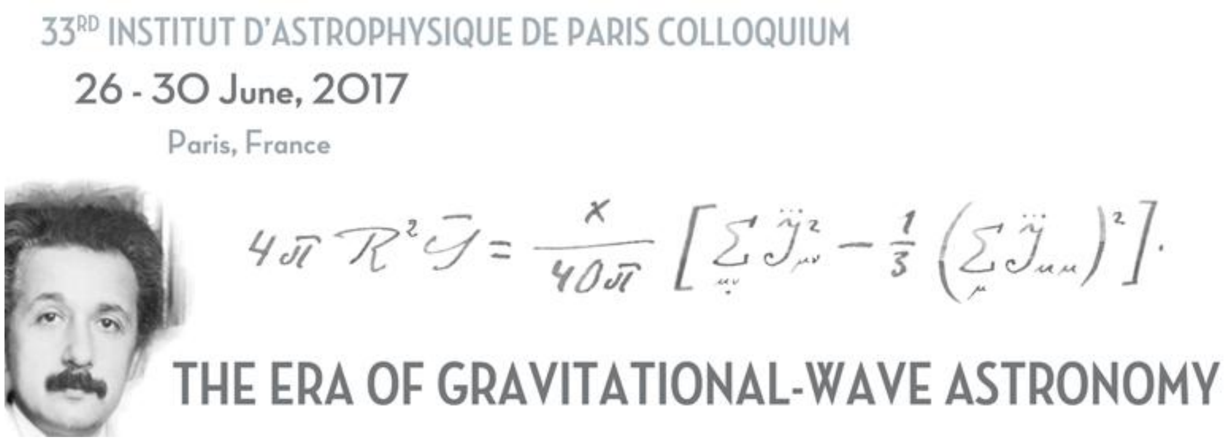


Gauge Dependence of Gravitational Waves Generated from Scalar Perturbations

J. Hwang & H. Noh

Juin 30, 2017
IAP

Perturbation method:

- Perturbation expansion
- All perturbation variables are small
- Weakly nonlinear
- Strong gravity; fully relativistic
- Valid in all scales
- Fully nonlinear and Exact perturbations

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time
- Newtonian equations of motion with GR corrections
- Expansion in strength of gravity $\frac{\delta\Phi}{c^2} \sim \frac{GM}{Rc^2} \sim \frac{v^2}{c^2} \ll 1$
- Fully nonlinear
- No strong gravity; weakly relativistic
- Valid far inside horizon
- Case of the Fully nonlinear and Exact perturbations

Fully NL & Exact Pert. Theory

JH, Noh, MN (2013) **433**, *3472*
JH, Noh, Park, MN (2016) **461**, *3239*
Gong, JH, Noh, Yoo, **arXiv: 0706.07753**

Metric convention without fixing temporal gauge (slicing) condition:

TT

$$\widetilde{g}_{00} = -a^2\left(1+2\alpha\right), \quad \widetilde{g}_{0i} = -a\chi_i, \quad \widetilde{g}_{ij} = a^2\left[\left(1+2\varphi\right)\delta_{ij} + 2h_{ij}\right].$$

raised and lowered using δ_{ij}

T

$$+\,2\gamma_{,ij} + 2C^{(v)}_{(i,j)} \equiv 0$$

Spatial Gauge taken

=spatial Harmonic to 1PN

Exact inverse metric:

$$\widetilde{g}^{00} = -\frac{1}{a^2\mathcal{N}^2}, \quad \widetilde{g}^{0i} = -\frac{\delta^{ij}+H^{ij}}{a^3\mathcal{N}^2(1+2\varphi+I)}\chi_j,$$

$$\widetilde{g}^{ij} = \frac{1}{a^2(1+2\varphi+I)}\left(\delta^{ij}+H^{ij} - \frac{(\delta^{ik}+H^{ik})(\delta^{j\ell}+H^{j\ell})}{a^2\mathcal{N}^2(1+2\varphi+I)}\chi_k\chi_\ell\right).$$

$$H^{ij} \equiv -2\frac{(1+2\varphi)h^{ij} - 2h^{ik}h^j_k}{(1+2\varphi)^2 - 2h^{k\ell}h_{k\ell}}, \quad I \equiv \frac{8}{3}\frac{h_{k\ell}h^k_m h^{\ell m}}{(1+2\varphi)^2 - 2h^{k\ell}h_{k\ell}}$$

$$N = a\sqrt{1+2\alpha+\frac{\delta^{ij}+H^{ij}}{a^2(1+2\varphi+I)}\chi_i\chi_j} \equiv a\mathcal{N}$$

Temporal gauge (slicing, hypersurface):

$$
\begin{aligned}
&\text{comoving gauge :} && v \equiv 0, \\
&\text{zero}-\text{shear gauge :} && \chi \equiv 0, \\
&\text{uniform}-\text{curvature gauge :} && \varphi \equiv 0, \\
&\text{uniform}-\text{expansion gauge :} && \kappa \equiv 0, \\
&\text{uniform}-\text{density gauge :} && \delta \equiv 0, \\
&\text{synchronous gauge :} && \alpha \equiv 0.
\end{aligned}
$$

Perturbed trace of extrinsic curvature (→ κ)
~Maximal Slicing

Synchronous gauge: → **Remnant gauge mode**

Applicable to fully NL orders!

→

Except for synchronous gauge, complete gauge fixing.
Remaining variables are gauge-invariant to fully NL order!

Post-Newtonian Approximation

Chandrasekhar, ApJ (1965) **142**, 1488: **1PN, Minkowski**
JH, Noh, Puetzfeld, JCAP (2008) **03**, 010: **cosmological**
Noh, JH, JCAP (2013) **08**, 040: **as a limit of FNL PT**

1PN convention: (Chandrasekhar 1965)

$$ds^2 = -\left[1 - \frac{1}{c^2}2U + \frac{1}{c^4}\left(2U^2 - 4\Phi\right)\right]c^2dt^2 - \frac{1}{c^3}2aP_i c dt dx^i + a^2\left(1 + \frac{1}{c^2}2V\right)\gamma_{ij}dx^i dx^j,$$

$$\widetilde{\mu} \equiv \mu \equiv \varrho c^2\left(1 + \frac{1}{c^2}\Pi\right), \quad \widetilde{p} = p, \quad \widetilde{u}^i \equiv \frac{1}{c}\frac{1}{a}\overline{v}^i\widetilde{u}^0,$$

Identification:

PT 1PN

$$\alpha = -\frac{1}{c^2}\left[U - \frac{1}{c^2}\left(U^2 - 2\Phi\right)\right], \quad \varphi = \frac{1}{c^2}V, \quad \kappa = -\frac{1}{c^2}\left(3\frac{\dot{a}}{a}U + 3\dot{V} + \frac{1}{a}P^k{}_{,k}\right),$$

$$\chi_i = \frac{1}{c^3}aP_i, \quad v_i = \frac{1}{c}\left\{\overline{v}_i + \frac{1}{c^2}\left[\overline{v}_i\left(\quad U + 2V\right) - P_i\right]\right\},$$

1PN equations,
without taking temporal gauge

JH, Noh, Puetzfeld, JCAP (2008)

1PN Equations:

Tracefree ADM propagation: $V = U.$

Covariant energy-conservation:

$$\frac{1}{a^3}\left(a^3\widetilde{\varrho}\right)^{\cdot} + \frac{1}{a}\left(\widetilde{\varrho}\overline{v}^i\right)_{,i} = -\frac{1}{c^2}\left[\widetilde{\varrho}\left(\frac{\partial}{\partial t} + \frac{1}{a}\overline{\mathbf{v}}\cdot\nabla\right)\left(\frac{1}{2}\overline{v}^2 + 3U + \widetilde{\Pi}\right) + \left(3\frac{\dot{a}}{a} + \frac{1}{a}\nabla\cdot\overline{\mathbf{v}}\right)\widetilde{p}\right],$$

Covariant momentum-conservation:

$$\frac{1}{a}\left(a\overline{v}_i\right)^{\cdot} + \frac{1}{a}\overline{v}_{i,k}\overline{v}^k - \frac{1}{a}U_{,i} + \frac{1}{a}\frac{\widetilde{p}_{,i}}{\widetilde{\varrho}} = \frac{1}{c^2}\left[\frac{1}{a}\overline{v}^2U_{,i} + \frac{2}{a}\left(\Phi - U^2\right)_{,i} + \frac{1}{a}\left(aP_i\right)^{\cdot} + \frac{1}{a}\overline{v}^k\left(P_{i,k} - P_{k,i}\right)\right.$$

$$\left. + \frac{1}{a}\left(\overline{v}^2 + 4U + \widetilde{\Pi} + \frac{\widetilde{p}}{\widetilde{\varrho}}\right)\frac{\widetilde{p}_{,i}}{\widetilde{\varrho}} - \overline{v}_i\left(\frac{\partial}{\partial t} + \frac{1}{a}\overline{\mathbf{v}}\cdot\nabla\right)\left(\frac{1}{2}\overline{v}^2 + 3U\right) - \overline{v}_i\frac{1}{\widetilde{\varrho}}\left(\frac{\partial}{\partial t} + \frac{1}{a}\overline{\mathbf{v}}\cdot\nabla\right)\widetilde{p}\right],$$

Trace of ADM propagation:

$$\frac{\Delta}{a^2}U + 4\pi G\left(\widetilde{\varrho} - \varrho\right) = -\frac{1}{c^2}\left\{\frac{1}{a^2}\left[2\Delta\Phi - 2U\Delta U + \left(aP^i{}_{,i}\right)^{\cdot}\right] + 3\ddot{U} + 9\frac{\dot{a}}{a}\dot{U} + 6\frac{\ddot{a}}{a}U\right.$$

$$\left. + 8\pi G\left[\widetilde{\varrho}\overline{v}^2 + \frac{1}{2}\left(\widetilde{\varrho}\widetilde{\Pi} - \varrho\Pi\right) + \frac{3}{2}\left(\widetilde{p} - p\right)\right]\right\},$$

ADM momentum-constraint:

$$0 = \frac{1}{a^2}\left(P^k{}_{,ki} - \Delta P_i\right) - 16\pi G\widetilde{\varrho}\overline{v}_i + \frac{4}{a}\left(\dot{U} + \frac{\dot{a}}{a}U\right)_{,i},$$

$$\overline{v}_i = \left(1 - \frac{3}{c^2}U\right)v_i + \frac{1}{c^2}P_i,$$

JH, Noh, Puetzfeld , JCAP (2008); Noh, JH, JCAP (2013)

General gauge conditions:

$$\frac{1}{a}P^i{}_{|i} + n\dot{U} + m\frac{\dot{a}}{a}U = 0,$$

$$\begin{aligned}
&\text{Harmonic gauge :(Weinberg 1972)} && n = 4,\ m = \text{arbitrary},\\
&\text{Chandrasekhar's gauge :} && n = 3,\ m = \text{arbitrary},\\
&\text{Uniform-expansion gauge :} && n = 3 = m,\\
&\text{Transverse-shear gauge :} && n = 0 = m.
\end{aligned}$$

Propagation speed of potential $= \dfrac{c}{\sqrt{n-3}}$

Propagation speed of Weyl tensor = c

JH, Noh, Puetzfeld, JCAP (2008)

1PN Hydrodynamics (Minkowski):

$$\dot{\overline{\varrho}} + \nabla \cdot (\overline{\varrho \mathbf{v}}) = -\frac{1}{c^2}\overline{\varrho}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U\right),$$

$$\dot{\overline{\varrho}} + \nabla \cdot (\overline{\varrho \mathbf{v}}) = -\frac{1}{c^2}\left[\overline{\varrho}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U + \Pi\right) + p\nabla\cdot\overline{\mathbf{v}}\right],$$

$$\dot{\overline{\mathbf{v}}} + \overline{\mathbf{v}}\cdot\nabla\overline{\mathbf{v}} - \nabla U + \frac{1}{\overline{\varrho}}\nabla p = \frac{1}{c^2}\Big[-2\nabla\left(U^2 - \widetilde{\Phi}\right) + \dot{P}_i + \overline{v}^j\left(P_{i,j} - P_{j,i}\right)$$

$$-\overline{\mathbf{v}}\frac{d}{dt}\left(\frac{1}{2}\overline{v}^2 + 3U\right) + \overline{v}^2\nabla U + \left(\overline{v}^2 + 4U + \Pi + \frac{p}{\overline{\varrho}}\right)\frac{1}{\overline{\varrho}}\nabla p - \overline{\mathbf{v}}\frac{1}{\overline{\varrho}}\frac{d}{dt}p\Big],$$

$$\Delta U + 4\pi G\overline{\varrho} = -\frac{1}{c^2}\left[3\ddot{U} - 2U\Delta U + 2\Delta\widetilde{\Phi} + \dot{P}^i{}_{,i} + 8\pi G\left(\overline{\varrho v^2} + \frac{1}{2}\overline{\varrho}\Pi + \frac{3}{2}p\right)\right],$$

$$0 = \frac{1}{4}\left(P^j{}_{,ji} - \Delta P_i\right) + \nabla\dot{U} - 4\pi G\overline{\varrho\mathbf{v}},$$

$$0 = U - V.$$

General gauge: $$P^i{}_{,i} + n\dot{U} = 0.$$

Harmonic gauge: n ≡ 4
Maximal Slicing: n ≡ 3
Zero-shear Slicing: n ≡ 0

$$g_{00} = -\left[1 - \frac{1}{c^2}2U + \frac{1}{c^4}\left(2U^2 - 4\widetilde{\Phi}\right)\right], \quad g_{0i} = -\frac{1}{c^3}P_i, \quad g_{ij} = \left(1 + \frac{1}{c^2}2V\right)\delta_{ij}.$$

$$u^i \equiv u^0\frac{\overline{v}^i}{c} \qquad v_i = \overline{v}_i + \frac{1}{c^2}\left[(U + 2V)\,\overline{v}_i - P_i\right]$$

Special Relativistic Hydrodynamics with Gravity

Special Relativistic Hydrodynamics + ~0PN

Weak gravity and Action-at-a-distance

With relativistic pressure, velocity, stress

JH, Noh, ApJ (2016) ***833***, 180

Minkowski background:

Metric:

$$\chi_i \equiv c\chi_{,i} + \chi_i^{(v)} \text{ with } \chi^{(v)i}{}_{,i} \equiv 0$$

$$ds^2 = -\left(1 - \frac{2\Phi}{c^2}\right) c^2 dt^2 - 2\chi_i c dt dx^i + \left(1 + \frac{2\Psi}{c^2}\right) \delta_{ij} dx^i dx^j$$

Assumptions:

Weak Gravity

Action-at-a-distance

$$\frac{\Phi}{c^2} \ll 1, \quad \frac{\Psi}{c^2} \ll 1, \quad \gamma^2 \frac{t_\ell^2}{t_g^2} \ll 1$$

$$t_g \sim 1/\sqrt{G\varrho}$$

$$t_\ell \sim \ell/c \sim 2\pi/(kc)$$

SR Hydrodynamics with Gravity

Maximal Slicing: K ≡ 0

Continuity:

$$\frac{d\overline{\varrho}}{dt} + \overline{\varrho}\nabla\cdot\mathbf{v} = \frac{\overline{\varrho}}{c^2}\frac{1}{\varrho + p/c^2}\left(\frac{dp}{dt} - \frac{1}{\gamma^2}\dot{p}\right),$$

E conservation:

$$\frac{d\varrho}{dt} + \left(\varrho + \frac{p}{c^2}\right)\nabla\cdot\mathbf{v} = \frac{1}{c^2}\left(\frac{dp}{dt} - \frac{1}{\gamma^2}\dot{p}\right),$$

M conservation:

$$\frac{d\mathbf{v}}{dt} = \boxed{\nabla\Phi} - \frac{1}{\gamma^2}\frac{1}{\varrho + p/c^2}\left(\nabla p + \frac{1}{c^2}\mathbf{v}\dot{p}\right),$$

Poisson eq:

$$\Delta\Phi + 4\pi G\left(\varrho \boxed{+ 3\frac{p}{c^2}\right) = -8\pi G\left(\varrho + \frac{p}{c^2}\right)\gamma^2\frac{v^2}{c^2}}$$

$$\varrho \equiv \overline{\varrho}(1 + \Pi/c^2), \quad u_i \equiv \gamma\frac{v_i}{c} \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

With Anisotropic Stress:

$$\frac{d\overline{\varrho}}{dt} + \overline{\varrho}\left(\nabla\cdot\mathbf{v} + \frac{d}{dt}\ln\gamma\right) = 0,$$

$$\frac{d\varrho}{dt} + \left(\varrho + \frac{p}{c^2}\right)\left(\nabla\cdot\mathbf{v} + \frac{d}{dt}\ln\gamma\right) = -\frac{1}{c^2}\Pi_i^j\nabla_j v^i - \frac{1}{c^4}\Pi_{ij}v^i\dot{v}^j,$$

$$\frac{d\mathbf{v}}{dt} = \nabla\Phi - \frac{1}{\varrho + p/c^2}\frac{1}{\gamma^2}\left(\nabla p + \frac{1}{c^2}\mathbf{v}\dot{p}\right)$$

$$+\frac{1}{\varrho + p/c^2}\frac{1}{\gamma^2}\left\{-\Pi_{i,j}^j + \frac{1}{c^2}\left[\mathbf{v}\left(\Pi_j^k v^j\right)_{,k} - \frac{1}{\gamma^2}\left(\Pi_{ij}v^j\right)^{\cdot}\right] + \frac{1}{c^4}\mathbf{v}\left(\Pi_{jk}v^j v^k\right)^{\cdot}\right\},$$

$$\Delta\Phi + 4\pi G\left(\varrho + 3\frac{p}{c^2}\right) = -\frac{8\pi G}{c^2}\left[\left(\varrho + \frac{p}{c^2}\right)\gamma^2 v^2 + \Pi_i^i\right],$$

$$\Delta\Psi + 4\pi G\varrho = -\frac{4\pi G}{c^2}\left[\left(\varrho + \frac{p}{c^2}\right)\gamma^2 v^2 + \Pi_i^i\right].$$

GW generated from scalar pert. Gauge dependence

TT perturbation generated from Galaxy Clustering

JH, Jeong, Noh, ApJ (2017) **842**, 46

Tracefree ADM propagation

$$\frac{1}{a^2}\left(\nabla_i\nabla_j-\frac{1}{3}\gamma_{ij}\Delta\right)\left[\frac{1}{a}\left(a\chi\right)^{\cdot}-\alpha-\varphi-8\pi G\Pi\right]+\frac{1}{a}\nabla_{(i}\left\{\frac{1}{a^2}\left[a^2\left(B^{(v)}_{j)}+a\dot{C}^{(v)}_{j)}\right)\right]^{\cdot}-8\pi G\Pi^{(v)}_{j)}\right\}$$
$$+\ddot{h}_{ij}+3H\dot{h}_{ij}-\frac{\Delta-2K}{a^2}h_{ij}-8\pi G\Pi^{(t)}_{ij}=n_{ij},$$

Non-linear contributions

$$\ddot{h}_{ij}+3H\dot{h}_{ij}-\frac{\Delta-2K}{a^2}h_{ij}-8\pi G\Pi^{(t)}_{ij}=s_{ij}$$

TT projection

$$s_{ij}\equiv\mathcal{P}_{ij}{}^{k\ell}n_{k\ell}\equiv n_{ij}-\frac{1}{3}\gamma_{ij}n^k_k+\frac{1}{2}\left(\nabla_i\nabla_j-\frac{1}{3}\gamma_{ij}\Delta\right)\left(\Delta+3K\right)^{-1}\left[n^k_k-3\Delta^{-1}\left(n^{k\ell}{}_{|k\ell}\right)\right]$$
$$-2\nabla_{(i}\left(\Delta+2K\right)^{-1}\left[n^k_{j)|k}-\nabla_{j)}\Delta^{-1}\left(n^{k\ell}{}_{|k\ell}\right)\right],$$

Scalar contribution to second order

$$n_{ij}=\frac{1}{a^3}\left[a\left(2\varphi\chi_{,i|j}+\varphi_{,i}\chi_{,j}+\varphi_{,j}\chi_{,i}\right)\right]^{\cdot}+\frac{1}{a^2}\left(\kappa\chi_{,i|j}-4\varphi\varphi_{,i|j}-3\varphi_{,i}\varphi_{,j}\right)+\frac{1}{a^4}\left(\chi^{,k}{}_{|i}\chi_{,j|k}-K\chi_{,i}\chi_{,j}\right)$$
$$+\frac{1}{a^2}\left[2\dot{\chi}_{,i|j}\alpha-H\chi_{,i|j}\alpha+\chi_{,i|j}\dot{\alpha}-2\left(\alpha+\varphi\right)\alpha_{,i|j}-\alpha_{,i}\alpha_{,j}-2\alpha_{,(i}\varphi_{,j)}\right]+8\pi G\left(\mu+p\right)v_{,i}v_{,j}$$
$$-\frac{1}{3}\gamma_{ij}\left\{\frac{1}{a^3}\left[a\left(2\varphi\Delta\chi+2\varphi^{,k}\chi_{,k}\right)\right]^{\cdot}+\frac{1}{a^2}\left(\kappa\Delta\chi-4\varphi\Delta\varphi-3\varphi^{,k}\varphi_{,k}\right)+\frac{1}{a^4}\left(\chi^{,k|\ell}\chi_{,k|\ell}-K\chi^{,k}\chi_{,k}\right)\right.$$
$$\left.+\frac{1}{a^2}\left[2\alpha\Delta\dot{\chi}-H\alpha\Delta\chi+\dot{\alpha}\Delta\chi-2\left(\alpha+\varphi\right)\Delta\alpha-\alpha^{,k}\alpha_{,k}-2\alpha^{,k}\varphi_{,k}\right]+8\pi G\left(\mu+p\right)v^{|k}v_{,k}\right\}.$$

without taking temporal gauge

$$h_{ij}(\mathbf{x},t) = \frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot\mathbf{x}}\left[h(\mathbf{k},t)e_{ij}(\mathbf{k}) + \overline{h}(\mathbf{k},t)\overline{e}_{ij}(\mathbf{k})\right],$$

$$e_{ij}(\mathbf{k}) \equiv \frac{1}{\sqrt{2}}\left[e_i(\mathbf{k})e_j(\mathbf{k}) - \overline{e}_i(\mathbf{k})\overline{e}_j(\mathbf{k})\right], \quad \overline{e}_{ij}(\mathbf{k}) \equiv \frac{1}{\sqrt{2}}\left[\overline{e}_i(\mathbf{k})e_j(\mathbf{k}) + e_i(\mathbf{k})\overline{e}_j(\mathbf{k})\right],$$

$$h(\mathbf{k},t) = e^{ij}(\mathbf{k})\int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}}h_{ij}(\mathbf{x},t), \quad \overline{h}(\mathbf{k},t) = \overline{e}^{ij}(\mathbf{k})\int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}}h_{ij}(\mathbf{x},t).$$

$$h_{\mathrm{x}}(\mathbf{k},\eta) = \frac{6}{5}\frac{1}{k^2}\frac{1}{(2\pi)^3}\int d^3q\left[e^{ij}(\mathbf{k})q_iq_j\right]C(\mathbf{q})C(\mathbf{k}-\mathbf{q})W_{\mathrm{x}}(\mathbf{k},\mathbf{q},\eta)$$

Gauge (slicing)

$$W_\chi = g(k\eta), \quad W_v = g(k\eta) - \frac{1}{15}\frac{k^2}{a^2H^2}, \quad W_\varphi = g(k\eta) - \frac{3}{20}\frac{k^2}{a^2H^2},$$

$$W_\kappa = g(k\eta) - \frac{1}{15}\frac{k^2}{a^2H^2}\left(1+\frac{2}{9}\frac{q^2}{a^2H^2}\right)^{-1}\left(1+\frac{2}{9}\frac{|\mathbf{k}-\mathbf{q}|^2}{a^2H^2}\right)^{-1},$$

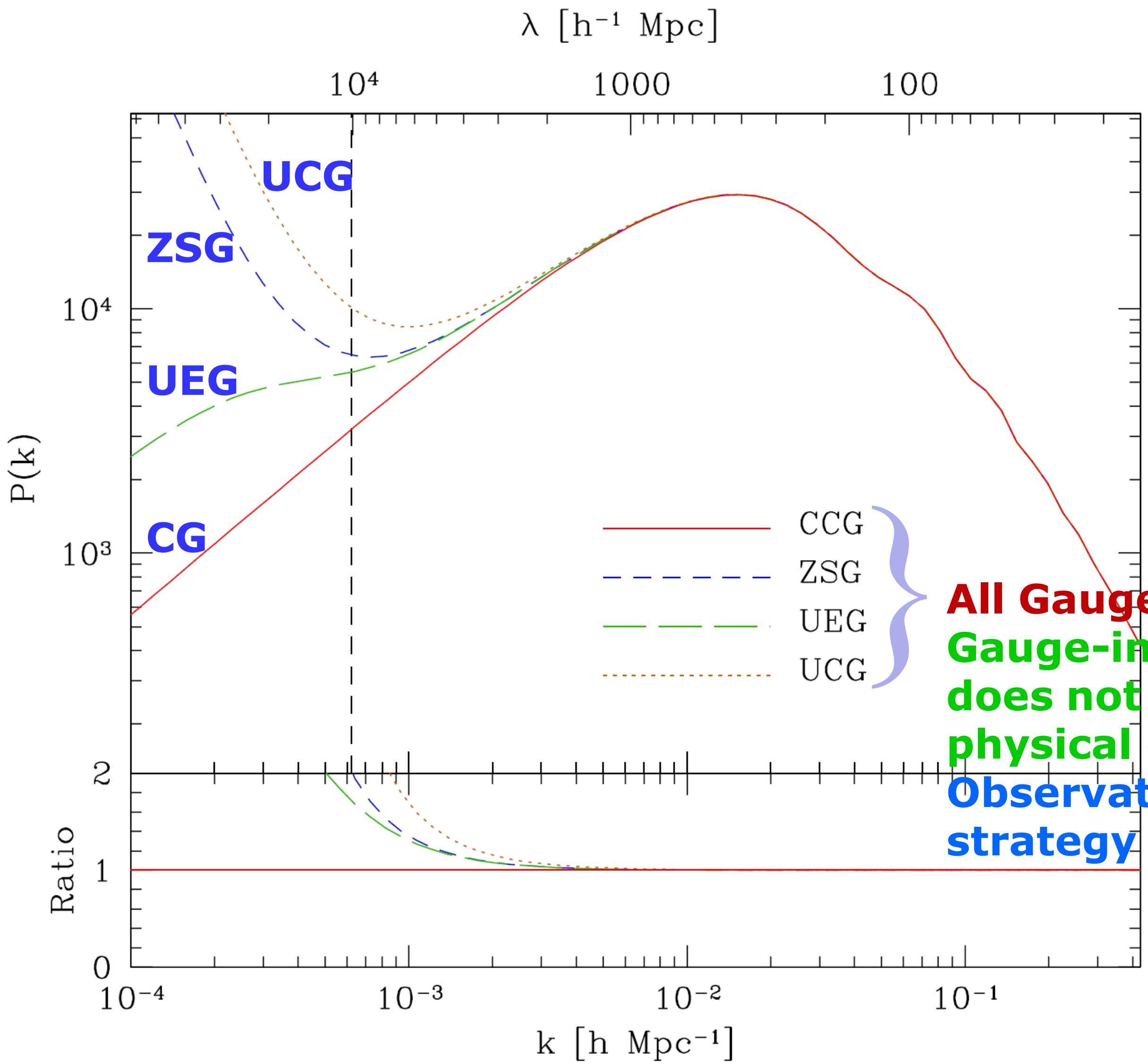
$$W_\delta = g(k\eta) - \frac{1}{15}\frac{k^2}{a^2H^2}\left(1+\frac{1}{3}\frac{q^2}{a^2H^2}\right)\left(1+\frac{1}{3}\frac{|\mathbf{k}-\mathbf{q}|^2}{a^2H^2}\right).$$

$$\langle C(\mathbf{k})C(\mathbf{k}')\rangle \equiv (2\pi)^3\delta^D(\mathbf{k}+\mathbf{k}')P_C(k) \qquad \varphi_v = C, \qquad \delta_v = -\frac{2}{5}\frac{\Delta}{a^2H^2}C.$$

$$\langle h_{\mathrm{x}}(\mathbf{k},\eta)h_{\mathrm{x}}(\mathbf{k}',\eta)\rangle \equiv (2\pi)^3\delta^D(\mathbf{k}+\mathbf{k}')\frac{1}{2}P_{h_{\mathrm{x}}}(k,\eta)$$

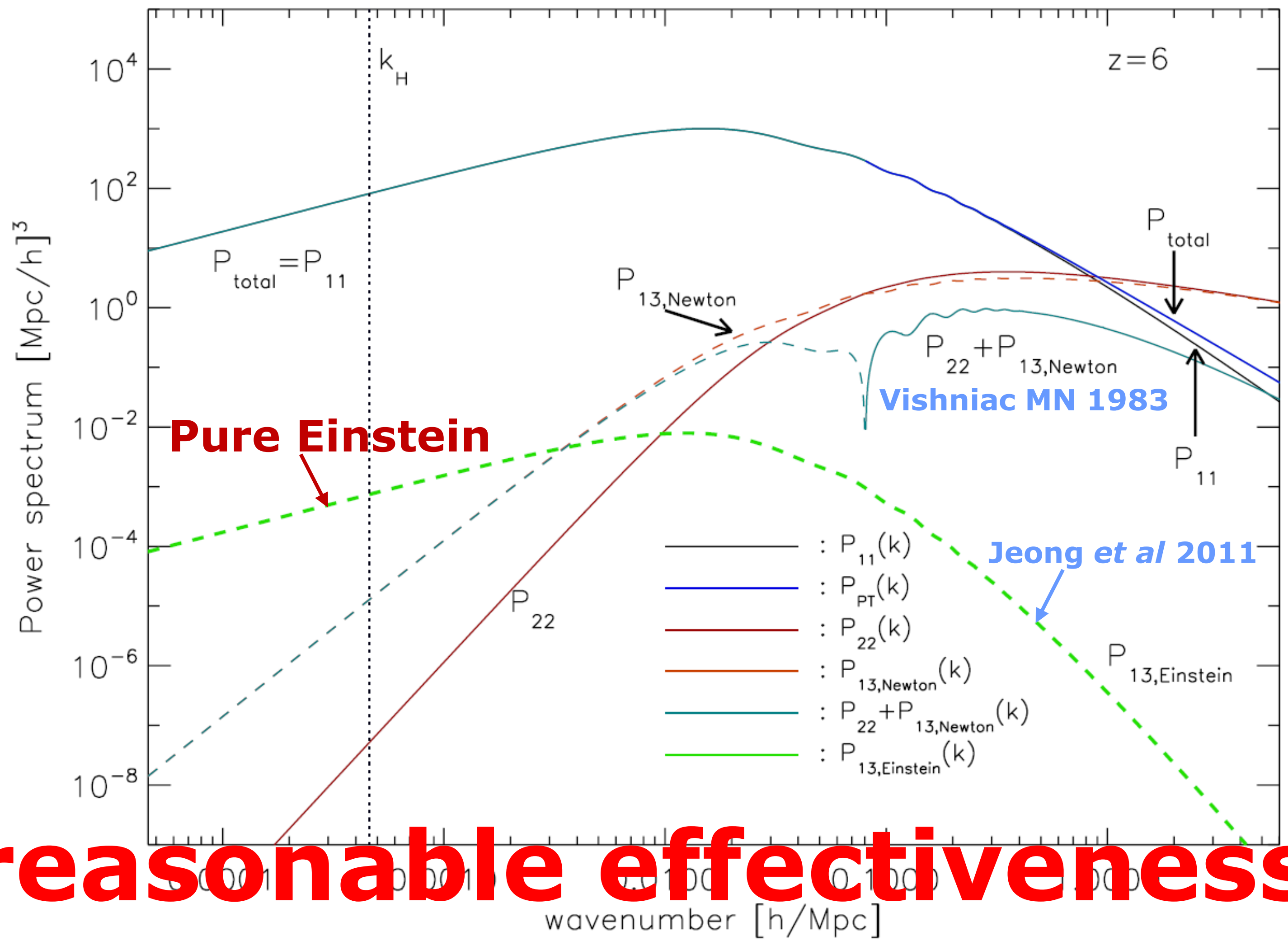
$$P_{h_{\mathrm{x}}}(k,\eta) = \frac{144}{25}\frac{1}{k^4}\frac{1}{(2\pi)^3}\int d^3q\left[e^{ij}(\mathbf{k})q_iq_j\right]^2 P_C(q)P_C(|\mathbf{k}-\mathbf{q}|)W_{\mathrm{x}}^2(\mathbf{k},\mathbf{q},\eta)$$

Baryonic matter power spectrum in the CDM model: linear theory



All Gauge-invariant!
Gauge-invariance does not guarantee physical observable!
Observational strategy may tell.

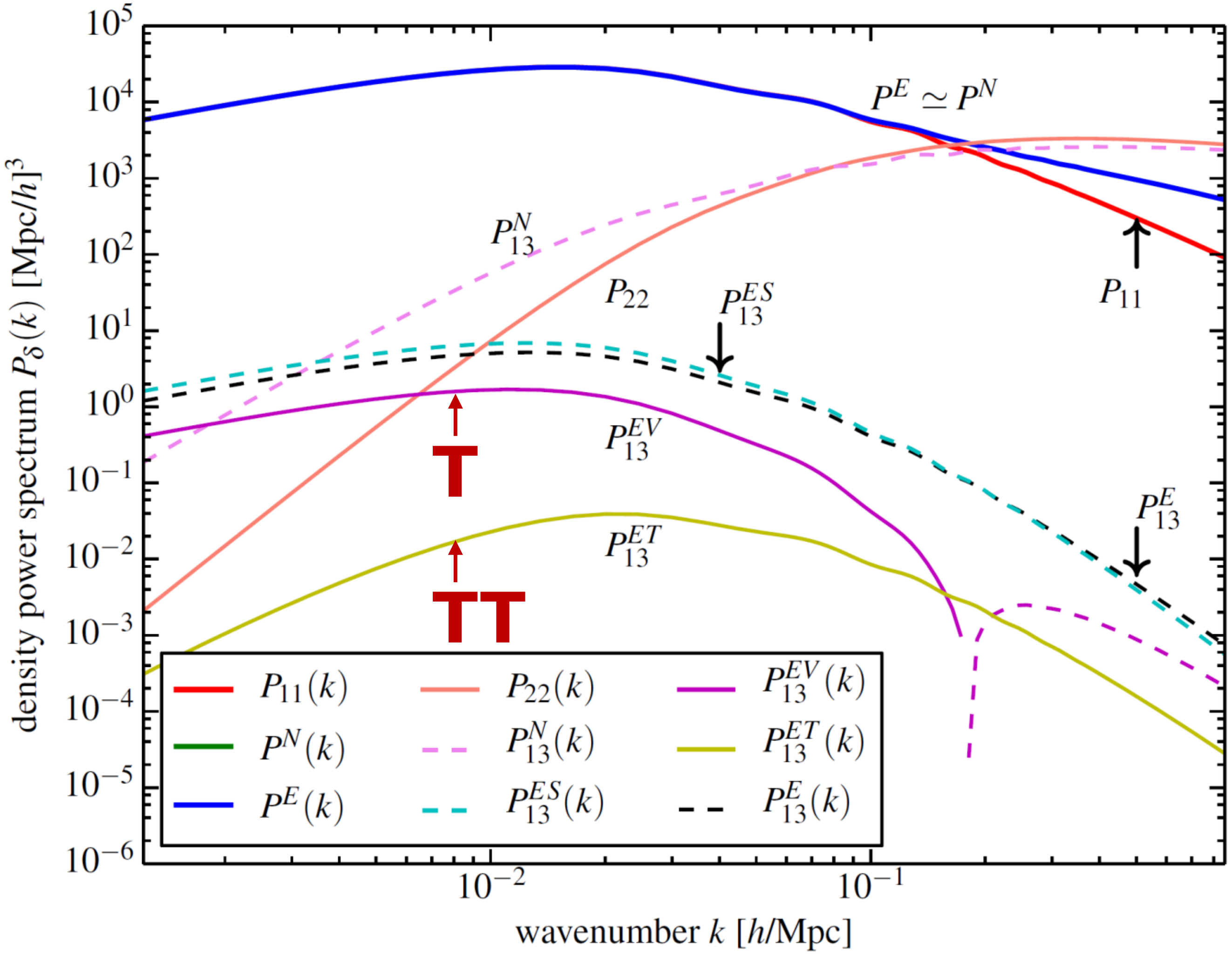
Leading Nonlinear Density Power-spectrum in the Comoving gauge:



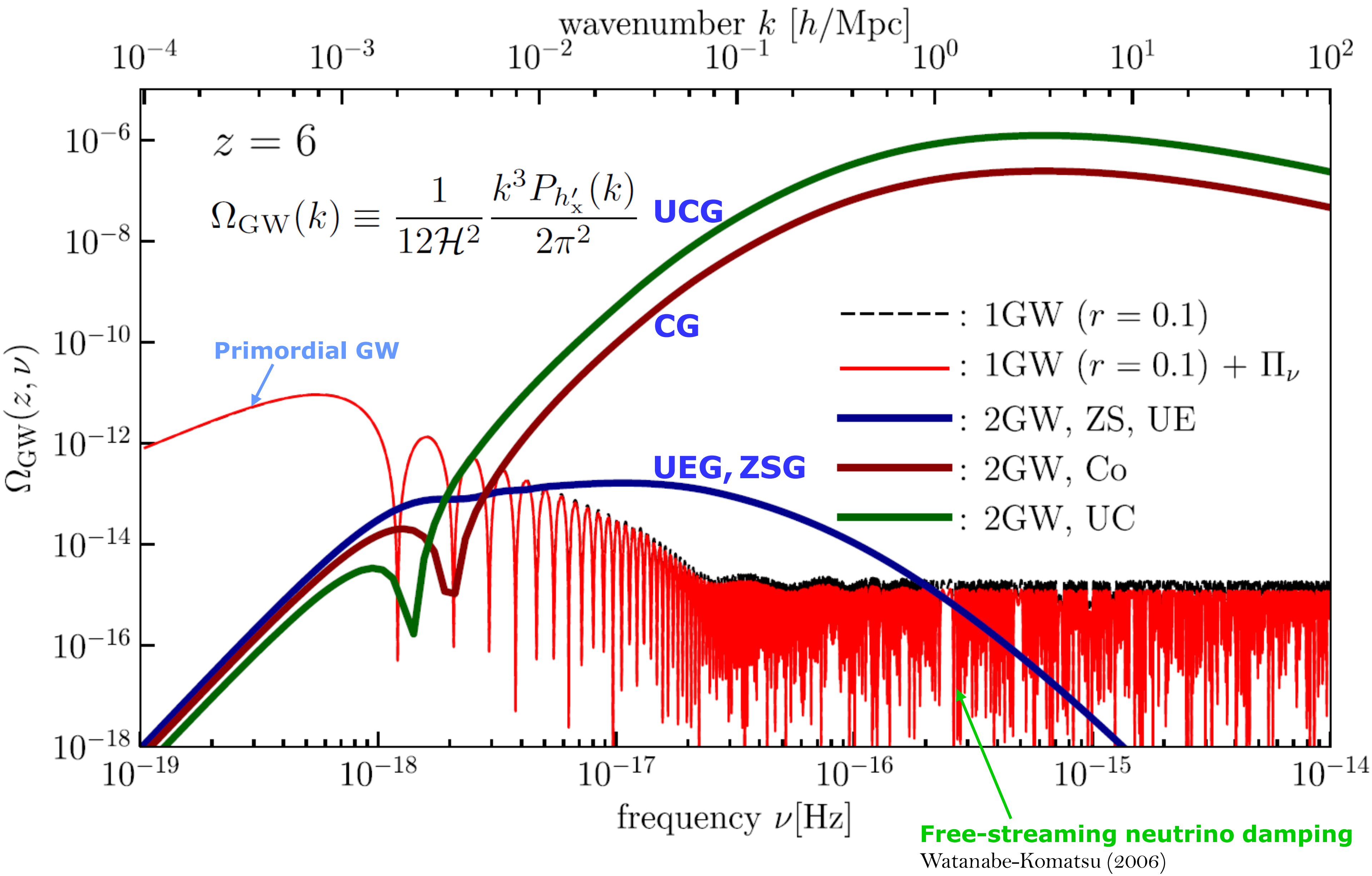
Unreasonable effectiveness of Newton's gravity in cosmology!

Jeong, Gong, Noh, JH, ApJ (2011) **722**, 1

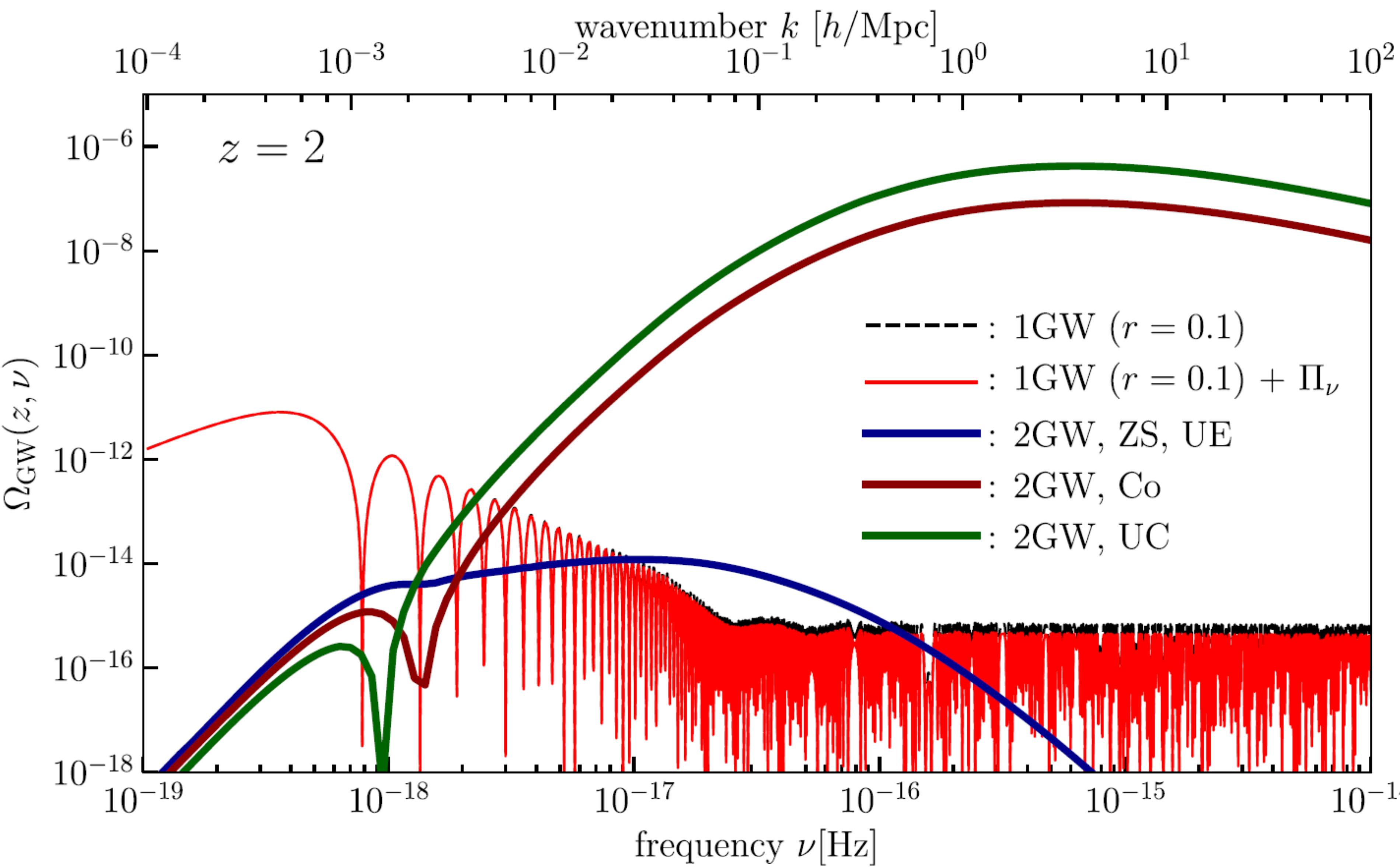
NL Density Power-spectrum in the CG with vector and tensor contributions:



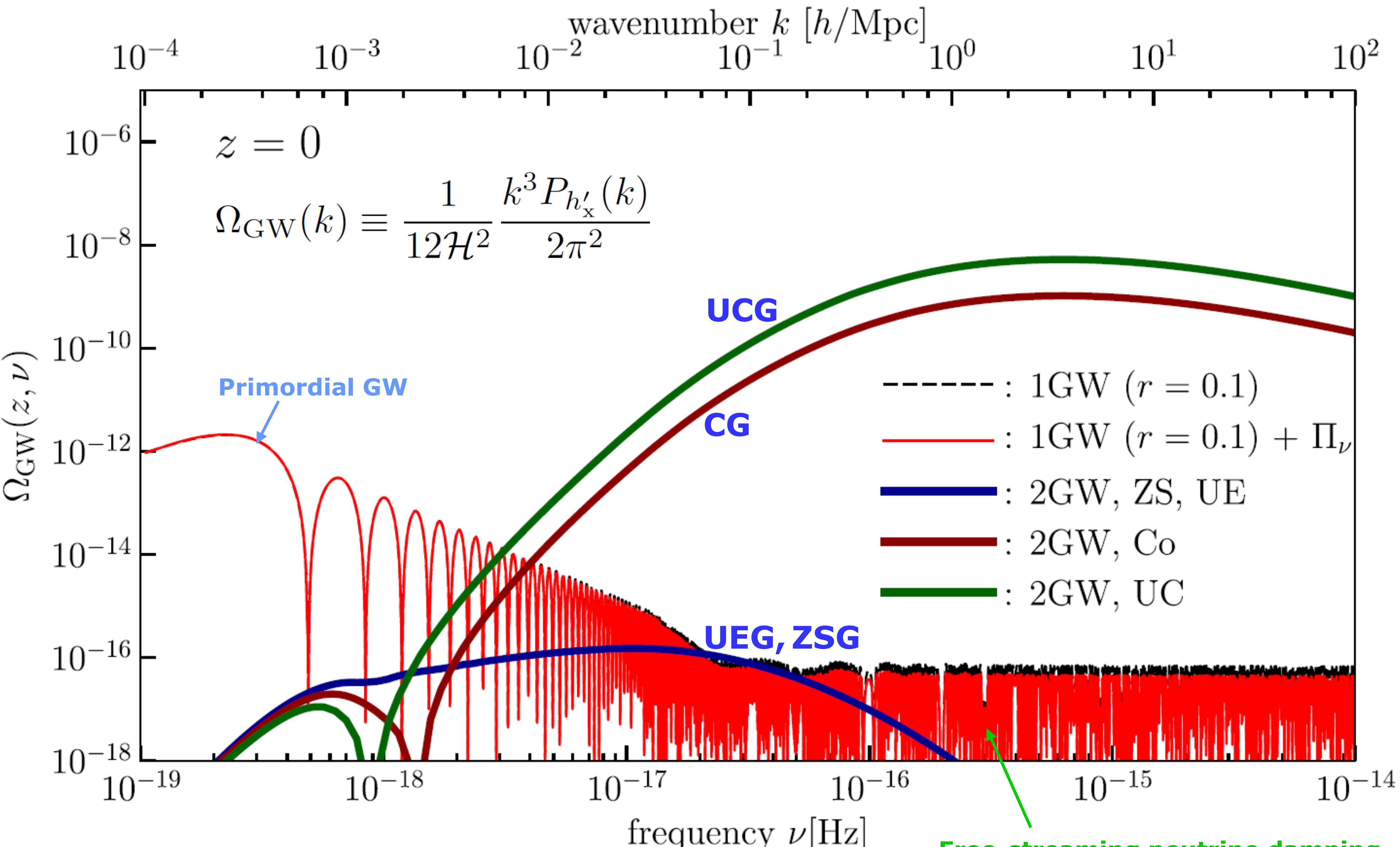
JH, Jeong, Noh, MN (2016) **459**, 1124



wavenumber k [h/Mpc]
$z = 6$
$\Omega_{GW}(k) \equiv \frac{1}{12\mathcal{H}^2} \frac{k^3 P_{h'_x}(k)}{2\pi^2}$
UCG
CG
UEG, ZSG
Primordial GW
: 1GW ($r = 0.1$)
: 1GW ($r = 0.1$) + Π_ν
: 2GW, ZS, UE
: 2GW, Co
: 2GW, UC
$\Omega_{GW}(z, \nu)$
frequency ν[Hz]
Free-streaming neutrino damping
Watanabe-Komatsu (2006)



wavenumber k [h/Mpc]
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
$z = 2$
$\Omega_{\rm GW}(z,\nu)$
10^{-6} 10^{-8} 10^{-10} 10^{-12} 10^{-14} 10^{-16} 10^{-18}
: 1GW ($r = 0.1$)
: 1GW ($r = 0.1$) + Π_ν
: 2GW, ZS, UE
: 2GW, Co
: 2GW, UC
10^{-19} 10^{-18} 10^{-17} 10^{-16} 10^{-15} 10^{-14}
frequency ν[Hz]



Watanabe-Komatsu, PRD (2006) **73**, 123515

Pulsar Timing Array: 10^{-11}~10^{-7}Hz,
LISA: 10^{-5}~1Hz, LIGO: 10~10^{4}Hz

Fully NL and exact cosmological pert.

1. Multi-component fluids
2. Minimally coupled scalar fields
3. 1PN hydrodynamics
4. Special Relativistic Hydrodynamics with gravity
5. Now including TT, most general!

Future extentions

1. Special Relativistic Magneto-hydrodynamics with gravity
2. Special Relativistic Hydrodynamics with 1PN gravity
3. Light propagation (geodesic, Boltzmann)
4. Higher order PN equations
5. Gauge-invariant combinations

Applications

1. Fitting and Averaging
2. Backreaction
3. Relativistic (cosmological) numerical simulation