Analytical Gravitational Self-force:

Non-circular motion+application to EOB

Chris Kavanagh

I.H.E.S

Donato Bini, Thibault Damour, Seth Hopper,

Adrian C. Ottewill & Barry Wardell

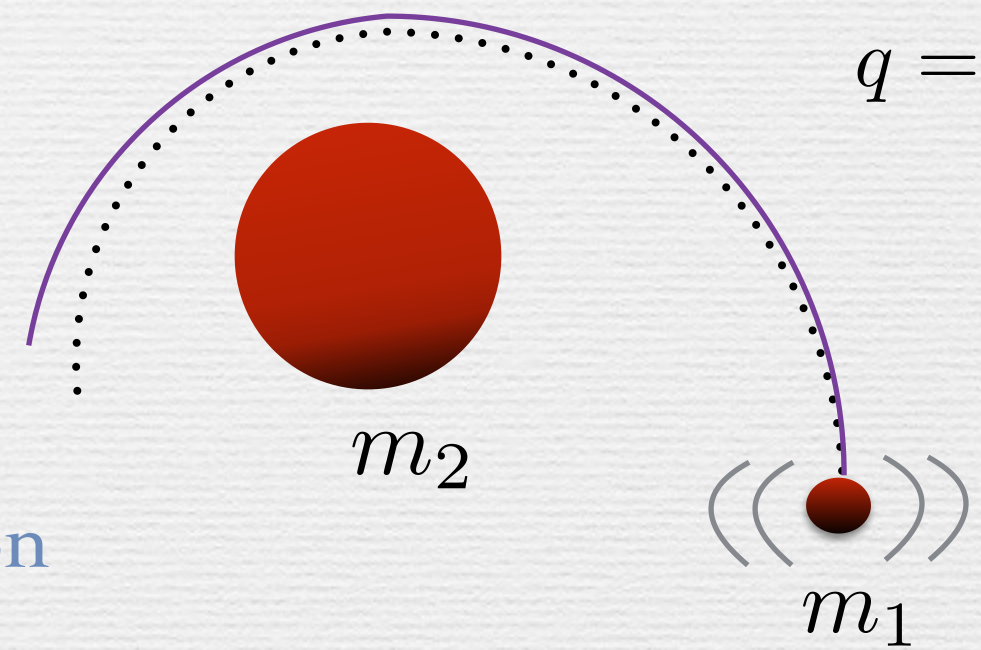
(mostly) based on arXiv:1706.00459 [gr-qc]

What do we mean? See M. Van de Meent tomorrow!

$$\boxed{\begin{aligned} m_1 \frac{Du^\alpha}{d\tau} &= q^2 F^\alpha[h] \\ \delta G_{\mu\nu}[h_{\mu\nu}] &= 8\pi T_{\mu\nu} \end{aligned}}$$

$$g_{\mu\nu} = g^0_{\mu\nu} + q h_{\mu\nu} + \mathcal{O}(q^2)$$

$$q = \frac{m_1}{m_2}$$



- Osculating assumption--for now, motion is geodesic.

<u>Aim</u>

- Calculate anything of interest in EMRI modelling without numerics--typically asymptotic expansions (think PN)

analytical SF $\longleftrightarrow$ post-Newtonian SF

Dissipative + conservative effects

$$F^\alpha[h] = F^\alpha_{\rm diss}[h] + F^\alpha_{\rm cons}[h] \longrightarrow \quad \text{e.g. shift in periastron adv.}$$

$$\text{e.g. } \langle \tfrac{1}{u^t} F_t \rangle = \langle \frac{dE^\infty}{dt} \rangle$$

- Fluxes are dominant, but 'easy'—exp. convergence

$$\frac{dE^\infty}{dt} = \sum_{\ell m} \frac{1}{4\pi\omega_m^2} |Z_{\ell m\omega}|^2 \qquad \psi_{0,\ell m\omega} \to Z_{\ell m\omega} r^3 e^{i\omega r^*}$$

– Teukolsky equation

$$\frac{dE^\infty}{dt} = -\frac{32}{5}\frac{1}{r^5}\left(1 - \frac{1247}{336r} + \frac{4\pi}{r^{1.5}} + \dots\right)$$

Finite PN order $\longrightarrow$ Finite ℓ contribution

- Their calculation is well developed analytically
  - Mano, Suzuki & Takasugi (~1996): analytic solns Teukolsky equation
  - eg. 22PN fluxes Schw & 11PN Kerr (Fujita 2015), Generic eccentric inclined Kerr (Sago, Fujita 2015).

Post-Newtonian SF: conservative sector

- Focus in recent years has been on linear in the mass ratio conservative effects, e.g. SF correction to classical redshift
    - Need to deal with regularisation - ~solved (Barack+collaborators, Detweiler-Whiting), e.g. mode-sum approach

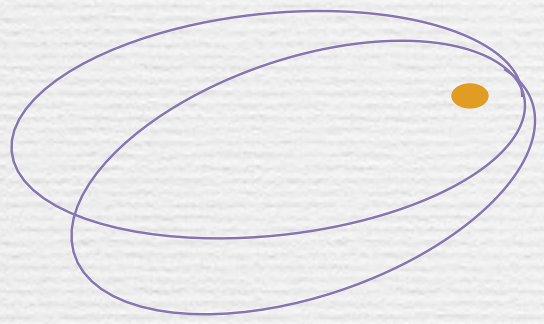
$$F^{\mu} = \sum_{l=0}^{\infty} \left(F^{\mu,\ell} \mp A^{\mu}(2\ell+1) - B^{\mu} + \mathcal{O}(\ell^{-2}) \right)$$

Polynomial convergence

— Not a problem for PN

Bini, Damour (2014)- Soln's Teukolsky equation valid for *all l's*

Non-circular case: Situation to date



$$r_p(\chi) = \frac{pm_2}{1 + e\cos\chi}, \qquad 0 < \chi < 2\pi$$

$$\frac{p}{1+e} < r_p/m_2 < \frac{p}{1-e} \qquad \Omega_r \neq \Omega_\varphi$$

$\frac{1}{p}, e \ll 1 :$

SF correction to u^t, encapsulated in $h_{\mu\nu}u^\mu u^\nu$

Generalized redshift $\langle U \rangle$:

- Bini et al (2016): 6.5PN, e^2 Schwarzschild
- Hopper et al (2016): 4PN, e^{10} Schw.
- Bini et al (2016): 8.5PN, e^2 Kerr.

we want more...

Non-circular case: next level

- Want self-force $F^\mu = \mathcal{P}^{\mu\nu\lambda\rho}(2h_{\nu\lambda;\rho} - h_{\lambda\rho;\nu})$
- Schwarzschild, but Radiation Gauge (eye on Kerr)

$$\cancel{\delta G_{\mu\nu}[h_{\mu\nu}] = 8\pi T_{\mu\nu}} \longrightarrow \boxed{{}_s\mathcal{O}\psi = \mathcal{T}[x^\mu(t)]}$$

Teukolsky equation

$$\longleftarrow$$

$$h_{\mu\nu}, h_{\mu\nu,\rho}$$

CCK reconstruction*

$$h^{\ell=2}_{tt,t} = (4\sin\chi\, e + 4\sin(2\chi)\, e^2 + \ldots)p^{-5/2} + (6 + 20\cos\chi e + (10 + 14\cos(2\chi))e^2 + \ldots)p^{-3} + \ldots$$

7 orders in $1/p$

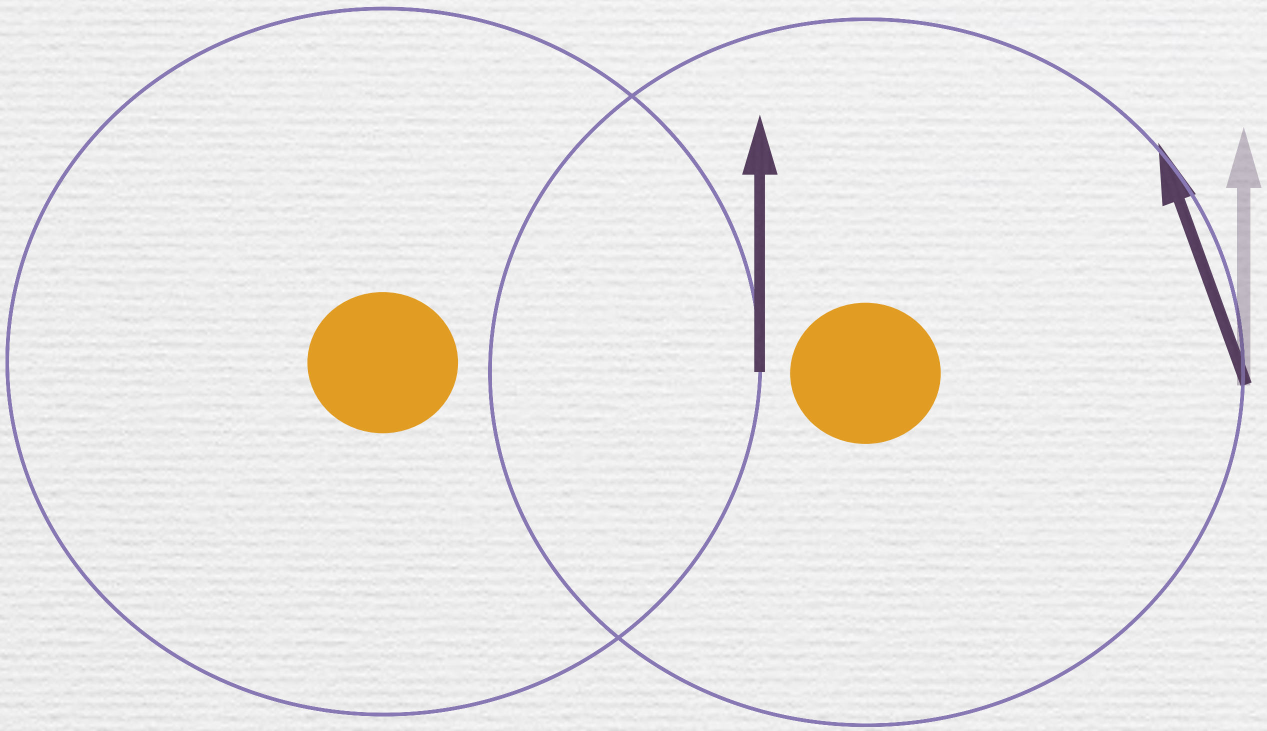
including to e^4

all ℓ

Can we do an inspiral? maybe, but first…

Spin-precession: Circular overview

The orientation of a test spin will precess on a curved background



$$u^a \nabla_a s^b = 0$$

$$\frac{d\mathbf{s}}{d\tau} = \omega_s \times \mathbf{s}$$

$$\psi = 1 - \frac{|\omega_s|}{u^\varphi}$$

- Dolan et al (2014)-- 1sf correction to this is gauge invariant - schw background

$$\psi = \psi_0 + q\Delta\psi + \dots$$

Spin-precession: Eccentric Schwarzschild

Akcay, Dolan & Dempsey (2016)

$$\psi = 1 - \frac{\Psi}{\Phi}$$

Ψ → Accumulated precession/ radial period

Φ → Acc. azimuthal phase/radial period

+prescription for calculating the 1-SF correction, explicitly requiring knowledge of the SF, and derivatives of the metric perturbation.

ADD give, strong field numerical data in Lorenz gauge, and a PN expansion from the full PN Hamiltonian:

Spin-precession: Eccentric Schwarzschild

Akcay, Dolan & Dempsey (2016)

ADD give, strong field numerical data in Lorenz gauge, and a 3PN expansion from the full PN hamiltionian:

- Add terms to 6PN
- Independent check of the gauge invariance.
- Complete the parameter space.
- Validate our code.

Spin-precession: Results

$$\Delta\psi = \Delta\psi_0 + \Delta\psi_2 e^2 + \mathcal{O}(e^4)$$

$$\Delta\psi_2 = \frac{1}{p^2} + \frac{\frac{341}{16} - \frac{123\pi^2}{256}}{p^3} + \frac{-\frac{164123}{480} + \frac{536\gamma}{5} - \frac{268\ln p}{5} - \frac{23729\pi^2}{4096} + \frac{11720\ln 2}{3} - \frac{10206\ln 3}{5}}{p^4}$$

$$+ \frac{-\frac{89576921}{57600} - \frac{22682\gamma}{15} + \frac{11341\ln p}{15} + \frac{21333485\pi^2}{49152} - \frac{4836254\ln 2}{105} + \frac{4430133\ln 3}{320} + \frac{9765625\ln 5}{1344}}{p^5} + \frac{\frac{319609\pi}{630}}{p^{11/2}}$$

$$- \frac{\frac{464068669129}{5080320} + \frac{2508913\gamma}{945} - \frac{2508913\ln p}{1890} - \frac{32088966503\pi^2}{2359296} + \frac{146026515\pi^4}{1048576} - \frac{273329813\ln 2}{945} + \frac{159335343\ln 3}{8960} + \frac{17193359375\ln}{145152}}{p^6}$$

$$+ \mathcal{O}(p^{-13/2})$$

- Leading orders agree
- Agreement within numerical error

Passed strenuous test of code.

Effective-one-body approach: See T. Damour tomorrow!

Proposal: Knowledge of the SF spin precession can determine terms in the EOB Hamiltonian (circ. case Bini,Damour...)

$$\mathcal{H}(R,P,S_1,S_2) = Mc^2\sqrt{1+2\nu\left(\frac{\mathcal{H}_{\rm eff}}{\mu c^2}-1\right)}$$

$$\mathcal{H}_{\rm eff} = \mathcal{H}^{\rm O}_{\rm eff} + \mathcal{H}^{\rm SO}_{\rm eff}$$

$$\mathcal{H}^{\rm SO}_{\rm eff} = \frac{G}{c^2R^3}\left(g_{\rm S}\mathbf{L}\cdot\mathbf{S} + g_{\rm S^*}\mathbf{L}\cdot\mathbf{S}^*\right)$$

ν-symmetric mass-ratio

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$\mathbf{S}^* = \frac{m_2}{m_1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2$$

The spin vectors satisfy a familiar equation..

$$\frac{d\mathbf{S}_a}{dt} = \{\mathcal{H},\mathbf{S}_a\} = \Omega_{\mathbf{S}_a}\times\mathbf{S}_a$$

$$\Omega_{\mathbf{S}_a} = \frac{\partial\mathcal{H}}{\partial\mathbf{S}_a}$$

EOB spin-precession

$$\frac{d\mathbf{S}_a}{dt} = \{\mathcal{H}, \mathbf{S}_a\} = \Omega_{\mathbf{S}_a} \times \mathbf{S}_a \qquad \Omega_{\mathbf{S}_a} = \frac{\partial \mathcal{H}}{\partial \mathbf{S}_a}$$

Strategy: 1) Set up same type situation..

$$m_1 \ll m_2, \quad S_1 \ll 1, \quad S_2 = 0, \quad P_R \neq 0, \quad \mathbf{L}\cdot\mathbf{S} = P_\varphi S$$

$$\longrightarrow \boxed{\psi \equiv \frac{\langle \Omega_{\mathrm{S}_1} \rangle}{\Omega_\varphi}}$$ is the same function!

2) Extract SF info by equating

$$\mathcal{O}(\nu) \text{ piece of } \psi^{\mathrm{EOB}} \longleftrightarrow \Delta\psi$$

via gauge inv. parameterisation.

(straightforward, but delicate)

EOB spin-precession: what does SF give?

e.g.

$$M\Omega_{S_1} = p_\phi(1-\nu\hat{\mathcal{E}}_{\rm eff})u^3[\nu g_S + (1-\nu)g_{S*}] + \mathcal{O}(\nu^2)$$

$$g_S = 2 + \nu g_S^1(u,p_r) + \mathcal{O}(\nu^2)$$
$$g_{S^*} = g_{S^*}^0(u,p_r) + \nu g_{S^*}^1(u,p_r) + \mathcal{O}(\nu^2)$$

$\Delta\psi$ picks out $g^1_{\rm S*}(u,p_r)$

$$g^1_{\rm S*}(u,p_r) = g^{1,0}_{\rm S*}(u) + g^{1,2}_{\rm S*}(u)p_r^2 + \mathcal{O}(p_r^4)$$

$$p_r^2 \sim \mathcal{O}(e^2)$$

EOB spin-precession: Results

$$g_{S^*}^{1,2}(u) = -\frac{9}{4} - \frac{9}{4}u - \frac{717}{32}u^2 + \left(\frac{1447441}{960} - \frac{4829}{256}\pi^2 - \frac{16038}{5}\ln(3) + \frac{46976}{15}\ln(2) - \frac{512}{5} + \gamma - \frac{256}{5}\ln(u)\right)u^3$$
$$-\left(\frac{185195453}{38400} + \frac{19162}{35}\gamma + \frac{2097479}{8192}\pi^2 + \frac{454167}{20}\ln(3) - \frac{1081966}{35}\ln(2) + \frac{9581}{35}\ln(u)\right)u^4 + \ldots$$

..The coefficient of p_r^2 in the linear in ν piece of

$$\mathcal{H}_{\text{eff}}^{\text{SO}} = \frac{G}{c^2 R^3}\left(g_{\text{S}}\mathbf{L}\cdot\mathbf{S} + g_{\text{S}^*}\mathbf{L}\cdot\mathbf{S}^*\right)$$

i.e. EOB can handle spin-orbit interactions with greater accuracy!

Conclusions and outlook

Results

1. $h_{\mu\nu}, h_{\mu\nu,\rho}, F^{\mu}$ – high order PN + eccentricity expansions
2. Extended knowledge of $\Delta\psi$, verified results of Akcay Dolan & Dempsey
3. Improved knowledge of gyrogravitomagnetic ratio g_{S^*}

What's next?

1. Investigate the SF in radiation gauge?
2. Application to osculating inspiral?
3. PN always needs resummation!
4. SF in Kerr spacetime