The detector response

At least when *L*<< *λ* (i.e. *T(f)* ~ *1*) an interferometer measures

$$h(t) = \frac{1}{2}(h_{ij}u^i u^j - h_{ij}v^i v^j)$$

$$= D^{ij}h_{ij}(t) = F_+ h_+(t) + F_\times h_\times(t)$$

$$D^{ij} = \frac{1}{2}\left(u^i u^j - v^i v^j\right)$$ Detector tensor

F_+ , $F_\times$ Beam pattern functions

Pattern functions

Exercise: derive pattern functions for detectors at 90 and 60 degrees, and plot them

$$F_+^{(90^\circ)} = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi,$$
$$F_\times^{(90^\circ)} = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi.$$

$$F_+^{(60^\circ)} = \frac{\sqrt{3}}{2}\left[\frac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi\right],$$
$$F_\times^{(60^\circ)} = \frac{\sqrt{3}}{2}\left[\frac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi\right].$$

GW vs EM astronomy

- GWs interact very weakly with matter, strain h decays as 1/r
  ⇨ GWs visible to very high z, eg SMBHs with LISA, stochastic backgrounds

- Gravitational wavelength >~ source's size (because GWs generated by bulk motion of matter) vs EM wavelengths << source's size (because EM waves generated by moving charges, atomic processes, etc)

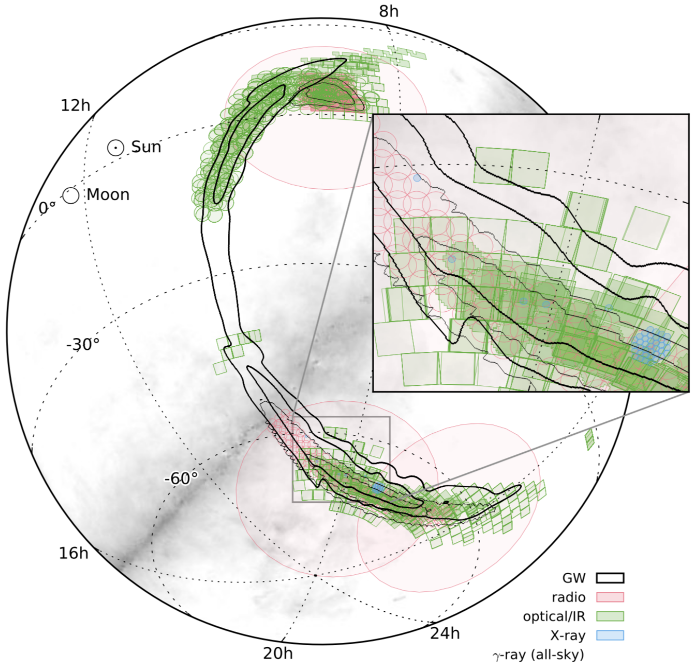
  ⇨ EM can be used for imaging, GWs do not have angular resolution (akin to sound)

  ⇨ EM surveys cover small areas, GWs cover whole sky

GW and EM waves are complementary tools for testing fundamental physics, astrophysics and cosmology

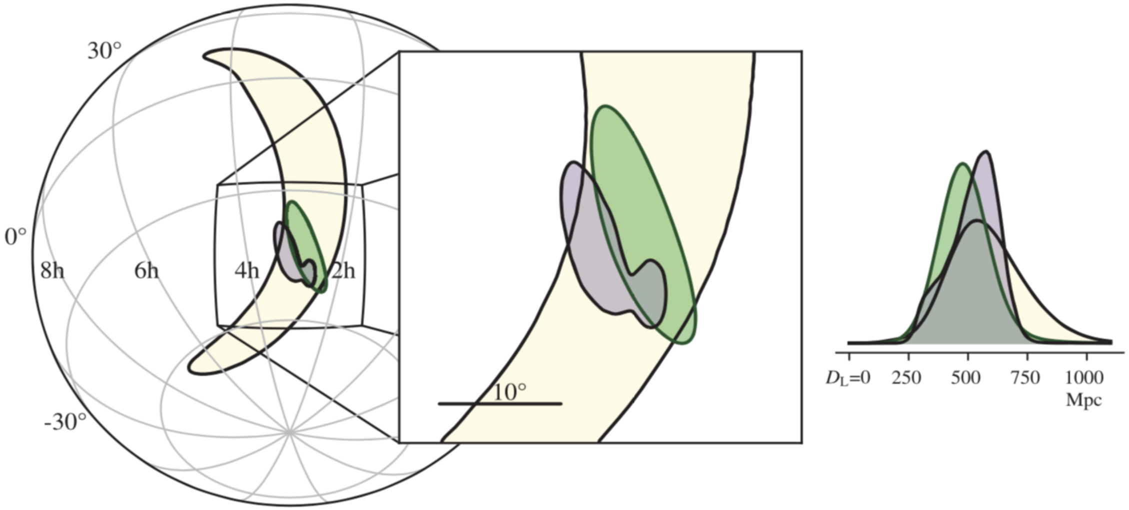
GWs alone have poor sky localization

Need network of detectors/many pulsars (also to enhance detection confidence and minimize downtime)



GW150914

GW170814



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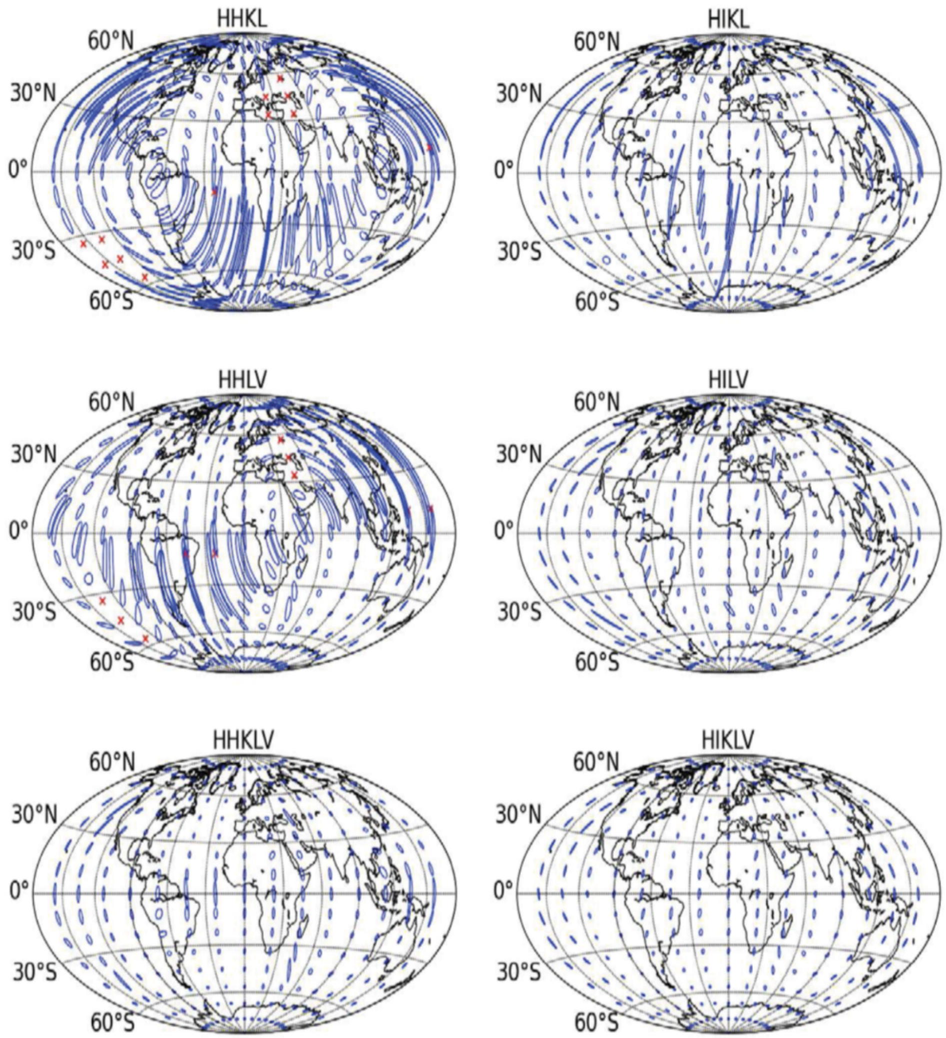
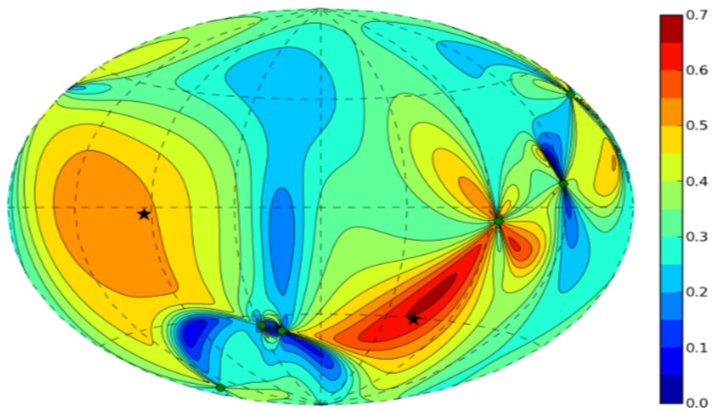
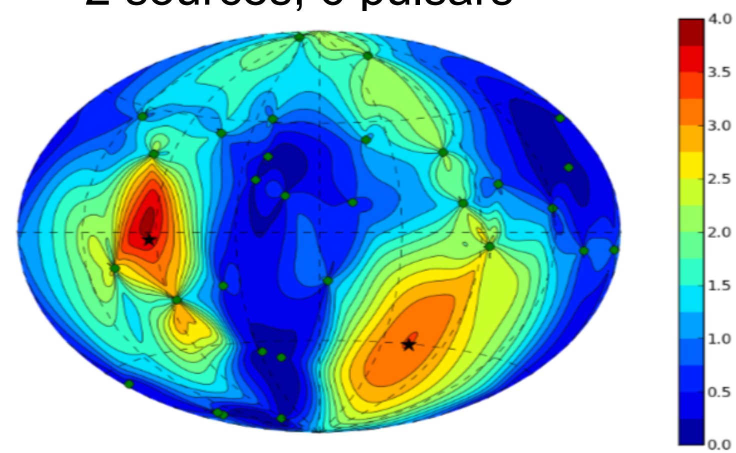




2 sources, 6 pulsars



2 sources, 30 pulsars

Babak & Sesana (2012)

PTAs: to estimate sky location of N sources, 3 x N pulsars are needed

BH-NS face-on at 160 Mpc (Fairhurst 2014)

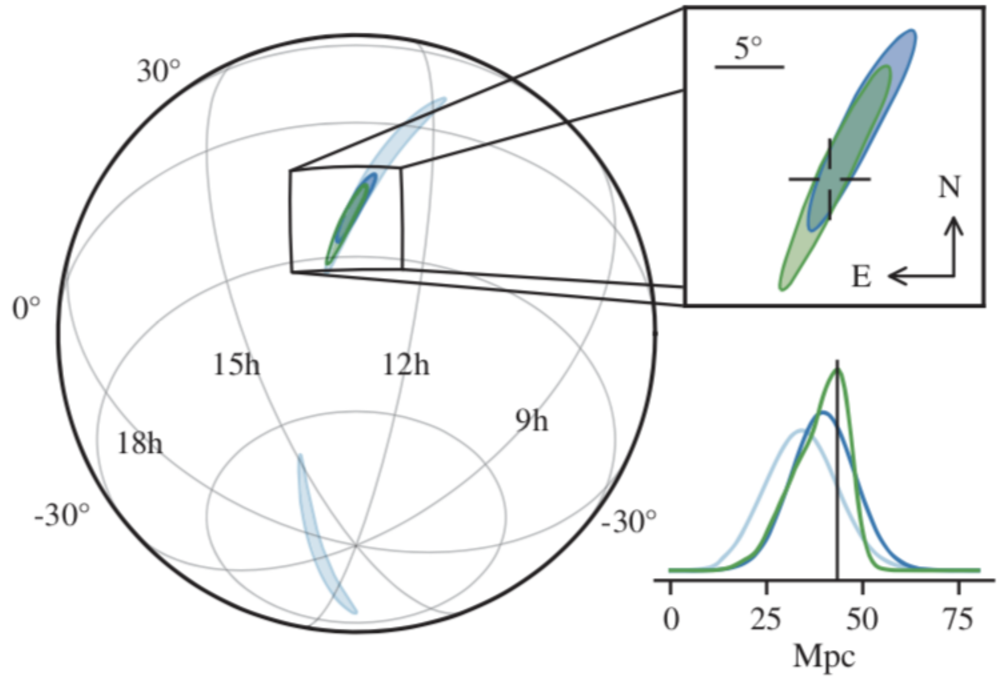
EM counterparts to GW sources?

- Allow for: - sky localization and detection confidence enhanced

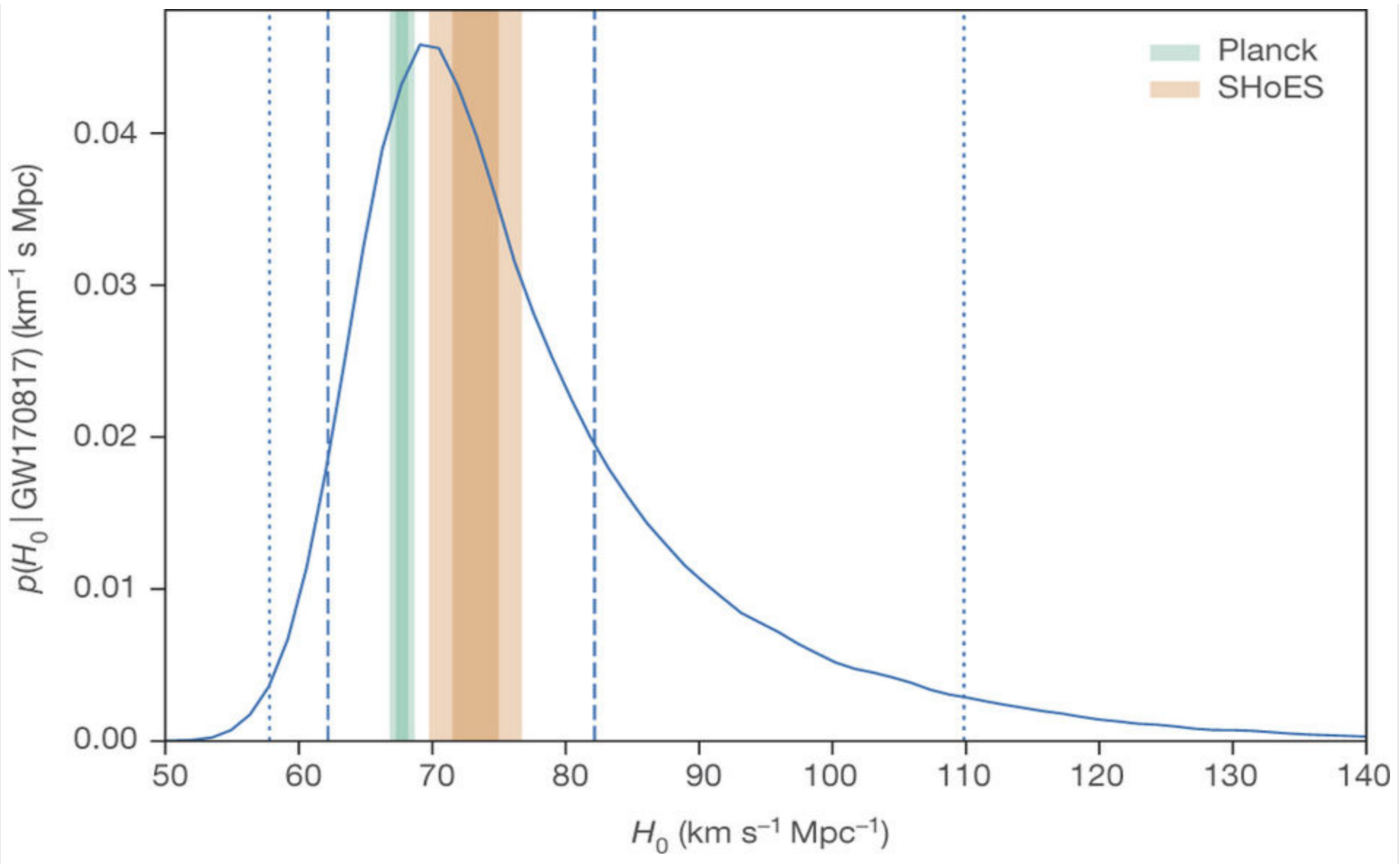
  - redshift measurement, unavailable with GWs alone (no intrinsic energy scale in GR)

- Goals: - GRB as triggers for GW searches

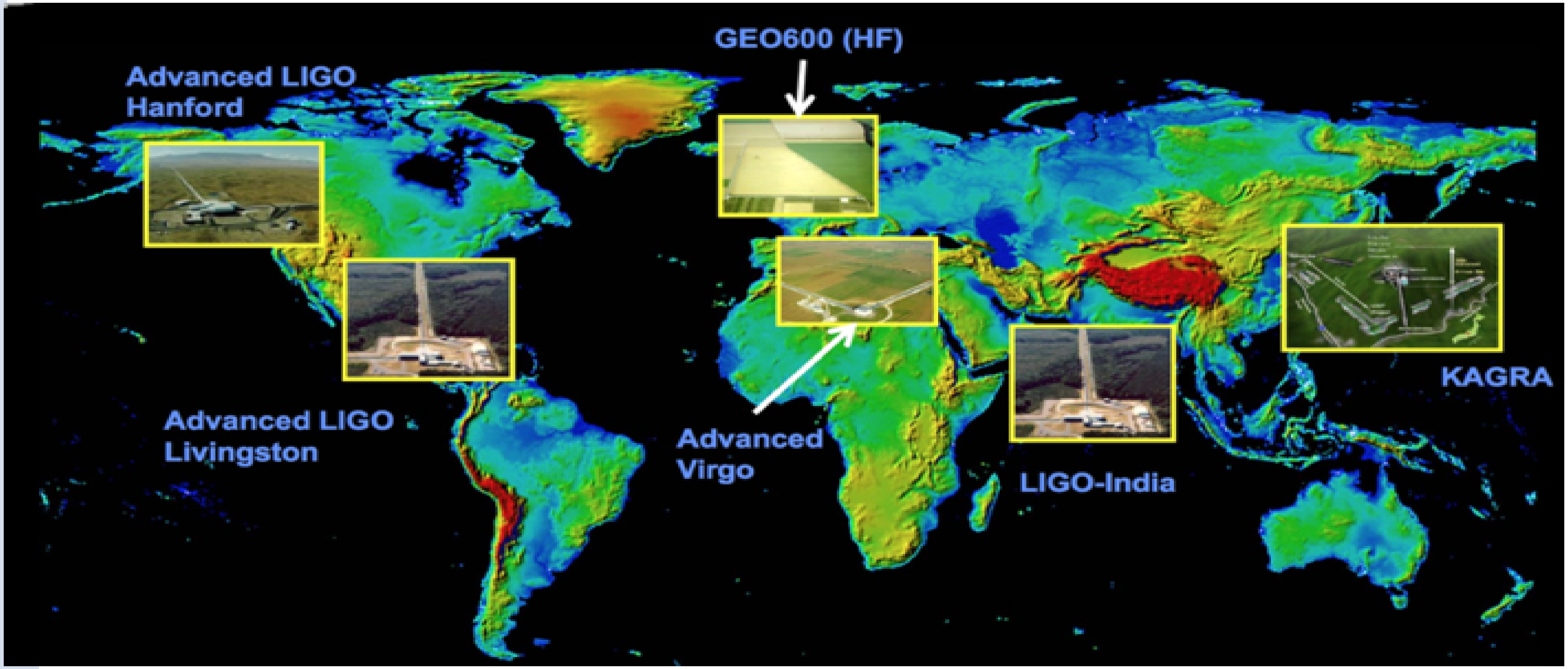
  - generate GW triggers to point telescopes in 10-100 sec to observe optical prompt emission, 100 sec-days for afterglow

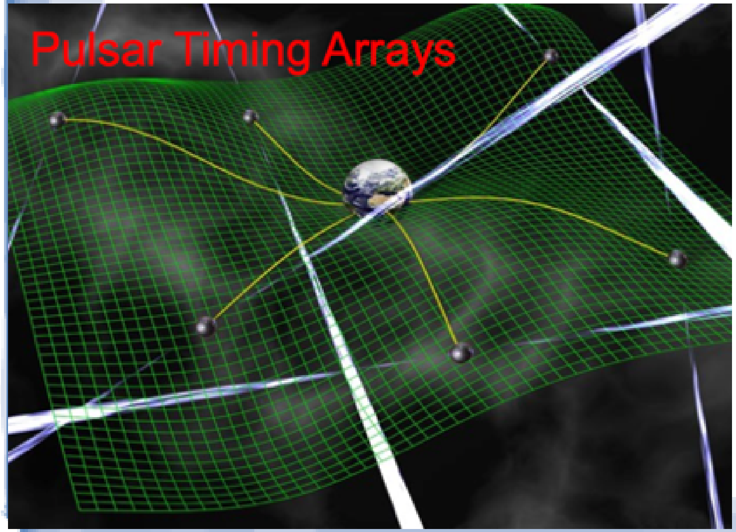


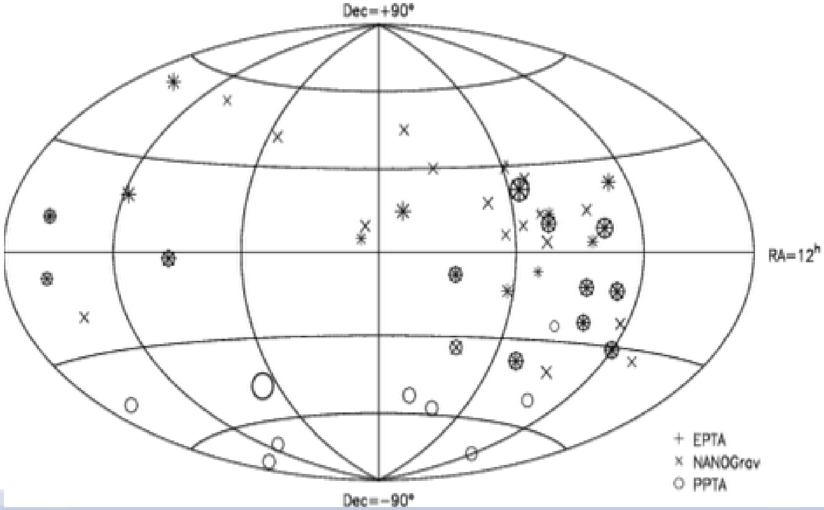
GW-based distance ladder



Existing detectors

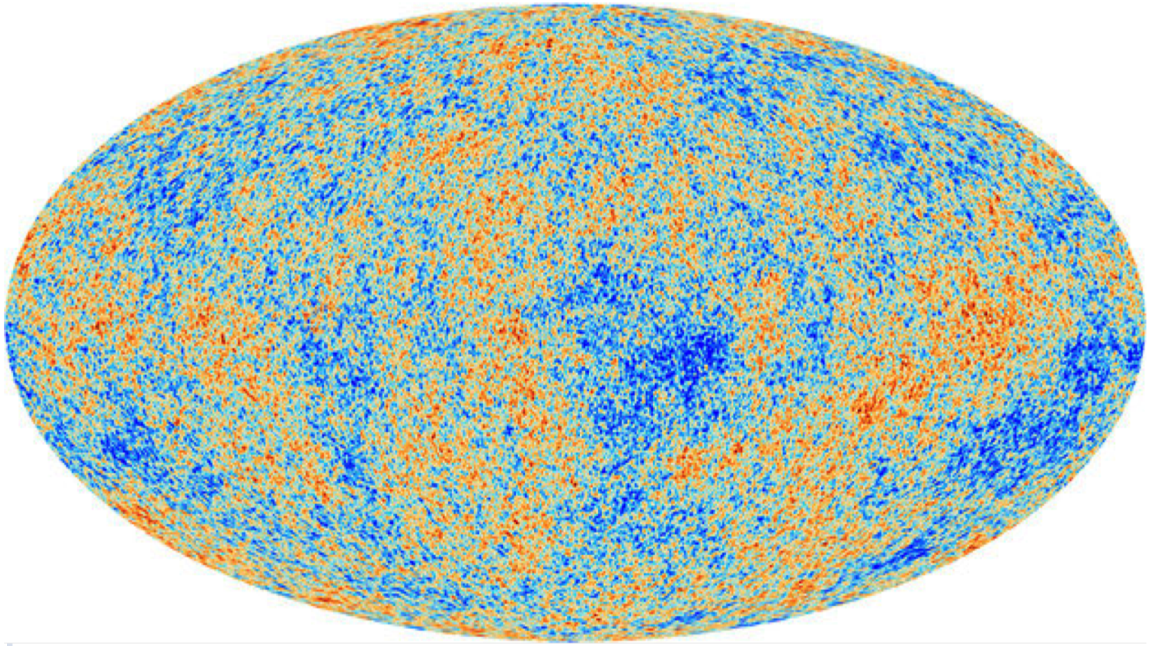


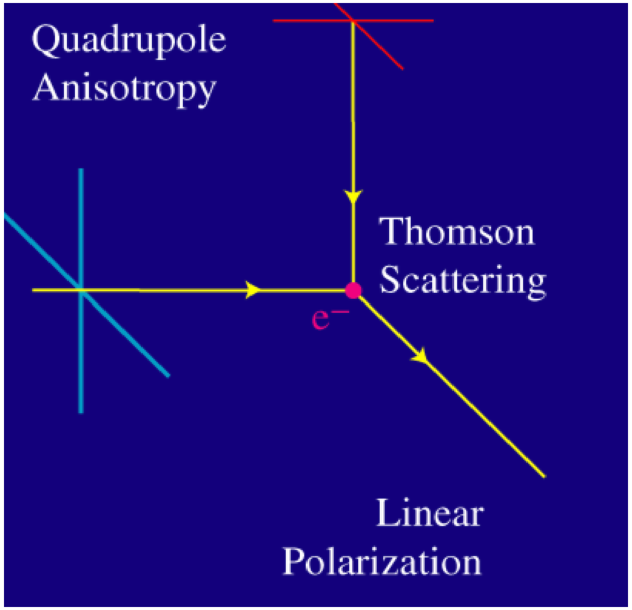




Existing detectors

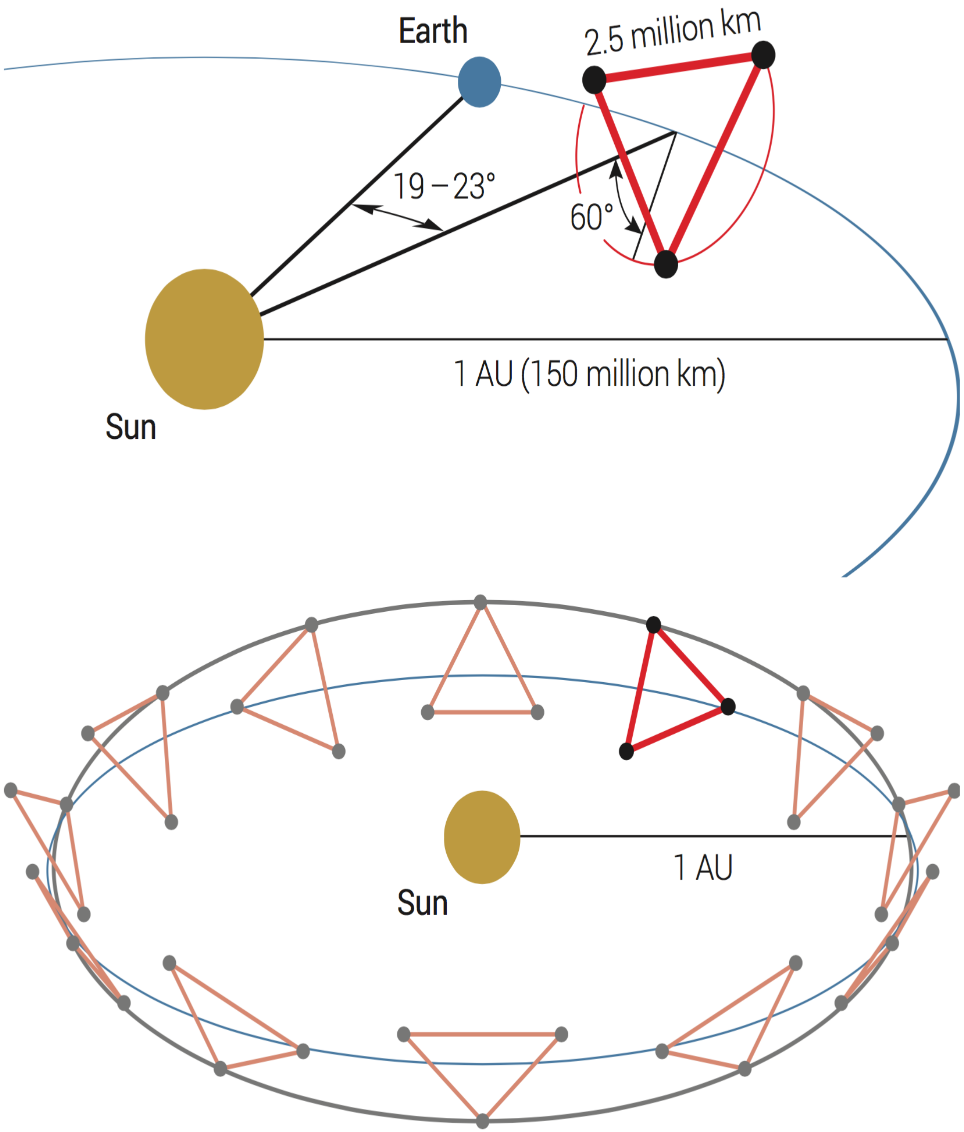
CMB B modes

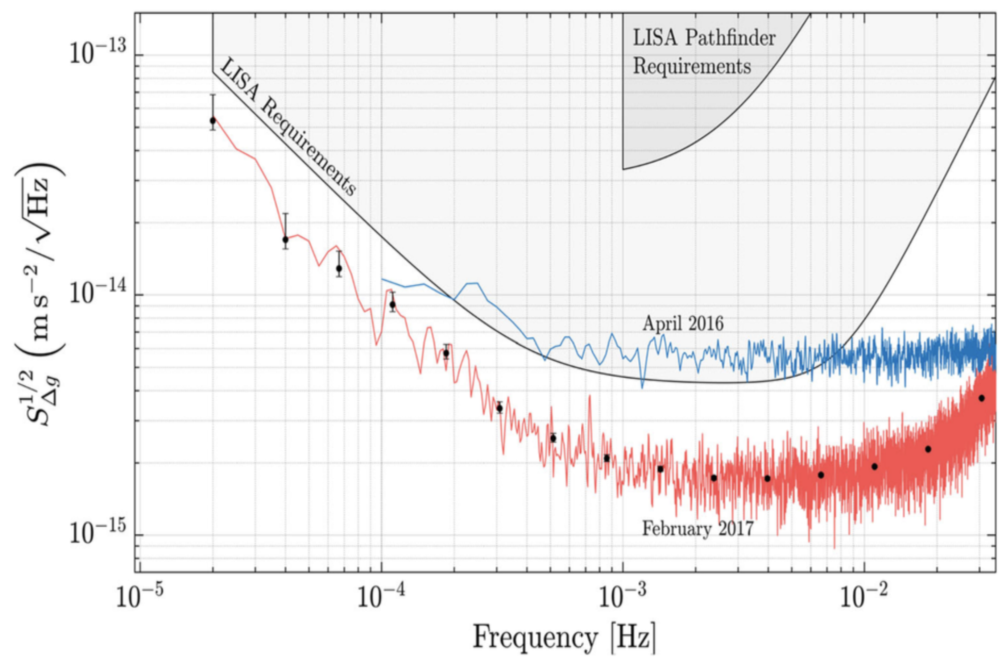




Animation from Hu 2001

Next-generation detectors



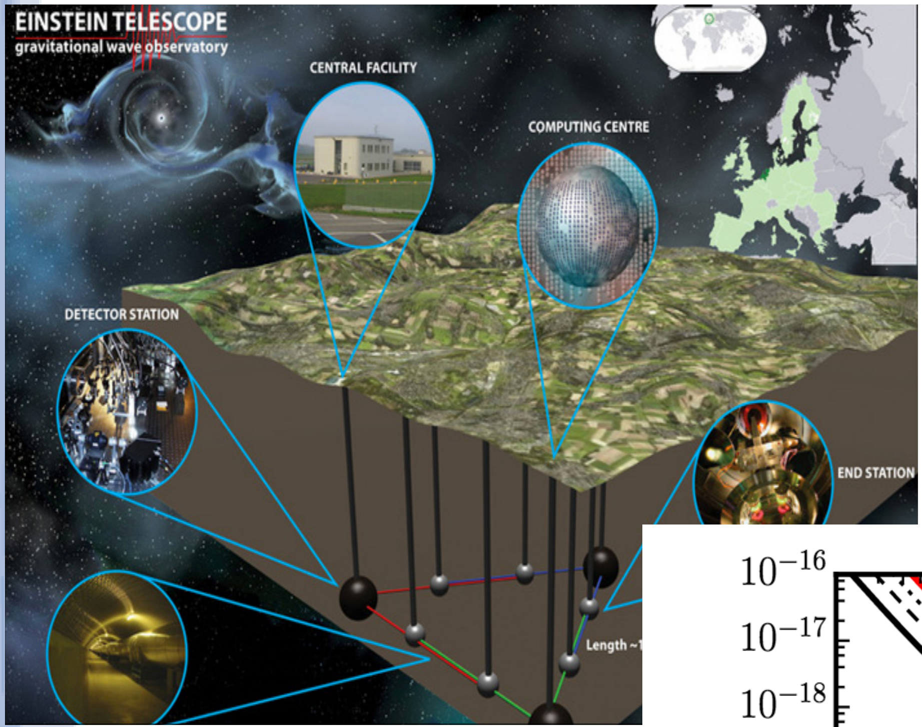


LISA: accepted for ESA's L3 launch Slot (Jan 2017)

Technology tested by LISA Pathfinder (2016)

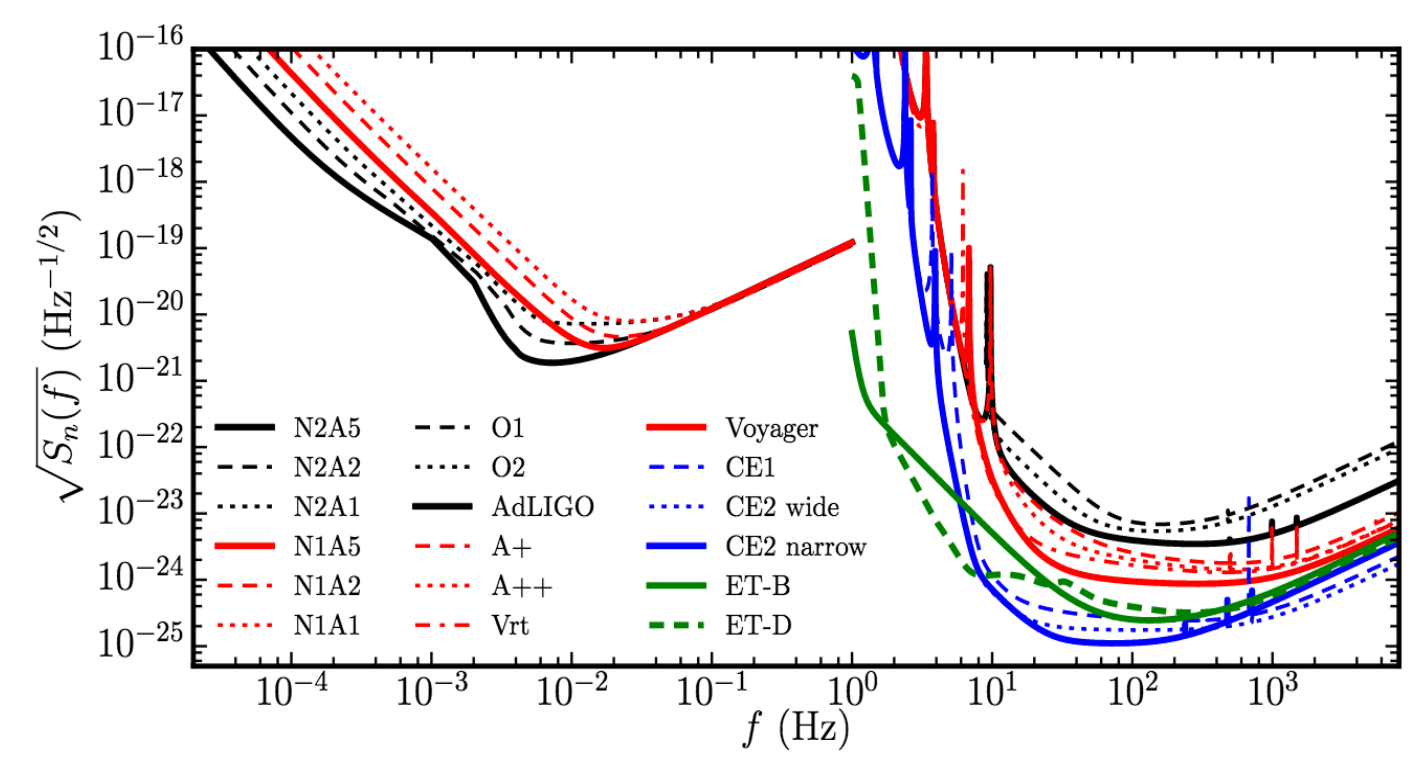
Launch in 2028-2030?

Next-generation detectors

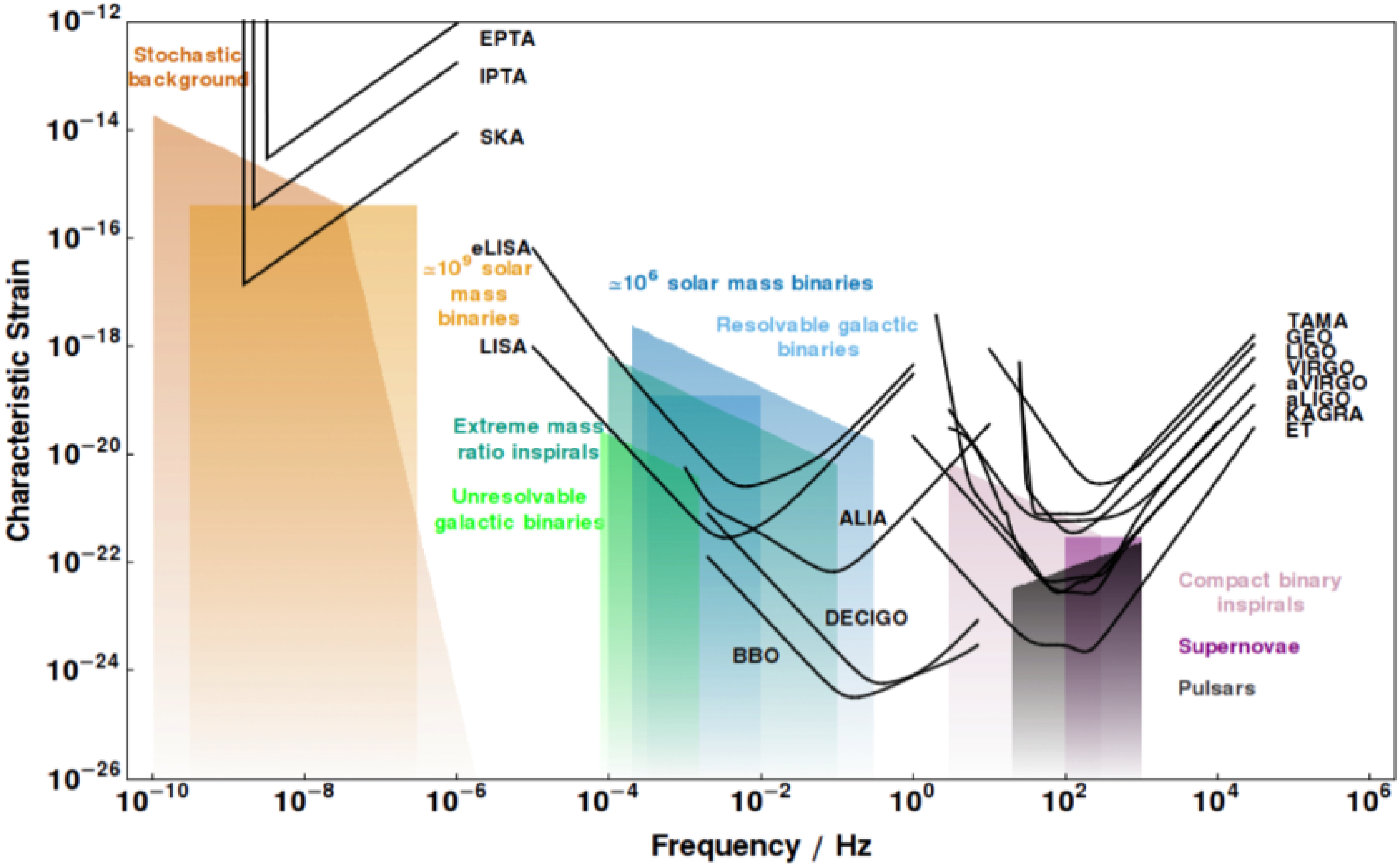


3rd generation ground detectors in Europe and US (ET, CE, Voyager...)

Longer arms (10-40 km?), underground, cryonigenic, squeezed laser states, etc



Frequency windows



GWs from binary systems

From quadrupole formula, GW frequency is twice orbital one

$$f_{\rm GW} = \frac{6 \times 10^4}{\tilde{m}\tilde{R}^{3/2}}{\rm Hz} \qquad \tilde{R} = R/m \qquad \tilde{m} = m/M_\odot$$

aLIGO/aVirgo:

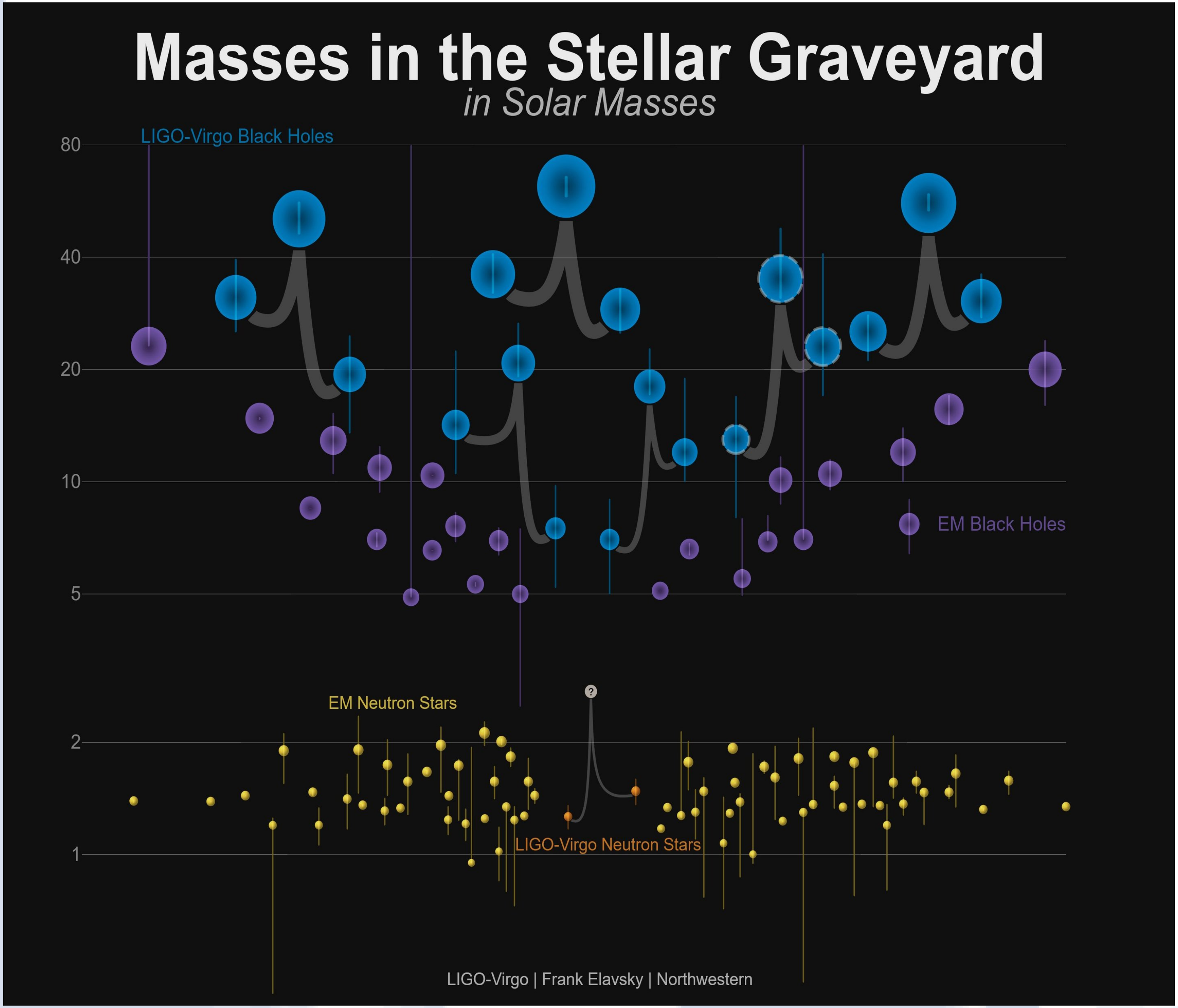
1) BH-BH late inspiral and merger, with masses up to 60-70 M_{sun}

2) NS-NS and possibly BH-NS: from few to hundreds of events per year

Binary pulsars observed with masses ~ 1.4 M_{sun}, but isolated NS can have masses   2 M_{sun}

3) If intermediate mass BHs exists, IMBH-BH/NS/WD and IMBH-IMBH observable with third generation ground detectors

LIGO/Virgo detections



Why important?

- First direct detection of GWs (indirect evidence from binary pulsars)
- Opens up era of multi-band EM+GW astronomy
- Evidence that sGRB=NS+NS
- High BH masses imply formation in weak-wind/low-metallicity environment
- Test GR for the first time in strong-field ($U \sim c^2$) and highly relativistic ($v \sim c$) regime

GWs from binary systems

LISA:

Supermassive BHs observed in center of galaxies with masses ~ 10^5 – 10^9 M_{sun}; believed to merge when galaxies merge (cf double AGNs)

1) Inspiral and merger of SMBH-SMBH (with masses ~ 10^5 – 10^6 M_{sun}): from a few to hundreds per year

2) Inspiral and merger of SMBH – BH/NS/WD (aka Extreme Mass Ratio Inspirals, EMRIs): rates uncertain, from a few to hundreds/thousands per year

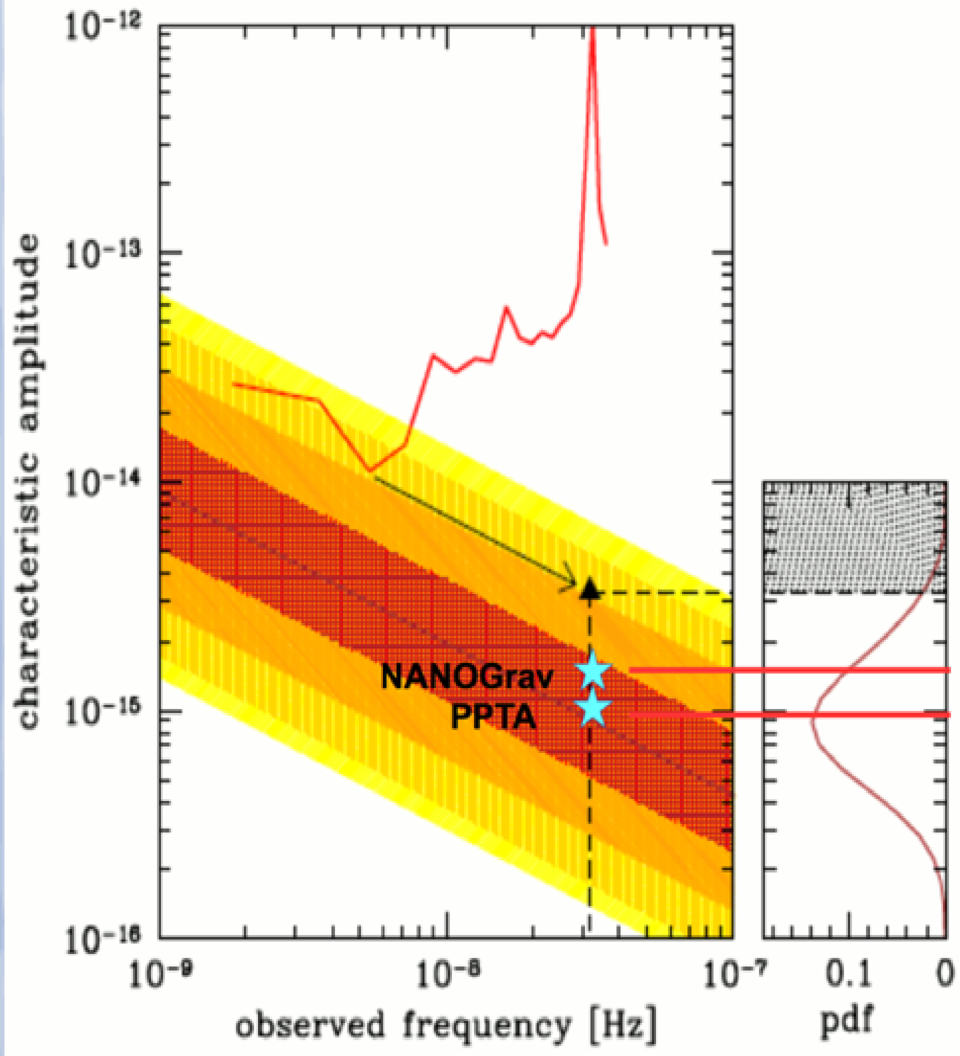
3) IMBH-SMBH: rates uncertain

4) WD-WD at separations of a few star radii (~ 10^5 km): thousands of resolved sources, a few guaranteed sources in the Galaxy

Pulsar timing array:

SMBH-SMBH at $0.2 < z < 1.5$, with masses л 5×10^8 M_{sun} and separations of hundreds gravitational radii

PTA limits on stochastic background from SMBH binaries



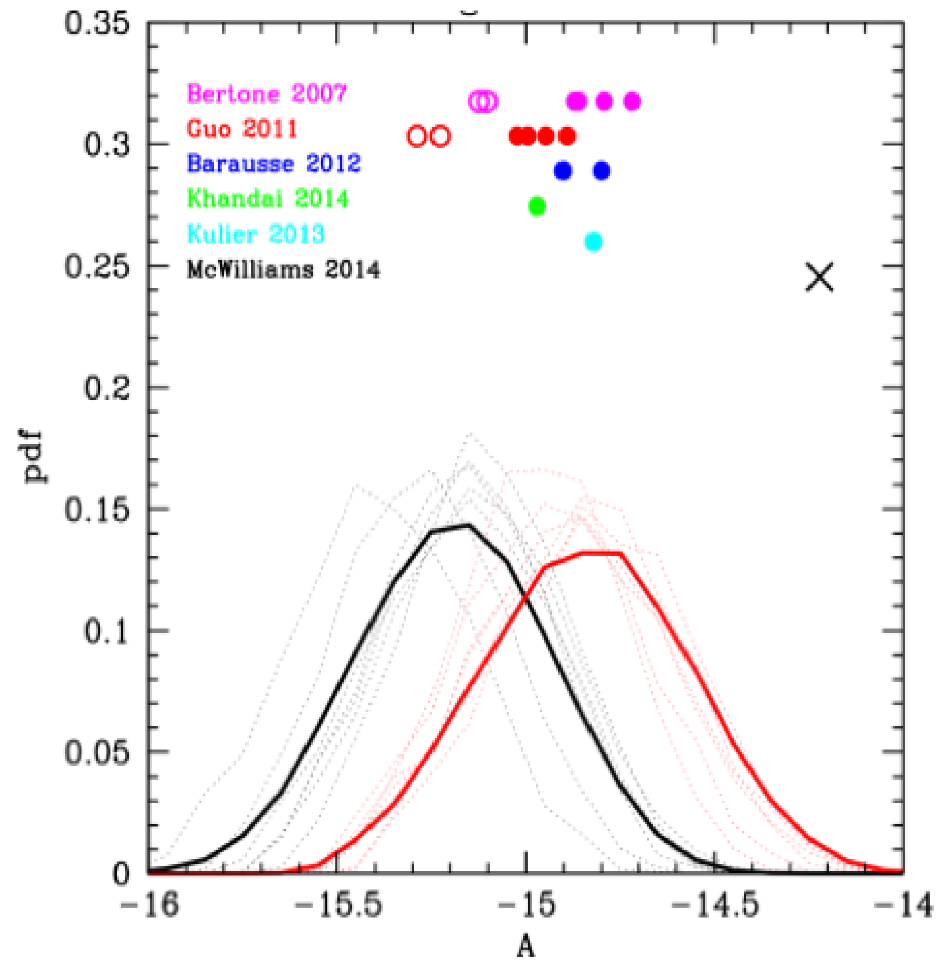


Figure courtesy of A. Sesana

GWs from isolated systems

- Rotating axisymmetric star/spherical collapse do not emit
- Core collapse supernovae (type II) produce bursts of GWs if instabilities develop due to high rotational velocities, or if asymmetries are present:

  possible sources for LIGO/Virgo/Einstein telescope
- Rotating pulsar can radiate monochromatically if rotation deviates from axisymmetry: possible sources for LIGO/Virgo/Einstein telescope but no good model for ϵ

$$h \sim \frac{G}{c^4}\frac{I f^2 \epsilon}{r} \qquad \epsilon = (I_{xx} - I_{yy})/I$$

LIGO/Virgo will constrain $\epsilon < 10^{-7}$

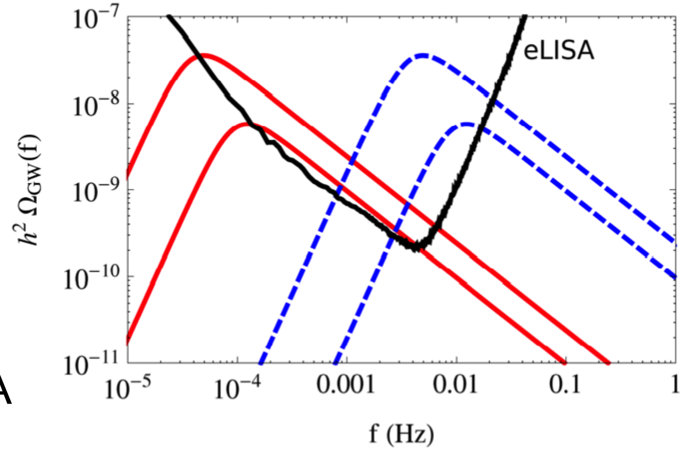
Stochastic backgrounds

- Astrophysical origin: superposition of many unresolved GW signals (eg from WD-WD binaries for LISA, or SMBH binaries for PTAs)
- Cosmological origin, eg inflationary or due to phase transitions
- Isotropic and homogenous (cosmological origin) or approximately so (astrophysical origin)
- Look like noise by can be detected by cross-correlating detectors
- Inflationary GWs depend on energy scale of inflation

$$\Omega_{gw}(f) \propto (E_{inflation}/M_P)^4 \approx \text{constant} \quad \Rightarrow \quad E_{inflation} < 1.9\times 10^{16}\,\text{GeV}$$

- GWs produced by phase transitions have peaked spectrum

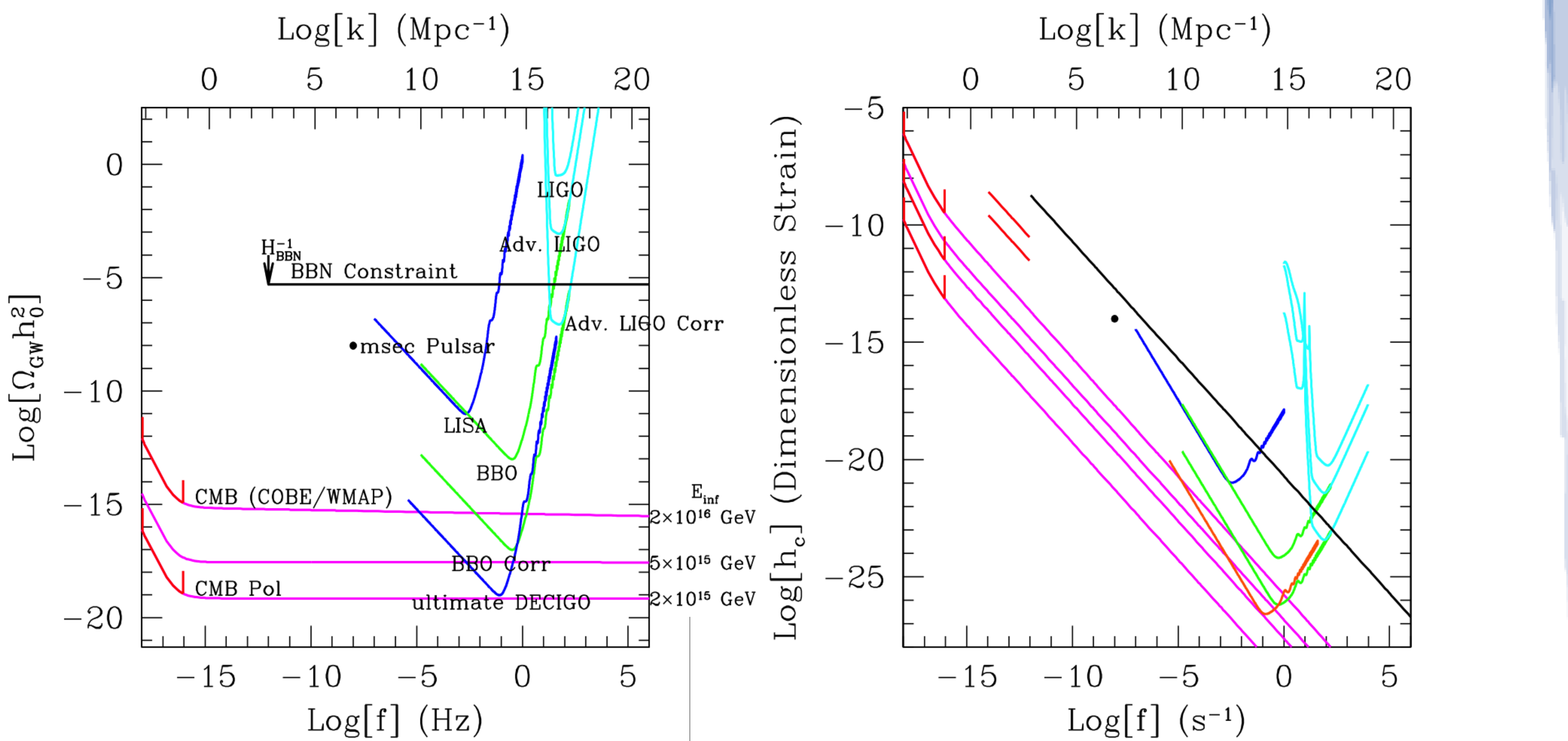
$$f_{\text{peak}} \sim 100\,\text{Hz}\left(\frac{T}{10^5\,\text{TeV}}\right)$$



E.g. some exotic models (eg extra dimensions, cosmic strings) could produce phase transitions observable by LISA (Dufaux 2012)

Frequency ranges

Figure from A. Cooray, astro-ph/0503118

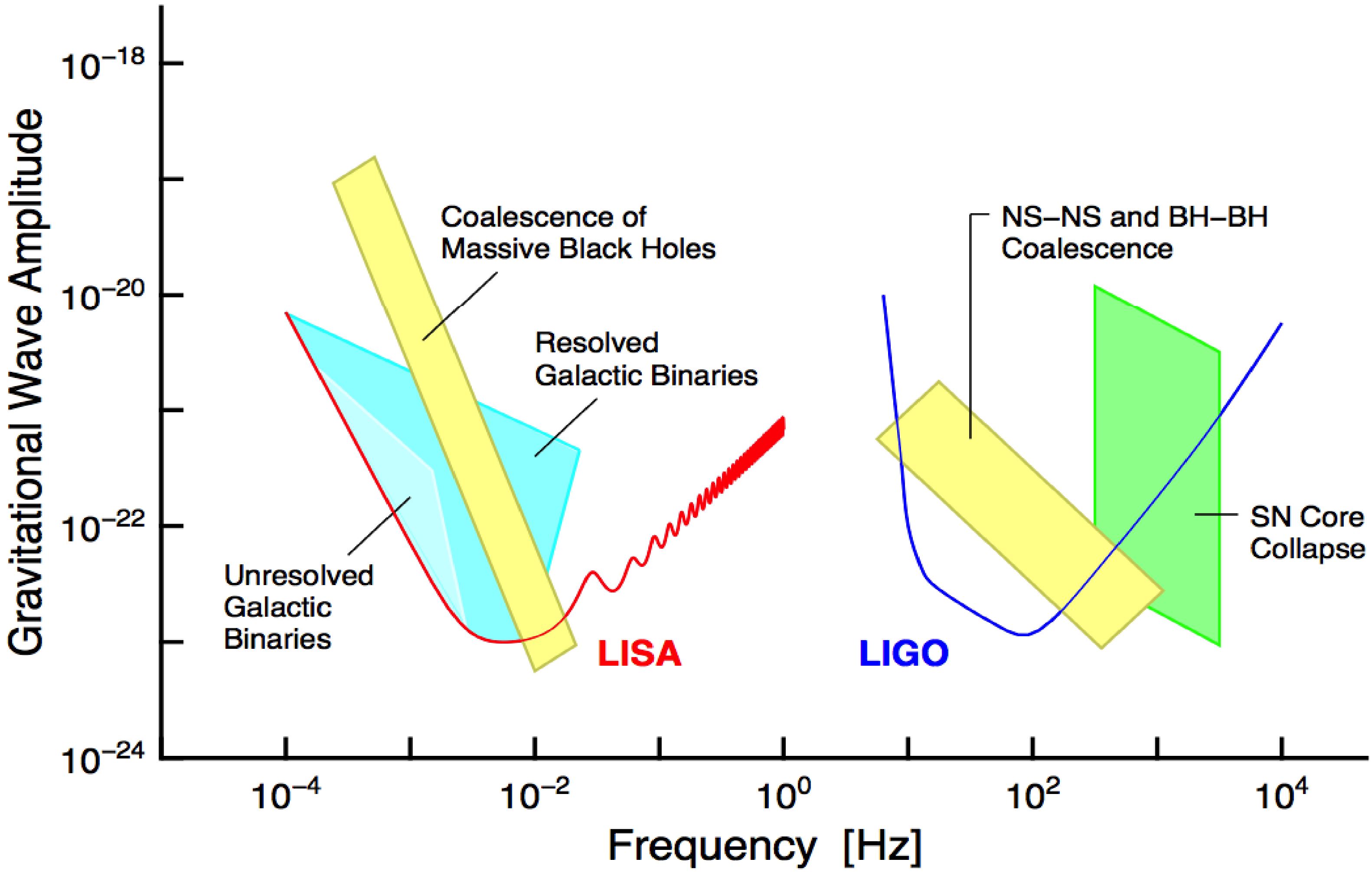


$$\Omega_{gw}(f) = \frac{1}{\rho_{crit}} \frac{d\rho_{gw}}{d\ln f}$$

Energy scale of inflation

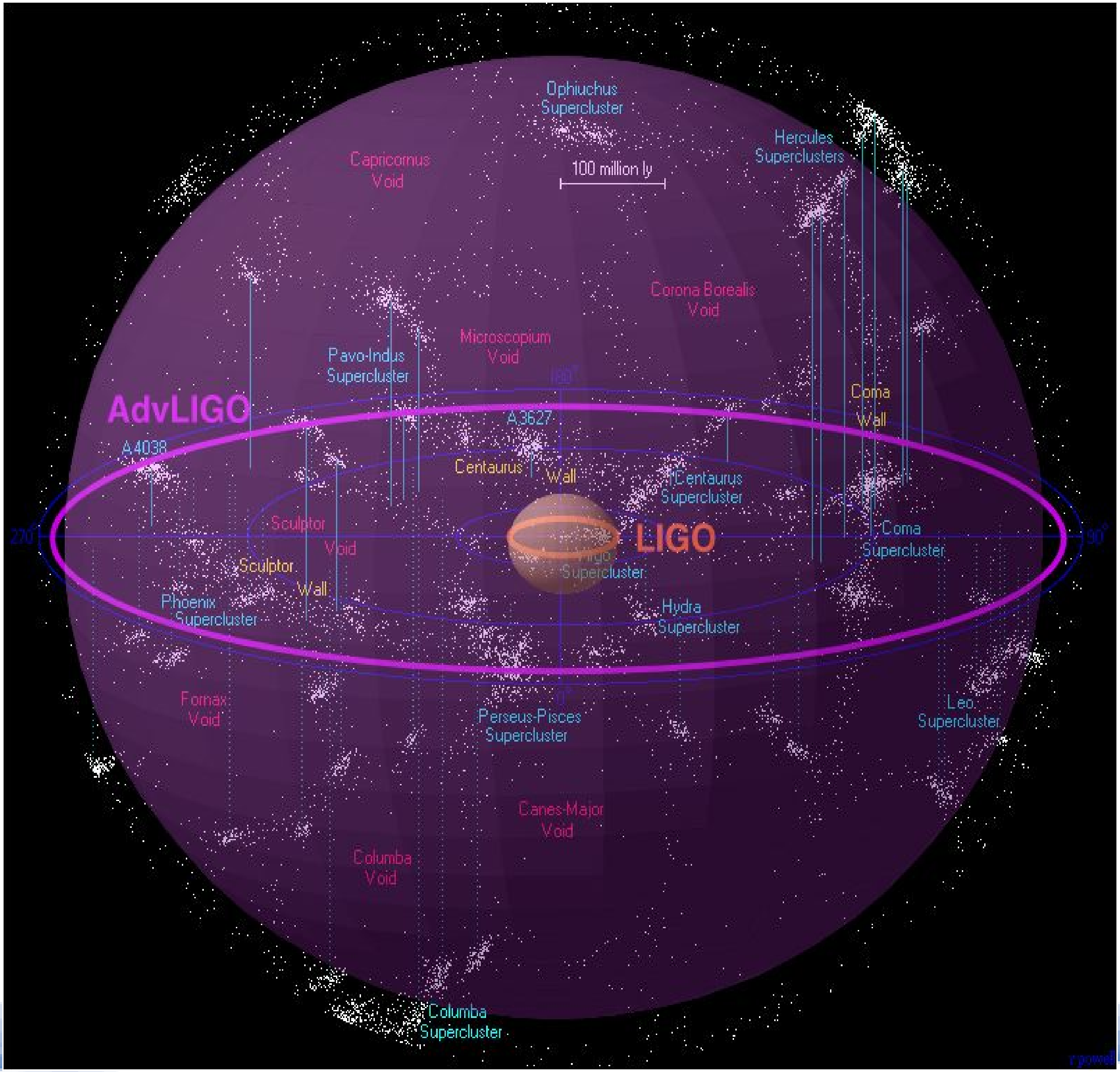
$$\Omega_{gw}(f) = 2\pi^2 \left[f h_c(f)\right]^2 / (3H_0^2)$$

LISA vs LIGO/Virgo



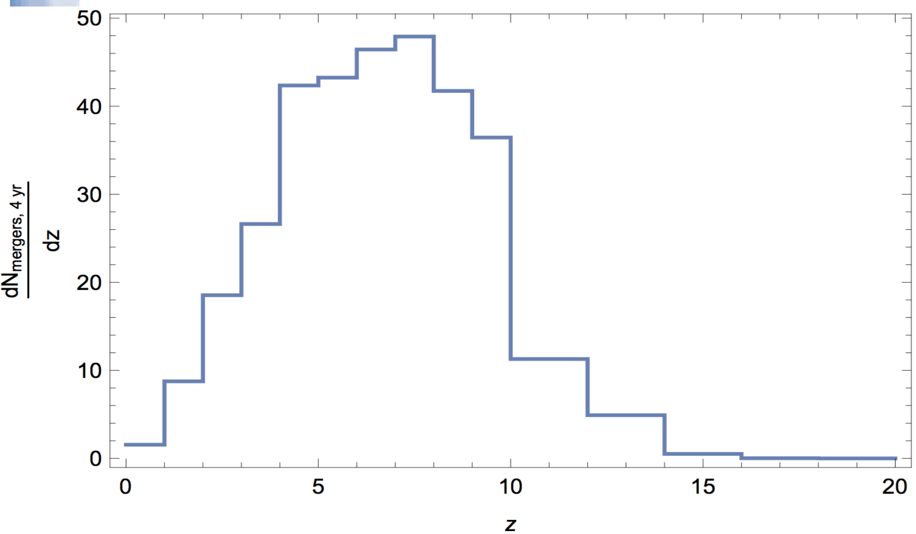
LISA vs LIGO/Virgo

Range depends on sources, but is at most z~0.1 for LIGO/Virgo...

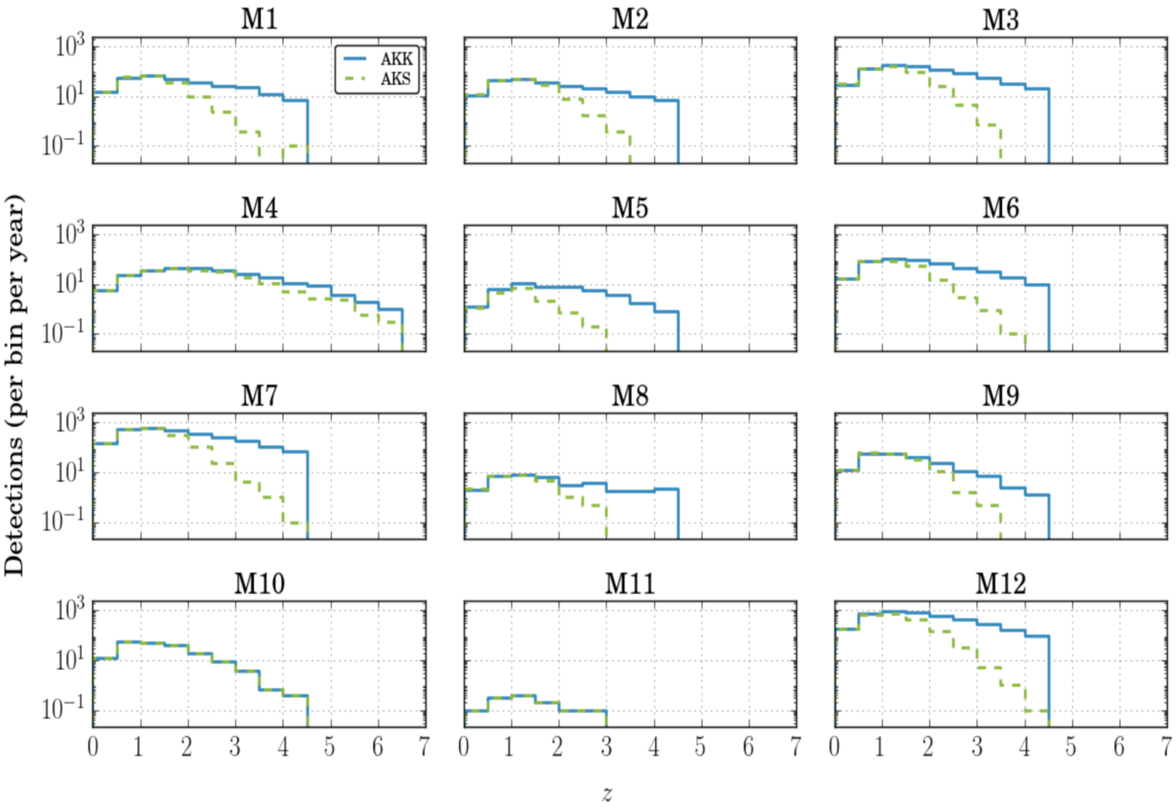


LISA vs LIGO/Virgo

... vs z>10 for LISA (for SMBH binaries)



Detectable SMBH binaries in a popIII seed formation model



Detectable EMRIs

Detector noise

- *s(t)=h(t)+n(t)*
- By central theorem limit, noise n(t) should be close to a Gaussian process, i.e. noise should be uncorrelated in Fourier but not in time domain

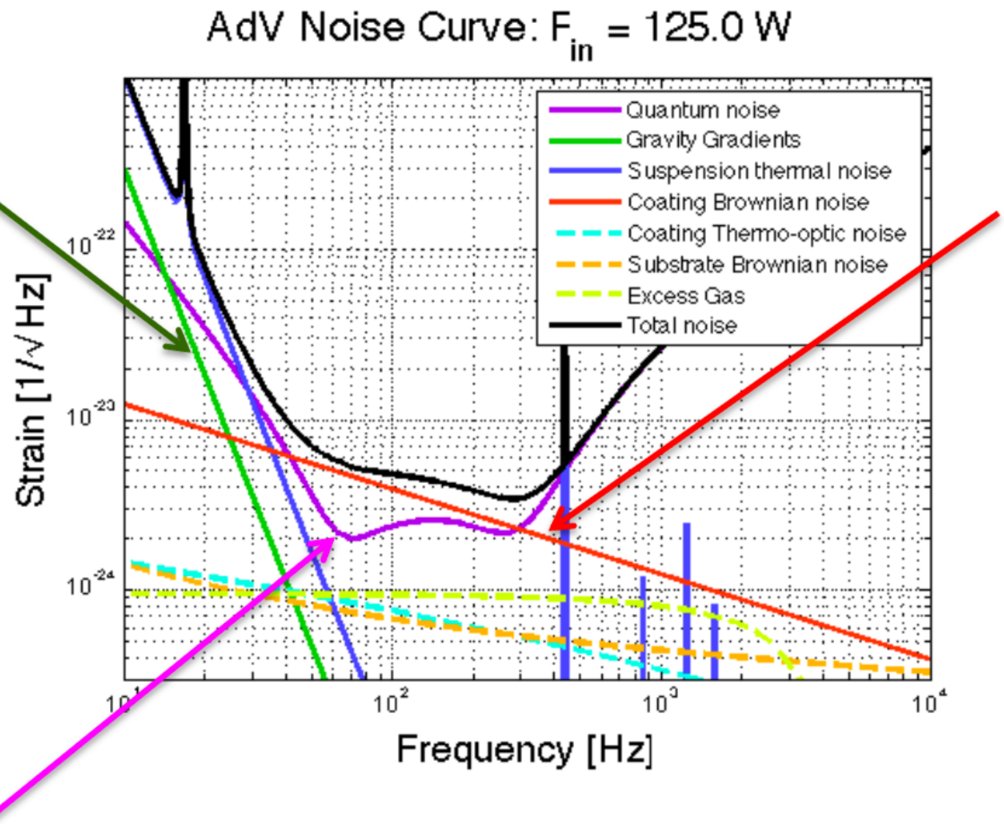
$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \frac{1}{2}S_n(f)\delta(f-f')$$

$$\langle n^2(t)\rangle = \int_0^\infty S_n(f)df$$

- $S_n(f)$ is called (single sided) noise spectral density

Detector noise

Seismic and gravity gradient noise
Geophysics

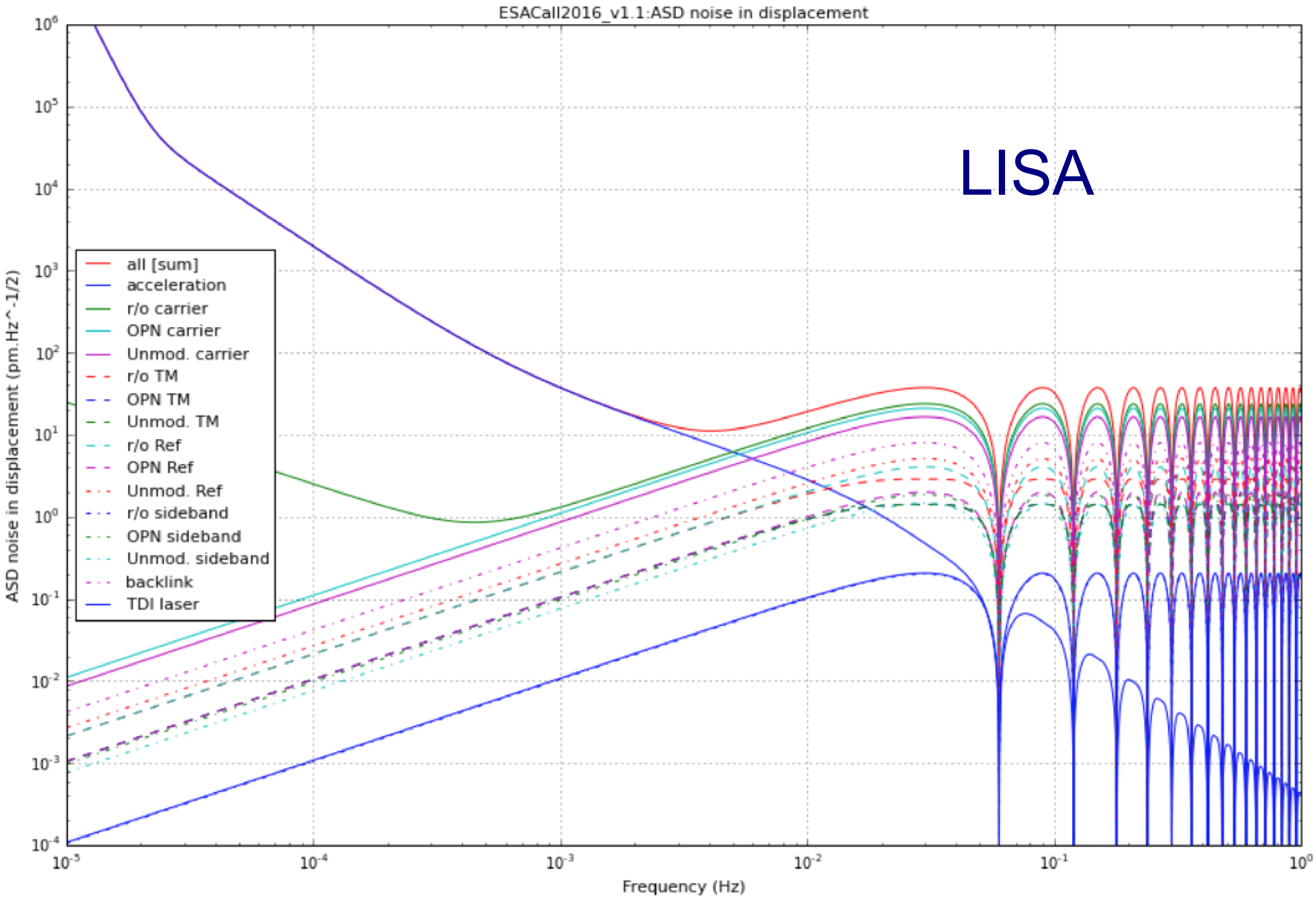


Thermal noise
Thermodynamics

Quantum noise
Quantum mechanics

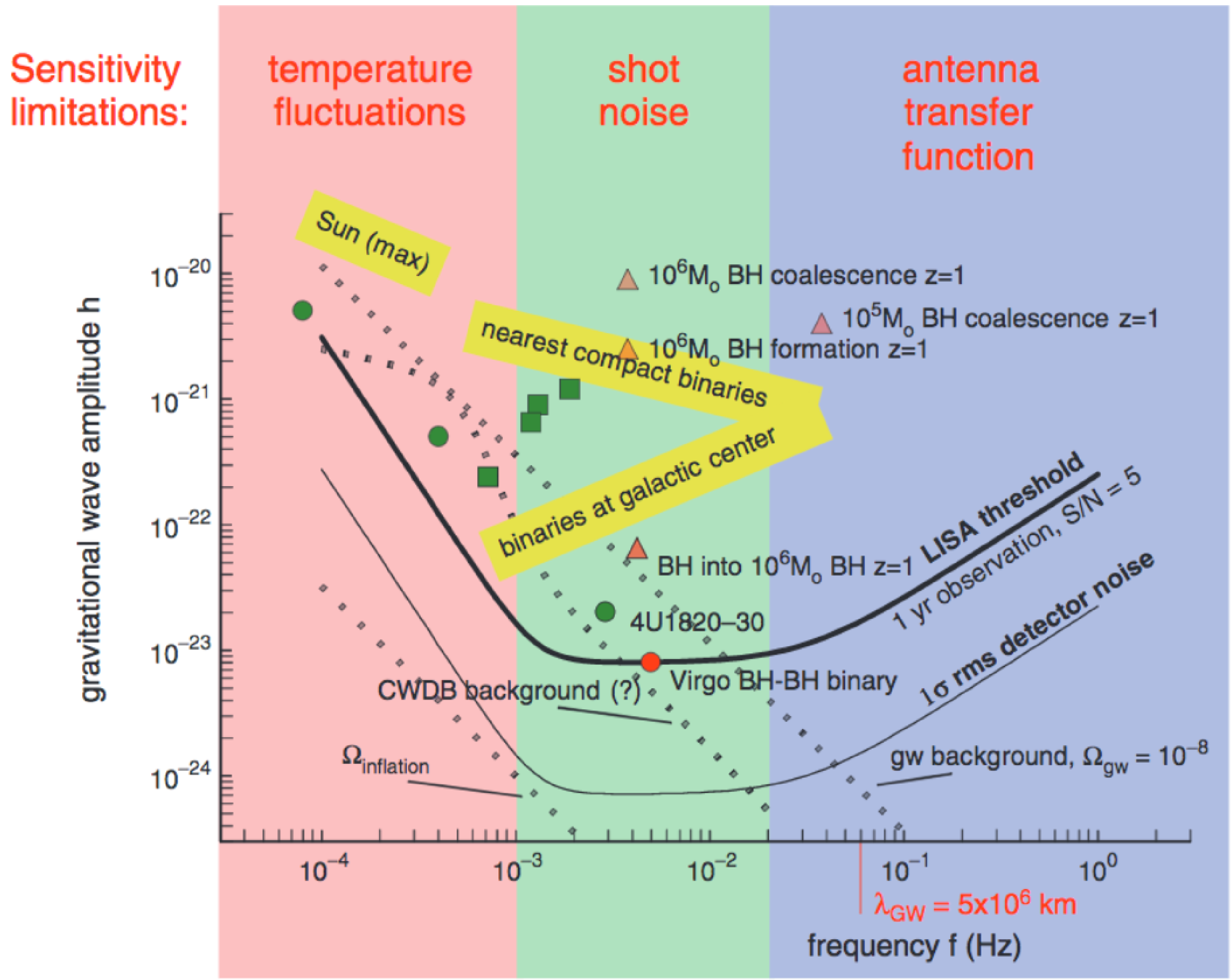
Figure courtesy Matteo Barsuglia

Detector noise



Detector noise

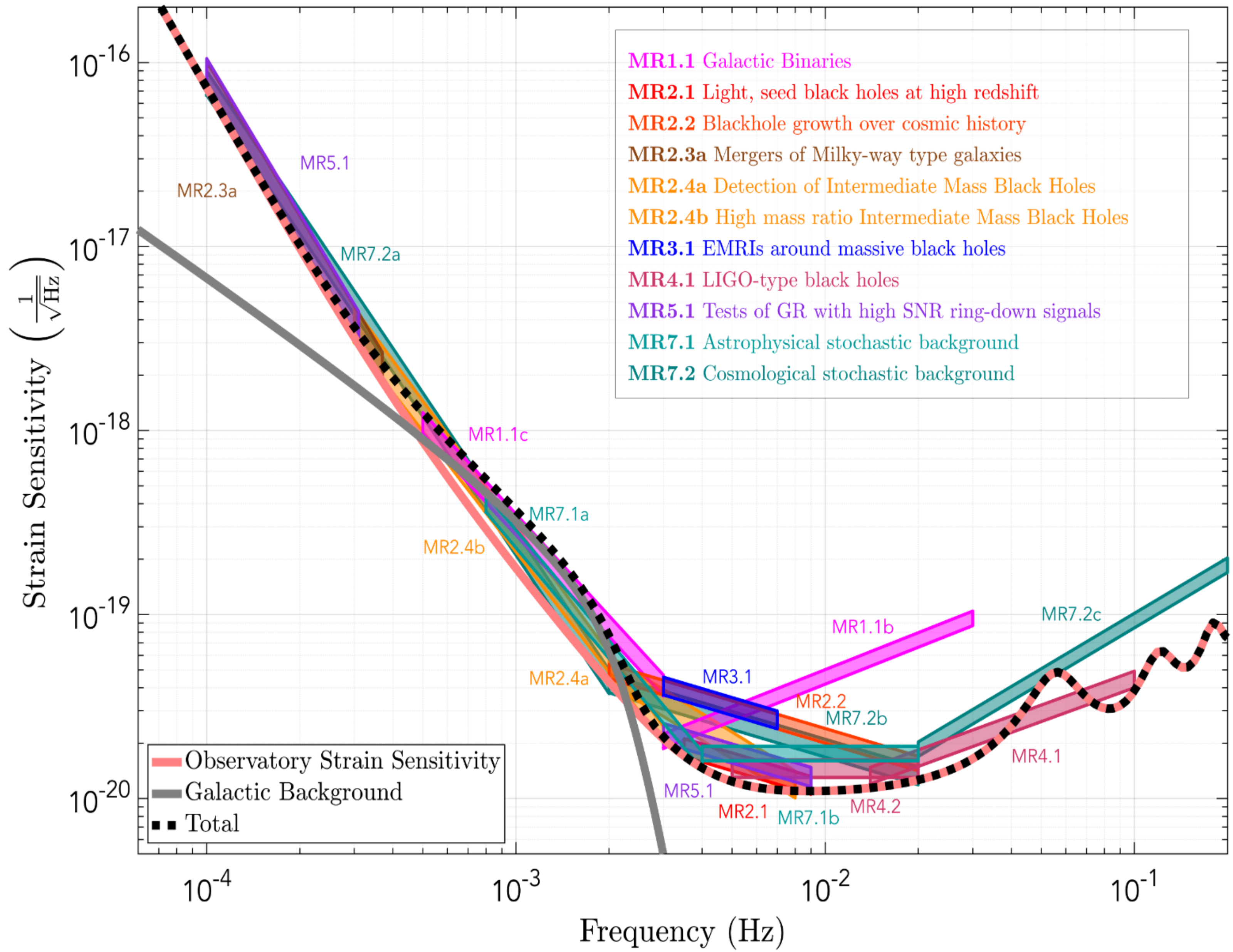
LISA



Rudiger et al 2008

Detector noise

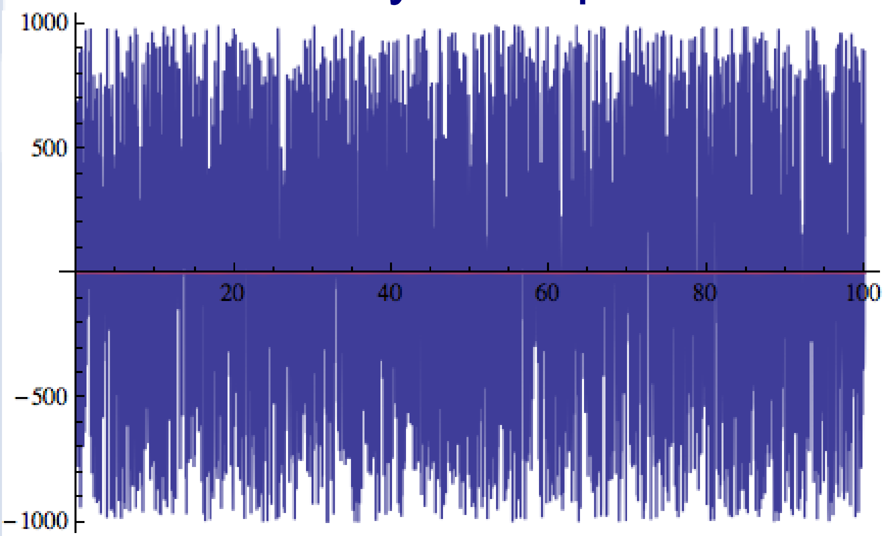
LISA

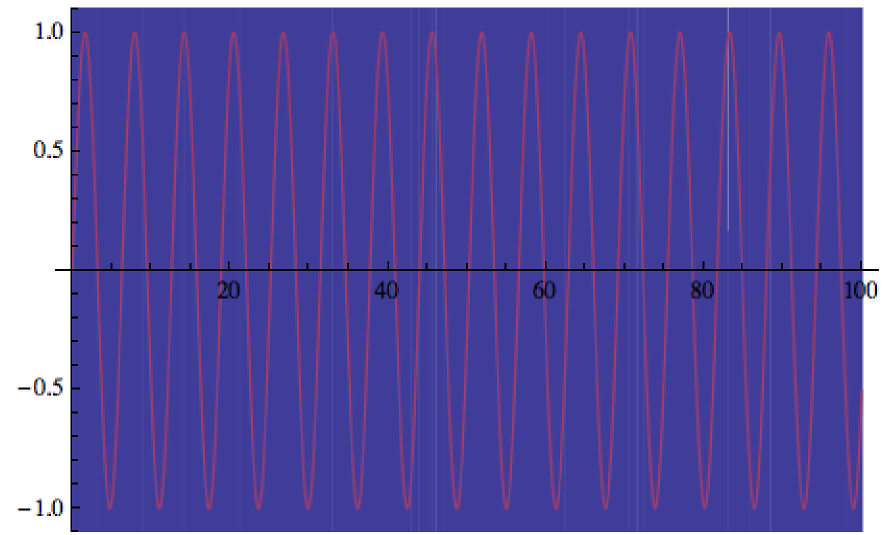


How to extract signal from noise?

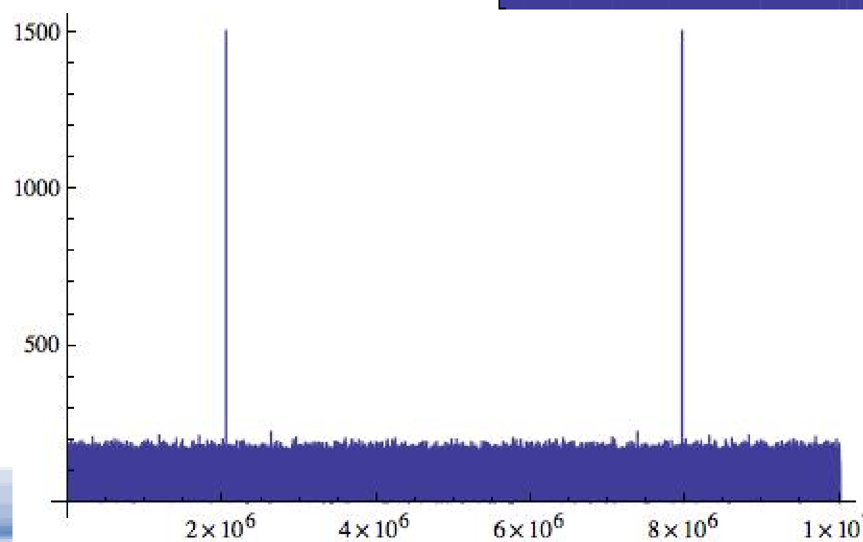
Can we extract GW signal even if it is much smaller than the instrumental noise?

Elementary example





Discrete FT



The matched filtering theorem

- *s(t)=h(t)+n(t)*
- Define filter $\hat{s} \equiv \int_{-\infty}^{+\infty} s(t)K(t)dt$
- Maximum signal-to-noise ratio S/N, with $S \equiv \hat{s}(h \neq 0), \; N \equiv \hat{s}(h = 0)$, is given by optimal filter

$$\tilde{K}(f) \propto \tilde{h}(f)/S_n(f) \quad \Rightarrow \quad \left(\frac{S}{N}\right)^2 = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}df$$

(proof on the blackboard, c.f. also Maggiore's book)

- *h(f)* is called template

SNR for compact binaries

- From quadrupole + pattern functions formula, $h(t) = F_+ h_+(t) + F_\times h_\times(t)$
- Use Newtonian dynamics (i.e. Kepler's law) and energy conservation
- Compute Fourier transform via stationary phase approximation, and account for propagation in cosmological background by replacing distance with luminosity distance and masses by redshifted masses

$$\tilde{h}(f) = \sqrt{\frac{5}{6}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{2\pi^{2/3} D_L} e^{i\psi} \frac{2Q}{2} . \qquad Q = \frac{1+\cos^2\iota}{2} F_+ + i\cos\iota F_\times \qquad \mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \times (1+z)$$

- If sky and orientation averaged, $\langle (1+\cos^2\iota)^2 F_+^2 + 4\cos^2\iota F_\times^2 \rangle^{1/2} = \frac{4}{5}$

$$\Rightarrow \quad \tilde{h}(f) = \sqrt{\frac{5}{6}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{2\pi^{2/3} D_L} e^{i\psi} \frac{2}{5} = \frac{1}{\sqrt{30}} \frac{\mathcal{M}^{5/6} f^{-7/6}}{\pi^{2/3} D_L} e^{i\psi}$$

$$\left(\frac{S}{N}\right)^2 = 4 \int_{f_{\rm in}}^{f_{\rm out}} \frac{|\tilde{h}(f)|^2}{S_n(f)} df$$

SNR for quasi-monochromatic sources

$$h(t) \; = \sqrt{2} h_0 \cos\left[\phi(t)\right] \qquad \phi(t) = 2\pi[f + \dot{f}(t - t_0) + ...](t - t0)$$

$$\tilde{h}(f) \simeq \frac{h_0}{\sqrt{2\dot{f}}}$$

$$\left(\frac{S}{N}\right)^2 = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df = \frac{2h_0^2 T_{\rm obs}}{S_n(f)}$$

For long-lived sources, SNR grows with sqrt of observation time

Parameter estimation

- With what accuracy can observations estimate the source parameters?
- Assuming Gaussian stationary noise,

$$p(n_0) \propto \exp\left[-\frac{1}{2}(n_0|n_0)\right], \qquad (A|B) \equiv 4\mathrm{Re}\int_0^\infty df\, \frac{\tilde{A}^*(f)\tilde{B}(f)}{S_n(f)},$$

$$s(t) = h(t;\boldsymbol{\theta}_t) + n_0(t) \qquad h_t \equiv h(\boldsymbol{\theta}_t)$$

- Extracted parameters maximize the likelihood $\Lambda(s|\theta_t) \propto \exp\left[(h_t|s) - \frac{1}{2}(h_t|h_t)\right]$

$$(\partial_i h_t|s) - (\partial_i h_t|h_t) = 0 \quad \partial_i \equiv \partial/\partial\theta_t^i$$

- Expanding to quadratic order near true parameters, and assuming large SNR

$$\Lambda(s|\theta) \propto \exp\left[-\frac{1}{2}\Gamma_{ij}\Delta\theta^i\Delta\theta^j\right] \qquad \theta_t^i = \hat{\theta}^i + \Delta\theta^i \qquad \Gamma_{ij} = (\partial_i h|\partial_j h)$$

Fisher Information Matrix=
Inverse of covariance matrix

- Errors on parameters:

$$\sqrt{\langle(\Delta\theta^i)^2\rangle} = \sqrt{(\Gamma^{-1})_{ii}}$$

- More advanced tecniques (MCMC) used to sample likelihood