Stochastic background of gravitational waves: analytic description of its anisotropies

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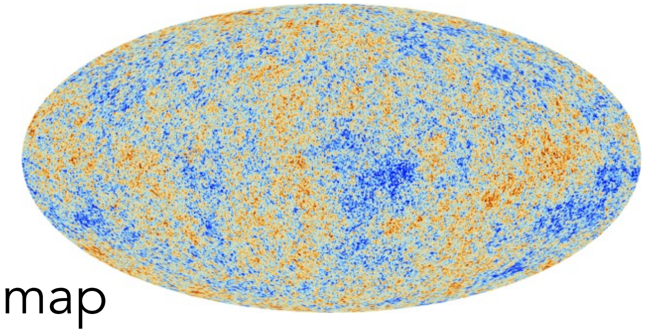
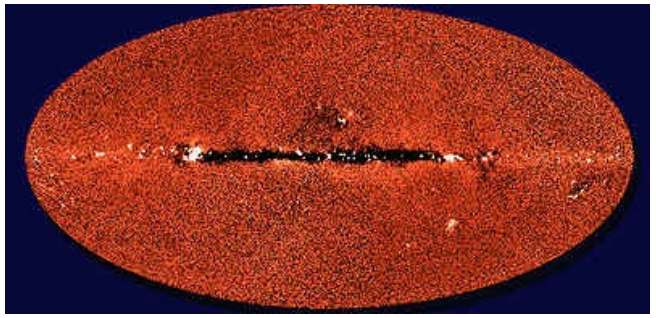
Stochastic backgrounds of radiation

Stochastic backgrounds of radiation: incoherent superposition of radiation emitted by unresolved sources

	cosmological origin	astrophysical origin
EM radiation	CMB	IR extragalactic background
GW radiation	cosmological background	**astrophysical background**



Plank CMB map

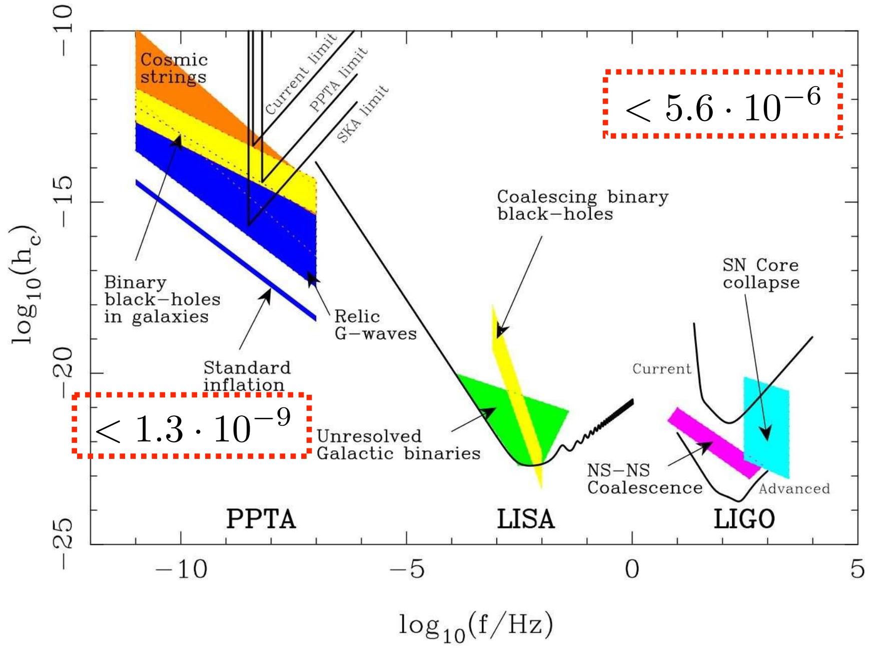


Plank IR map

Astrophysical background: status of the art

Observations

bounds on Ω_{GW} integrated over directions



… Ω_{GW} sky map (persistent sources)

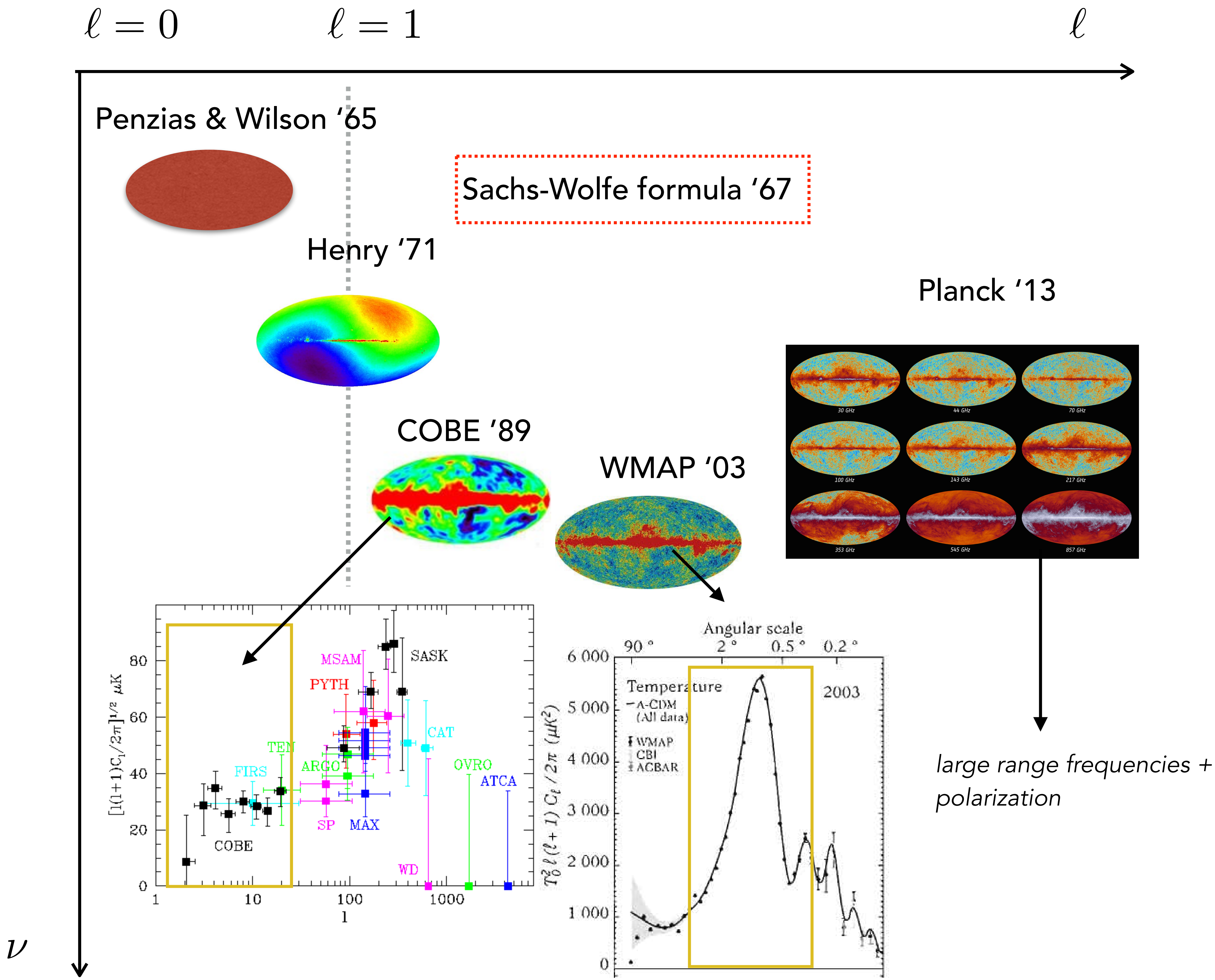
Theory

Ω_{GW} computed in FLRW

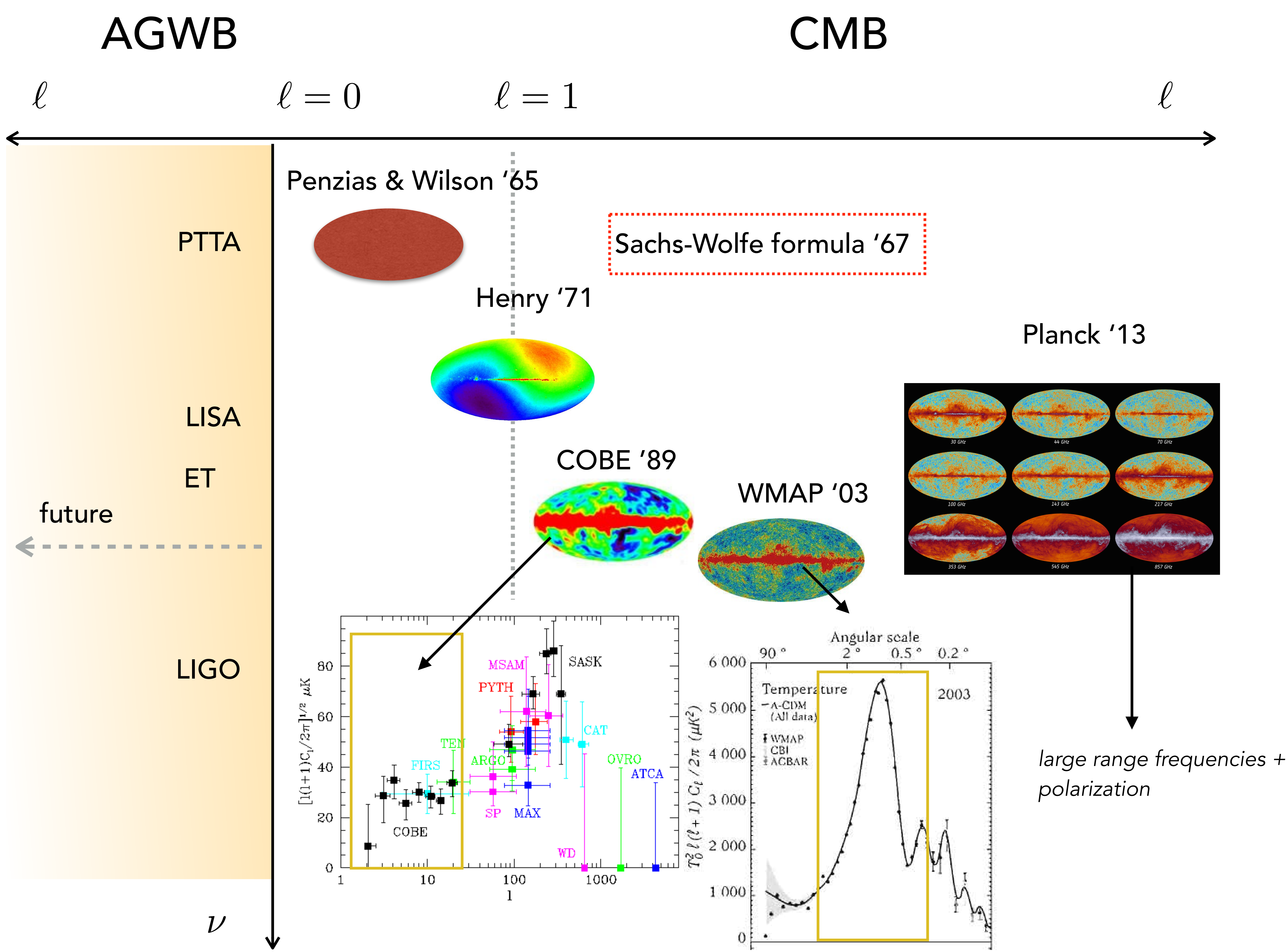
we go beyond this assumption!

Our goal: compute the total flux of GW from astrophysical sources, received per units of frequency and solid angle around the direction of observation

CMB



$\ell = 0$
$\ell = 1$
ℓ
ν
Penzias & Wilson '65
Sachs-Wolfe formula '67
Henry '71
COBE '89
WMAP '03
Planck '13
large range frequencies + polarization
COBE
SASK
MSAM
PYTH
CAT
TEN
FIRS
ARGO
OVRO
ATCA
SP
MAX
WD
$[l(l+1)C_l/2\pi]^{1/2}$ μK
l
Angular scale
Temperature
Λ-CDM
(All data)
WMAP
CBI
ACBAR
2003
$T_0^2\,\ell(\ell+1)\,C_\ell\,/\,2\pi\ (\mu K^2)$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What we can learn

We calculate the **angular power spectrum** of the GW stochastic background and its **correlations** with lensing and galaxy distribution

Two types of information:

(1) cosmology: cosmic structure at large scales, spacetime geometry…

(2) local physics: GW production, star formation rate, distribution sources…

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we learn mostly about the distribution of GW sources

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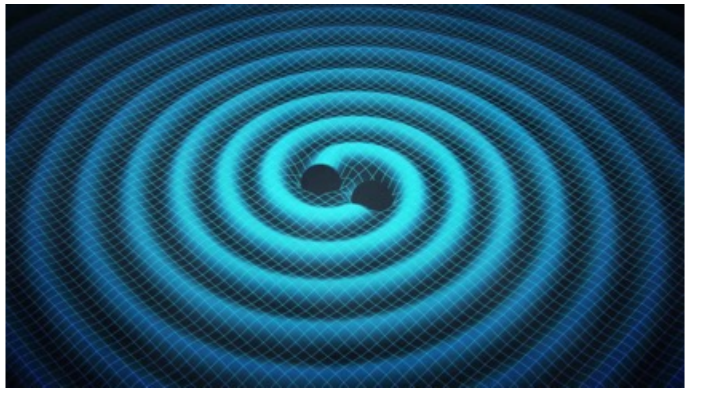
new cosmological observable robust to local physics unknown

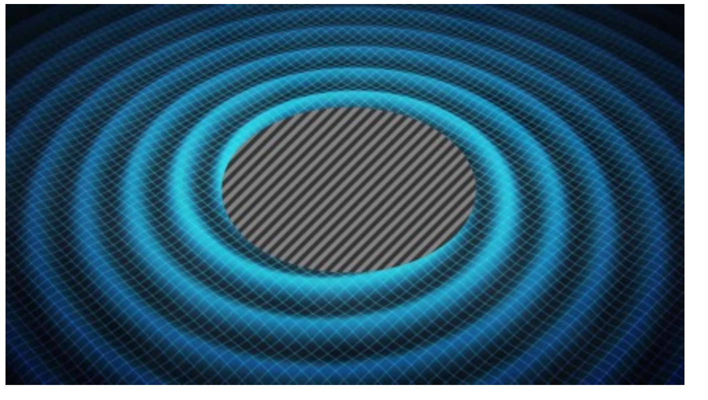
Unresolved sources of GW

We distinguish the following **sources of GW** (inside a galaxy), depending on the characteristic time of GW emission with respect to the life time system

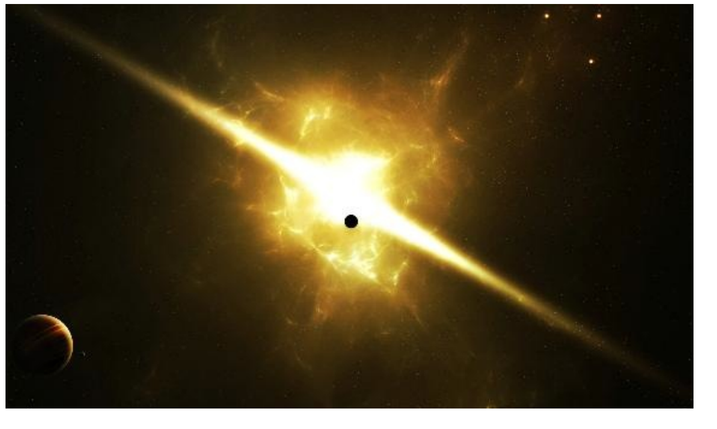
(1) binary inspiralling objects



(2) binary merging objects



(3) exploding supernovae



Scheme of our approach

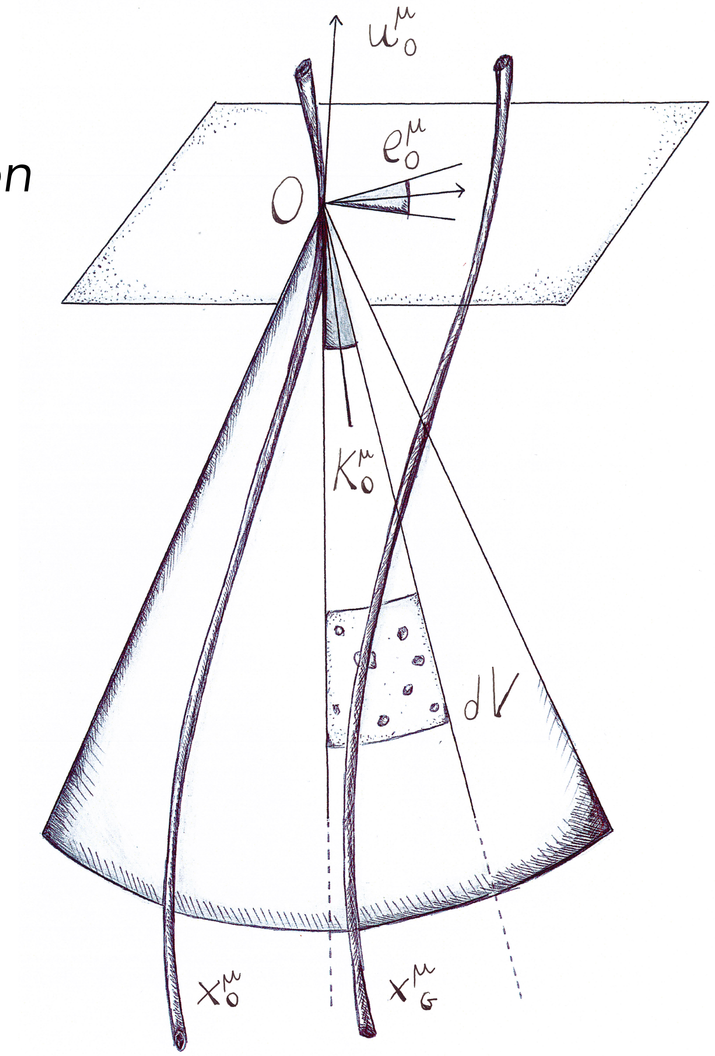
We consider the flux of GW from a galaxy seen in direction e^μ_O at redshift z_G.

4-velocity observer u^μ_O

direction observation e^μ_O

wave vector graviton $\frac{dx^\mu}{d\lambda} \equiv k^\mu = E(u^\mu - e^\mu)$

3+1 decomposition



e^μ_O in solid angle $d\Omega_O$ defines a null bundle along the past light cone

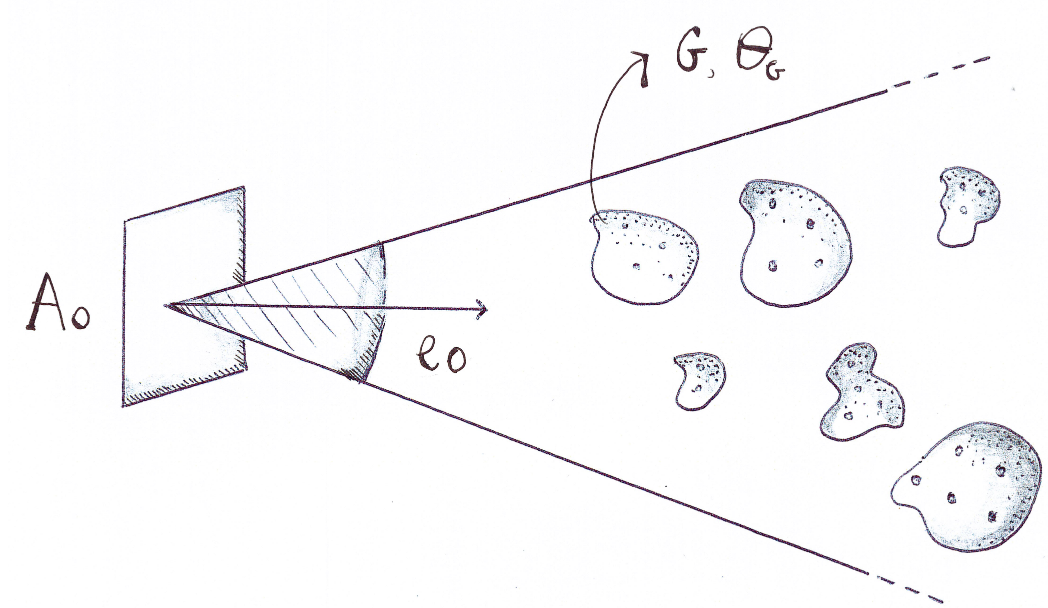
dV is 3D physical volume element given by the intersection of 4-volume with the observer past light cone

Some observational issues

Galaxy G, at z_G and observed in e_O. Associated flux:

$$\Phi(e_O, z_G, \theta_G) \equiv \frac{\text{Energy}}{A_O \Delta t_O}$$

θ_G parameters describing G (mass, metallicity…)

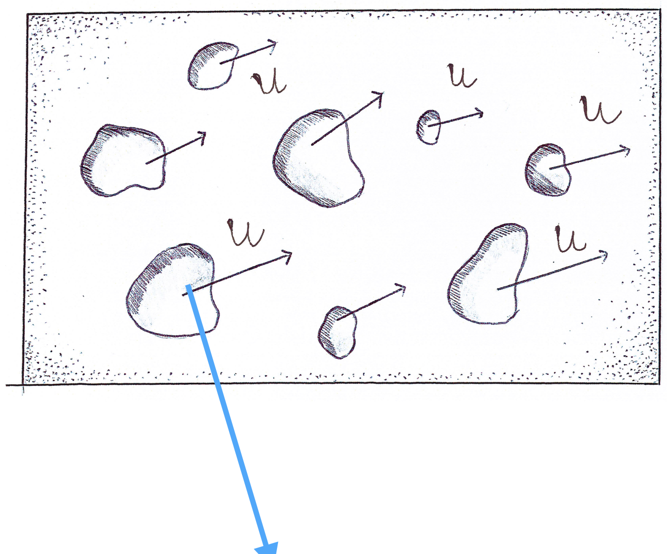


$$\int d\nu_O \Phi(e_O, \nu_O, z_G, \theta_G) \equiv \Phi(e_O, z_G, \theta_G)$$

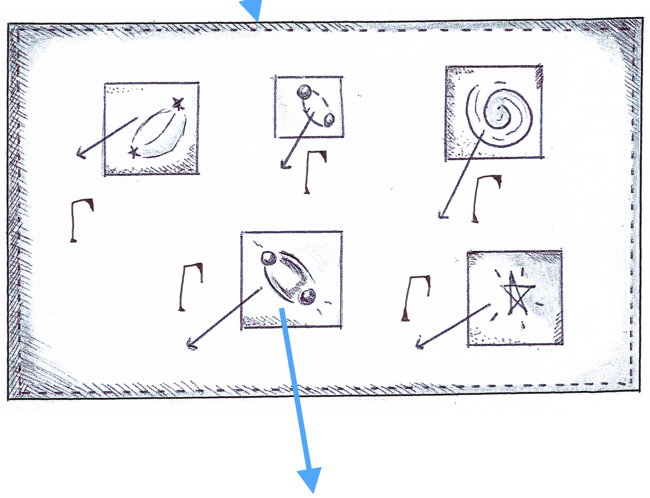
specific flux

To find the total flux received, we need to sum the contributions from all the galaxies in the solid angle $d\Omega_O$

Three scales in our problem



(1) **cosmological scale**. Galaxies: point-like sources moving with the cosmic flow



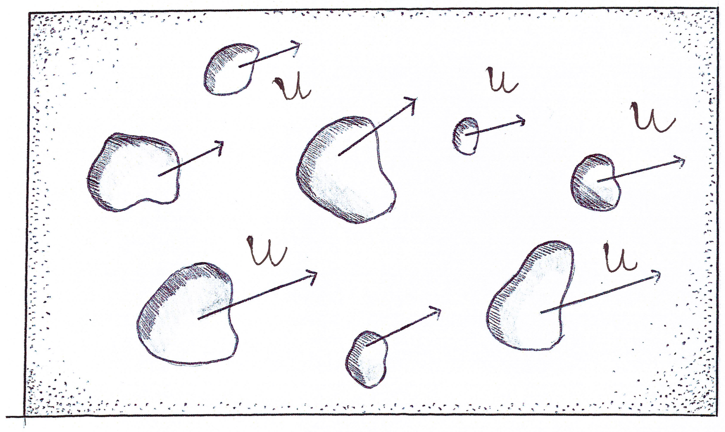
(2) **galactic scale**. A source -i inside a galaxy is characterized by parameters $\theta^{(i)}$ and is moving with velocity $\mathbf{\Gamma}$. Effective luminosity of a galaxy defined taking into account the various contributions of the sources

(3) **local scale**. Scale of single sources emitting GW inside a galaxy

(1) Cosmological scale

$$\int_0^\infty \mathcal{L}_G(\nu_G, \theta_G) d\nu_G = L_G(\theta_G)$$ effective luminosity

$$\nu_G = (1 + z_G)\nu_O$$ effective frequency



The observed flux from a galaxy, is related to its effective luminosity through

$$\Phi(z_G, e_O, \theta_G) \equiv \frac{1}{4\pi D_L^2(z_G, e_O)} L_G(\theta_G)$$

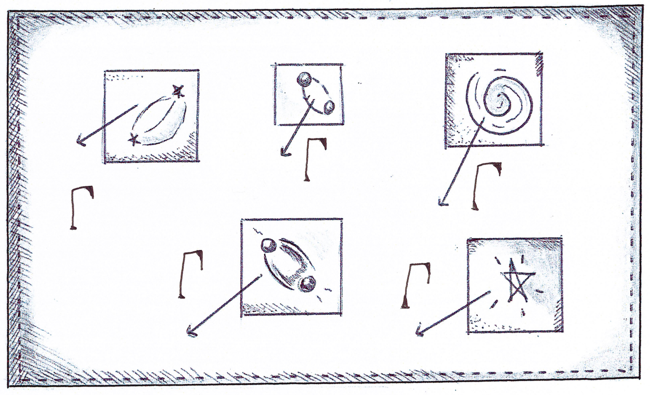
$$\Phi_\nu(z_G, e_O, \nu_O, \theta_G) d\nu_O \equiv \frac{(1 + z_G)}{4\pi D_L^2(z_G, e_O)} \mathcal{L}_G(\nu_G, \theta_G) d\nu_O$$

$$d\nu_G = (1 + z_G) d\nu_O$$

(2) Galactic scale

$\mathbf{\Gamma} = \mathbf{\Gamma}(\theta^{(i)}, \theta_G)$ with distribution function $f(\mathbf{\Gamma}, \theta_G)$

with $\int d^3\Gamma f(\mathbf{\Gamma}, \theta_G) = 1$



galaxy rest frame

Effective luminosity (per unit of effective frequency)

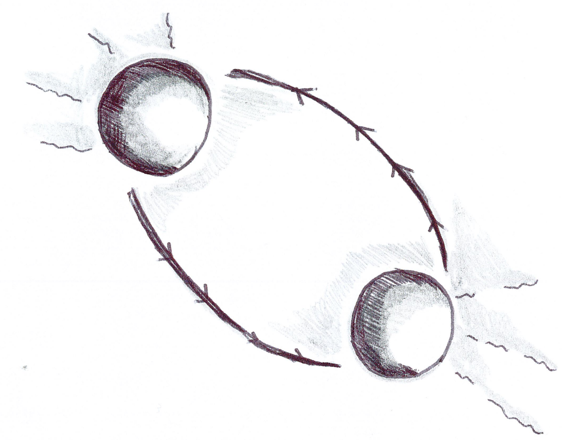
$$\mathcal{L}_G(\theta_G, \nu_G) = \mathcal{L}_G^I(\theta_G, \nu_G) + \mathcal{L}_G^M(\theta_G, \nu_G) + \mathcal{L}_G^{SN}(\theta_G, \nu_G)$$

$$\mathcal{L}_G^I(\theta_G, \nu_G) = \sum_{(i)}^{I} \int d\theta^{(i)} \mathcal{N}^{(i)}(\theta^{(i)}, \theta_G) \int d^3\mathbf{\Gamma}\, f(\mathbf{\Gamma}, \theta_G) \frac{dE_G^{(i)}}{dt_G d\nu_G}(\nu_G, \mathbf{\Gamma}, \theta_G)$$

$$\mathcal{L}_G^{M,SN}(\theta_G, \nu_G) = \sum_{(i)}^{M,SN} \int d\theta^{(i)} \frac{d\mathcal{N}^{(i)}}{dt_G}(\theta^{(i)}, \theta_G) \int d^3\mathbf{\Gamma}\, f(\mathbf{\Gamma}, \theta_G) \frac{dE_G^{(i)}}{d\nu_G}(\nu_G, \mathbf{\Gamma}, \theta_G)$$

(3) Local scale

e.g. BH+BH

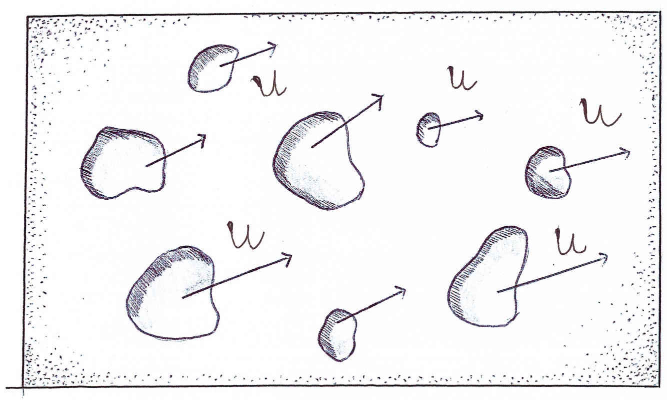


Spectra and powers in the galaxy rest frame are obtained Lorentz transforming the results in the literature (which are computed in the source local frame)

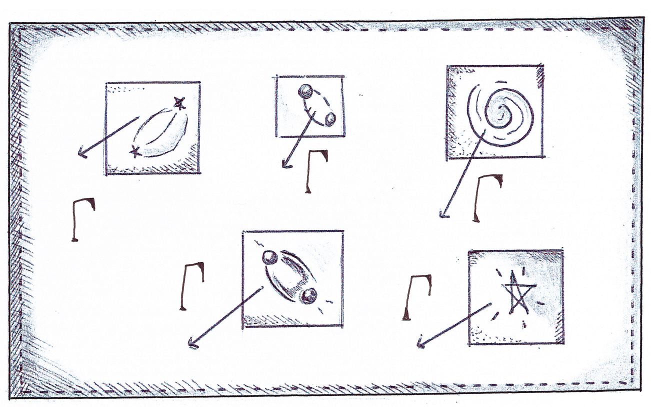
$$e.g. \quad \left.\frac{dE}{dt\, d\nu}\right|_G = boost\left(\left.\frac{dE}{dt\, d\nu}\right|_{\rm loc}, \mathbf{\Gamma}\right)$$

persistent background

Summary: from cosmological to local scale



cosmological scale



$$\Phi = \frac{(1+z_G)}{D_L^2} \boxed{\mathcal{L}_G}$$

function local
quantities at sources

local scale

Final parametrization of ρ_{GW}

$$\frac{d^2\rho_{GW}}{d\nu_O d\Omega_O}(\nu_O, e_O) = \int dz_G \int d\theta_G \, \Phi(z_G, \nu_O, \theta_G) \, \frac{d^2\mathcal{N}_G}{dz_G \, d\Omega_O}(z_G, \theta_G)$$

integration over redshift

change variable integration

$z_G \to \lambda$

integration over parameters galaxy

number of galaxies per solid angle per redshift bin

$$d\mathcal{N}_G(x^\mu(\lambda), \theta_G) \equiv n_G(x^\mu(\lambda), \theta_G) dV(x^\mu(\lambda))$$

$$dV(x^\mu(\lambda)) = d\Omega_O D_A^2(\lambda) \sqrt{p_\mu(\lambda) p^\mu(\lambda)} d\lambda$$

physical surface

spatial displacement

…covariant: valid in a generic cosmology!

FLRW with perturbations

We specialize to homogeneous & isotropic universe + scalar perturbations

$$ds^2 = a^2 \left[-(1+2\Psi) d\eta^2 + (1-2\Phi)\delta_{ij} dx^i dx^j \right] \qquad (u^\mu) \equiv \frac{1}{a}(1-\Psi\,, v^i)$$

We need to consider perturbations of the following quantities:

redshift perturbation $\quad z_G \equiv \bar{z} + \delta z$

density perturbation $\quad n_G(x^\mu) = \bar{n}_G(\bar{x}^\mu) + \delta x^\mu \nabla_\mu \bar{n}_G(\bar{x}^\mu) + \delta n_G(\bar{x}^\mu)$, with $\delta n_G(\bar{x}^\mu) \rightarrow b\,\delta_{CDM}(\bar{x}^\mu)$

perturbation spatial displacement $\quad p^\mu = \bar{p}^\mu + \delta p^\mu$

Results in cosmological context

$$\frac{d^2\rho_{GW}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O) = \frac{d^2\rho_{\overline{GW}}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \nu_O) + \boxed{\mathcal{E}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O)}$$

$$\frac{1}{4\pi H_0}\int d\bar{z}\frac{1}{E(\bar{z})}\frac{1}{(1+\bar{z})^4}\int d\theta_G\, \bar{n}_G(\bar{z}, \theta_G)\mathcal{L}_G(\nu_G, \theta_G)$$

background

first order perturbation. **Angular dependence!**

Results in cosmological context

$$\frac{d^2\rho_{GW}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O) = \frac{d^2\rho_{\overline{GW}}}{d\nu_O d\Omega_O}(\eta_O, \mathbf{x}_O, \nu_O) + \boxed{\mathcal{E}(\eta_O, \mathbf{x}_O, \mathbf{e}, \nu_O)}$$

[cosmological contribution]

$$\frac{1}{4\pi}\int d\eta\, a^4 \int d\theta_G\, \bar{n}_G(\eta, \theta_G)\, \mathcal{L}_G(\nu_G, \theta_G) \left[b\delta_{\rm CDM} + 4\Psi - 2\mathbf{e}\cdot\nabla v - 6\int d\eta' \dot{\Psi} \right]$$

[local physics contribution]

Doppler

contribution local overdensity

integrated contribution

$\sim$ **CMB temperature** in perturbed universe (SW, integrated SW and Doppler terms…)

In progress…

Our results have a **cosmological part** and a **local part**

we already have it

we are working it out in detail, for different source types

numerical prediction for the angular power spectrum of Ω_{GW} to test vs observations!

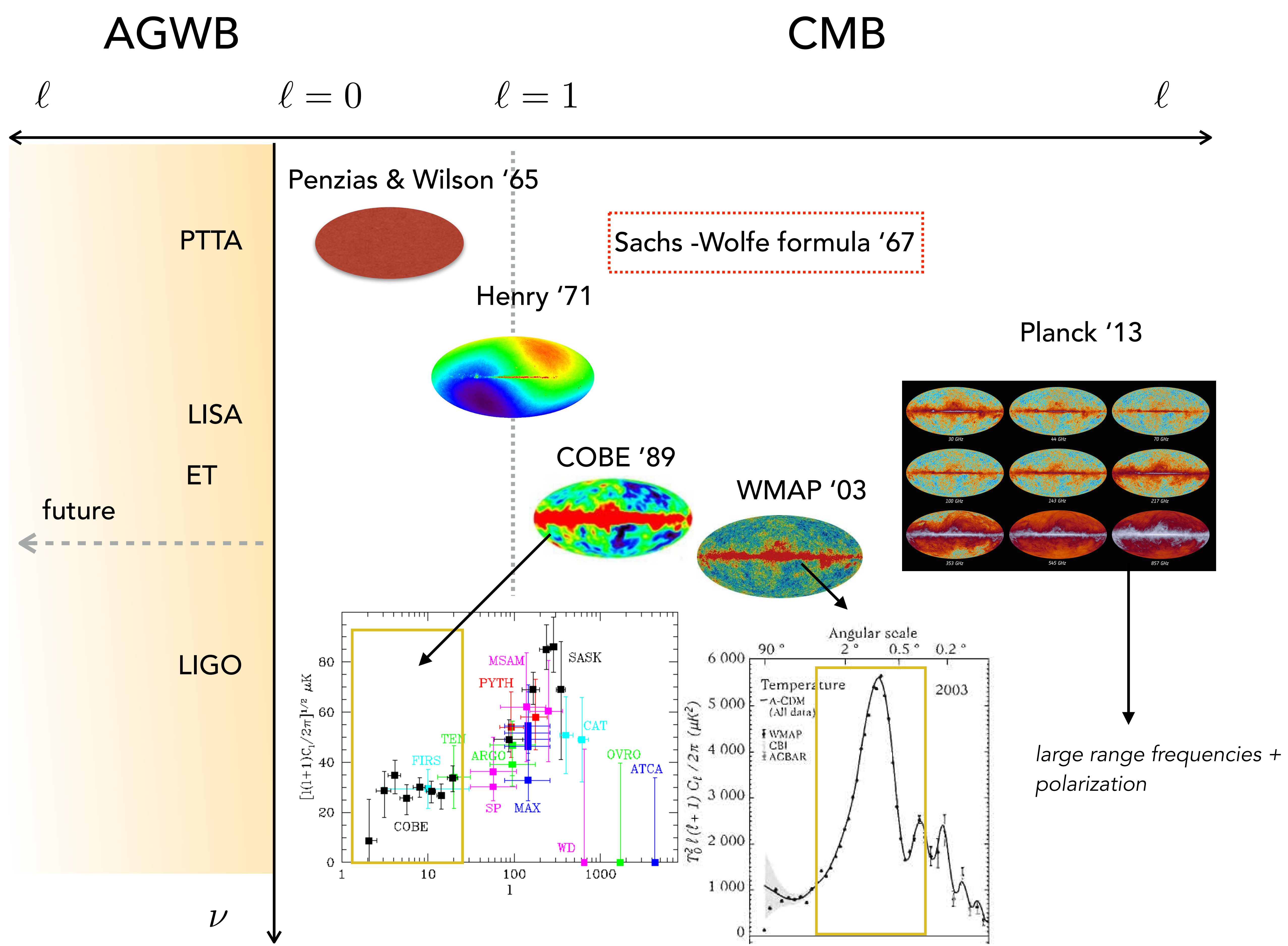
Impact of our work, how to continue it

We have proposed an analytic framework to **study the anisotropies** of the astrophysical background of GW

Accurate modeling needed to **disentangle** GW astrophysical background from primordial background and instrumental noise

New generation of GW experiments: astrophysical background of GW has a chance to be detected in the next few years *LIGO coll. PRL 116.131102*

Method developed to **extract angular information** from signal: radiometer method & spherical harmonic decomposition *(see Cornish & Romano review)*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$T_0^2\, \ell(\ell+1)\, C_\ell / 2\pi\ (\mu K^2)$

Thank you

Extracting angular information

Two ways of implementing the maximum likelihood approach for mapping GW power *(see Cornish & Romano)*

Radiometer method & spherical harmonic decomposition method

↓ optimized for a background dominated by point-like sources

↓ diffuse background

LIGO angular searches: 2 directional searches for persistent GW background, using the two methods above

SRN ratio maps obtained from the two methods consistent with detector noise

LISA angular resolution

LISA: all-sky monitor (not pointed instrument)

it measures simultaneously both polarizations of GW: 2 time series of data

Info on the **source position** is **encoded in the signal** in three ways:

(1) relative amplitude and phase of the two polarization components

(2) periodic Doppler shift

(3) modulation of the signal due to detector time-varying orientation

angular resolution $\Delta\Omega_S$ for a given source

(Cutler '97)

In any case, if we just had 3 LISA…

Future detection background

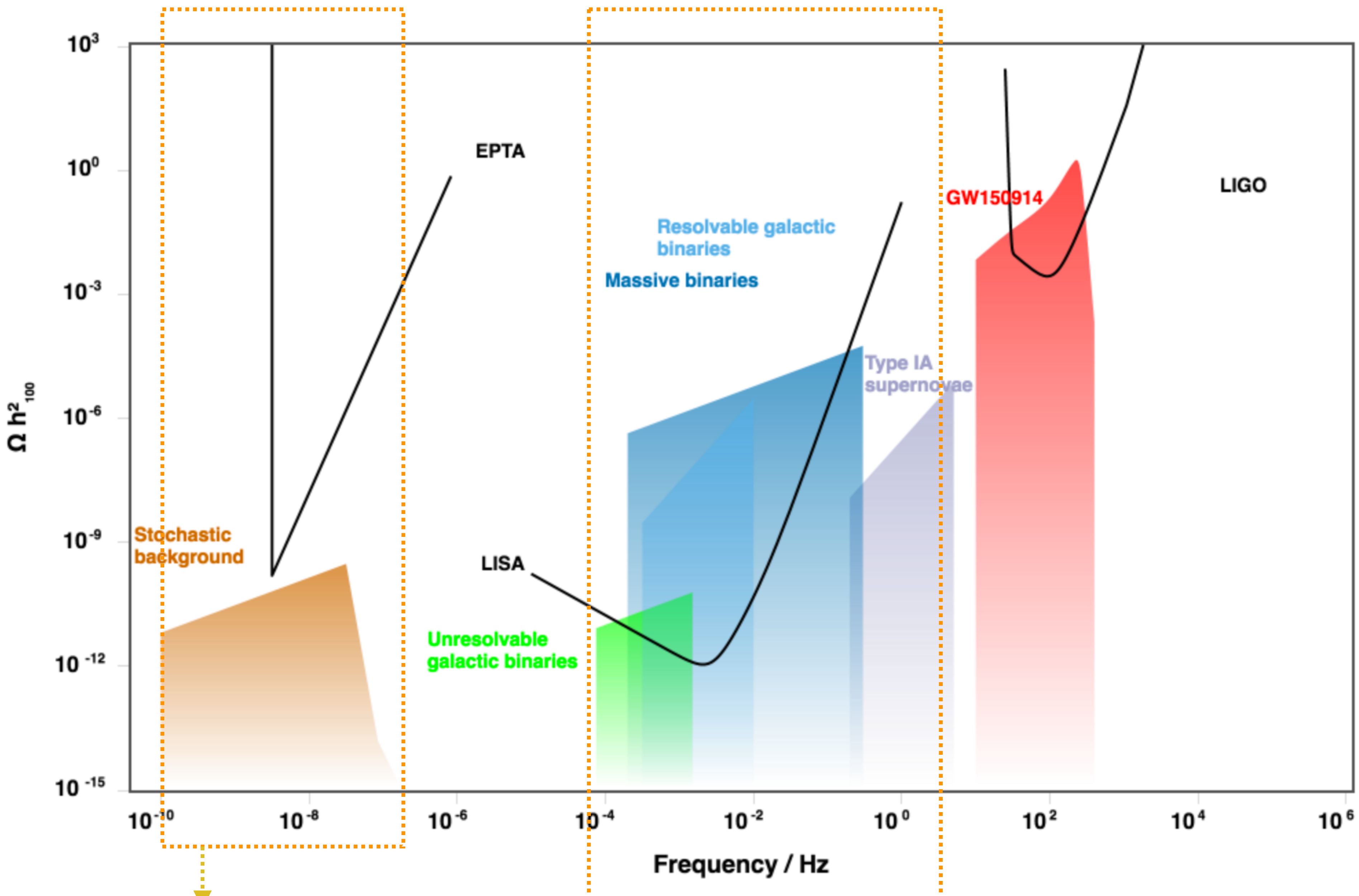
LIGO (PRL 116.131102)

Analysis characteristic event GW150914 —-> prediction for flux GW from BH+BH events

↓

Amplitude detectable by advanced VIRGO & LIGO (even before reaching the final designed sensitivity)

Sensitivity curves and expected fluxes



cosmological background (inflation, cosmic strings…)

expected regime of frequencies and strain for **astrophysical stochastic background**

Final parametrization of ρ_{GW}

After these substitutions, we get

$$\frac{d^2\rho_{\rm GW}}{d\nu_O d\Omega_O}(\nu_O, e_O) = \frac{1}{4\pi}\int d\lambda \int d\theta_G \frac{1}{(1+z_G(\lambda))^3}\sqrt{p_\mu(\lambda)p^\mu(\lambda)}\, n_G(x^\mu(\lambda), \theta_G)\mathcal{L}_G(\nu_G, \theta_G)$$

redshift

spatial displacement

galaxy density

effective luminosity of a galaxy

…covariant: valid in a generic cosmology!