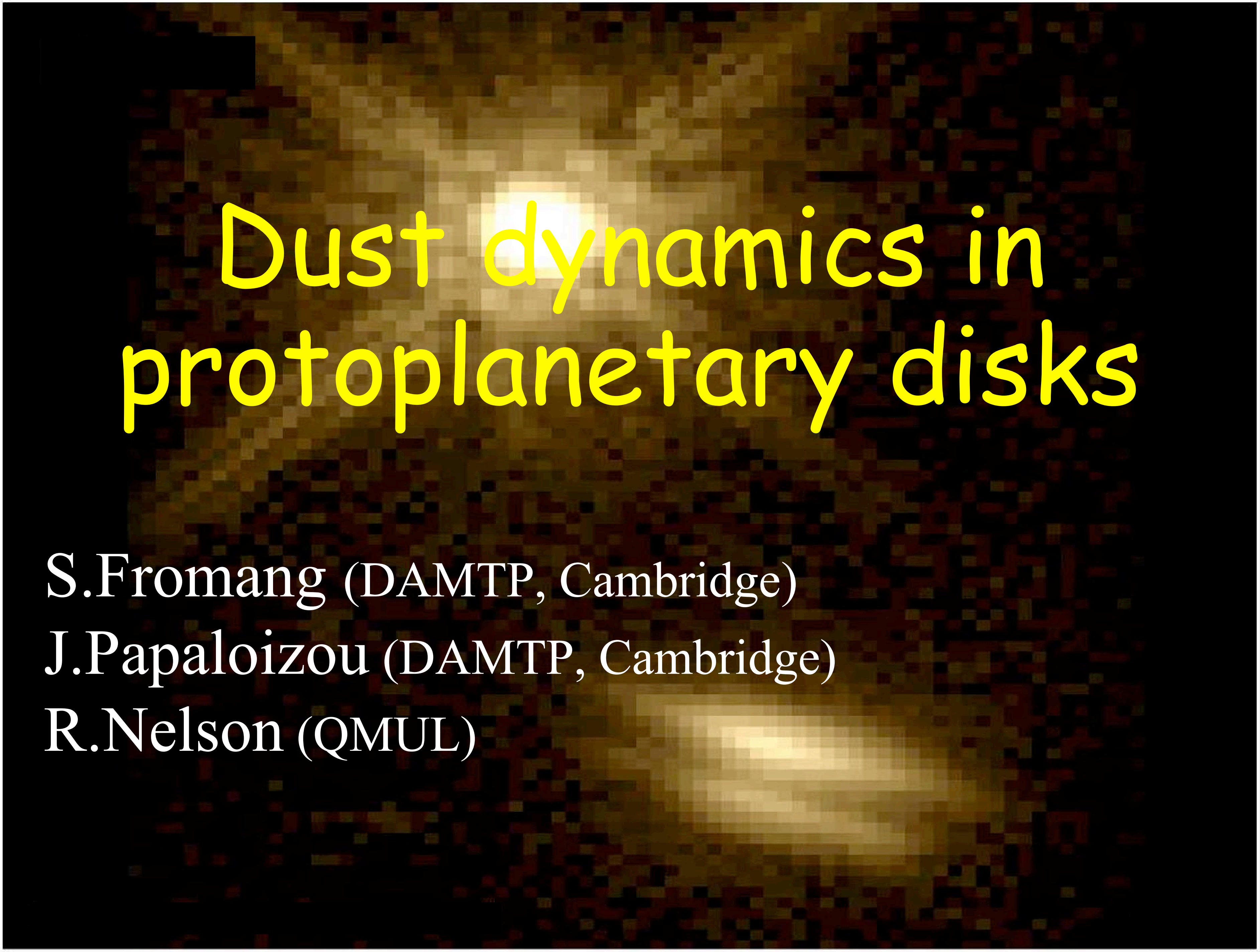


Dust dynamics in protoplanetary disks

S.Fromang (DAMTP, Cambridge)
J.Papaloizou (DAMTP, Cambridge)
R.Nelson (QMUL)

Motivations

Theory:

- Planet formation

  (Pollack et al. 1996, Goldreich & Ward 1973)

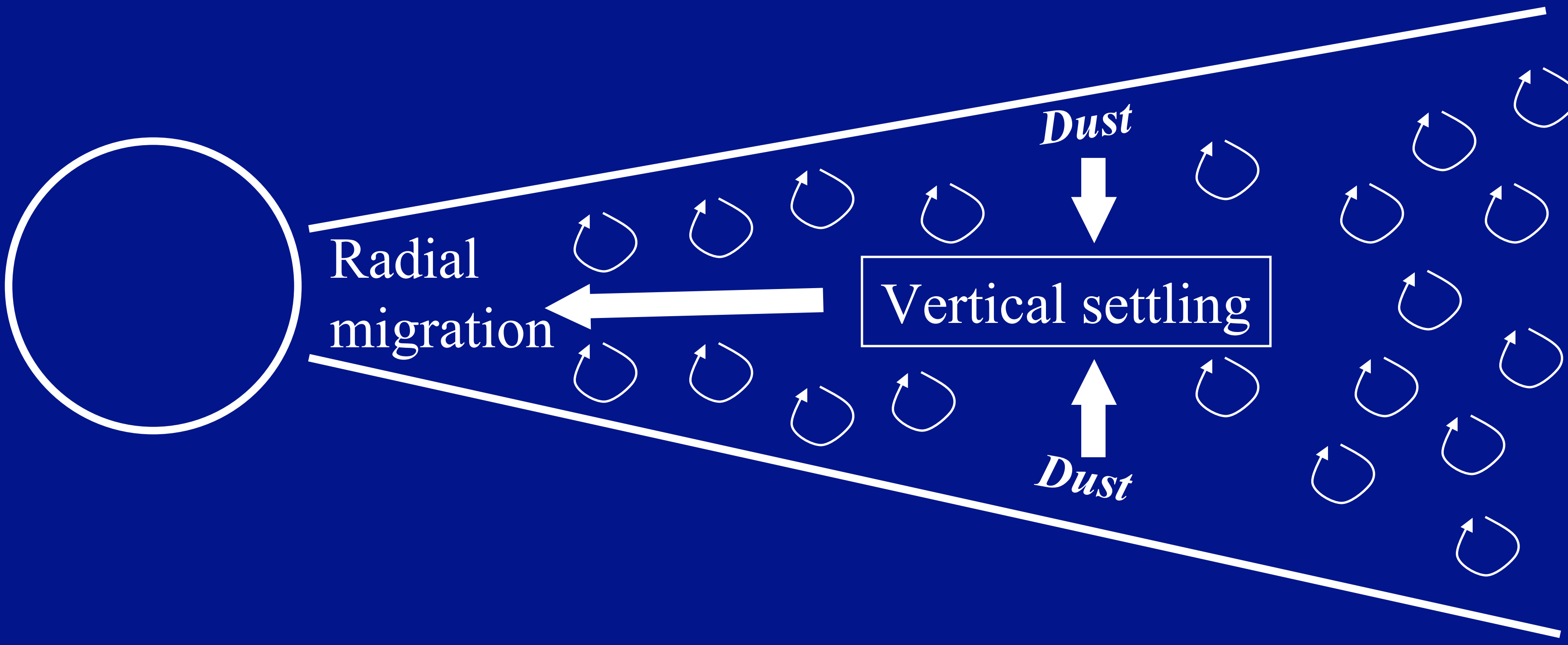
Observations:

- Scattering in the optical

  (Mc Caughrean et al. 2000, Duchene et al. 2004)
- Emission at millimeter wavelengths

  (Natta et al. 2006, revue PPV)

Overview



- How does MHD turbulence affect dust settling?
- How does MHD turbulence affect dust radial migration?

Drag forces

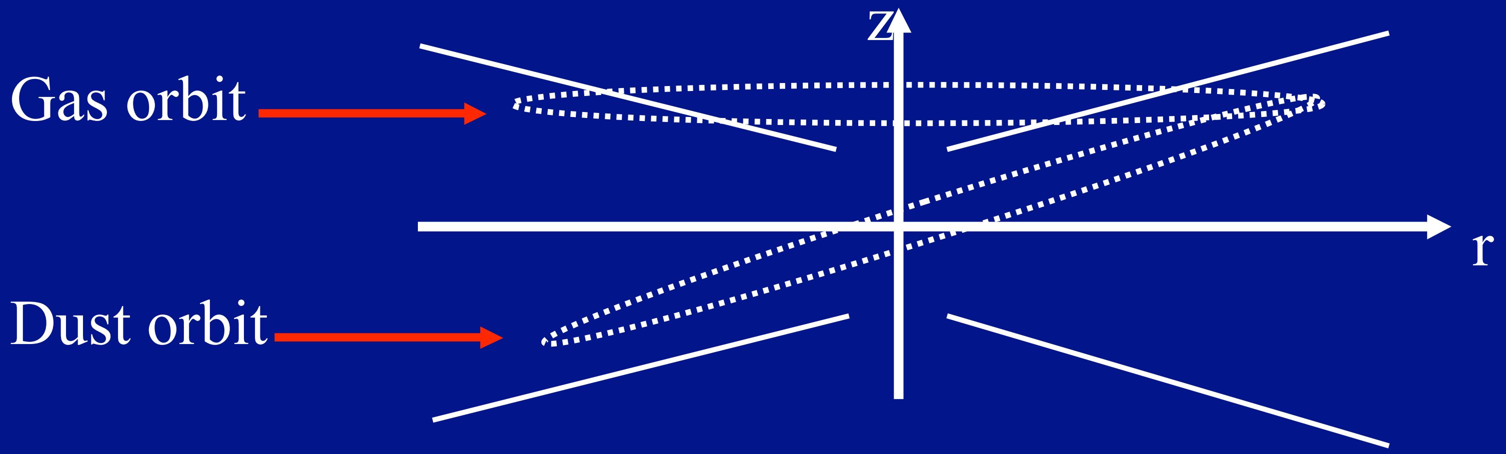
Force on the dust in the Epstein regime: $$F_{drag} = m_p \frac{\rho c_s}{\rho_{dust} a}(v_g - v_{dust})$$

Stopping time scale: $$\tau_s = \frac{\rho_{dust} a}{\rho c_s}$$

- If $\Omega\tau_s$<1 → good coupling, 2 fluid description (Garaud et al, 2004)

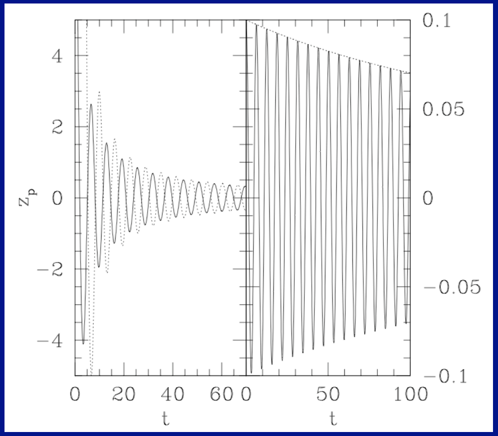
- If $\Omega\tau_s$~1 → weak coupling, N-body approach better

Vertical settling



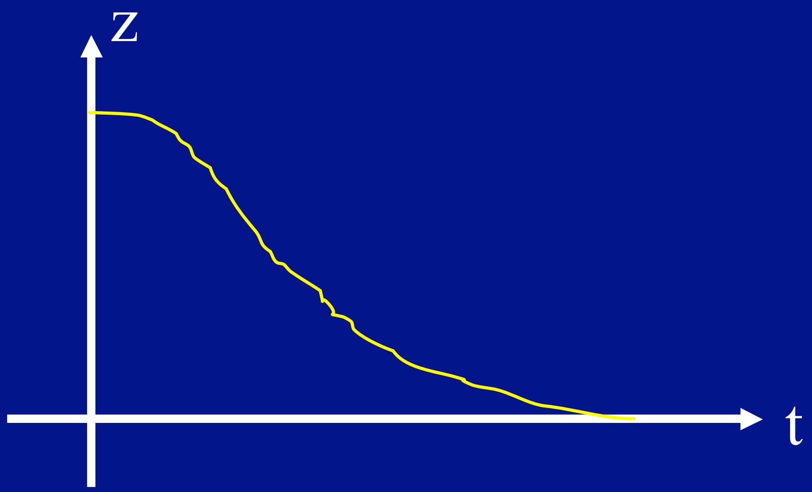
Gas orbit

Dust orbit



Weak coupling limit

(Garaud et al. 2004)



Strong coupling limit

Radial migration

Weak coupling limit:

- gas orbits at sub-Keplerian velocity
- dust orbits at Keplerian velocity
  - □ dust feels an head-wind
  - □ **migrate inward**

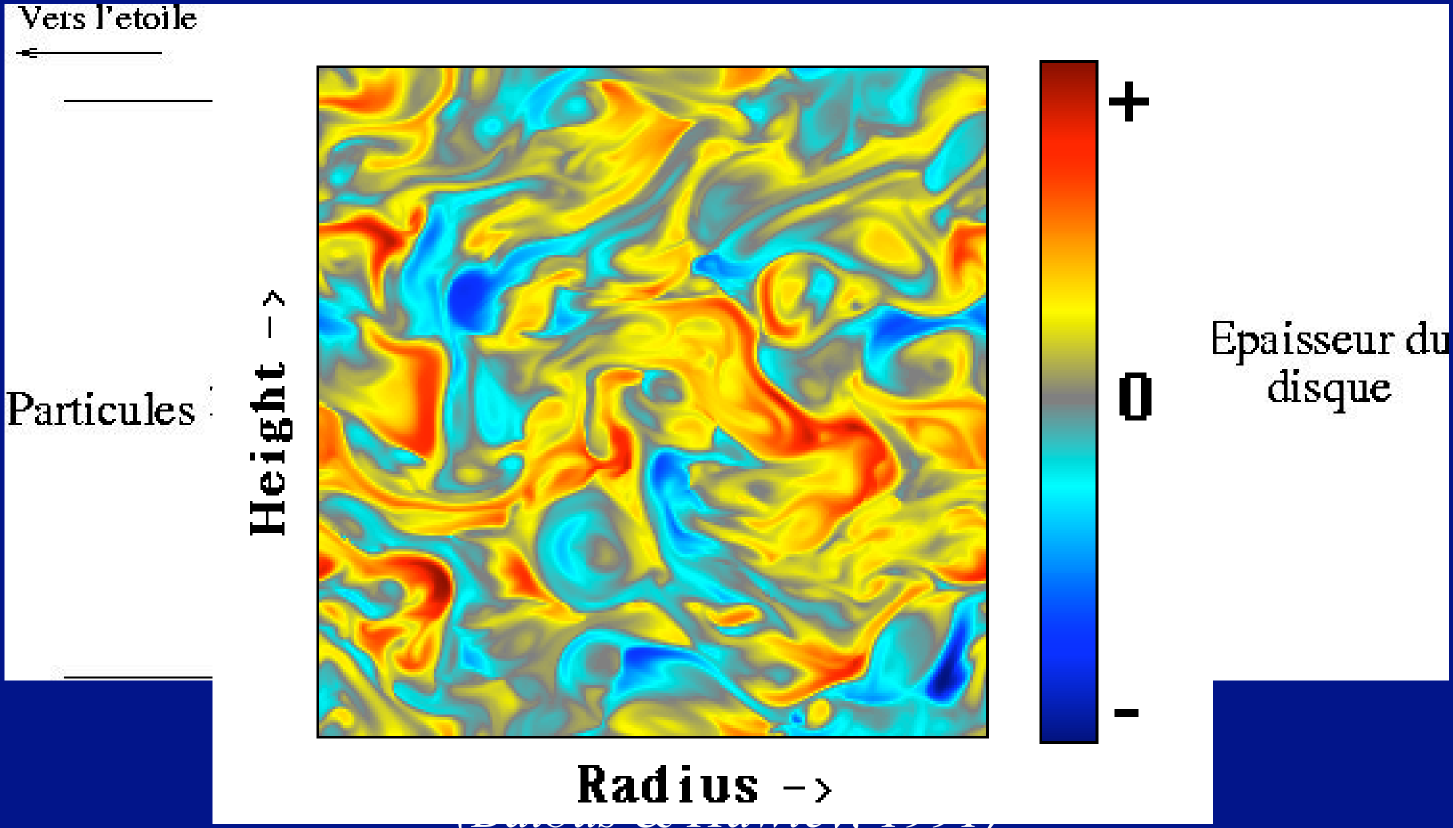
Strong coupling limit:

- gas & dust orbits at sub-Keplerian velocity
- dust does **NOT** feel radial pressure gradient
  - □ **migrate inward**

For 1 meter particles: $\square_{mig} \sim 10^{2\text{-}3}$ years

(Weidenschilling 1977)

Turbulence MHD



Simulation locale

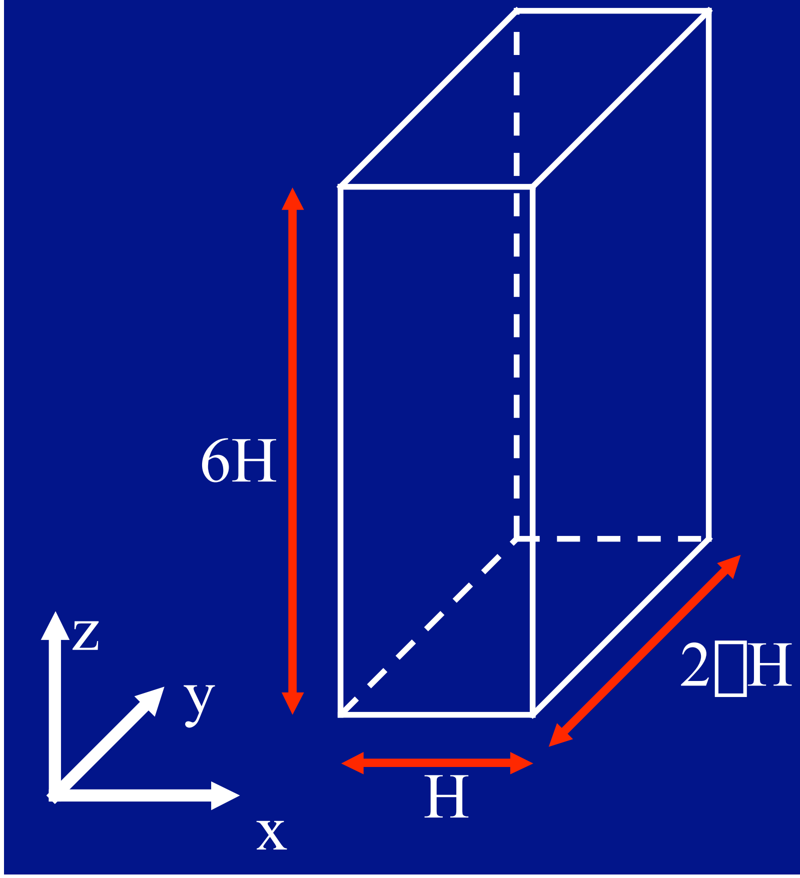
(Hawley & Balbus 1992)

Dust vertical settling

(Fromang & Papaloizou 2006)

Disk model: setup

- Stratified shearing box (Stone et al. 1996) with ZEUS
- Resolution: $(N_r,N_\phi,N_z)=(32,100,192)$

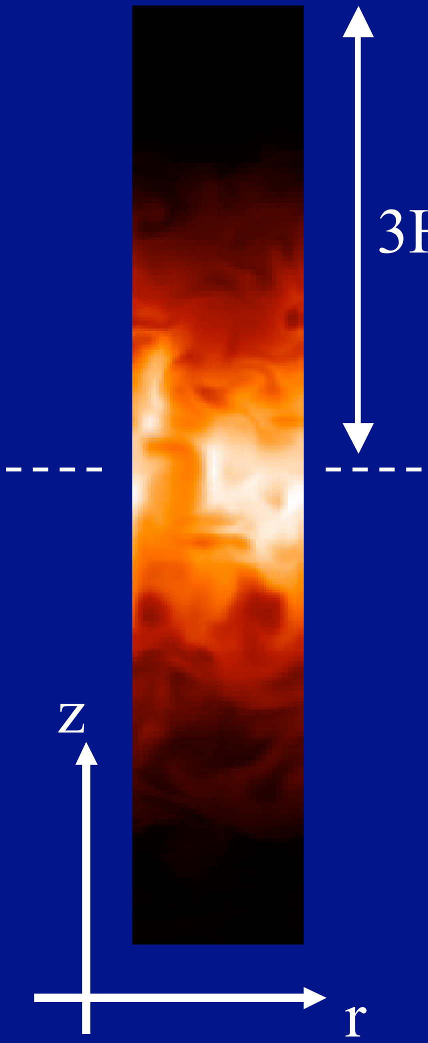


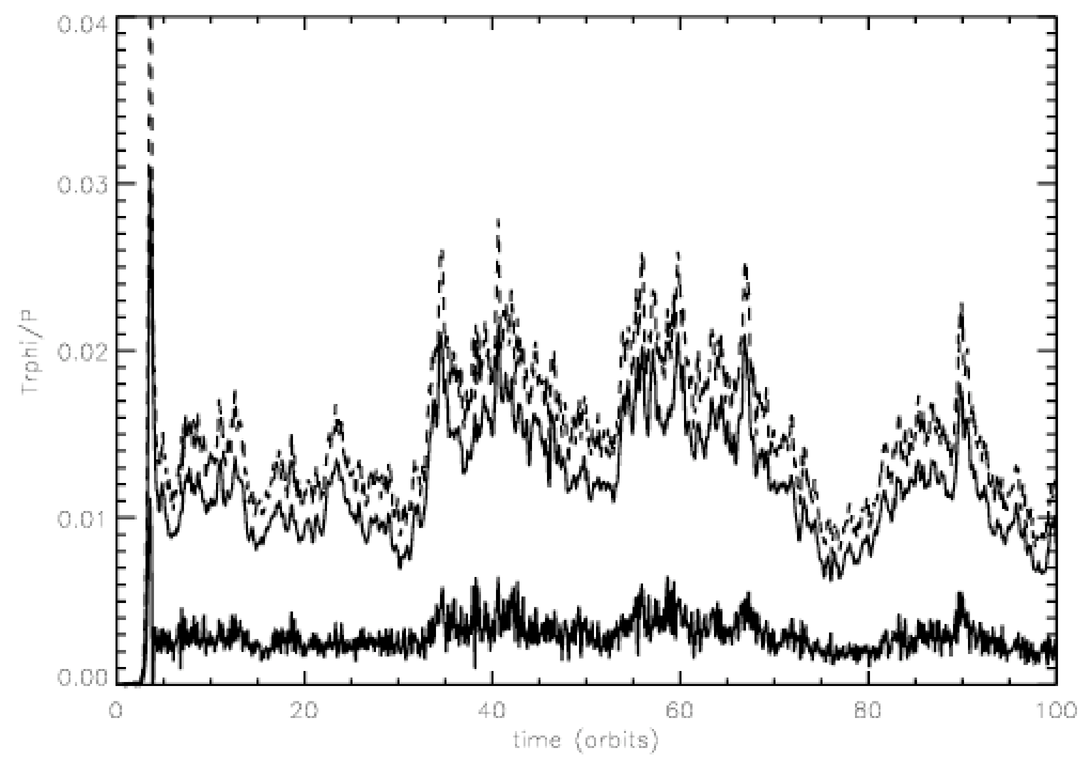
Champ magnétique initial:

$$B_z = B_0 \sin\left(\frac{2\pi x}{H}\right)$$

⇒ pas de flux à travers le disque

Disk model: properties

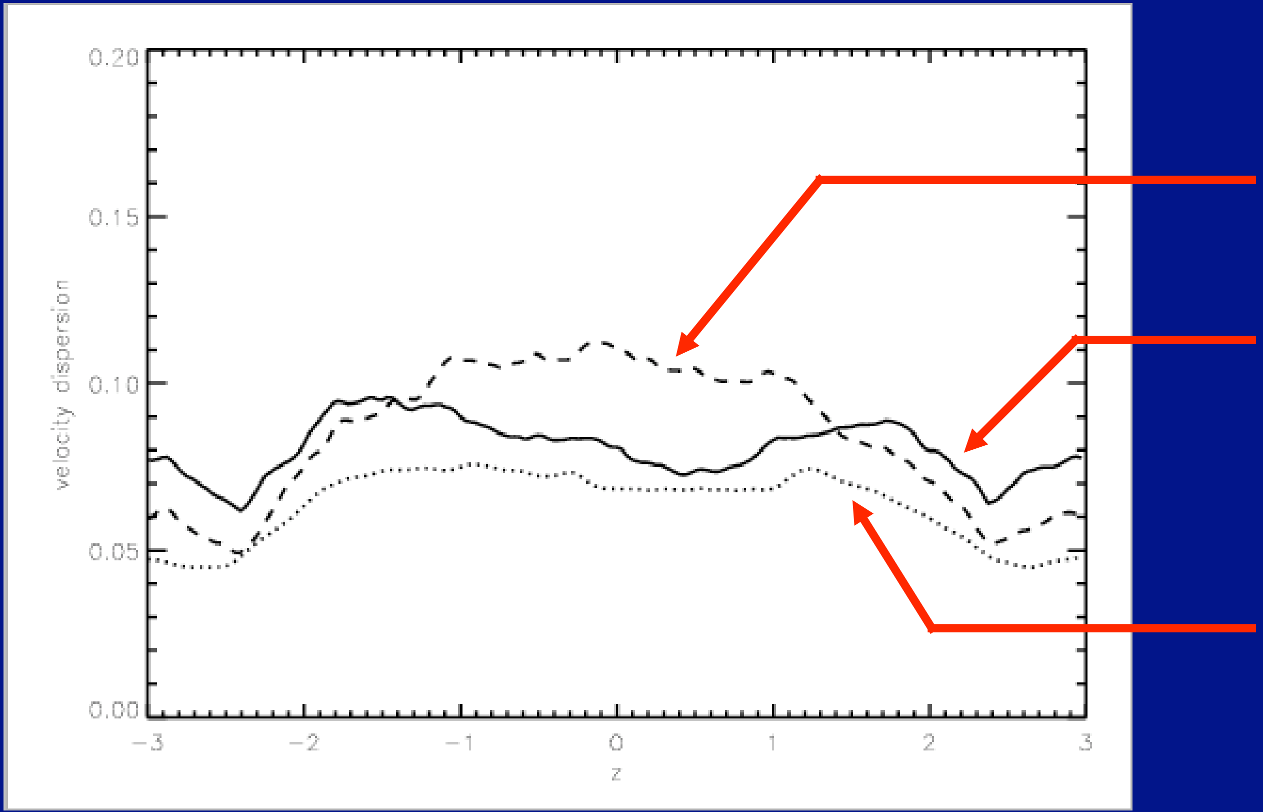




Time history for the Maxwell and Reynolds stress tensor

Similar to Stone et al. (1996)

Velocity fluctuations



$[\square(v_x{}^2)]^{1/2}$

$[\square(v_y{}^2)]^{1/2}$

$[\square(v_z{}^2)]^{1/2}$

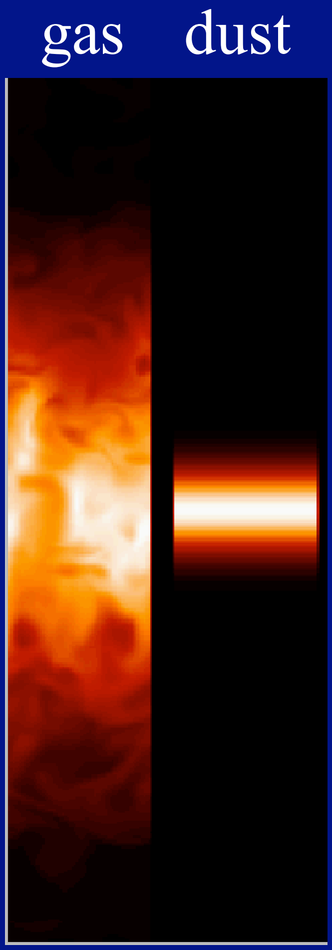
$[\square(v_x{}^2)]^{1/2} \sim 0.11 c_s$

$[\square(v_y{}^2)]^{1/2} \sim 0.08 c_s$

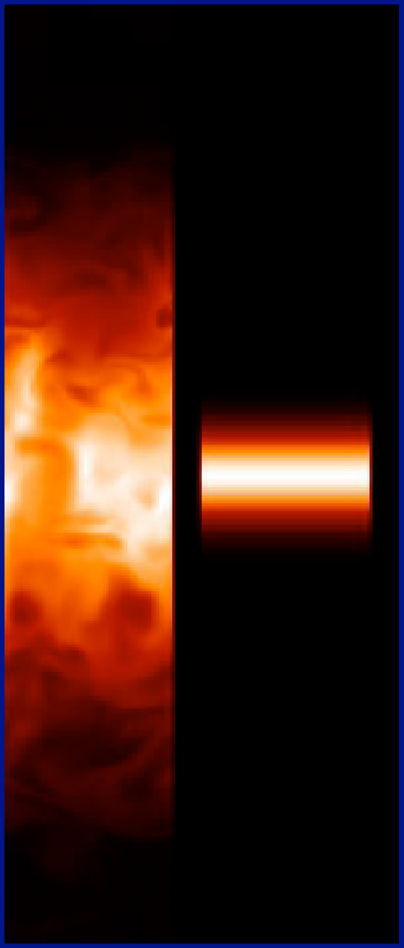
$[\square(v_z{}^2)]^{1/2} \sim 0.07 c_s$

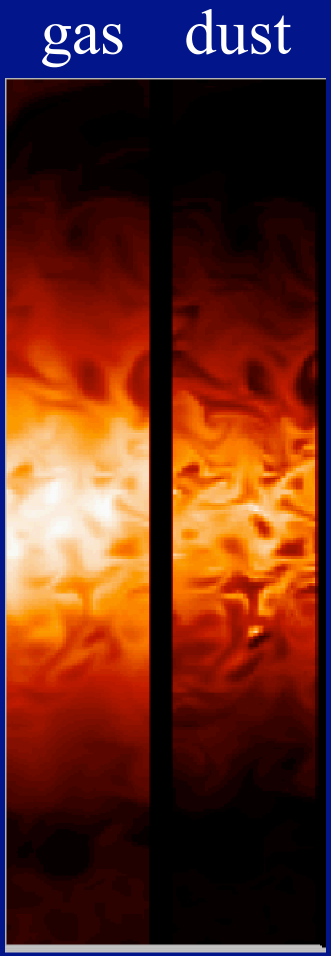
1 micron particles (□ □$_s$=10^{-6})

Early evolution



Initial state





10 orbits

15 orbits

Simple model (1/2)

- 3D, isotropic, steady state turbulence
- $Z(z_0,t)=z-z_0$ et $U(z_0,t)$, Lagrangian displacement and velocity
- $v_z(z,t)$, Eulerien velocity: $v(z,t)= U(z_0,t)$

$$\frac{\partial Z^2(z_0,t)}{\partial t} = 2Z(z_0,t)U(z_0,t)$$

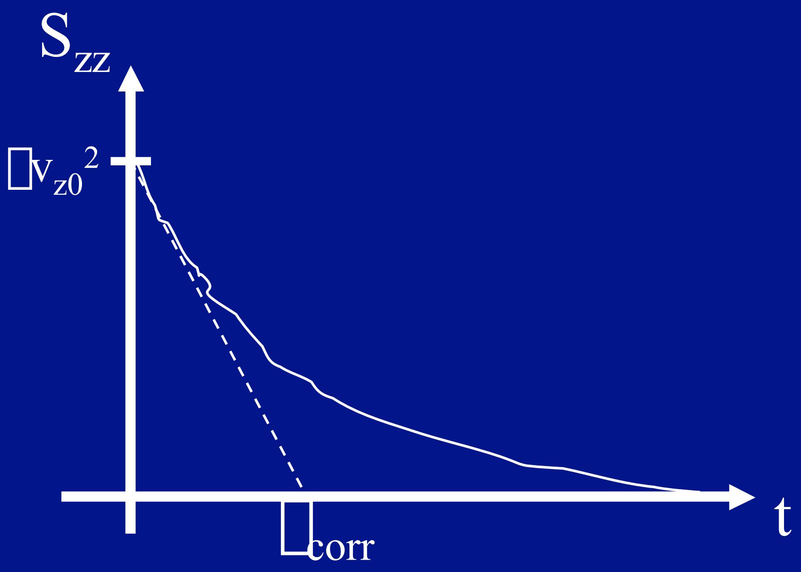
$$\frac{\partial < Z^2(z_0,t) >}{\partial t} = 2\int_0^t \underbrace{< v_z(z(z_0,t'),t')v_z(z(z_0,t),t) >}_{S_{zz}(t,t') = S_{zz}(t-t') = S_{zz}(\tau)} dt'$$

Then: $S_{zz}(\tau) =< v_z(z(z_0,\tau),\tau)v_z(z_0,0) >$

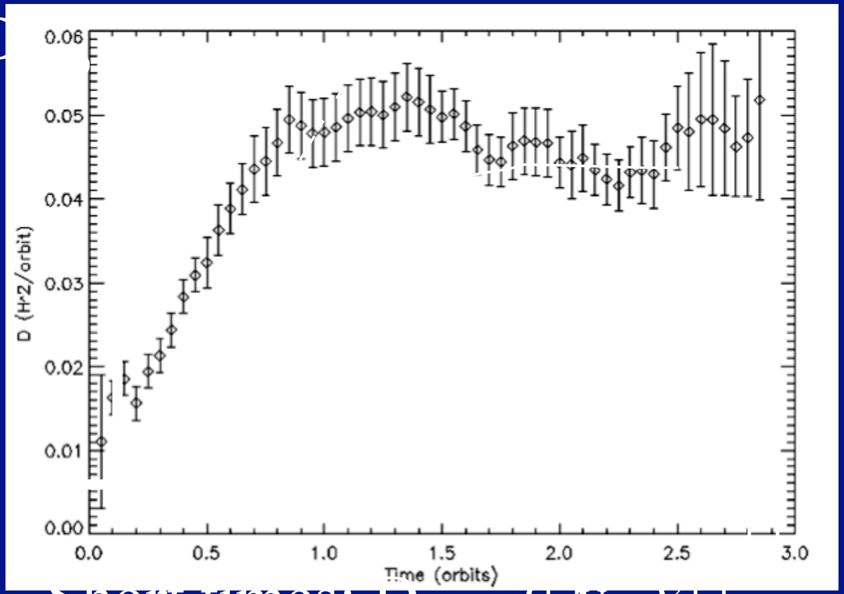
Simple model (2/2)

$$\frac{\partial < Z^2(z_0,t) >}{\partial t} = 2D_{turb}$$

$$D_{turb} = \int_0^t S_{zz}(\tau)d\tau = \int_0^t < v_z(0)v_z(\tau) > d\tau$$



D



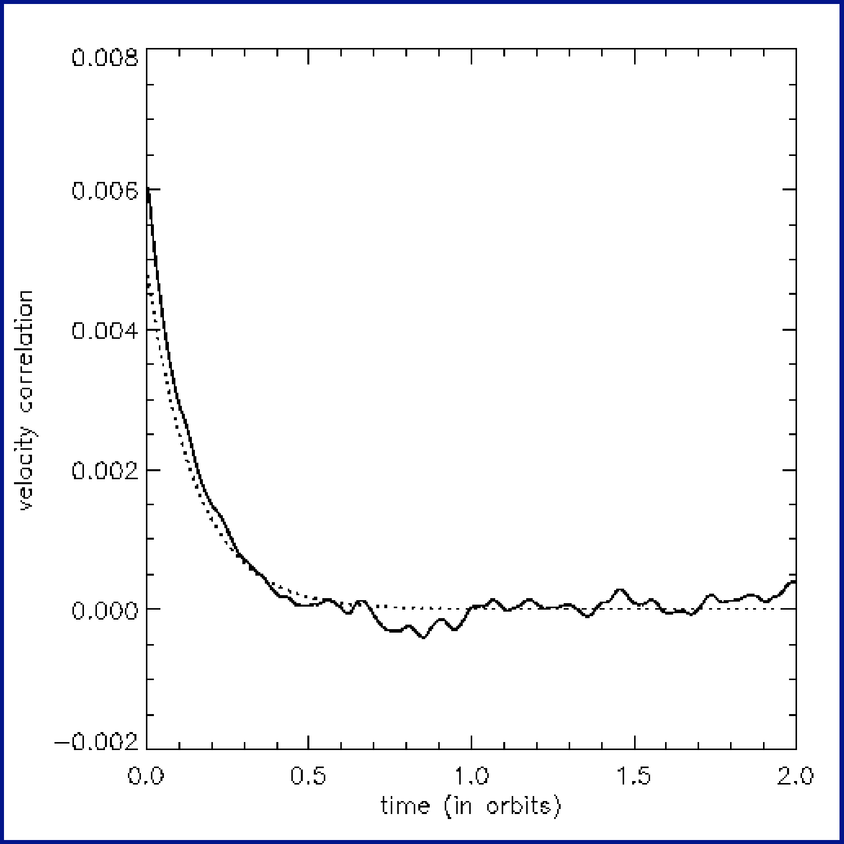
- Short times: $D_{turb} \sim (\delta v_{z0})^2 t$

(Carballido, Stone & Pringle 2005)

- Long times: $D_{turb} \sim (\delta v_{z0})^2 \tau_{corr}$

S_{zz}

- Start models at t=40,45,50,55,60,65,70,75,80 orbits
- Evaluate S_{zz} and D_{turb} (volume average for $|z|<H$)

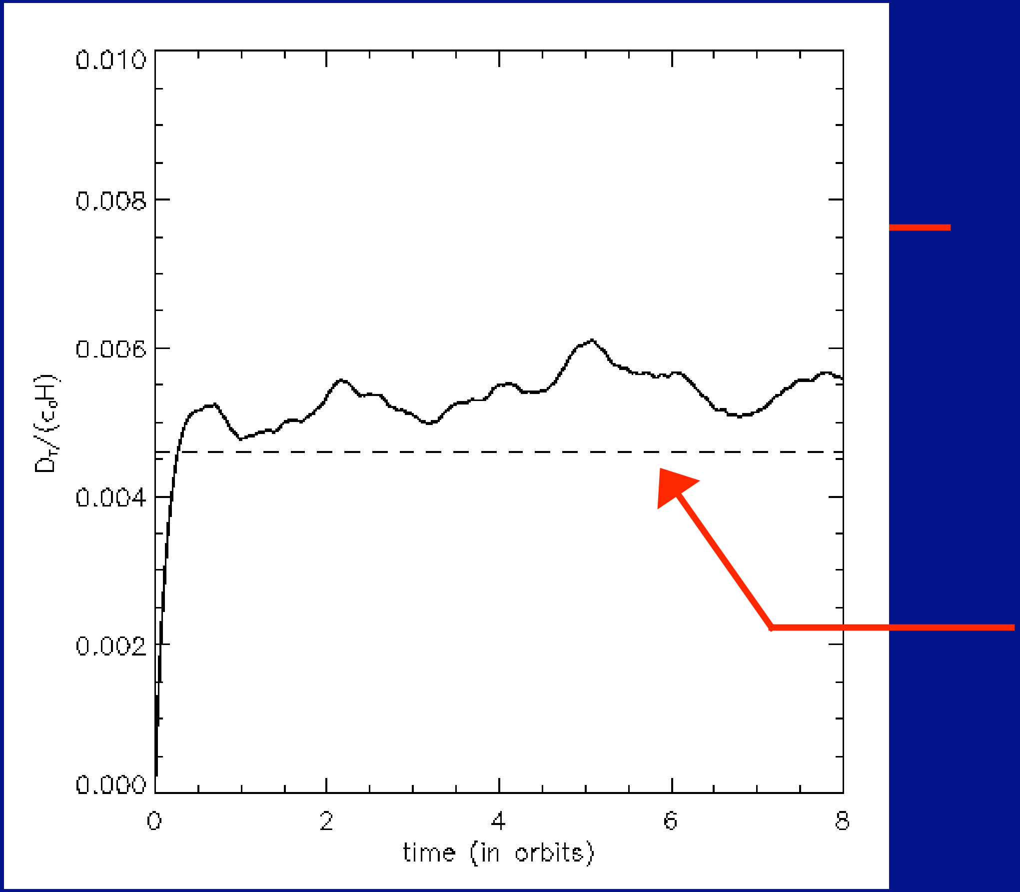


$$S_{zz}=(\delta v_z)^2 \cdot \exp(-t/\tau_{corr})$$

with:

- $\delta v_z=0.07 \cdot c_s$
- $\tau_{corr}=0.15$ orbits

Diffusion coefficient



Short time limit:

$D_{dust,th}=(\Box v_z)^2\Box\Box$

$D_{dust,th}=(\Box v_z)^2\Box_{corr}$

$=4.7 \Box 10^{-3}\ c_sH$

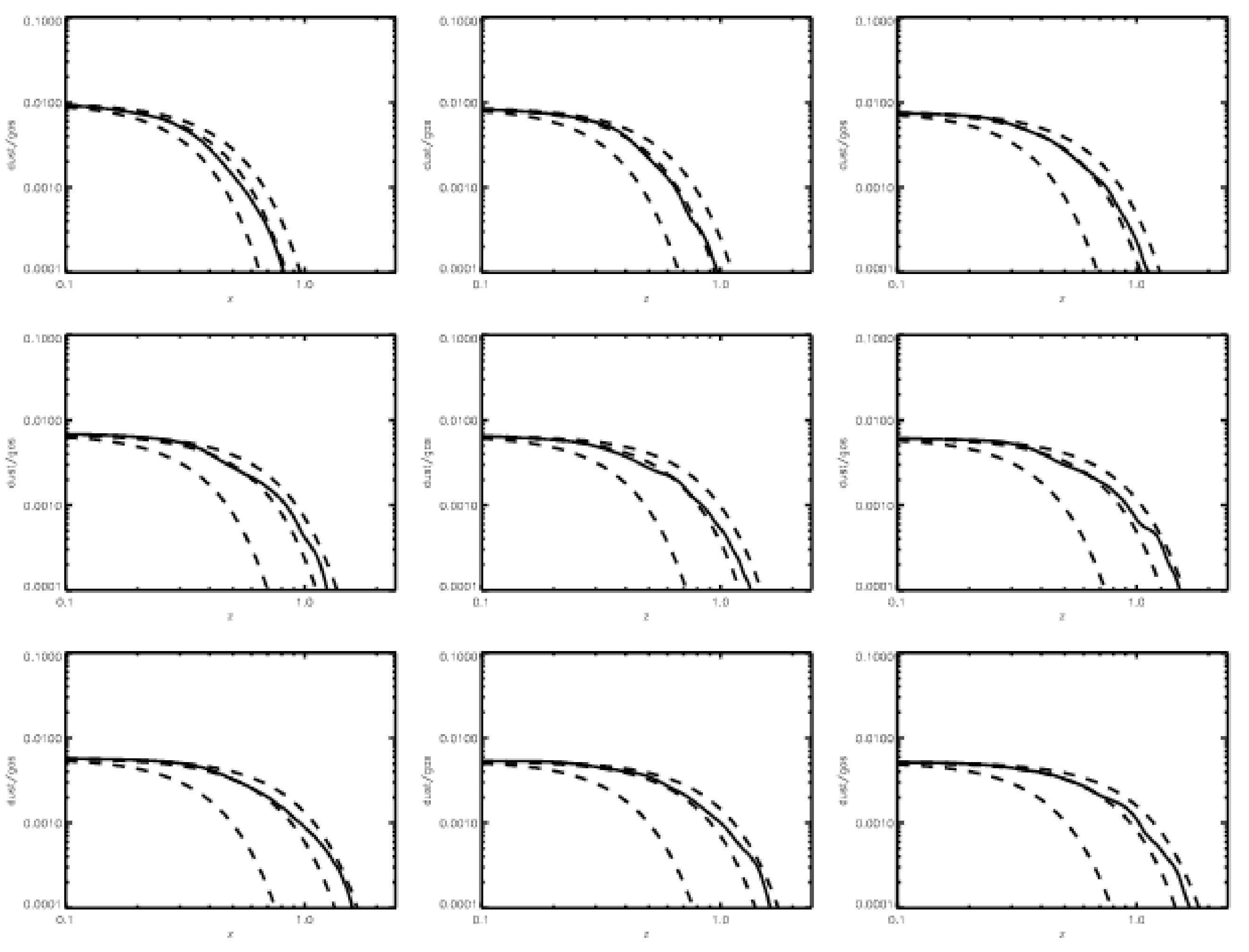
$<D_{dust}>=5.5\Box 10^{-3}c_sH$

Diffusive evolution

t=0.15 orbits
0.45 orbits
0.75 orbits

1.05 orbits
1.45 orbits
1.75 orbits

2.05 orbits
2.35 orbits
2.65 orbits



$D_{dust}/(c_sH)=10^{-3},5.5\square10^{-3},10^{-2}$

Schmidt number

$$S_c = \frac{\square_{turb}}{D_{dust}} = \frac{\square c_s H}{D_{dust}} = 2.8$$

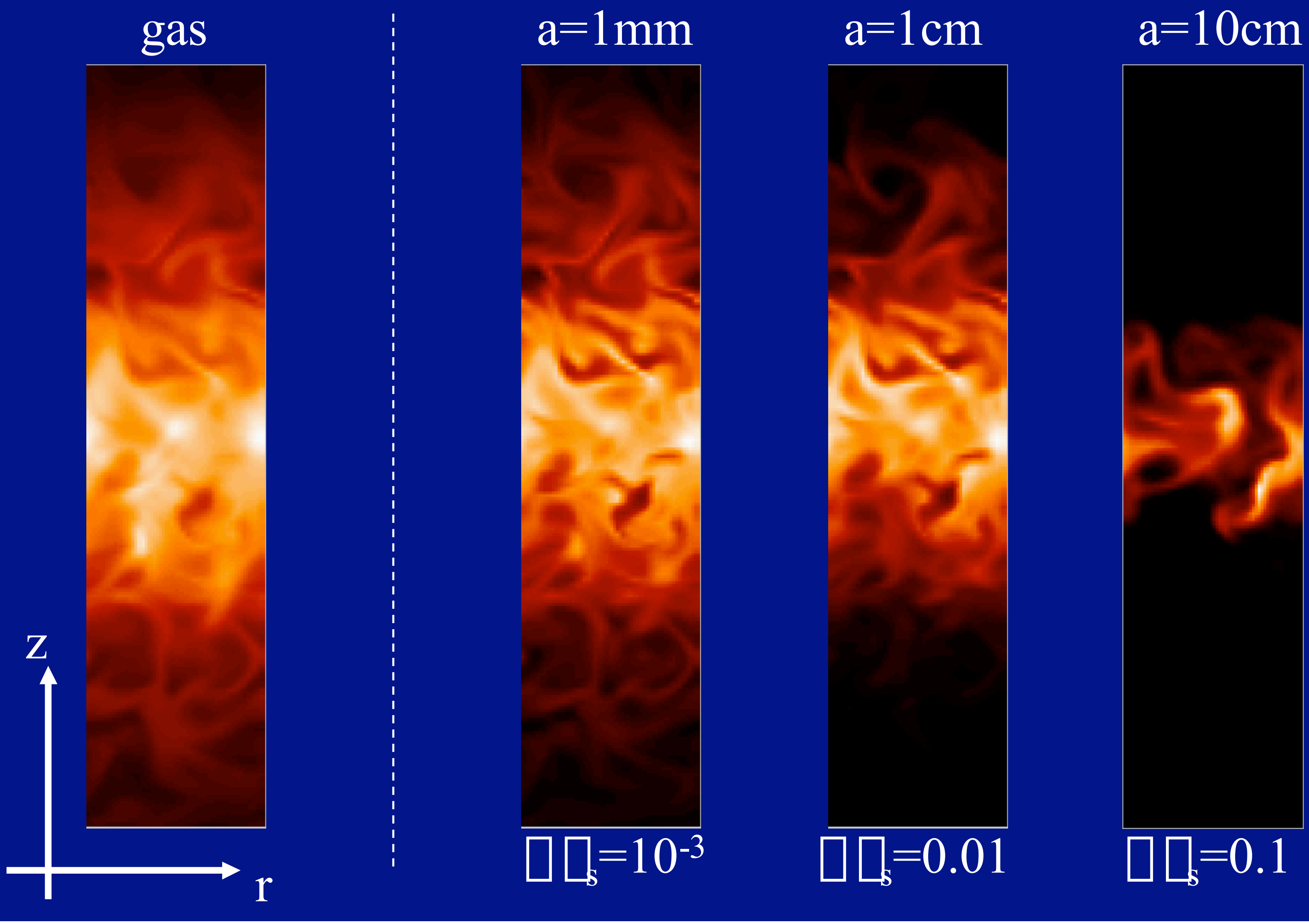
- Radial diffusion, with net flux (Carballido et al. 2005):

  S_c=11
- Vertical diffusion, no stratification (Johansen & Klahr 2005):

  S_c=1.8
- Diffusion in disks upper layers (Turner et al. 2006):

  S_c~1

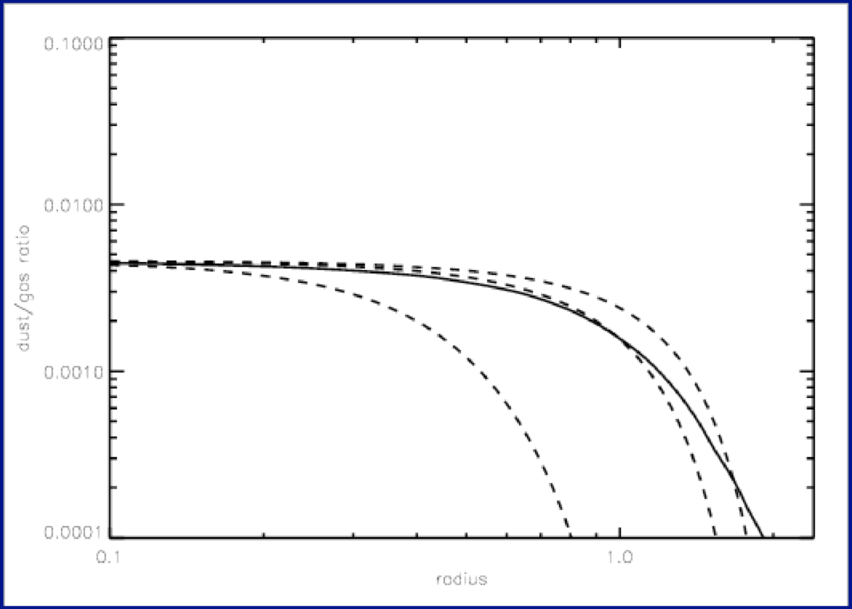
Larger particles



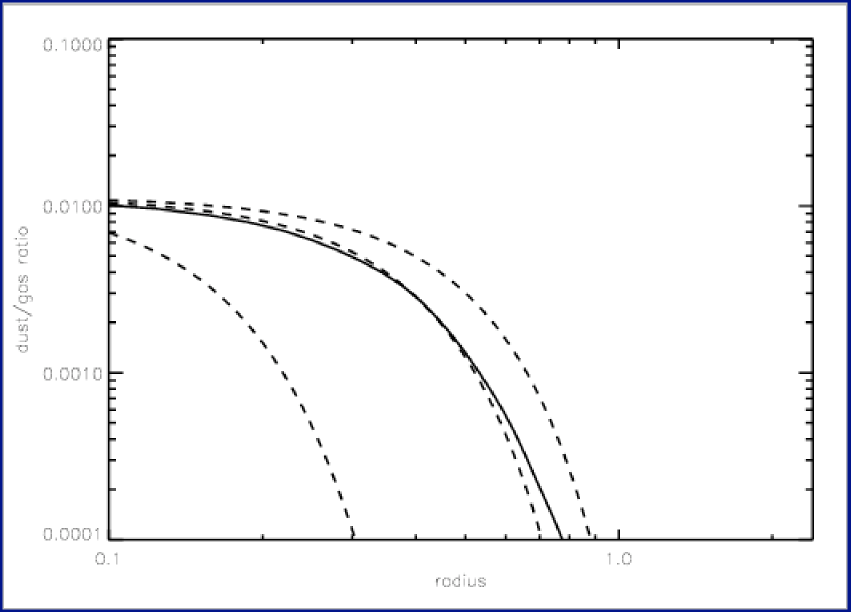
Steady state

$$\frac{\partial \rho_d}{\partial t} - \frac{\partial}{\partial z}(z\Omega^2 \tau_f \rho_d) = \frac{\partial}{\partial z}\left[D_{dust}\frac{\partial \rho_d}{\partial z}\right]$$

with $D_{dust}/(c_s H) = 10^{-3}, 5.5\times10^{-3}, 10^{-2}$

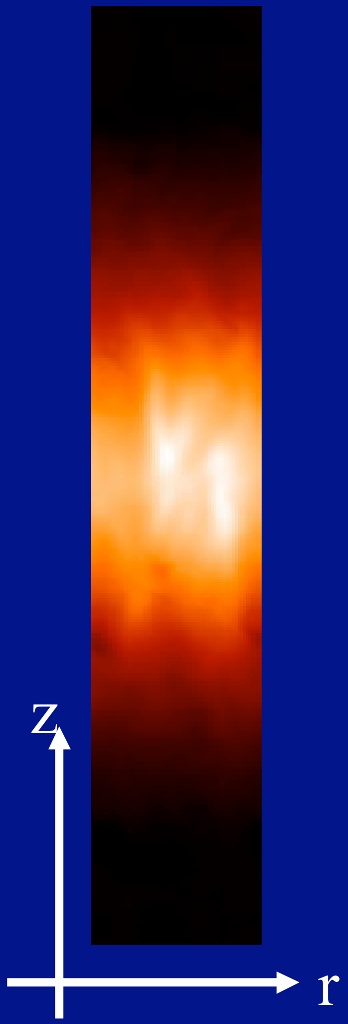


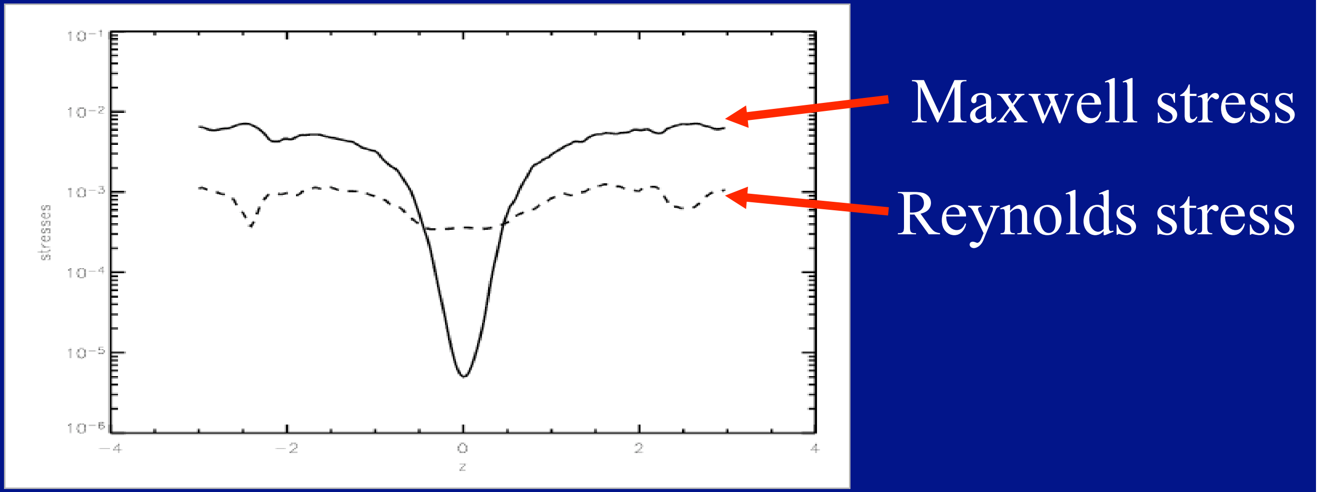
a=1cm

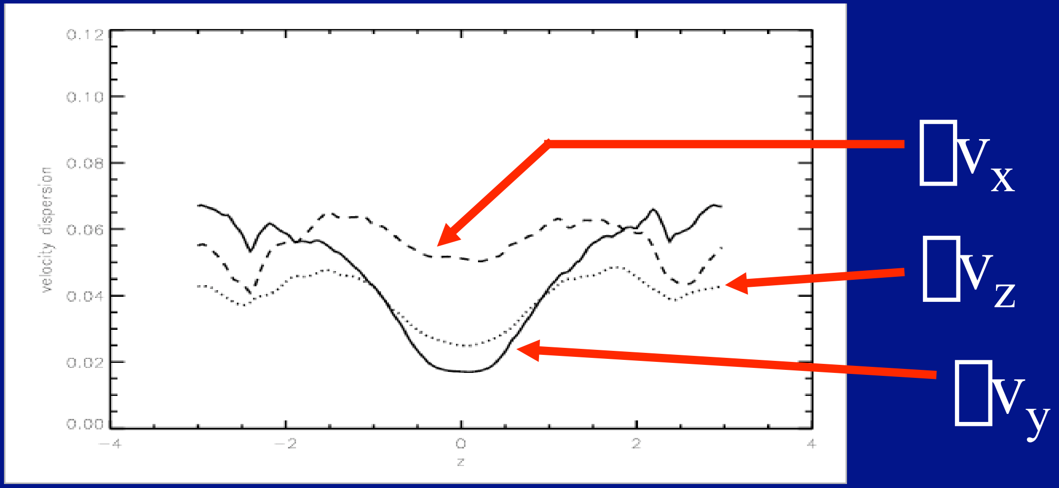


a=10cm

Dead zones

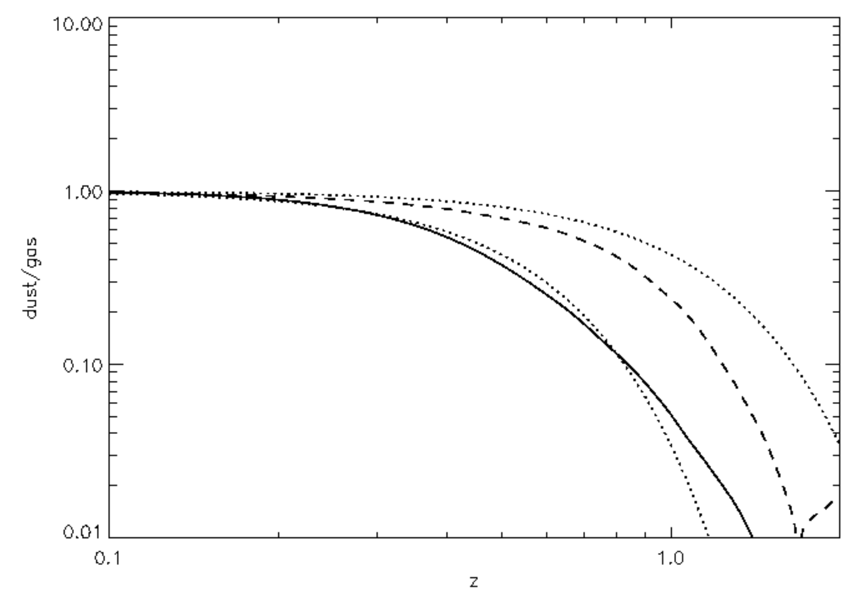






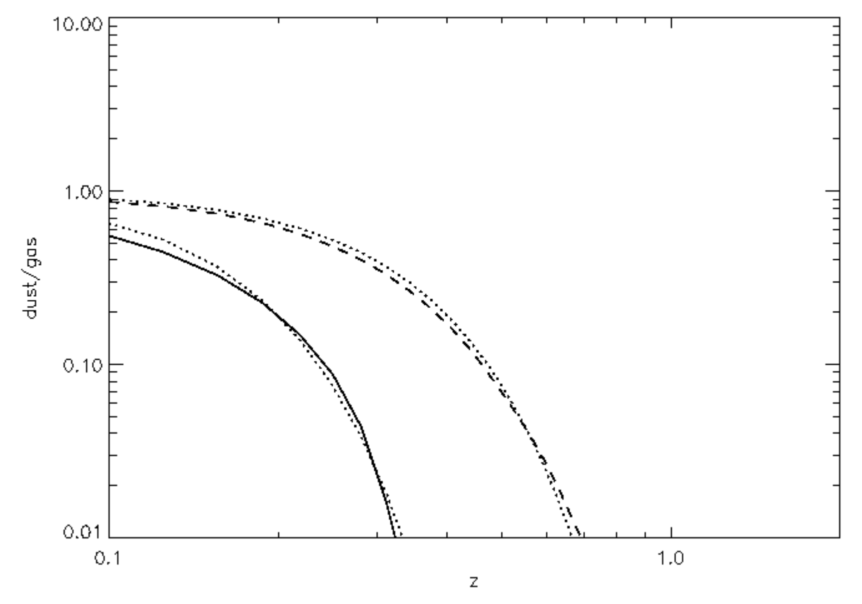
See also Fleming & Stone (2003)

Poussière et zone morte



a=1cm

a=10cm



— With a dead zone

- - - Without a dead zone

⋯ Analytical result

Solid bodies radial migration

(Fromang & Nelson 2005)

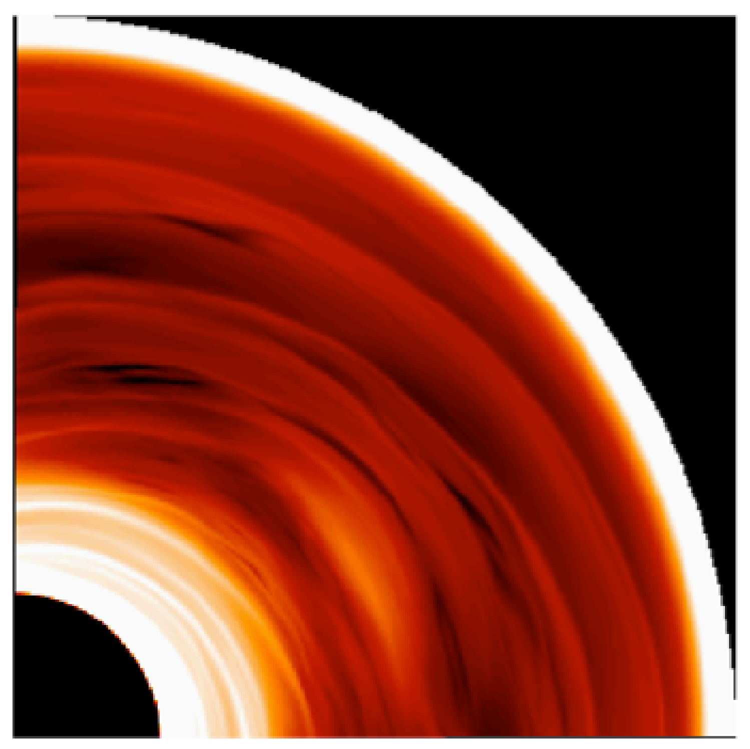
The disk model

- Cylindrical disk model, based on Steinacker & Papaloizou (2002), Papaloizou & Nelson (2003, 2004), Nelson (2005)

-Models computed with GLOBAL (Hawley & Stone 1995) and NIRVANA (Ziegler & Yorke 1997)

- Resolution: $(N_r,N_\phi,N_z)=(260,152,44)$
- r in [1,5]
- ϕ in $[0, \pi/2]$
- Toroidal field initially

$$\frac{T_{r\phi}^{Max} + T_{r\phi}^{\mathrm{Rey}}}{P} \sim 10^{-2}$$

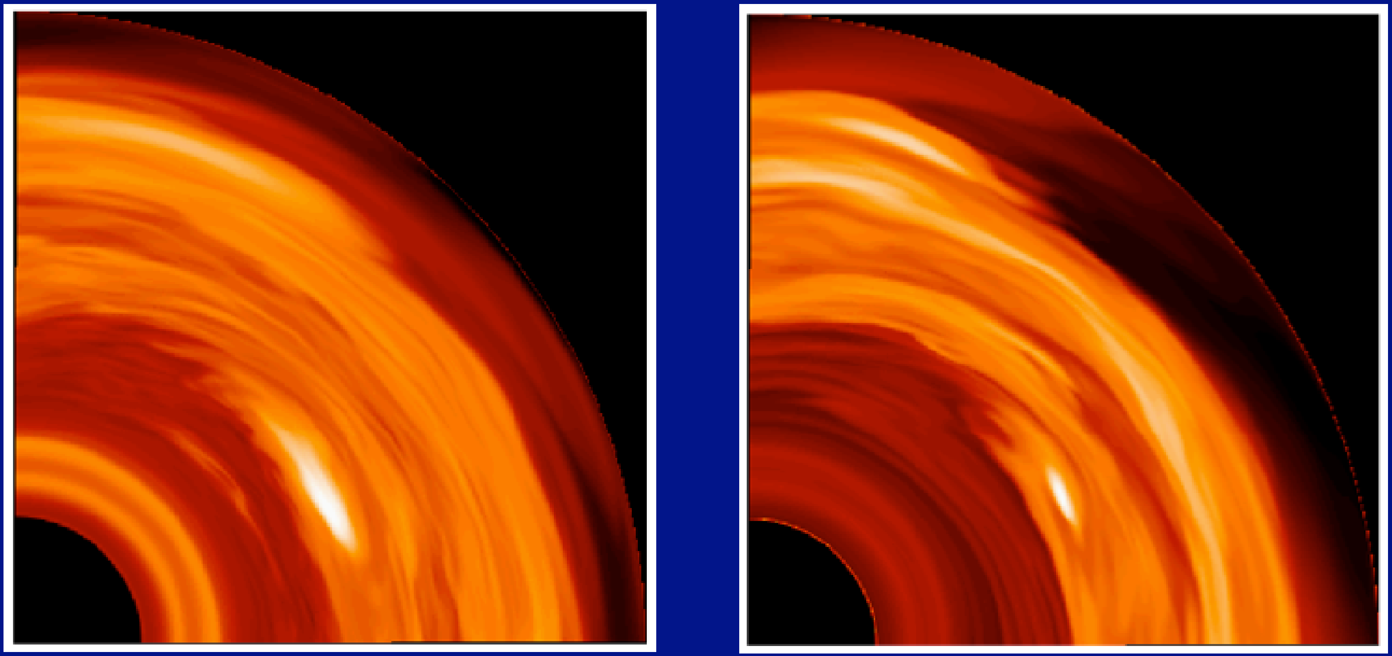


Density in the equatorial planet

Dust migration

2 methods: - Two fluids description (5 & 25 centimeters)
- N-body description (1 meter)

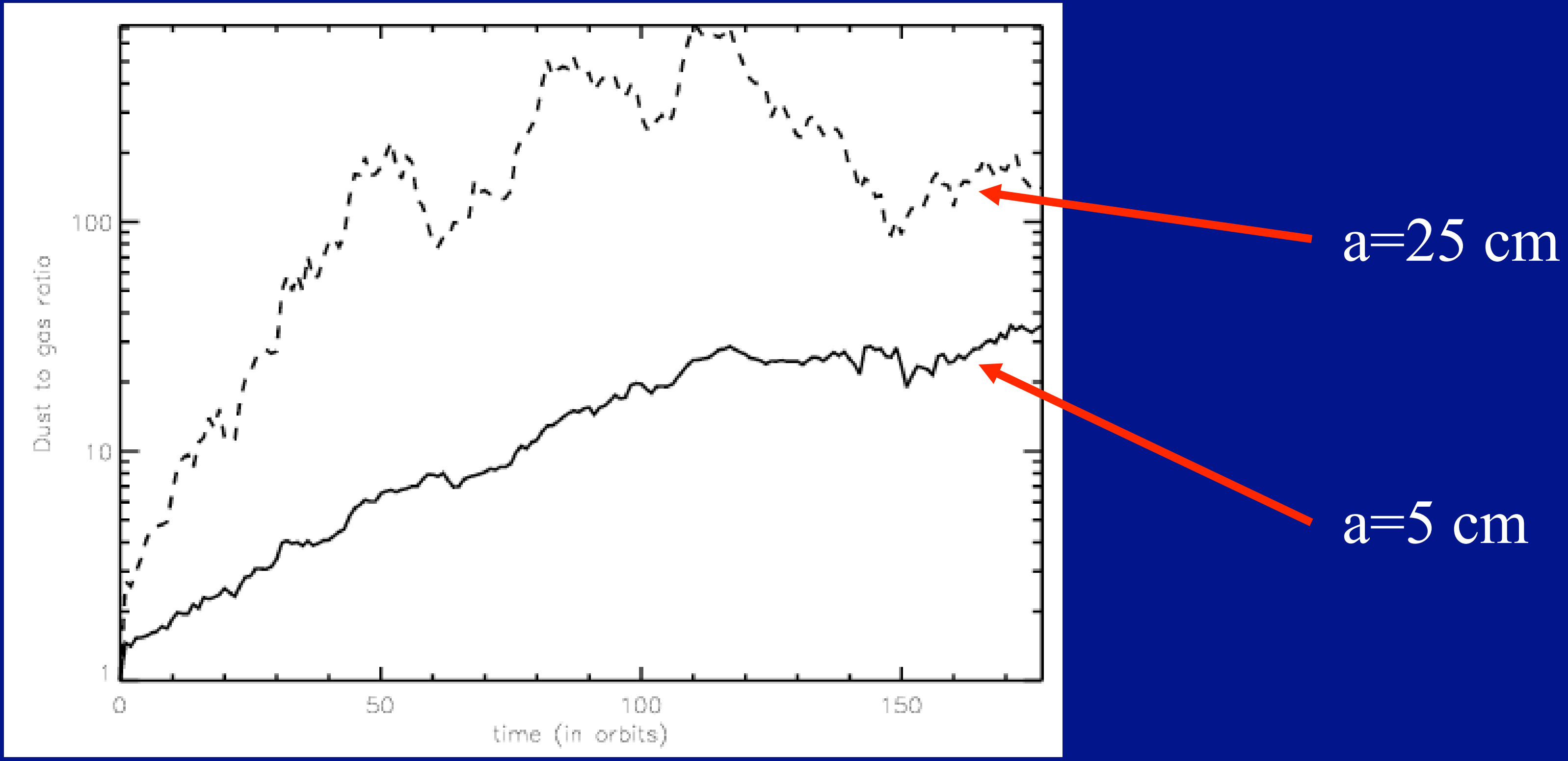
Dust density in the equatorial plane



5 centimeters

25 centimeters

Dust to gas ratio



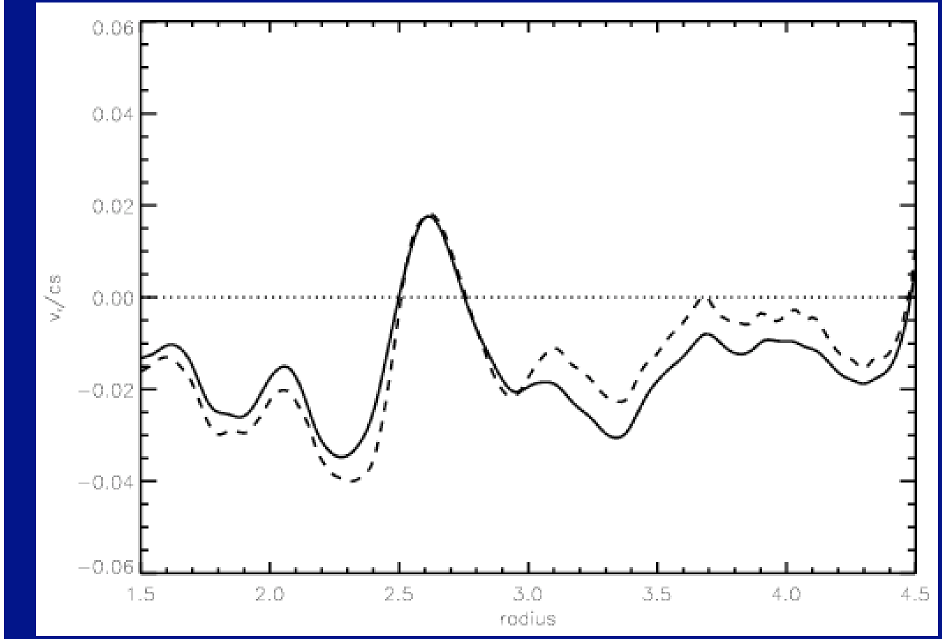
a=25 cm

a=5 cm

Drift velocity

(1) $v_r^{dust} = \frac{\square_s}{\square}\frac{\partial P}{\partial r}$ (Weidenschilling 1977)

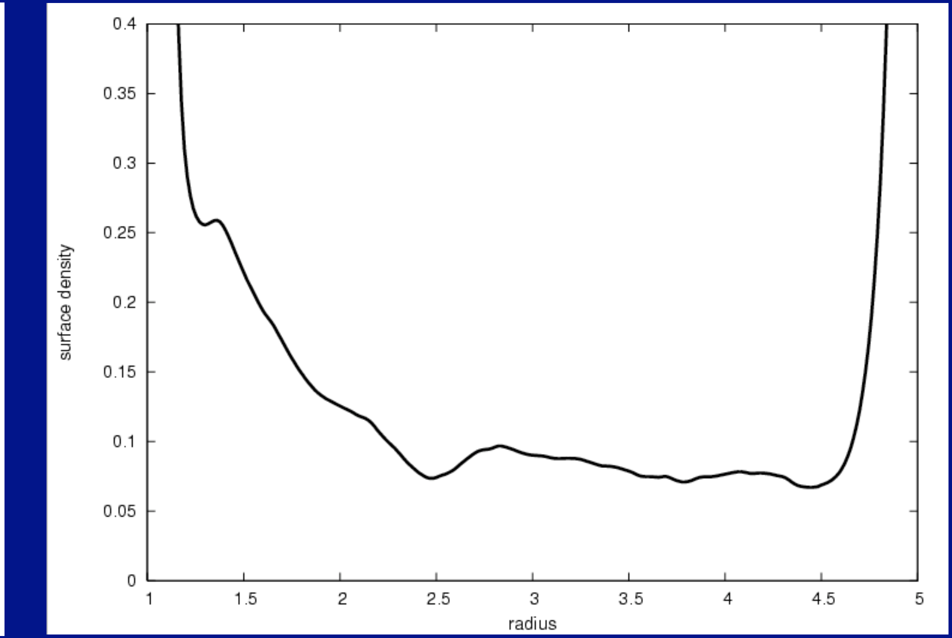
Radial velocity



——— Numerical simulations

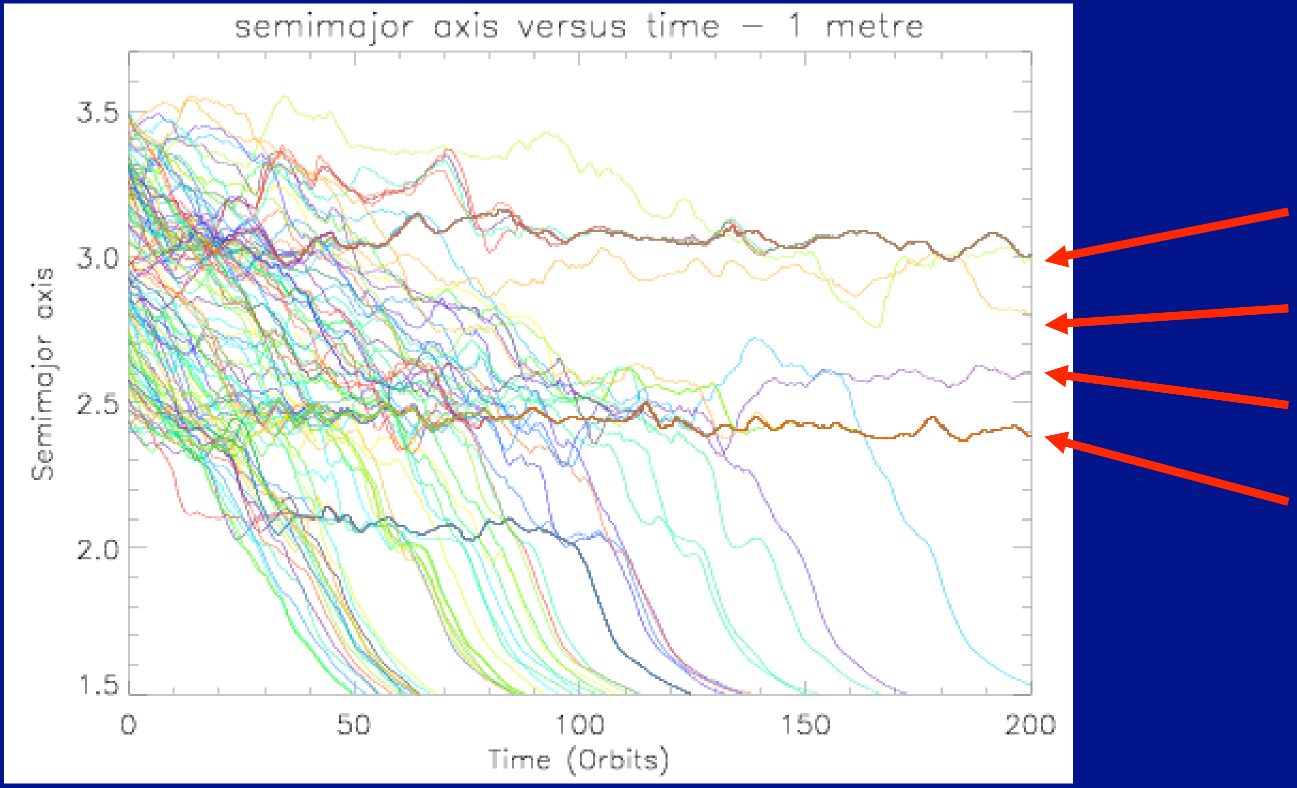
– – – Using equation (1)

Gas surface density



N-body approach

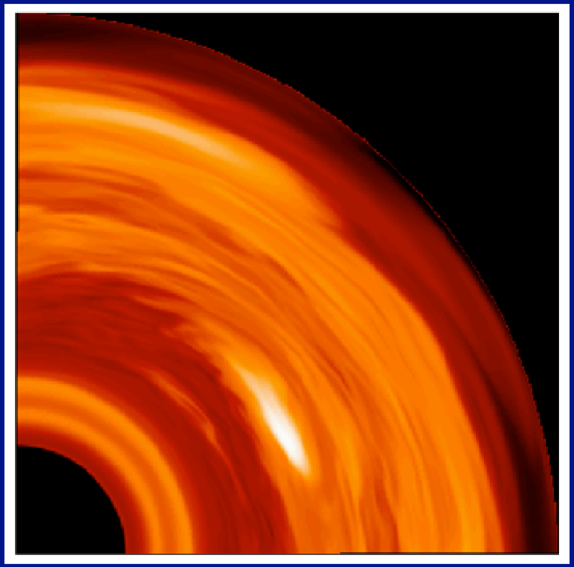
- Gas simulations + 3000 particles (a=1 meter) using NIRVANA



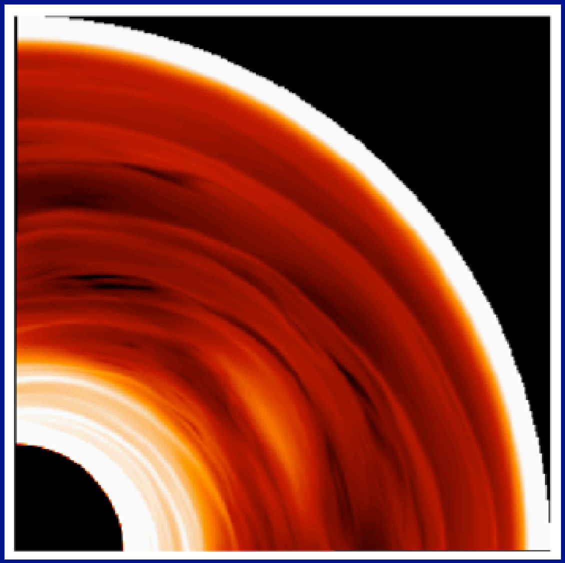
Solid body accumulation at a few radii

Vortex

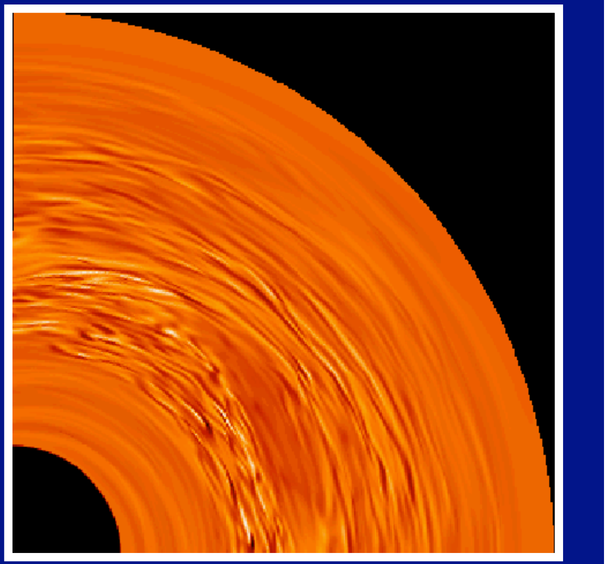
Vorticity: $\vec{\square} = \vec{\square} \; \square \; \overrightarrow{v_{\square}}$



Dust density



Gas density

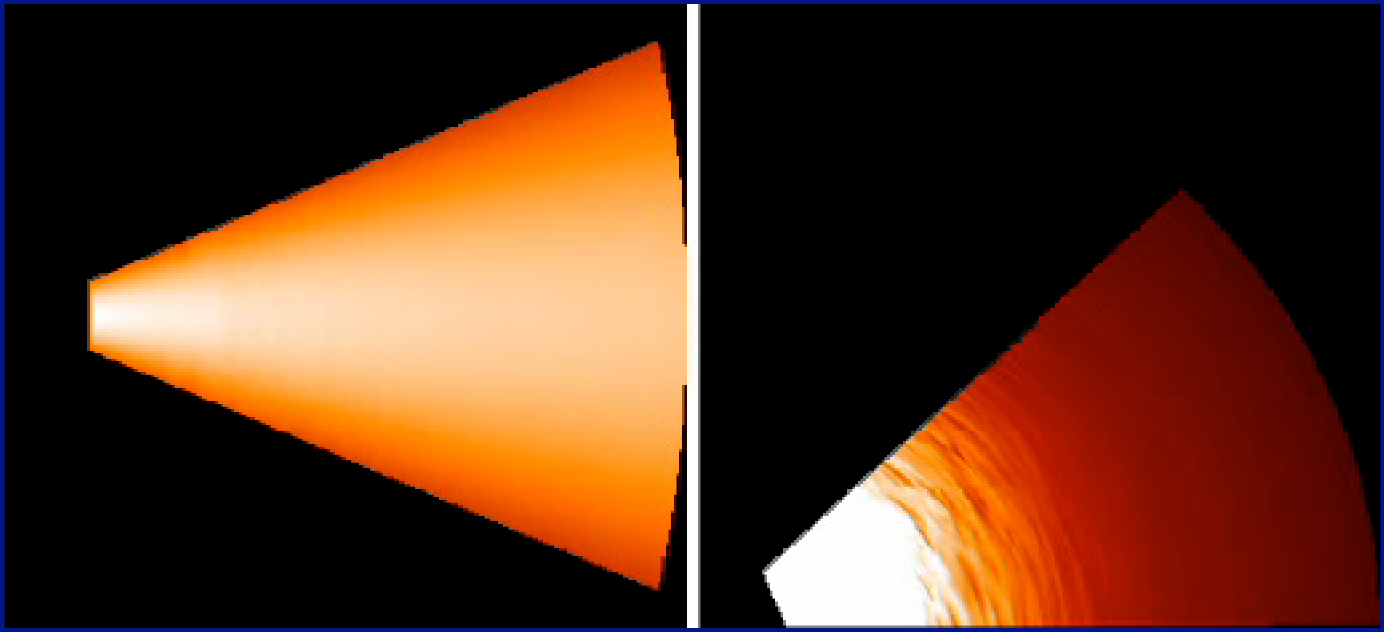


Vorticity

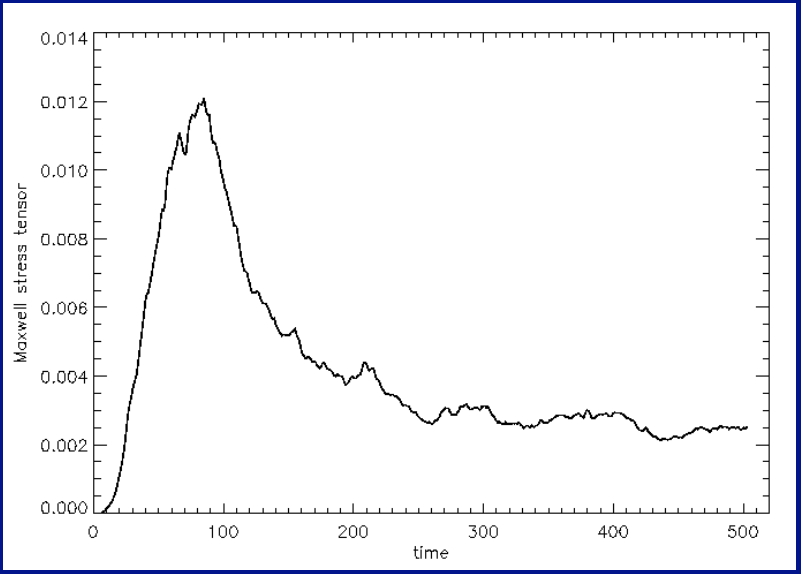
- Dust accumulates in anticyclonic vortices
- Effect of density stratification? (Barranco & Marcus 2005)

Stratified Disk

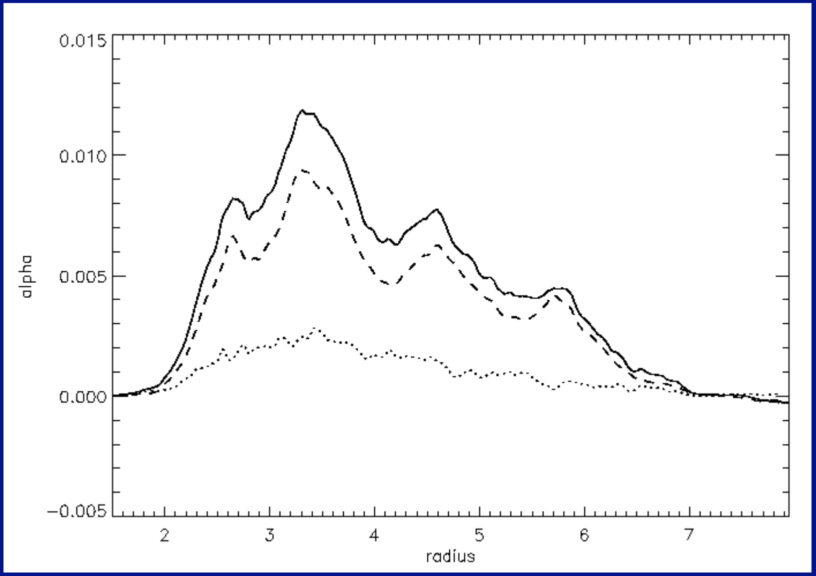
- Thin Disk: $H/R=0.07$, □ $r^{-1/2}$
- Isothermal EQS
- Toroidal magnetic field
- Resolution: $(N_r,N_{□},N_{□})=(455,150,213)$



Properties…



Tenseur de Maxwell



Profil radial alpha

Applications:

- Influence of the stratification
- Dust dynamics
- Planet/disk interaction
- Dead zones, radiative transfer

Conclusions

DUST SETTLING:

- MHD turbulence spreads the dust sub-disk, even for 10 cm sized particles, even in the presence of a dead zone.
- Diffusion coefficient can be calculated from the velocity fluctuations.
- Future study of large particules (in collaboration with A.Carballido, IoA)

DUST RADIAL MIGRATION:

-50-75% of the dust remains in the disk, trapped in local pressure maxima.

- Effect of vertical stratification???