The particle-without-particle approach to the self-force problem

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Introduction

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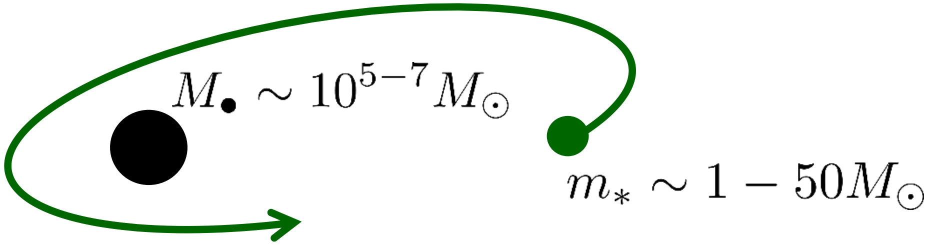

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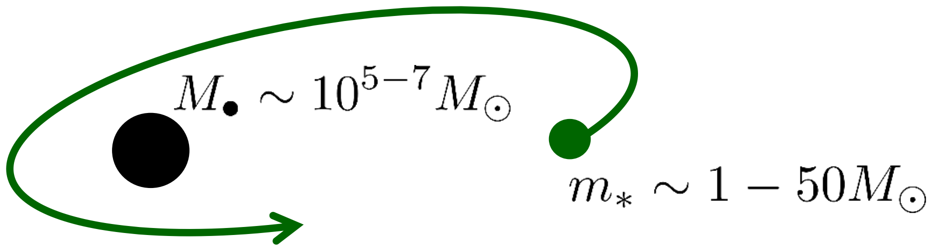

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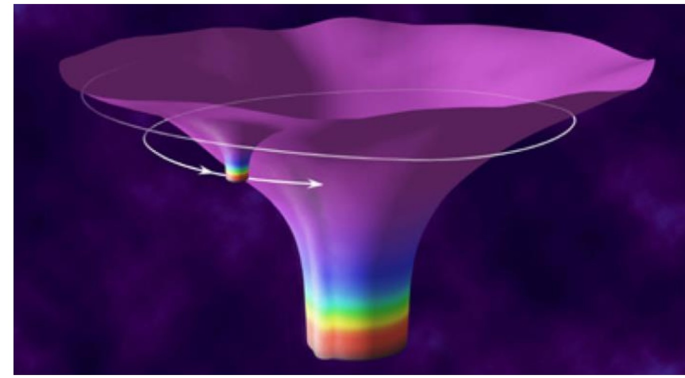

[arXiv:1305.5720]

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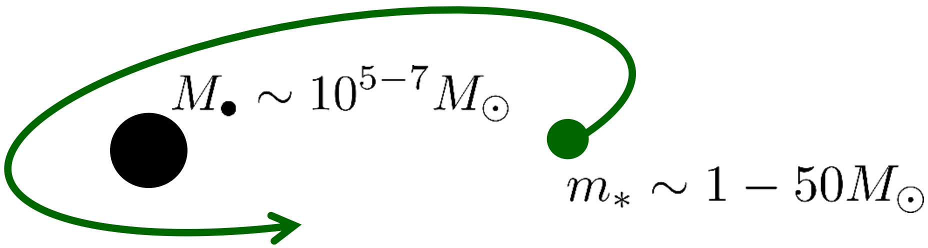

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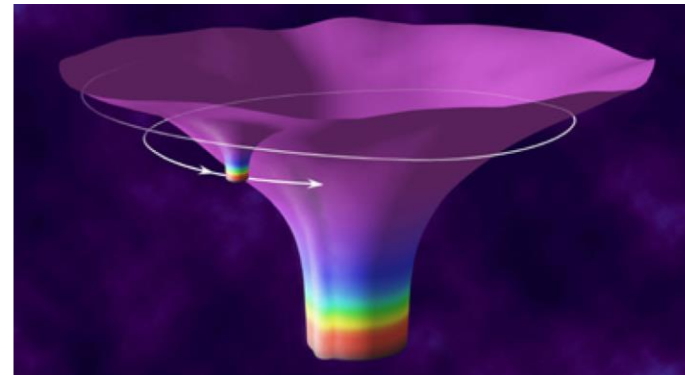

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- The theoretical formalism to compute the self-force has been largely established, e.g. [S. Gralla and R. Wald, CQG 25, 205009 (2008)], but its mathematical implementation is still under development; we use the *Particle-without-Particle (PwP)* method.

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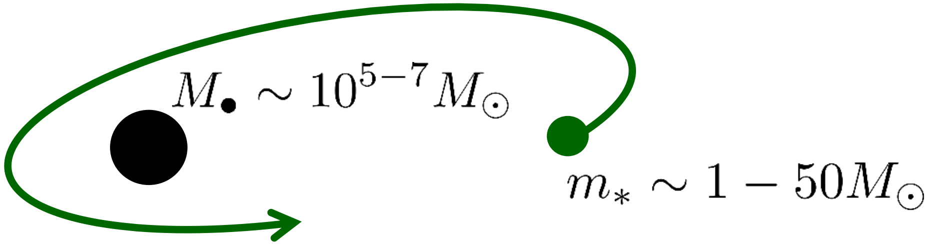

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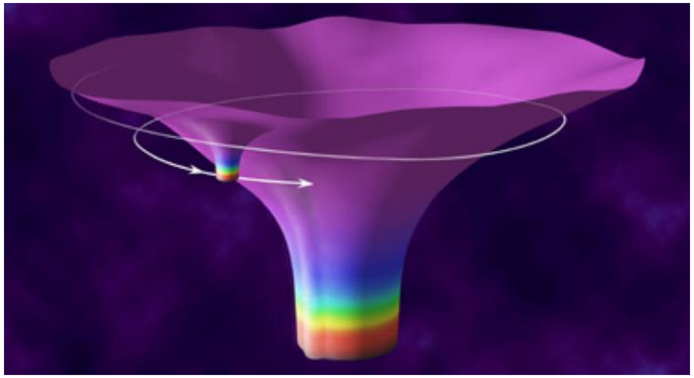

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- A helpful testbed for the gravitational self-force is the *scalar* self-force—we tackle this using the PwP method in the *frequency domain*.

Scalar self-force: a simplified EMRI model

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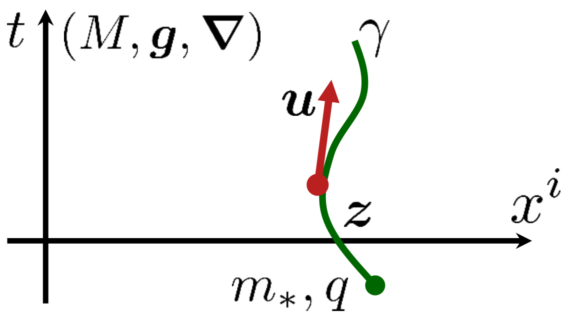
- **Setup**: m_* is a *charged scalar particle* (with charge q associated to a scalar field Φ) orbiting a *non-rotating black hole* (of fixed—Schwarzschild—geometry, $(M, \boldsymbol{g}, \boldsymbol{\nabla})$) along a geodesic γ with worldline $\boldsymbol{z}(\tau)$ and 4-velocity $\boldsymbol{u} = \dot{\boldsymbol{z}}$. The EOMs are:



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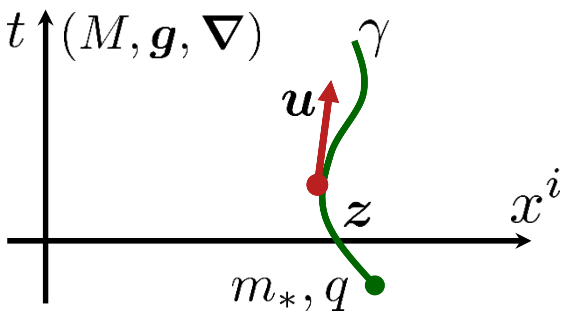

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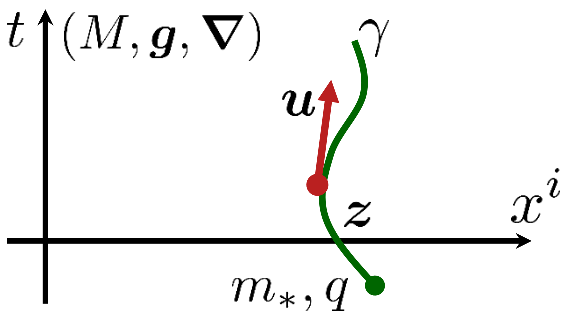

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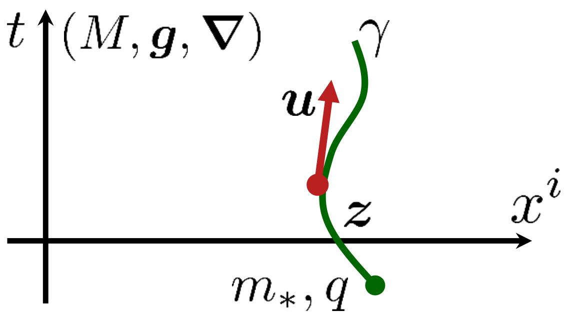

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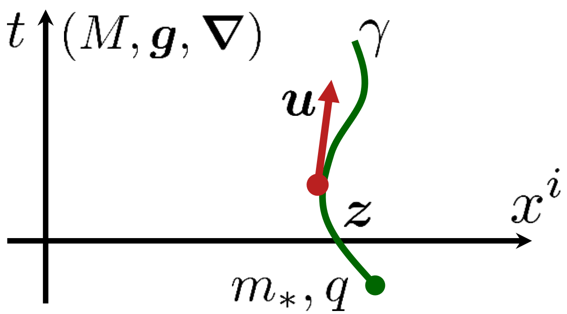

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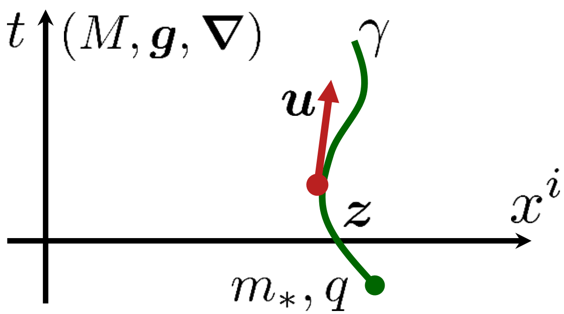

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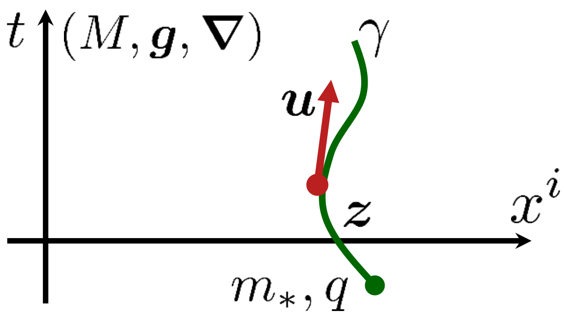

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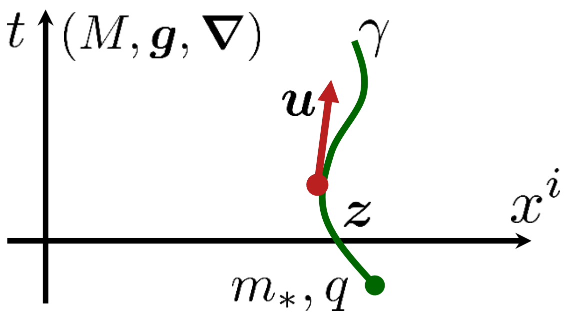

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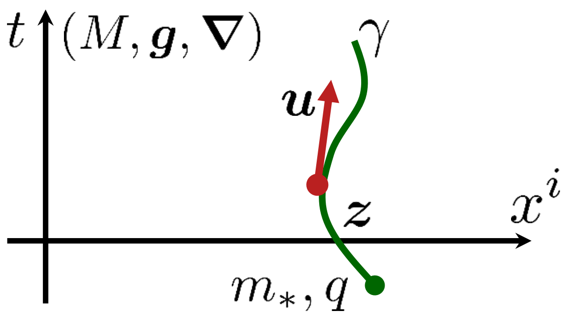

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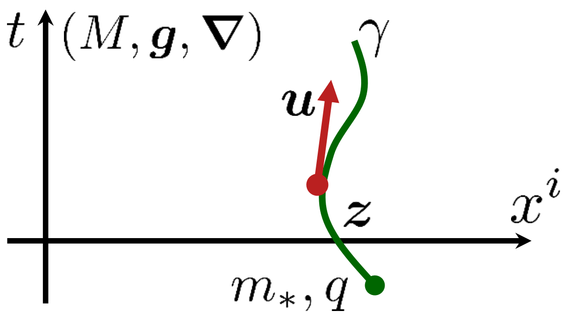

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- Once the field is solved for, its singular part must be subtracted (via "mode-sum regularization" [L. Barack and A. Ori, PRD 61, 061502 (2000)]).

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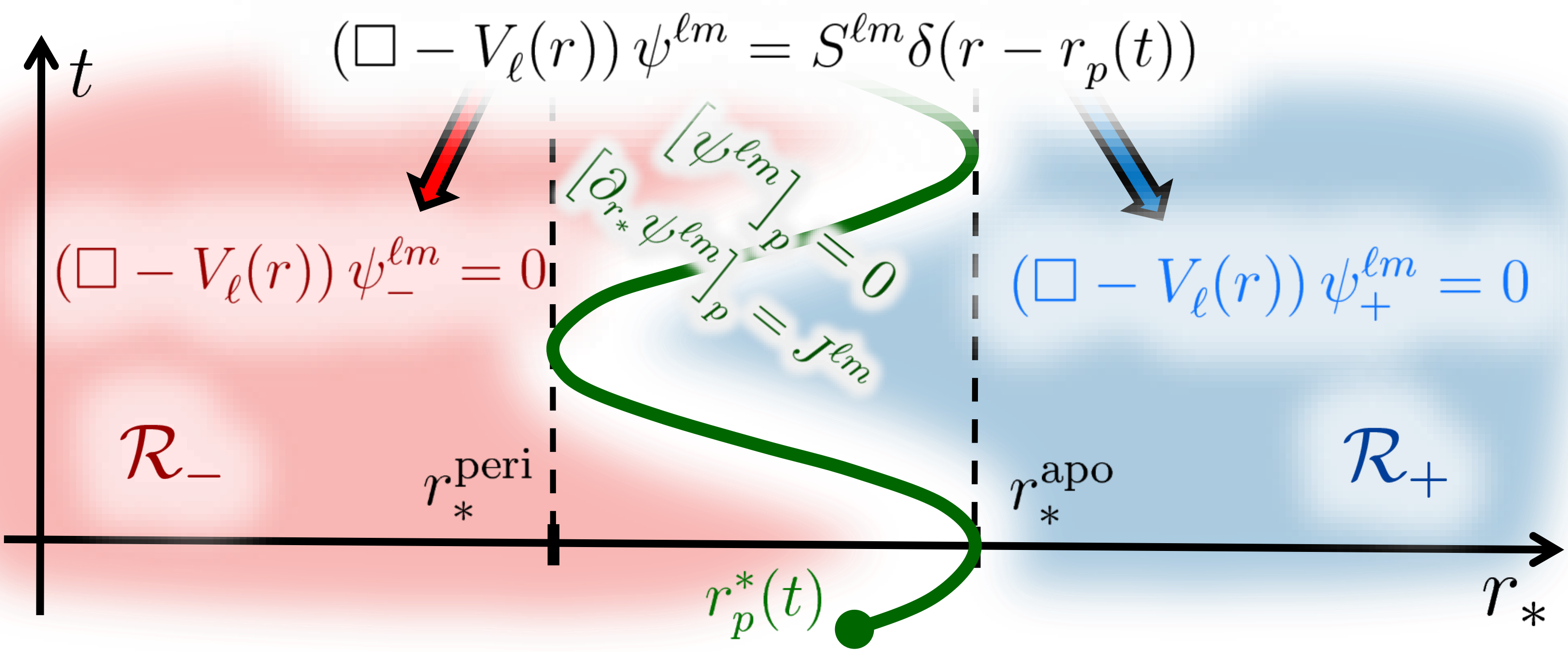

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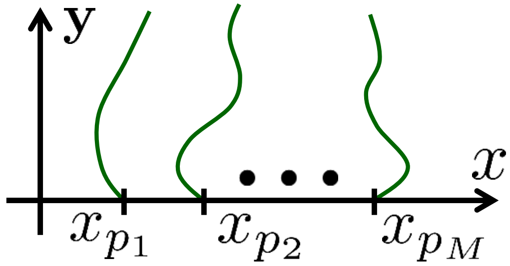

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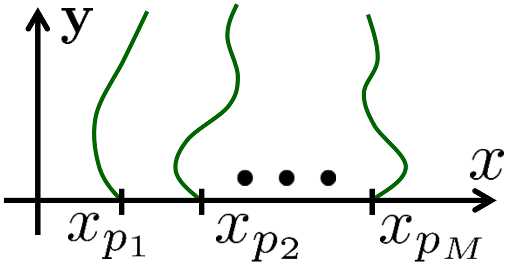

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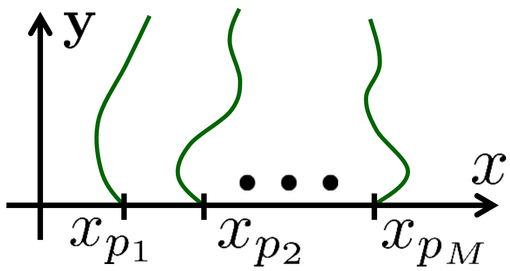

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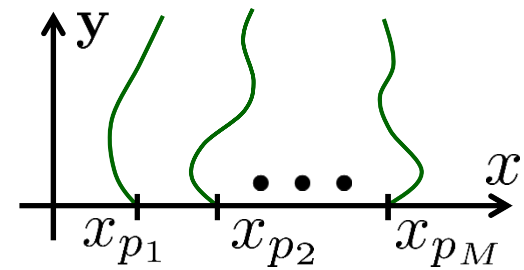

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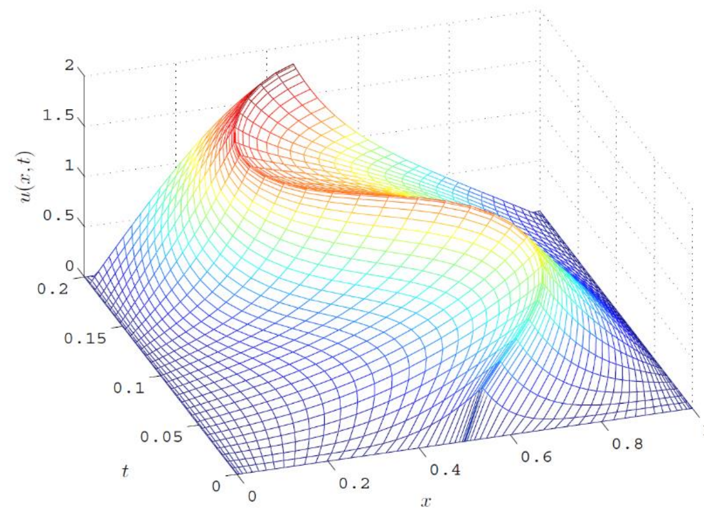
- **Example**: Heat equation with constant source supported at a sinusoidally moving particle (using a Chebyshev-Lobatto grid in space and a standard finite-difference scheme in time).



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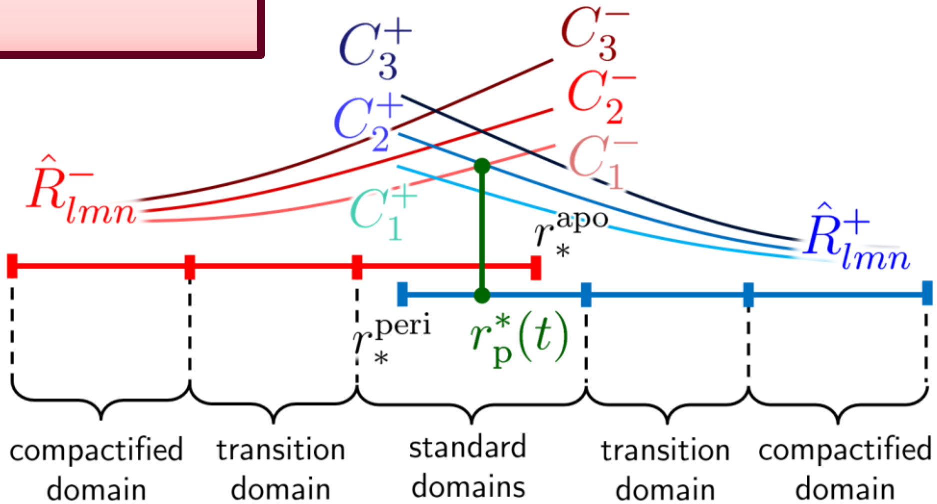
- We use a pseudospectral collocation method to find the homogeneous numerical solutions $\hat{R}^{-}_{lmn}$ and $\hat{R}^{+}_{lmn}$ for arbitrary BCs, then use the jump conditions to get the true solution, $R_{lmn} = C^{-}_{lmn}\hat{R}^{-}_{lmn}\Theta^{-}_{\mathrm{p}} + C^{+}_{lmn}\hat{R}^{+}_{lmn}\Theta^{+}_{\mathrm{p}}$



Results and work in progress...

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- Thus far, using this method, the known value of the self-force has been recovered for *circular orbits* (in agreement with the time-domain PwP and the results of other methods in the literature).

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- For future work, another objective is to also extend this method to *rotating black holes (Kerr)*.

Thanks for your attention!