The Stellar Initial Mass Function

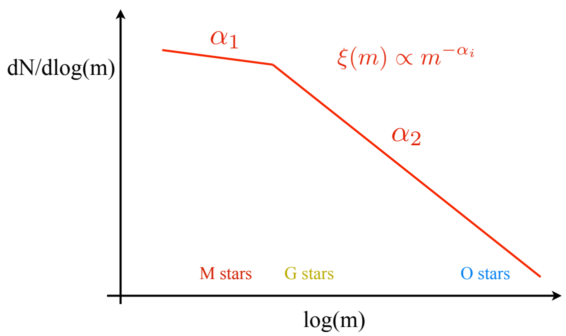
of simple and composite stellar populations

Pavel Kroupa
Argelander Institute for Astronomy (AIfA)
University of Bonn

O star	G star	M star
$N = 1$stars	$N = 10^{3.8}$ stars	$N = 10^{5.2}$ stars
$M = 63\, M_\odot$	$M = 1\, M_\odot (= 2 \times 10^{33}\, g)$	$M = 0.15\, M_\odot$
$L = 10^6\, L_\odot$	$L = 1\, L_\odot (= 3.8 \times 10^{33}\, erg/s)$	$L = 10^{-2.5}\, L_\odot$
$\tau = 10^{6.2}\, yr$	$\tau = 10 \times 10^9\, yr$	$\tau > 10^{11}\, yr$
$R = 20\, R_\odot$	$R = 1\, R_\odot (= 7 \times 10^{10}\, cm)$	$R = 0.20\, R_\odot$
$T_e = 35000\, K$	$T_e = 5000\, K$	$T_e = 3300\, K$

IMF = the distribution of stellar masses born together.

$$\xi(m)\, dm = dN = \text{Nr. of stars in interval } [m, m + dm]$$

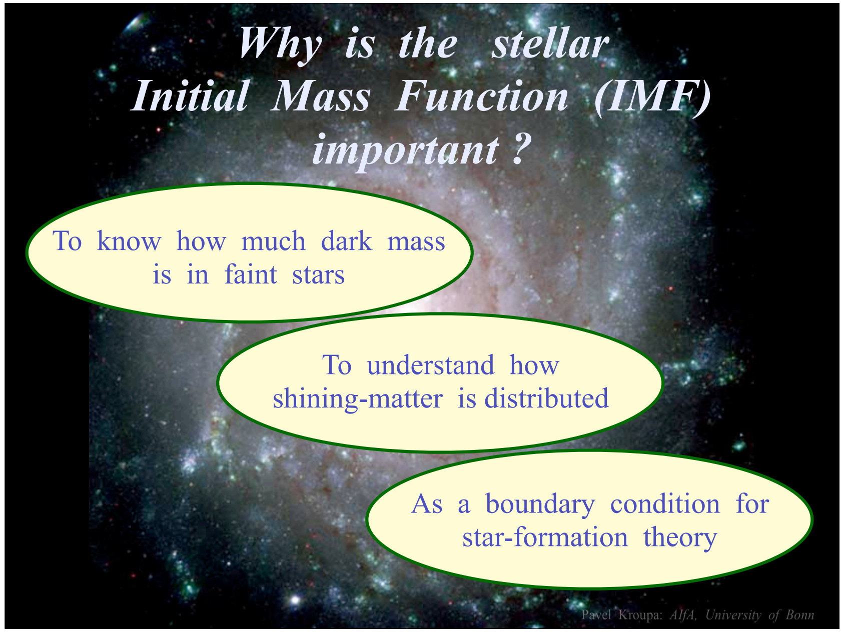


1 000 000 M stars combine to

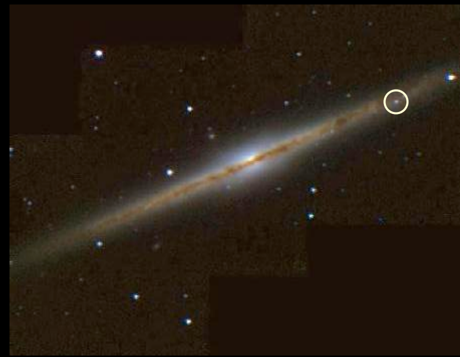
100 000 $M_\odot$ but only

$10^{3.5}\, L_\odot \ll L_{\mathrm{O\ star}}$

One O star ($60\, M_\odot$) out-shines one million M stars ($10^5\, M_\odot$).



The solar neighbourhood



Contents

- ***Solar-neighbourhood***
  - the average / standard IMF
- ***Massive stars***
  - a fundamental upper stellar mass limit
  - unresolved multiples
- ***Clusters*** : noise or true variations?
- A ***variable IGIMF*** and implications !
- ***Origin*** of the IMF.

The distribution of stars

We have $dN = \Psi \, dM_V$ = # of stars with $M_V \in [M_V, M_V + dM_V]$

$dN = \xi(m) \, dm$ = # of stars with $m \in [m, m + dm]$

thus
$$\frac{dN}{dM_V} = -\frac{dm}{dM_V} \frac{dN}{dm}$$

i.e.
$$\boxed{\Psi(M_V) = -\frac{dm}{dM_V} \, \xi(m)}$$

There are ***two luminosity functions*** for the solar neighbourhood

I. Count stars nearby to Sun

Obtain M_V and d from trigonometric parallax

$\longrightarrow$ Well observed individual stars but *small numbers at faint end* ($\Psi_{\rm near}$)

II. Deep (100 - 300 pc) pencil-beam photographic/CCD surveys

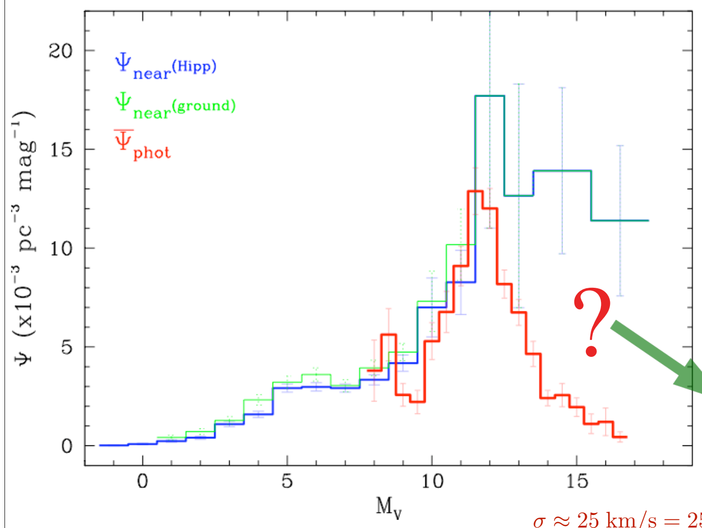
Formidable data reduction (10^5 images $\longrightarrow$ $\approx$ 100 stars)

Obtain M_V and d from photometric parallax

$\longrightarrow$ Large # of stars but *poor resolution* (2"-3") ($\Psi_{\rm phot}$)

Two solar-neighbourhood samples:

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

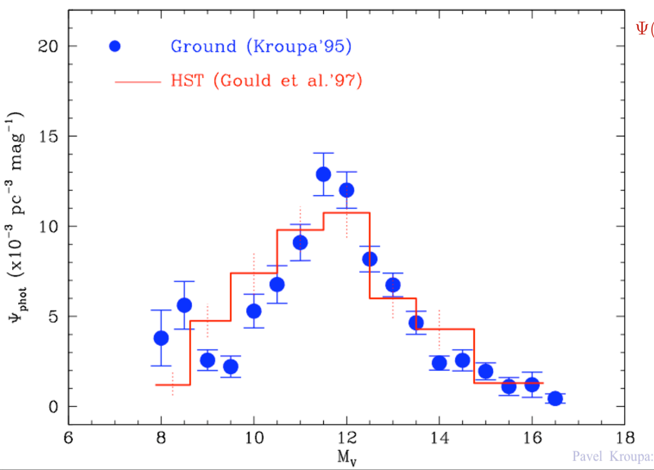


Stellar samples should be *well-mixed*:

$\sigma \approx 25$ km/s $= 250$ pc/10Myr

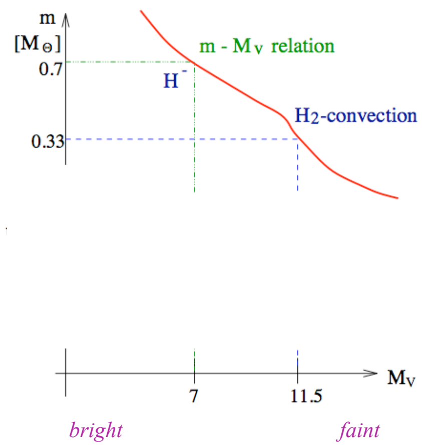
$\longrightarrow$ $\Psi_{\rm phot}$ { - independent of direction; - maximum (peak) at $M_V \approx 12$

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



Understand ***detailed shape*** of LF from fundamental principles :

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



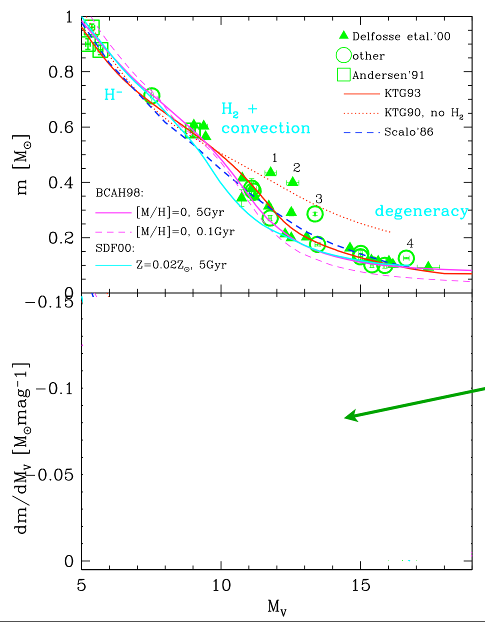
Points of inflection in *m(Mv)* relation

+

$$\xi(m) = c$$

$\Downarrow$

$$\Psi(M_V) = -c\,\frac{dm}{dM_V}$$



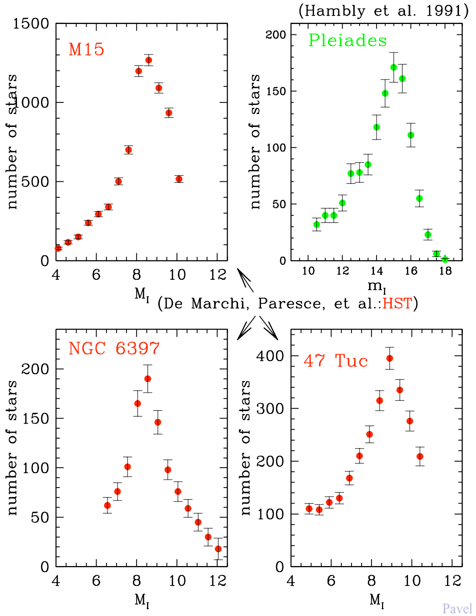
The mass-luminosity relation of low-mass stars

$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$

Kroupa, 2002, *Science*

The maximum near
$M_{\rm V} \approx 12;\ M_{\rm I} \approx 9$
is universal
and well understood.

But we are still trying to understand the
local LF discrepancy...

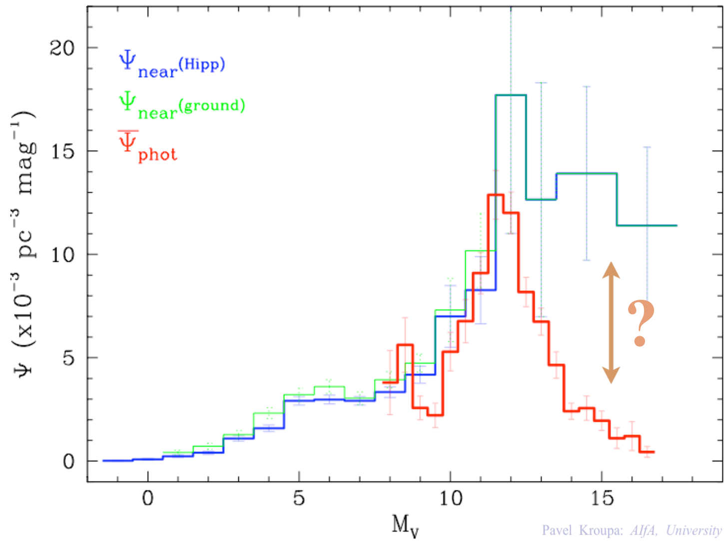


$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$

Two solar-neighbourhood samples:

$$\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)$$

. . . understand the peak, but a problem remains . . .



Multiple systems :

Counting example: Observer sees 100 systems:

unknown is that
40 are *binaries*
15 are *triples*
5 are *quadruples*

$$f_{\mathrm{mult}} = \frac{40+15+5}{100} = 0.60$$

but 85 (= 40 + 2x15 + 3x5) stars are missed.

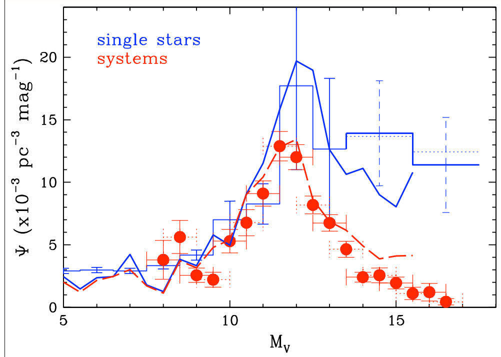
Correct treatment of this important bias *solves* the LF discrepancy !

(Kroupa, Tout & Gilmore 1991, 1993)

Dynamical Population Synthesis:

(Kroupa 1995)

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



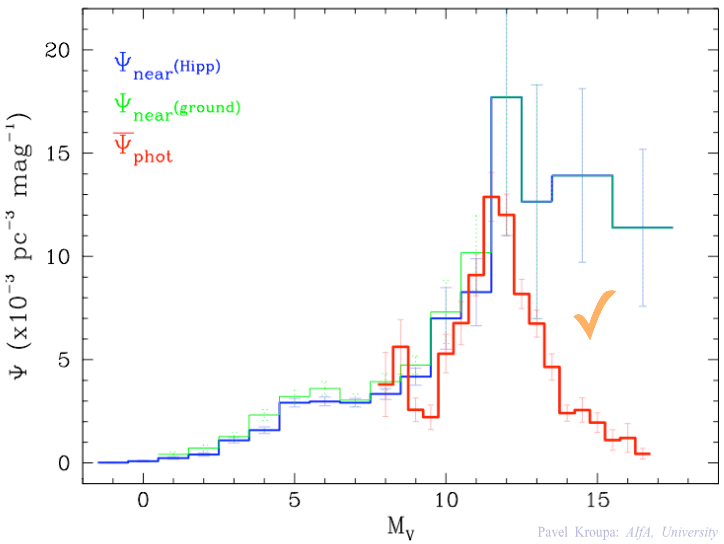
Assume all stars form as binaries in typical open clusters:

$R \approx 0.8$ pc, $N \approx 800$ stars

(Using KTG93 MLR)

Two solar-neighbourhood samples:

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$



Resultant Galactic-field MF

for low-mass stars :

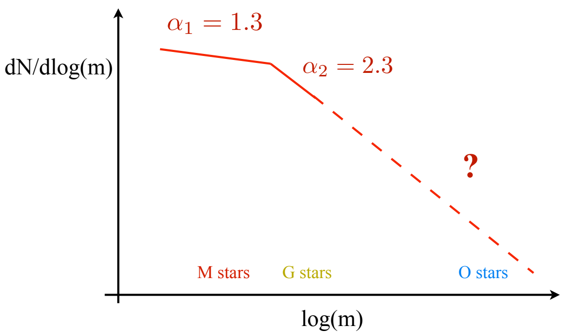
$$\xi(m) \propto m^{-\alpha_i}$$

$$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \le m/M_\odot < 0.5$$
$$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \le m/M_\odot < 1$$

. . . *unifies* the two LFs.

(Kroupa, Tout & Gilmore 1993; Kroupa 1995;
Reid, Gizis & Hawley 2002)

$$\xi(m) \propto m^{-\alpha_i}$$



No diverging mass in faint stars !!

Massive stars

Scalo (1986): A very detailed study of local star-counts together with assumptions about the SFH, spatial structure of the MW disk and stellar evolution corrections

$\longrightarrow$ $\alpha_3 \approx 2.7, \quad 1 \lesssim m/M_\odot$

Thus, the *standard Galactic-field IMF* (KTG93) becomes

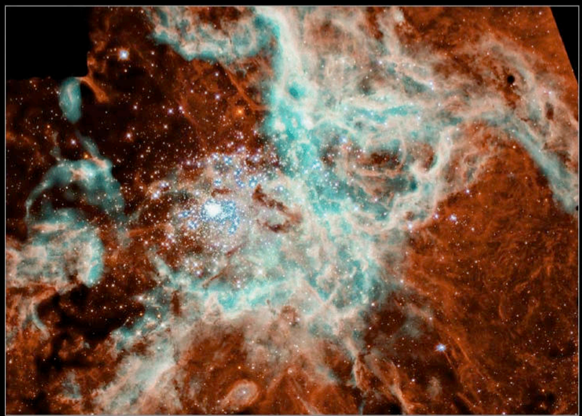
$$\xi(m) \propto m^{-\alpha_i}$$

$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \le m/M_\odot < 0.5$

$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \le m/M_\odot < 1$

$\alpha_3 = 2.7, \quad 1 \le m/M_\odot$

Massive stars



30 Doradus in the Large Magellanic Cloud
Hubble Space Telescope • WFPC2

NASA, N. Walborn (STScI), J. Maíz-Apellániz (STScI), and R. Barbá (La Plata Observatory, Argentina) • STScI-PRC01-21

Massey (various papers): A rigorous *spectroscopic* study of OB associations and clusters in the MW, LMC, SMC

Steps: - *spectroscopy* and *photometry* to get

$T_{\rm eff}, \quad BC, \quad L_{\rm bol},$ redenning

$\downarrow$

- HRD + isochrones $\longrightarrow$ ***initial stellar masses***

Massey (1995) finds for OB associations

(IMF only determined for stars with $\tau_{\rm ms} \geq \tau =$ age of population)

1)

SMC	LMC	MW
Z = 0.002	0.008	0.02
$\alpha_3 = 2.3 \pm 0.1$	2.3 ± 0.1	2.1 ± 0.1
$\xi(m) =$	$\xi(m) =$	$\xi(m)$

$\rightarrow$ $\underset{2.3}{\alpha_{3,\rm OB\ ass}} < \underset{2.7}{\alpha_{3,\rm Scalo}}$?

Thus, the *standard Galactic-field IMF* is

$$\xi(m) \propto m^{-\alpha_i}$$

$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \leq m/M_\odot < 0.5$

$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \leq m/M_\odot < 1$

$\alpha_3 = 2.7, \quad 1 \leq m/M_\odot$ (*Scalo*)

KTG93

But, the *standard stellar IMF* is

?

$$\xi(m) \propto m^{-\alpha_i}$$

$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \leq m/M_\odot < 0.5$

$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \leq m/M_\odot < 1$

$\alpha_3 = 2.3, \quad 1\,M_\odot \leq m$ (*Massey*)

Kroupa 2001

2) Independence of density:

Massey 1995

Example: R136 in LMC

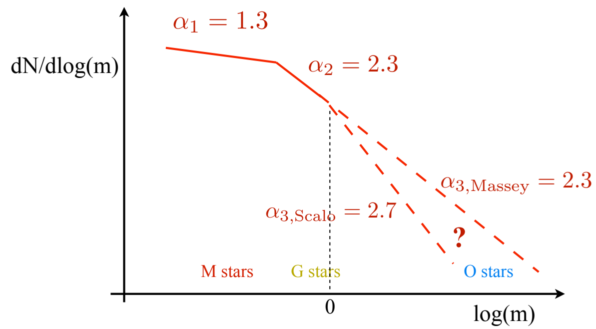
central density $\rho_C \approx 10^5$ stars/pc^3

> 39 O3 stars !

$$\alpha_3 = 2.35 \pm 0.15$$

$\rightarrow$ $\underset{2.3}{\alpha_{3,\rm OB\ ass}} < \underset{2.7}{\alpha_{3,\rm Scalo}}$?

$\xi(m) \propto m^{-\alpha_i}$



N.B: 10 times fewer O stars for Scalo value

Not One

but

Two

or even

Many More

Example:

The **Orion Nebula Cluster**

Multiplicity of massive stars

(towards understanding the "*Scalo vs Massey problem*")

Most massive stars are in
binary ($\mathcal{B}$),
triple ($\mathcal{T}$) or
quadruple ($\mathcal{Q}$) systems.

Def: The companion star fraction:

$$CSF = \frac{\mathcal{B} + 2\,\mathcal{T} + 3\,\mathcal{Q}}{\mathcal{S} + \mathcal{B} + \mathcal{T} + \mathcal{Q}}$$

(*cf* Reipurth & Zinnecker 1993)

Example: The **Orion Nebula Cluster** (≈ 1 Myr)

The multiplicity of the 8 most massive stars:
(Preibisch et al. 1999)

θ^1 Ori A	$\mathcal{T}$	
θ^1 Ori B	$\mathcal{Q}$	
θ^1 Ori C	$\mathcal{B}$	– the exciting star
θ^1 Ori D	$\mathcal{S}$	
θ^2 Ori A	$\mathcal{T}$	
θ^2 Ori B	$\mathcal{S}$	
LP Ori	$\mathcal{T}$	
ν Ori	$\mathcal{T}$	

$$\longrightarrow \qquad CSF = \frac{1+8+3}{2+1+4+1} = 1.5$$

separation $\lesssim 1$ AU – 1000 AU
mass-ratios $0.1 \lesssim q \leq 1$

Multiplicity of massive stars

$$CSF = \frac{\mathcal{B} + 2\,\mathcal{T} + 3\,\mathcal{Q}}{\mathcal{S} + \mathcal{B} + \mathcal{T} + \mathcal{Q}}$$

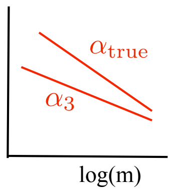
Compare

ONC OB stars: $CSF = 1.5$

pre-main-sequence low-mass stars: $CSF = 1$

Galactic-field late-type stars $CSF = 0.5$

But $CSF \geq 1$ → $\alpha_{\rm true} > \alpha_3$ **?** (true steeper IMF)



Sagar & Richtler 1991: $\alpha_{\rm true} \geq 2.7\,(2 - 14\,M_\odot)$

Clusters

Thus, we have a *consistent formulation* of the *field-IMF*, and we begin to see a possible solution for the ***Scalo*** *vs* ***Massey*** discrepancy (unresolved multiples?).

Next, turn to clusters:

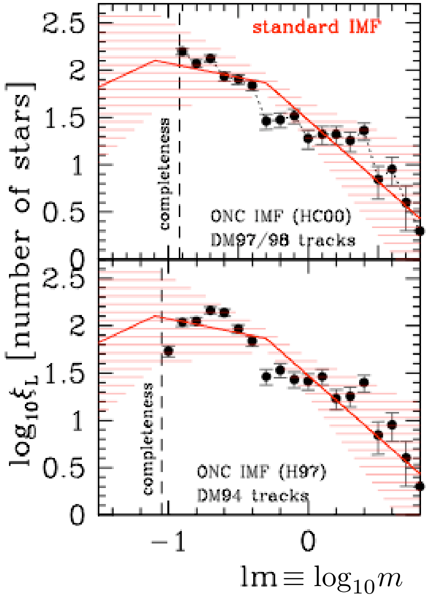
Clusters

Advantages: Stars have same d, τ, z .

Disadvantages: If young (to avoid dynamical evolution) need *pre-main sequence models,* and $f_{\rm mult}$ *high*.

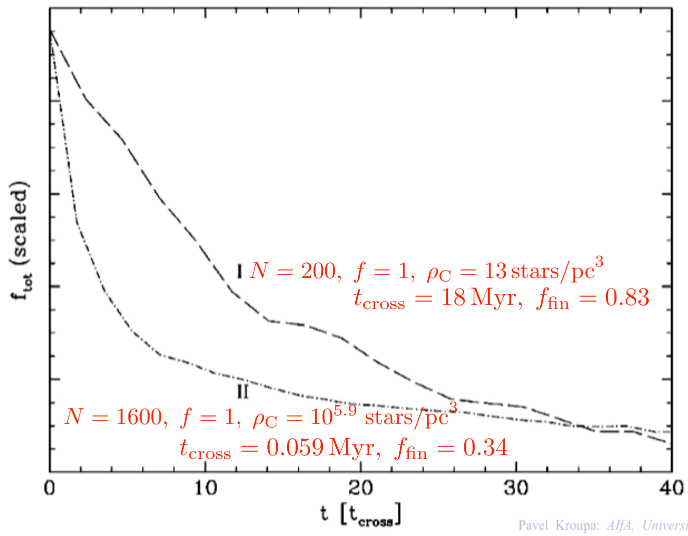
If main-sequence age then have *substantial dynamical evolution*.

Uncertainties due to pre-main sequence tracks



N-body Models of Binary-Rich Clusters

(Kroupa 1995, Kroupa 2000)



Binaries in clusters and dynamical evolution

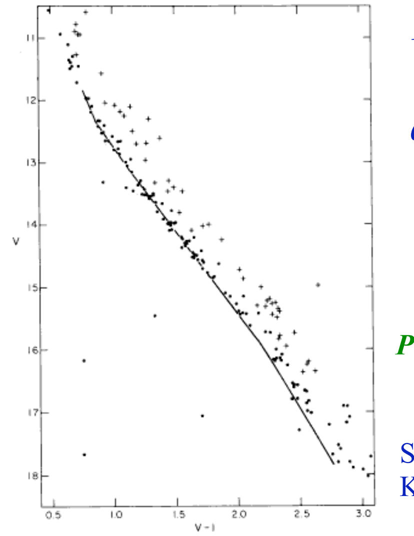


FIG. 7.—Same as Fig. 1, except that the main-sequence line has been shifted downward by 0.25 mag. The larger symbols in this diagram denote stars identified as photometric binaries.

Pleiades (Stauffer 1984)

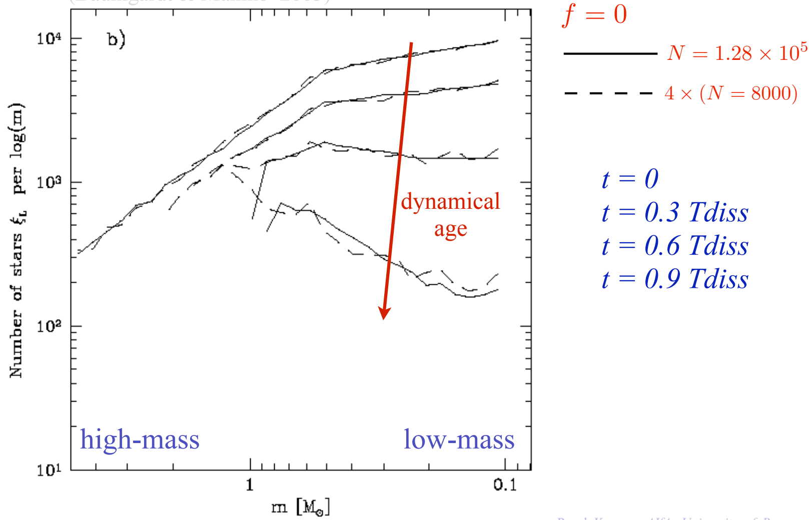
d = 126pc, age = 100Myr

Stauffer $f_{\mathrm{phot}} \approx 0.26$

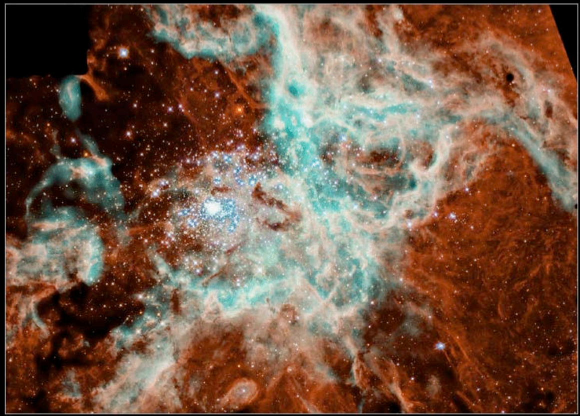
Kaehler $f_{\mathrm{phot}} \approx 0.6 - 0.7$ possible

MF(t) due to cluster evolution

(Baumgardt & Makino 2003)



Massive stars in clusters



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Hubble Space Telescope • WFPC2

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Note:

46 % of all O stars are runaways (v>30 km/s);

4 % of B stars are runaways. (Stone 1991)

10% of all runaway O stars are binaries.

(Gies & Bolton 1986)

→ Qualitative consistency with ***dynamical ejections*** from cluster cores. (Clarke & Pringle 1995; Pflamm-Altenburg & Kroupa 2006)

OB stars in clusters / HII regions

Two competing processes:

Mass segregation ***e.g.*** $t_{\rm relax} \approx 0.6\,{\rm Myr}$ for pre-exposed ONC

$$t_{\rm msgr} \approx 2 \left(\frac{m_{\rm av}}{m_{\rm massive}}\right) t_{\rm relax}$$

$$t_{\rm relax} = \frac{21}{\ln(0.4N)} \left(\frac{M_{\rm cl}}{100\,M_\odot}\right)^{\frac{1}{2}} \left(\frac{1\,M_\odot}{m_{\rm av}}\right) \left(\frac{R_{0.5}}{1\,{\rm pc}}\right)^{\frac{3}{2}}$$

↓

$t_{\rm msgr} \approx 0.12\,{\rm Myr} \ll$ age of ONC

Core decay ***e.g.*** $R_{\rm core} \approx 0.02\,{\rm pc},\ M_{\rm core} \approx 150\,M_\odot$

$t^{\rm core}_{\rm cross} \approx 1.2 \times 10^4\,{\rm yr}$

$$t_{\rm decay} \approx N_{\rm m} \times t_{\rm core,cross}$$

$$t^{\rm core}_{\rm cross} \approx 5 \left(\frac{M^{\rm core}}{100\,M_\odot}\right)^{-\frac{1}{2}} \left(\frac{R^{\rm core}_{0.5}}{1\,{\rm pc}}\right)^{\frac{3}{2}}$$

↓

$t_{\rm decay} \approx 10^4 - 10^5\,{\rm yr} \ll$ age of ONC

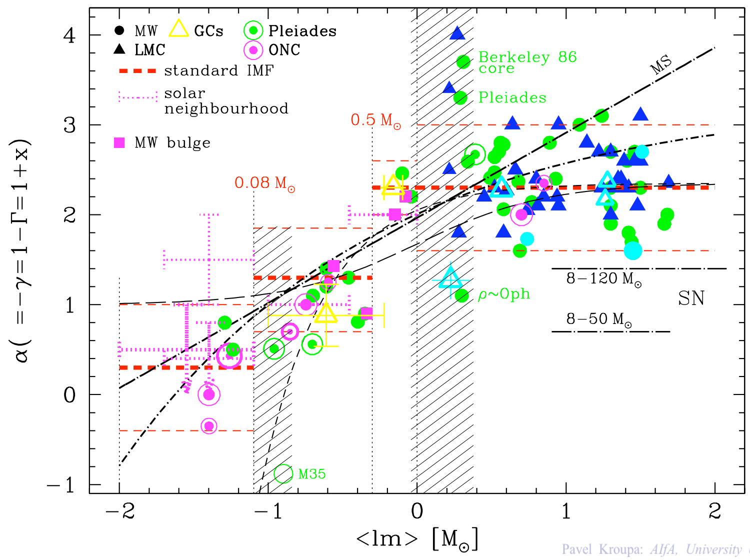
↓

Therefore, for *massive stars* there are *terrible biases* (unresolved multiplicity, mass-segregation, ejections) that *prohibit* an interpretation of a measured MF as being a *straightforward estimate* of the IMF.

Need to perform *N*-body modelling of *completely realistic* clusters on the *individual object basis* to place confident constraints on the *individual-cluster IMF*.

Clusters of any age are quite a *horrible place* to study the IMF.

The α-plot $\quad \xi(m) \propto m^{-\alpha(m)}$



The alpha-plot

$$\xi(m) \propto m^{-\alpha(m)}$$

$$\alpha(m)$$

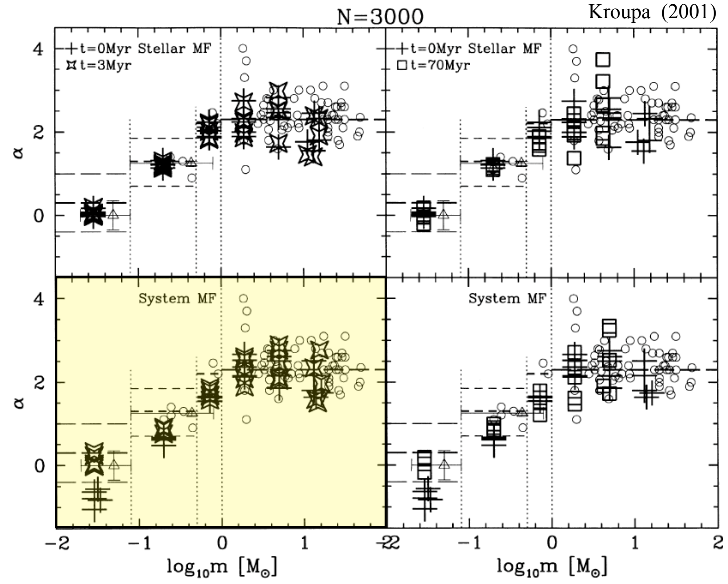
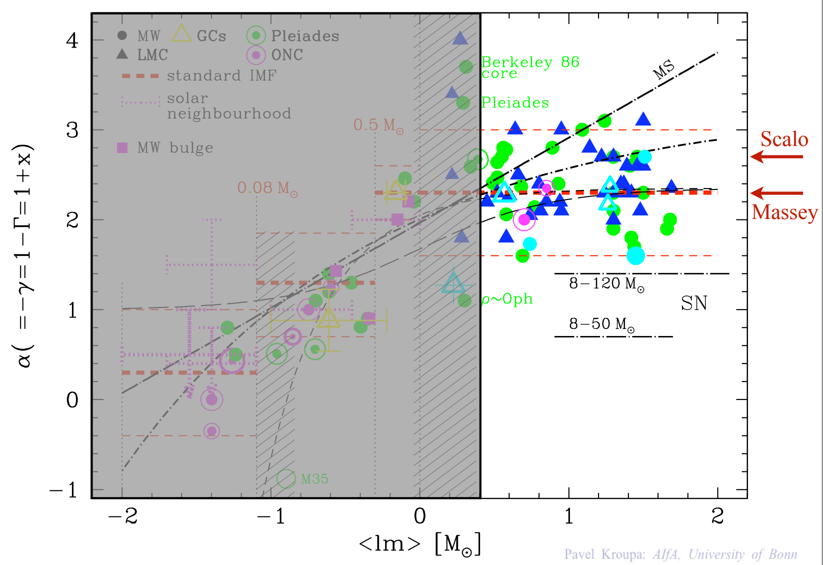
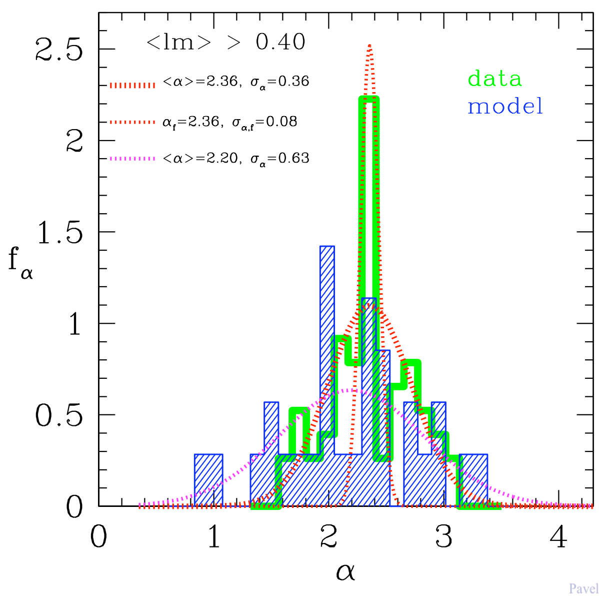


Figure 8. As in Fig. 7, but for five B3000 models.

The α - plot $\xi(m) \propto m^{-\alpha(m)}$



The ansatz
for binaries
to possibly solve the
Scalo/Massey discrepancy
fails.



Kroupa 2002

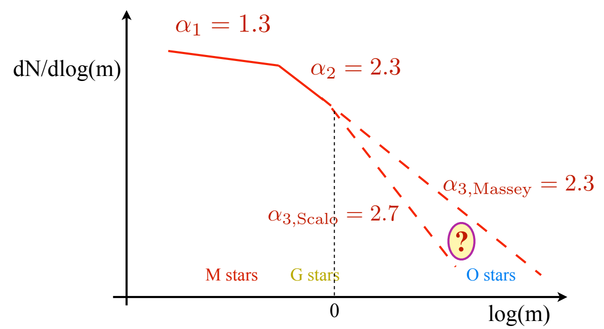
1. No asymmetry

binaries cannot resolve the *Scalo vs Massey* discrepancy.

2. Model *worse* than data **!?**

Return to the Massey / Salpeter discrepancy :

$\xi(m) \propto m^{-\alpha_i}$



Composite stellar populations

Composite Stellar Populations

Stars form in clusters (Lada & Lada 2003).
Thus, the **I**ntegrated **G**alaxial **IMF**

$$\xi_{\rm IGIMF}(m,t) = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}(SFR(t))} \xi(m \le m_{\rm max}(M_{\rm ecl}))\; \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)

The embedded-cluster MF (ECMF) **:**

$\xi_{\rm ecl} \propto M_{\rm ecl}^{-\beta}$; $\beta \approx 2-2.4$

solar-neighbourhood (Lada & Lada 2003)	few 10 $M_\odot$ – 1000 $M_\odot$
LMC & SMC (Hunter et al. 2003)	$10^3\,M_\odot - 10^4\,M_\odot$
Antennae (Zhang & Fall 1999)	$10^4\,M_\odot - 10^6\,M_\odot$

Composite Stellar Populations

Stars form in clusters (Lada & Lada 2003).
Thus, the **I**ntegrated **G**alactic **IMF**

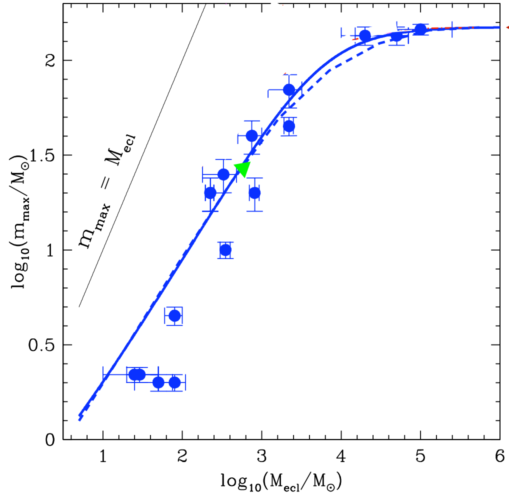
$$\xi_{\rm IGIMF}(m,t) = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}(SFR(t))} \xi(m \le m_{\rm max}(M_{\rm ecl}))\; \xi_{\rm ecl}(M_{\rm ecl})\, dM_{\rm ecl}$$

Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)

Add-up all IMFs
in all clusters **!**

The $m_{\rm max}(M_{\rm ecl})$ *relation*

Weidner & Kroupa 2005, 2006

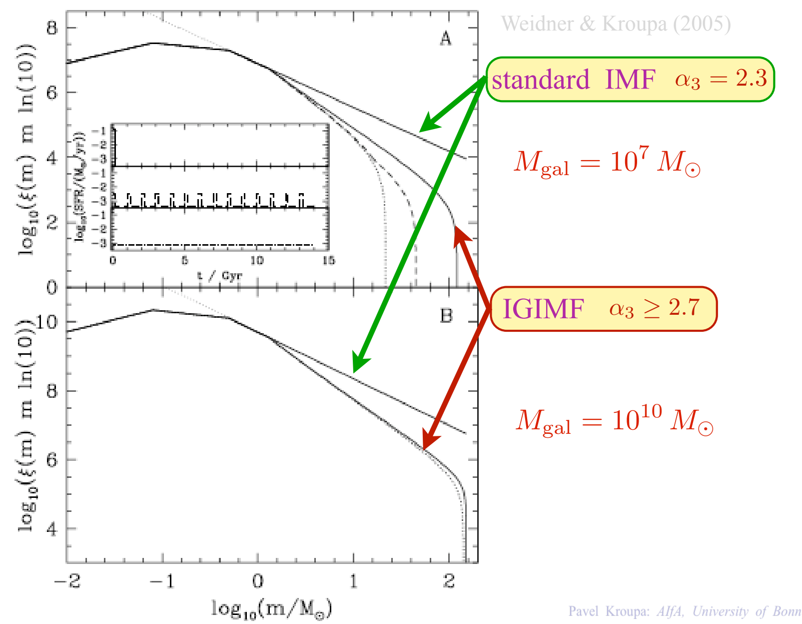


physical maximum stellar mass ?

$m_{\rm max,*} \approx 150\,M_\odot$

(Weidner & Kroupa 2004; Figer 2005; Oey & Clarke 2005, Koen 2006)

▲ Theoretical point (Bonnell et al. 2003)



Origin of the stellar IMF
& its required variation :

$\alpha \longrightarrow \Omega$

Thus, the *standard Galactic-field IMF* is

$$\xi(m) \propto m^{-\alpha_i}$$

$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \leq m/M_\odot < 0.5$

$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \leq m/M_\odot < 1$

$\alpha_3 = 2.7, \quad 1 \leq m/M_\odot$ *(Scalo)*

KTG93

But, the *standard stellar IMF* is

$$\xi(m) \propto m^{-\alpha_i}$$

$\alpha_1 = 1.3 \pm 0.5, \quad 0.08 \leq m/M_\odot < 0.5$

$\alpha_2 = 2.3 \pm 0.3, \quad 0.5 \leq m/M_\odot < 1$

$\alpha_3 = 2.3, \quad 1\, M_\odot \leq m$ *(Massey)*

✓

Kroupa 2001

Different theories on origin of stellar masses :

- The *Jeans mass* depends on *temperature* and *density* : $M_{\rm Jeans} \propto T^{\frac{3}{2}}\, \rho^{-\frac{1}{2}}$ (e.g. Bonnell, Larson & Zinnecker 2006)
- Stars define their own masses through *accretion* and *feedback* . (Adams & Fatuzzo 1996; Adams & Laughlin 1996)

The different theoretical approaches have in common that *higher-metallicity environments* should produce *lighter stars* on average.

$Z \uparrow \quad \longrightarrow \quad m \downarrow$

Can this be seen in the measured IMF ?

Different theories on origin of stellar masses :

- The *Jeans mass* depends on *temperature* and *density* : $M_{\rm Jeans} \propto T^{\frac{3}{2}}\ \rho^{-\frac{1}{2}}$ (e.g. Bonnell, Larson & Zinnecker 2006)
- Stars define their own masses through *accretion* and *feedback* . (Adams & Fatuzzo 1996; Adams & Laughlin 1996)

No empirical evidence of this has been found !

Origin of IMF

(Motte, Andre et al. 2001)

850 μm and 450 μm mapping of NGC 2068 and 2071.

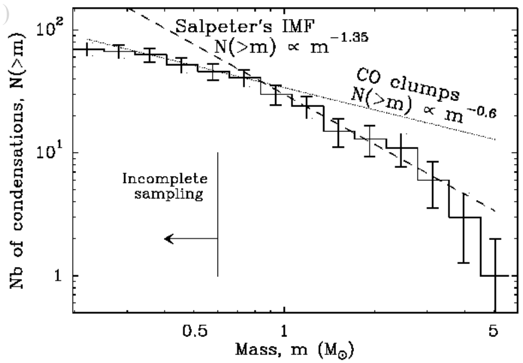


Fig. 3. Cumulative mass distribution of the 70 pre-stellar condensations of NGC 2068/2071. The dotted and dashed lines are power-laws corresponding to the mass spectrum of CO clumps (Kramer et al. 1996) and to the IMF of Salpeter (1955), respectively. The error bars correspond to $\sqrt{N}$ counting statistics.

Origin of IMF

(Motte, Andre & Neri 1998)

1.3 mm continuum mapping of Oph ρ.

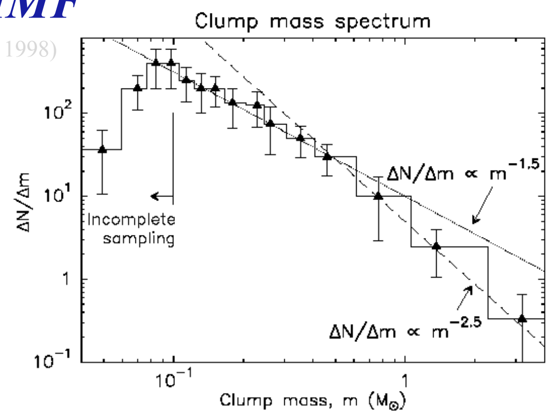


Fig. 5. Frequency distribution of masses for 60 small-scale clumps extracted from the mosaic of Fig. 1 (solid line). The dotted and long-dashed lines show power laws of the form $\Delta N/\Delta m \propto m^{-1.5}$ and $\Delta N/\Delta m \propto m^{-2.5}$, respectively. The error bars correspond to $\sqrt{N}$ counting statistics.

Schematic IMF (Bonnell, Larson & Zinnecker 2006, PPV)

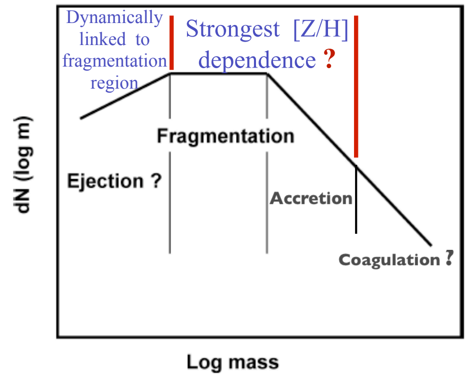


Fig. 11.— A schematic IMF showing the regions that are expected to be due to the individual processes. The peak of the IMF and the characteristic stellar mass are believed to be due to gravitational fragmentation, while lower mass stars are best understood as being due to fragmentation plus ejection or truncated accretion while higher-mass stars are understood as being due to accretion.

Elmegreen (2004) also proposes a *three-part IMF*.

Is a *consensus* emerging on the *fundamental physics* active in the *three mass regimes* ?

But then, what about the [Z/H] dependence ?

Conclusions

- ***LF*** structure understood (universal peak at Mv=12).
- ***IMF*** reasonably-well constrained for $m \lesssim 1\,M_\odot$.
- ***OB stars:*** IMF remains an uncertain issue (mass-segregation, ejections, unresolved multiples).
- $m_{\mathrm{max}*} \approx 150\,M_\odot$?
- ***Universality:*** evident but not understood fully.
- ***Composite stellar populations:*** A *variable IGIMF*, resolution of *Scalo vs Massey* controversy, and implications !?
- ***Origin of IMF:*** frozen-in from pre-stellar cores ?

According to the standard stellar IMF :

mass range $[M_\odot]$	% by number	% by mass
0.01 - 0.08	37.2	4.1
0.08 - 0.5	47.8	26.6
0.5 - 1	8.9	16.1
1 - 8	5.7	32.4
8 - 120	0.40	20.8
$<m>$	0.38 $M_\odot$	

THE END

mass range $[M_\odot]$	η_N [per cent] α_3 2.3	η_N [per cent] α_3 2.7	η_N [per cent] α_3 4.5	η_M [per cent] α_3 2.3	η_M [per cent] α_3 2.7	η_M [per cent] α_3 4.5	$\rho^{\rm st}$ $[M_\odot/{\rm pc}^3]$ α_3 4.5	$\Sigma^{\rm st}$ $[M_\odot/{\rm pc}^2]$ α_3 4.5
0.01–0.08	37.15	37.69	38.63	4.08	5.39	7.39	3.21×10^{-3}	1.60
0.08–0.5	47.81	48.50	49.71	26.61	35.16	48.21	2.09×10^{-2}	10.45
0.5–1	8.94	9.07	9.30	16.13	21.31	29.22	1.27×10^{-2}	6.35
1 – 8	5.70	4.60	2.36	32.38	30.30	15.09	6.54×10^{-3}	1.18
8 – 120	0.40	0.14	0.00	20.80	7.83	0.08	3.63×10^{-5}	6.53×10^{-3}
$\overline{m}/M_\odot =$	0.380	0.292	0.218				$\rho^{\rm st}_{\rm tot} = 0.043$	$\Sigma^{\rm st}_{\rm tot} = 19.6$

$m_{\rm max}$ $[M_\odot]$	$\alpha_3 = 2.3$ $N_{\rm cl}$	$\alpha_3 = 2.3$ $M_{\rm cl}$ $[M_\odot]$	$\alpha_3 = 2.7$ $N_{\rm cl}$	$\alpha_3 = 2.7$ $M_{\rm cl}$ $[M_\odot]$	$m_{\rm to}$ $[M_\odot]$	$\Delta M_{\rm cl}/M_{\rm cl}$ [per cent] $\alpha_3 = 2.3$	$\Delta M_{\rm cl}/M_{\rm cl}$ [per cent] $\alpha_3 = 2.7$
1	16	2.9	21	3.8	80	2.1	0.5
8	245	74	725	195	60	3.8	0.9
20	806	269	3442	967	40	6.5	1.6
40	1984	703	1.1×10^4	2302	20	12	3.5
60	3361	1225	2.2×10^4	6428	8	21	7.8
80	4885	1812	3.6×10^4	1.1×10^4	3	24	9.7
100	6528	2451	5.3×10^4	1.5×10^4	1	36	24
120	8274	3136	7.2×10^4	2.1×10^4	0.7	39	28

general	$dN = \xi(m)\,dm = \xi_{\rm L}(m)dlm$		
	$\xi_{\rm L}(m) = (m\ln 10)\,\xi(m)$		*gen*
Scalo's IMF index (*111*)	$\Gamma(m) \equiv \frac{d}{dlm}\,(\log_{10}\xi_{\rm L}(lm))$		*Gam*
	$\Gamma = -x = 1+\gamma = 1-\alpha$		*ind*
	e.g. for power-law form:	$\xi_{\rm L} = A\,m^{\Gamma} = A\,m^{-x}$ $\xi = A'\,m^{\alpha} = A'\,m^{-\gamma}$ $A' = A/\ln 10$	
Salpeter(1955) (*108*)	$\xi_{\rm L}(lm) = A\,m^{\Gamma}$	$\Gamma = -1.35\,(\alpha = 2.35)$	*S*
	$A = 0.03\,{\rm pc}^{-3}\log_{10}^{-1}M_\odot;\quad 0.4 \le m/M_\odot \le 10$		
Miller-Scalo(1979) (*90*)	$\xi_{\rm L}(lm) = A\exp\left[-\frac{(lm-lm_o)^2}{2\,\sigma_{lm}^2}\right]$	$\Gamma(lm) = -\frac{(lm-lm_o)}{\sigma_{lm}^2}\log_{10}e$	*MS*
thick long-dash-dotted line	$A = 106\,{\rm pc}^{-2}\log_{10}^{-1}M_\odot;\quad lm_o = -1.02;\quad \sigma_{lm} = 0.68$		
Larson(1998) (*74*)	$\xi_{\rm L}(lm) = A\,m^{-1.35}\exp\left[-\frac{m_o}{m}\right]$	$\Gamma(lm) = -1.35 + \frac{m_o}{m}$	*La*
thin short-dashed line	$A = -;\qquad m_o = 0.3\,M_\odot$		
Larson(1998) (*74*)	$\xi_{\rm L}(lm) = A\,\left[1+\frac{m}{m_o}\right]^{-1.35}$	$\Gamma(lm) = -1.35\,(1+\frac{m_o}{m})^{-1}$	*Lb*
thin long-dashed line	$A = -;\qquad m_o = 1\,M_\odot$		
Chabrier(2001) (*22, 23*)	$\xi(m) = A\,m^{-\delta}\exp\left[-\left(\frac{m_o}{m}\right)^{\beta}\right]$	$\Gamma(lm) = 1-\delta+\beta\left(\frac{m_o}{m}\right)^{\beta}$	*Ch*
thick short-dash-dotted line	$A = 3.0\,{\rm pc}^{-3}\,M_\odot^{-1};\quad m_o = 716.4\,M_\odot;\quad \delta = 3.3;\quad \beta = 0.25$		

Table 1: Summary of different proposed analytical IMF forms (the modern power-law form, the standard IMF, is presented in eq. 4). Notation: $lm \equiv \log_{10}(m/M_\odot) = \ln(m/M_\odot)/\ln 10$; dN is the number of single stars in the mass interval m to $m+dm$ and in the logarithmic-mass interval lm to $lm+dlm$. The mass-dependent IMF indices, $\Gamma(m)$ (eq. *Gam*), are plotted in Fig. 5 using the line-types defined here. Eq. *MS* was derived by Miller&Scalo assuming a constant star-formation rate and a Galactic disk age of 12 Gyr (the uncertainty of which is indicated in the lower panel of Fig. 5a). Larson (*74*) does not fit his forms (eqs. *La* and *Lb*) to solar-neighbourhood star-count data but rather uses these to discuss general aspects of likely systematic IMF evolution; the m_o in eq. *La* and *Lb* given here are approximate eye-ball fits to the standard IMF.

THE END

Which IMF form is "best" ?

log -normal

✓ $\xi_L(\mu) = C\,e^{-\frac{(\mu-\bar{\mu})^2}{2\sigma_l^2}}$, $\mu = log_{10}\,m$

$\bar{\mu} = -1.1 \quad m_c = 0.08 M_\odot$

$\sigma_l = 0.69 \quad 10^{\sigma_l} = 5 M_\odot$

Larson (1998)

✓ $\xi_L^{L_1} = C\,m^{-\alpha_L}\,e^{-\frac{m_1}{m}}$

$\alpha_L = 1.5$

$m_1 = 0.3 M_\odot$

✓ $\xi_L^{L_2} = C\,(1+\frac{m}{m_1})^{-\alpha_L}$

$\alpha_L = 1.3$

$m_1 = 1.0 M_\odot$

Chabrier (1999)

✓ $\xi_L^{C} = C\,m^{-\alpha_C}\,e^{-\left(\frac{m_1}{m}\right)^{\beta}}$

None results from some "IMF theory".

Personal opinion: the 3-part power-law form

- has been demonstrated to fit a large number of populations using a consistent approach;
- is mathematically very easy to handle;
- nice for experiments (eg.varying # Mdwarf / #Ostars).