Tweaking general relativity: MOND relativistic gravity theory as a substitute for dark matter

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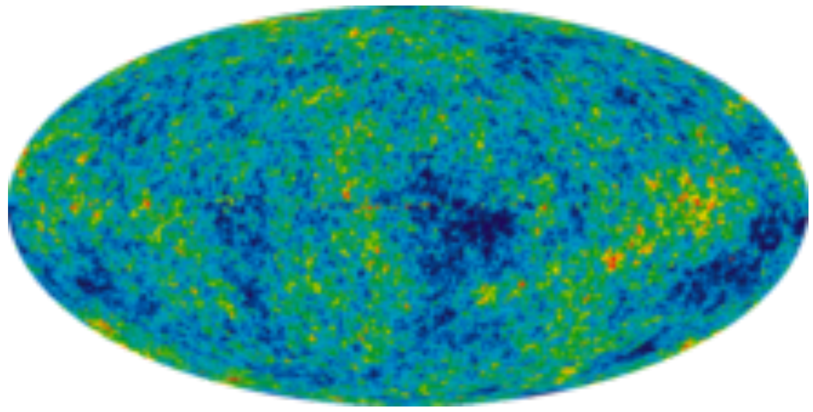
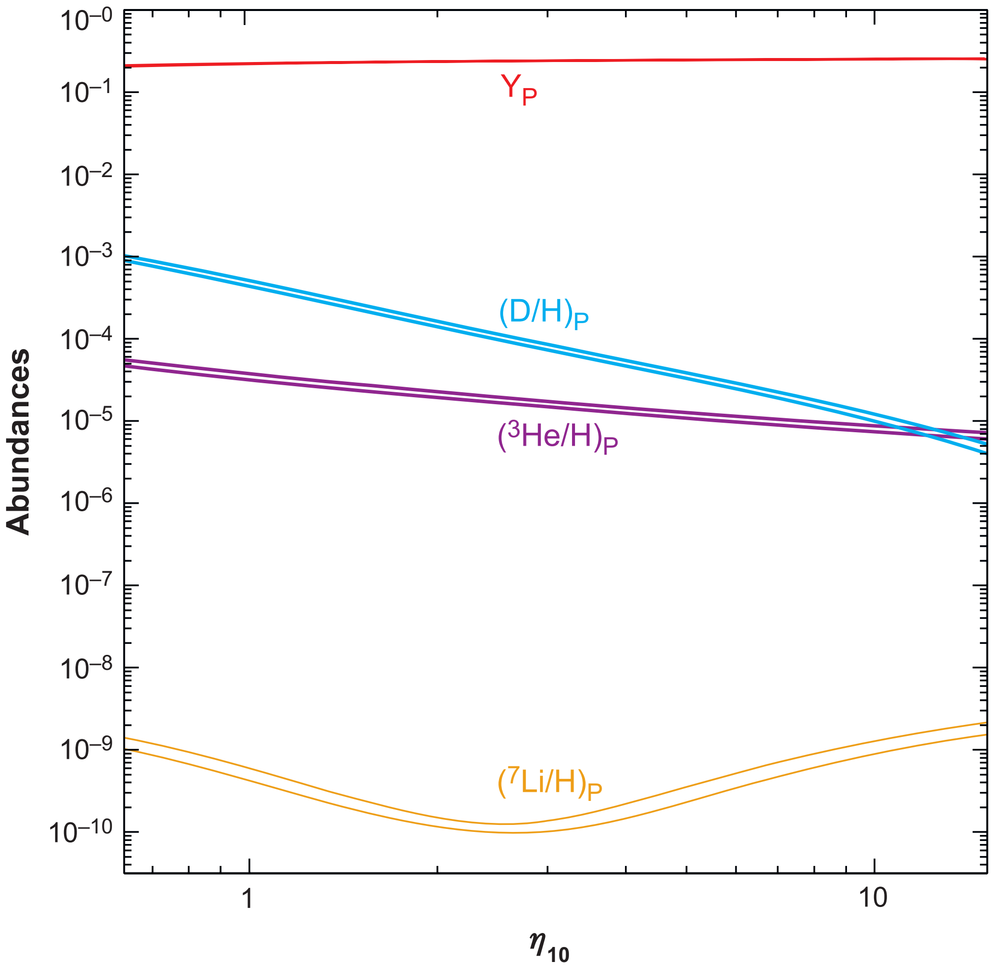
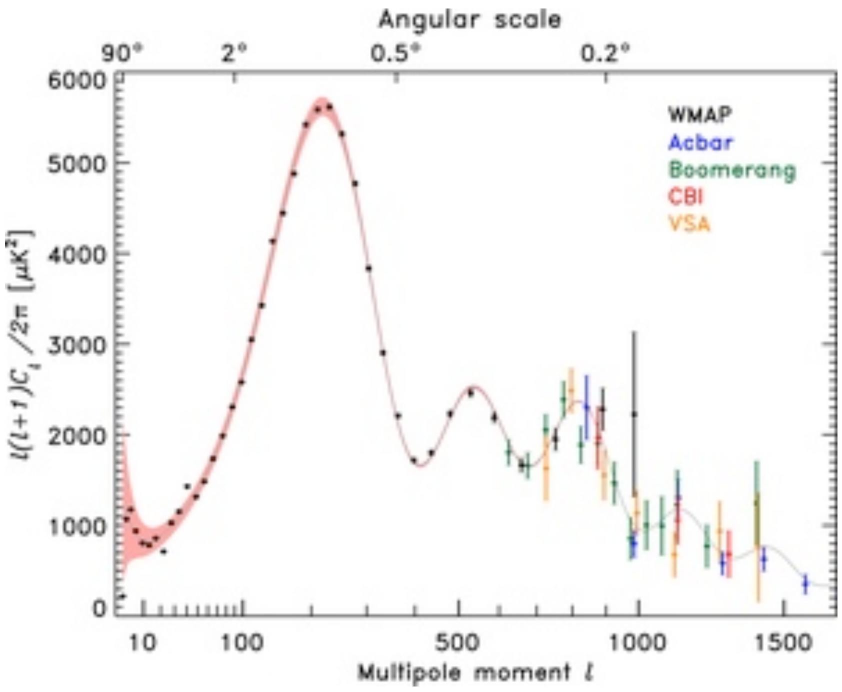
Successes of standard cosmological model





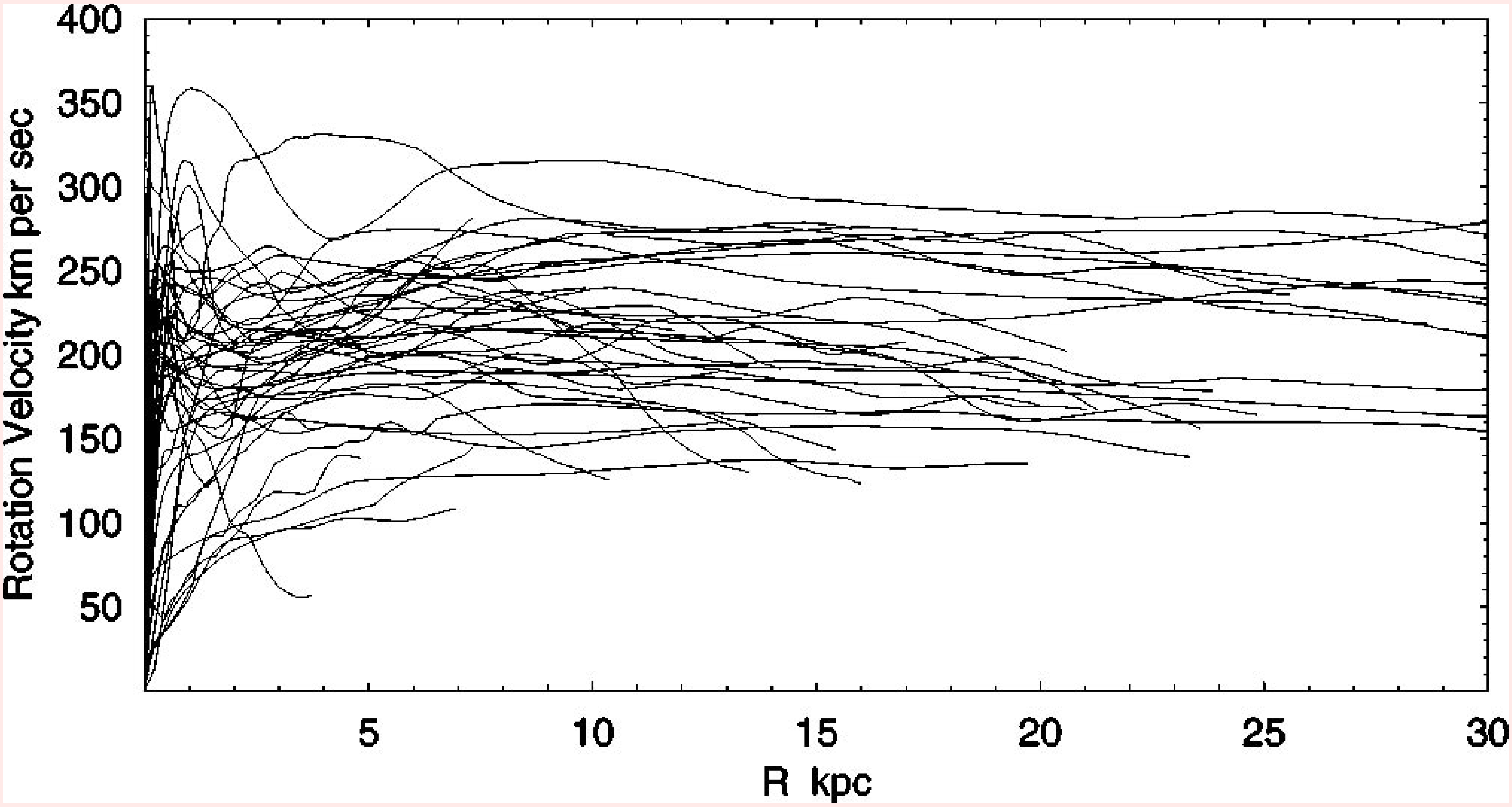


The predictions of the theory of big bang nucleosynthesis provide a good fit for the measured cosmic abundances of the light elements. Likewise, the measured angular spectrum of fluctuations of the cosmic microwave background (CMB) can be well fit by the general relativistic theory of growth of cosmological perturbations. Successes like these have convinced many that the standard concordance cosmological model must be near the mark.

But now suppose we attempt to deduce properties of galaxies from the elements of the standard cosmological model. Then some problems arise. This talk is about how early perception of these problems at small scales led to MOND---the modified Newtonian dynamics, about MOND's possible physical basis, its successes and failures, how possibly to temper the latter, and about the effort to make MOND into a relativistic theory of gravity.

Rotation curves of disk galaxies are flat

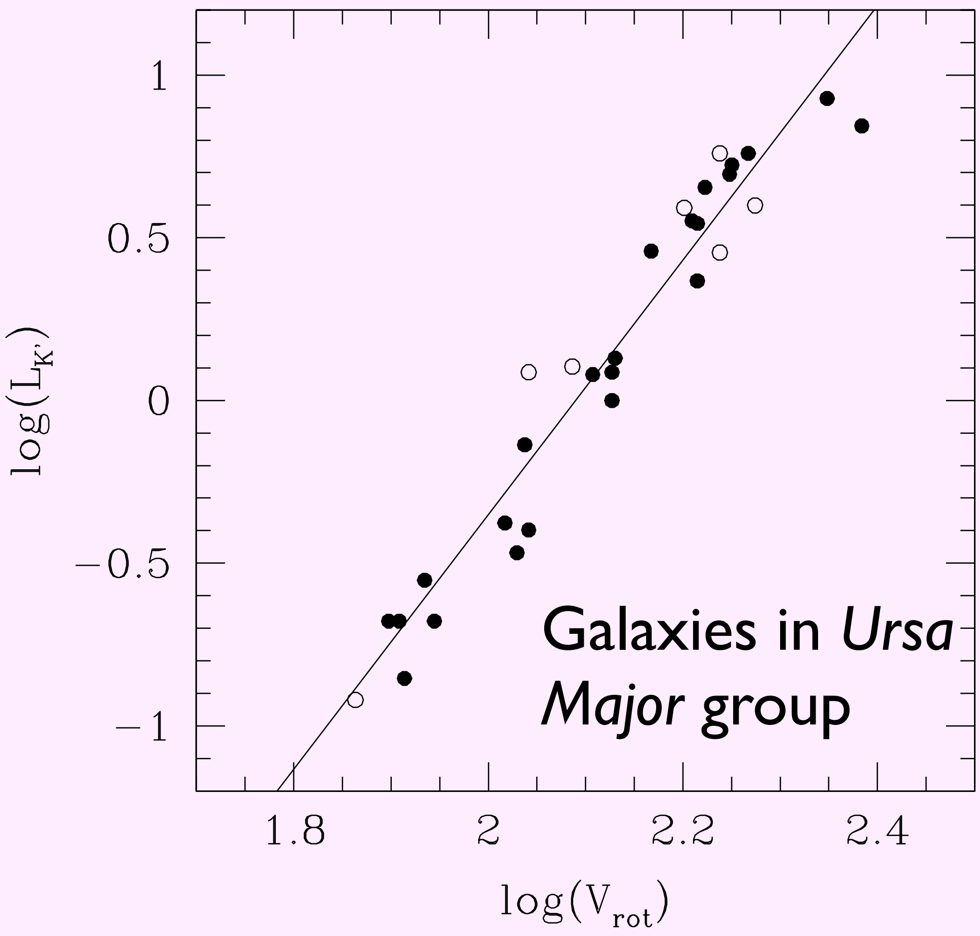
Sofue and Rubin (2001)



One of the basic facts about small scale structure is the flat RCs of disk galaxies. This collage of RCs of nearby disk galaxies was obtained by combining Doppler data from CO molecular lines for the central regions, optical lines for the disks, and HI 21 cm line for the outer (gas) disks. It shows that RCs are flat to well beyond the edges of the optical disks (~ 10 kpc). The flatness has always been a striking fact.

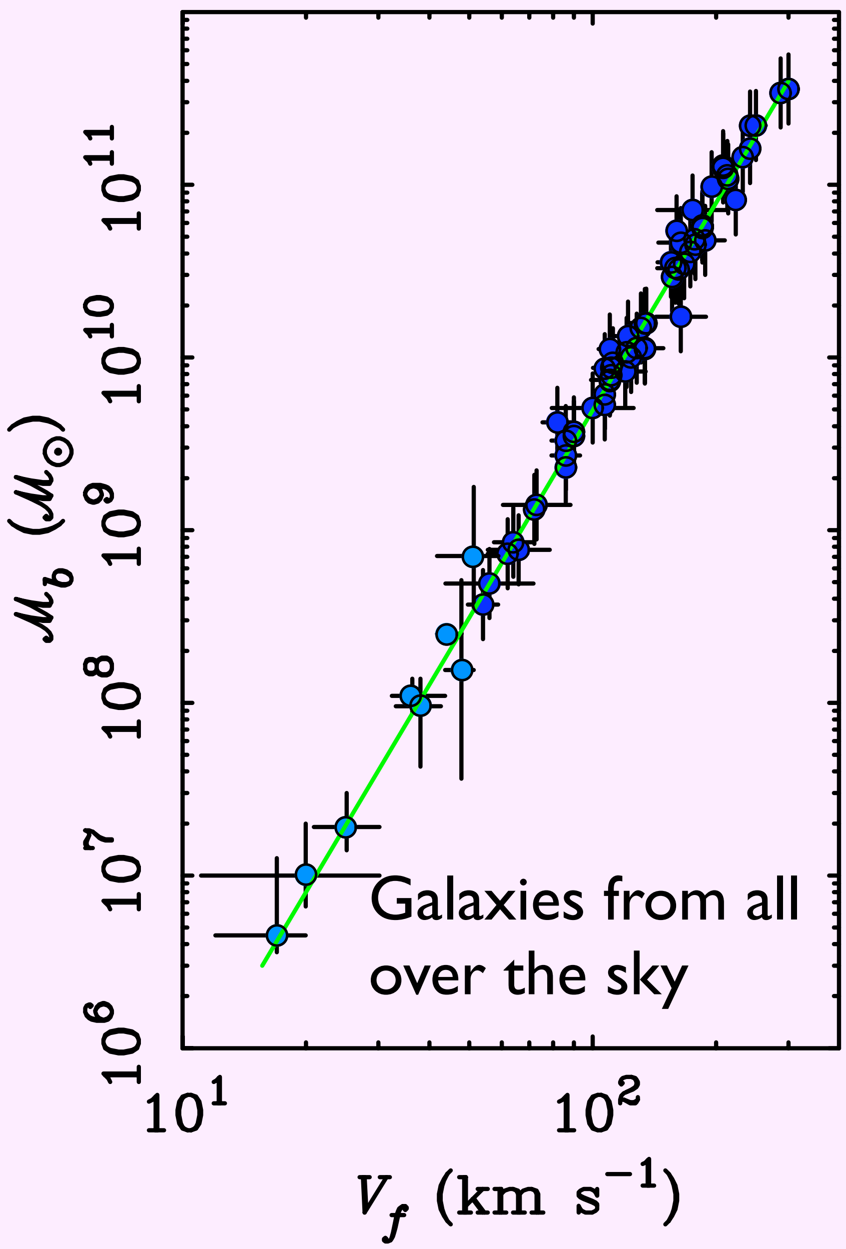
Tully-Fisher Law (1977)

Aaronson et al. (1982)



Plot: Sanders and Verheijen (1998)

McGaugh (2005)



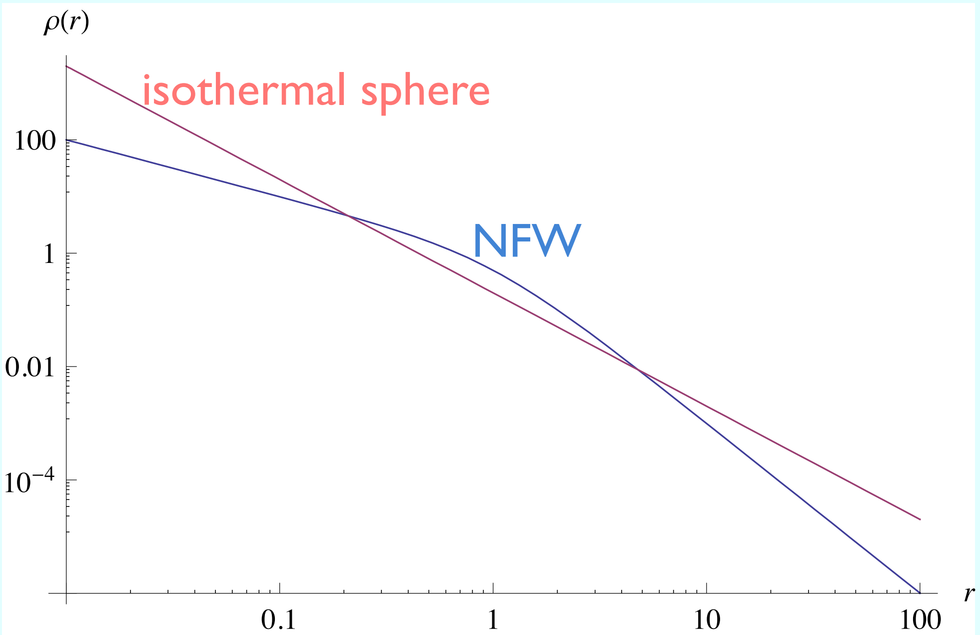
The amplitude of the RC (the flat part) is controlled by the empirical Tully-Fisher law.
Left: 1980's version, where the infrared K band luminosity of a galaxy is seen to be accurately proportional to the fourth power of the circular velocity.
Right: 21th century's version, where the mass in baryons, both in stars and in gas, is found to be accurately proportional to the fourth power of the circular velocity.
Quite aside from its usefulness in measuring intergalactic distances, the TF law is one of those basic facts that begs for explanation in physical terms.

Dark halo paradigm

Each galaxy nested in a roundish extended dark halo

$$\rho \propto 1/r^2 \longrightarrow v = \text{const.}$$

Navarro, Frenk and White's CDM simulations (1996)



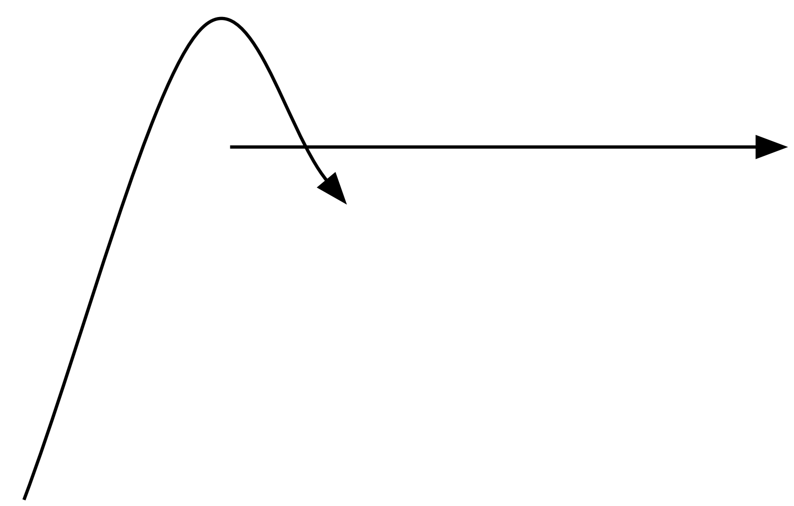
The standard explanation of the flatness of RC's harks back to Ostriker and Peebles' (1972) scenario that has disks set within massive dark halos to suppress rotational instability. A flat rotation curve follows if the halo in question has a $r^(-2)$ density profile (isothermal sphere) and also dominates the gravitational field outside the central regions of the galaxy.

But well known cosmological simulations of the growth of structure in dark matter (eg Navarro, Frenk and White) come up with halos which have a r^(-1) density profile in the inner parts and a r^(-3) one in the outer ones. Here is a log-log plot of density of a NFW halo compared with that of an isothermal sphere. NFW will only give a flat RC in the intermediate regime; it will not give flat extended rotation curves.

Dark clouds over dark halos

- No extended flat rotation curves
- Need to fine tune

$$L_{K'} \propto {\rm V_{rot}}^4$$

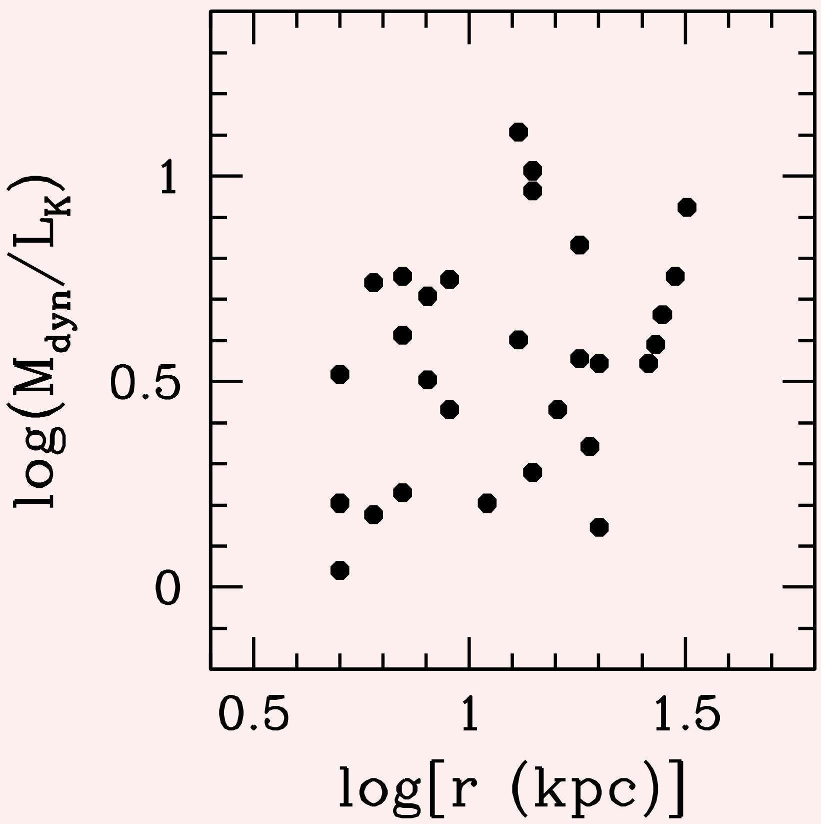


- Too many satellites
- Cuspy halos

Halo model parameters: $\sigma^2,\ R_c, \Upsilon = M/L$

Fine tunning present. There is need to fine tune halo to disk to avoid a feature in the RC. In addition, TF requires fine tuning because L_K is a disk property, while V_{rot} is mostly halo.
Problem of the satellites: NFW simulations predict too many satellites per galaxy, that is too many small halos accompany a big halo. It has been a pious hope of dark halo pundits that the gas dynamics responsible for populating the halos with baryons will impose the said the fine tuning, and also prevent a certain fraction of minihalos from becoming populated by baryons so that they stay invisible.
Problem of the cusps: The NFW halo has a 1/r cusp in the mass density of its inner parts. At least in dwarf galaxies, where the halo is supposed to strongly dominate, kinematic evidence rules out cusps in the mass density.
That is why modern halo models of RCs replace the NFW profile by various analytic cored profiles. Usually these models have 3 parameters: the velocity dispersion of halo constituents, the radius of the halo core (which comes in place of cusp), and the M/L ratio for the luminous matter.

Not length scale but acceleration is the key !



Data: Tully et al. (1996); Verheijen and Sancisi, (2001)
Plots: Sanders and McGaugh (2003)

In the early 80's Milgrom realized that the mass discrepancy in disks, which halos are supposed to resolve, is not tied to a particular mass or length scale, but to an acceleration scale. You can see this well in the modern data. Left a log-log plot of the dynamical M/L_K (a measure of the mass discrepancy) vs. the radius at the last measured point of RCs for a uniform sample of spiral galaxies in the Ursa Major group. The dynamical M is calculated from Newtonian dynamics. The fact that M/L_K is considerably larger than unity, the value typical of stars, shows, again, that there is a mass discrepancy. There is not much of a correlation of M/L with size. Right the Newtonian M/L plotted against centripetal acceleration (v^2/r) at the last measured point: there is now a correlation in the sense that M/L ∝1/a for a < 10^{-10} m/s^2. For accelerations higher than 10^{-10} m/s^2 the mass discrepancy disappears. This characteristic acceleration became central to MOND.

What physics is behind MOND ? (Milgrom 1983)

- a modification of inertia ?
- a modification of Newtonian gravitation ?
- other (dipolar dark matter, ...)

$$\mathbf{a} = \mathbf{F}_N \longrightarrow \tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = \mathbf{F}_N$$

- requires nonlocal physics
- hard to work with

$$\mathbf{a} = \tilde{\nu}(|\mathbf{F}_N|/a_0)\,\mathbf{F}_N = \mathbf{F}$$

Even before outlining the modified dynamics, one should ask, what is the physics behind it. Two options, really. A modification of the inertia or of the form of the gravity law.
In the first case this newtonian relation a=F is replaced by the well known MOND nonlinear relation where the inertia becomes a function of acceleration. In the 1990's Milgrom made a try at a theory of modified inertia: No lagrangian exists, so he worked with a nonlocal action. This is eminently hard to work with. He did show that circular orbits will have the desired behavior, but could say no more.
In the second interpretation acceleration is a some nonlinear function of the Newtonian force, which function can be interpreted as modified gravity. This interpretation is much easier to work with. It is easy to make a NR gravitational theory with MOND form, and, in fact, this was done quite early.

Newtonian theory in Lagrangian form

$$\mathbf{a} = -\nabla\Phi \qquad \mathcal{L} = -\frac{|\nabla\Phi|^2}{8\pi G} - \rho\Phi$$

$$\nabla \cdot \left(\frac{\partial \mathcal{L}}{\partial \nabla\Phi} \right) = \frac{\partial \mathcal{L}}{\partial \Phi}$$

$$\nabla \cdot \nabla\Phi = 4\pi G\rho$$

Let us first look at Newtonian gravity. First is the relation between acceleration of a test mass and the gradient of a gravitational potential This because we know gravitation is a conservative force. What would be the natural equation to determine \Phi for a give mass distribution? It is useful for what is coming to start from a Lagrangian density. The simplest one has the square of the derivatives of \Phi in this form. Why? Because that combination is rotationally invariant - space looks isotropic, so the Lagrangian density should be a rotational scalar. Then we have a coupling of Phi to the mass density \rho; this, again, is rotationally invariant as both \rho and \Phi are scalars at this level of the physics. The constant 1/8\Pi G is put in with the benefit of hindsight. Then we apply Lagrange's equation to get the Poisson equation which with suitable boundary conditions is equivalent to the inverse squared law of Newton.
Now we pass to a modification of this standard gravity. It is this part of the Lagrangian which is susceptible to change.

AQUAL modified gravity (B and Milgrom 1984)

$$\mathbf{a} = -\nabla\Phi \qquad \mathcal{L} = -\frac{a_0^2}{8\pi G} f\left(\frac{|\nabla\Phi|^2}{a_0^2}\right) - \rho\Phi$$

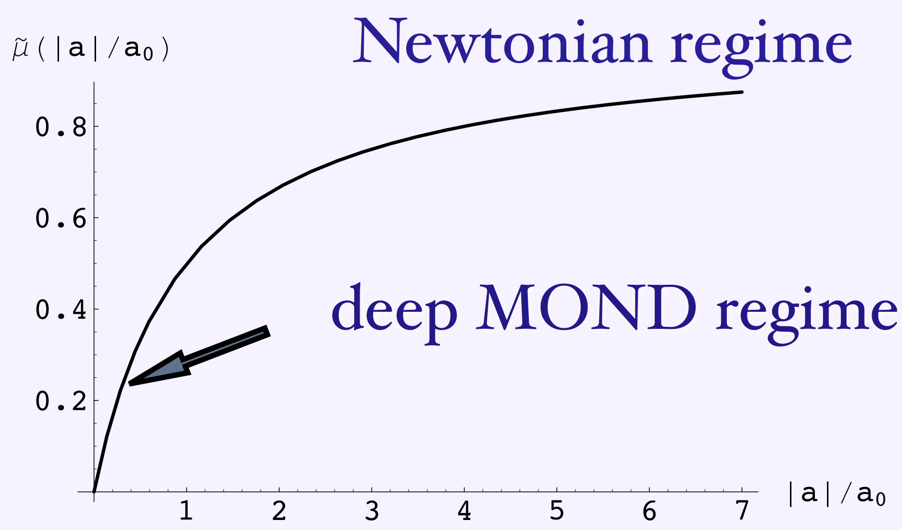
$$\nabla \cdot [\tilde{\mu}(|\nabla\Phi|/a_0)\nabla\Phi] = 4\pi G\rho$$

$$\tilde{\mu}(\sqrt{y}) \equiv df(y)/dy$$

$$\nabla \cdot \nabla\Phi_N = 4\pi G\rho$$

$$\tilde{\mu}(|\nabla\Phi|/a_0)\nabla\Phi = \nabla\Phi_N + \cancel{\nabla \times \mathbf{h}}$$

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_N$$



$$a_0 \approx 10^{-8}\ \mathrm{cm\ s}^{-2}$$

Suppose we replace the Newtonian Lagrangian by a more general one which is still rotationally invariant. Call this AQUAL theory for AQUadratic LAgrangian. We get a modified Poisson equation. Compare with the Poisson equation; recall that divergence of a curl is zero. Thus find a first integral of the modified Poisson equation. The curl disappears with high symmetry. Otherwise is seems to be relatively small. So we drop it. Substitute grad\Phi by -a. This is the celebrated MOND relation. So we take the scale a0 to be Milgrom's. We adjust f so that \tilde\mu has one of the suitable MOND forms. The shape is like this. A Newtonian limit; an extreme MOND regime.

Conceptual defficiencies of the MOND formula

(Milgrom 1983)

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_{\mathrm{N}}$$

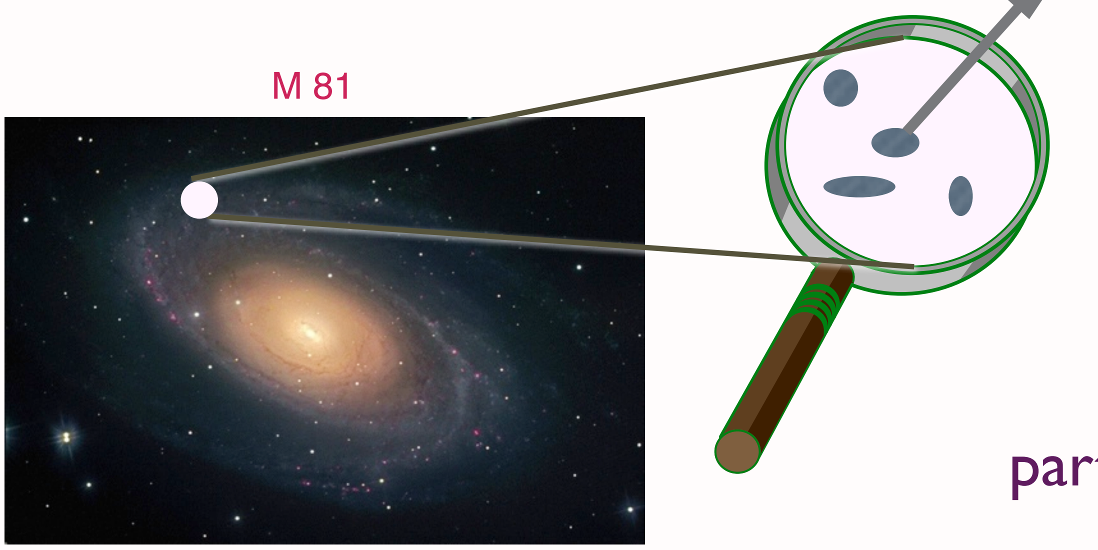
2

$$\tilde{\mu}(|\mathbf{a}_2|/a_0)\mathbf{a}_2 = -(Gm_1/r^3)\mathbf{r}$$

$$0 = m_1\tilde{\mu}(|\mathbf{a}_1|/a_0)\mathbf{a}_1 + m_2\tilde{\mu}(|\mathbf{a}_2|/a_0)\mathbf{a}_2$$

1

$$\tilde{\mu}(|\mathbf{a}_1|/a_0)\mathbf{a}_1 = (Gm_2/r^3)\mathbf{r}$$



$\tilde{\mu} \approx 1$

part-whole paradox

We see that the famous MOND formula is a limit of a more more complete theory. Can use it with confidence in highly symmetric systems. One should always remember that without symmetry the curl term makes \nabla \Phi not parallel to \nabla \Phi_N. Using the MOND formula uncritically can lead to difficulties. For one it does not respect the conservation laws.
Consider two masses according to MOND. These are the equations of motion. Combine them to get zero. This is different from conservation of momentum. Momentum not conserved if the curl term is dropped.
Another problem. In a galaxy this star, being in the low acceleration regime, orbits in a MOND orbit.
But this fluid element in the star is subject to strong acceleration; so are its neighbors. They are individually in the Newtonian regime. Reasonable that their CM should move Newtonially in the ambient gravitational field. Thus we have a clash, the part-whole paradox. It comes from ignoring the curl term, which takes care of conservation of momentum.

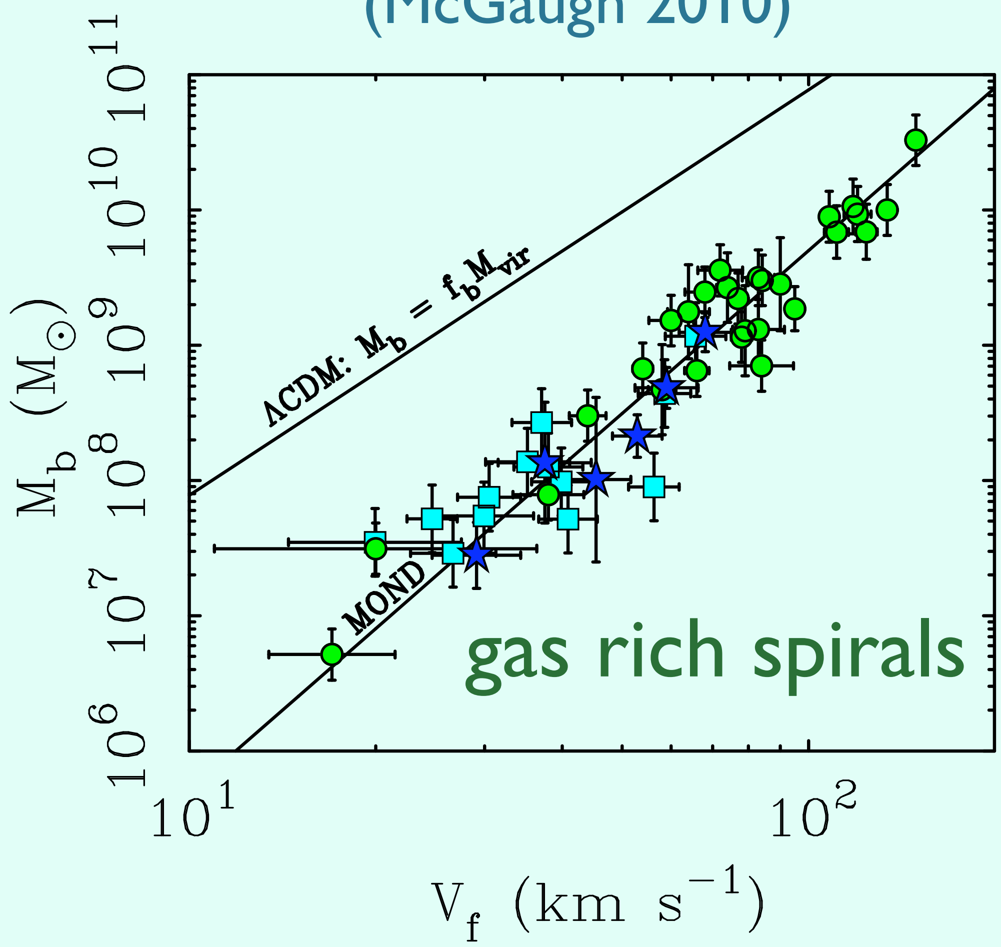
Tully-Fisher relation explained

$$|\mathbf{a}|\mathbf{a}/a_0 = -\nabla\Phi_{\rm N}$$

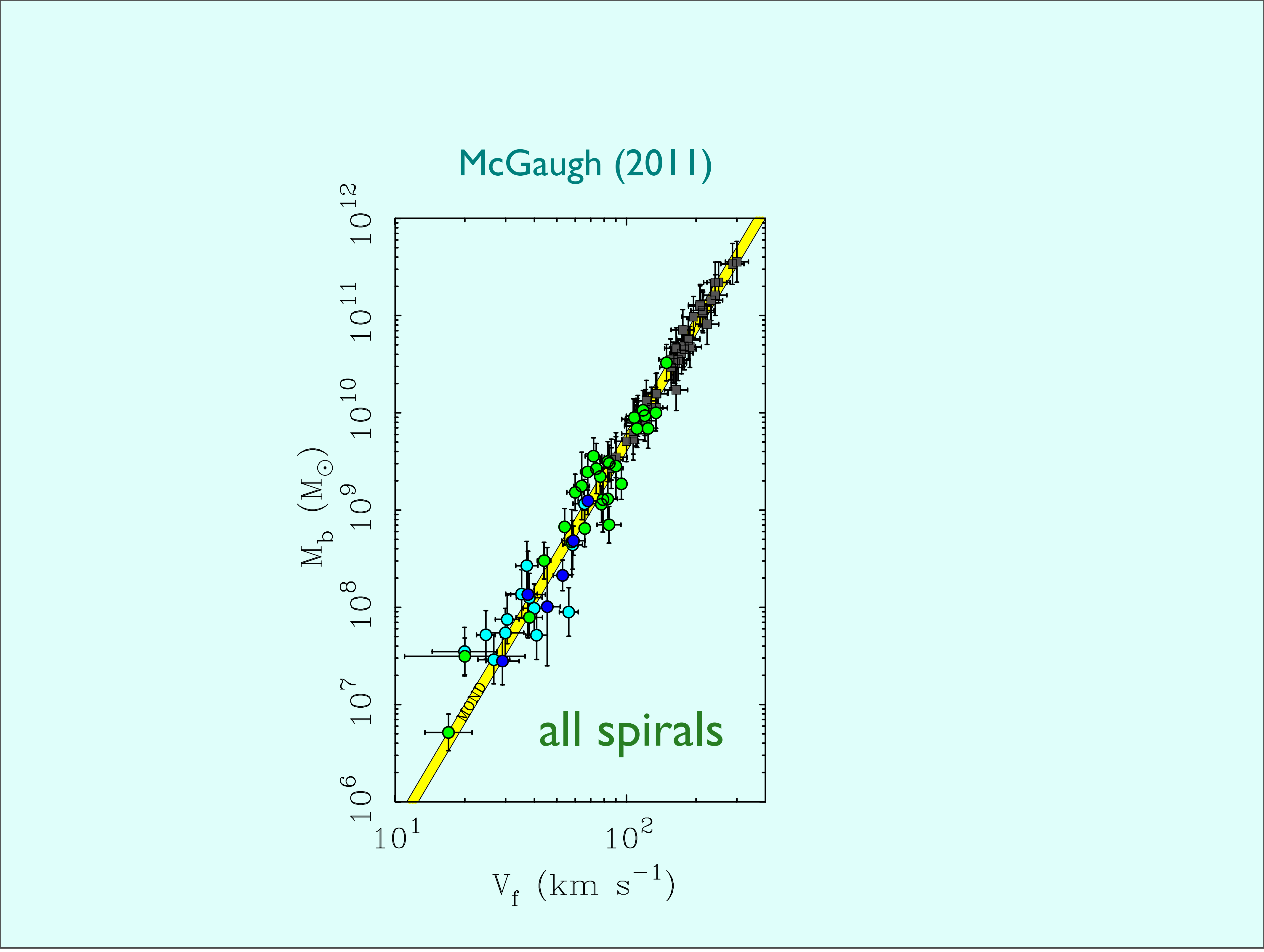
$$|\mathbf{a}| = v^2/r \qquad |\nabla\Phi_N| = GM/r^2$$

$$M = v^4/Ga_0$$

(McGaugh 2010)



Even with naive MOND one can give a good explanation of TF law. TF refers to the asymptotic behavior of the velocity, that is to the outer parts where accelerations are weak (extreme MOND regime). The form of the MOND equations is like this. For acceleration use the centripetal force due to circular motion with velocity v. For the Newtonian gravitational field use that of a point mass like the galaxy (we are considering outer parts). Get that the rotation curve is flat. Obtain modern form of the TF relation.
The MOND line (slope=4) is theoretical. There are no fitting parameters. The breadth of the distribution reflects measurement errors (Gaussian-distributed). The underlying law is a sharp one.
The LCDM prediction (slope=3) comes from NFW-type simulations together with the assumption that the baryonic mass falling into a halo should be a definite fraction of the virial mass.



The previous concentrated on gas rich spirals. This one looks at all spirals. Also a very good fit.

MOND RCs for disk galaxies in *Ursa Major* group

$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_{\rm N}$$

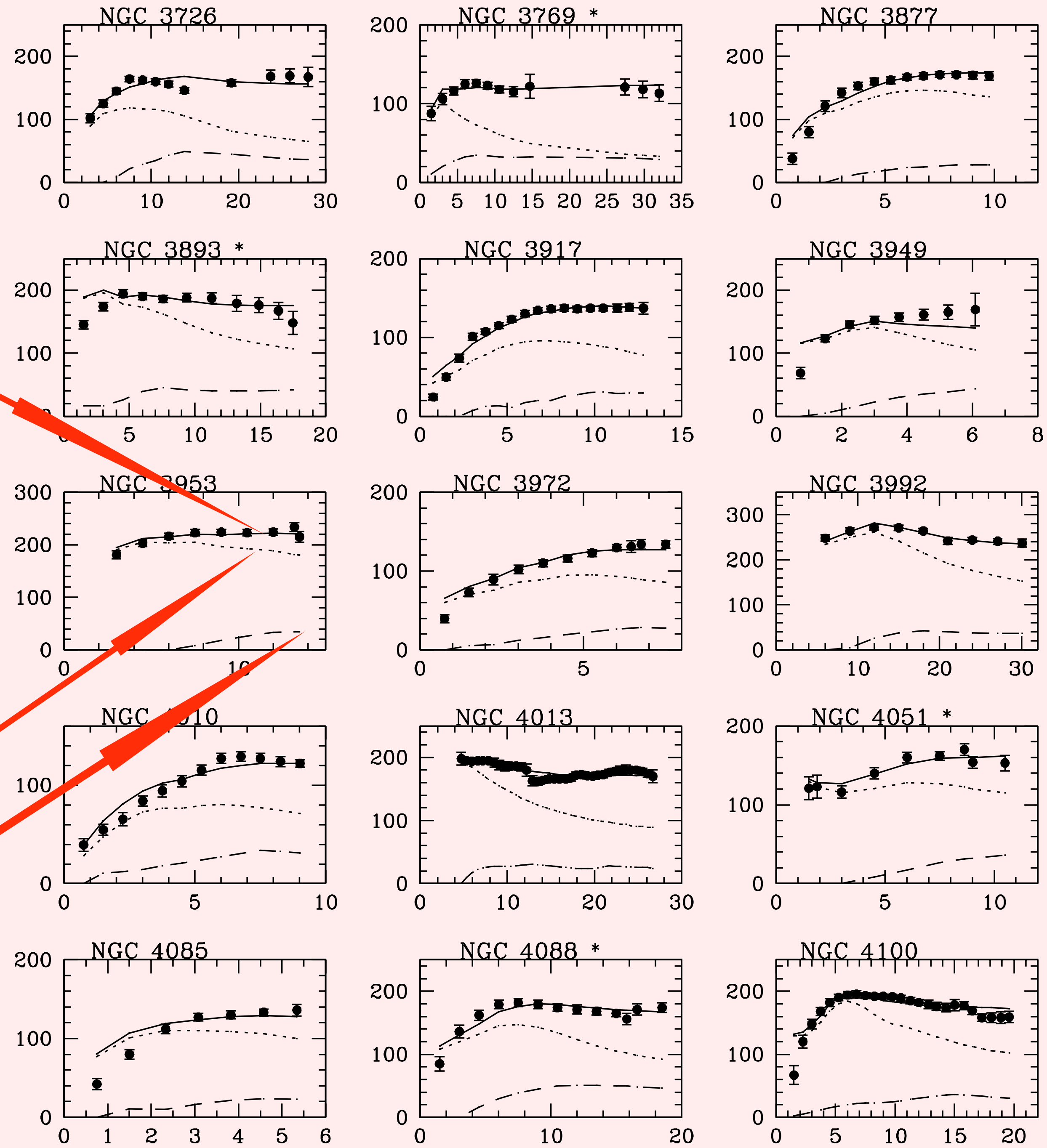
$$\tilde{\mu}(x) = \frac{x}{1+x}$$

Data: Verheijen (1997)

Fit: Sanders and Verheijen (1998)

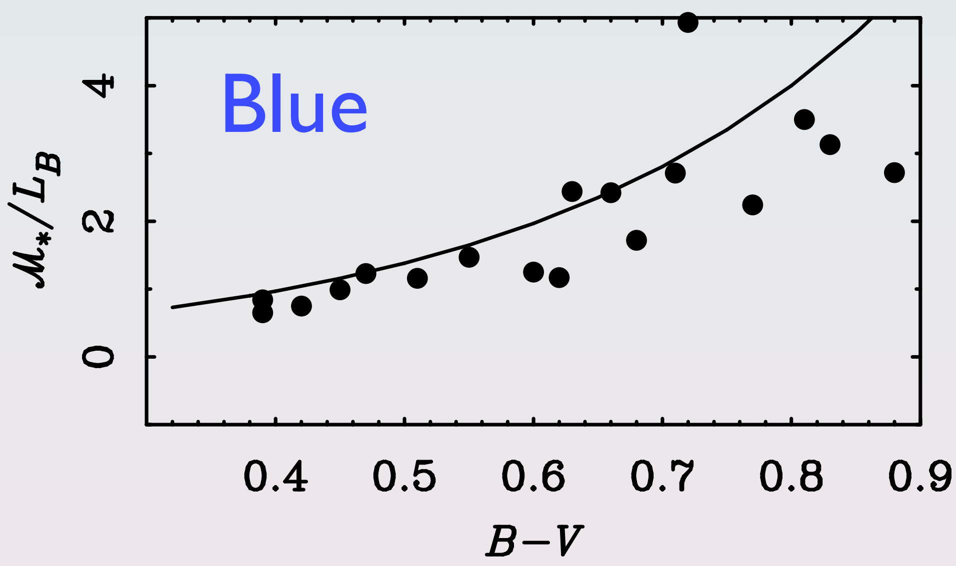
stars

gas



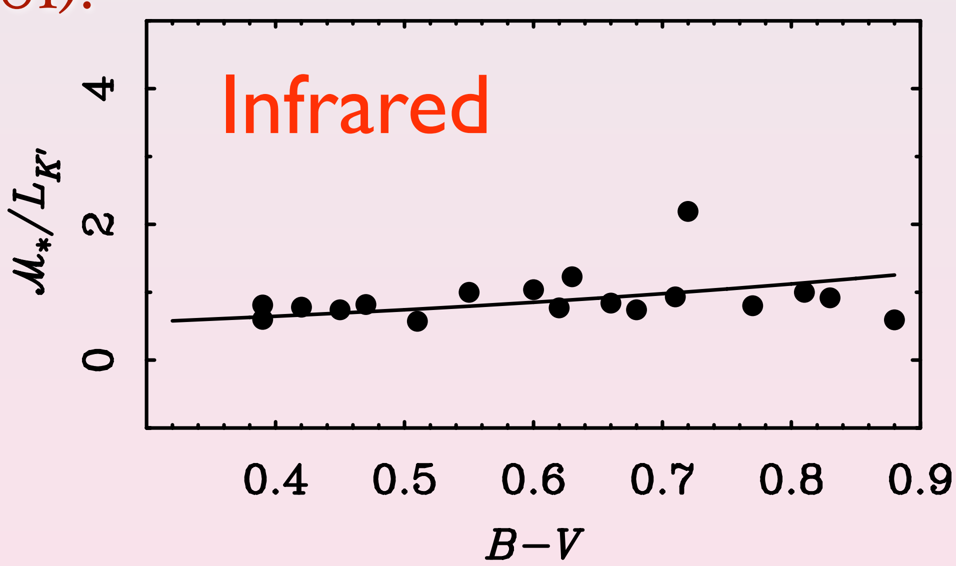
MOND also fits well the shape of RC even in the inner regions. Shown are naive MOND fits to the rotation curves of 15 Ursa Major galaxies. The radius (horizontal axis) is given in kiloparsecs and in all cases the rotation velocity is in kilometers/second. The dotted and dashed lines are the Newtonian rotation curves of the stellar and gaseous components of the disk, and the solid line is the MOND rotation curve (using the shown "simple" \mu function) with ao = 1.2 × 10^-8 cm/s^2 -the value derived from the rotation curves of 10 nearby galaxies (Begeman et al. 1991). The distance to all galaxies is assumed to be 15.5 Mpc. The free parameter of the fitted curve is the mass of the stellar disk. If the distance to UMa is taken to be 18.6 Mpc, as suggested by the Cepheid-based re-calibration of the Tully-Fisher relation (Sakai et al. 2000), then ao must be reduced to 10-8 cm/s 2 .

The determined M/L ratios



Dots: MOND fits by Sanders & Verheijen (1998)

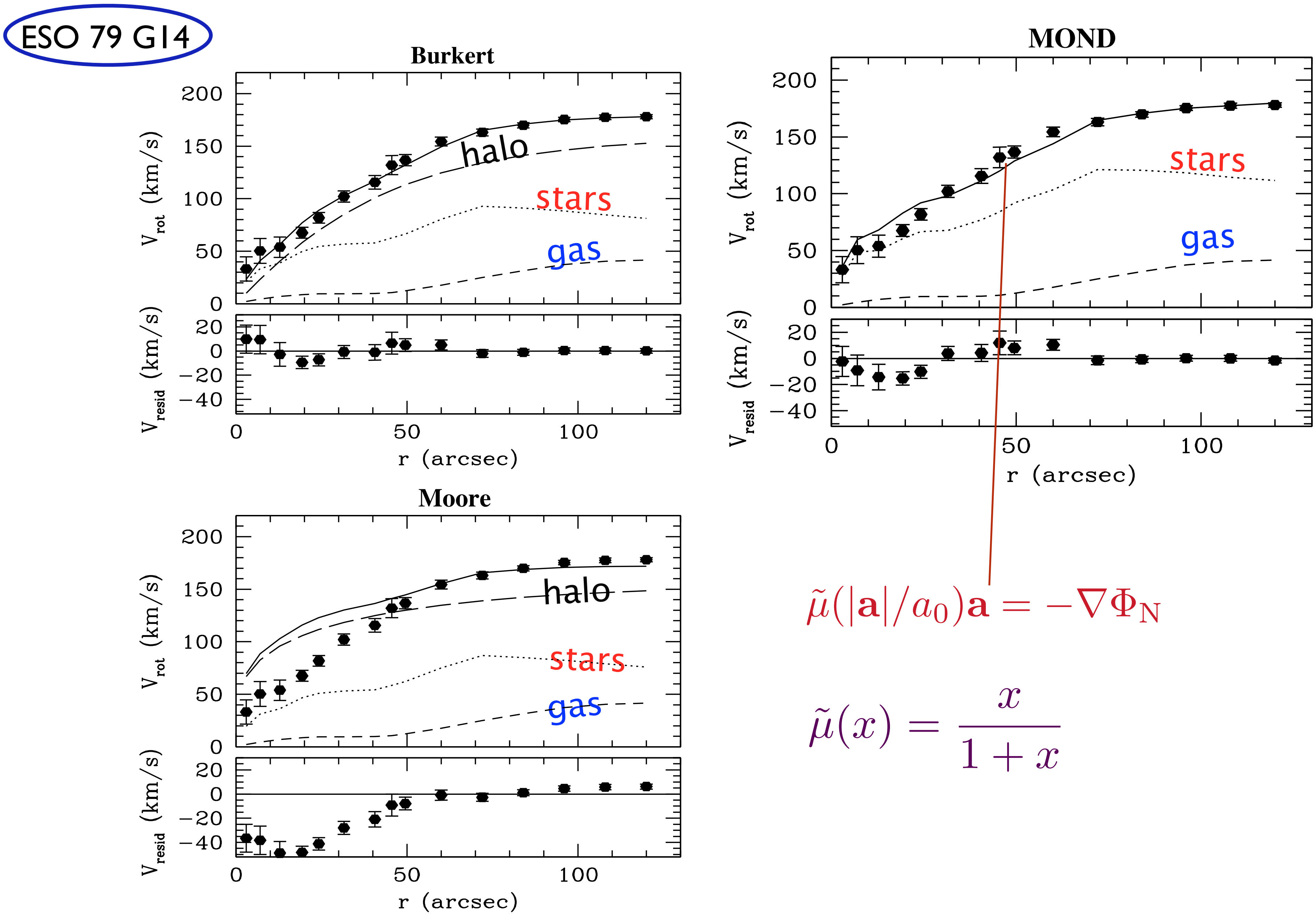
Curves: populations synthesis models by Bell & de Jong (2001).



The only fitting parameter for each galaxy of the above is the M/L for the stellar component. Here plotted are the Inferred M/L for the UMa spirals (Sanders & Verheijen) in the B-band (top) and the K'-band (bottom) plotted against B-V (blue minus visual) color index. The solid lines show predictions from populations synthesis models by Bell and de Jong (2001). You see that MOND predicts both the value of M/L and its trend with stellar population (morphology). And the fact that M/L is almost constant in the K' band means that the observed K' band luminosity TF law is also a consequence of MOND.

Comparison of dark halos with MOND

Gentile, Salucci et al (2003)



$$\tilde{\mu}(|\mathbf{a}|/a_0)\mathbf{a} = -\nabla\Phi_N$$

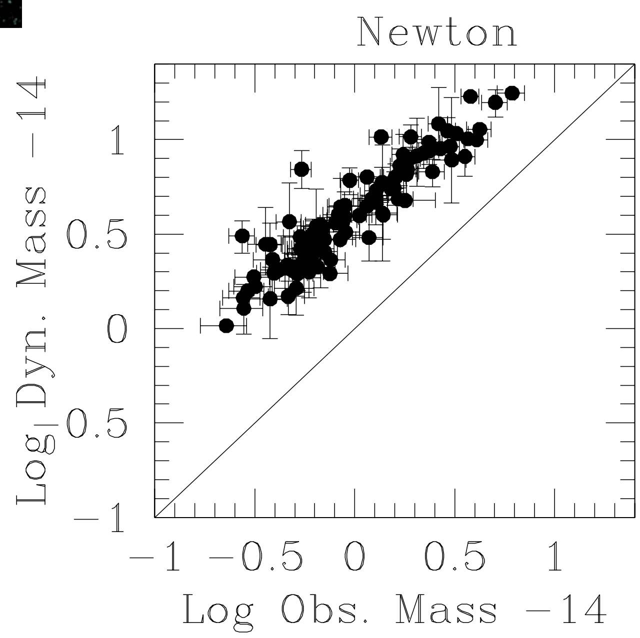
$$\tilde{\mu}(x) = \frac{x}{1+x}$$

Just for comparison, here are some dark halo fits to a well measured RC. The galaxy is ESO 79 G14. Shown are the Newtonian RCs established by the gas and the stars, and the assumed halo contribution. This last comes from a 3 free-parameter halo model. Burkert's model is usually the best fit. Now look at the MOND fit using the naive MOND equation, the now popular simple \mu function, with stellar M/L as the only free parameter. The fit is just as good as Burkert's and with fewer adjustable parameters. So MOND works very well for disks.

MOND does not do away with all dark matter

Coma

Milgrom (1983): galaxy clusters must contain lots of gas



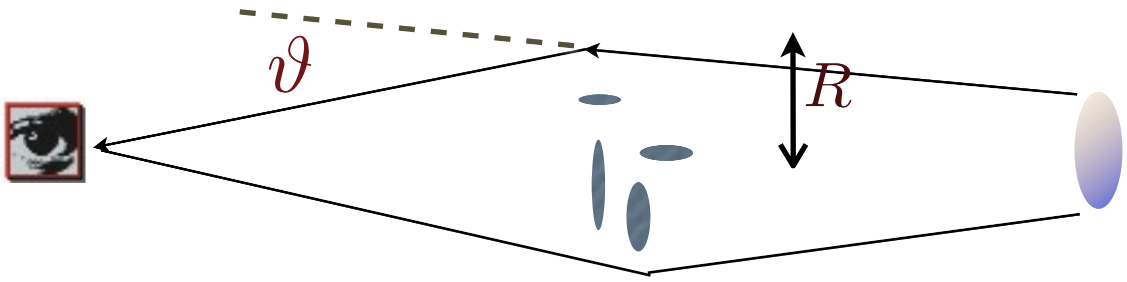
White et al

3C 295

In his original papers Milgrom noticed that Zwicky's mass discrepancy in large clusters of galaxies is not fully resolved by MOND. This led him to predict that galaxy clusters must contain much unobserved gas. In the mid-1980s a lot of X-ray emitting gas was discovered in clusters. But it did not resolve the whole problem. For 93 X-ray emitting clusters of galaxies: Left – the dynamical mass of clusters of galaxies (from Newtonian virial theorem) within an observed cutoff radius vs. the total observable baryonic mass – White et al. 1997. The solid line corresponds to Mdyn = Mobs (no discrepancy). Right the dynamical mass within r_out (from MOND version of the virial theorem) vs. the total observable mass for the same clusters (Sanders 1999). The swath is broader because the MOND mass estimator goes like v^4 and not v^2 like with Newtonian theory. MOND still leaves us with a mass discrepancy by a factor of 2-3.

Arches and weak gravitational lensing

Abell 1689



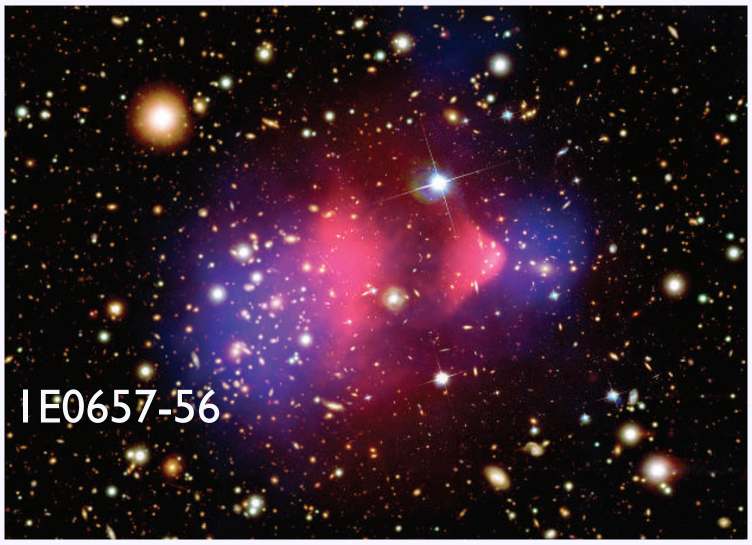
$$M \approx \frac{Rc^2\vartheta}{4G}$$

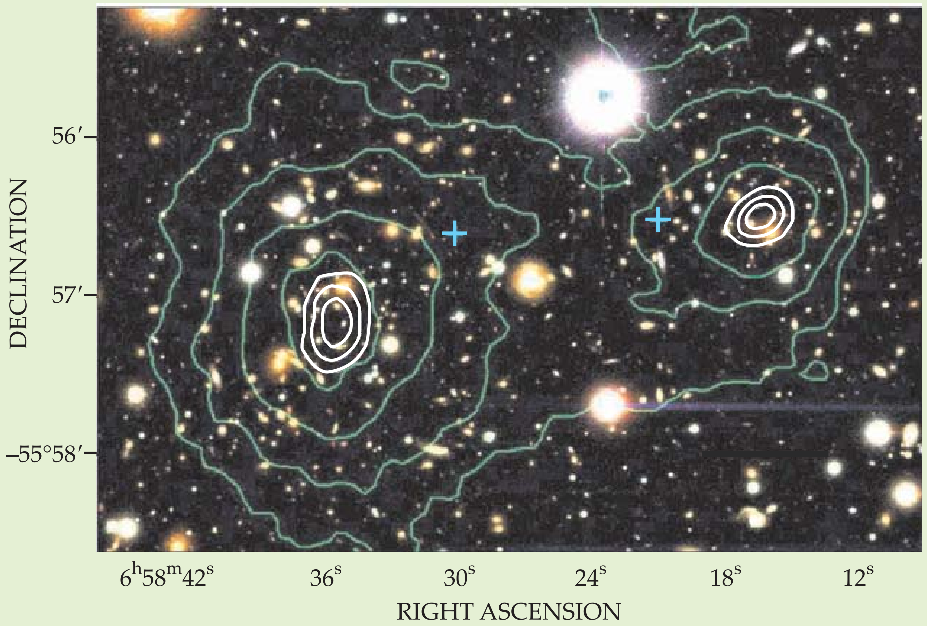
anomalously large mass

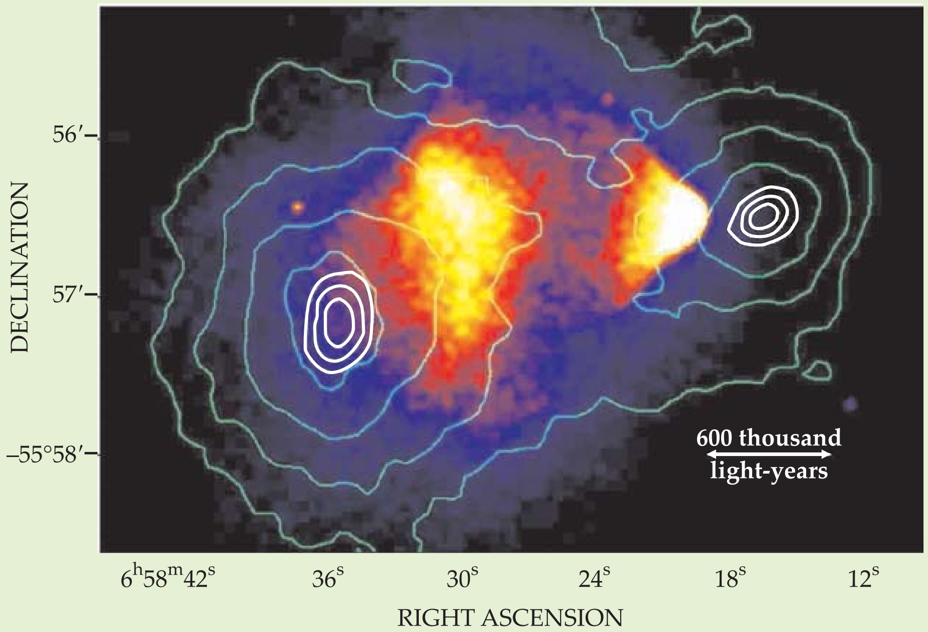
Gravitational lensing, such as the arcs of a galaxy lensed by cluster Abell 1689, reveal the same problem. In examples like these the lens masses can be determined by the usual GR recipe. Those masses, come out anomalously large, and are compatible with the dynamical estimates from Newtonian theory and those from analysis of the X-ray emission. This, of course, is consistent with the dark matter paradigm.

The bullet cluster (1E0657-56)

Clowe, Badrac, et al., Ap. J. **648**, L109 (2006)







On the same line, the bullet cluster--- two clusters colliding at some 4000 km/s with consequent expulsion of most of the intracluster hot gas (gas - red, galaxies-blue)--- was hailed as a proof of DM. Why? The second panel shows constant shear contours of the lensing map: the strongest shearing is where the galaxies are densest. The composite third panel confirms that the strongest lensing is not spatially associated with the hot gas, which happens to contain the lion's share of the visible baryonic mass. Of course, collisionless DM would not be expected to be expelled with the gas. What is new here is the delocalization of the DM vs the baryons. The fact that there is a mass discrepancy which MOND does not do away with was, as I mentioned, known already in the 1980's.

Some possible ways out

- Milgrom (2007): baryonic clouds may provide the still missing mass contributing 0.025 to Ω
- Sanders (2003, 2007): massive neutrinos
  - requires ~1.9 eV neutrino masses for three species
  - ~1 eV range sterile neutrinos a guarded possibility
- MOND is an incomplete paradigm
  - It is possible to resolve most of the problem by requiring a bigger a_0 in clusters
  - Bigger a_0 is better for CMB fits
  - AQUAL can be generalized in this direction

There are several ways to resolve this quandary. Milgrom has suggested the presence of unobserved baryonic matter in clusters in the form of small gas clouds. For some ranges of parameters these could survive for an Hubble time. These invisible cluster baryons would contribute about 0.025 to \Omega; this is consistent with the quantity of the still unobserved baryons required by primordial nucleosynthesis arguments. Not clear how these baryon clouds avoid being swept out with the gas in the bullet cluster example.

Even earlier, Sanders had suggested neutrinos as an extra source of gravity. Of course this is invoking dark matter to help MOND, but at least we know neutrinos from the laboratory. Again, one would need all types of neutrinos in clusters to contribute about 0.025 to \Omega. Sanders gets this with the 3 neutrino masses of about 1.9 eV, a value which is (still) consistent with the present upper bound of 2 eV (Russian tritium experiment) and with the lower bound of 0.04 eV from neutrino oscillations. I t is not consistent with gravitational lensing or cosmological bounds on neutrino masses (sum=0.5 eV). However, these last are within a standard Lambda CDM cosmological model. Also, one hears that the miniBoone experiment at Fermilab seems to require a couple sterile neutrino species with masses in the 1 eV scale. Maybe the right kind of neutrino exists for our quandary.

Another possibility is that our MOND paradigm is incomplete. A good part of the cluster problem goes away is one uses a bigger a_0, say 2 x 10^{-8} cgs, than when fitting galaxy RCs. And, as I will mention later, when one uses a brand of relativistic MOND to fit the CMB spectrum, an even bigger a_0 comes in handy. It is as if a_0 grows with length or mass scale! One can actually generalize the AQUAL theory in this direction.

AQUAL with scale dependent critical acceleration

$$\mathcal{L} = -\frac{a_0^2}{8\pi G} f\left(\frac{|\nabla\Phi|^2}{a_0^2}\right) - \rho\Phi$$

$$\mathcal{L} = -\frac{a_0^2\, e^{-2\Phi/s^2}}{8\pi G} f\left(\frac{|\nabla\Phi|^2\, e^{2\Phi/s^2}}{a_0^2}\right) - \rho\Phi$$

physical acceleration scale $a_0\, e^{-\Phi/s^2}$ $\qquad s \sim 10^3\,\mathrm{km\,s^{-1}}$

$$\nabla\cdot(\nabla\Phi\, f') - \frac{{a_0}^2}{s^2}\, e^{-2\Phi/s^2} f + \frac{|\nabla\Phi|^2}{s^2} f' = 4\pi G\rho$$

Newton's iron sphere theorem no longer valid

Recall the AQUAL lagrangian density? Suppose we divide each a0 by an exponential in \Phi/s^2 where
s is a fixed scale of velocity. The more bound the part of the system we consider, the deeper or more negative\Phi is, so that a0 is effectively jacked up by a factor which depends on the region in the system.
The physical acceleration scale is this. If we take s to be about 10^3 km/s, then in a rich cluster of galaxies, the scale would be ~ a0 e^1 \approx 3 a0, which is about what is required to remove MOND's problem. In a galaxy the scale would be a0 e^{0.04} which is very close to a0 and pretty constant to boot, consistent with the uniformity of the critical acceleration for galaxies.
But why an exponential? The form of the lagrangian should not depend on the zero point of the field, that is should remain unchanged when one adds a constant to the field. If you tamper with AQUAL with an arbitrary function of \Phi, this will not be true. But with the exponential, if you add a constant to \Phi, the change can be absorbed in a redefinition of the scale a0. Nothing wrong with this since the measurable scale is a0 e^{-\Phi/s^2} which remains unchanged.
A significant property: the Newton "iron sphere theorem" is transcended. No longer true that the gravitational field at a surrounding surface fixes uniquely the mass within it; matter elsewhere can influence the result. Put another way, part of the the sources of a gravitational field distribution could be outside the region it straddles. This can address the problem of the bullet cluster.

Need for relativistic MOND

Relativistic MOND needed to give a correct account of gravitational lensing.

Other questions

- How to describe pulsars and black holes in MOND?
- How to construct MOND cosmological models?
- How to describe gravitational radiation in MOND?

But I am really getting ahead of myself; a relativistic form of MOND is needed to give a correct account of gravitational lensing, just because propagation of light is a relativistic phenomenon.

For years the absence of a relativistic theory for MOND was used by DM advocates as an argument against MOND itself. With the advent of TeVeS in 2004, and additional theories like MOND-like Einstein Aether, we have passed that stage.

First try .... (B and Milgrom 1984)

$$c = 1$$

$$\Phi \Longrightarrow \phi$$

$$\left[f'\left(\frac{g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}{{a_0}^2}\right)g^{\alpha\beta}\phi_{,\alpha}\right]_{;\beta} = -4\pi G T$$

Einstein's equations

$$ds^2 = e^{2\phi}\, g_{\mu\nu}\, dx^\mu\, dx^\nu$$

$$\Phi = \Phi_N + \phi$$

no anomalously large light deflection

superluminal propagation

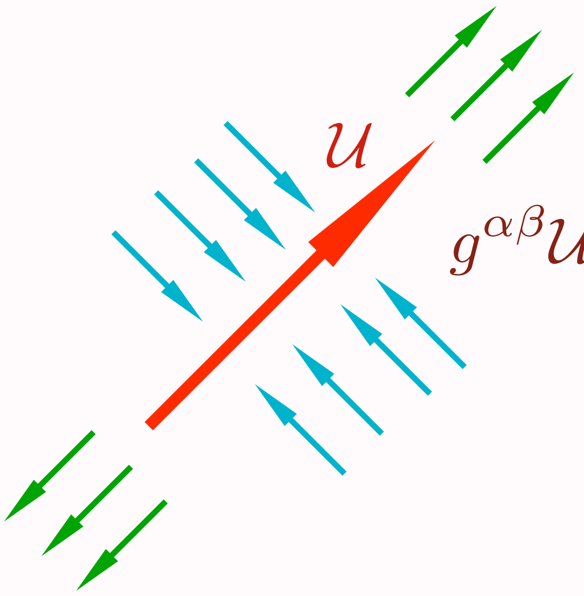
Must break in some way with GR because GR leads inexorably to Newtonian gravity in the nonrelativistic limit. The simplest theory is obtained by making the AQUAL potential a scalar field which obeys a covariant version of the AQUAL equation. The scalar is then used as a conformal factor to pass from Einstein's metric, which comes, as in GR, from Einstein's equations, to a new line element or metric. In this theory \Phi \approx Phi_N +\phi. Under broad conditions can get MOND behavior nonrelativistically.

Two failings 1) No way to get the light bending attributed to DM. Conformal relation --> light rays are null geodesics in Einstein's metric But scalar field contributes little energy for large scale systems, so light rays deflected by visible matter alone.

2) Perturbations of the scalar field in a time independent background can propagate faster than light.

The stratified theory (Sanders 1997)

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



$$g^{\alpha\beta}\mathcal{U}_\alpha \mathcal{U}_\beta = -1$$

$$ds^2 = \left[e^{-2\phi}(g_{\alpha\beta} + \mathcal{U}_\alpha \mathcal{U}_\beta) + e^{2\phi}\mathcal{U}_\alpha \mathcal{U}_\beta\right] dx^\alpha\, dx^\beta$$

$\perp$ $\quad \parallel$

$\tilde{g}_{\alpha\beta}$

$$\left[f'\left(\frac{g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}}{{a_0}^2}\right) g^{\alpha\beta}\phi_{,\alpha}\right]_{;\beta} = -4\pi G T$$

How break out of this impasse? Imagine spacetime is pervaded by a timelike unit 4-vector U. Squeeze spacetime orthogonal to it and stretch it by the same factor along it. This is spacetime orthogonal to U; it is being squeezed. This is spacetime along U; it is being stretched by the same factor. The stretching (squeezing) factor defines a scalar field. We now construct a second metric, g twiddle, and take it to be the metric in which matter fields, and you and me live. In this stratified theory, the vector U is constant and \phi has AQUAL covariant dynamics. The *a priori* status of U breaks covariance, though.

To get an acceptable theory we must rearrange things.

Tensor Vector Scalar theory

B, Phys. Rev. D **70**, 083509 (2004); JHEP PoS (jhw2004) 012

Gravitation $$S_g = (16\pi G)^{-1}\int R(-g)^{1/2}d^4x$$

Matter $$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^A, f^A{}_{|\mu}, \cdots)(-\tilde{g})^{1/2}d^4x$$

Vector $$S_v = -\frac{1}{32\pi G}\int \big(K g^{\alpha\beta}g^{\mu\nu}U_{[\alpha,\mu]}U_{[\beta,\nu]} + \mathcal{K}(g^{\alpha\beta}U_{\alpha;\beta})^2\big)(-g)^{1/2}d^4x$$

Constraint $$S_c = \frac{1}{16\pi G}\int \lambda\big(g^{\mu\nu}U_\mu U_\nu + 1\big)(-g)^{1/2}d^4x$$

Scalar $$S_s = -\frac{1}{2k^2\ell^2 G}\int \mathcal{F}\Big(k\ell^2(g^{\alpha\beta} - g^{\alpha\mu}g^{\beta\nu}U_\mu U_\nu)\phi_{,\alpha}\phi_{,\beta}\Big)(-g)^{1/2}\,d^4x$$

TeVeS makes the U dynamical; so the theory is covariant. Here are the parts of the TeVeS action. The Einstein metric is subject to an Einstein-Hilbert action, as in GR. Matter's action is, however, written with the second, or physical, metric. The two metrics are related as in the previous slide. The vector field in question is given this action. In the original TeVeS there was only one term. A dynamical problems pointed out by Contaldi, Wiseman and Withers make it advisable to add further quadratic terms. This one is probably enough. The U was a constant unit vector in the stratified theory. Here unit norm is enforced with a Lagrange multiplier \lambda. Finally the scalar \phi, which is absolutely required to distinguish between the metrics, is given a relativistic AQUAL action, with an addition which precludes superluminal propagation of \phi. In all the theory has a scale of length \ell, and three dimensionless parameters, k, K and $\mathcal{K}$.

Some features of TeVeS

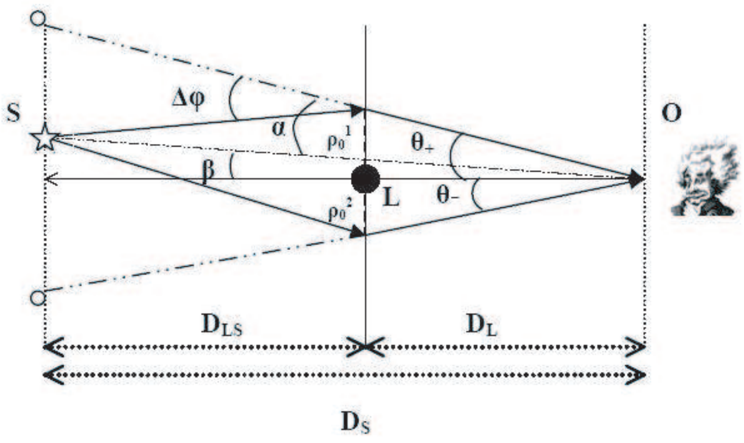
- Has GR as limit when k, K and $\mathcal{K}$ vanish and ℓ large
- Admits a one-metric reformulation with no scalar field (Ferreira, Starkman and Zlosnik 2006)
- Reproduces the AQUAL theory nonrelativistically
- a_0 is constructed out of k and ℓ
- Has a Newtonian limit at strong gravitational field
- Most post-Newtonian coefficients agree with GR's; the two that do not can be made compatible with bounds from solar system experiments (Sagi 2009)

In the interim several other relativistic MOND theories have been proposed, each imitating some aspects of TeVeS. Lots of activity in last 5 years in confronting TeVeS with observations.

Strong gravitational lensing *a la* TeVeS: theoretical

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)(dx^2+dy^2+dz^2)$$

$$\Phi = \Phi_N + \phi$$



Chiu, Ko & Tian, Ap. J. **636**, 565 (2006)

Point mass lens approximation

Deep MOND regime: deflection fairly constant for range of impact parameters

For two image lensing: difference of magnifications > 1

Time delay gives information on lens' mass independent of that from image separation or relative amplification

Consequences of the famous gravitational lensing diagram are calculated in GR using this weak field form of the the metric. \Phi is the Newtonian potential, and it occurs in two different contexts. With TeVeS in weak fields we get the same metric, except that here \Phi=\Phi_N+\phi, the last which comes from an AQUAL-like equation and has only ordinary matter as source. Thus for any local system where MOND gives a correct description of the observed dynamics, TeVeS predicts the same gravitational lensing as would be gotten from a GR+ dark matter model which also accounts for the dynamics. The only way to distinguish TeVeS from GR is to go far out, namely into the deep MOND regime, because there \Phi is very different from the DM's Newtonian potential.
Chiu et al: In deep MOND regime (impact parameter in region where field is < a_0) the light deflection angle \delta\varphi is pretty constant for a range of impact parameters, Sqrt[G M a0}/c^2, and interpreted by GR gives anomalously large mass. Difference of magnifications of two images>1 (1 is the GR value). Time delay by lensing gives information on "dark mass" which is independent of that obtained from image splitting or amplification.

Doubly imaged quasars and TeVeS

- Zhao, Bacon, Taylor & Horne, MNRAS **368**, 171 (2006)

  Studied 17 galaxy-lensed doubly imaged quasars from the CASTLES catalogue (CfA-Arizona Space Telescope Lens Survey) modeled both as point lenses and Hernquist spheres

  Lens masses determined from image positions are consistent with those from amplifications

  Mass (so determined) to luminosity ratios are reasonable

- Shan, Feix, Famaey & Zhao MNRAS **387**, 1303 (2008)

  Departures from spherical symmetry taken into account

  Agreement for 10 doubly imaged quasars (CASTLES) and one quadruply imaged quasar, all not in clusters. But some others problematic (extra dark matter ?)

Zhao et al. looked at a sample of 17 doubly imagined quasars from CASTLES (CfA–Arizona Space Telescope Lens Survey). They modeled the lenses (elliptical galaxies invariable) both as point sources and as Hernquist spheres. These served as sources for the scalar equation in TeVeS. For most of the pairs the lens mass determined using exclusively image positions agreed with that derived from the relative amplifications, and the consequent mass to light ratios were reasonable for ellipticals (ranging 0.5 – 2).

Shan et al. looked at effects of lens asphericity. For 15 doubly imagined quasars not in clusters (and a few quadruples) they found good agreement with predictions for most.

Conflicting opinions

- Mavromatos, Sakellariadou, & Yusaf, Phys. Rev. D **79**, 081301 (2009)

Ferreras, Mavromatos, Sakellariadou & Yusaf, Phys. Rev. D **80**, 103506 (2009)

conflict between their TeVeS mass estimates for 18 lenses in CASTLES with the masses inferred from luminosities

- Chiu, Ko, & Tian & Zhao ArXiv/1008.3114

for 10 of the lenses in question, which can be modeled as spherical, TeVeS with my toy $\mathcal{F}$ or with the $\mathcal{F}$ which yields the "simple" $\tilde{\mu}$, works adequately without dark matter

Sakellariadou, et al.: TeVeS will not work for galaxy lenses in the CASTLES catalogue without help from invisible matter. Early they actually calculated lensing using a mixture of MOND and GR instead of TeVeS (misleading). In the second attempt, they reiterate the above claim based on comparison of their TeVeS lensing mass estimates for 18 lenses in CASTLES with the masses inferred from luminosity.

Chiu et al. carefully repeated the comparison for 10 of the lenses in question which can be modeled as spherical. They conclude that with the $\tilde\mu$ deriving from my toy $\mathcal{F}$ or the simple $\tilde\mu$, TeVeS works adequately without requiring invisible mass. They ascribe the opposite conclusion by Mavromatos et al. to improper comparison of the inferred lensing mass with the baryonic stellar mass implied by optical intensity through a fixed aperture.

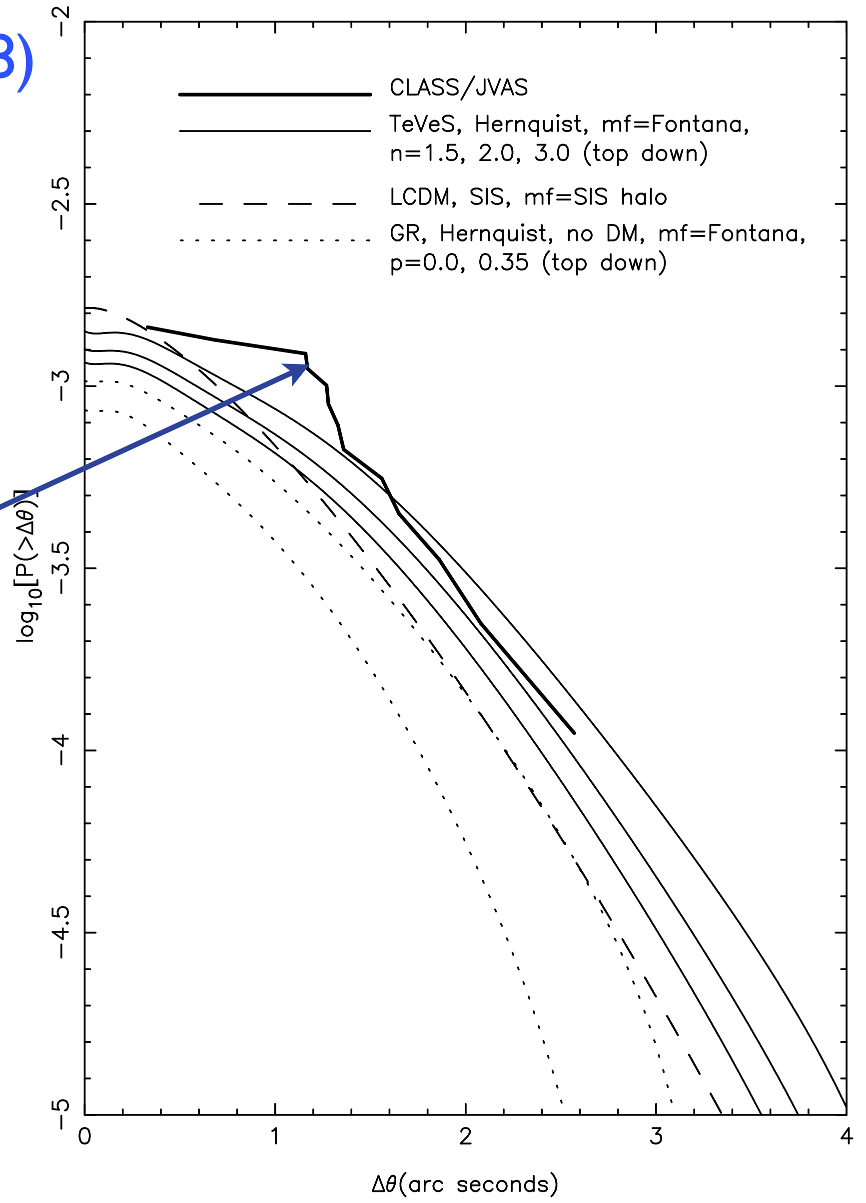
Statistics of image separation and TeVeS

Chen & Zhao, Ap. J. 650, L9 (2006)

Chen, J. Cos .Astropart. Phys. 01, 006 (2008)

CLASS - Cosmic Lens All-Sky Survey

JVAS - Jodrell / VLA Astrometric Survey



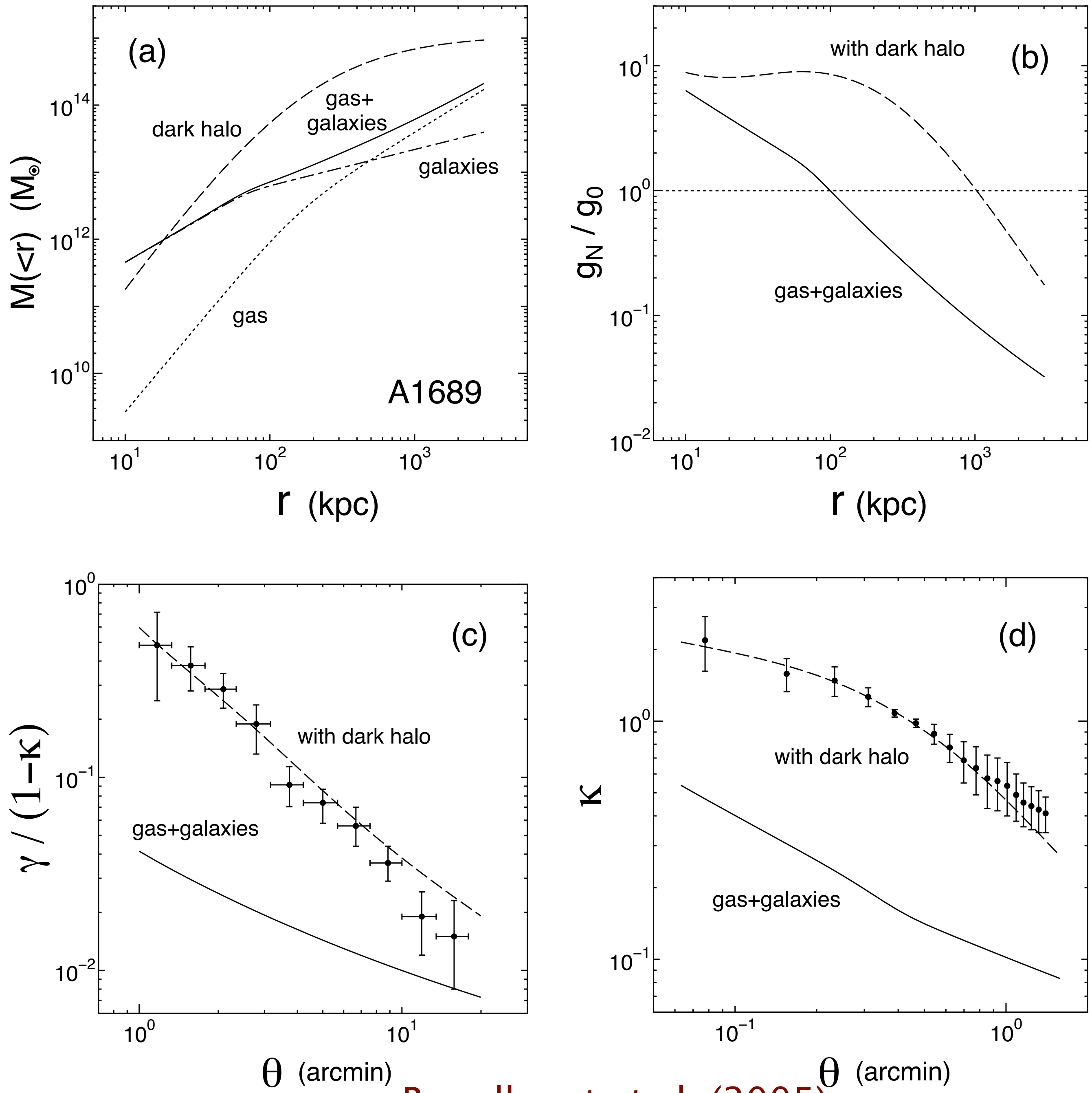
The statistics of images separation in lensed quasars has been problematic for the DM paradigm. In TeVeS it has been investigated by Chen & Zhao and lately by Chen. Shown here is a histogram from latter paper (thick broken line) of the observed frequency of pair separations as a function of angle from the CLASS and JVAS surveys (containing 9000 quasars with 13 cases of multiply lensed ones) The 3 thin solid lines are TeVeS predictions for three interpolating mu's. The 2 dotted lines are predictions from GR sans dark matter for two assumed M/L. The above assume an open cosmology with $\Omega_b = 0.04$ and $\Omega_\Lambda = 0.5$, model the lenses (mostly elliptical galaxies) with Hernquist profiles, and described their space distribution with Fontana's mass function. The dashed line is the ΛCDM prediction for a flat cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, with the lenses modeled as singular isothermal spheres and populated according to the Fontana mass function. As you can see here TeVeS comes out on top. The upcoming big lens surveys should allow a more enlightening confrontation between the DM and TeVeS approaches. CLASS – Cosmic Lens All-Sky Survey JVAS – Jodrell/VLA Astrometric Survey

Spherical clusters

Takahashi & Chiba,
Ap. J. 671, 53 (2007)

A1689

Natarajan & Zhao,
MNRAS 389, 250 (2008)



Broadhurst et al. (2005)

What about clusters of galaxies? Takahashi & Chiba analyzed weak lensing by three quasispherical clusters. For each they obtained the gravitational field with TeVeS using several acceptable $\tilde\mu$ functions and a model mass profile inferred from a large cluster sample from the SDSS survey. Their predicted shear and convergence of the lensed light failed to fit the observations unless they added a large mass in neutrinos, as proposed by Sanders. Here are typical results for Abel 1689. Top left (a): The mass profiles of the gas (dotted line), the galaxies (dot-dashed line), the gas+galaxies (solid line), and the dark halo (dashed line Topright (b): The Newtonian gravitational acceleration g_N normalised to a_0 for only baryonic components (gas+galaxies) (solid line) and including DM (dashed line). Left bottom (c) and Right bottom (d): The measured reduced shear $\gamma/(1-\kappa)$ and the convergence field κ as a function of the angular radius. (Data from Broadhurst et al. 2005). The solid lines are the pure TeVeS predictions; DM had to be added to produce the dashed line so MOND/TeVeS alone cannot explain the data a dark halo. Similar conclusions are reached by Natarajan \& Zhao. Conclusions not surprising given MOND's impotence in regard to the dynamical data for clusters.

The bullet cluster - a postscript

* Angus, Shan, Zhao, & Famaey, Ap. J. **654**, L13 (2006)
* Feix, Fedeli & Bartelmann, A&A. **480**, 313 (2008)
* Ferreira and Starkman, Science **326**, 812 (2009)

Angus et al: able to model bullet lensing with TeVeS, but need the baryons to supplemented with neutrinos. Feix et al confirmed that the source of gravity in the bullet must include an invisible component. They asserted that nonlinearity of the AQUAL-TeVeS equation will not generate the observed correlation between lensing and the galaxies (whose mass is subdominant). But Ferreira and Starkman, in their **Science** review, make the point that the absence of a Birkhoff theorem could lead to just such nonintuitive correlations.

It is time now to go to the large scale

Cosmology with TeVeS

B, Phys. Rev. D **70**, 083509 (2004)

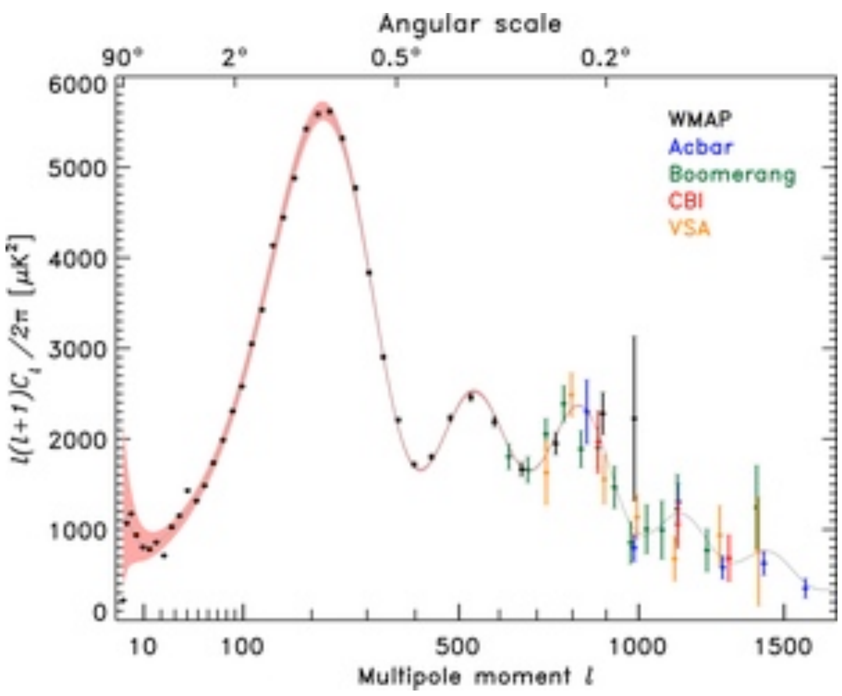
Chiu, Ko & Tian, Ap. J. **636**, 565 (2006)

Zhao, Bacon, Taylor & Horne, MNRAS **368**, 171 (2006)

Bourliot, Ferreira, Mota and Skordis, Phys. Rev. D**75** (2007) 063508

Sagi & B, Phys. Rev. D **77**, 103512 (2008)

ΛCDM

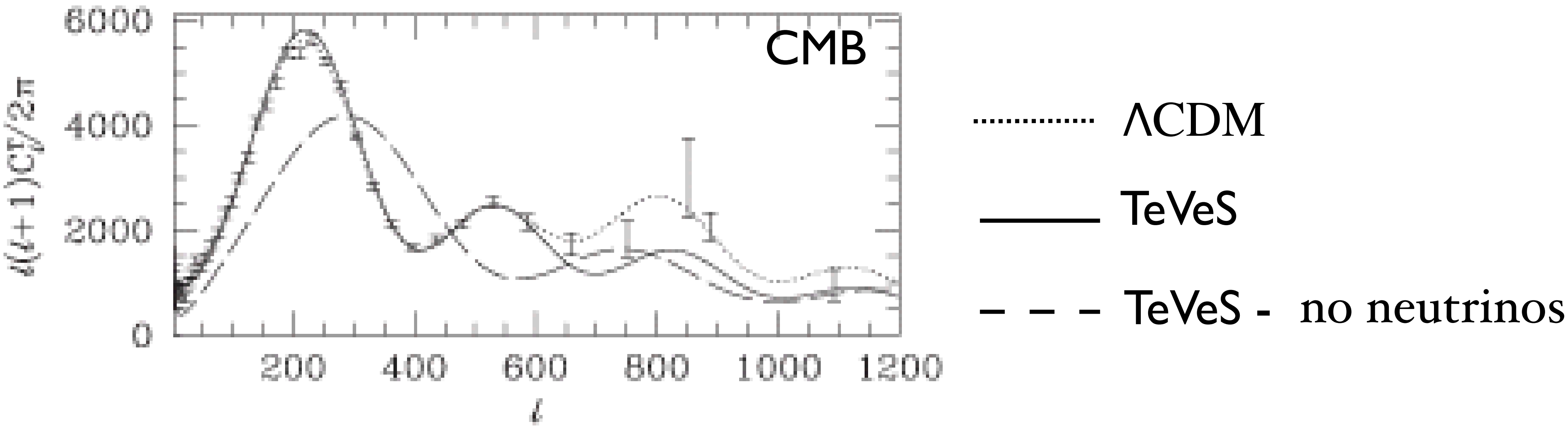


The simpler issues of cosmology are easily clarified. Bekenstein: Friedmann models can be transplanted to TeVeS.
Chiu et al. and Zhao et al.: Angular distance within an isotropic cosmological model.
Bourliot et al.: study of isotropic cosmological models in TeVeS narrowing the range of \mu functions suitable in face of the cosmological facts.
Sagi and Bekenstein: despite TeVeS's scalar sector, G is constant during the expansion. However, a_0 decreases, though slowly. RC's of large z galaxies are not yet good enough to put this to the test.
But no question about it, in cosmology the principal challenge to a theory like TeVeS is to compete with ΛCDM 's parameterized fit of the angular CMB power spectrum. DM aficionados regard this as absolute proof of the presence of dark matter globally, and an impossible challenge for something like MOND.

Cosmological fluctuations in TeVeS

Skordis, Phys. Rev. D**74**, 103513 (2006)

Skordis, Mota, Ferreira, and Boehm, Phys. Rev. Letters **96**, 011301 (2006)



Dodelson and Liguori, Phys. Rev. Letters **97**, 231301 (2006)

The theory of cosmological perturbations in TeVeS was first worked out by Skordis. Even before that paper came out, Skordis et al made use of the results to provide a fit of the CMB angular power spectrum with TeVeS. Data as of 2006. The Λ-CDM model (dotted line). The solid line is for a TeVeS cosmology with $\Omega B = 0.05$, $\Omega\Lambda = 0.78$ and $\Omega\nu = 0.17$ (very massive sterile neutrino) and a0 ≃ 4.2 ×10^(−8) cm/s^2. The dashed line is for for a TeVeS cosmology with $\Omega\Lambda = 0.95$ and $\Omega B = 0.05$. TeVeS with $\Omega\nu = 0.17$ fits well the first two peaks. There was a similar good fit for the baryon spatial spectrum obtained by SDSS.

Dodelson and Liguori concurred that TeVes is successful at this level, but differed from Skordis in ascribing to the vector field primarily responsability for supplanting DM.

A statistic to tell theories apart

Zhang, Liguori, Bean & Dodelson, Phys. Rev. Letters **99**, 141302 (2007)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)d\vec{x}^2$$

Ratios of the potentials to the mass overdensity are theory dependent.

From galaxy surveys measure the cross-correlation $\langle \delta n_g \nabla \cdot \mathbf{v}/H \rangle$

Mass conservation leads to the connection $\nabla \cdot \mathbf{v}/H = -(d\ln D/d\ln a)\delta_\rho$

Combine them to get $\langle \delta n_g \, \delta_\rho \rangle$

Lensing convergence $\Longrightarrow \nabla^2(\Psi - \Phi)$

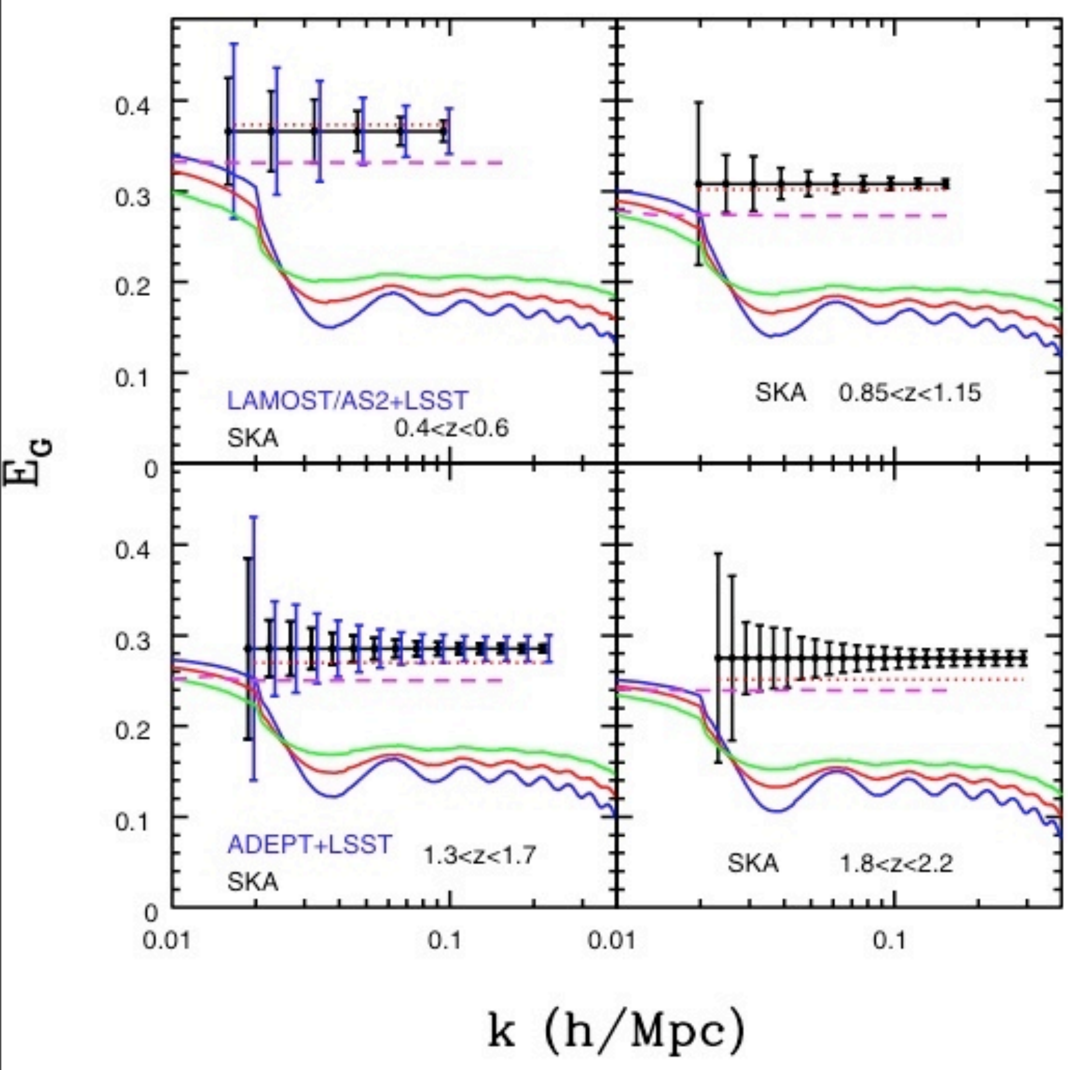
Cross-correlate lensing shear and galaxy density to get $\langle \delta n_g \nabla^2(\Psi - \Phi) \rangle$

$$\hat{E}_G; \qquad \langle E_G \rangle = \frac{\nabla^2(\Psi - \Phi)}{\delta_\rho}$$

The near degeneracy between the predictions of GR with DM and TeVeS (with neutrinos) led Zhang et al. to suggest a better way to distinguish between the theories. Small perturbations of isotropic cosmology are described, both in GR and modified gravities, by this metric. The predicted ratios between the depths of the two gravitational potentials and the mass overdensity are theory dependent; could thus distinguish between theories. The problem is that the mass overdensity is not directly measurable; it depends on assumptions about biasing. Zhang et al's procedure sidesteps the problem in five easy steps: 1) Galaxy surveys ---> cross-correlation of galaxy number overdensity with the expansion of proper velocities. 2) Mass conservation relates the expansion with the mass overdensity via the variation of the growth factor with expansion factor.

3) Combine the above to get correlation of galaxy number overdensity with mass overdensity. 4) Recall that convergence in a lensing map is a measure of Lapl(\Psi-\Phi). So 4) use a lensing survey together with a galaxy survey to cross-correlate galaxy number overdensity with the Lapl. 5) "Divide" the two correlations to "factor out" the galaxy number overdensity. In practice this is done with
E_G, an estimator for the ratio.

Expectations



	Redshift	deg^2	N_{gal}	Band	Operation
LAMOST	$z < 0.8$	10 000	$\sim 10^6$	optical	2008
AS2	$z < 0.8$	10 000	$\sim 10^6$	optical	$\geq$2009
ADEPT	$1 < z < 2$	28 600	$\sim 10^8$	infrared	$\geq$2009
SKA	$z \lesssim 5$	22 000	$\sim 10^9$	radio	2020
LSST	$z \lesssim 3.5$	10 000	$\sim 10^9$	optical	2012

In GR E_G is constant with scale, but in TeVeS it varies with it. In both, in fact, in any theory, E_G varies with redshift. The four graphs from Zhang et al., for 4 different redshift ranges, show predictions for E_G as function of scale for three TeVeS models (solid colored lines), a Dvali-Gabadadze-Porrati modified gravity theory (red dotted lines), a f(R) gravity model (dashed magenta lines) and the ΛCDM model (solid black line). The theories in question would be hard to distinguish by differences in the expansion history.

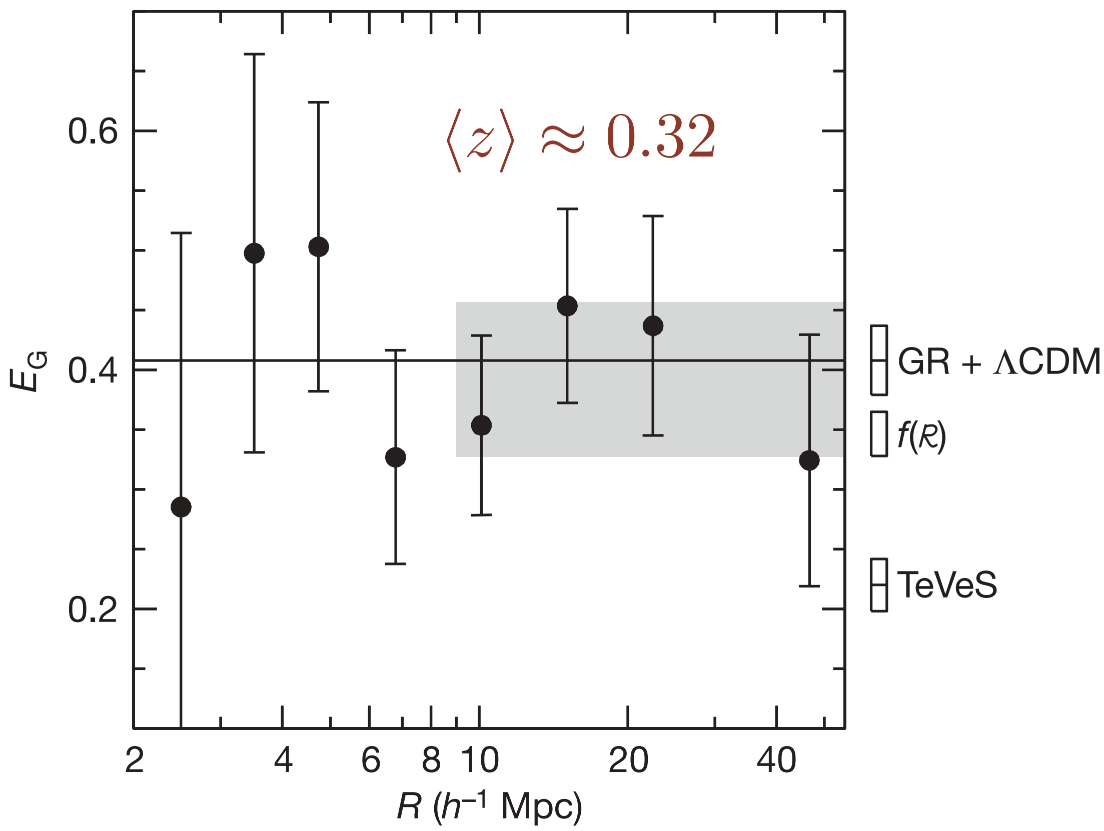
Error bars, all calculated with ΛCDM, are actually the expected sensitivities to become available from various surveys. These span the extant optical LAMOST and AS2 million galaxy surveys, to the billion galaxy Large Synoptic Survey Telescope optical and Square Kilometer Array radio surveys.

You can see that there is a good chance to tell TeVeS and ΛCDM apart, a smaller chance to tell ΛCDM from DGP or f(R) theories.

an actual test already ?

Reyes, Mandelbaum, Seljak, et al. Nature **464**, 256 (2010)

"Confirmation of general relativity on large scales from weak lensing and galaxy velocities"



Last year a group jumped the gun and claimed to have finished the problem. Look at the pretentious title of the paper. They used sample of 70,000 luminous red galaxies from the SDSS with a mean z=0.32. The theoretically predicted ranges of E_G are shown on right. Claim a statistically significant difference between the predicted E_G for a TeVeS' model and the measured one, whereas GR's E_G fit well. But 1) Error bars are large; a much larger sample is really required for a statistically significant discrimination between the theories (Zhang et al). 2) The cited TeVeS prediction for E_G is actually for a redshift interval 0.4-0.6; TeVeS's predictions for E_G actually converge to those of GR as z goes down. 3) The TeVeS cosmological model in question (Zhang et al) comes from TeVeS with only the K term in vector action, the version known to develop caustics (Contaldi eta al), and not to fit all the measured PPN coefficients (Sagi). So the claim of the title is surely premature.

But the fact remains that massive galaxy surveys seem to be the way to clarify many issues involving modified gravity.

Conclusions

- MOND unifies disparate facts about dynamics of galaxies at all scales; but has trouble with clusters
- MOND can be implemented as a nonrelativistic modified gravity (AQUAL)
- MOND may be an overly narrow paradigm; AQUAL with running a_0 may be in order - work in progress
- MOND can be implemented as relativistic TeVeS (and variants of it)
- TeVeS works well for lensing by galaxies and can supplant DM (and perhaps DE) in cosmology; it does less well for clusters of galaxies