

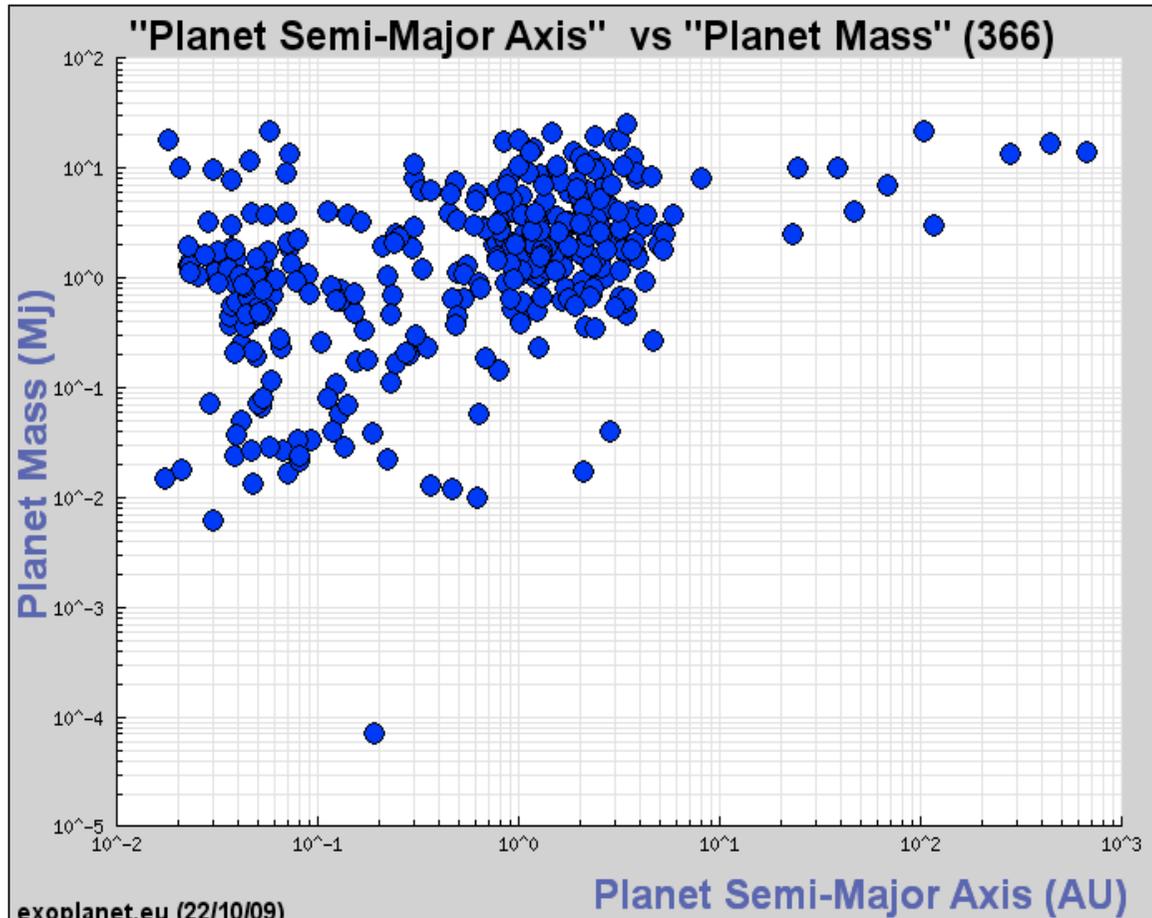
# Planetary migration in gaseous disks: some recent results

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AIM, CEA Saclay



# Some properties of extra-solar planets



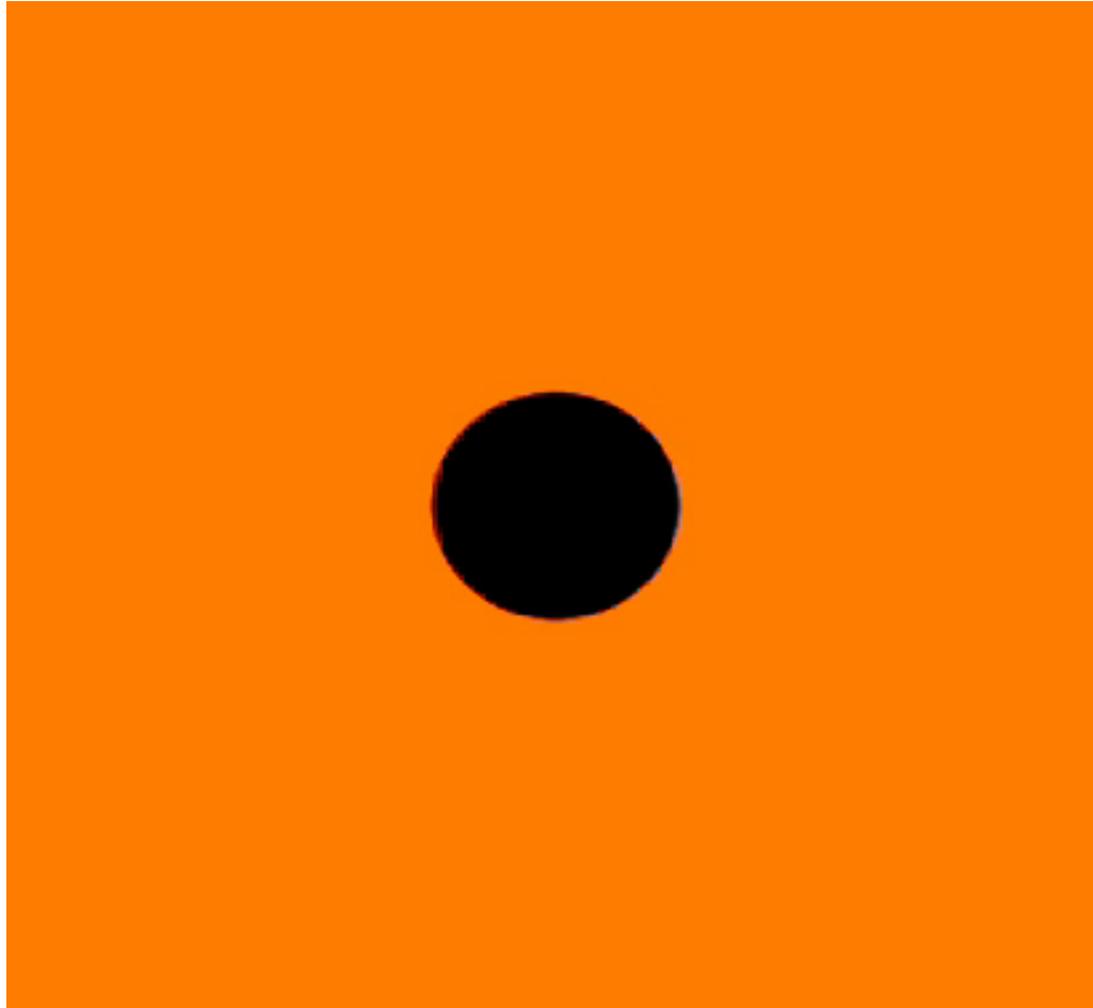
A significant fraction of extra-solar planets orbits very close to its host star ( $<0.2$  AU)

These planets cannot have formed so close from their host star: they must have formed further out and they must have *migrated*.

How do planets migrate ? Can migration account for the statistics of orbital properties of extrasolar planets ?

# Disk response to an embedded planet

Disk response to a point-like mass on a circular orbit



→ The planet tidally excites a one-armed spiral wake that propagates both inwards and outwards.

# What causes planetary migration ?

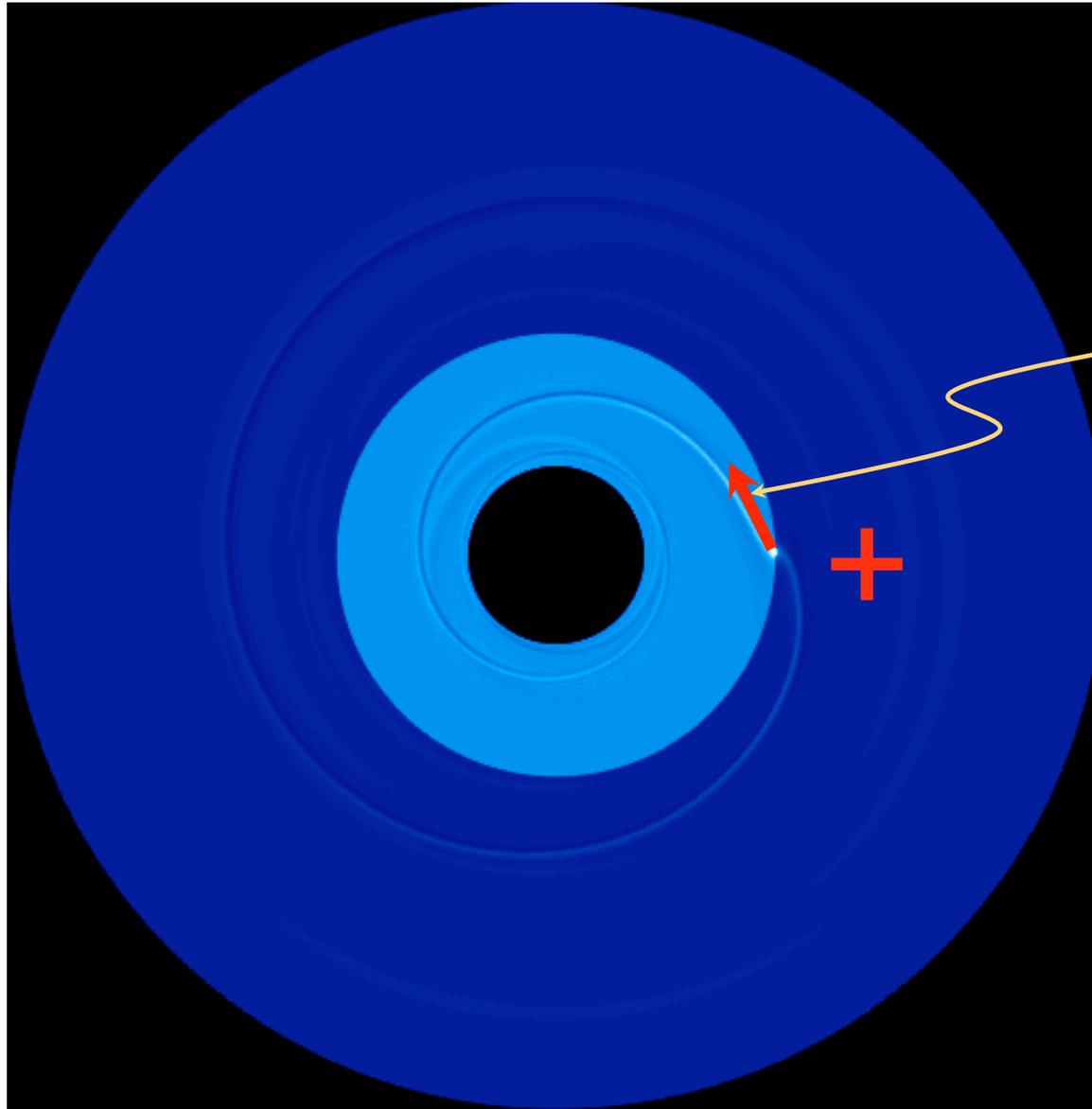
The protoplanet tidally excites a spiral wake.

This wake exerts a gravitational force on the planet

The semi-major axis, the eccentricity, the inclination vary with time.

The most spectacular effect is the variation of the semi-major axis: it is called *planetary migration*.

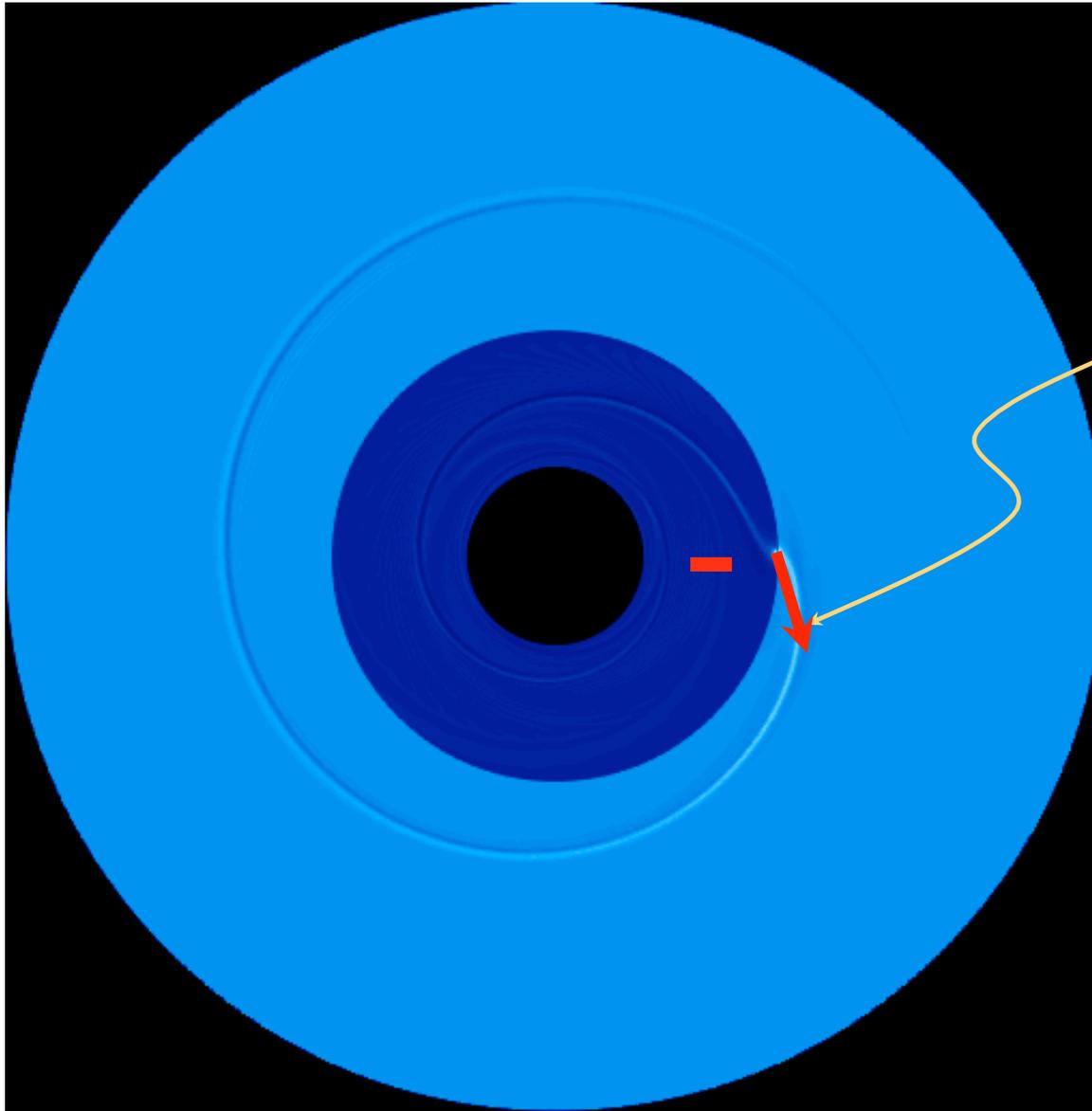
# Inner disk's torque



The inner wake leads the planet.

It therefore exerts a positive torque on the latter

# Outer disk's torque



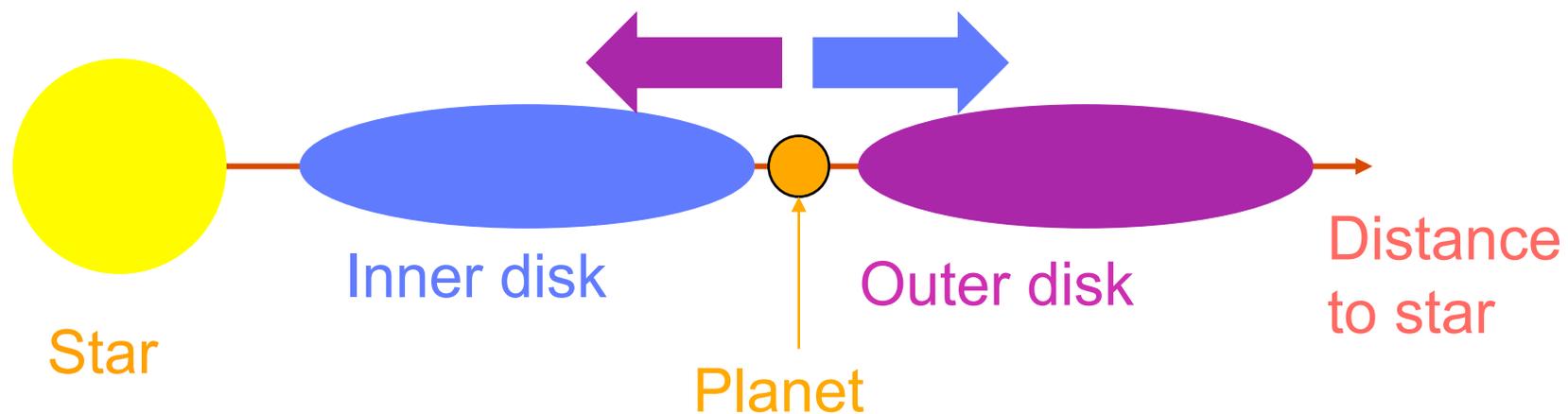
The wake in the outer disk lags the planet.

It therefore exerts a negative torque on the planet.

# Imbalance between inner and outer torques

The specific angular momentum increases with the radius, therefore:

- The inner disk, which exerts a positive torque, tends to impose an outwards migration.
- The outer disk, which exerts a negative torque, tends to impose an inwards migration.

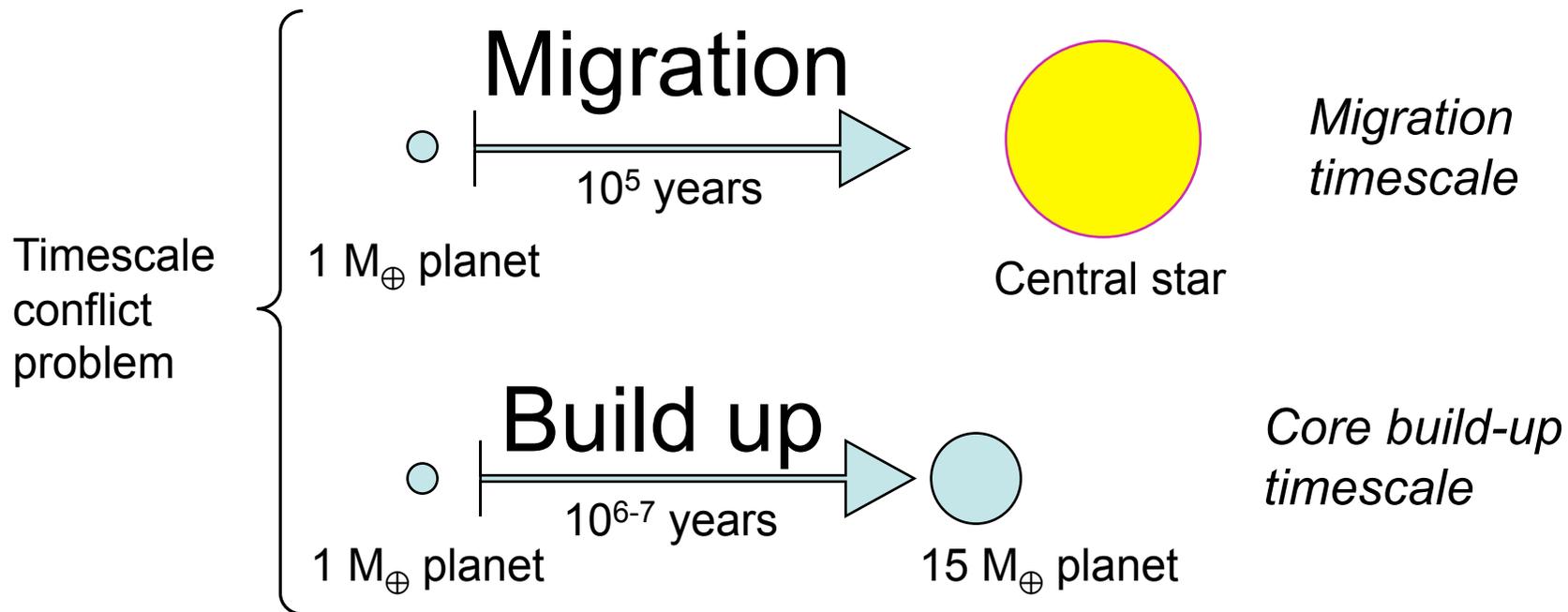


# Type I migration is inwards

- A (slight) imbalance between these two torques yields a migration of the planet.
- The net torque is called the *differential Lindblad torque*.
- In the linear regime (low-mass planet), the differential Lindblad is a negative quantity (Ward 1986).  
→ *Type I* migration is inwards.

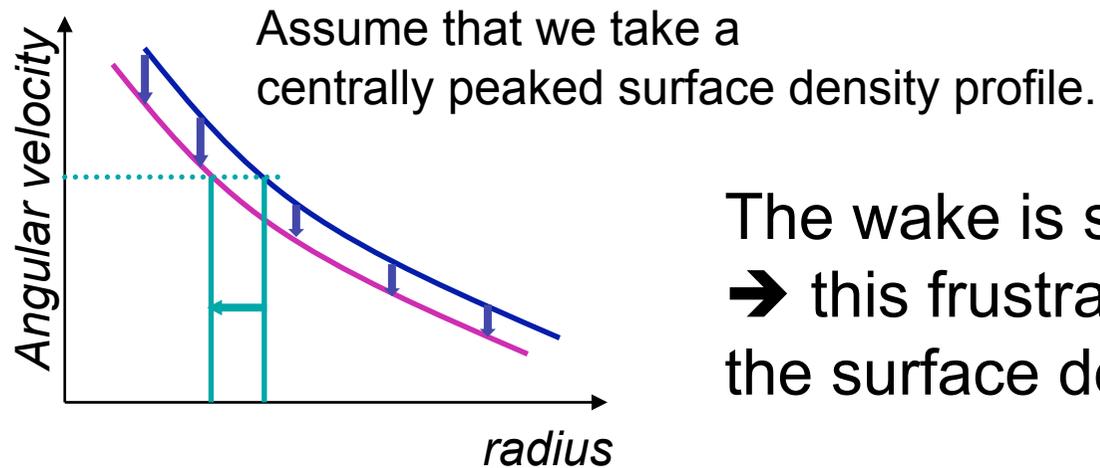
# Type I migration is fast

A one-Earth mass planet, dropped in a MMSN (*Minimum Mass Solar Nebula*) at 1 AU, should be flushed onto the central object in  $\sim 10^5$  yrs.



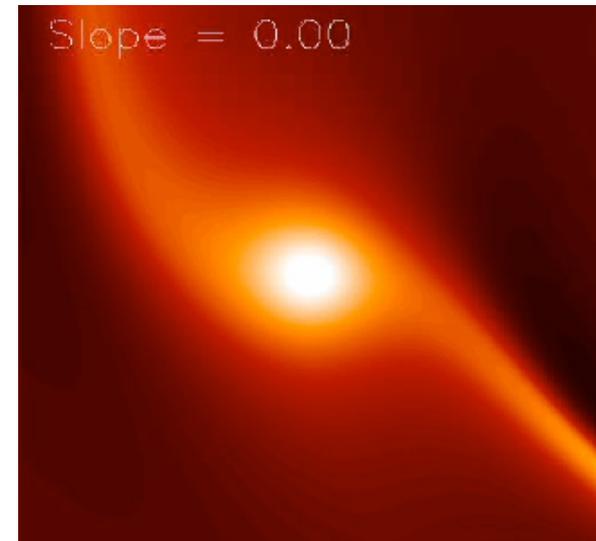
# Type I migration is *really* inwards

The drift rate is quite insensitive to the disk profiles (surface density and temperature).

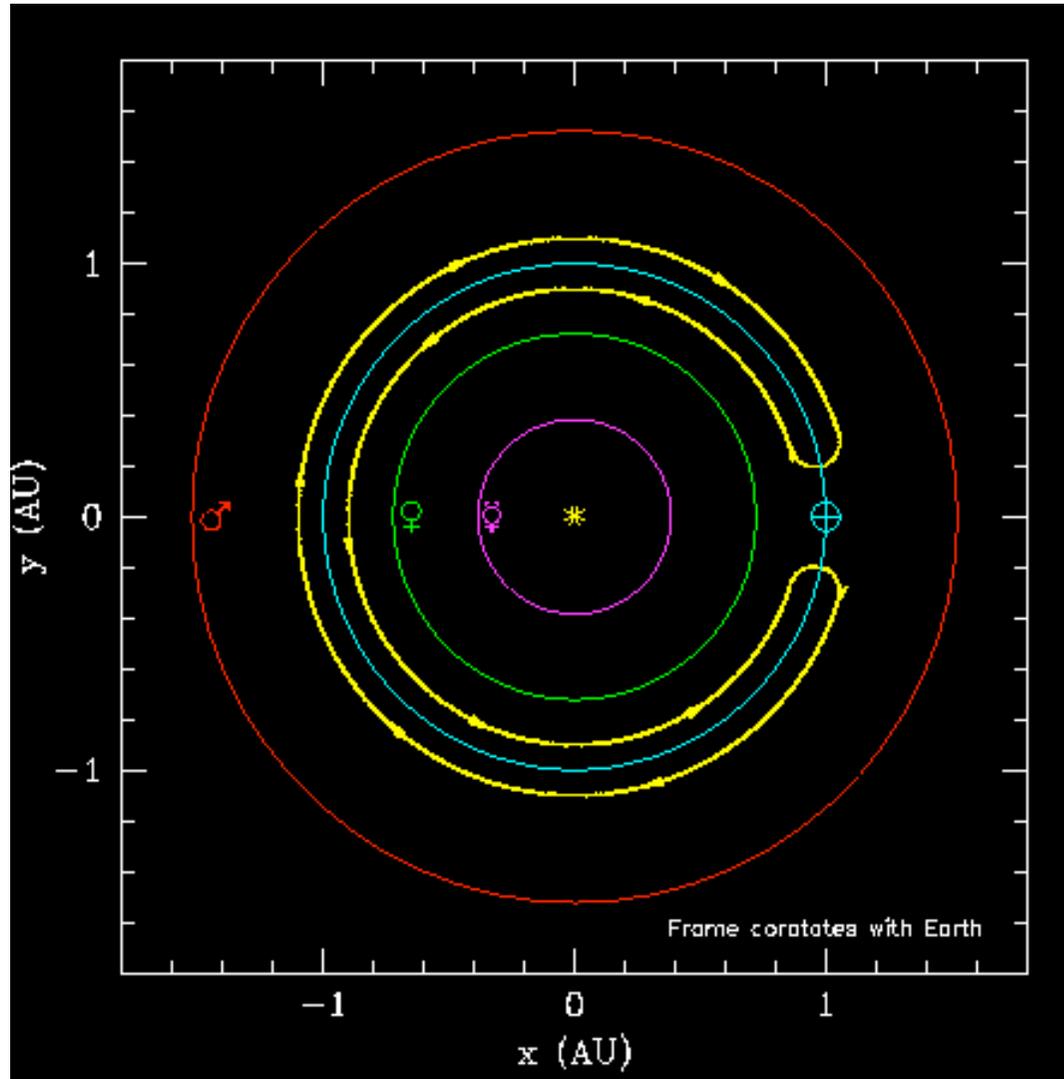


The wake is shifted inwards  
→ this frustrates the effect of the surface density variation.

Pressure buffer effect  
(Ward, 1997)

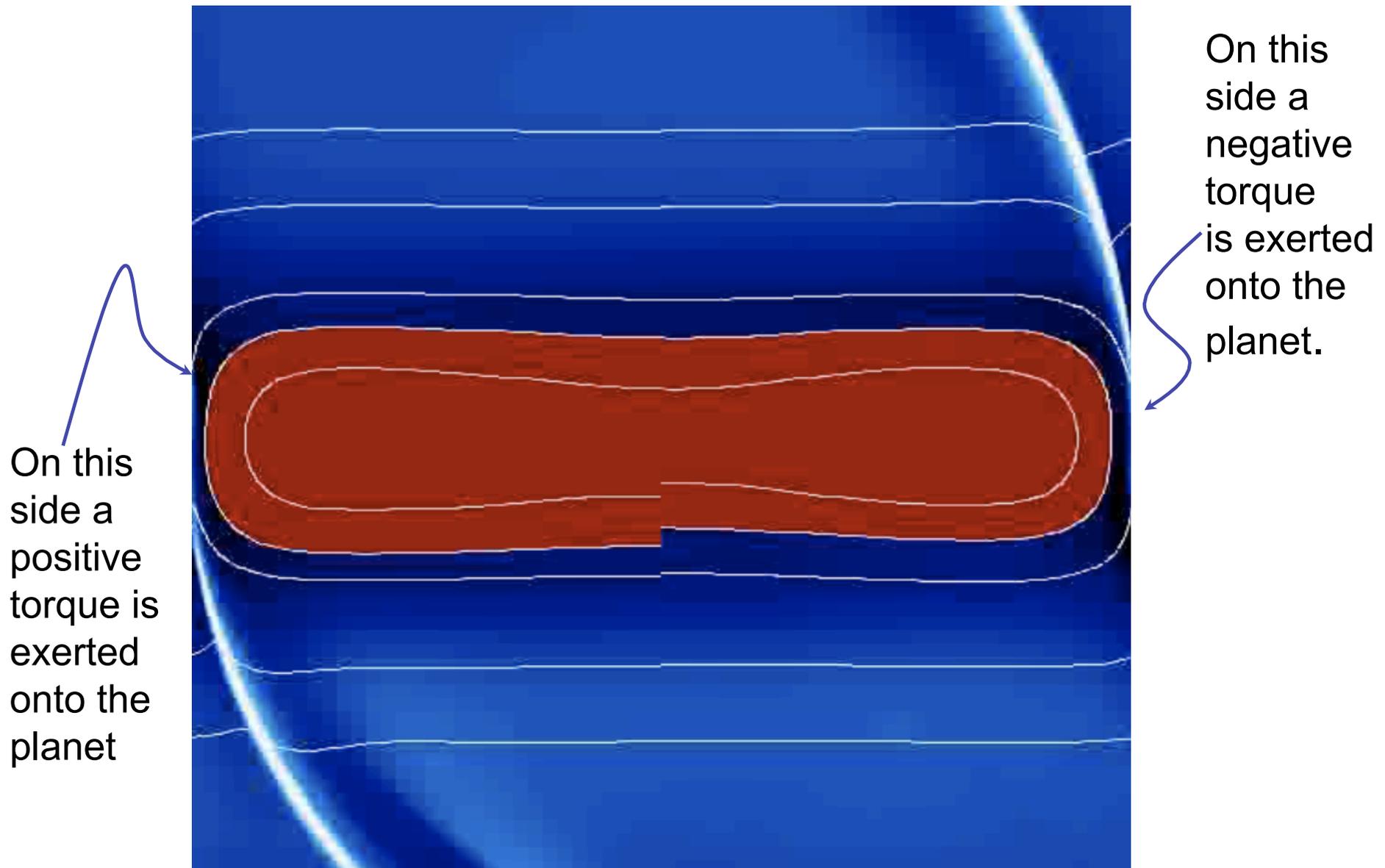


# Coorbital dynamics



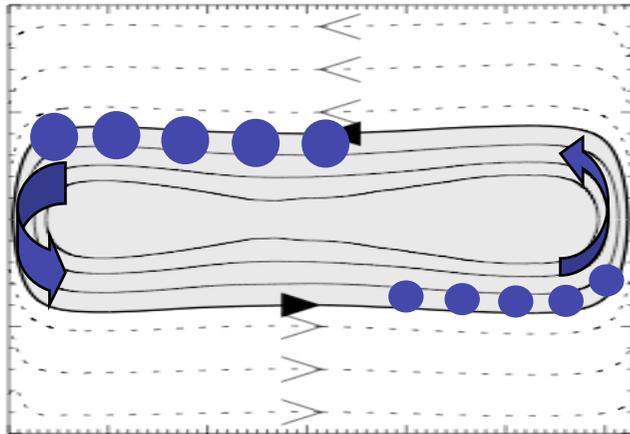
Horseshoe trajectories in the Earth's corotating frame.

# Horseshoe drag and corotation torque



# Horseshoe drag and corotation torque

The horseshoe drag scales with the vortensity gradient (ratio of vorticity and surface density).

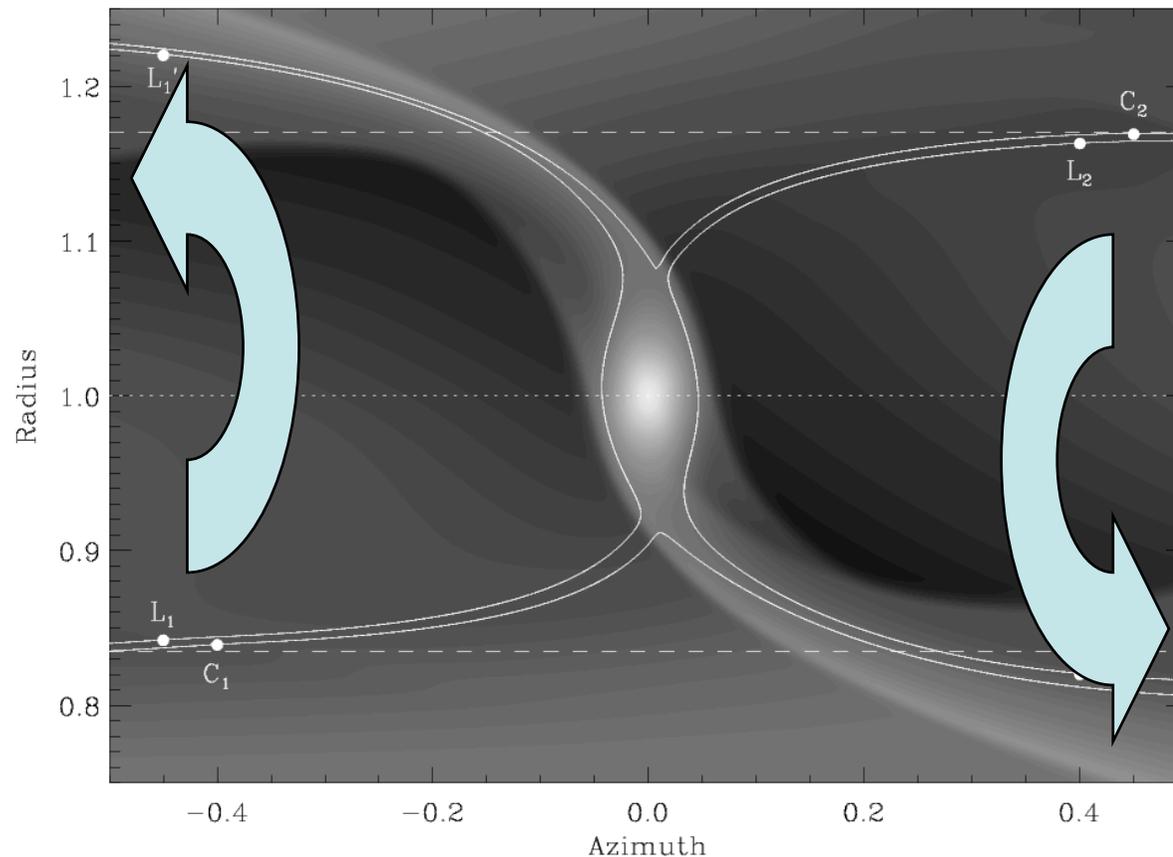


$$\Gamma_c \propto \frac{d(\Sigma/B)}{dr}$$

It is too weak to counteract the differential Lindblad torque in the linear regime.

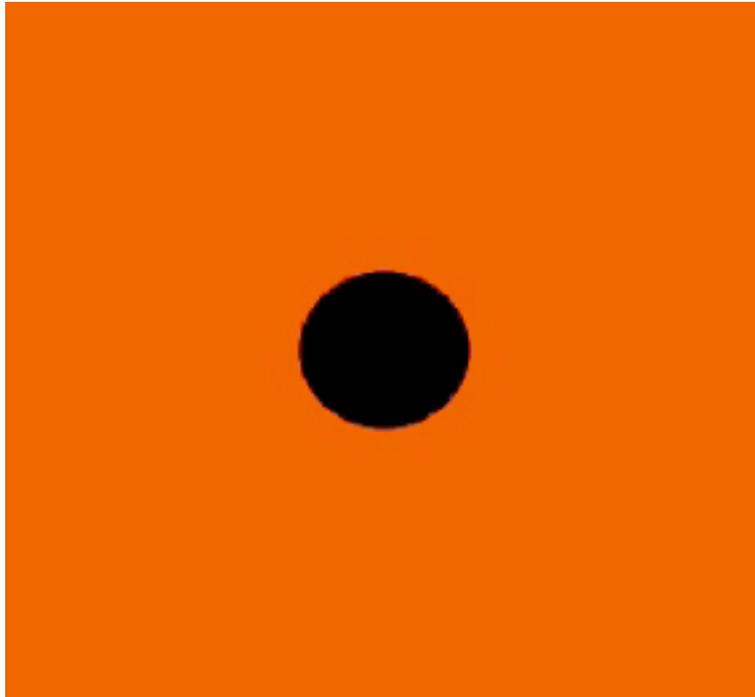
# Gap clearance at large planetary mass

When the planetary mass becomes sufficiently large, the wake degenerates into a shock in the vicinity of the planet. Horseshoe U-turns then become asymmetric.



The gas leaves the horseshoe region in a few libration times and a gap is excavated around the orbit.

# Gap clearance at large planetary mass

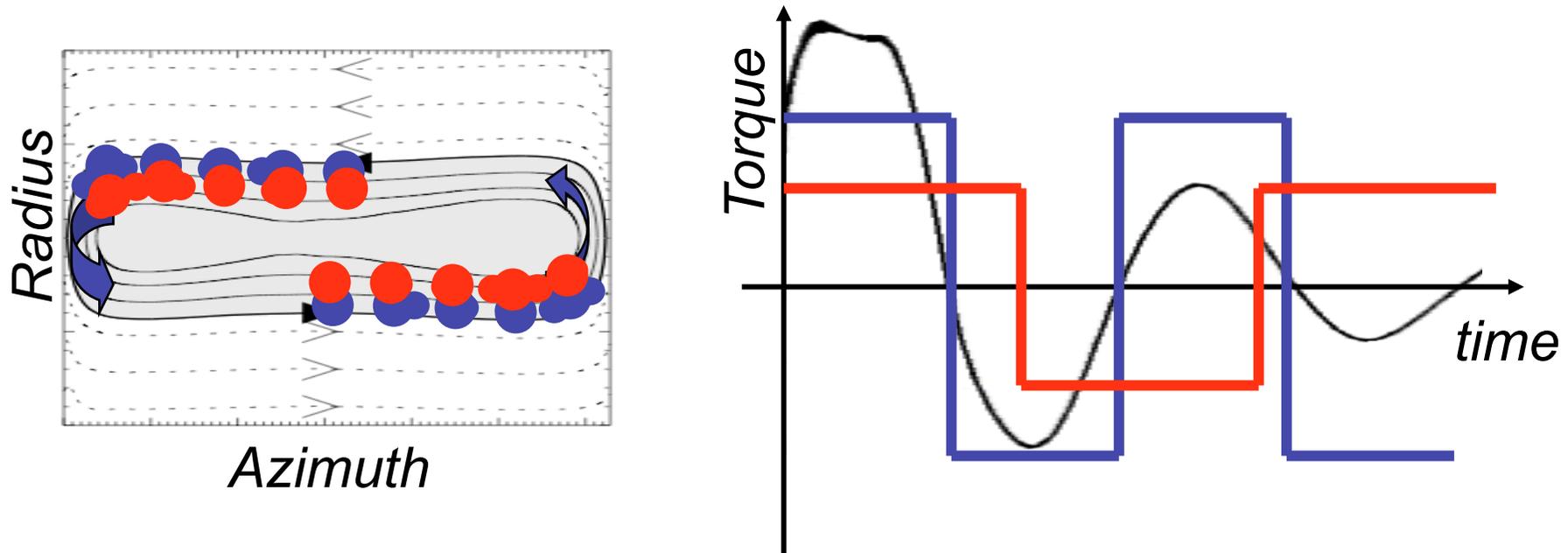


A Jovian planet clears a gap in approximately 100 orbits.

Migration with gap = Type II migration

The planet is locked in the disk's viscous evolution  
→ it is then much slower than type I migration.

# Saturation of corotation torque



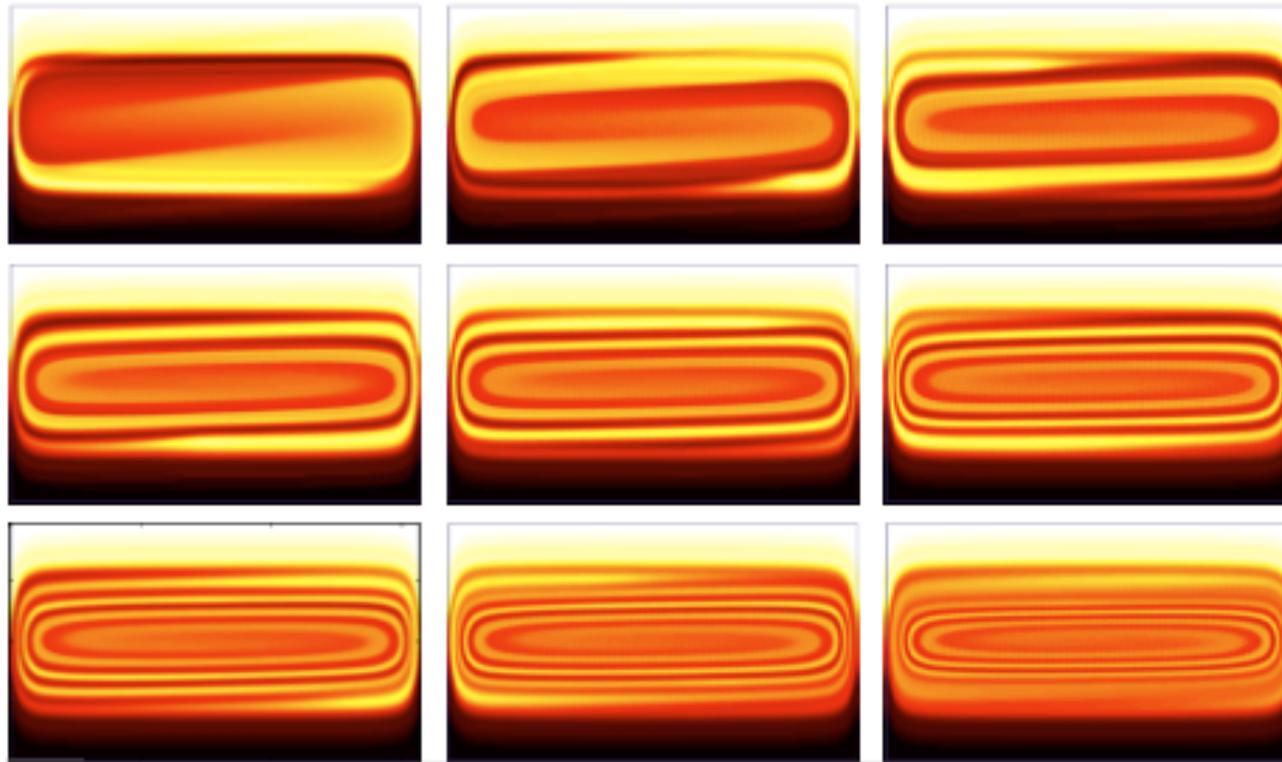
Each streamline gives a periodic contribution to the CR torque

Each streamline has a different libration time

→ CR torque tends to 0 as a result of phase mixing

# Saturation of corotation torque

The vortensity is conserved by an inviscid 2D isothermal flow



Balmforth & Korycansky, 2001

Horseshoe dynamics tends to cancel out the source of the corotation torque.  
Viscous diffusion, if sufficient, can maintain an unsaturated torque.

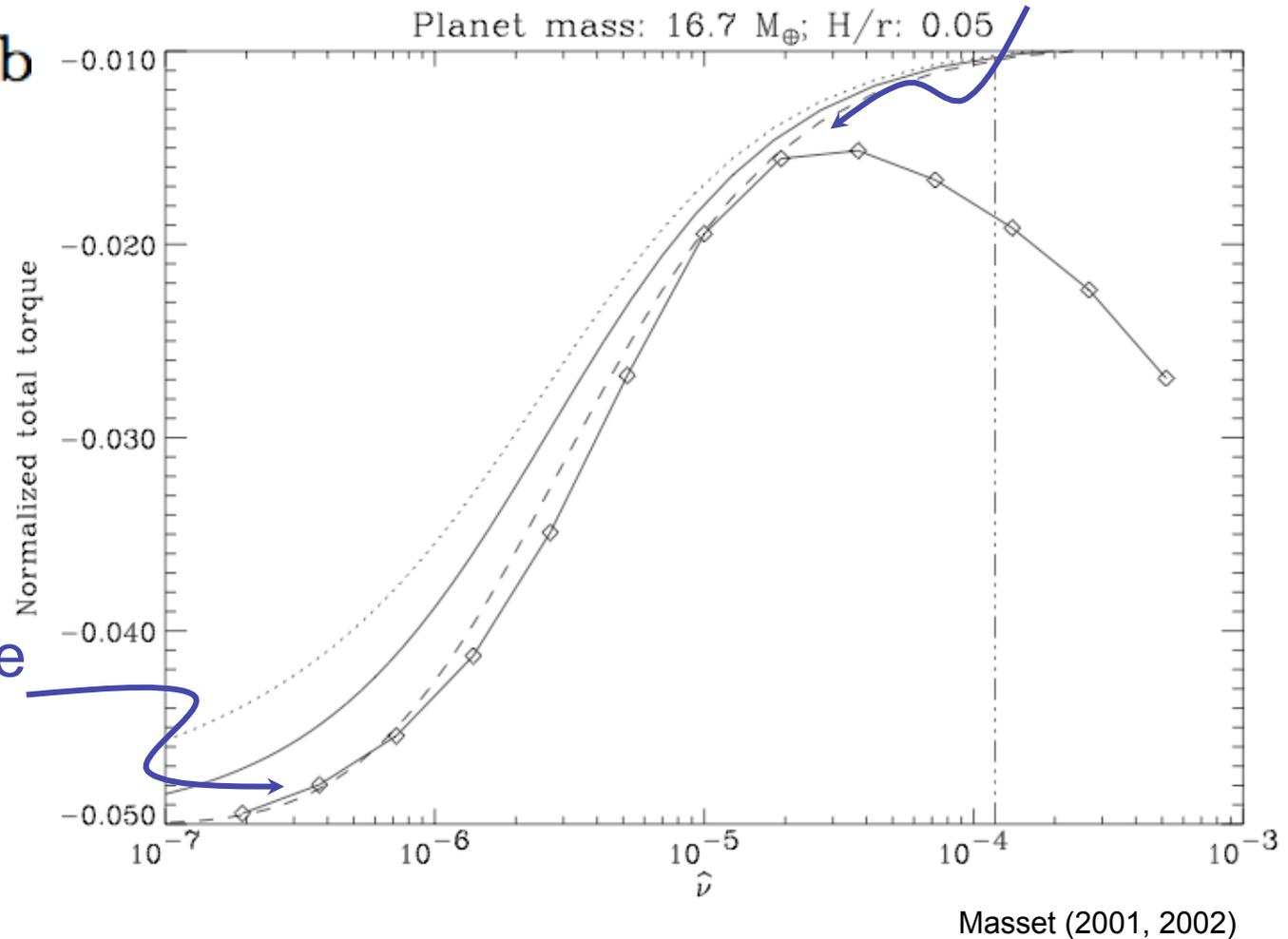
# Viscous diffusion desaturates the CR torque

Viscous diffusion is efficient

if  $\tau_{\text{visc}} < \tau_{\text{lib}}$

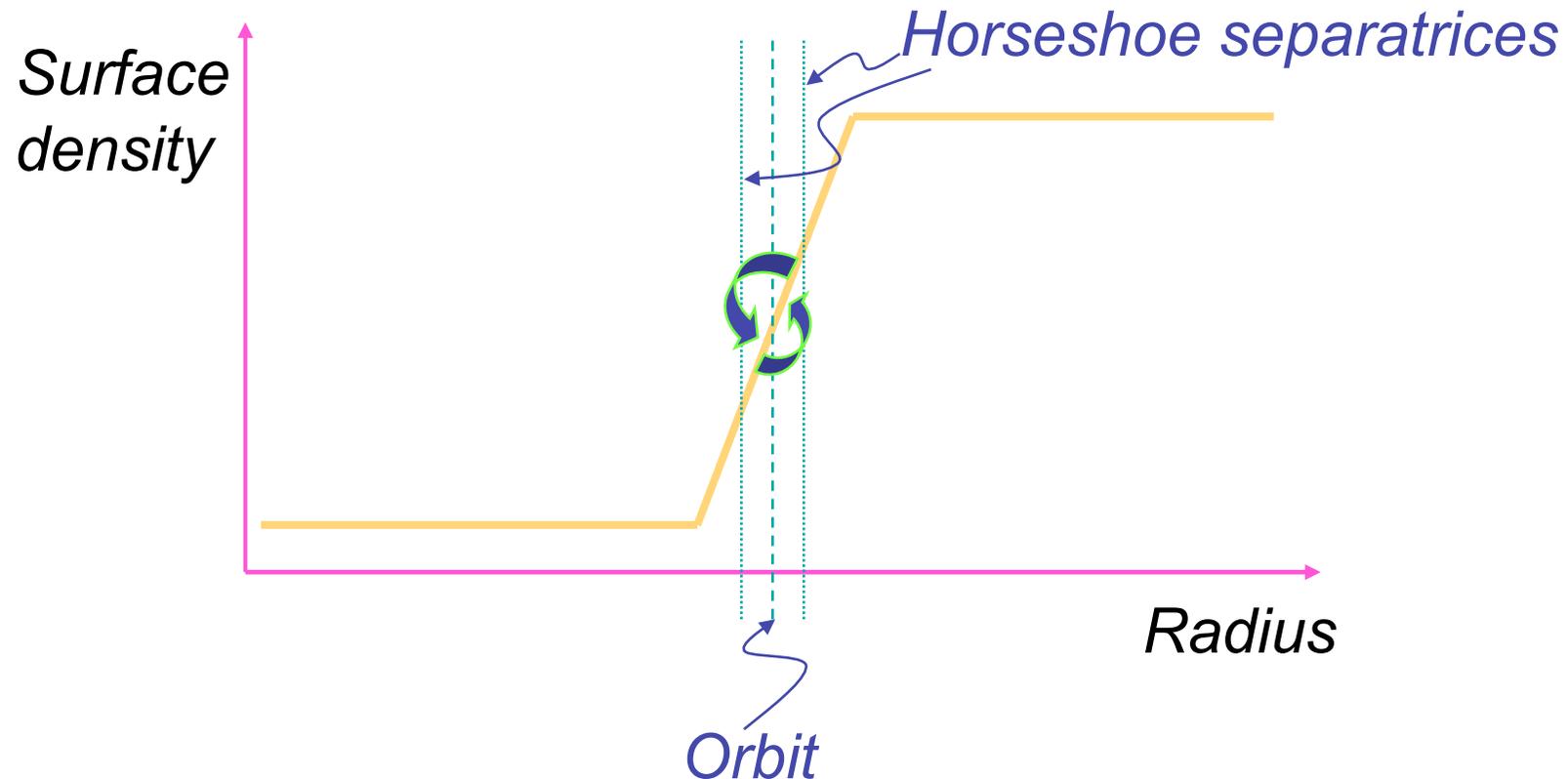
Unsaturated torque  
at large viscosity

Saturated torque  
at low viscosity



# Planetary trap at a cavity edge

The corotation torque scales with the vortensity gradient, hence is sensitive to a strong surface density gradient.



The CR torque can be arbitrarily large and positive at a cavity edge.

# Torques on the cavity edge

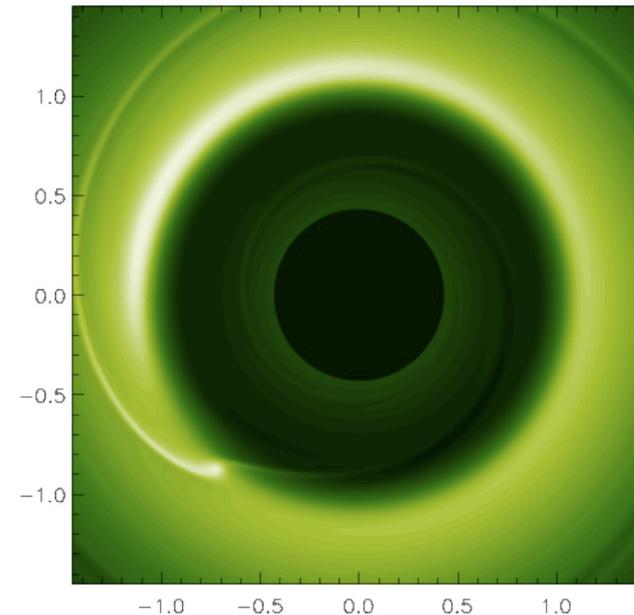
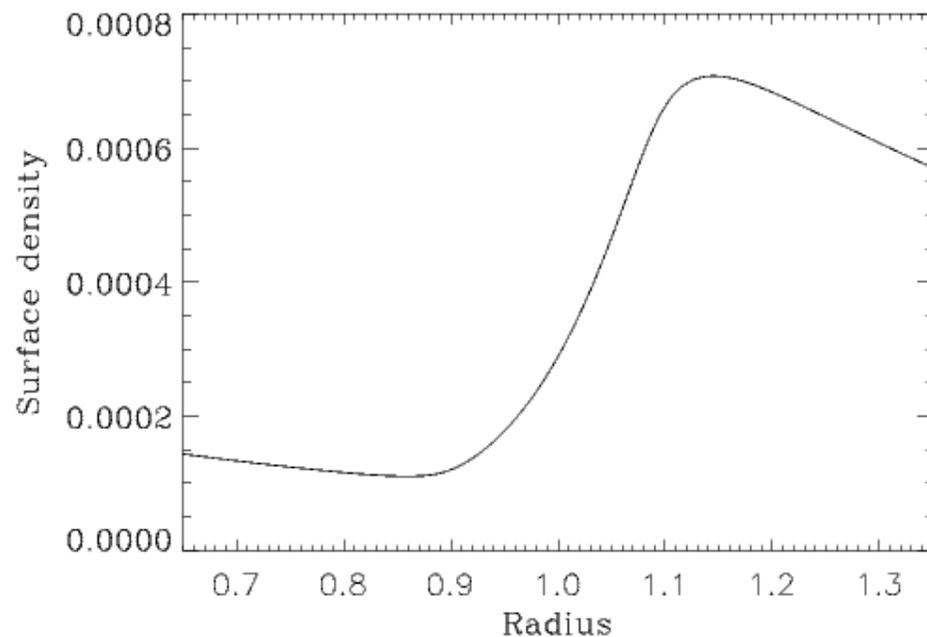
	Power law disk		Cavity edge
Differential Lindblad torque	-1	$\xrightarrow{\text{x } 2 - 4}$	-4
Corotation torque	+0.5	$\xrightarrow{\text{x } 10 - 20}$	+5
TOTAL	< 0		> 0

# Numerical simulations with a cavity

Surface density profile which includes a surface density jump.

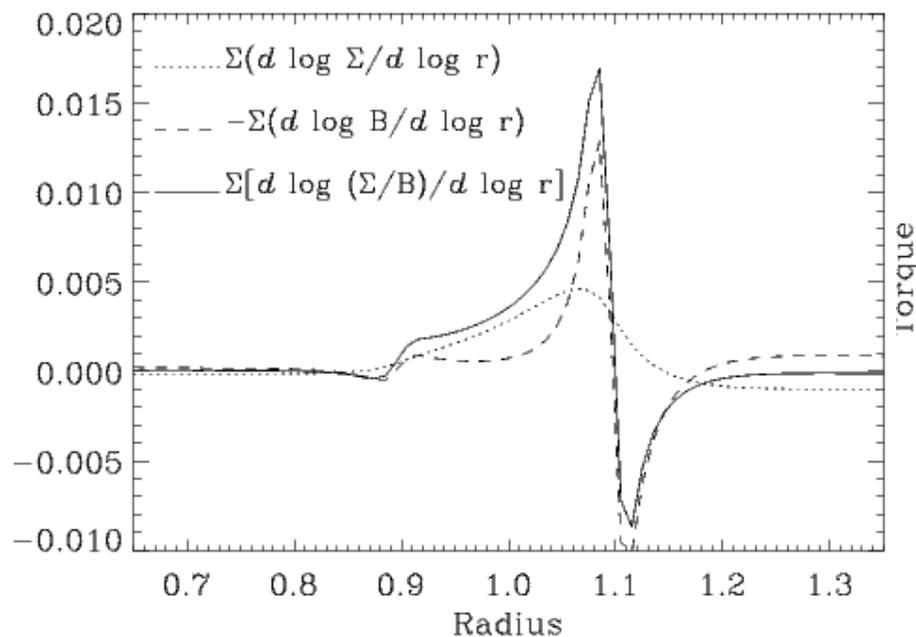
\* Realized with a jump of kinematic viscosity.

\* Relaxed on  $10^5$  orbits



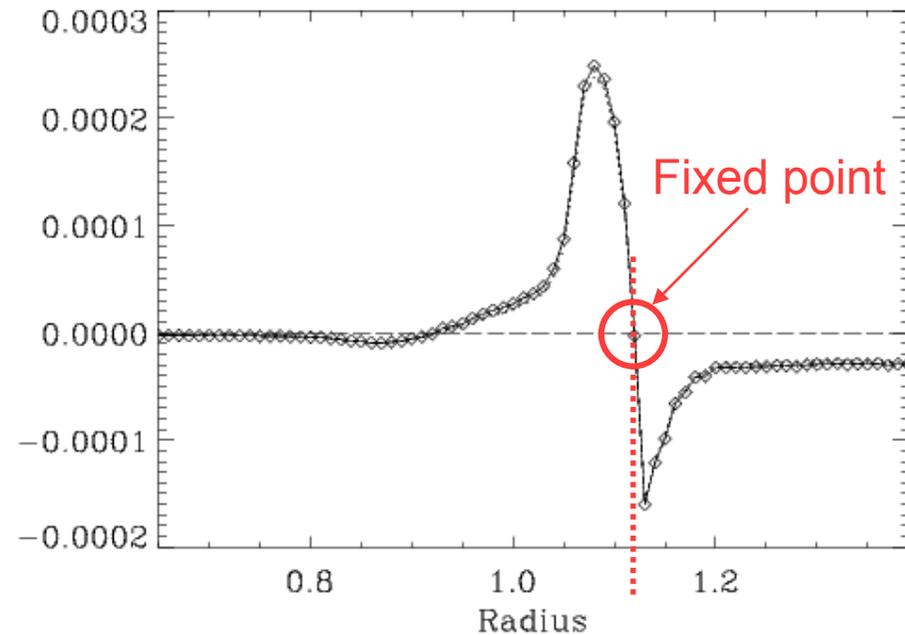
# Numerical simulations with a cavity

Vortensity gradient as a function of radius.

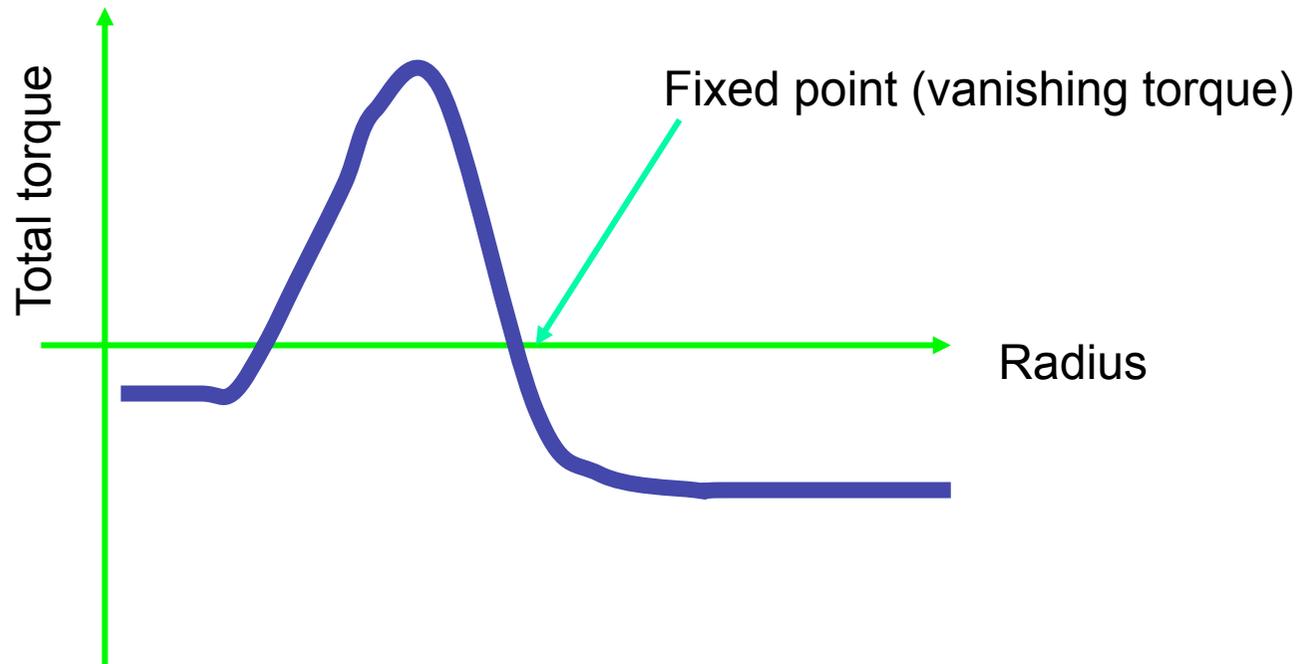


Masset, Morbidelli, Crida & Ferreira (2006)

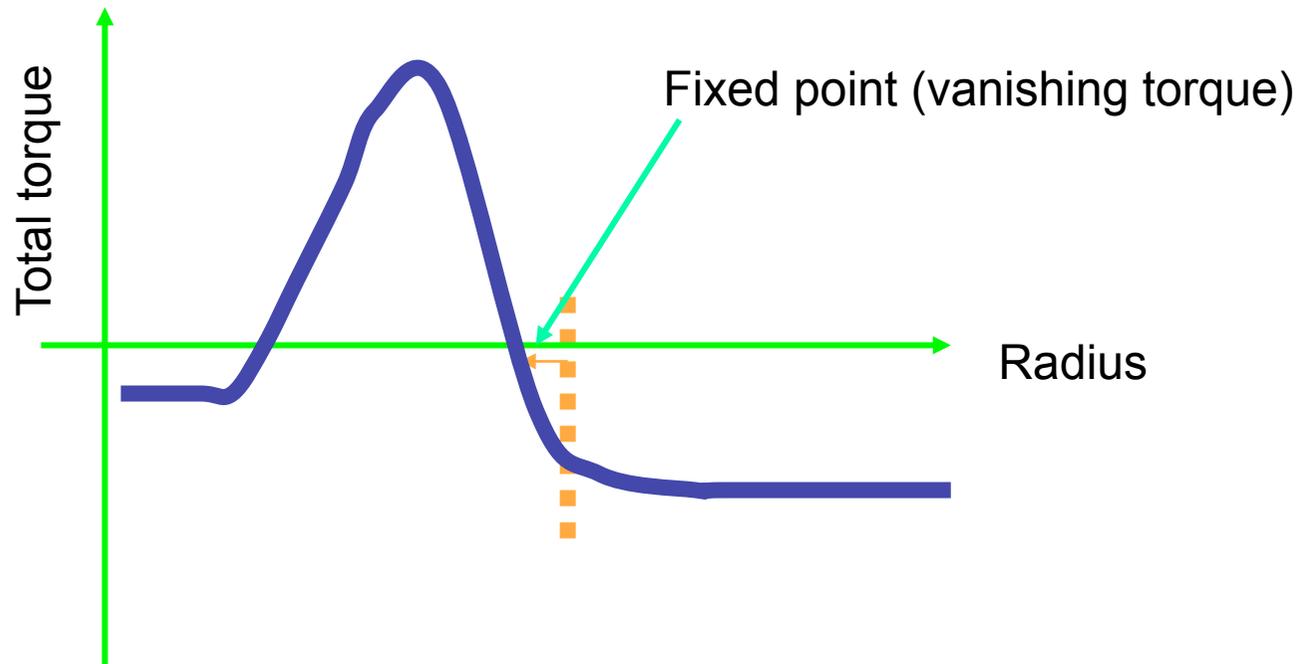
Total torque measured in 75 different calculations with 75 different semi-major axis.



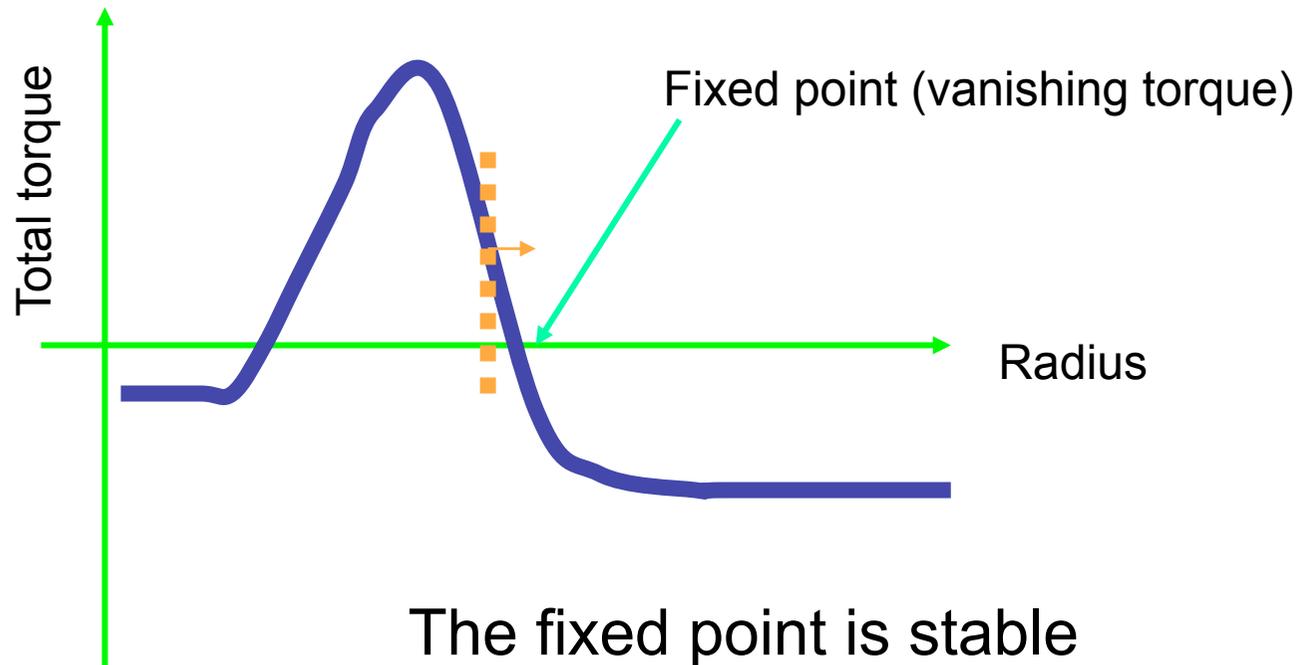
# Fixed point



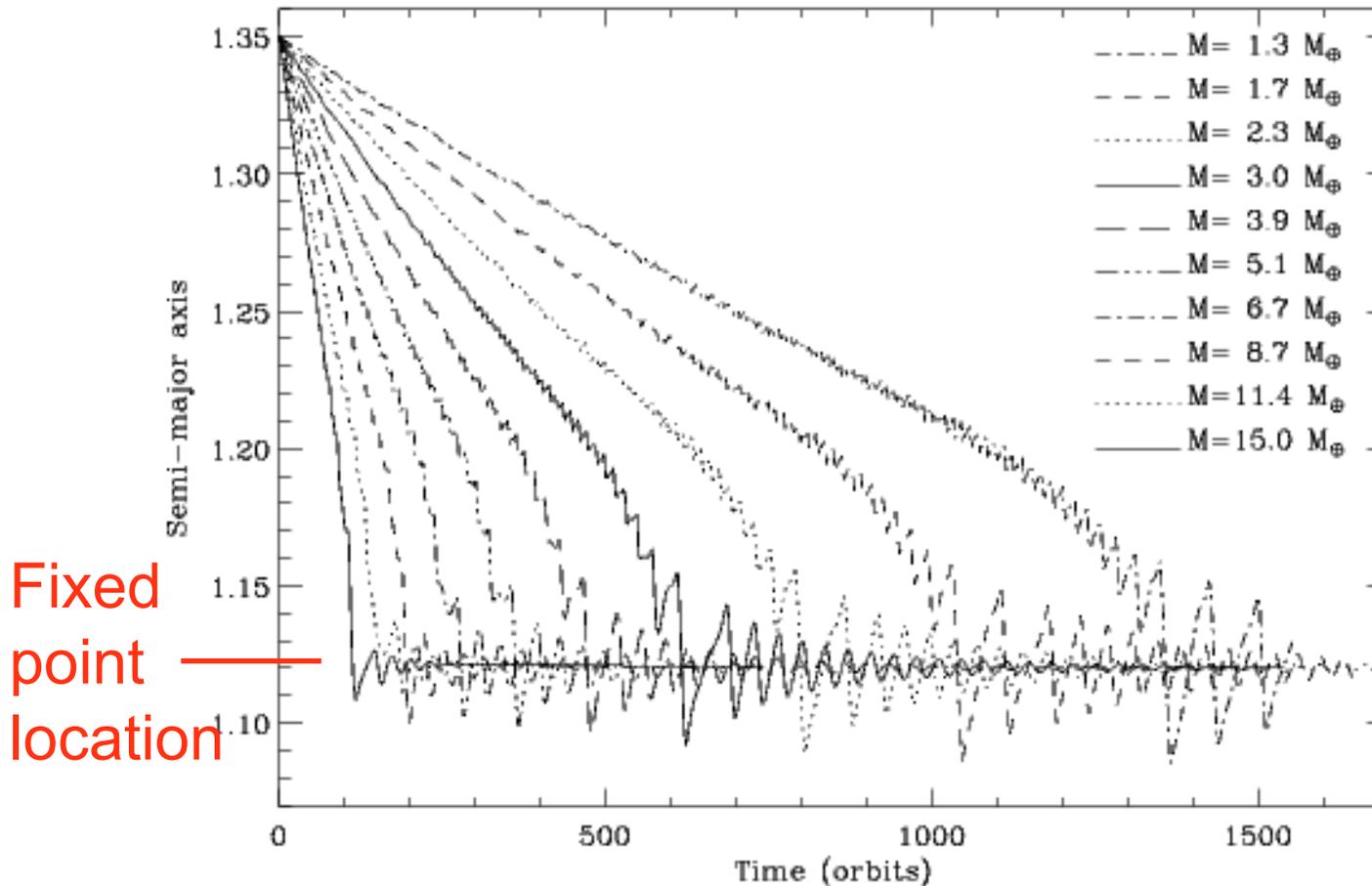
# Fixed point



# Fixed point

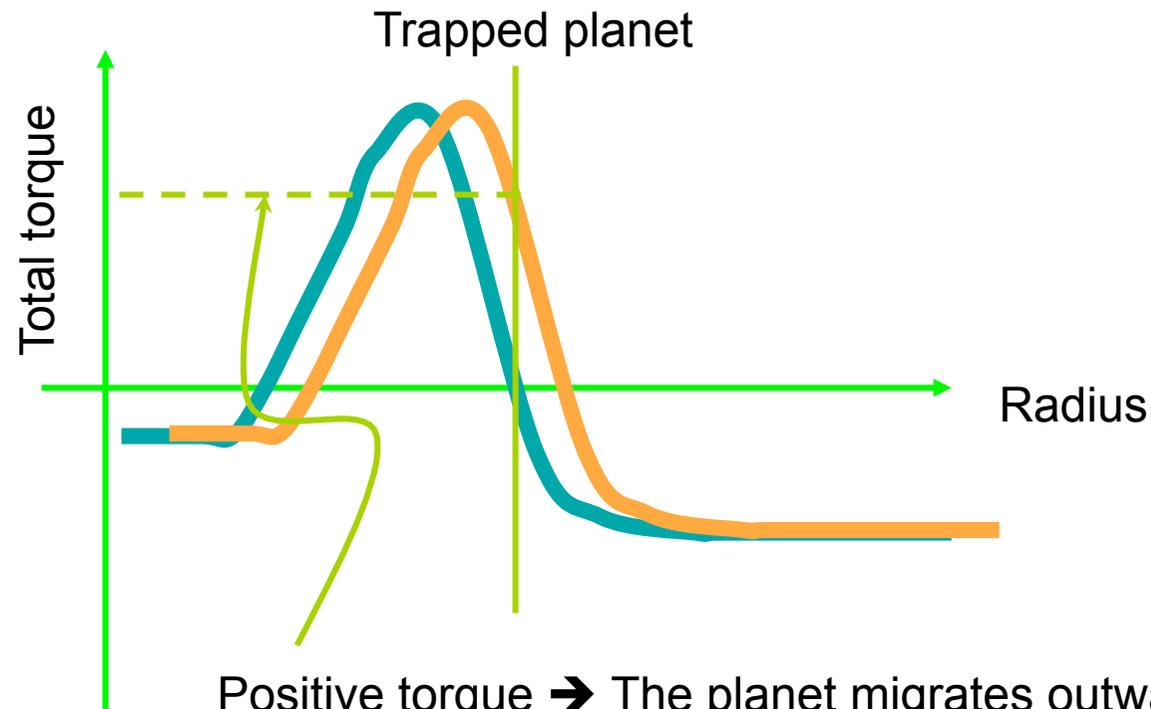


# Planetary trap at work



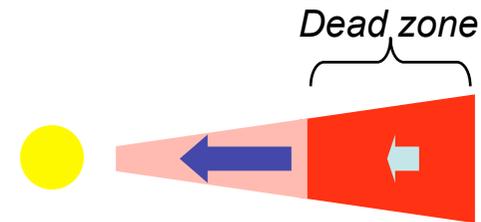
# Trapping on the cavity edge

If the cavity radius varies with time, the trapped embryos follow the cavity edge.



Positive torque → The planet migrates outwards and follows the edge.

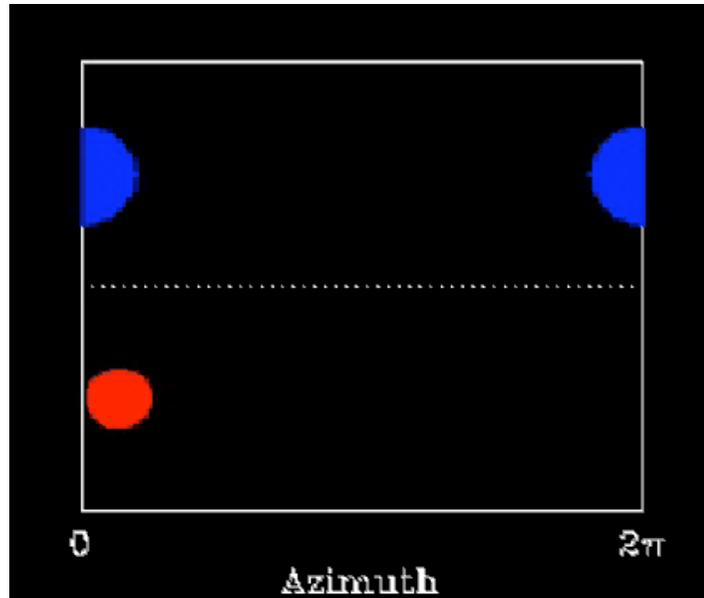
# Example of cavities



- Inner edge of the *dead zone*.
- Transition between a Standard Accretion Disk (SAD) and a jet emitting disk (SED).
- Disc truncated by tidal effects (e.g. at the outer edge of a gap, or disk truncated by a companion star -Pierens & Nelson, 2007-)

Note that some degree of turbulence is requested in order to avoid the CR torque's saturation.

# Corotation torque on a migrating planet



● Planet

● Fluid element

Dotted line : horseshoe separatrix

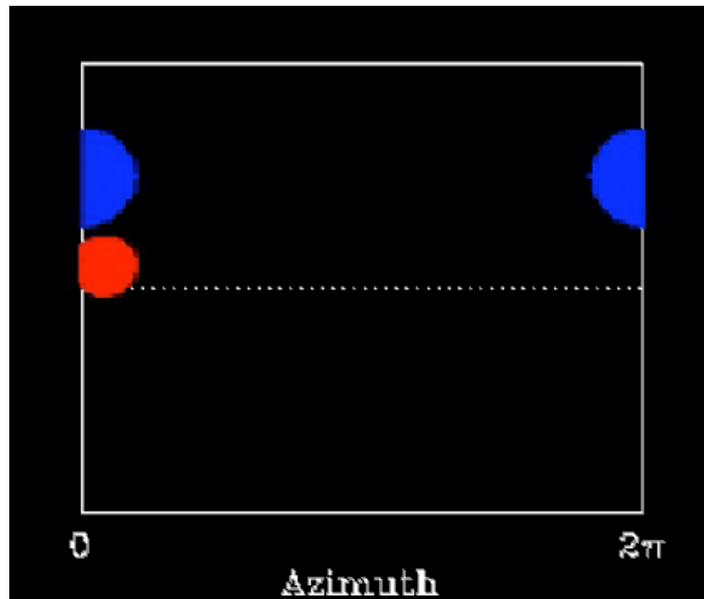
The planet migrates inwards



Fluid elements of the inner disk exert a negative torque

Positive feedback

# Corotation torque on a migrating planet



● Planet

● Fluid element

Dotted line : horseshoe separatrix

The planet  
migrates  
inwards



The horseshoe region exerts  
a positive torque on the planet

Negative feedback

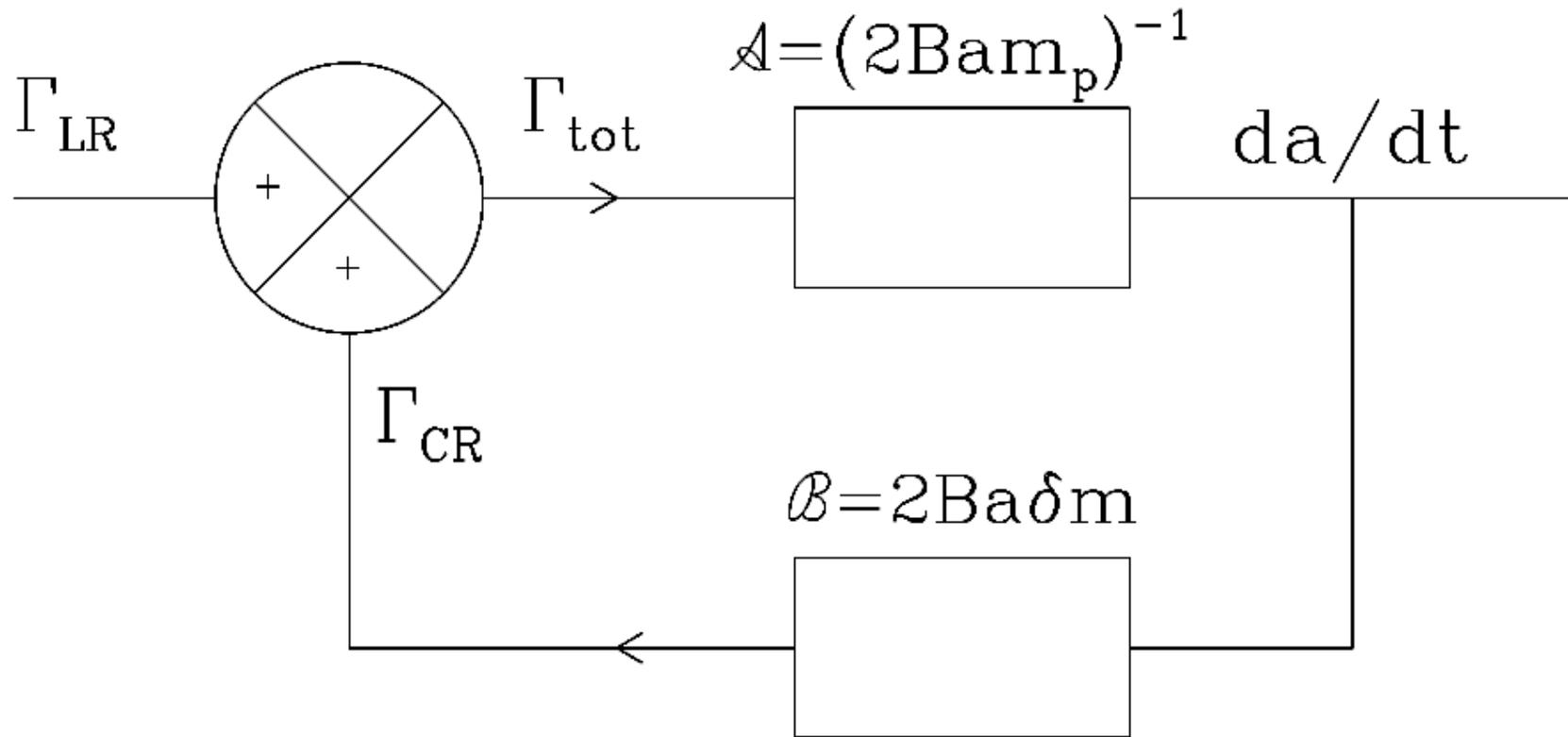
# Corotation torque on a migrating planet

Both feedbacks cancel out if the disk profile is unperturbed.

If the horseshoe region is depleted, the net feedback is positive.

This can therefore happen for planets which deplete their coorbital region (giant planets).

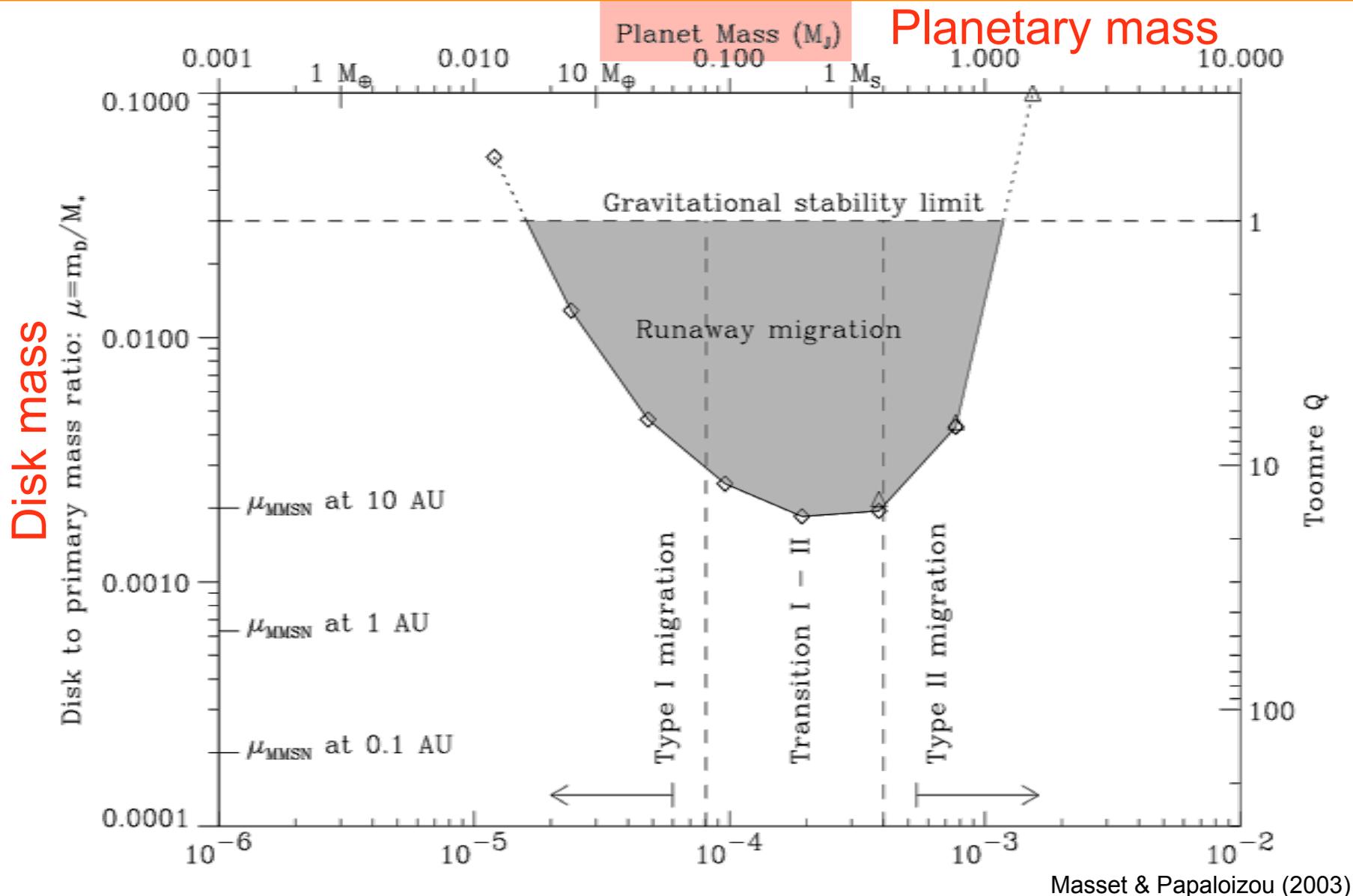
# Positive feedback loop



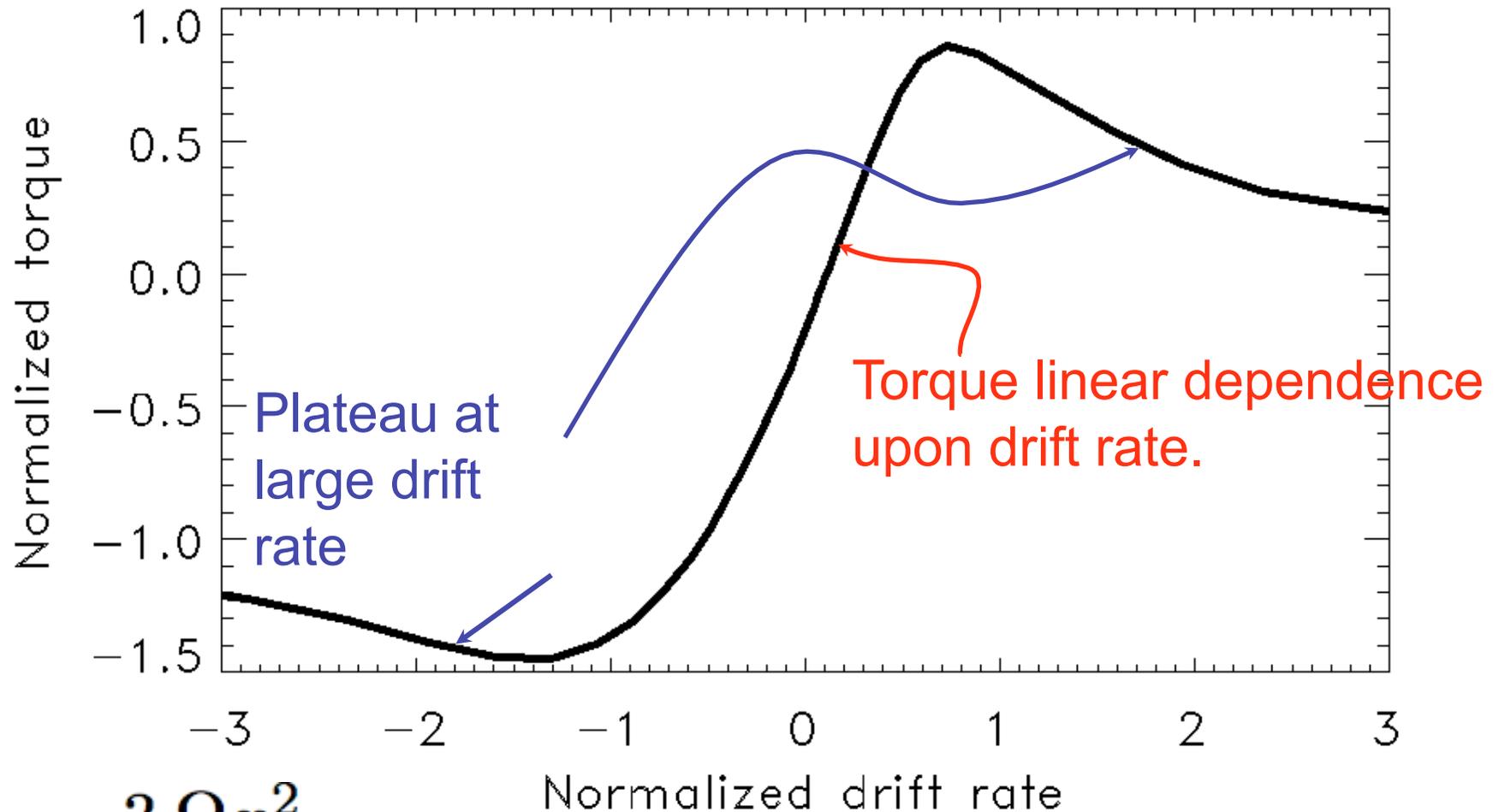
Positive feedback → runaway?

Yes, if the disk is massive enough.

# Type III (runaway) migration domain

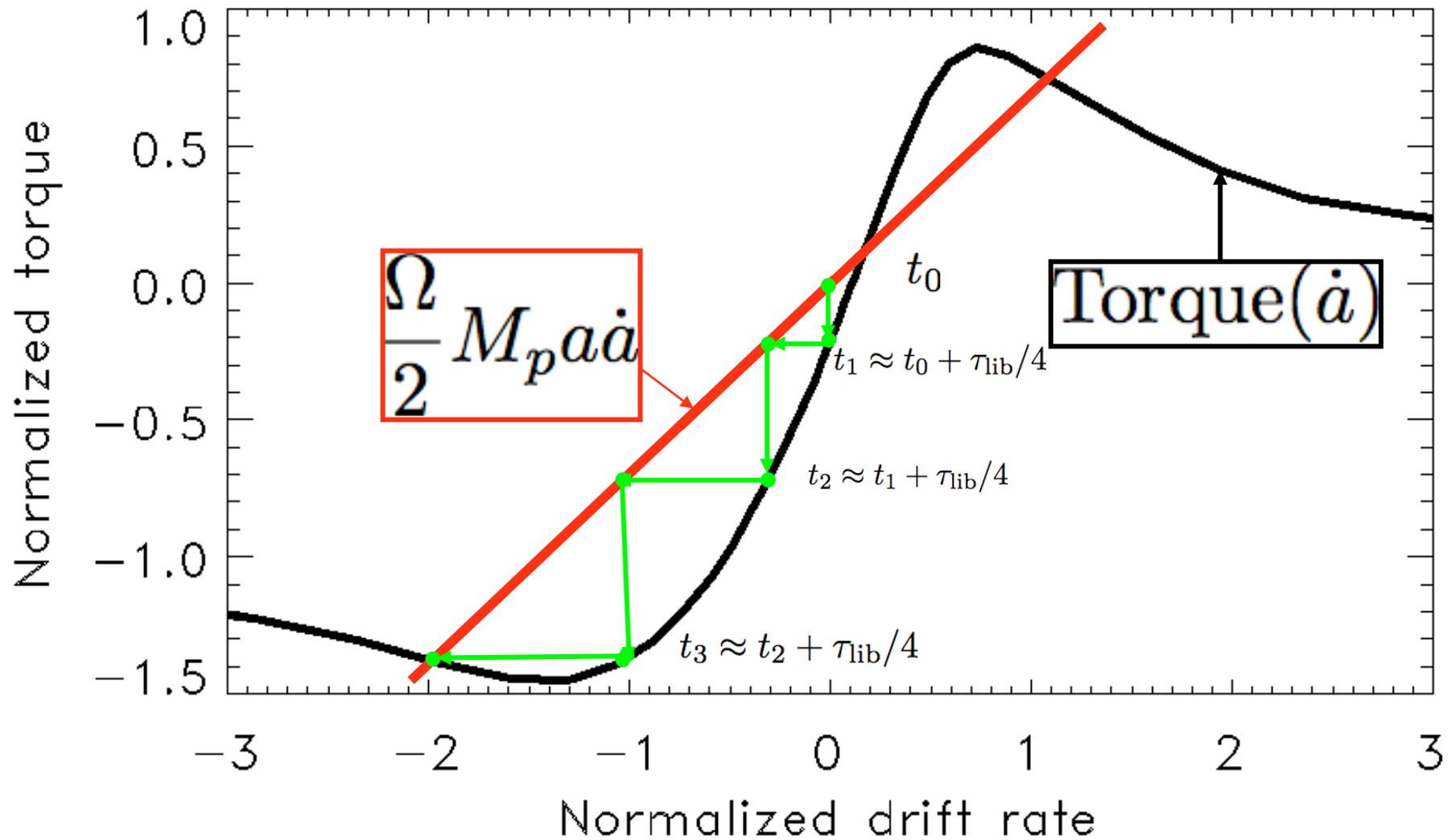


# Maximal type III drift rate

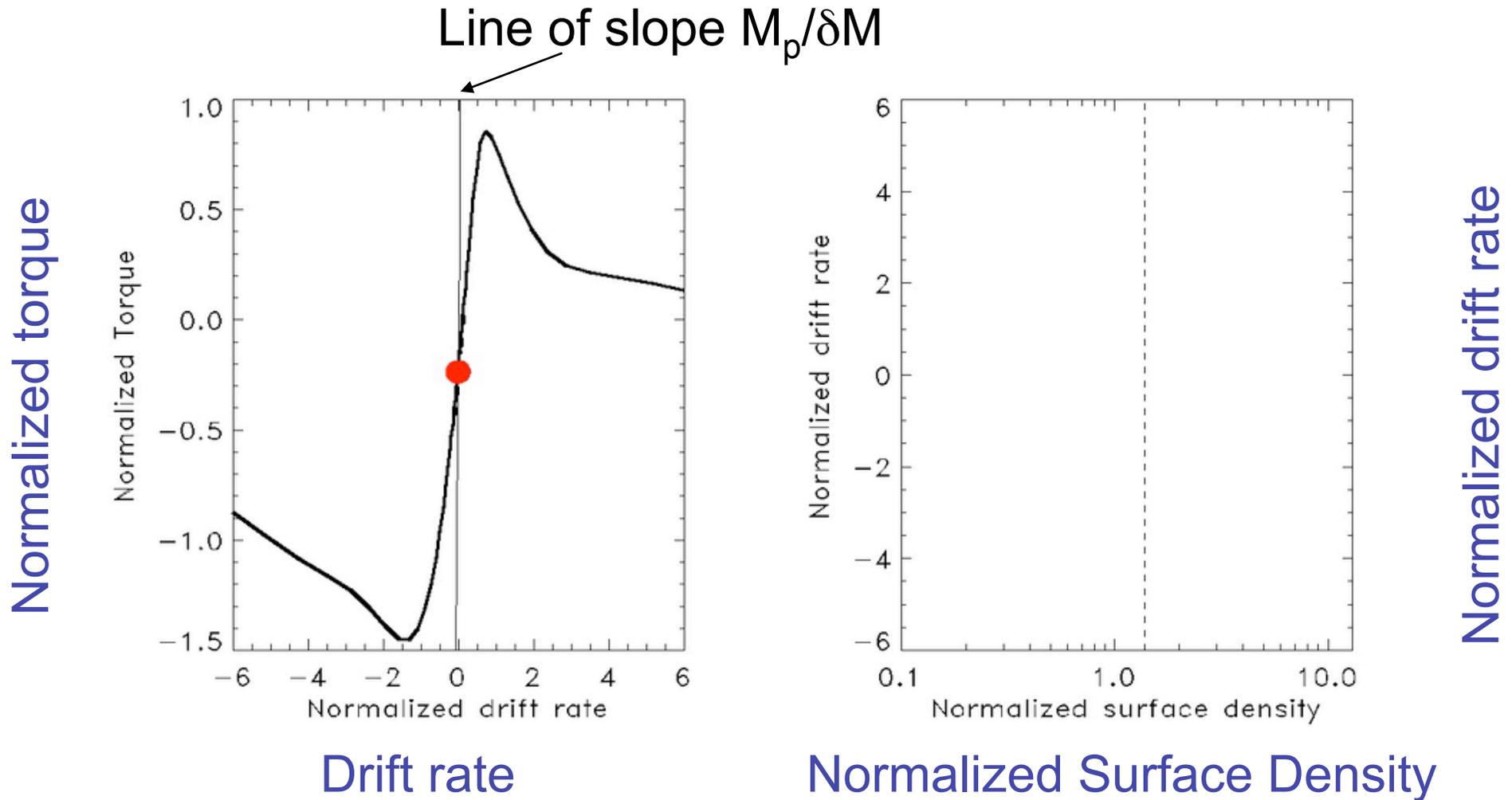


$$\dot{a}_c = \frac{3 \Omega x_s^2}{4 \pi a}$$

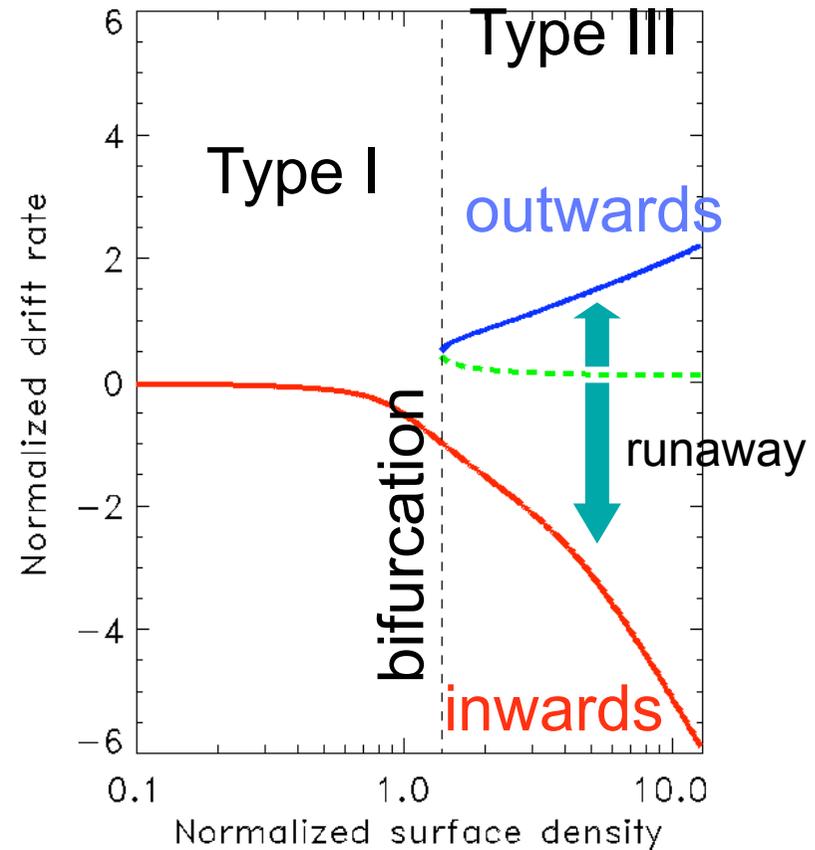
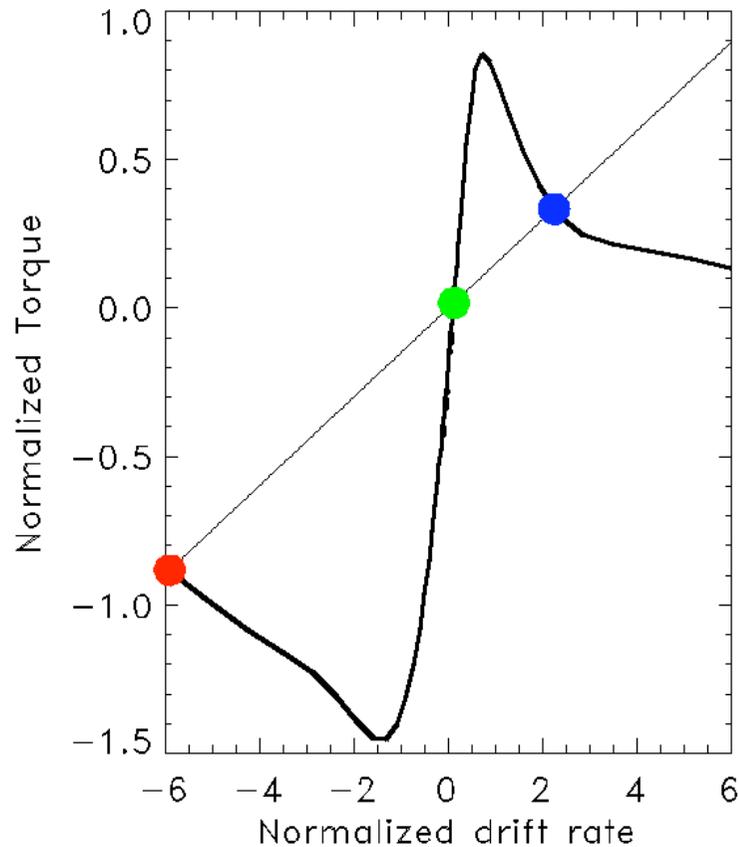
# Maximal type III drift rate



# Transition to type III migration

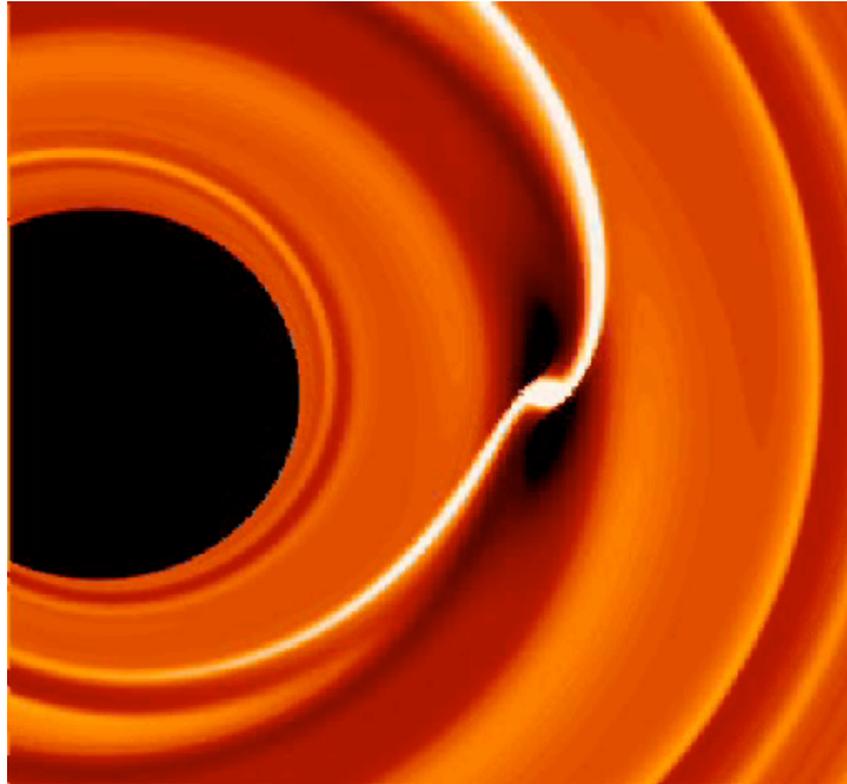


# Transition to type III migration



Type III migration is inwards or outwards: potentially a rich variety of outcomes.

# Type III migrating Saturn

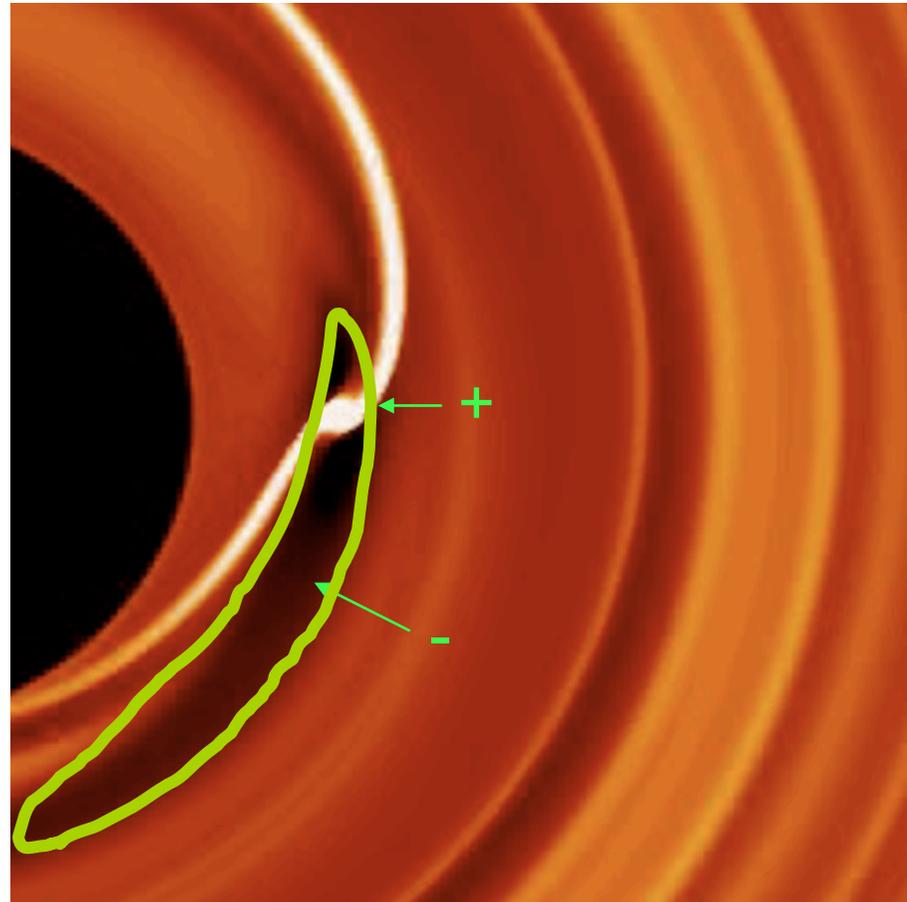


# Type III migrating Saturn

What migrates  
has an « effective  
mass »  $M_p - \text{CMD}$   
(*Coorbital Mass Deficit*)

Migration is type III  
when  $\text{CMD} > M_p$

Masset & Papaloizou (2003)



# Migration of two giant planets

Example of a « Jupiter-Saturn » system embedded in a nebula of uniform surface density.

What is the time evolution of such a system ?

# Migration of two giant planets

Jupiter adopts a slow, type II migration.

Saturn migrates much faster (actually type III).

Saturn therefore catches Jupiter up.

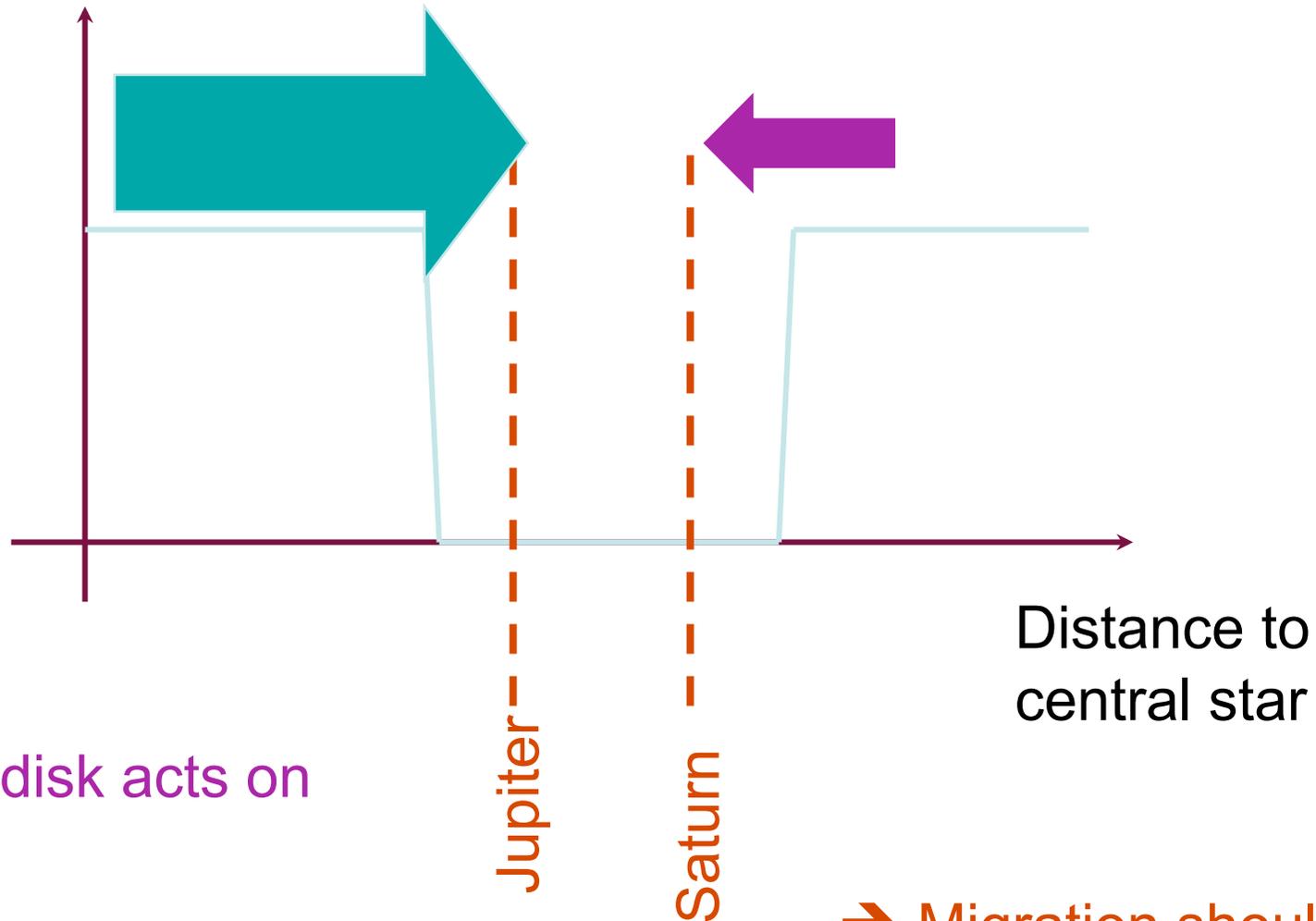
It get locks on a mean motion resonance with Jupiter.

The gaps of both objects overlap and the two planets therefore share a wide, common gap.

What is the evolution of this system of planets locked in MMR and possessing a common gap ?

# Migration of two giant planets

Surface density



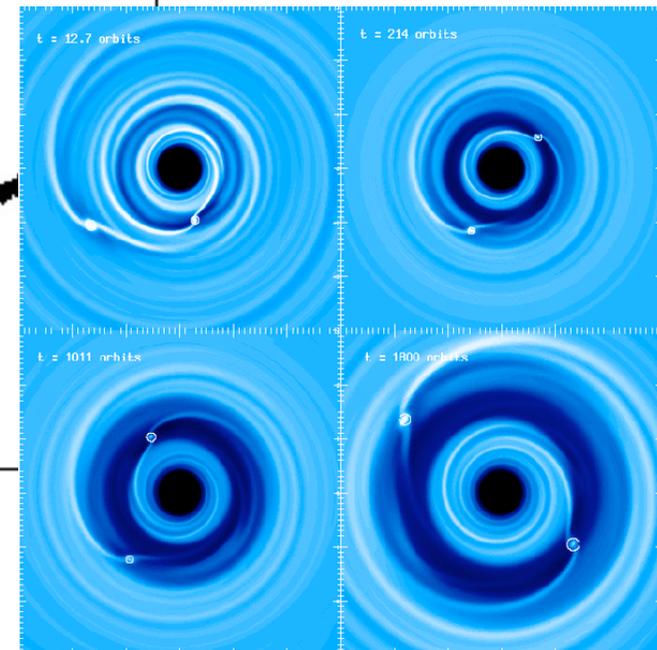
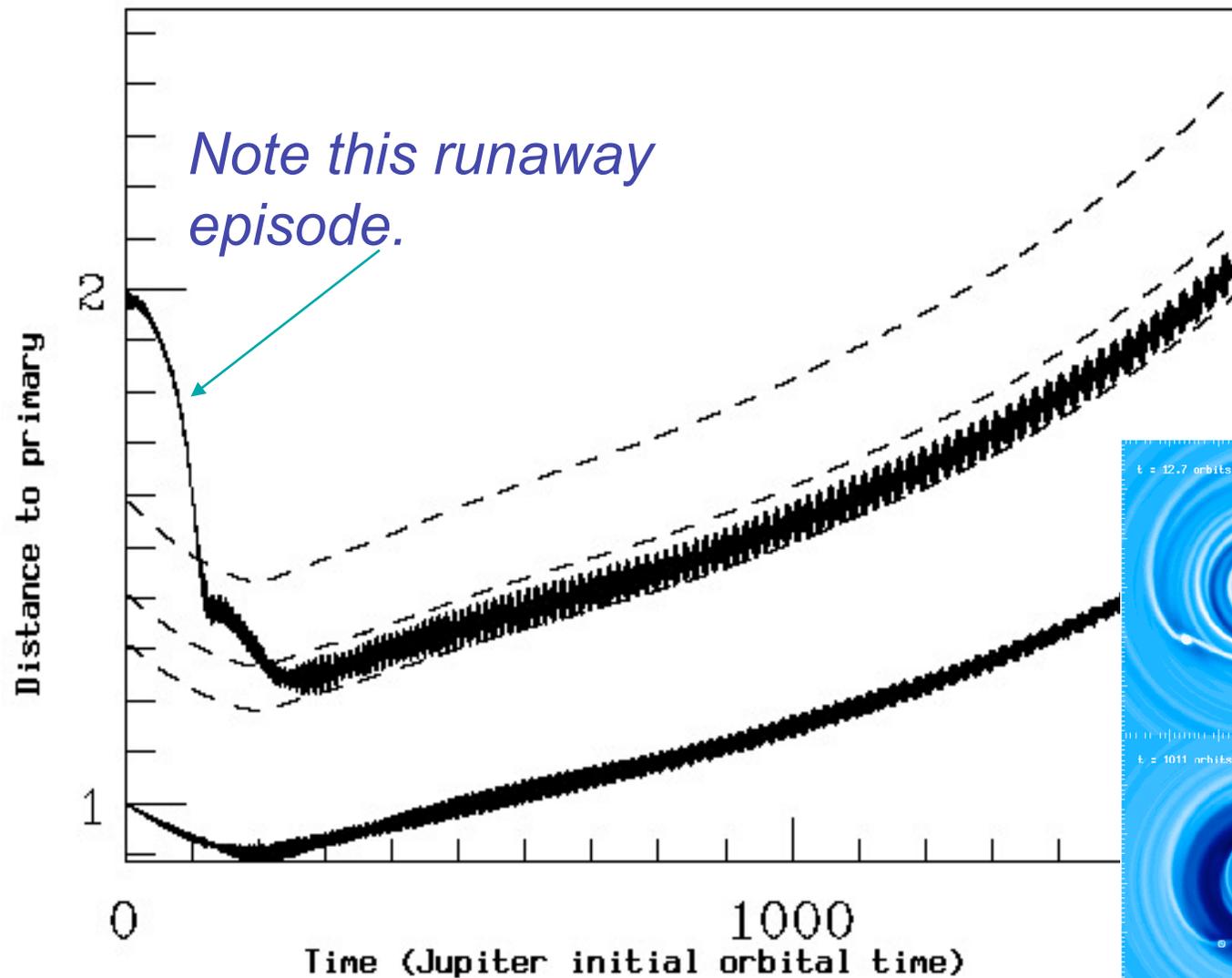
The outer disk acts on Saturn

The inner disk acts on Jupiter

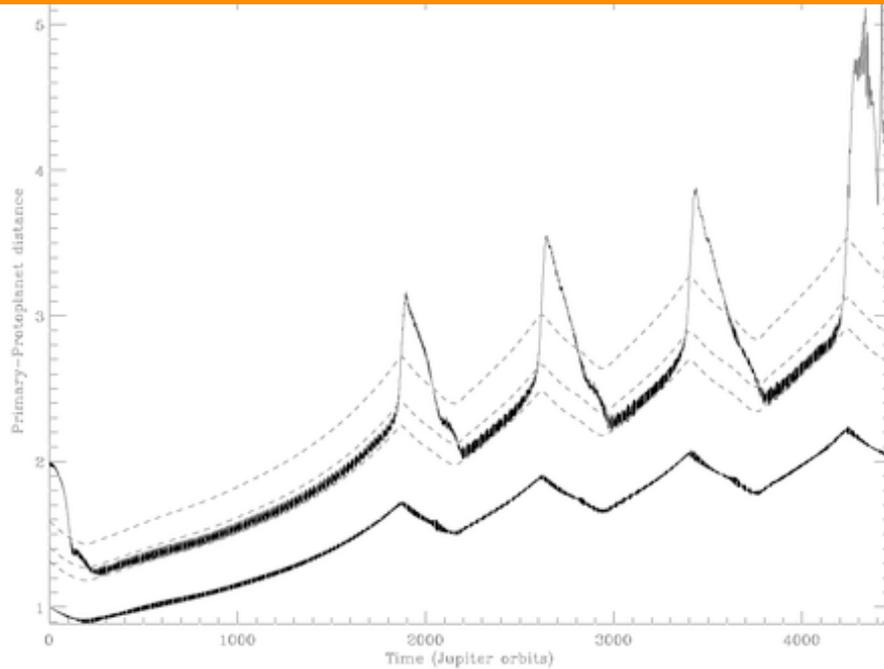
→ Migration should occur outwards

# Migration of two giant planets

Masset & Snellgrove (2001)

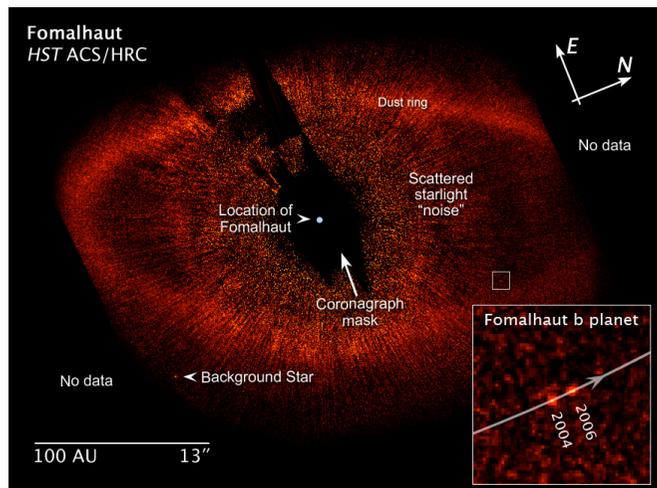


# Long term behavior



Sequence of outwards and inwards type III.

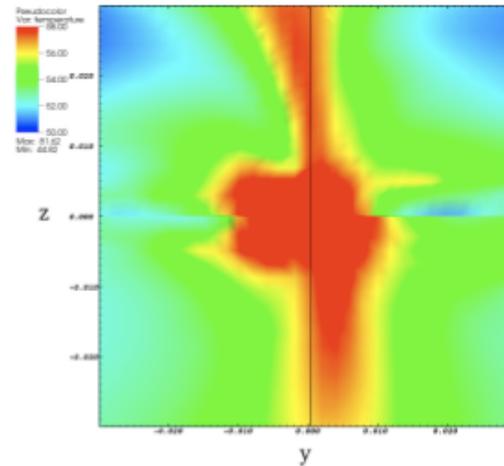
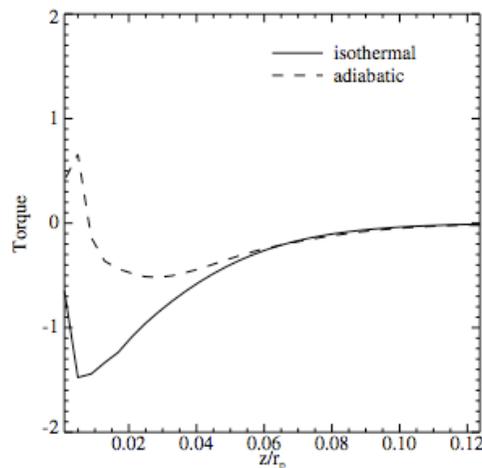
Slight outwards drift on the average.



This mechanism has recently been suggested to account for the presence of Fom b, detected by Kalas et al. (2008) at 115 AU from its central star (Crida et al. 2009)

# Corotation torque in non-barotropic disks

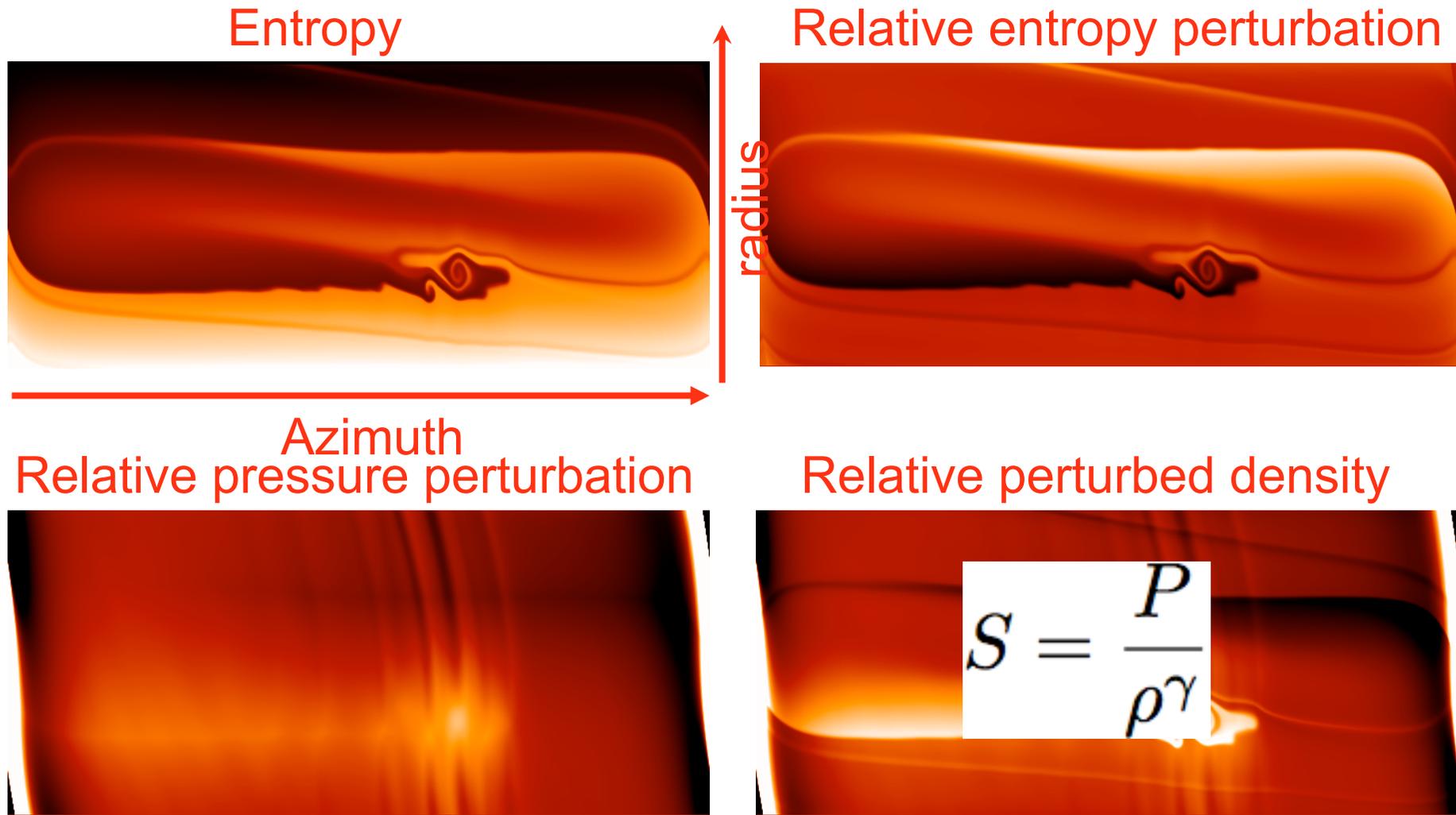
Paardekooper & Mellema (2006) find with nested grid 3D calculations and radiative transfer that a low-mass planet can actually *migrate outwards*, if the disk is sufficiently opaque, under the action of the CR torque !



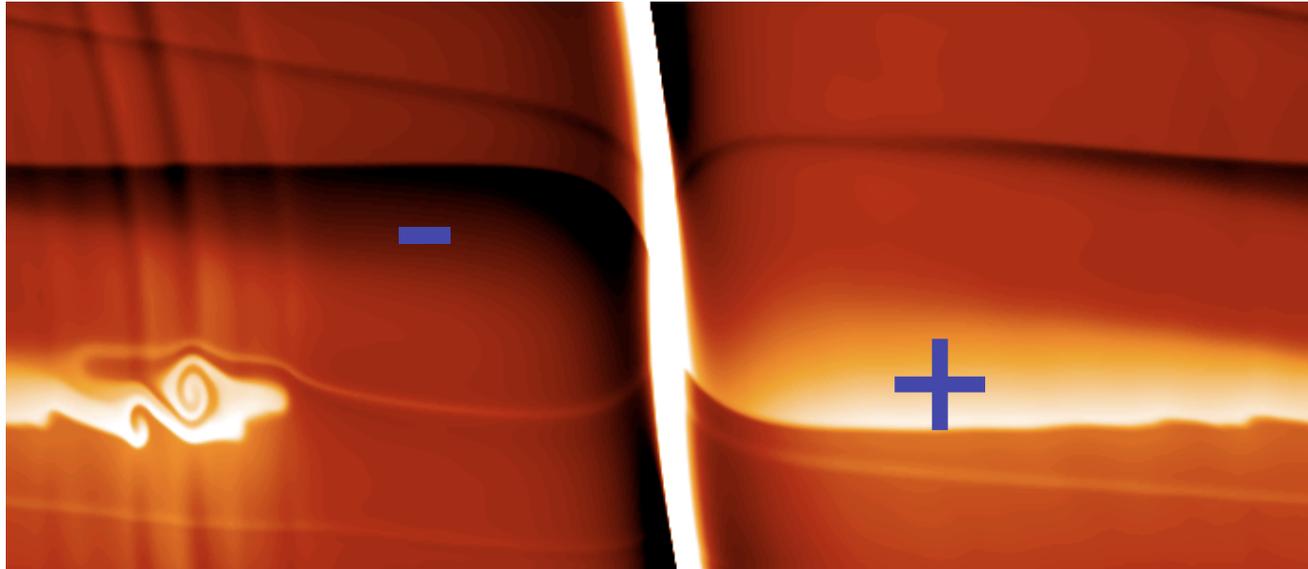
In order to understand these results → 2D adiabatic simulations

In an adiabatic flow, the entropy is conserved along a fluid element path (provided no irreversibility is introduced). This is true for the coorbital flow of a low mass planet.

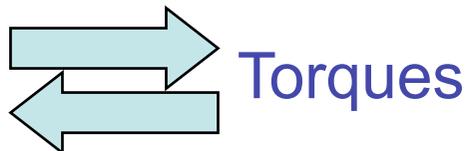
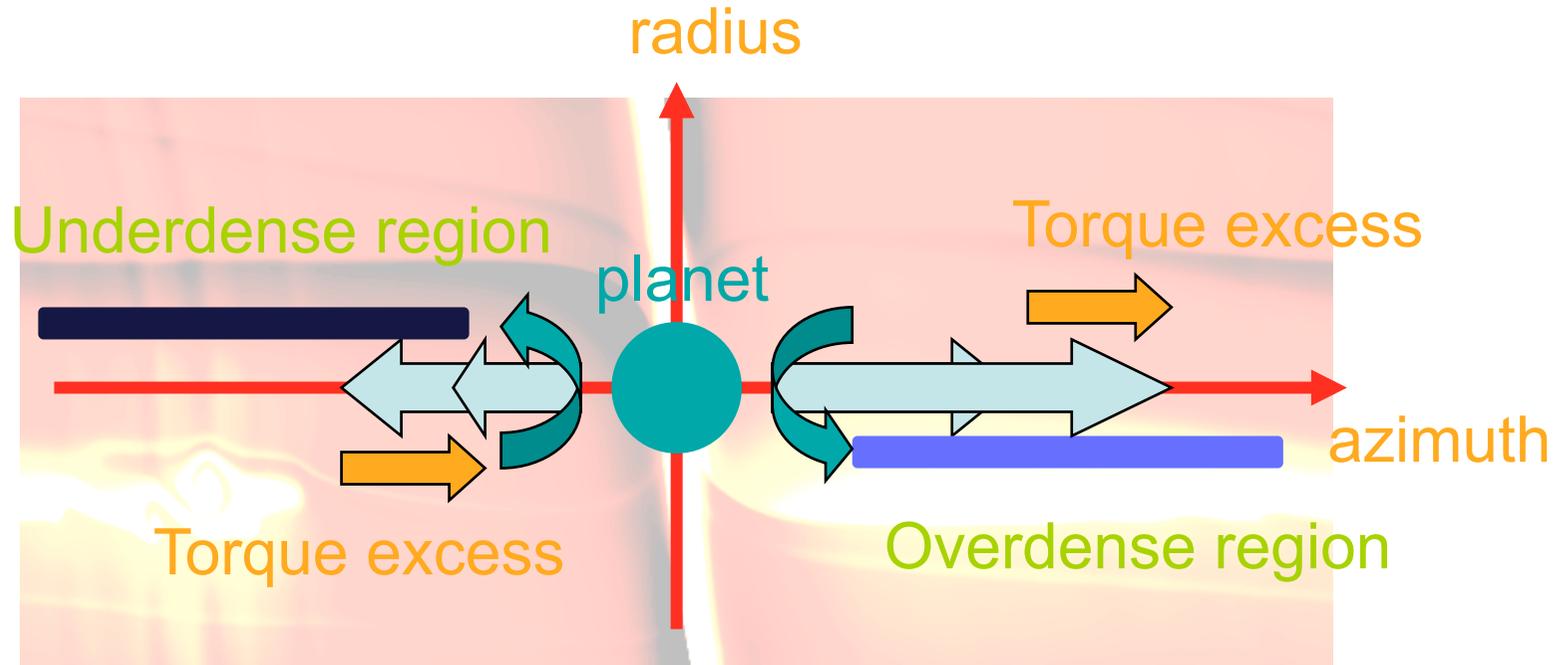
# Corotation torque in a 2D adiabatic disk



# Impact on corotation torque



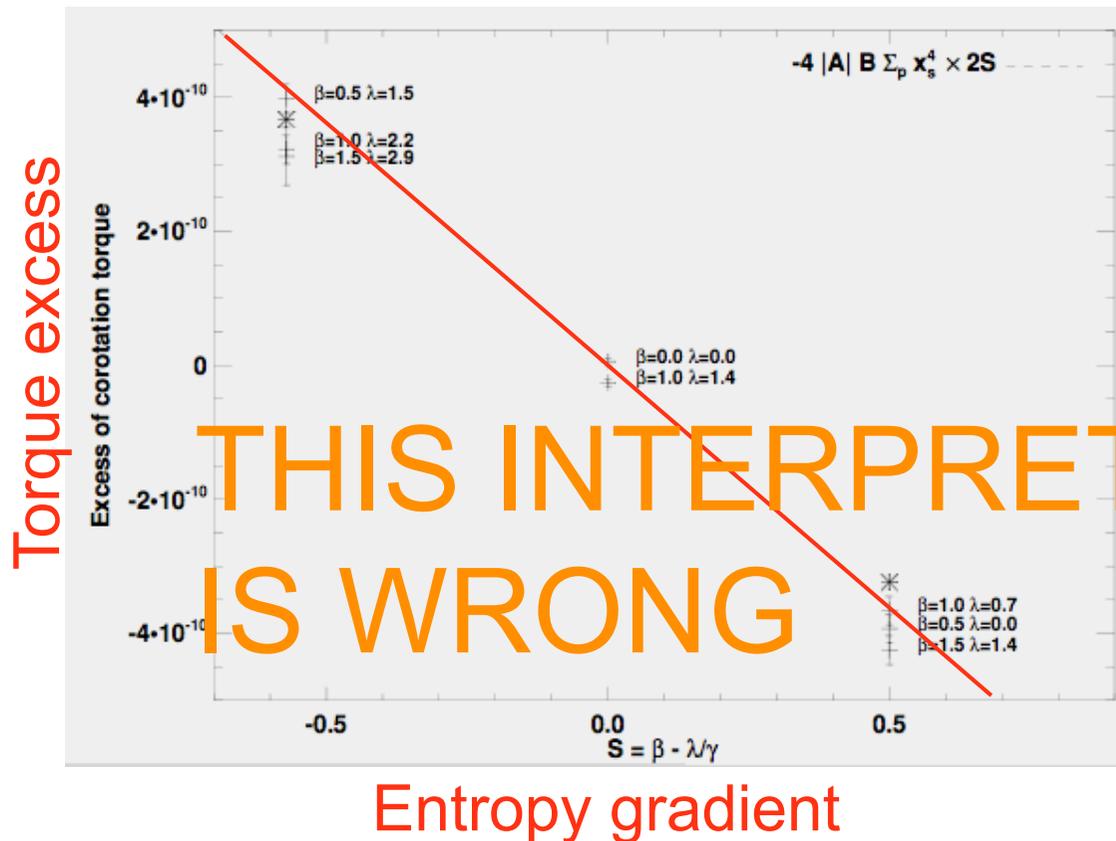
# Impact on corotation torque



Torque excesses add up and are both positive.

# CR torque in a radiatively inefficient disk

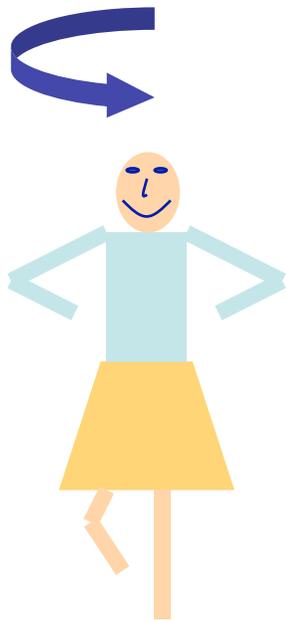
The torque excess therefore scales with the entropy gradient.



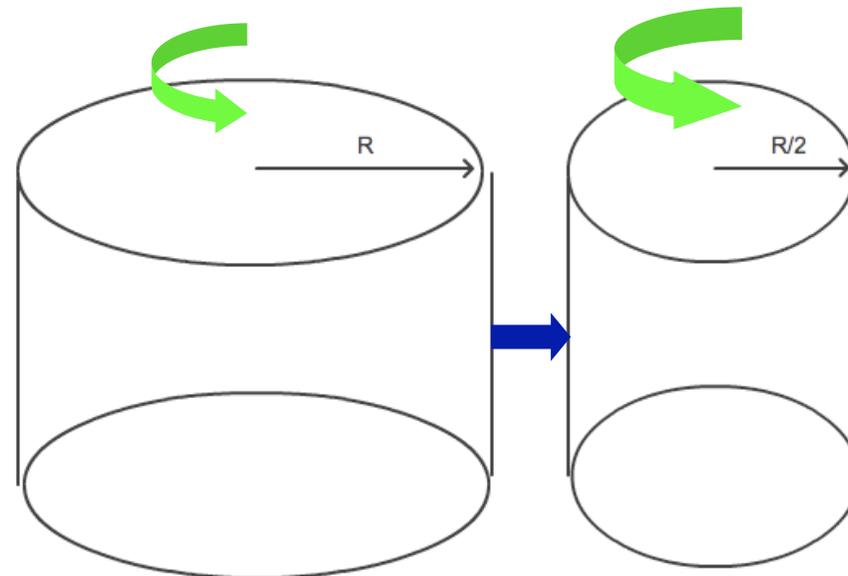
The torque excess is measured by comparing the results to a calculation with an isothermal disk.

This excess can be sufficient to revert migration

# Vortensity advection



rotation speeds up when the dancer retracts arms or legs



$$\omega, \Sigma$$

$$\frac{4\omega}{4\Sigma}$$

Vortensity is conserved

# Vortensity conservation for 2D flows

The fluid elements have their vortensity conserved if the force acting on them is curl free.

⇒ The fluid must be inviscid

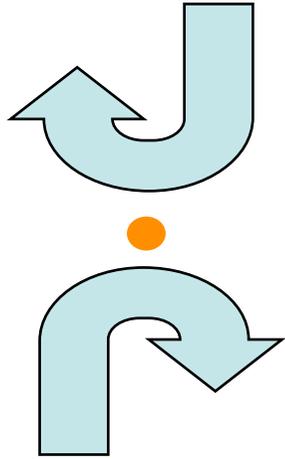
⇒ The fluid must be barotropic:  $P(\Sigma)$

$$\frac{D\vec{v}}{Dt} = -\frac{\vec{\nabla}P}{\Sigma} - \vec{\nabla}\Phi$$

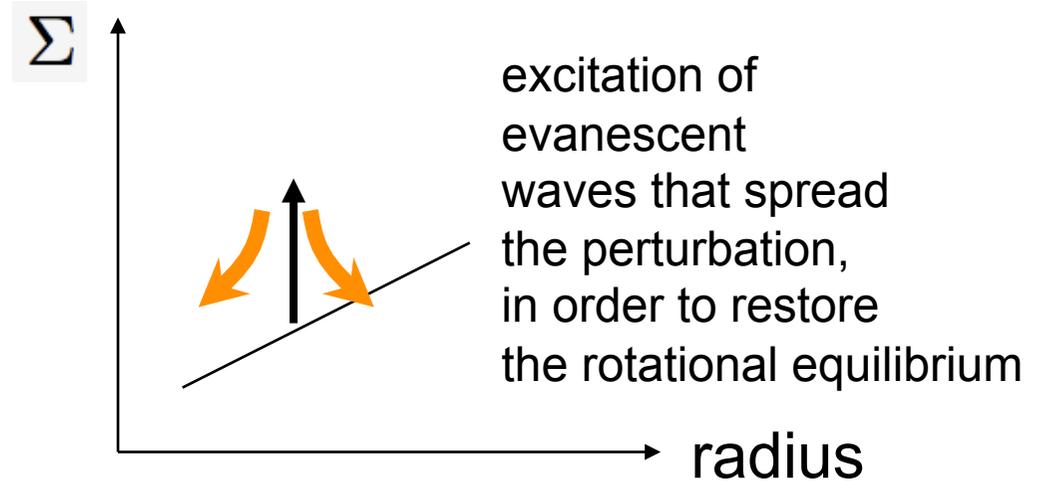
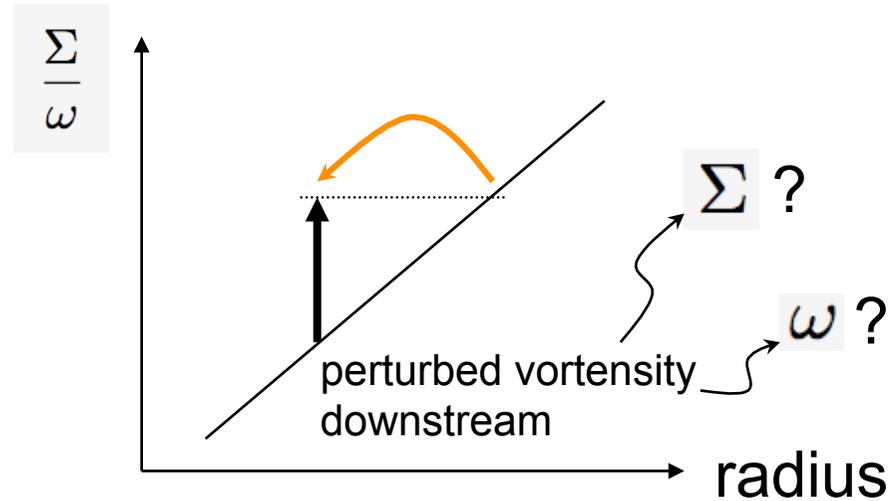
If the fluid is barotropic, then:

$$-\frac{\vec{\nabla}P}{\Sigma} = -\vec{\nabla}H \quad \text{where} \quad H = \int \frac{dP}{\Sigma} \quad \text{is the enthalpy}$$

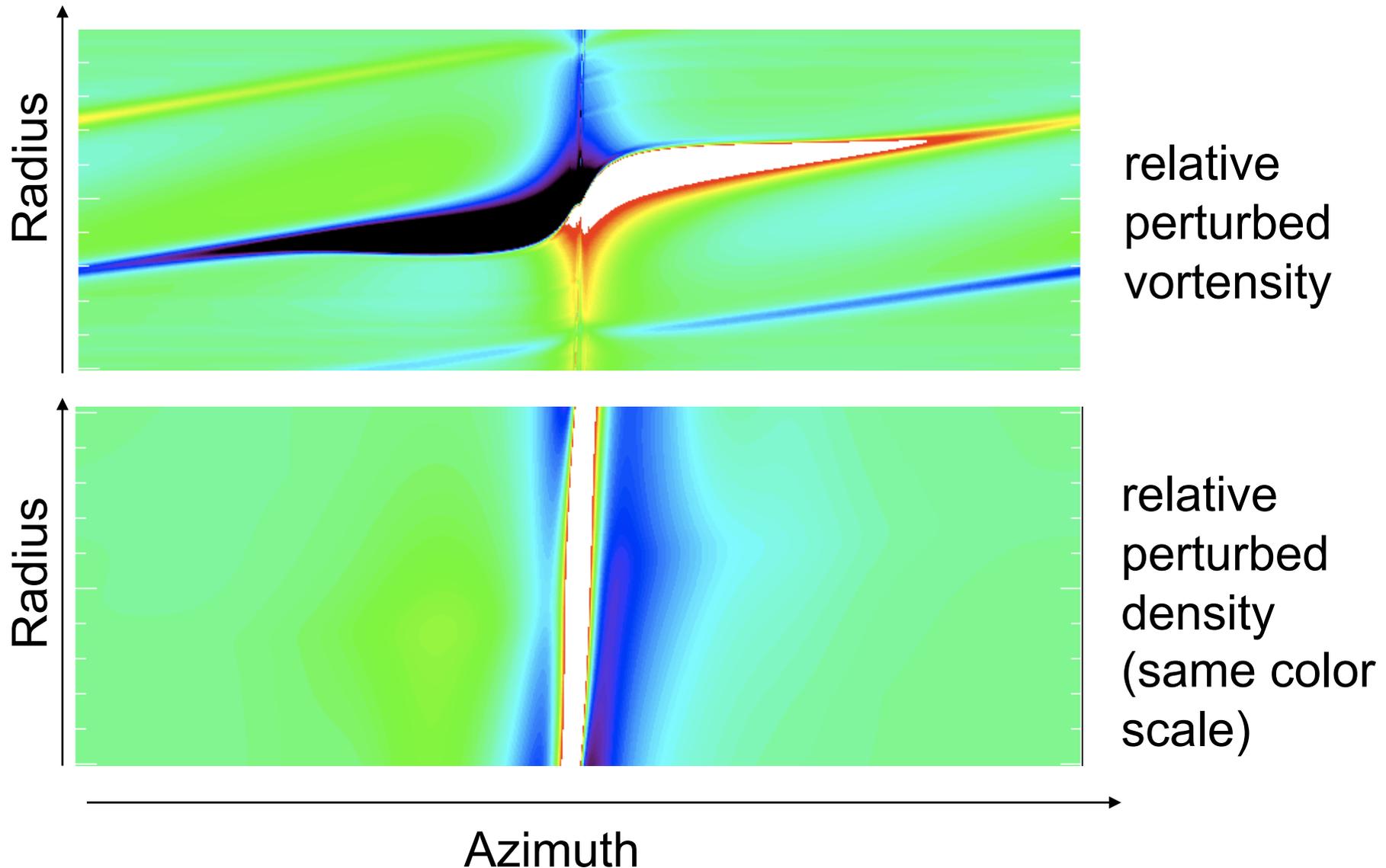
# Back to isothermal (*ie* barotropic) disks



Vortensity is conserved along a U-turn



# Radial spread of density disturbance

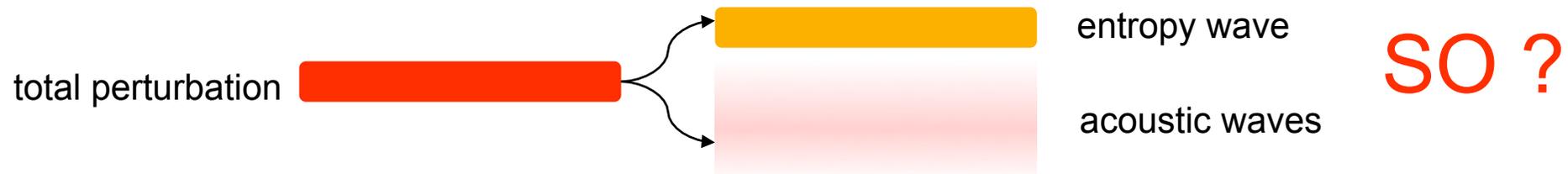


# Baroclinic disk

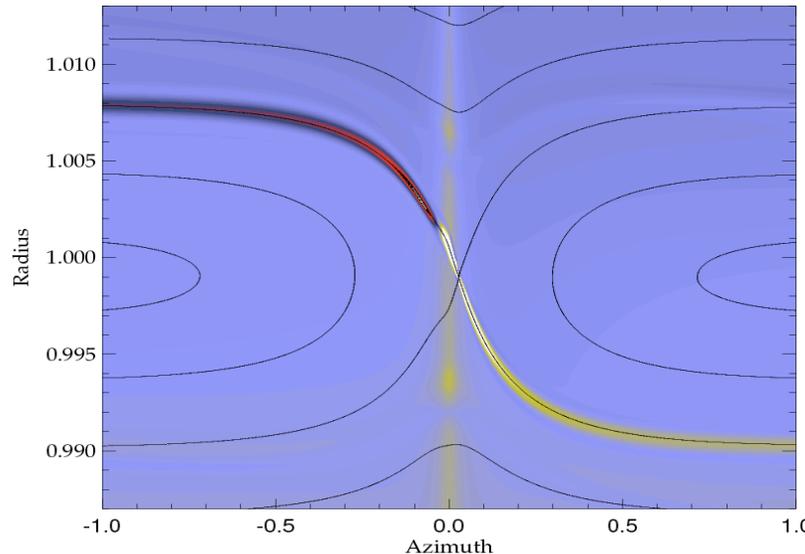


Pressure disturbance: radially spread, but plays a role like in an isothermal disk. This has been overlooked so far.

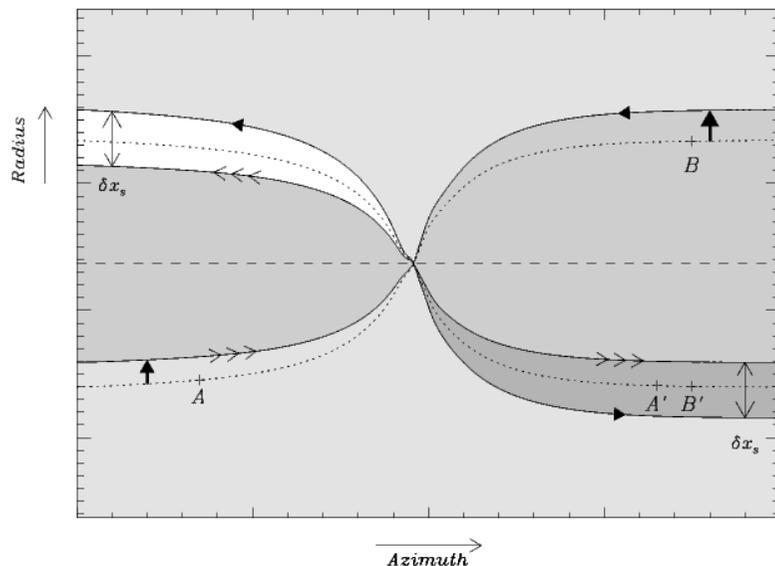
General picture: vortensity **almost** conserved everywhere. The perturbed vortensity downstream of the flow triggers a disturbance which projects onto acoustic waves and onto an entropy wave, but the linear mass of the disturbance scales with the vortensity gradient, as always...



# Downstream singular vortensity



Vortensity is created at the separatrix, downstream of the flow. Scales with the entropy gradient.



In steady state, can be interpreted in terms of an asymmetry of the horseshoe region.

Very efficient. Associated perturbed density rather discreet...

# Saturation of entropy related torque

The origin of the entropy related torque is now well understood.

This allows to proceed towards a study of the saturation of this torque component.



Entropy related torque function of thermal diffusion and viscosity

Expression very useful for population synthesis models

# Conclusions

Theories of planetary migration have considerably evolved over the last 10 years.

The coorbital region has proven to yield a rich variety of new mechanisms.

The lingering problem of Type I migration is about to be solved.

Planetary migration is entering a more quantitative phase, and should take into account more physical processes of the disk.