Gravitational waveforms from numerical simulations of binary-black-hole mergers

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The Era of Gravitational-Wave Astronomy
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SXS

SIMULATING EXTREME SPACETIMES

Black holes, neutron stars, and beyond...

Outline

- Motivation
- Numerical relativity techniques
  - two different approaches
  - extracting the physics
- Results
  - direct comparison to observations
  - testing, improving, and building analytic waveform models
  - remnant properties: large kicks!
  - spacetime dynamics
  - astrophysical implications and EM counterparts will be covered by Manuela Campanelli on Friday
- Future work
- Conclusions

Why model compact binaries?

- Two-body problem is a fundamental problem in gravity
- No known exact solution in theory of general relativity
- Want large compactness $\frac{GM}{Rc^2}$ to probe strong gravity
    - Sun $\sim 10^{-6}$
    - White Dwarf $\sim 10^{-4}$
    - Neutron star ~ 0.1
    - Black hole ~ 1
- One of the most promising sources of detectable gravitational waves
- Mergers of black holes are detectable across a wide frequency range
- Black hole mergers "only" require solving the vacuum Einstein equations

Payoffs from binary black hole simulations

What do we learn?

- Dynamics of strongly warped spacetime
- Gravitational waveforms

How can they be used?

- Directly compare theory to observations
- Improve analytic waveform models
- Determine remnant black hole properties
- Explore nonlinear behavior of gravity
- Produce visualizations for public outreach

Solving the vacuum Einstein equations on a computer

Goal: determine the spacetime metric describing the inspiral, merger, and ringdown of a binary black hole system

- Solve as an initial-boundary value problem
- Slice spacetime into spatial hypersurfaces
    - Constraint equations
    - Evolution equations
    - Coordinate freedom
- Specify initial conditions that describe a binary black hole system and satisfy the constraint equations
- Choose the computational domain on which to do the evolution
    - deal with singularities inside the black holes
    - introduce artificial outer boundary
- Choose a formulation of the evolution equations
- Choose a numerical algorithm
- Specify coordinate conditions
- Specify boundary conditions
- Decide how to control of constraint violations

Brief History of BBH simulations

- 1964 – First attempt: 2D head-on equal-mass
  [Hahn, Lundquist; Ann. Phys. 29, 304 (1964)]
- mid-1970s – First success: 2D head-on equal-mass
  [Smarr, Cadez, DeWitt, Eppley; PRD 14, 2443 (1976)]
- mid-1990s – Computational Grand Challenge
- 2005 – First successful inspiral and merger (unique methods)
  [Pretorius; gr-qc/0507014]
- 2005 – Moving punctures approach
  [Campanelli, Lousto, Marronetti, Zlochower; gr-qc/0511048]
  [Baker, Centrella, Choi, Koppitz, van Meter; gr-qc/0511103]
  Adopted by many groups: RIT, GSFC, GaTech, Illinois, FAU, LSU, Maryland, AEI, Jena, Vienna, Palma, ...
- 2006 (inspiral); 2008 (merger) – Spectral Einstein Code (SXS)
  [Boyle et al.; 0710.0158] [Scheel et al.; 0810.1767]

Independent approaches, multiple codes

- Puncture codes:
  - Robust
  - Fairly straightforward to implement
  - Open-source infrastructure
  - Good for short inspirals (< 10 orbits)
  - Many codes, most share common infrastructure (BAM independent)
  - LazEv(RIT), Hahndol(GSFC), Maya(GaTech), CCATIE(AEI), BAM(Jena,Cardiff,Palma,Vienna), Llama(AEI,Palma), Lean, UIUC, ...
- Spectral Einstein Code (SXS)
  - more accurate and efficient
  - can do long inspirals (20 - 40 orbits in reasonable time)
  - Black-box for $1 \leq q \leq 4$, $\chi < 0.8$

Independent approaches, multiple codes

Initial data:

- Formulation of Einstein constraint equations
  - SpEC: Conformal thin sandwich [York (1999)] [Pfeiffer, York (2003)]
  - LazEv, Maya, BAM: conformal [Bowen, York (1989)]]
- Singularity treatment
  - SpEC: excision [Cook (2002)] [Cook, Pfeiffer (2004)]
  - LazEv, Maya, BAM: puncture data [Brandt, Brügmann (1997)]
- Numerical method
  - SpEC: pseudospectral [Pfeiffer, Kidder, Scheel, Teukolsky (2003)]
  - LazEv, Maya, BAM: TwoPuncture (pseudospectral) [Ansorg, Brügmann, Tichy (2004)]
- Achieving low orbital eccentricity
  - SpEC: iterative eccentricity removal [Pfeiffer et al. (2007)] [Buonanno et al. (2011)]
  - LazEv: post-Newtonian inspiral [Healy et al. (2017)]
  - BAM: [Púrrer et al. (2012)]

Independent approaches, multiple codes

Evolution:

- Numerical algorithm
  - SpEC: multi-domain pseudo-spectral methods [Kidder et al. (2002)]
  - LazEv: high-order finite-differences [Zlochower et al. (2005)]
  - Maya: high-order finite-differences [Hermann, Shoemaker, Laguna (2006)] [Vaishnav et al. (2007)] [Healy, Levin, Shoemaker (2009)] [Pekowsky et al. (2013)]
  - BAM: high-order finite-differences [Brügmann et al. (2008)] [Husa et al. (2008)]
- Formulation of Einstein evolution equations
  - SpEC: First-order generalized harmonic [Friedrich (1985)] [Pretorius (2005)] [Lindblom et al. (2006)]
  - LazEv, Maya, BAM: BSSNOK [Nakamura, Oohara, Kojima (1987)] [Shibata, Nakamura (1995)] [Baumgarte, Shaprio (1999)]
- Singularity treatment
  - SpEC: Excision [Kidder et al. (2002)]
  - LazEv, Maya, BAM: Moving punctures [Campanelli et al. (2006)] [Baker et al. (2006)]

Independent approaches, multiple codes

Evolution:

- Mesh refinement
  - SpEC: hpr-refinement, multiple coordinate frames [Hemberger et al. (2013)] [Szilágyi (2014)]
  - LazEv, Maya: Moving boxes mesh refinement (Carpet/EinsteinToolkit/Cactus) [Schnetter, Hawley, Hawke (2004)]
- Coordinate (gauge) conditions
  - SpEC: Damped harmonic [Szilágyi, Lindblom, Scheel (2009)]
  - LazEv: evolved lapse and shift [Bona et al. (1997)] [Alcubierre et al. (2003)] [van Meter et al. (2006)]
- Boundary conditions
  - SpEC: minimally-reflective, constraint-preserving [Lindblom et al. (2006)] [Rinne, Lindblom, Scheel (2007)]
  - LazEv: Sommerfeld

Extracting physics

Measuring the mass and spin of the black holes:

- determined from the apparent horizon
  - SpEC: fast flow algorithm [Gundlach (1998)]
  - LazEv: AhFinderDirect [Thornburg (2004)]
- definition of spin
  - SpEC: quasilocal (eigenvalue problem) [Cook, Whiting (2007)] [Owen (2007)]
  - LazEv: isolated horizon [Dreyer et al. (2003)] [Campanelli et al. (2007)]
- mass determined from area of horizon and spin

Extracting physics

Extracting the gravitational waveform:

- extract information at finite radii
  - Newman-Penrose scalar Ψ_4
    - SpEC: [Pfeiffer et al. (2007)] [Scheel et al. (2009)] [Boyle et al. (2007)
    - LazEv: [Campanelli, Lousto (1997)] [Lousto, Zlochower (2007)]
    - Maya: [Reisswig (2009)]
    - BAM: [Brügmann et al. (2008)]
  - RWZ extraction of h
    - SpEC: [Buchman, Sarbach (2007)] [Rinne et al. (2009)]
- extrapolate to infinity
  - SpEC: [Boyle, Mroué (2009)]
  - LazEv: perturbative extrapolation [Nakano et al. (2015)]
- Cauchy-characteristic extraction
  - SpEC: [Bishop et al. (1997)] [Gomez et al. (2007)] [Reisswig et al. (2013)] [Handmer, Szilágyi (2015)]
- spin-weighted spherical harmonic decomposition

Running a BBH simulation

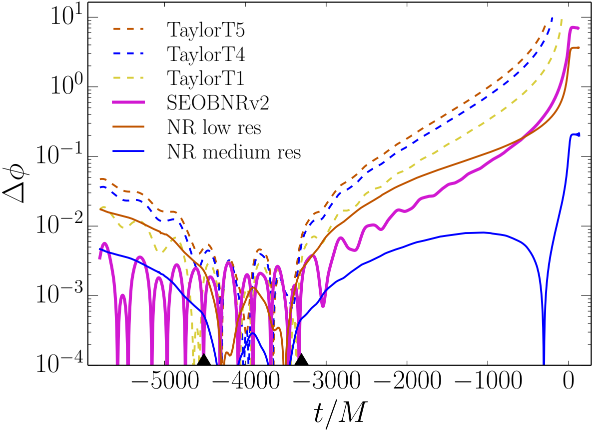
- Choose desired physical configuration q, S_1, S_2, e at some initial orbital parameters ω_{orb}, d, v_r
- Iterative initial data solve to get desired parameters
- Evolve for several orbits, measure eccentricity and adjust initial orbit parameters
- Also adjust physical parameters as black holes relax
- Once desired setup is achieved, evolve through merger and ringdown until waves reach extraction surfaces
- Extrapolate/Evolve extracted waves to null infinity

What can be simulated?

- Number of orbits before merger
  - Desired orbits for testing analytic models?
  - Desired orbits for parameter estimation?
  - For low mass systems, need to hybridize to cover detector frequency band
- Parameter space
  - Total mass m scales out of the problem
  - Mass ratio: $1 \leq q \lesssim 20$
    $q = 100$ [Lousto, Zlochower; 1009.0292]
  - Spins: $0 \leq \chi \lesssim 0.9$
    higher spin requires improved initial data
  - Precession: no problem
  - Eccentricity: no problem

SXS Large Spins 0.994

Waveform phase error:



[Scheel et al.; 1412.1803]

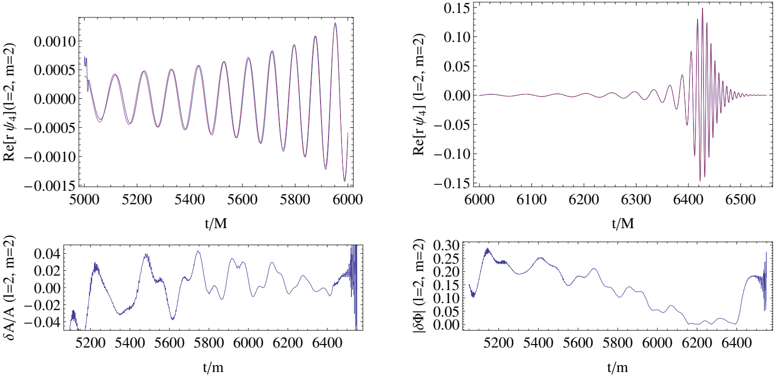
- Equal mass
- aligned spins
- $\chi_a = \frac{S_a}{m_a^2} = 0.994$
- 25.4 orbits.
- $\chi_f = 0.949931(5)$
- $E_r = 0.11351(5)M$

RIT Large spins compared to SpEC

Equal-mass, aligned spins, $\chi_a = \frac{S_a}{m_a^2} = 0.99$, 10 orbits.

Initial data: Superposed Kerr-Schild [Ruchlin, Healy, Lousto, Zlochower; 1410.8607]

Evolution: CCZ4 evolution system [Alic et al.; 1106.2254]



Top: (2,2) mode for SpEC vs RIT [Zlochower, Healy, Lousto, Ruchlin; 1706.01980]

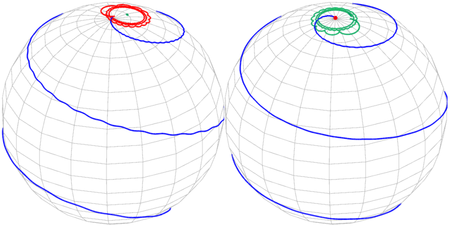
Bottom Left: fractional amplitude difference

Bottom Right: phase difference (oscillation likely due to eccentricity)

RIT Flip Flop

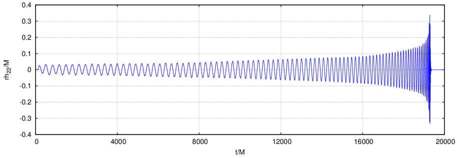
Equal mass, $\chi_1 = \frac{S_1}{m_1^2} = 0.2$ aligned, $\chi_2 = 0.8$ z-component anti-aligned, in-plane towards companion

- 48 orbits, 3 precession cycles and flips direction
- $\hat{L}$, $\hat{J}$, $\hat{S}_1$

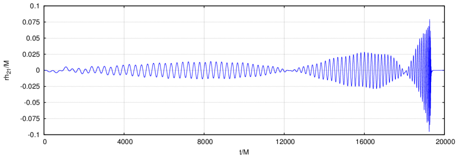


Left: in orbital frame
Right: in coordinate frame

- h_{22}



- h_{21}



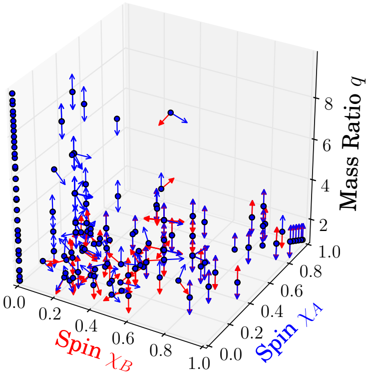
[Lousto, Healy; 1410.3830]

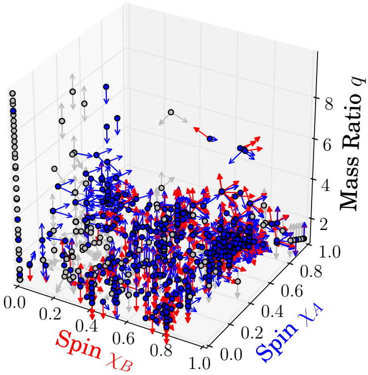
Waveform catalogs: SpEC

[Mroue et al.; 1304.6077] [Chu et al.; 1512.06800]

Publicly available at www.black-holes.org/waveforms

- 8-dimensional parameter space: mass-ratio, spins, eccentricity
- Left: Currently 316 waveforms (93 precessing) Median: 22 orbit
- Right: Soon another 1000+ waveforms (800+ precessing)



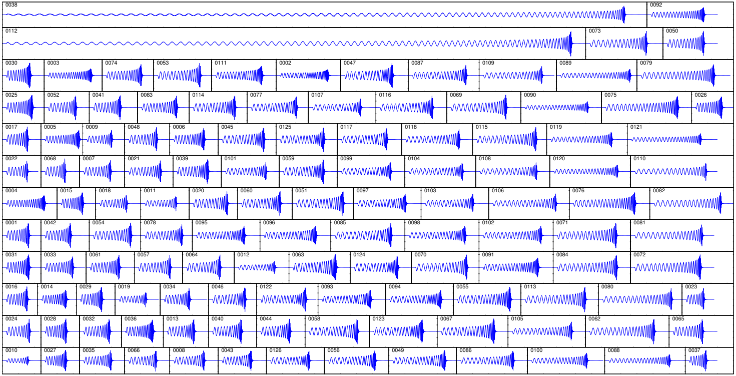


Waveform catalogs: RIT

[Healy, Lousto, Zlochower, Campanelli; 1703.03423]

Publicly available at `http://ccrg.rit.edu/~RITCatalog`

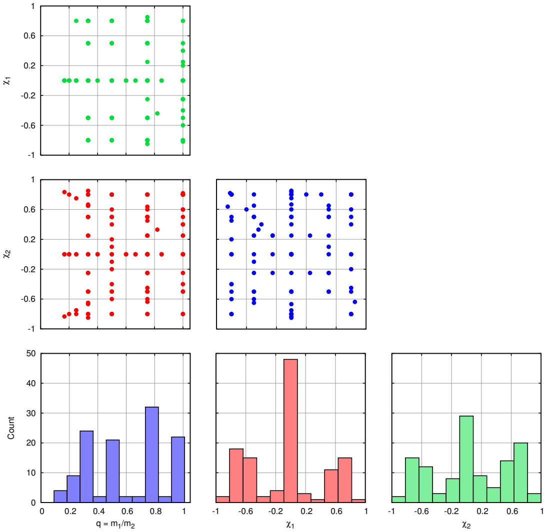
- Currently 126 simulations
- Two longest; equal-mass non-spinning, flip flop



Waveform catalogs: RIT

Top: χ_m
Middle: χ_M
Bottom: Count

Left: $\frac{1}{6} \leq q \leq 1$
Middle: χ_m
Right: χ_M

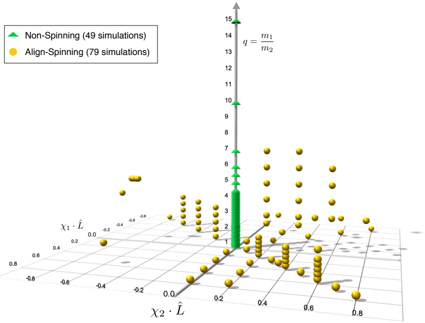


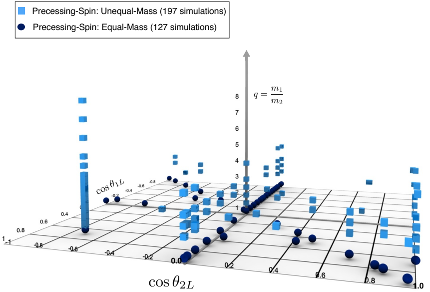
Waveform catalogs: GaTech

[Jani, Healy, Clark, London, Laguna, Shoemaker; 1605.03204]

Publicly available at www.einstein.gatech.edu/catalog

- Currently 452 waveforms, a few to 10 orbits



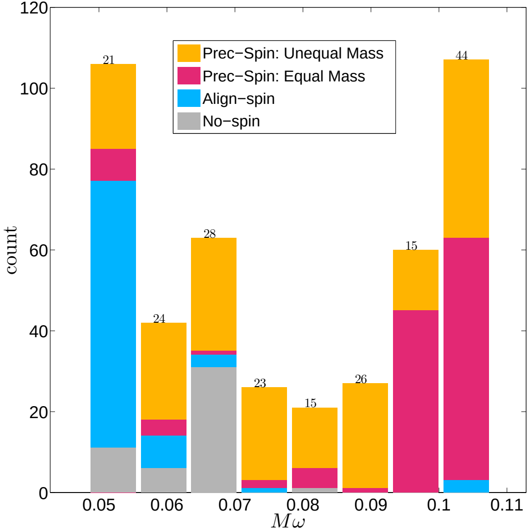


Vertical axes: mass ratio   Left: aligned spin   Right: precessing

Waveform catalogs: GaTech

Count vs $M\omega_{orb}$

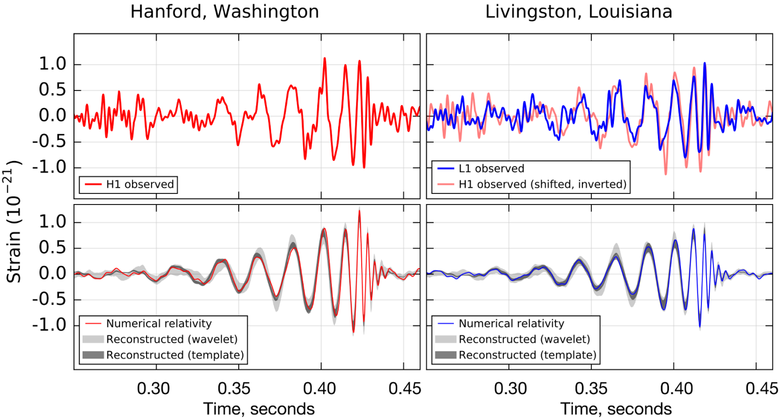
For AdvLIGO this limits minimum mass to $[50, 110]M_\odot$



Comparison with observations : GW150914

[Abbot et al.; 1602.03837]

- BH masses: $36 \pm 4M_\odot$ and $29 \pm 4M_\odot$
- Final mass: $62 \pm 4M_\odot$
- GW Energy: $3.0 \pm 0.5M_\odot c^2$
- Distance: $410 \pm 170Mpc$



Comparison with observations: GW150914

[Abbott et al.; 1606.01262]

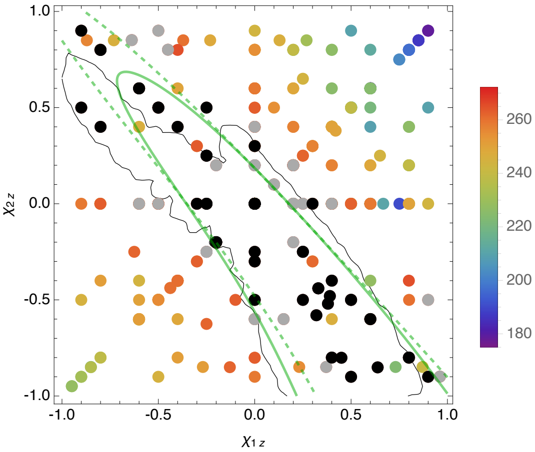
Many different systems consistent

Initial spins poorly constrained

Excellent Match

Fair match

Poor match

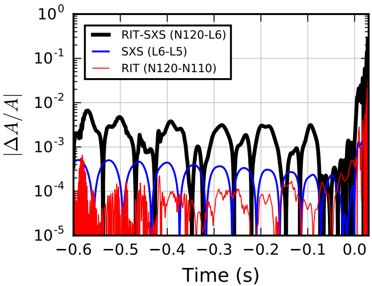


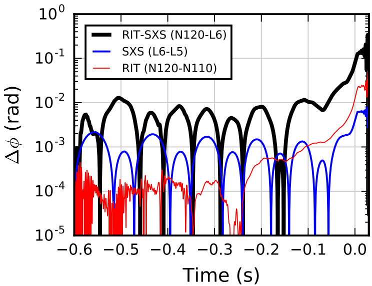
Code comparisons: GW150914

SpEC (SXS) waveform compared to LazEv (RIT) waveform

$q = \frac{m_2}{m_1} = 1.22$, $\chi_1 = \frac{S_1}{m_1^2} = -0.44$, $\chi_2 = 0.33$

[Lovelace et al.; 1607.05377]





Red: RIT(N120-N110) Blue: SXS(L6-L5) Black: RIT - SXS

Left: Relative amplitude difference Right: phase difference

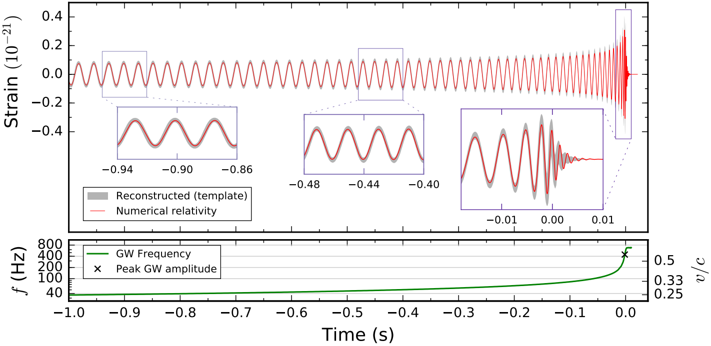
Comparison with observations: GW151226

[Abbot et al.; 1606.04855]

NR: SpEC waveform, 46 orbits, 2.5 months

$q = 3.32$, $\chi_1 = 0.5226$, $\chi_2 = -0.4482$

‘



Community NR projects

- Samurai code-comparison of equal-mass, non-spinning waveforms between SpEC, BAM, CCATIE, Hahndol, Maya
  [Hannam et al.; 0901.2437]
- Numerical INJection Analysis (NINJA) NR - data analysis project
  [Aylott et al.; 0901.4399]
- NINJA-2 constructed 63 NR-PN hybrids [Ajith et al.; 1201.5319] and did blind-injection test of 7 waveforms [Aasi et al.; 1401.0939]
- NR-AR NR - analytic modelers project compared 25 NR waveforms by 9 codes to 5 analytic models [Hinder et al.; 1307.5307]

Informing and testing analytic models

- Effective-one-body model
  - SEOBNRv1 (spinning, non precessing) [Taracchini et al.; 1202.0790]
    calibrated with 2 spinning, 5 non-spinning SpEC simulations
  - SEOBNRv2 (spinning, non-precessing, used in O1)
    [Taracchini et al.; 1311.2544]
    calibrated with 8 non-spinning, 30 spinning SpEC simulations
  - SEOBNRv3 (spinning, precessing) [Pan et al.; 1307.6232]
    based on v2, tested with 2 precessing SpEC waveforms
    further tested with 70 precessing SpEC waveforms
    [Babak, Taracchini, Buonanno; 1607.05661]
  - SEOBNRv4 (spinning, non-precessing) [Bohe et al.; 1611.03703]
    calibrated with 140 SpEC, 1 BAM waveform
  - IHES-EOB (non-spinning) [Damour, Nagar, Bernuzzi; 1212.4357]
    calibrated to 5 SXS waveforms
  - EOB (spinning, non-precessing) [Nagar et al.; 1506.08457]
    calibrated with 40 SpEC, 10 Llama waveforms

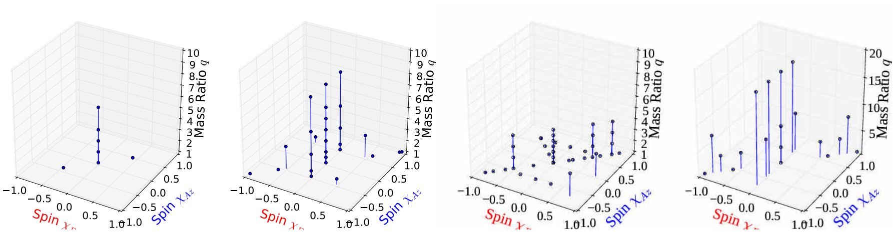
Informing and testing analytic models

- Phenom models
  - PhenomB (non-precessing) [Ajith et al.; 0909.2867]
    calibrated against 24 BAM waveforms; tested with BAM, Llama, CCatie, LazEv, SpEC waveforms
  - PhenomC (non-precessing) [Santamaria et al.; 1005.3306]
    calibrated against BAM, Llama, LazEv, SpEC waveforms
  - PhenomP (precessing) [Hannam et al.; 1308.3271]
    based on PhenomC, tested with 4 BAM waveforms
  - PhenomD (non-precessing) [Khan et al.; 1508.07253]
    calibrated against 9 SpEC and 10 BAM waveforms
    [Husa et al.; 1508.07250]

Informing and testing analytic models

[Kumar et al.; 1601.05396]

- Test waveform models with 84 non-precessing SpEC simulations with $1 \leq q \leq 3$ and $\chi \leq 0.9$
- PhenomD and SEOBNRv2 both perform better than their predecessors



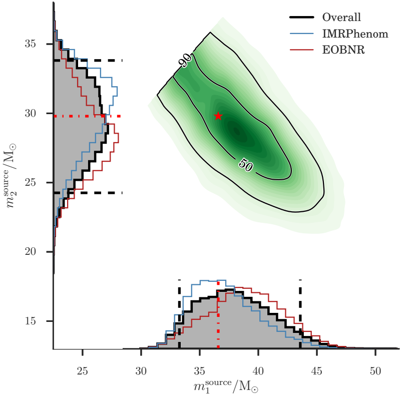
From left to right: Waveforms used to calibrate SEOBNRv1, SEOBNRv2, IMRPhenomC, IMRPhenomD

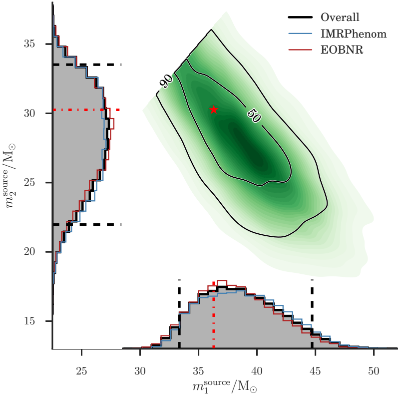
Informing and testing analytic models

Is there a systematic bias in analytic waveform models?

- inject numerical waveforms into zero noise detector
- estimate parameters with analytic waveform models

[Abbott et al.; 1611.07531]



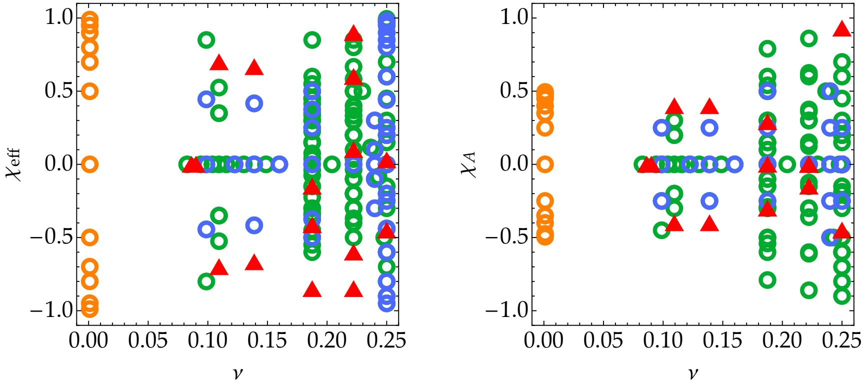


Left: Inject SpEC

Right: Inject BAM

Informing and testing analytic models

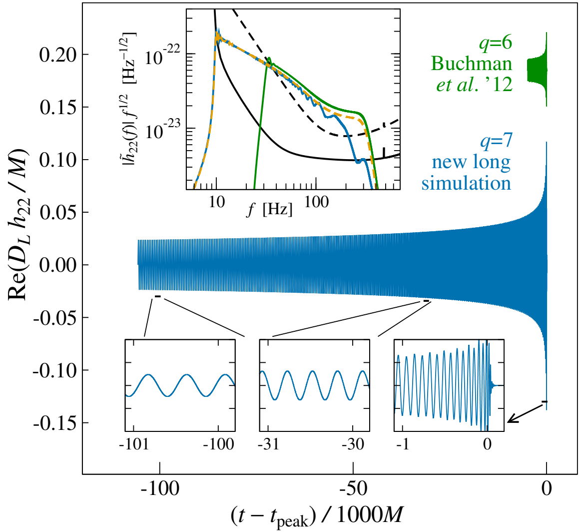
Calibrate SEOBNRv4 to 141 NR waveforms (140 SpEC, 1 BAM)
Tested with 4 SpEC, 2 EinsteinTookit waveforms. [Bohe et al.; 1611.03703]



Left: $\chi_{eff} = (m_1\chi_1 + m_2\chi_2)/(m_1 + m_2)$
$\nu = (m_1 m_2)/(m_1 + m_2)^2$

Right: $\chi_A = (\chi_1 - \chi_2)/2$

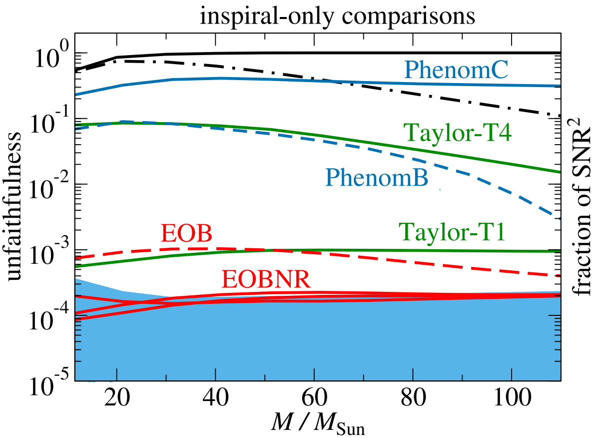
SXS 170 Orbits



- $3\times$ frequency range
- $40 M_\odot$ entire Advanced LIGO spectrum

[Szilagyi et al.; 1502.04953]

Waveform models vs 170 Orbits NR

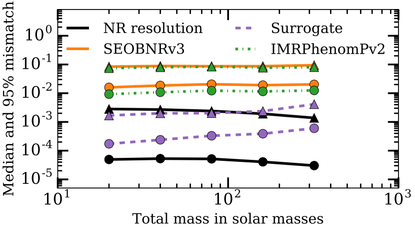


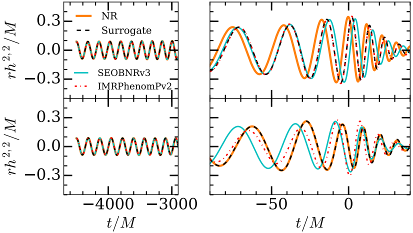
[Szilagyi et al.; 1502.04953]

- Standard PN $\sim$ 10%
- Phenom $\sim$ 30%
- uncalibrated EOB $\sim$ 0.1%
- calibrated EOBNR $\sim$ 0.02%

NR surrogate models

- built from 744 NR waveforms [Blackman et al.; 1705.07089]
- 7-dimensional (full precession), but $1 \leq q \leq 2$ and $0 \leq \chi \leq 0.8$
- Covers 20 orbits before merger
- surrogate can be evaluated in $\approx 50ms$



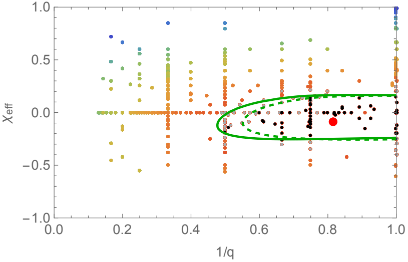


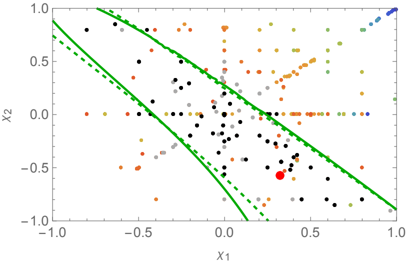
top: worst surrogate waveform
bottom: worst SEOBNRv3 waveform

Parameter estimation using NR waveforms

- Bayesian method that directly compares GW data to NR simulations
- Using $\ell = 3$ modes gain more information from the signal and can better constrain the parameters

[Lange et al.; 1705.09833]





Numerical Relativity Injection Infrastructure

[Schmidt, Harry, Pfeiffer; 1703.01076]

- include in LIGO Algorithms Library (LAL)
- NR groups provide data in given format
- NR waveforms can be used as simulated signals
  - parameter estimation
  - searches
  - hardware injections
- handles subdominant modes and precession

Properties of the remnant

- Simulations provide remnant mass, spin, velocity

  [Zlochower, Lousto; 1503.07536]

  [Hofmann, Barausse, Rezolla; 1605.01938]

  [Jimńez-Forteza et al.; 1611.00332]

  [Healy, Lousto; 1610.09713]

  [Bohé et al.; 1611.03703]

  [Healy, Lousto, Zlochower; 1705.07034]

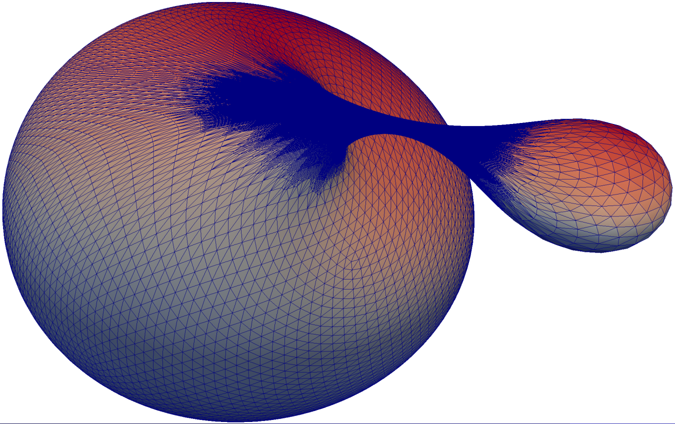
Large Kicks

- Anisotropic GW emission leads to net linear momentum flux
- Final black hole is kicked with respect to initial center of mass
- Non-spinning BHs $v_{max} \approx 175 \pm 11 \frac{km}{s}$ at mass-ratio 2.77
  [Gonzalez et al.; gr-qc/0610154]
- Spinning BHs give higher kicks
  - Aligned spins $v_{max} \approx 500 \frac{km}{s}$
    [Herrmann et al.; gr-qc/0701143] [Koppitz et al.; gr-qc/0701163]
  - Super kicks for spins equal and opposite in orbital plane
    v_{max} up to $4000 \frac{km}{s}$! [Campanelli et al.; gr-qc/0701164]
  - Hangup kicks, generic orientation v_{max} up to $5000 \frac{km}{s}$!
    [Lousto, Zlochower; 1108.2009]
- $v_{recoil} > 1000 \frac{km}{s}$ is between $0.1 - 17\%$
  [Zlochower, Lousto; 1503.07536]
- More on Friday from Manuela Campanelli

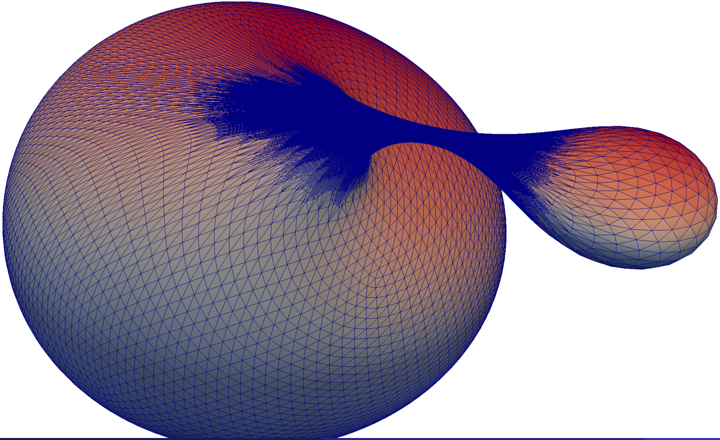
Evolution of the event horizon

[Bohn, Kidder, Teukolsky; 1606.00437]

- Need entire spacetime to find event horizon
- Integrate geodesics backwards in time
- Adaptive triangulation
- $q = 6$, $\chi_1 = 0.9$, $\chi_2 = 0.3$ precessing



Evolution of the event horizon

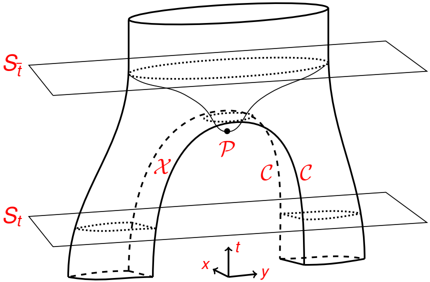


Toroidal horizons

[Bohn, Kidder, Teukolsky; 1606.00436]

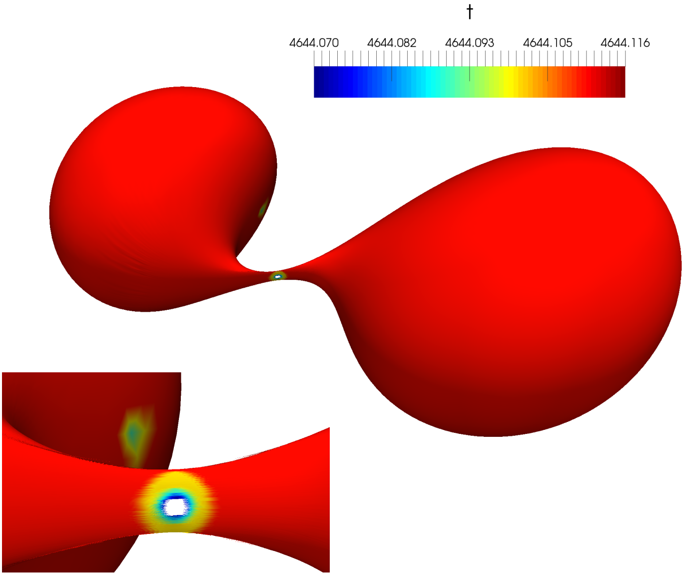
- Event horizon of a dynamical black hole can have either spherical or dynamical topology [Gannon (1976)]
- Torii must collapse faster than light-crossing time
  [Friedrich, Schleich, Witt (1993)] [Galloway (1995)] [Jacobsen, Venkataramani (1995)]
- Equivalently, can find another foliation where topology is spherical
  [Siino (1998)]

- Claim: during a generic merger, the topology of the event horizon can go through a toroidal phase. [Siino (1998)]
  [Husa, Winicour (1999)]
- Not seen in BBH evolutions
- Idea: reslice space time



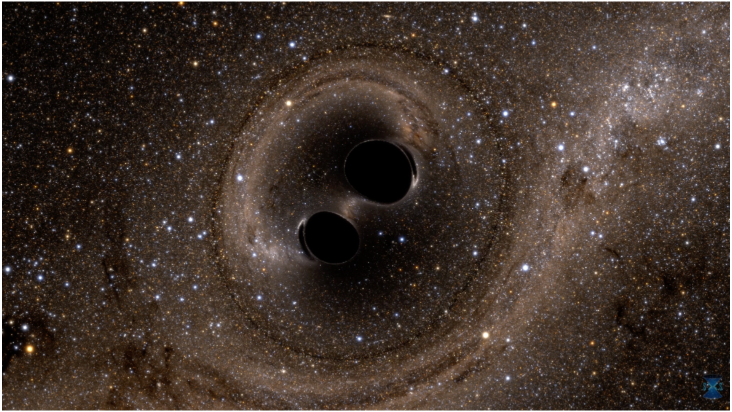
Toroidal horizons

[Bohn, Kidder, Teukolsky; 1606.00436]



What does a binary black hole merger look like?

[Bohn et al.; 1410.7775]



What does a binary black hole merger look like?

Future work

- Expand parameter space of catalogs
  - React to gravitational wave observations
  - Test and improve analytic models
- Improve the accuracy and efficiency of simulations
  - Longer simulations to cover frequency band for low-mass systems
  - Louder signals provide better measurements
  - Exploit parallelism to improve turn around time
- Simulating alternate theories of gravity
  - f(R) [Cao, Galaviz, Li; 1608.0781]
  - dynamical Chern-Simons [Okounkova et al.; 1705.07924]
  - effective theories of gravity [Cayuso, Ortiz, Lehner; 1706.07421]
- Getting the details correct
  - definitions of masses and spins
  - waveform extraction

Summary

Gravitational waveforms from numerical simulations of binary black hole mergers are invaluable tools for data analysts and waveform modelers.

- Directly compare with observations
- Inform and test analytic waveform models
- Construct surrogate models

In addition numerical simulations of binary black holes:

- provide properties of the remnant black hole
- explore highly dynamical spacetimes
- produce movies for public outreach