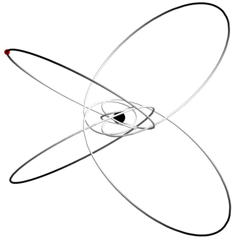
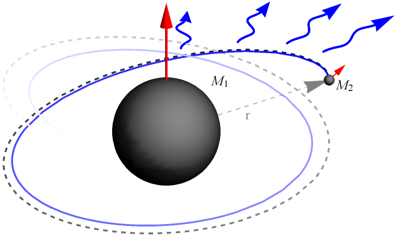
Modelling EMRIs using Self-Force

Maarten van de Meent

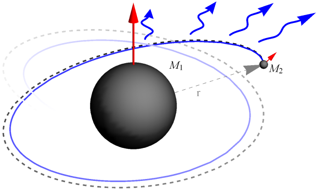
Albert Einstein Institute, Golm



The Era of Gravitational-Wave Astronomy
XXXIIIth international Colloquium of the IAP, Paris, 28 June 2017



Introduction: the challenge of modelling EMRI evolution



Extreme mass ratio inpiral (EMRI)

binary BH with $\mu = M_2/M_1 \ll 1$

Typical (LISA) source [Babak et al., 2017]

$M_1 \sim 10^{5-6} M_\odot$	$M_2 \sim 10 - 30 M_\odot$
$a_1 \sim 0.98 M_\odot$	$z \sim 2 - 3$
$e \sim 0 - 0.2$	generic inclination

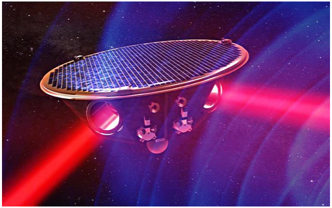
Event rate [Babak et al., 2017]

Expect ~ 1 per MYr per galaxy.
$1 - 5000$ detectable LISA events per year.

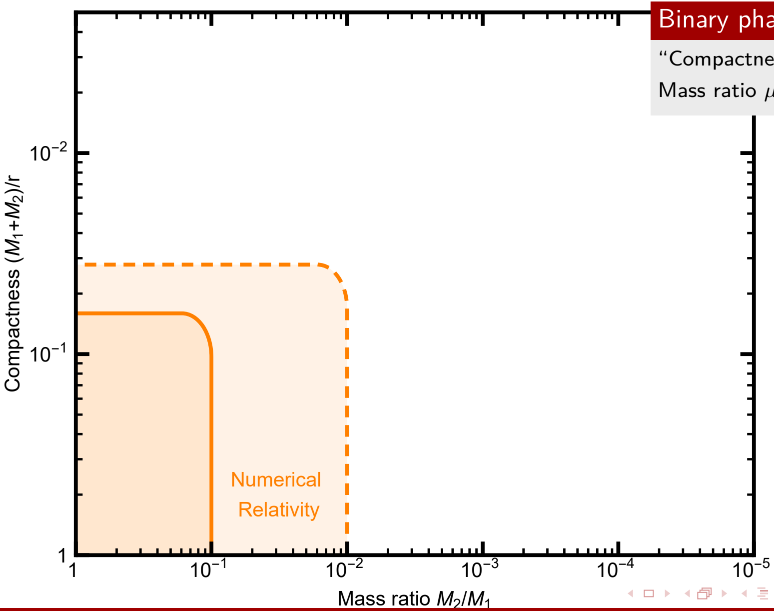
High precision measurements

Quantity	accuracy
M_1, M_2, a_1, e, ι	10^{-5}
quadrupole	10^{-3}
position	~ 2 sq. deg.
distance	3%

[Babak et al., 2017]



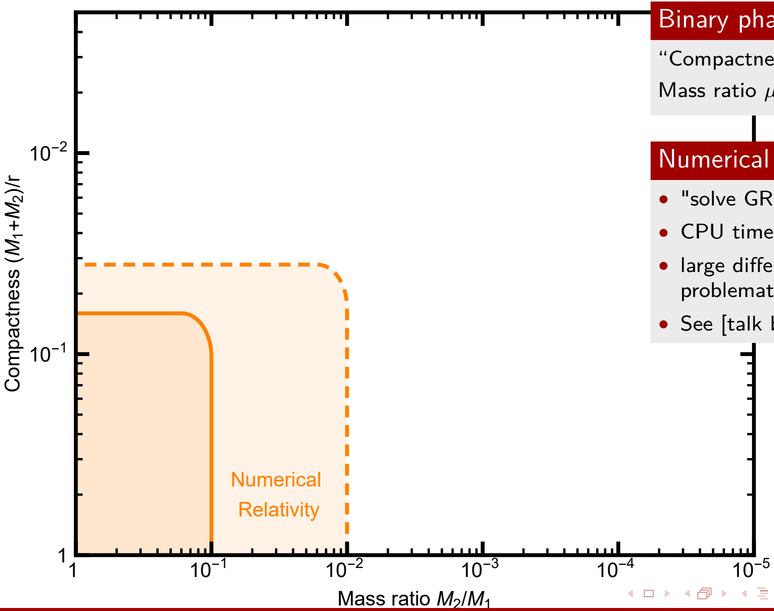
Modelling BH binaries



Binary phase space

"Compactness": $\frac{M_1+M_2}{r}$

Mass ratio $\mu = \frac{M_2}{M_1}$



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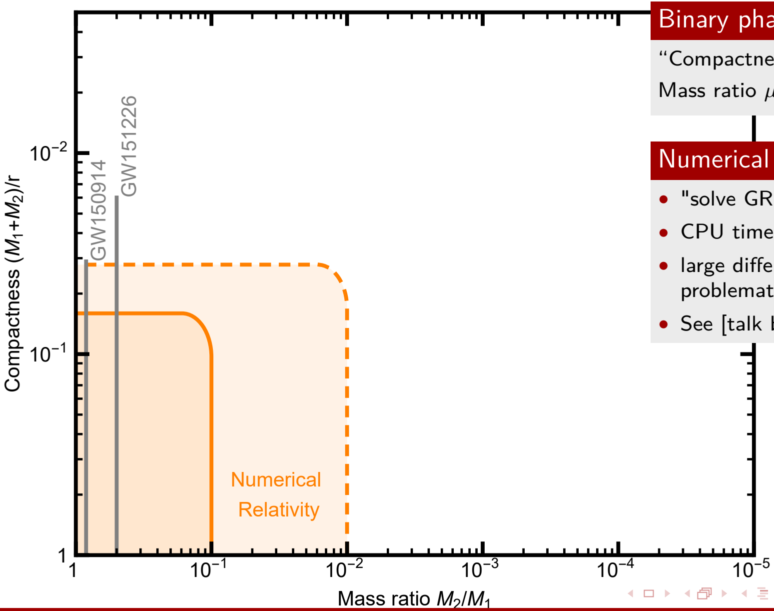
Numerical relativity

- "solve GR on a grid"
- CPU time intensive
- large difference in scale problematic
- See [talk by Kidder]



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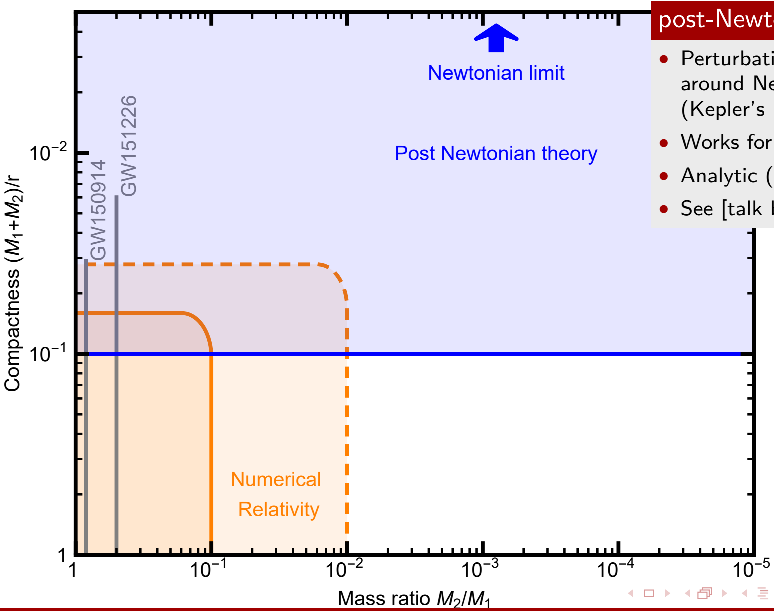
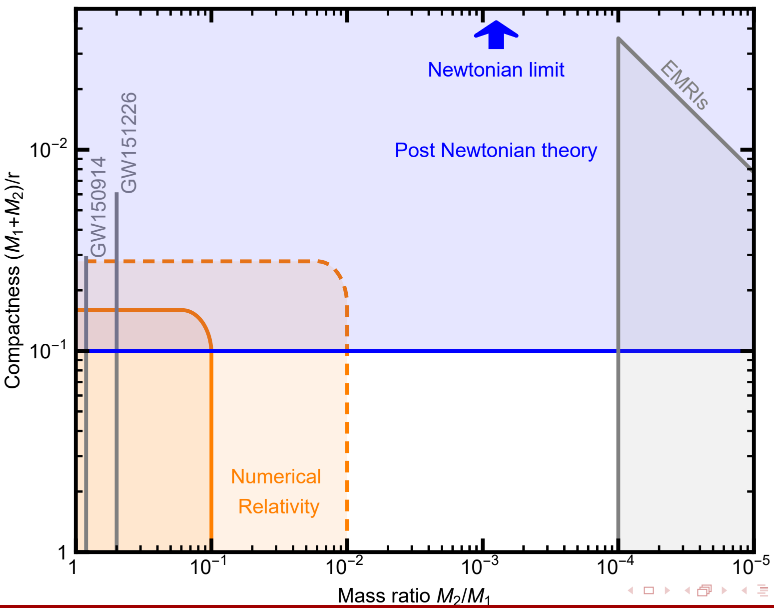


post-Newtonian

- Perturbative expension around Newtonian limit. (Kepler's laws)
- Works for large separations
- Analytic (fast!)
- See [talk by Blanchet]

Modelling BH binaries



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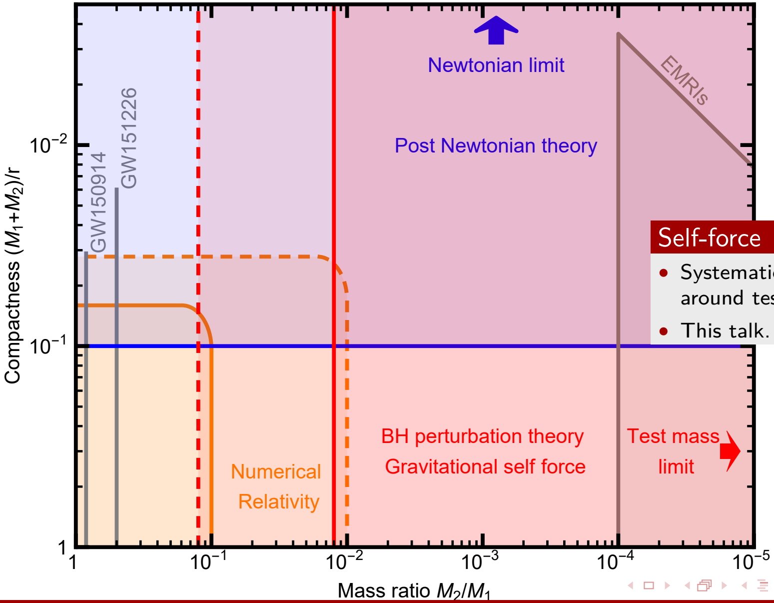


Self-force

- Systematic expansion in μ around test particle limit.
- This talk.

The "Capra" programme

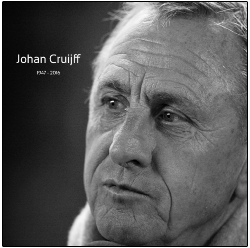
20th Capra meeting (2017) participants

Goals

- model EMRI evolution accurate to $\lesssim .1$ radian over $\sim 10^5$ orbits.
- include general eccentricities
- include general inclination
- include effects of spin on primary object
- include effects of spin on secondary object

Strategy: perturbation theory

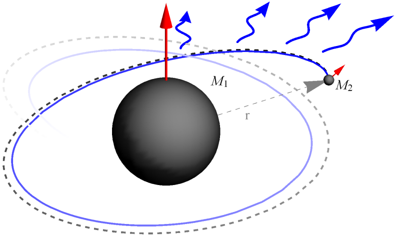
"Elk nadeel hep z'n voordeel."
[Johan Cruijff]



Strategy

Use the smallness of the mass-ratio $\mu := \frac{M_2}{M_1}$ to our advantage and use it as an expansion parameter using:

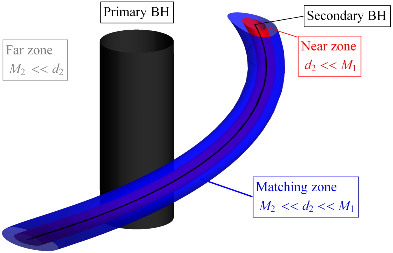
- Black hole perturbation theory
- Multi length scale analysis (matched asymptotics)
- Multi time scale analysis



Theory: perturbative expansion

Equations of Motion: Matched asymptotic expansions

[Mino, Sasaki & Tanaka, 1997] [Poisson, 2003][Pound, 2008-]



far zone

Kerr geometry of primary plus perturbation generated by secondary.

near zone

Kerr geometry of secondary (in rest frame) plus perturbation generated by primary.

Summary of results from matching:

- 0th order: Secondary follows geodesic in Kerr background generated by primary.
- 1st order: Motion of secondary is corrected by effective force term (the Gravitational Self Force) obtained from retarded metric perturbation generated by a point particle with mass M_2.
- Equivalently, (Detweiler-Whiting): Secondary follows geodesic in some effective perturbed vacuum spacetime.
- Similar results are obtained at higher-order.

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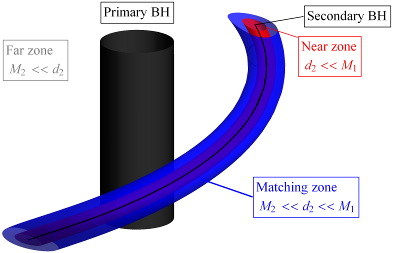

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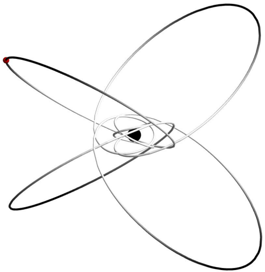


Constants of Motion

Geodesics in Kerr spacetime are characterized by three constants of motion: [Carter, 1968]

1. Energy, E
2. Angular momentum, L_z
3. Carter constant, Q, (related to total angular momentum)

Orbital phases

Position along the orbit is described by three independently evolving phases:

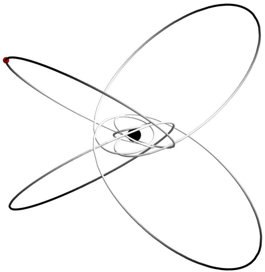
1. q_ϕ: related to azimuthal position
2. q_r: related to radial motion
3. q_z: related to oscillations around equator

Analytic solutions

Analytic solutions are available:

- [Fujita&Hikida, 2009]
- [Hackmann et al., 2008,2010]

Zeroth order: Geodesics in Kerr spacetime



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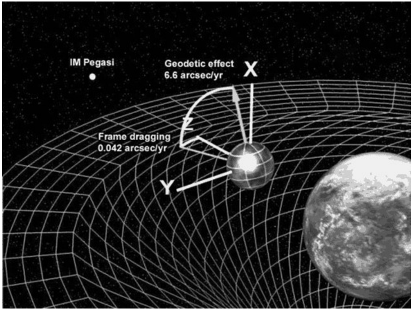
Zeroth order: Motion of secondary spin

Motion of test spin

- de Sitter precession (geodetic effect)
- Lense-Thirring effect



Parallel transport

- Motion of test spin is governed by parallel transport.
- Analytic solution in terms of generic orbit is known. [Marck, 1983]

Equation of Motion; order reduction

$$\frac{D^2}{d\tau^2}x^\alpha = \mu F_1^\alpha(\gamma_\tau;\tau) + \mu^2 F_2^\alpha(\gamma_\tau;\tau) + \mathcal{O}(\mu^3)$$

Action-angle variables

$$\dot{q}^i = \Omega^i(\mathbf{J}) + \mu g_1^i(\mathbf{J},\mathbf{q}) + \mu^2 g_2^i(\mathbf{J},\mathbf{q}) + \mathcal{O}(\mu^3)$$
$$\dot{J}_i = 0 + \mu G_j^1(\mathbf{J},\mathbf{q}) + \mu^2 G_j^2(\mathbf{J},\mathbf{q}) + \mathcal{O}(\mu^3)$$

Inspiral Evolution 1:

Equation of Motion; order reduction

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Inspiral Evolution 2:

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Near identity averaging transform

$$\tilde{q}^i = q^i + \mu X_1^i(\vec{J},\vec{q}) + \mu^2 X_1^i(\vec{J},\vec{q}) + \mathcal{O}(\mu^3)$$
$$\tilde{J}_j = J_j + \mu Y_j^1(\vec{J},\vec{q}) + \mu^2 Y_j^2(\vec{J},\vec{q}) + \mathcal{O}(\mu^3)$$

Averaged equations [Hinderer&Flanagan,2006,2012][MvdM, 2013]

$$\dot{\tilde{q}}^i = \Omega^i(\vec{\tilde{J}})$$
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Averaged equations (if $\vec{k}\cdot\vec{\Omega} \neq 0$) [Hinderer&Flanagan,2006,2012][MvdM, 2013]

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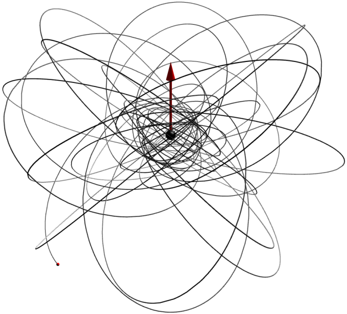
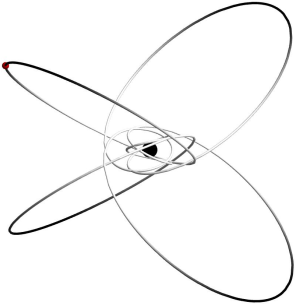
Resonant orbits

Generic



vs

Resonant



Resonances $\vec{k} \cdot \vec{\Omega} = 0$

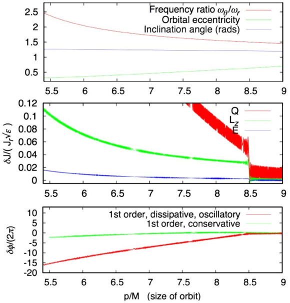
- Phase synchronization allows coherent build up of otherwise oscillatory effects.
- Resonances involving just 2 phases occur generically in EMRIs in LISA band.

Effects of Resonances

rz-resonances

$$\mu^{\frac{3}{2}} \sum_{\vec{k}} G_j^{res}[\langle \vec{G}^1 \rangle](\vec{J}, \vec{k} \cdot \vec{q})\delta(\vec{k} \cdot \vec{\Omega})$$

- Coherent build of oscillatory effects leads to jumps in constants of motion.[Flanagan& Hinderer, 2012]
- Jump is sensitive to resonant phase, $\vec{k} \cdot \vec{q}$.
- Can be obtained from averaged fluxes on resonant geodesics.[MvdM, 2013]
- "Resonant locking" unlikely.[MvdM, 2013]



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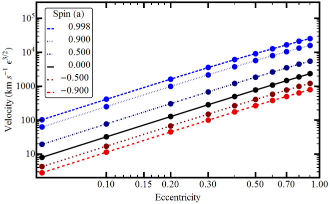

[MvdM, 2014]

$r\phi$- and $z\phi$- resonances[Hirata, 2012][MvdM, 2014]

Resonances involving ϕ motion:

- Cannot affect evolution of "intrinsic" orbital parameters.
- Can affect "extrinsic" parameters of EMRI systems such as CoM velocity ("Kicks")

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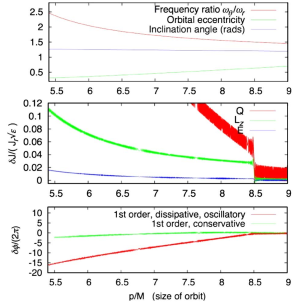

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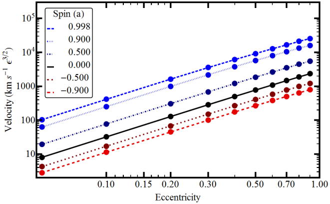

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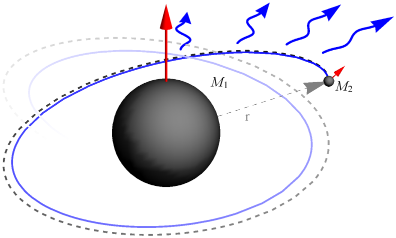

Numerical calculation

Adiabatic approximation

$$\dot{J}_j = \mu \langle G^1_j \rangle (\mathbf{J}) + \mathcal{O}(\mu^{3/2})$$

Averaged fluxes $\langle G^1_j \rangle (\mathbf{J})$

The (long term) average rate of change of the constants of motion can be obtained from the GW flux towards infinity and into the primary black hole.

- $\langle \dot{E} \rangle$ from the energy flux.
- $\langle \dot{L}_z \rangle$ from the angular momentum flux.
- $\langle \dot{Q} \rangle$ see [Sago et al., 2006].

State-of-the-art

- Flux calculations sourced by generic orbits in Kerr spacetime. [Drasco & Hughes, 2006][Fujita, Hikida & Tagoshi, 2009].

To Do:

- Fill orbital parameter space with numerical flux data (and find suitable interpolation/surrogate).

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g_1,G^1

1. (first order) Gravitational Self Force (independent of secondary spin)
2. spin-force (independent of self-field)

$\langle G^2\rangle$ second order "flux"

1. Correction to 1st order flux due to secondary spin.
2. Correction to 1st order flux due to inspiral deviation from geodesic.
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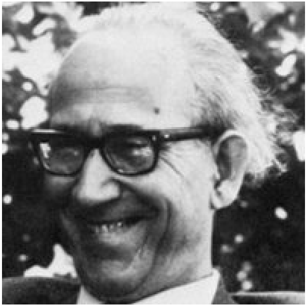
Spin force

Myron Mathisson



Achilles Papapetrou

Mathisson-Papapetrou spin-force

- force induced on geodesic by the presence of spin on test object.
- first order correction in μ (linear in a_2!)
- first derived by Papapetrou [Papapetrou, 1951].
- analytic expression in terms of position, velocity, and spin-vector.
- spin supplementary condition follows from asymptotic match procedure.

Gravitational self-force: formalism

MiSaTaQuWa formula

[Mino,Sasaki&Tanaka,1996][Quinn&Wald,1996]

$$\frac{D^2}{d\tau^2}x^\alpha = \mu F^\alpha[h^R]$$

h^R is "regular" part of (retarded) metric perturbation produced by point particle.

Methods for obtaining regular part

1. Mode-sum regularization [Barack&Ori,2001]
2. Effective source methods [Barack&Golbourn,2008]
3. Green's function methods [Mino, Sasaki & Tanaka, 1996]

Calculating the GSF: Schwarzschild

Time domain

- Decompose field equations in spherical harmonics.
- Numerically solve system of $1+1$D PDEs on a grid.
- [Barack, Lousto, Sago]
- 2+1D and 3+1D methods also explored

Frequency domain

- Further decompose equations in Fourier modes.
- Numerically solve system of ODEs.
- [Barack, Burko, Detweiler, Warburton, Akcay, Kavanagh, Ottewill, Evans, Hopper, ...]

State-of-the-art

- Self-force calculations using a wide variety of methods (Time domain, frequency domain, mode-sum, effective source, etc.)
- eccentricities up to $\lesssim 0.8$. [Osburn, Warburton& Evans, 2016]

Calculating the GSF: Kerr

The problem with Kerr

No spherical symmetry. Field equations do not decouple in "spherical" harmonics.

Time domain

- Decompose field equations in azimuthal m-modes.
- Numerically solve system of $2+1$D PDEs on a grid.
- [Dolan, Wardell, Barack, Thornburg]
- Issues with numerically unstable gauge modes

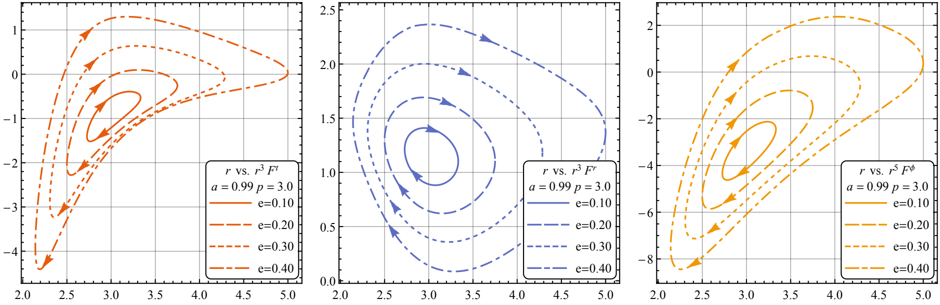
Frequency domain

- Teukolsky equation for Weyl scalars ψ_0 and ψ_4 does decouple in Fourier modes. [Teukolsky,1972]
- Can be solved using semi-analytical methods. [Mano,Suzuki&Tagasugi,1996]
- Metric perturbation can be reconstructed from ψ_0 and ψ_4 in radiation gauge. [Chrzanowski,Cohen,Kegeles, 1970s]
- [Friedman, Keidl, Shah, MvdM, ...]

State-of-the-art

- GSF on eccentric equatorial orbits [MvdM, 2016]
- Generic orbits... (coming soon)

Results: Gravitational self-force



Range of capabilities

- Any value of the spin parameter a.
- Any semilatus rectum p (including fairly high whirl numbers)
- Eccentricities upto $e \lesssim 0.8$
- Equatorial orbits (inclined orbits in the works)

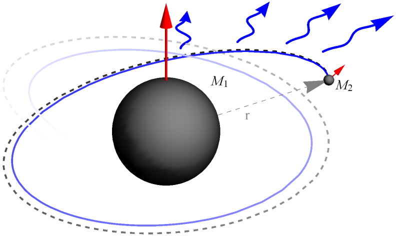
Status of second order calculations

Second order challenge

- Second order GSF essential ingredient for 1PA evolution.
- Technical formalism in place [Pound, Rosenthal, Gralla, Detweiler,...]
- Challenges in “UV”
- Challenges in “IR”

Status

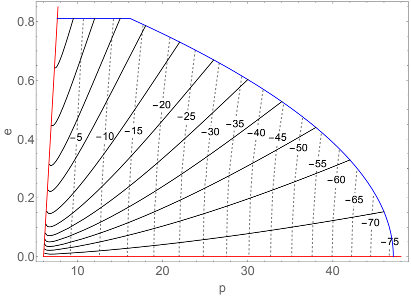
First numerical calculations (Schwarzschild circular orbits) “under evaluation”.

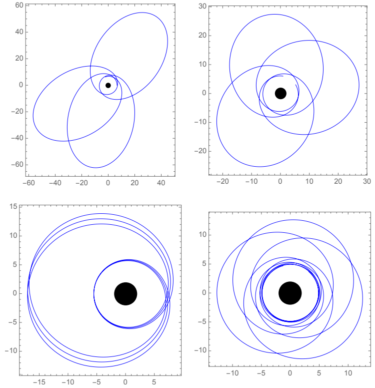


Inspiral evolution

Self-forced inspirals: Schwarzschild

[Warburton, Akcay, Barack, Gair & Sago, 2012] [Osburn, Warburton & Evans, 2016]

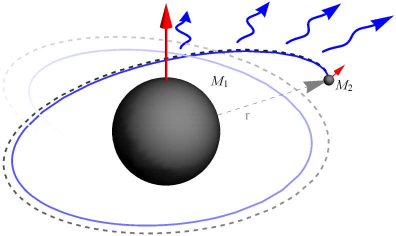




Osculating geodesics

- GSF sourced by instantaneously tangent geodesic.
- No second order GSF included.
- Conservative GSF effects add phase difference of several tens of radians over inspiral.

$\mu = 10^{-5}$
initial data: $p = 12$, $e = 0.81$
2115.5, 500, 100, and 1 day(s) before plunge.



Validation using gauge invariants

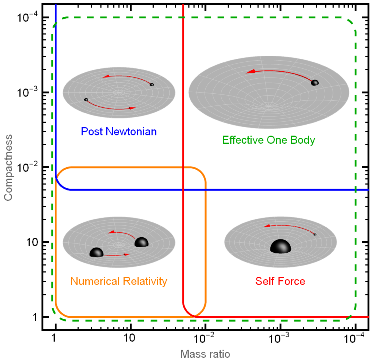
Validation

Gauge dependence

Self-force is fundamentally gauge dependent. Therefore need to calculate invariant quantities for comparison with other methods.

Invariants

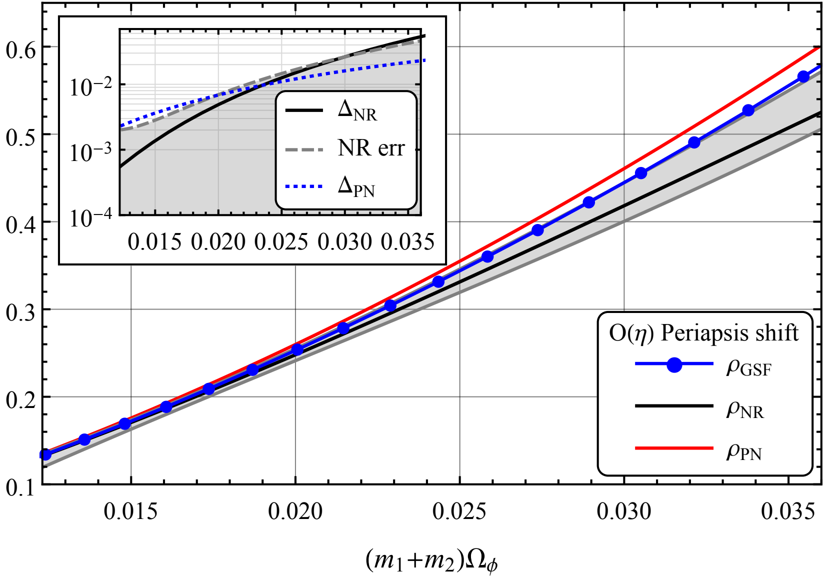
- Energy & angular momentum fluxes
  Kerr, eccentric equatorial
- Detweiler-Barack-Sago redshift
  Kerr, eccentric equatorial
- **Periapsis precession**
  Schwarzschild, Kerr
- **ISCO shift**
  Schwarzschild, Kerr
- Spin precession ("self-torque")
  Schwarzschild eccentric
- Tidal invariants
  Schwarzschild, Circular

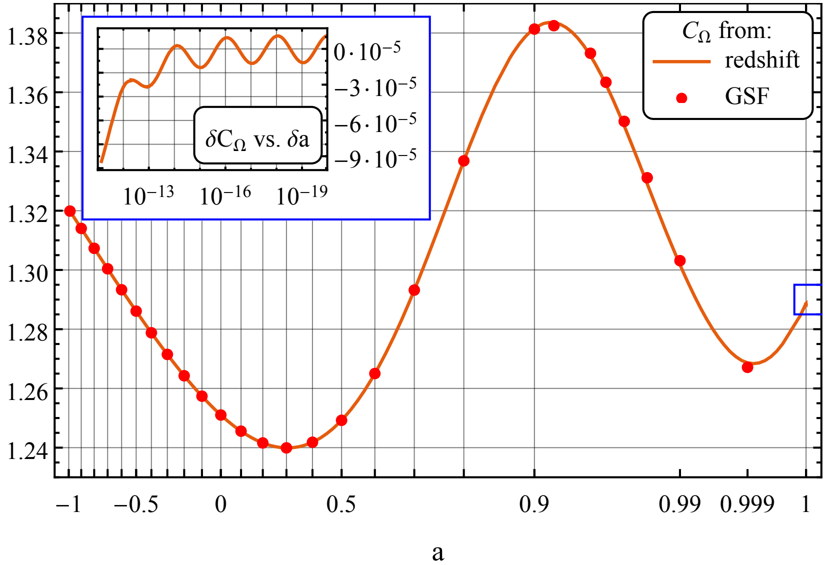


Crosschecks with ...

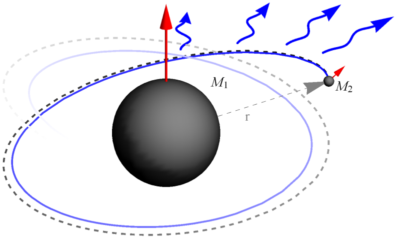
- Other self-force calculations (different method, gauge, etc.)
- post-Newtonian theory
- Numerical relativity
- Effective-One-Body models

Periapsis advance of circular orbits [MvdM,2016]





1.38
1.36
1.34
1.32
1.30
1.28
1.26
1.24
−1
−0.5
0
0.5
0.9
0.99
0.999
1
a
C_Ω from:
redshift
GSF
$0\cdot10^{-5}$
$-3\cdot10^{-5}$
$-6\cdot10^{-5}$
$-9\cdot10^{-5}$
10^{-13}
10^{-16}
10^{-19}
δC_Ω vs. δa



Status overview

Status of calculations

		Schwarzschild		Kerr		
		circ	ecc	circ	ecc	incl
Geodesics (analytic)		[Hackmann & Lammerzahl, 2008]		[Fujita & Hikida, 2009] [Hackmann et al, 2010]		
Adiabatic	orbit	[Cutler et al, 1994]		[Shibata et al, 1994]		[Hughes, 1999]
				[Drasco & Hughes, 2006]		
	evolution				[Glamp. & Ken., 2002]	[Hughes, 2001]
				to do		
$\frac{1}{2}$ PA: resonances				[Flanagan & Hinderer, 2012] [Flanagan, Hughes & Ruangsri, 2014] [MvdM, 2014]		
1PA:	1GSF	[Barack & Sago, 2007]	[Barack & Sago, 2010]	[Shah et al, 2012]	[MvdM & Shah,2015] [MvdM, 2016]	in progress
	2GSF	in progress	to do	to do		
	spin force	[Papapetrou, 1951]		[Papapetrou, 1951]		
	evolution	[Warburton et al, 2012] [Osburn et al, 2015]		to do		

Summary

Status

- Formalism mostly in place
- 1GSF calculations in Schwarzschild now routine
- 1GSF in Kerr now available for equatorial orbits
- First self-forced inspirals

To do...

- Numerical 2GSF calculations (soon...)
- 1GSF on Kerr generic orbits (soon...)
- self-forced inspirals in Kerr
- include secondary spin effects & 2GSF
- waveforms

The End

Thank you for listening!

Acknowledgments

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