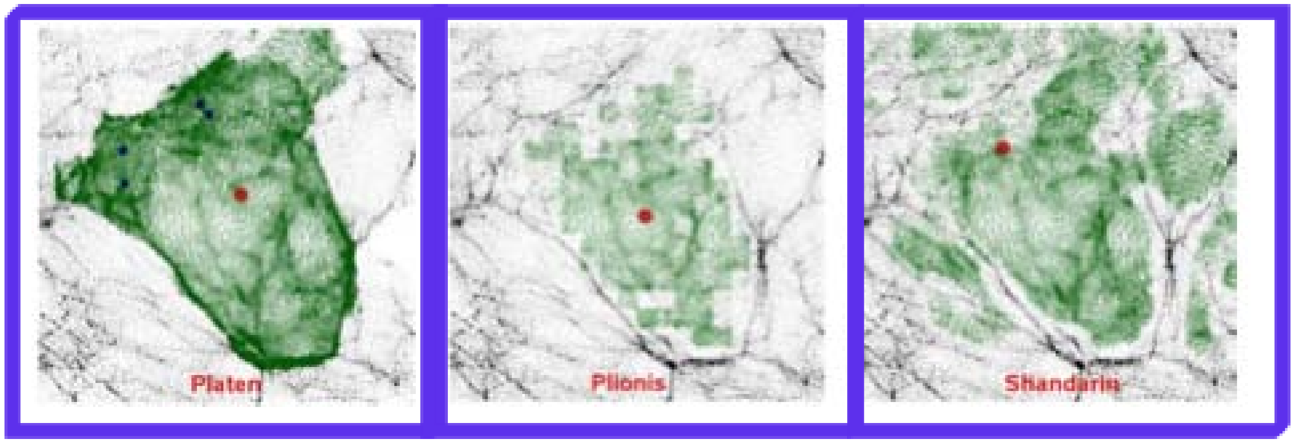


1/h Mpc, 0.25/h Mpc, mesh resolution 0.062/h Mpc

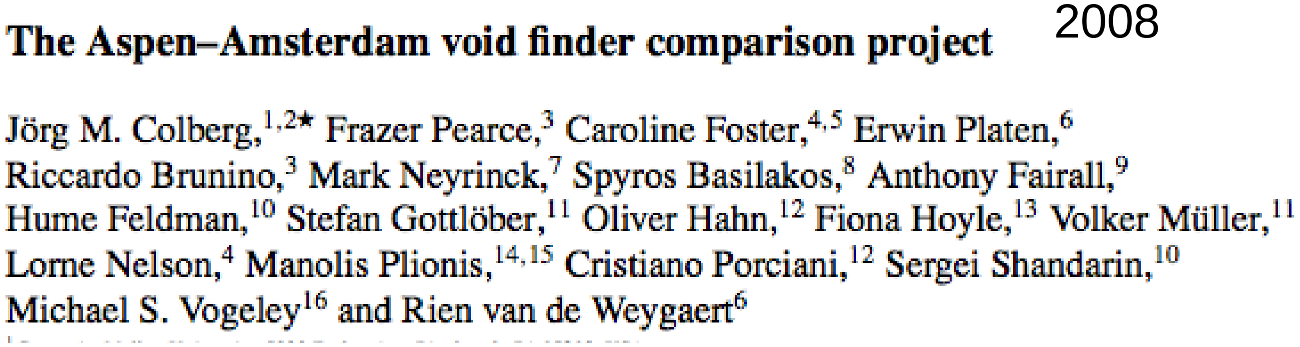
SHH -11,12

Voids





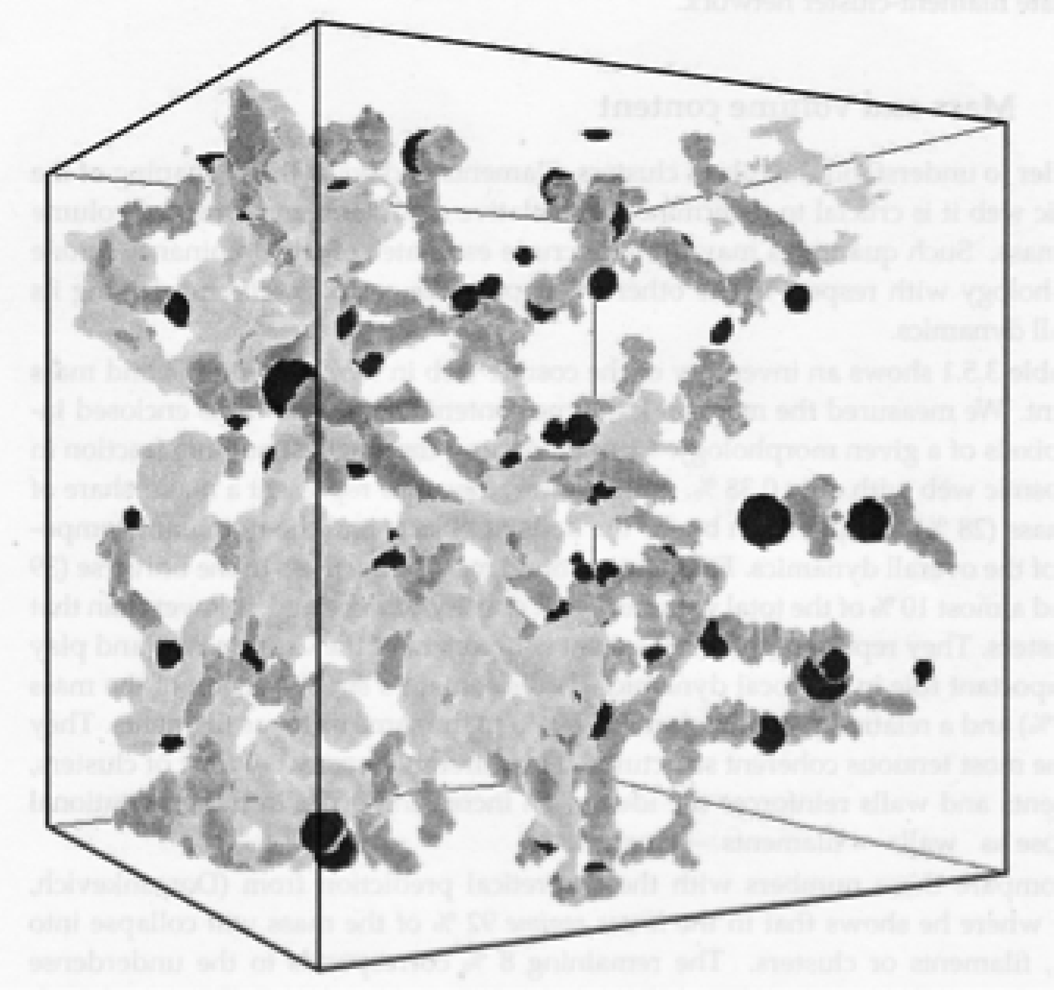
2008



The Aspen–Amsterdam void finder comparison project

Jörg M. Colberg,[1,2★] Frazer Pearce,[3] Caroline Foster,[4,5] Erwin Platen,[6] Riccardo Brunino,[3] Mark Neyrinck,[7] Spyros Basilakos,[8] Anthony Fairall,[9] Hume Feldman,[10] Stefan Gottlöber,[11] Oliver Hahn,[12] Fiona Hoyle,[13] Volker Müller,[11] Lorne Nelson,[4] Manolis Plionis,[14,15] Cristiano Porciani,[12] Sergei Shandarin,[10] Michael S. Vogeley[16] and Rien van de Weygaert[6]

Mass, volume and density of cosmic web



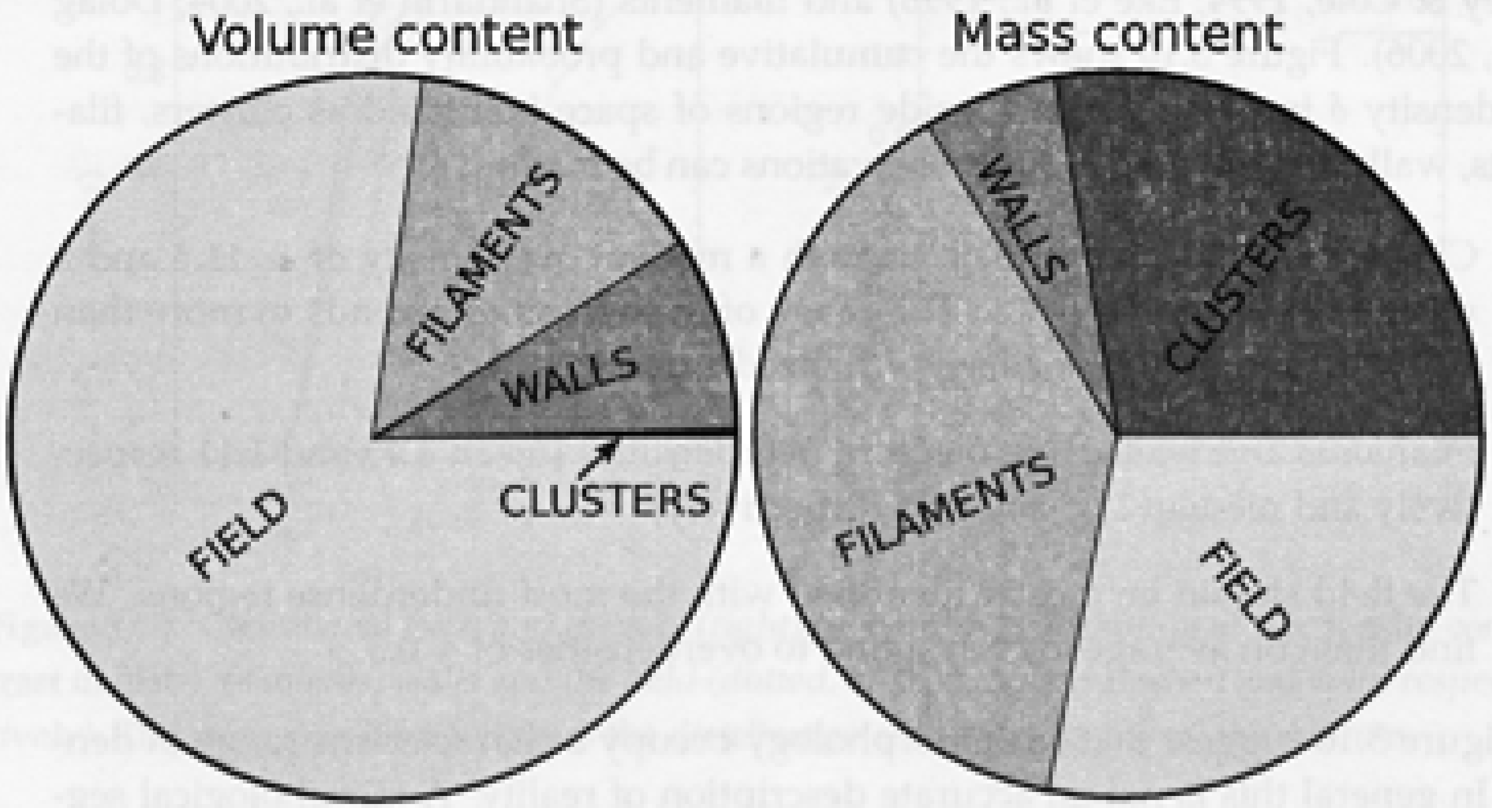
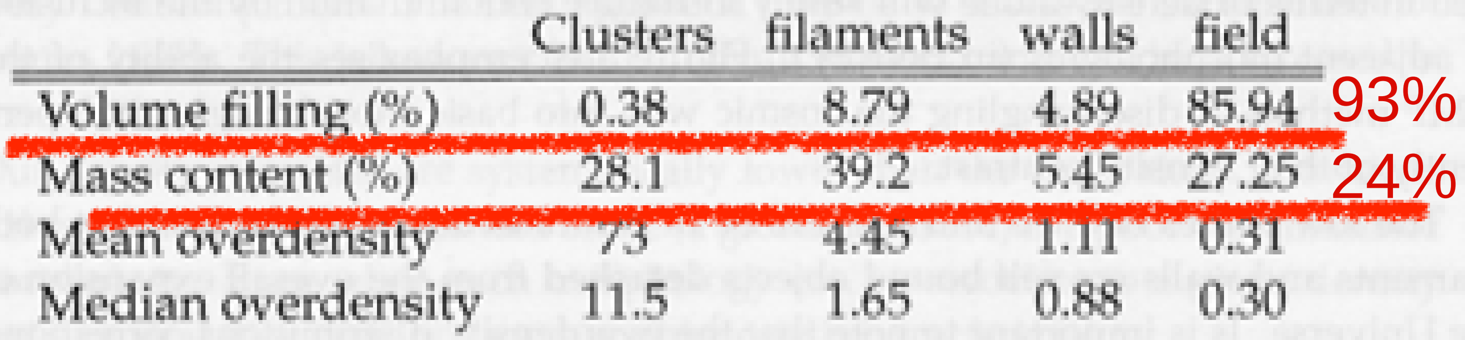


Figure 3.9: *Pie diagram showing an inventory of the Cosmic Web in terms of volume (left) and mass (right).*



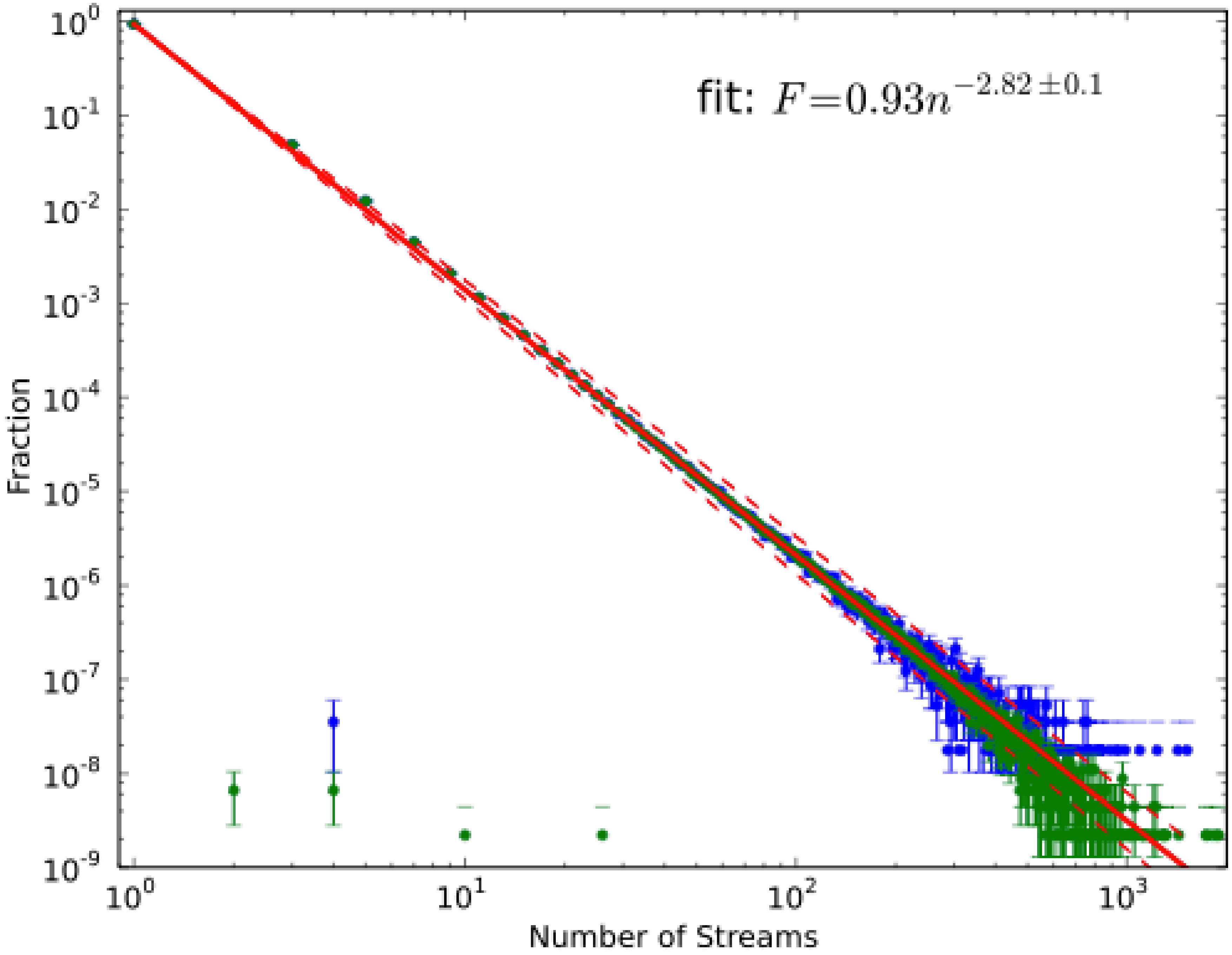
	Clusters	filaments	walls	field
Volume filling (%)	0.38	8.79	4.89	85.94
Mass content (%)	28.1	39.2	5.45	27.25
Mean overdensity	73	4.45	1.11	0.31
Median overdensity	11.5	1.65	0.88	0.30

93%

24%

Aragon-Calvo, van de Weygaert, Jones 2010

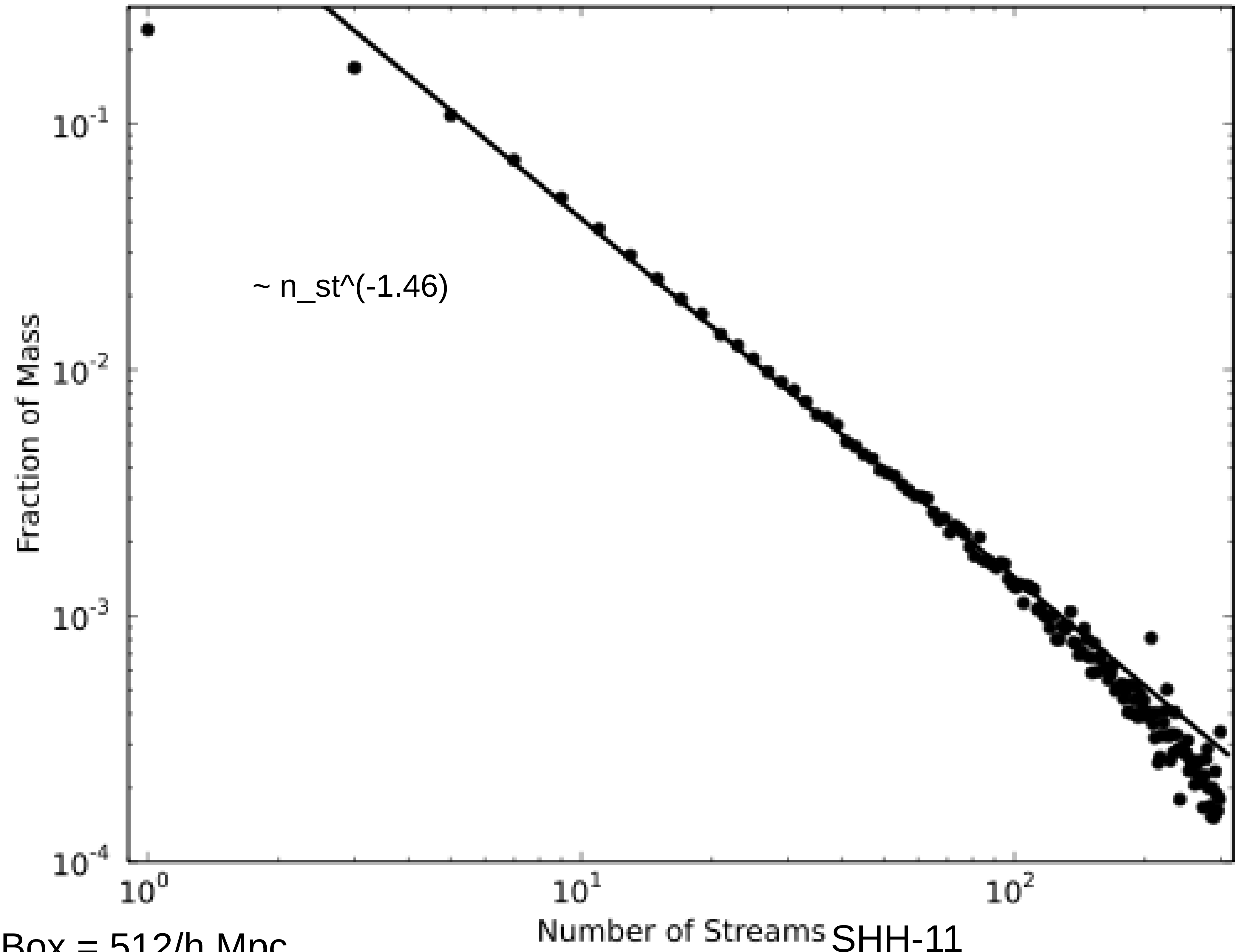
Volume fractions of multi-stream field



Box = 512/h Mpc,
N_p=512^3

SHH-11

Mass fraction of N_streams



~ n_st^(-1.46)

SHH-11

Box = 512/h Mpc,
N_p=512^3

Total volume in voids

		SHH-12	AWJ-10	FHGKY-09	HPCD-07
1 stream	Void(s)	~93% ;	~86%;	~13 - 82%	~17%;

HPCD-07 = Hahn, Porciani, Carollo, Dekel 2007

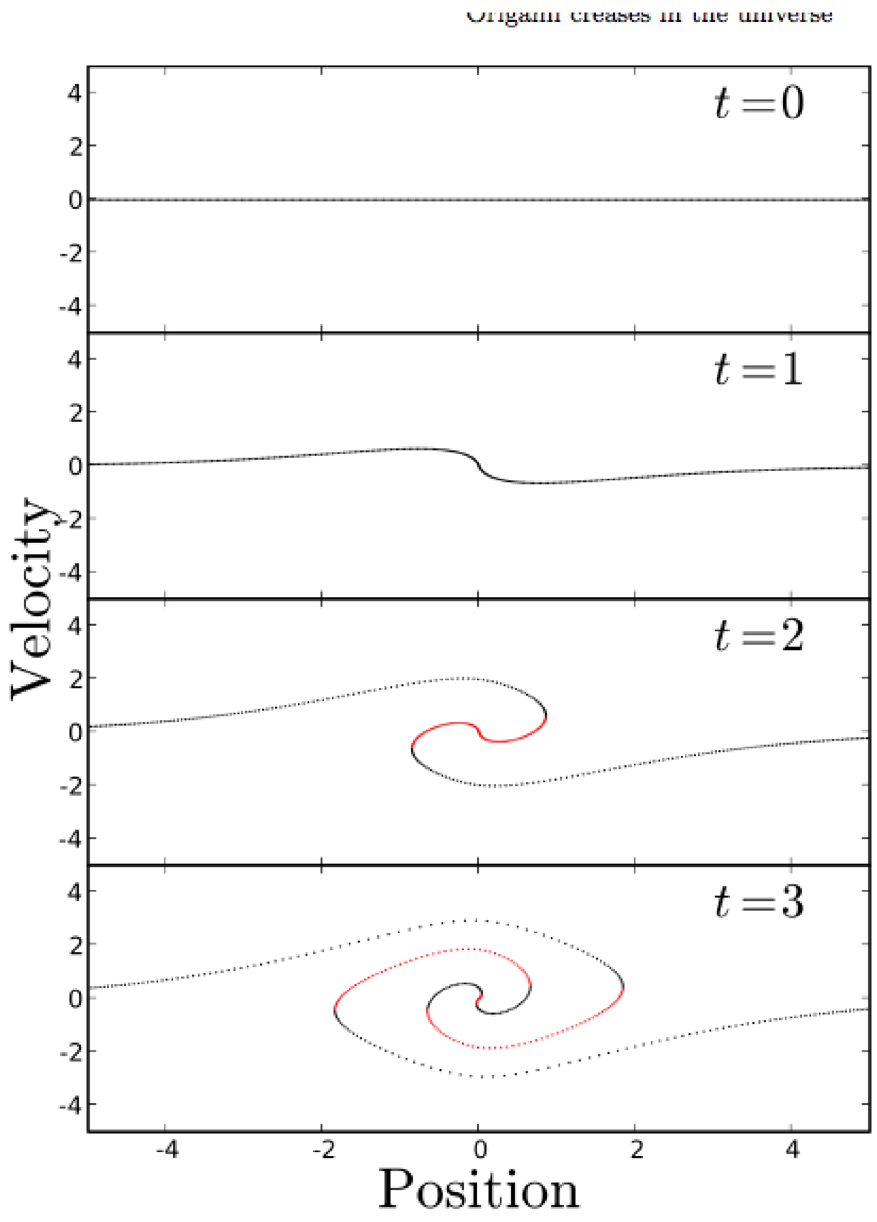
FHGKY-09 = Forero-Romero, Hoffman, Gottlober, Klypin, Yepes 2009

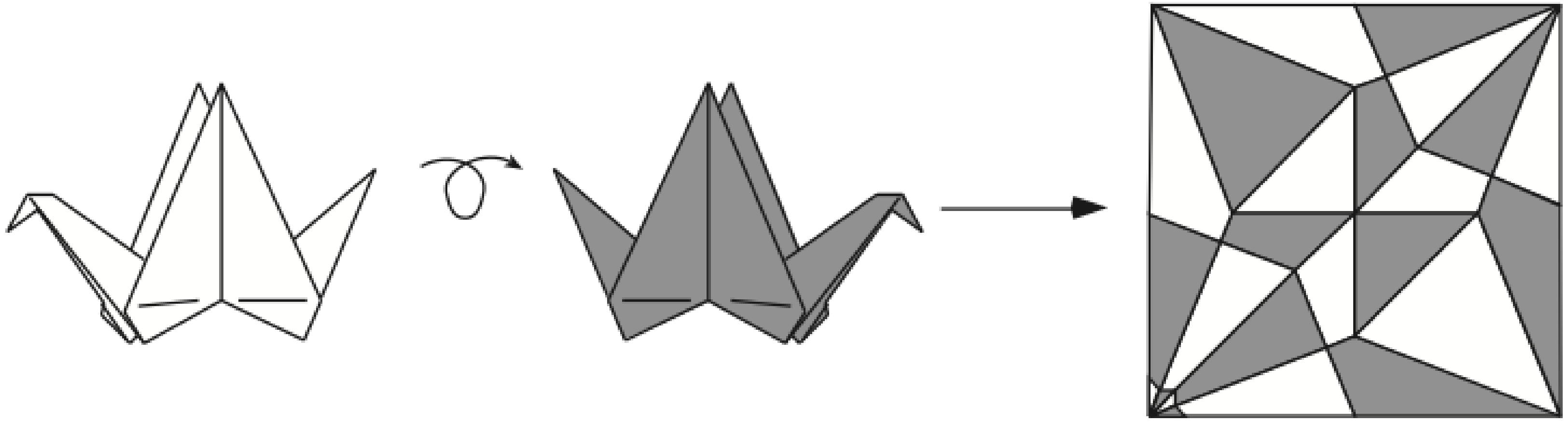
AWJ-10 = Aragón-Calvo, van de Weygaert, Jones, 2010

SHH-12 =Shandarin, Habib, Heitmann, 2012

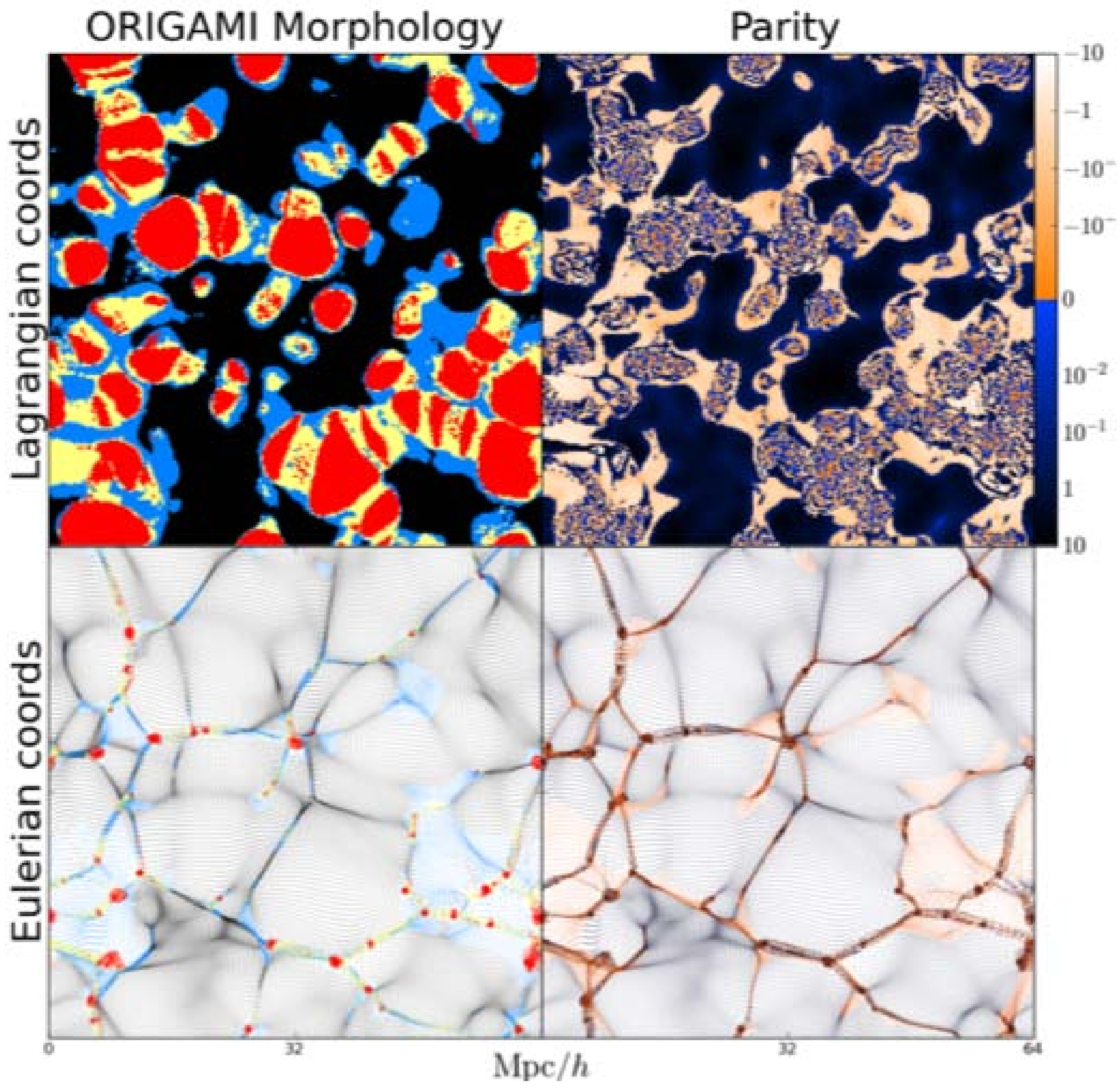
Physical Voids = One Stream Flow Regions

Parity = sign of nD volume of nD simplex



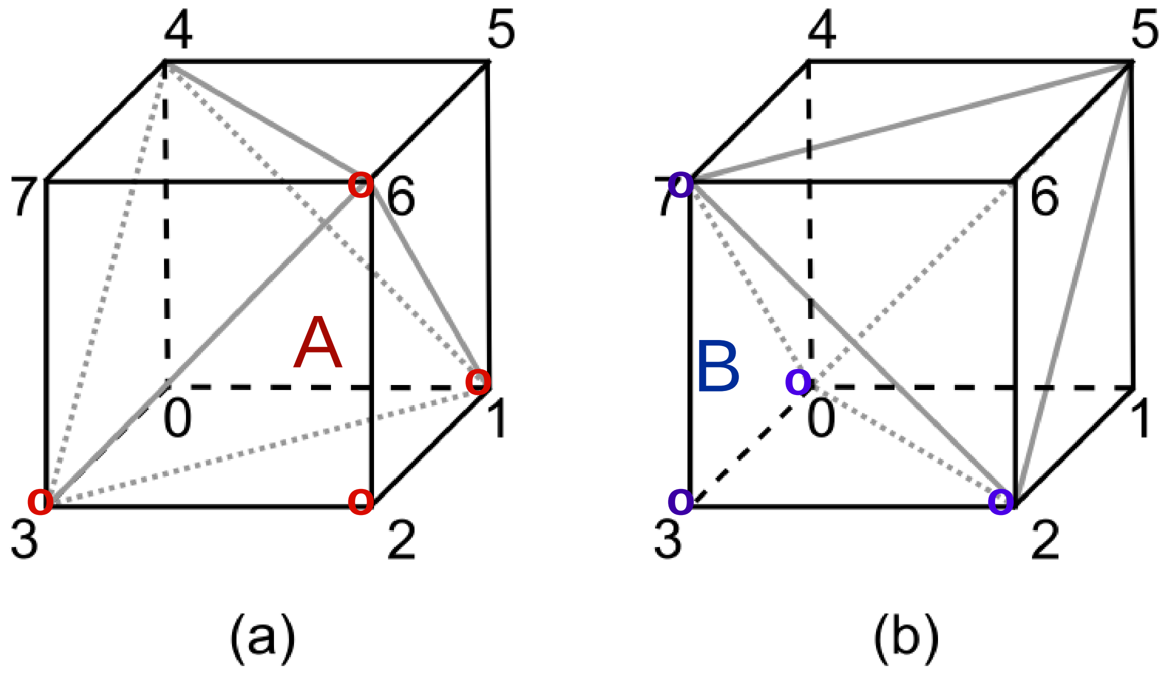


Origami



Neyrinck 12

Computing Caustics in N-body Simulations



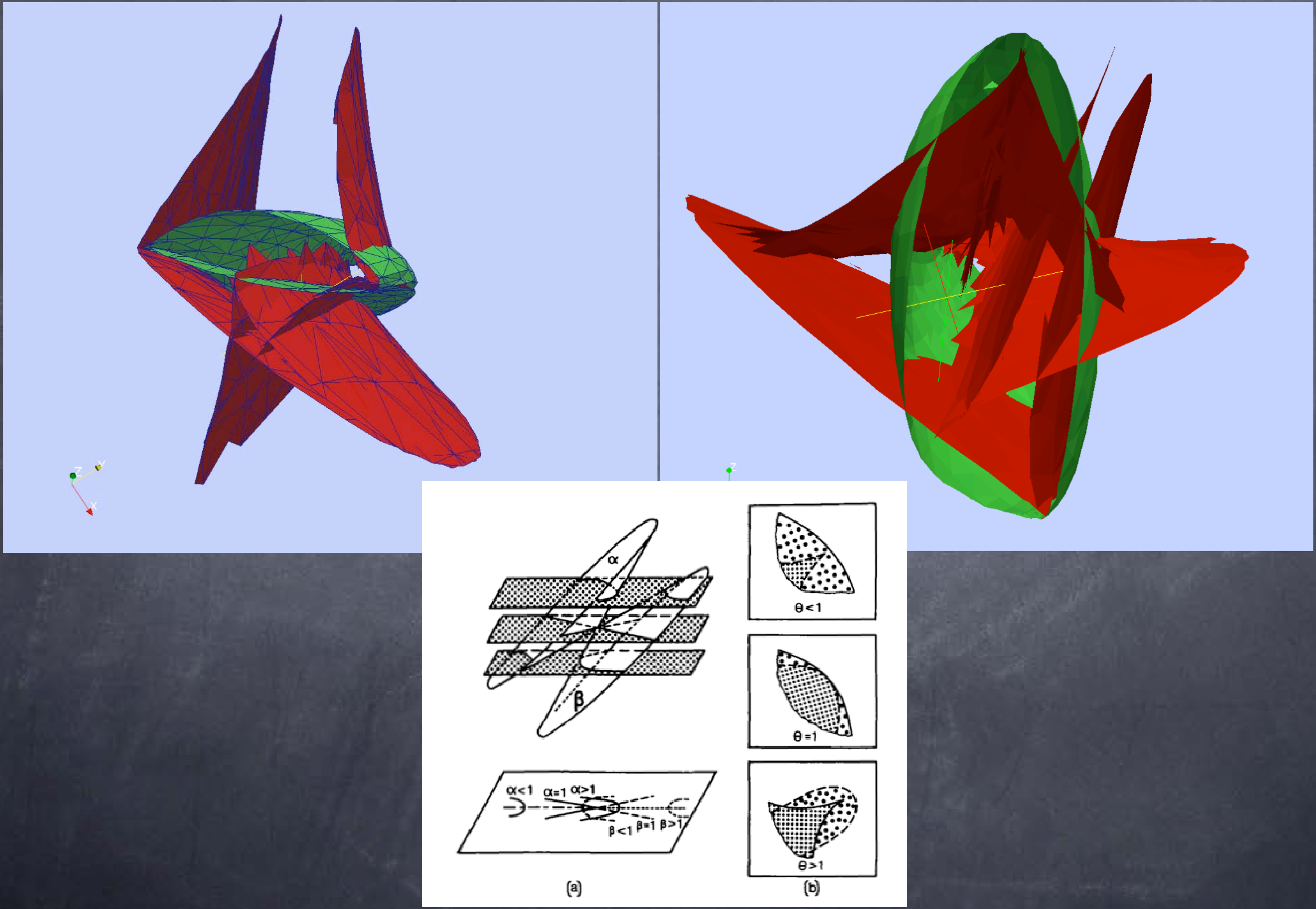
Neighboring tetrahedra A=[a1,a2,a3,a6] and B=[b0,b2,b3,b7]

share a common face [a1,a2,a6] = [b0,b3,b7].

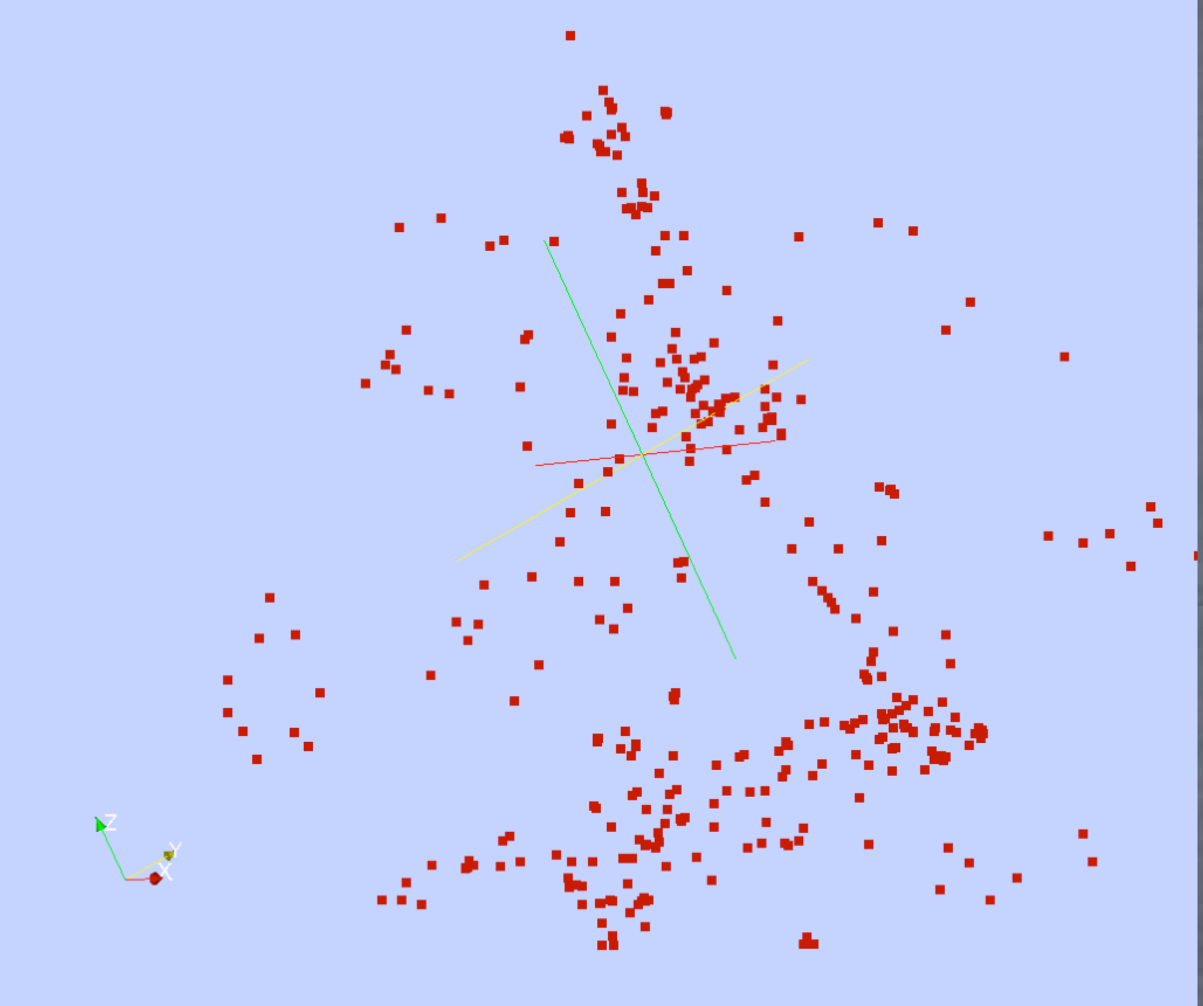
When the parity of A is opposite of the parity of B

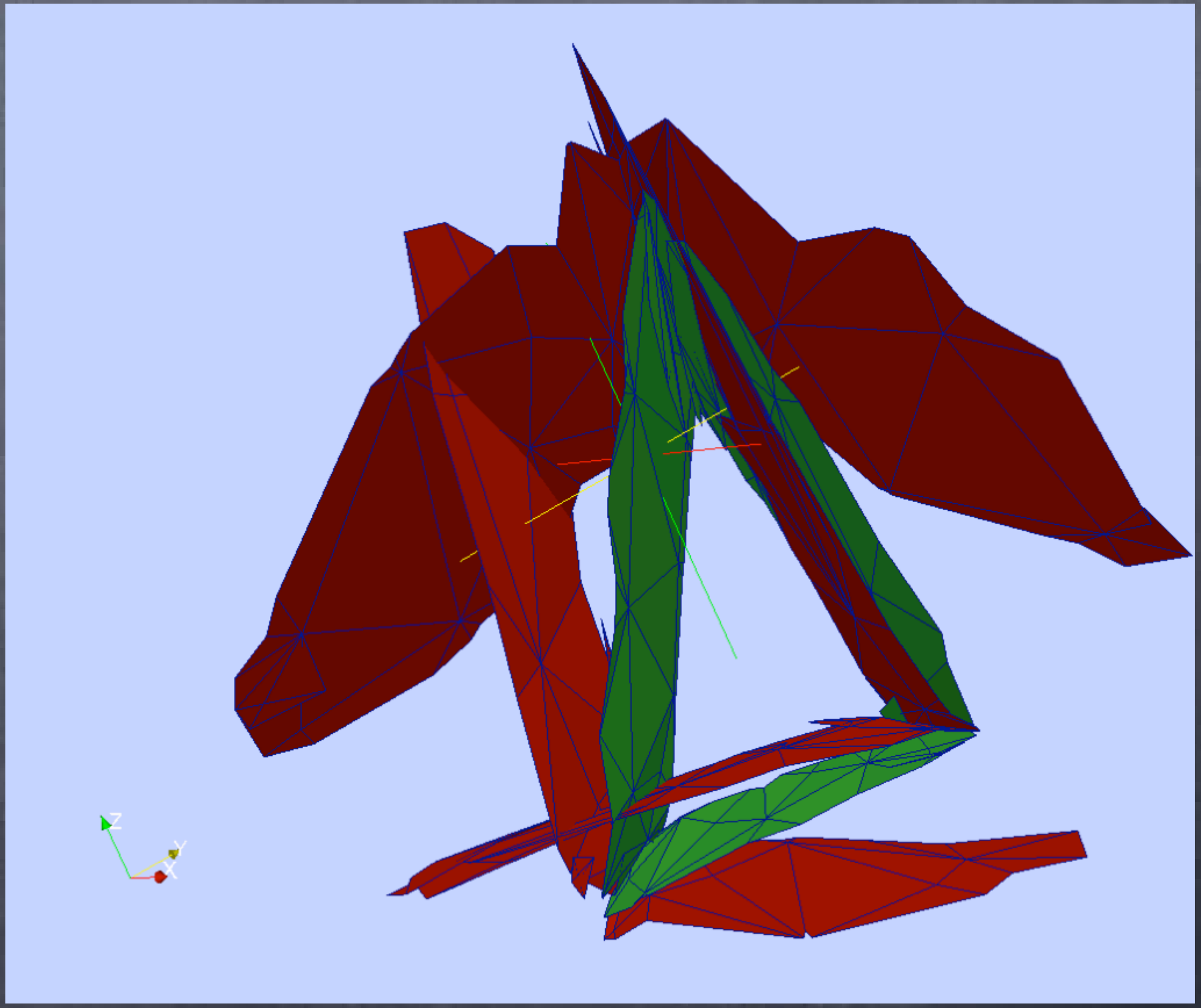
the common face is an element of a caustic surface

D_4 singularity (Arnold et al 1982)



Particles VS. Tessellation of PSS





Summary

- Tessellation of PSS allows to compute density and other fields in configuration, velocity and phase spaces with much greater spatial resolution than currently used methods including adaptive SPH.

- Currently only tessellation of PSS allows to compute caustics directly.

- Number of streams is well defined physical quantity reflecting dynamical stage of the evolution.
  Density thresholds (except virial threshold) has no physical significance.

- The PSS tessellation provides unique definition of “physical” voids: as the regions with one stream flow. Astronomical voids are regions devoid of galaxies brighter than some magnitude.

‘Voids’ devoid of halos with M >~ 3E11 M_ sun occupy ~93% of volume.

- Potential change of paradigm in N-B simulations:

currently standard: both mass and flow tracers are N-body particles;
new: flow tracers are tetrahedra vertices,
mass tracers are tetrahedra themselves (e.g. centroids, ...)