Recalibrated waveforms for EMRIs in the EOB frame

HAN Wenbiao (韩文标)
@Shanghai Astronomical Observatory, CAS;上海天文台

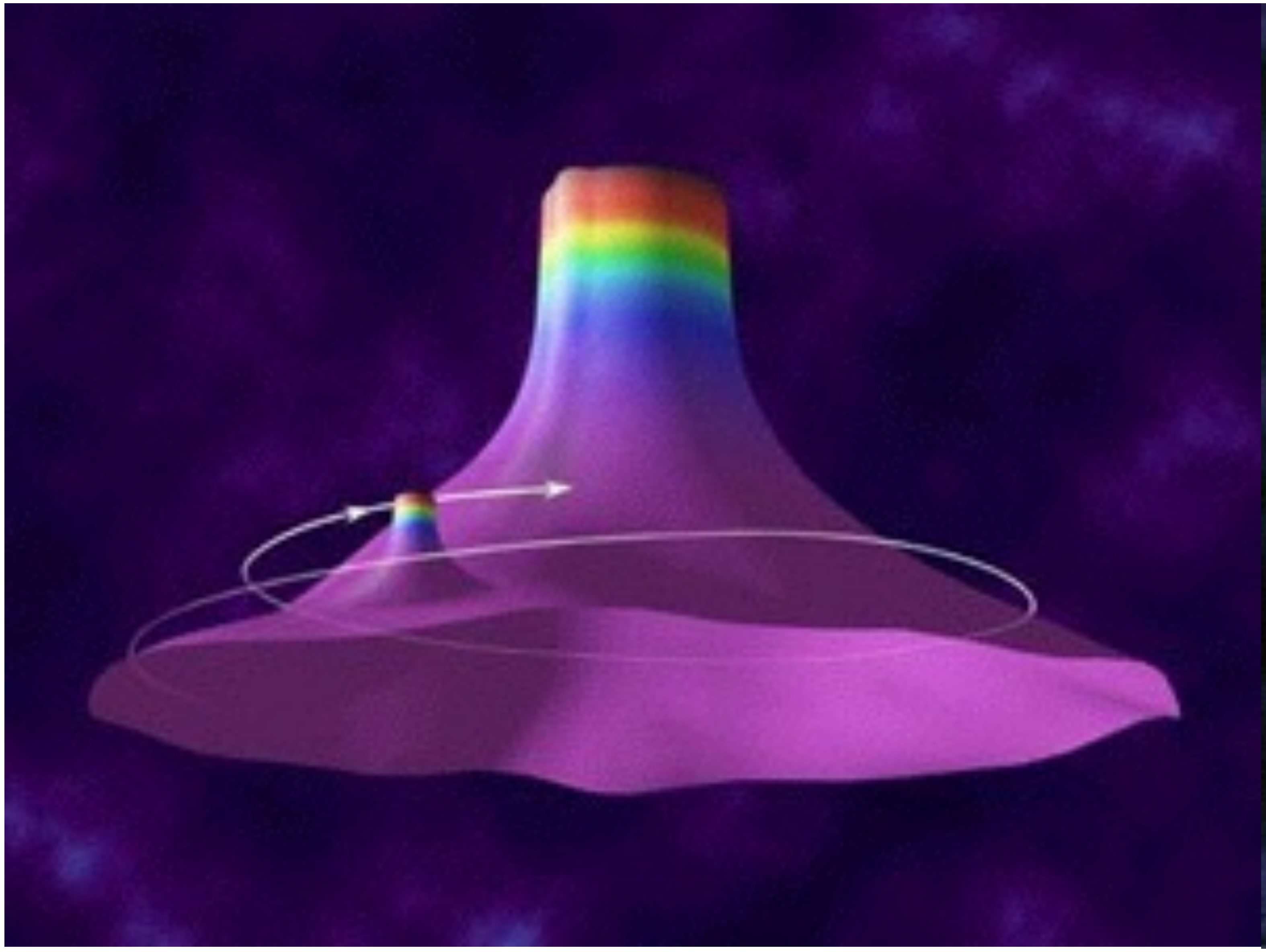
The Era of Gravitational-wave Astronomy@Paris

2017.6.28

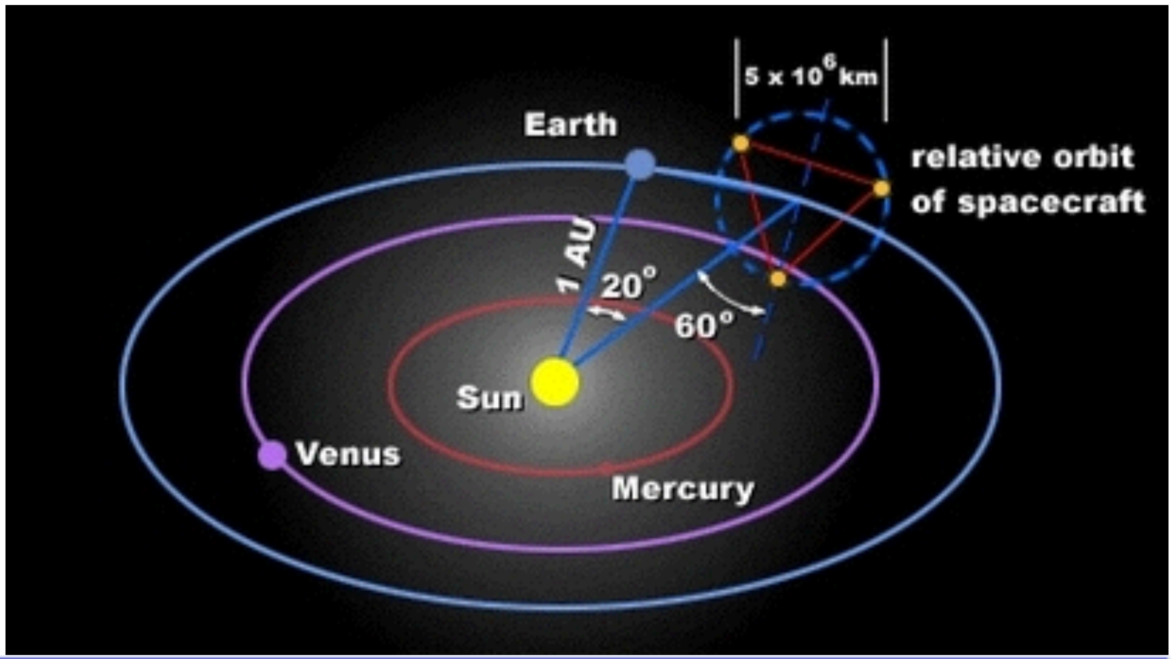
Outline

- Background and motivations;
- Numerical simulation for EMRIs;
- Effective-one-body formalism;
- Recalibrated waveforms in the EOB frame;
- Conclusions and prospect;

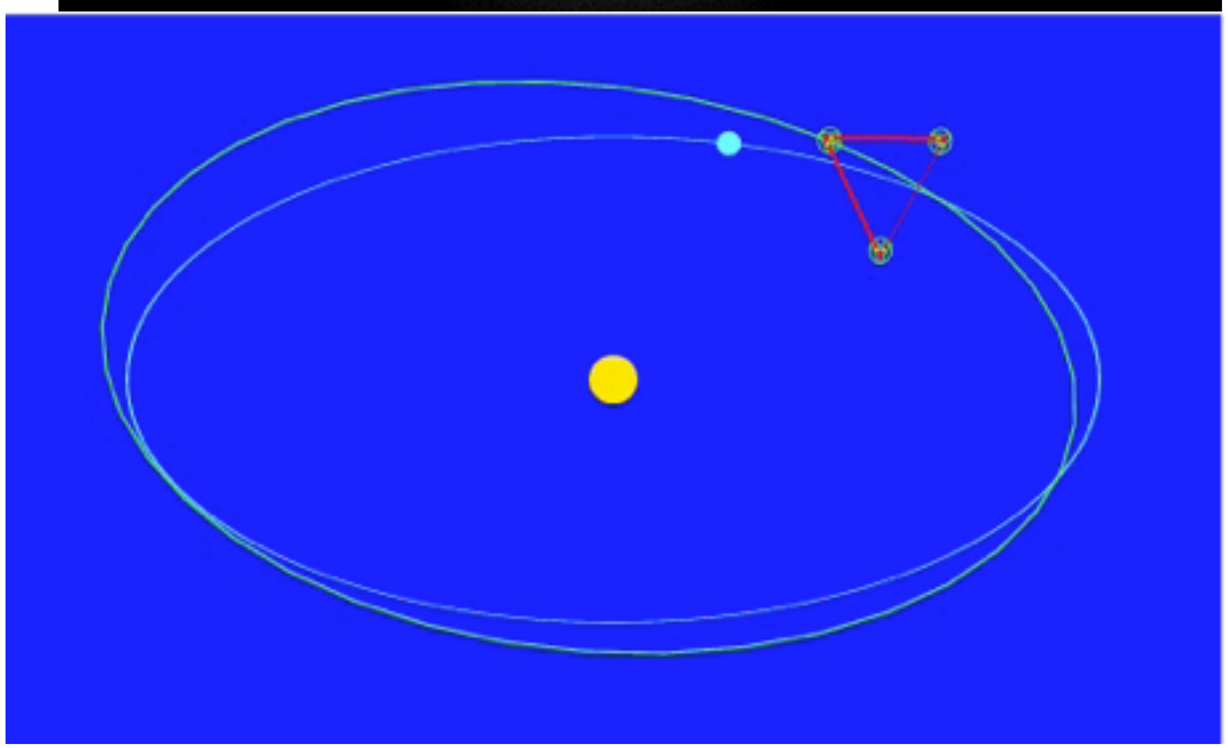
Extreme-mass-ratio inspirals: EMRIs

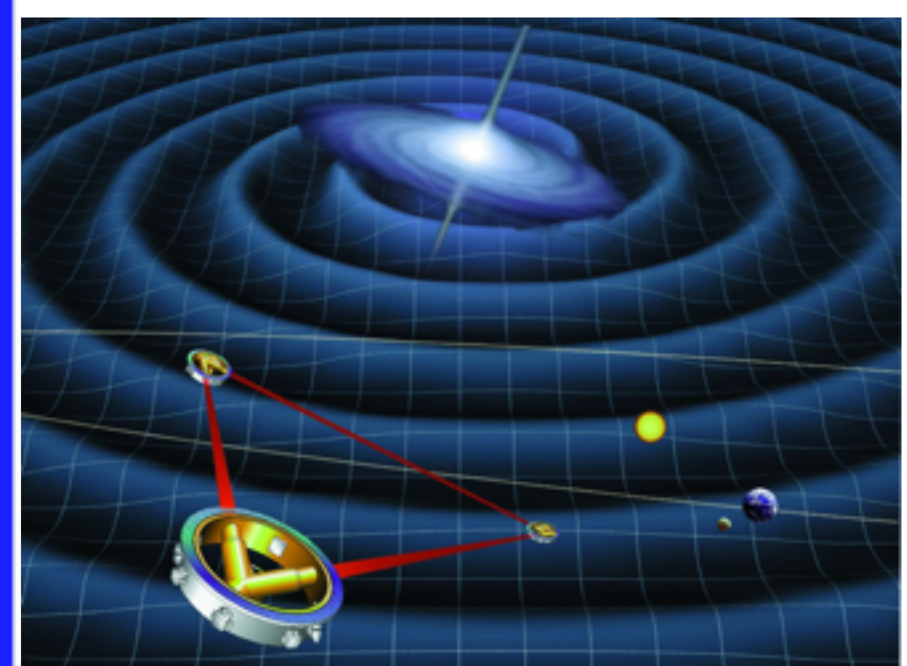


LISA

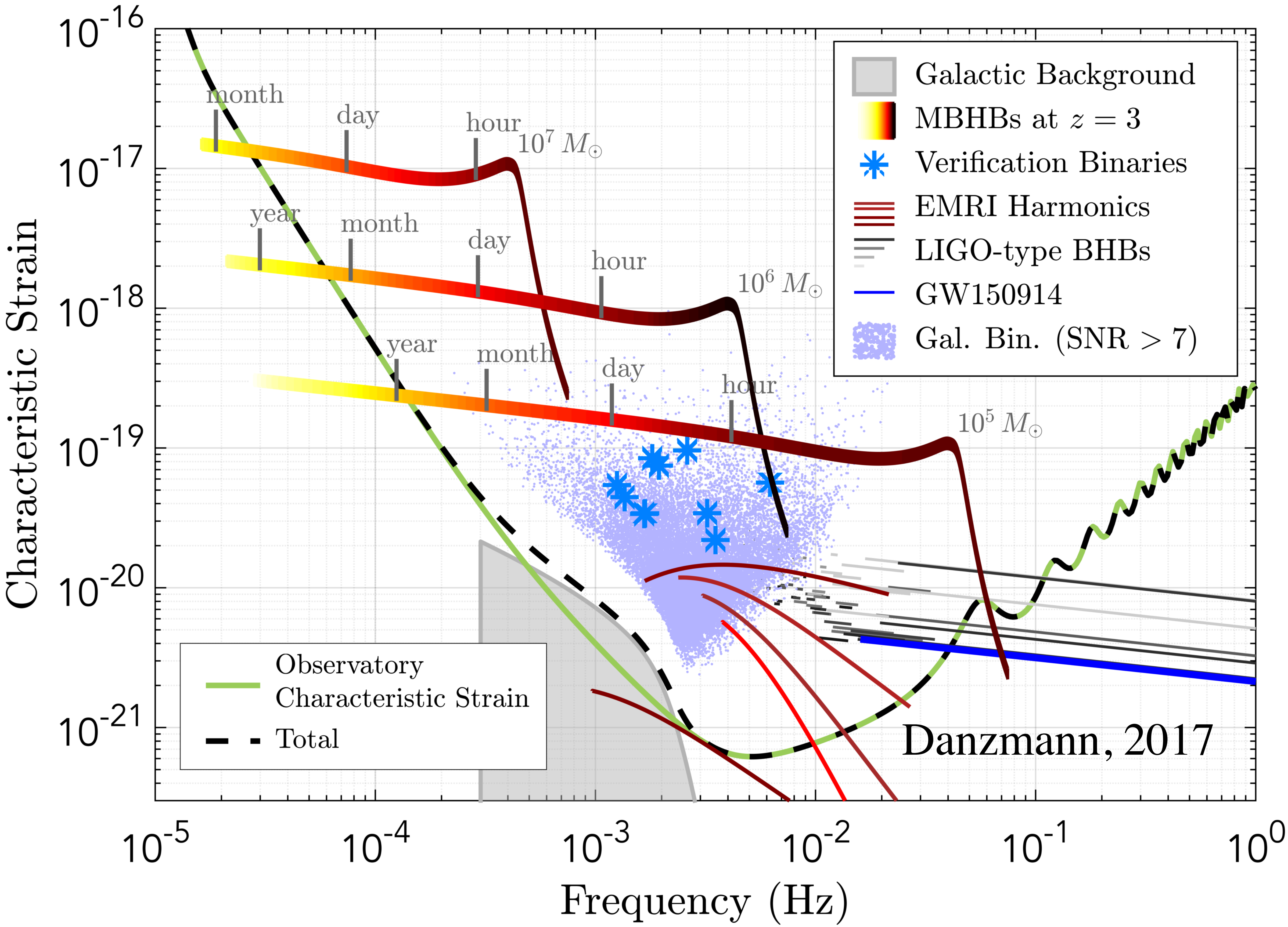


Chinese space-based projects:Taiji, Tianqin





eLISA pathfinder has gotten good results



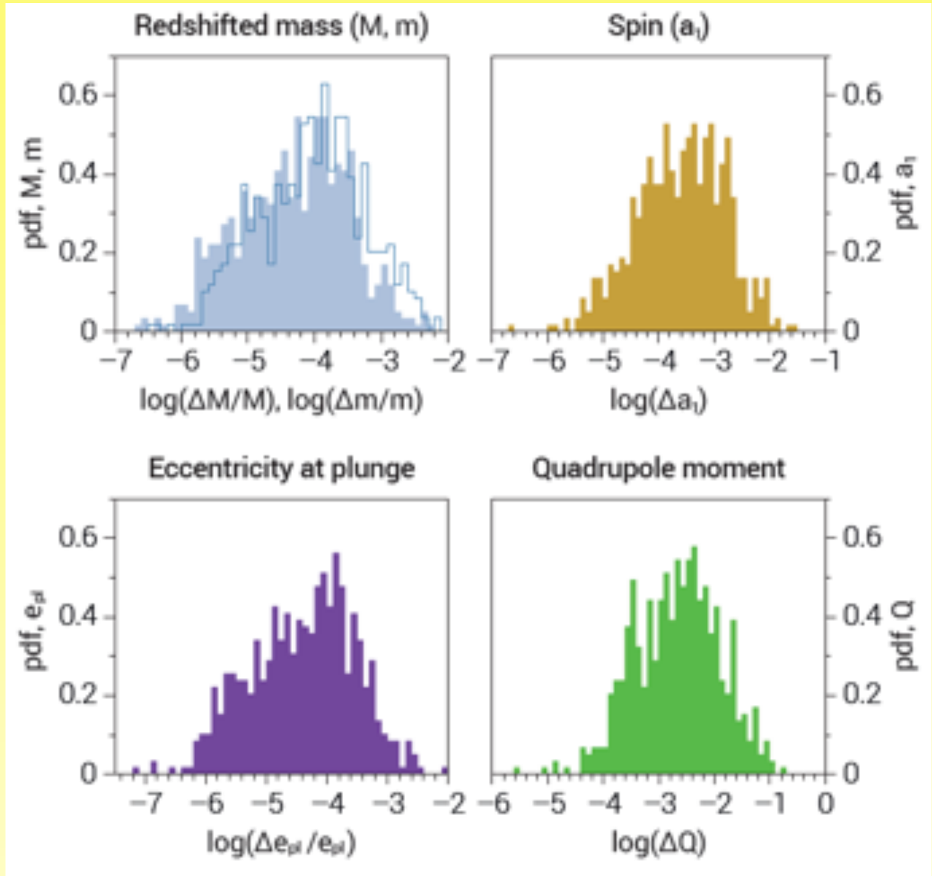
Galactic Background
MBHBs at $z = 3$
Verification Binaries
EMRI Harmonics
LIGO-type BHBs
GW150914
Gal. Bin. (SNR > 7)
Observatory Characteristic Strain
Total
Danzmann, 2017
month
day
hour
$10^7\ M_\odot$
year
month
day
hour
$10^6\ M_\odot$
year
month
day
hour
$10^5\ M_\odot$
Characteristic Strain
Frequency (Hz)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

Why studying EMRIs

- very long time scale
- SNR 20 up to z=0.7, dozens/year
- studying physics near horizon of SMBH
- testing gravitation theory
- Cosmology

Extreme Mass Ratio Inspirals

- SNR 20 up to $z \approx 0.7$ for 10^5-$10^6\ M_\odot$
- Dozens of events per year
- Mass, spin to 0.1% – 0.01 %
- Quadrupole moment to $< 0.001\ M_\odot^3 G^2/c^4$



- Do Black Holes have hair?
  - New objects in General Relativity
    - Boson Stars, Gravastars, non-Kerr solutions (e.g. Manko-Novikov)
  - Deviations from General Relativity
    - Chern-Simons, Scalar-Tensor, light scalar fields (axions) and black hole bomb instabilities
- Each has specific GW fingerprint!

From Danzmann, 2017 May 25, Beijing

Challenge:

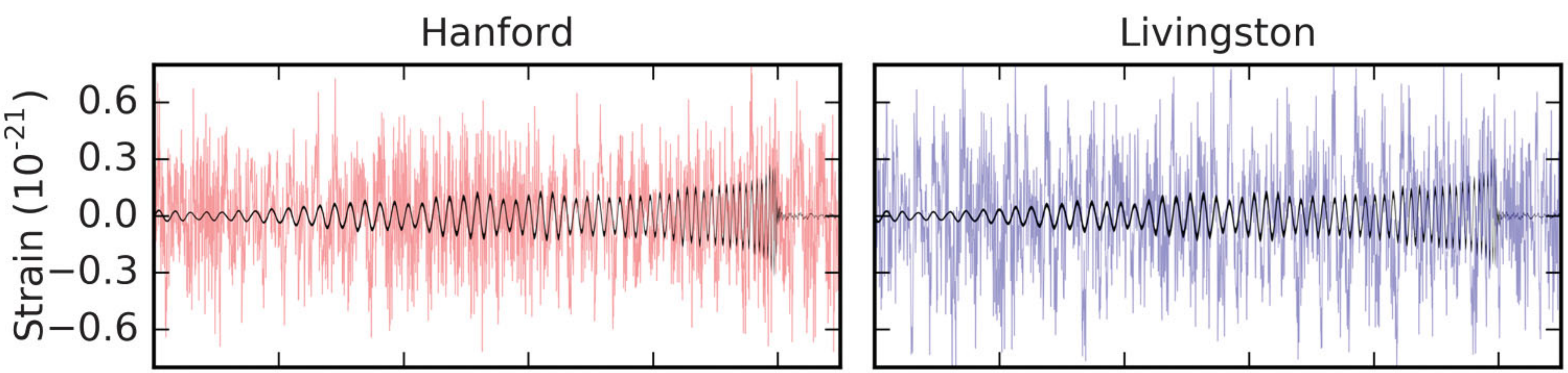
Firstly, we should have huge numbers of waveform templates of EMRIs with high accuracy.

The role of waveform templates
Matched filtering: find GWs from noise

GW150914: 20, 4.6σ → 24, 5.1σ

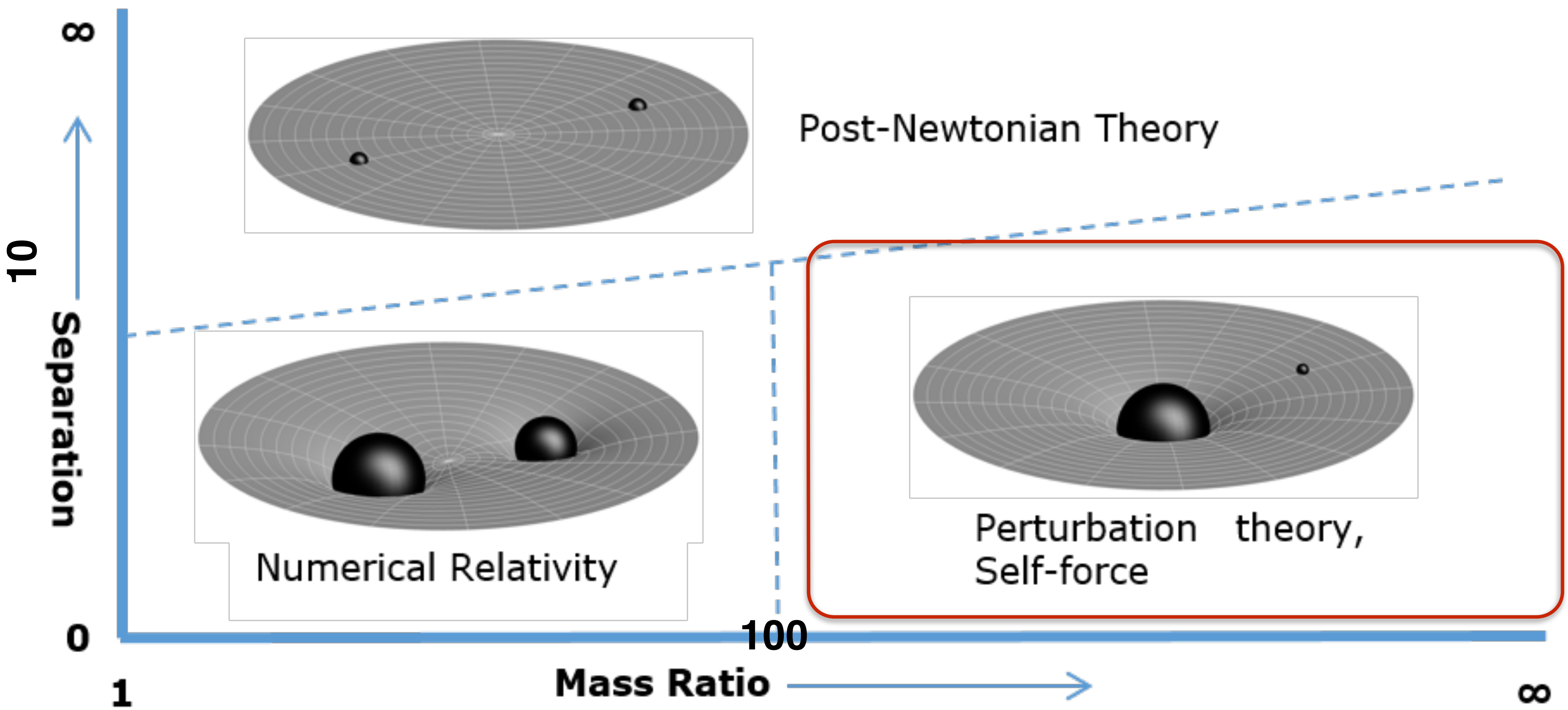
EOBNR

GW151226: ? → 13, 5.0σ



Recognize the parameters of binaries

EMRIs : Calculation method of waveforms



Teukolsky equation

$$\left[\frac{\left(r^2+a^2\right)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\Psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\Psi}{\partial t\partial\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\Psi}{\partial\phi^2} -$$
$$-\Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\Psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) - 2s\left[\frac{a\,(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\Psi}{\partial\phi} -$$
$$-2s\left[\frac{M\,(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\Psi}{\partial t} + \left(s^2\cot^2\theta - s\right)\Psi = 0\,.$$

energy flux, waveform

$$\frac{dE}{dt} = \lim_{r\to\infty}\left[\frac{1}{4\pi}\int_\Omega\left|\int_{-\infty}^t \psi d\tilde{t}\right|^2 d\Omega\right],$$
$$\frac{dP_i}{dt} = \lim_{r\to\infty}\left[\frac{1}{4\pi}\int_\Omega l_i\left|\int_{-\infty}^t \psi d\tilde{t}\right|^2 d\Omega\right],$$
$$\frac{dL}{dt} = -\lim_{r\to\infty}\left\{\frac{1}{4\pi}\mathrm{Re}\left[\int_\Omega\left(\partial_\phi\int_{-\infty}^t \psi d\tilde{t}\right)\left(\int_{-\infty}^t\int_{-\infty}^t \bar{\psi} d\tilde{t} d\hat{t}\right) d\Omega\right]\right\}$$

$$\psi \approx \frac{1}{2}\left(\frac{\partial^2 h_+}{\partial t^2} - i\frac{\partial^2 h_\times}{\partial t^2}\right)$$

Describe the orbit of small body

Effective-one-body (EOB) dynamics

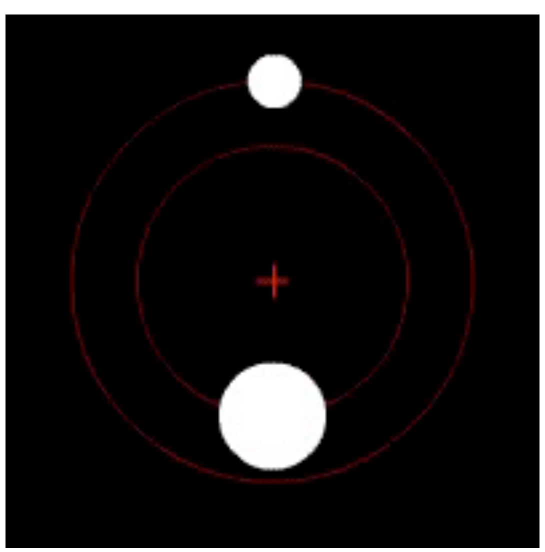
Buonanno & Damour, 1999, 2000, ……

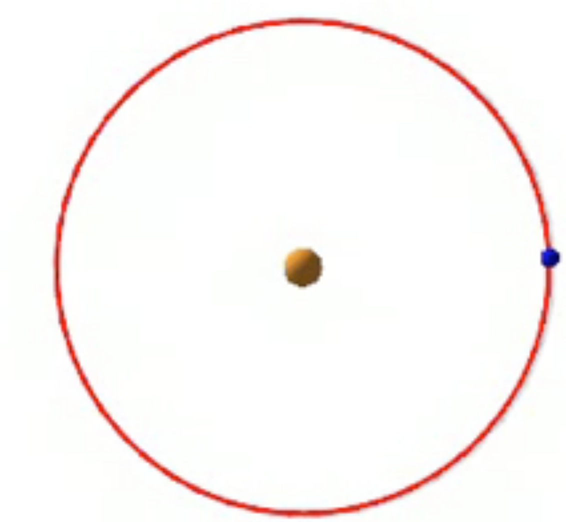
Two-body problem → one-body problem

$m_1 \quad m_2$

$$M = m_1 + m_2$$

$$\mu = m_1 m_2 / M$$





EOB formalism: dynamics

$$\dot{r} = \frac{\partial H_{\rm EOB}}{\partial p_r},$$

$$\dot{\phi} = \frac{\partial H_{\rm EOB}}{\partial p_\phi},$$

$$\dot{p_r} = -\frac{\partial H_{\rm EOB}}{\partial r} + \mathcal{F}_\phi \frac{p_r}{p_\phi},$$

$$\dot{p_\phi} = \mathcal{F}_\phi,$$

radiation reaction

$$H_{\rm EOB} = M\sqrt{1 + 2\nu(H_{\rm eff}/\mu - 1)}$$

$$H_{\rm eff} = H_{\rm NS} + H_{\rm S} - \frac{\mu}{2Mr^3} S_*^2$$

$$H_{\rm NS} = \beta^i p_i + \alpha\sqrt{\mu^2 + \gamma^{ij} p_i p_j}$$

$$H_{\rm S} = H_{\rm SO} + H_{\rm SS}$$

$$\alpha = \frac{1}{\sqrt{-g^{tt}}},$$

$$\beta^i = \frac{g^{ti}}{g^{tt}},$$

$$\gamma^{ij} = g^{ij} - \frac{g^{ti} g^{tj}}{g^{tt}},$$

$$\hat{\boldsymbol{\mathcal{F}}} = \frac{-1}{\nu\hat{\Omega}|\boldsymbol{r} \times \boldsymbol{p}|} \frac{dE}{dt} \boldsymbol{p},$$

$$\frac{dE}{dt} = \frac{\hat{\Omega}^2}{8\pi} \sum_{\ell=2}^{8} \sum_{m=0}^{\ell} m^2 \left| \frac{\mathcal{R}}{M} h_{\ell m} \right|^2$$

EOB formalism: waveforms of circular orbits

$$h_{\ell m} = h_{\ell m}^{\mathrm{insp-plunge}}\,\theta(t_{\mathrm{match}}^{\ell m} - t) + h_{\ell m}^{\mathrm{merger-RD}}\,\theta(t - t_{\mathrm{match}}^{\ell m})$$

$$h_{\ell m}^{\mathrm{insp-plunge}} = h_{\ell m}^{\mathrm{F}} N_{\ell m}$$

$$h_{\ell m}^{\mathrm{F}} = h_{\ell m}^{(N,\epsilon)}\,\hat{S}_{\mathrm{eff}}^{(\epsilon)}\,T_{\ell m}\,e^{i\delta_{\ell m}}(\rho_{\ell m})^{\ell}$$

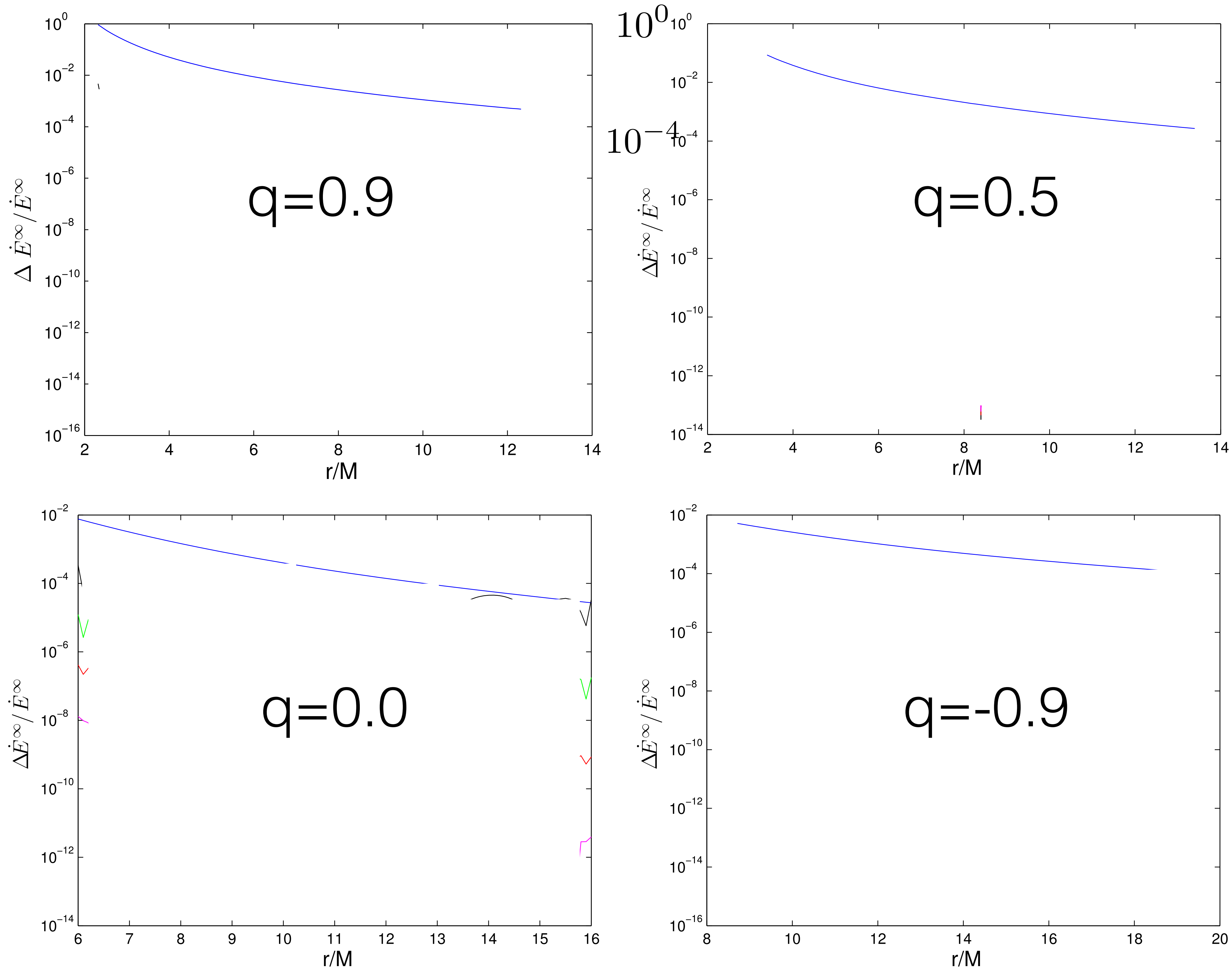
factorized waveform

Pan et al., PRD 84, 124052 (2011)

$$h_{\ell m}^{\mathrm{merger-RD}}(t) = \sum_{n=0}^{N-1} A_{\ell m n}\,e^{-i\sigma_{\ell m n}(t - t_{\mathrm{match}}^{\ell m})}$$

$$\begin{aligned}\rho_{22} = {} & 1 + \left(\frac{55\nu}{84} - \frac{43}{42}\right)v_\Omega^2 + \left(\frac{19583\nu^2}{42336} - \frac{33025\nu}{21168} - \frac{20555}{10584}\right)v_\Omega^4 + \left(\frac{10620745\nu^3}{39118464} - \frac{6292061\nu^2}{3259872} + \frac{41\pi^2\nu}{192} - \frac{48993925\nu}{9779616}\right. \\ & \left. - \frac{428\,\mathrm{eulerlog}_2(v_\Omega^2)}{105} + \frac{1556919113}{122245200}\right)v_\Omega^6 + \left(\frac{9202\,\mathrm{eulerlog}_2(v_\Omega^2)}{2205} - \frac{387216563023}{160190110080}\right)v_\Omega^8 \\ & + \left(\frac{439877\,\mathrm{eulerlog}_2(v_\Omega^2)}{55566} - \frac{16094530514677}{533967033600}\right)v_\Omega^{10}, \qquad \text{(B9a)}\end{aligned}$$

Accuracy of factorized PN waveform applying to EMRI



Accuracy requirement of EMRIs

- A typical EMRIs have M/μ cycles
- During the inspirals, for dephase less than 2pi, one asks

$$\Delta \dot{E}/\dot{E} < \mu/M$$

- So for typical mass-ratio 10^5, the relative precision should be 10^-5.
- F-R PN waveforms break down even for circular orbits

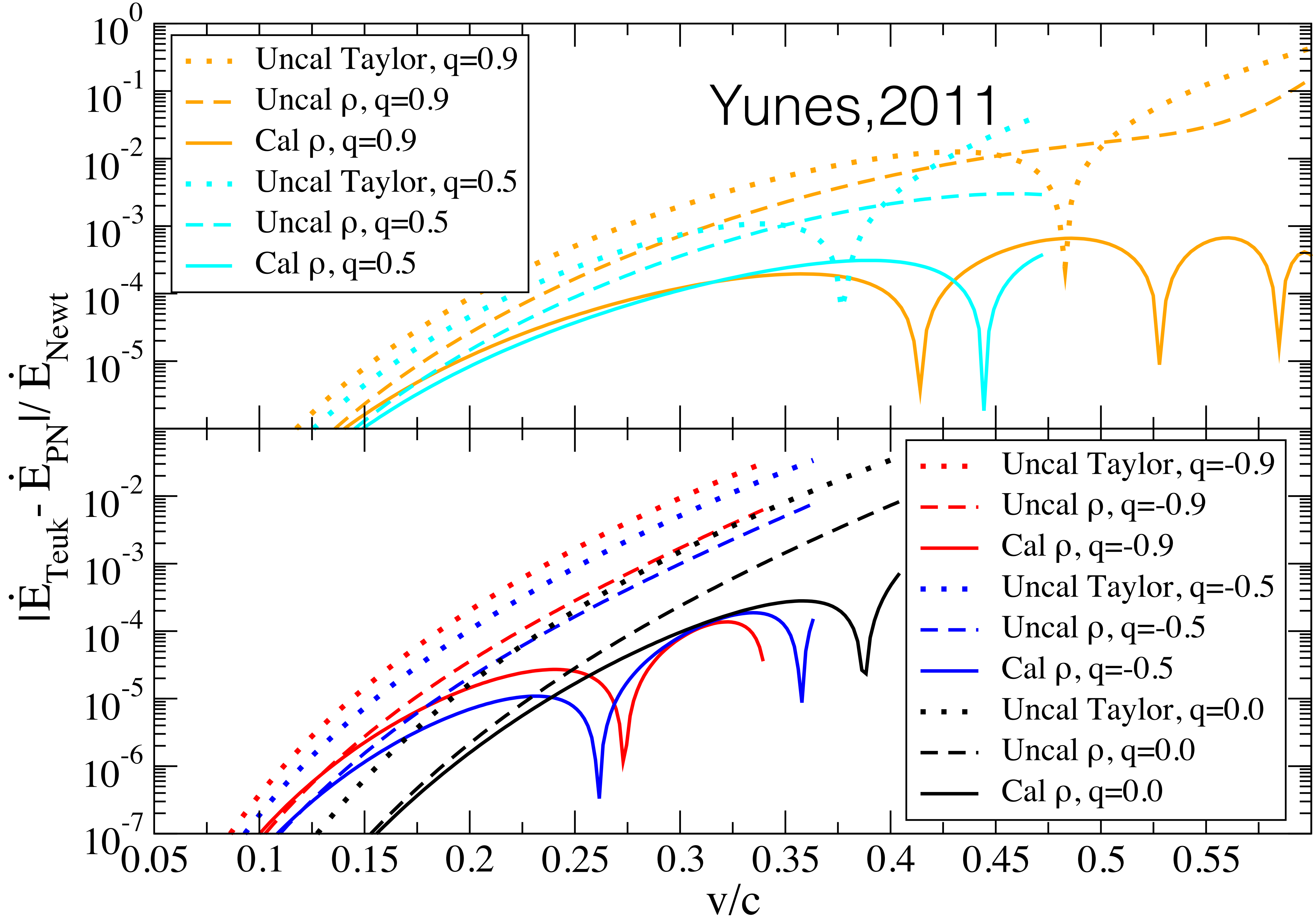
Highly accurate and efficient waveforms

- Numerical simulations are inefficient;
- We try to work on semi-analytical models
- Yunes et al., 2011

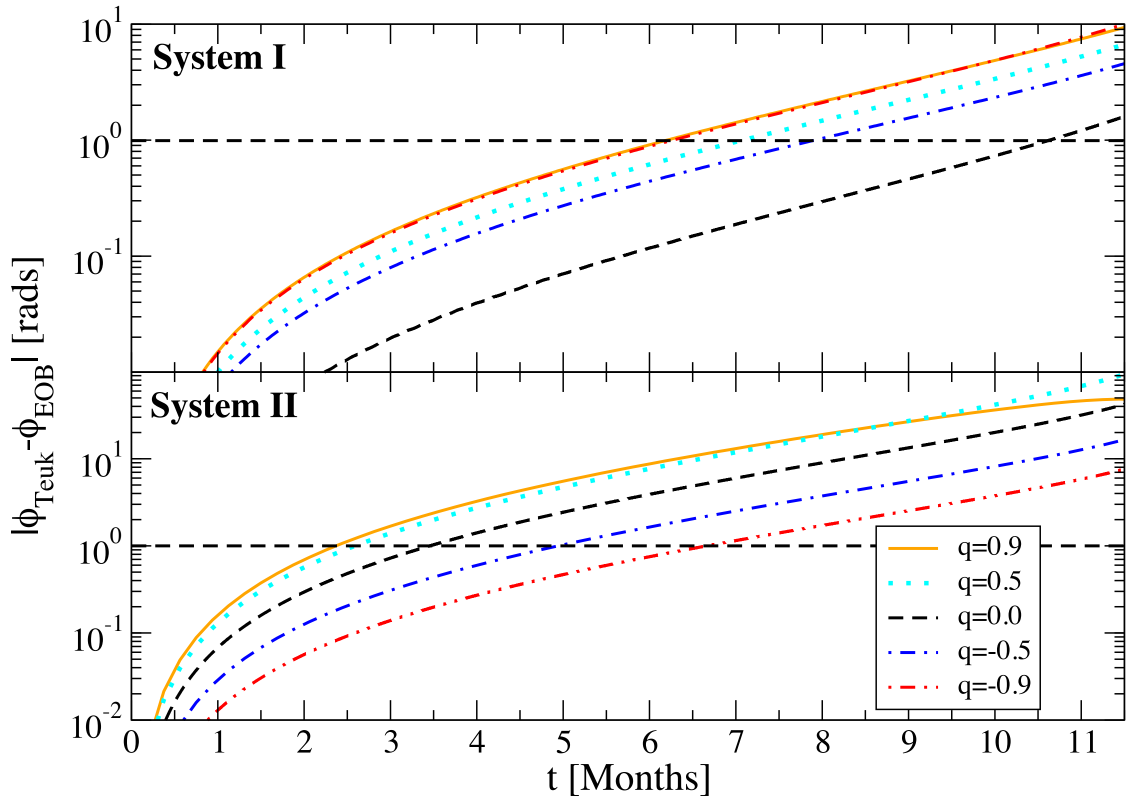
$$\rho^{22}_{\rm Cal} = \rho^{22} + \left[a^{(9,1)}_{22} + b^{(9,1)}_{22}\,\text{eulerlog}_2 v^2\right]\,\bar{q}\,v^9 + \left[a^{(12,0)}_{22} + b^{(12,0)}_{22}\,\text{eulerlog}_2 v^2\right] v^{12}\,,$$

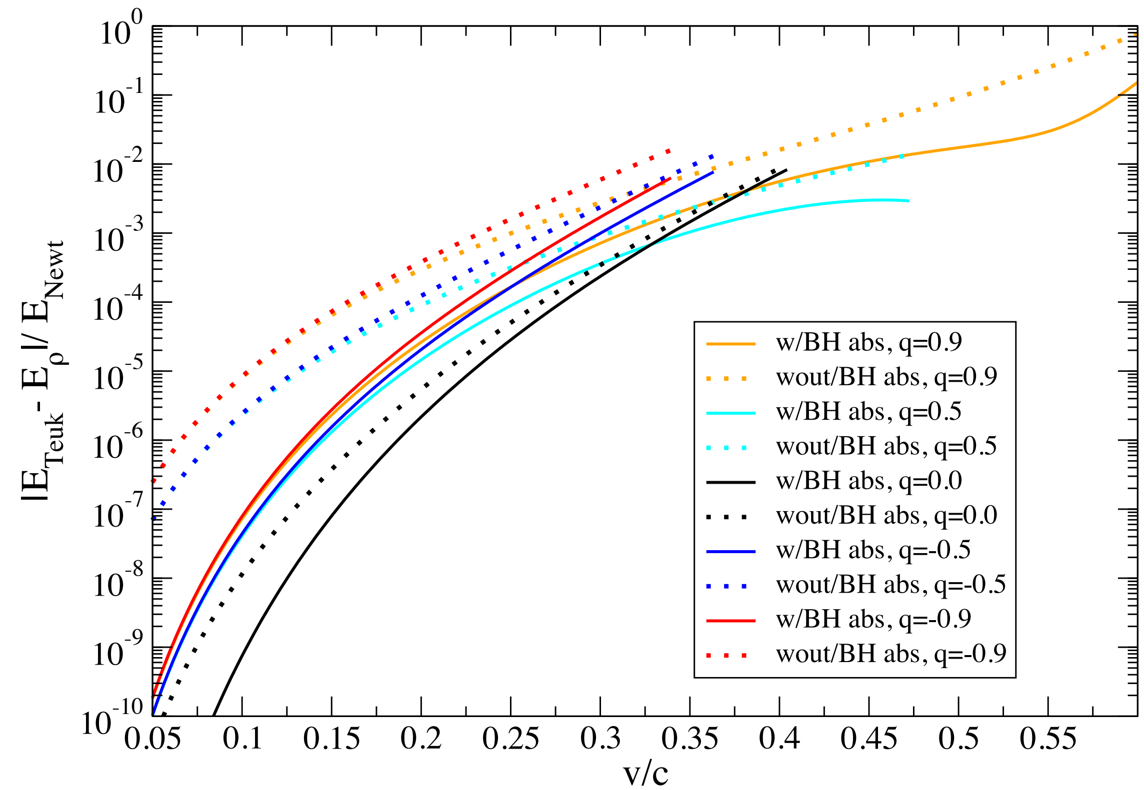
$$\rho^{33}_{\rm Cal} = \rho^{33} + \left[a^{(8,2)}_{33} + b^{(8,2)}_{33}\,\text{eulerlog}_3 v^2\right]\,\bar{q}^2\,v^8 + \left[a^{(10,0)}_{33} + b^{(10,0)}_{33}\,\text{eulerlog}_3 v^2\right] v^{10}\,,$$

Yunes’ s results:



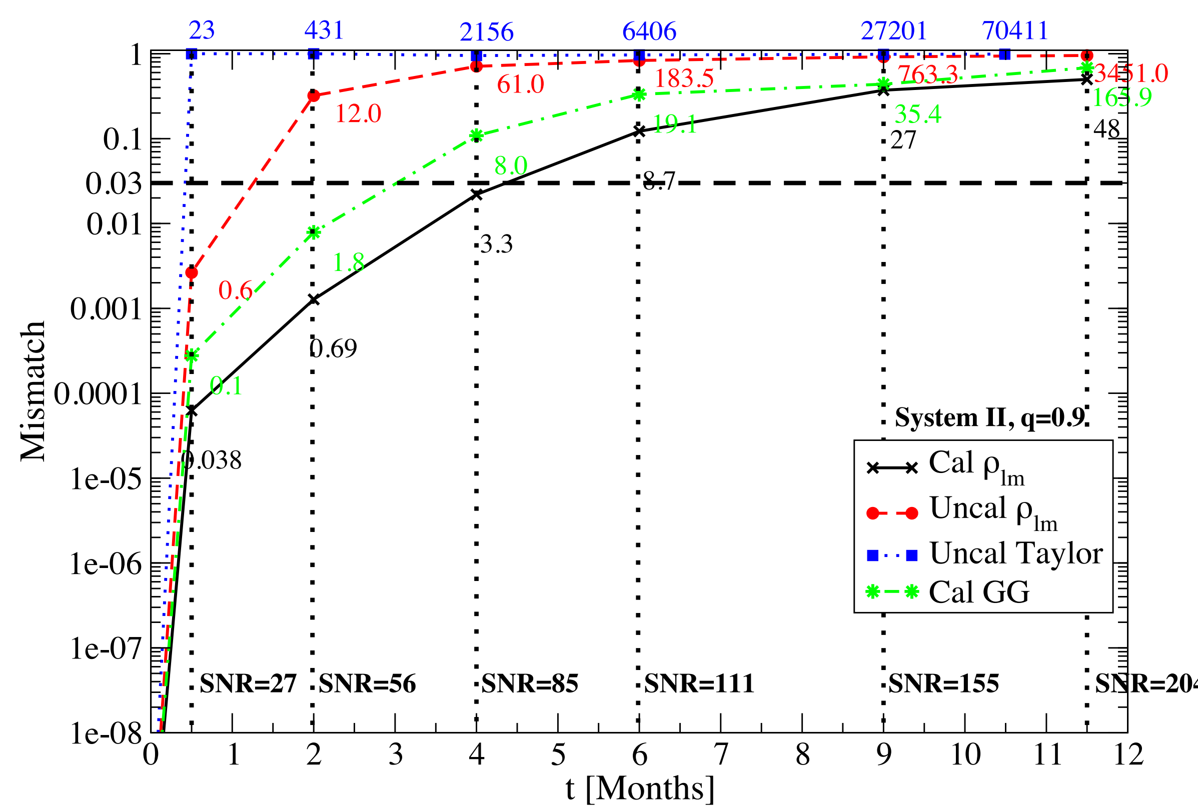
Yunes' s results:





SysI: 10 vs 10^5 solar mass

SysII: 10 vs 10^6 solar mass



Our model: fully calibrated version1-polynomials

$$\dot{E}^{\infty} = \sum_{i=0}^{n} a_i' x^i, \quad \dot{E}^{\mathrm{H}} = \sum_{i=0}^{n} b_i' x^i,$$

$$\mathrm{Re}[H_{lm}] = \sum_{i=0}^{n} R_{lm}'^{i} x^i, \quad \mathrm{Im}[H_{lm}] = \sum_{i=0}^{n} I_{lm}'^{i} x^i.$$

Han, CQG, 2016

Table 1. polynomial parameters for infinity fluxes.

	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
$a = 0.9$	1.51e-7	-1.22e-5	4.40e-4	-9.43e-3	1.36e-1	4.90e0	-5.27e0	-1.05e1	2.98e1	-4.34e1	3.99e1
$a = 0.7$	6.61e-7	-5.98e-5	2.41e-3	-5.69e-2	8.76e-1	-2.90e0	5.22e1	-2.75e2	8.81e2	-1.64e3	1.44e3
$a = 0.0$	1.46e-6	-1.72e-4	9.08e-3	-2.83e-1	5.76e0	-7.39e1	7.63e2	-5.02e3	2.21e4	-5.77e4	7.19e4
$a = -0.9$	1.26e-6	-1.85e-4	1.22e-2	-4.78e-1	1.22e1	-2.09e2	2.63e3	-2.21e4	1.25e5	-4.20e5	6.78e5

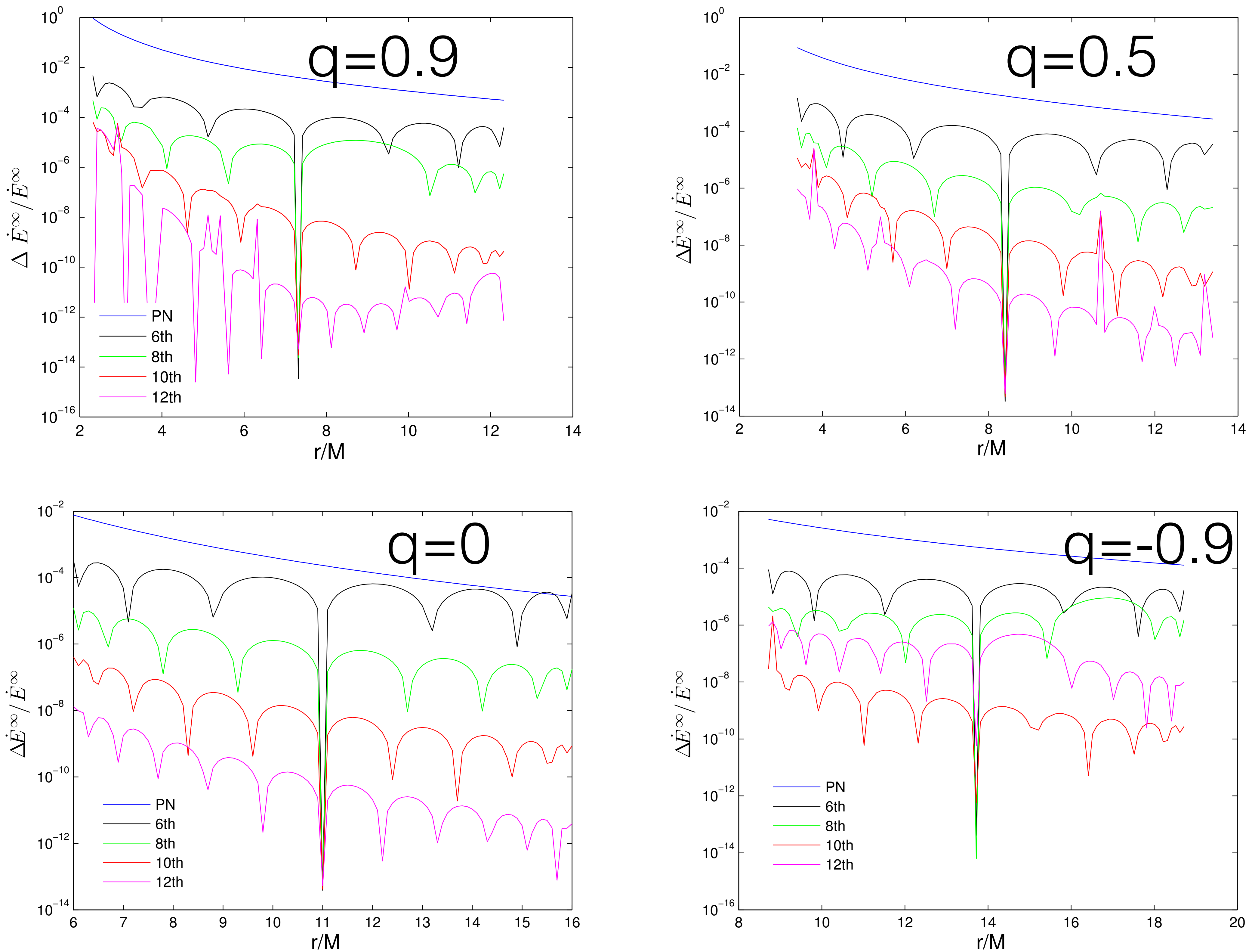
Table 2. polynomial parameters for horizon fluxes.

	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}
$a = 0.9$	-3.44e-9	2.93e-7	-1.09e-5	2.25e-4	-2.66e-3	1.34e-2	1.07e-1	-3.19e0	8.84e0	-8.33e0	2.37e0
$a = 0.7$	8.88e-8	-7.97e-6	3.17e-4	-7.39e-3	1.11e-1	-1.14e0	8.00e0	-3.89e1	1.17e2	-2.04e2	1.58e2
$a = 0.0$	4.84e-7	-5.70e-5	3.00e-3	-9.28e-2	1.88e0	-2.59e1	2.48e2	-1.62e3	6.97e3	-1.79e4	2.11e4
$a = -0.9$	5.30e-7	-7.76e-5	5.11e-3	-1.99e-1	5.08e0	-8.90e1	1.08e3	-9.10e3	5.05e4	-1.69e5	2.63e5

Table 3. polynomial coefficients for waveform (2,2) mode: real part.

	R^0_{22}	R^1_{22}	R^2_{22}	R^3_{22}	R^4_{22}	R^5_{22}	R^6_{22}	R^7_{22}	R^8_{22}	R^9_{22}	R^{10}_{22}
$a = 0.9$	4.00e-5	4.96e-1	-9.20e-1	-3.43e-1	-6.23e0	1.73e2	-1.02e3	3.20e3	-6.14e3	6.78e3	-3.27e3
$a = 0.7$	2.88e-5	4.96e-1	-9.20e-1	1.03e0	-1.98e1	2.75e2	-1.57e3	5.33e3	-1.18e4	1.54e4	-9.09e3
$a = 0.0$	1.78e-5	4.97e-1	-8.67e-1	5.44e0	-6.22e1	6.73e2	-4.23e3	1.89e4	-5.58e4	9.83e4	-7.12e4
$a = -0.9$	1.38e-5	4.97e-1	-7.79e-1	1.17e1	-1.17e2	1.34e3	-9.37e3	4.84e4	-1.46e5	2.09e5	1.13e5

Our model: Fully calibrated version1



Our model: Fully calibrated version1

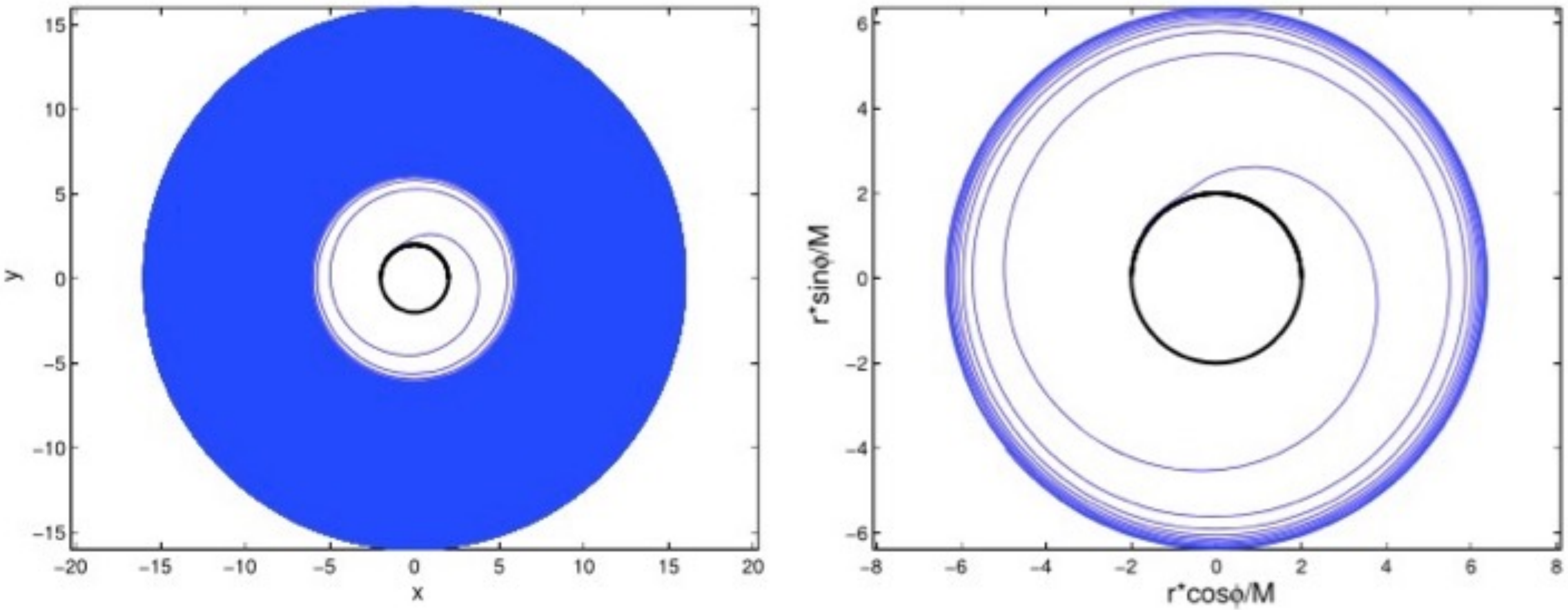
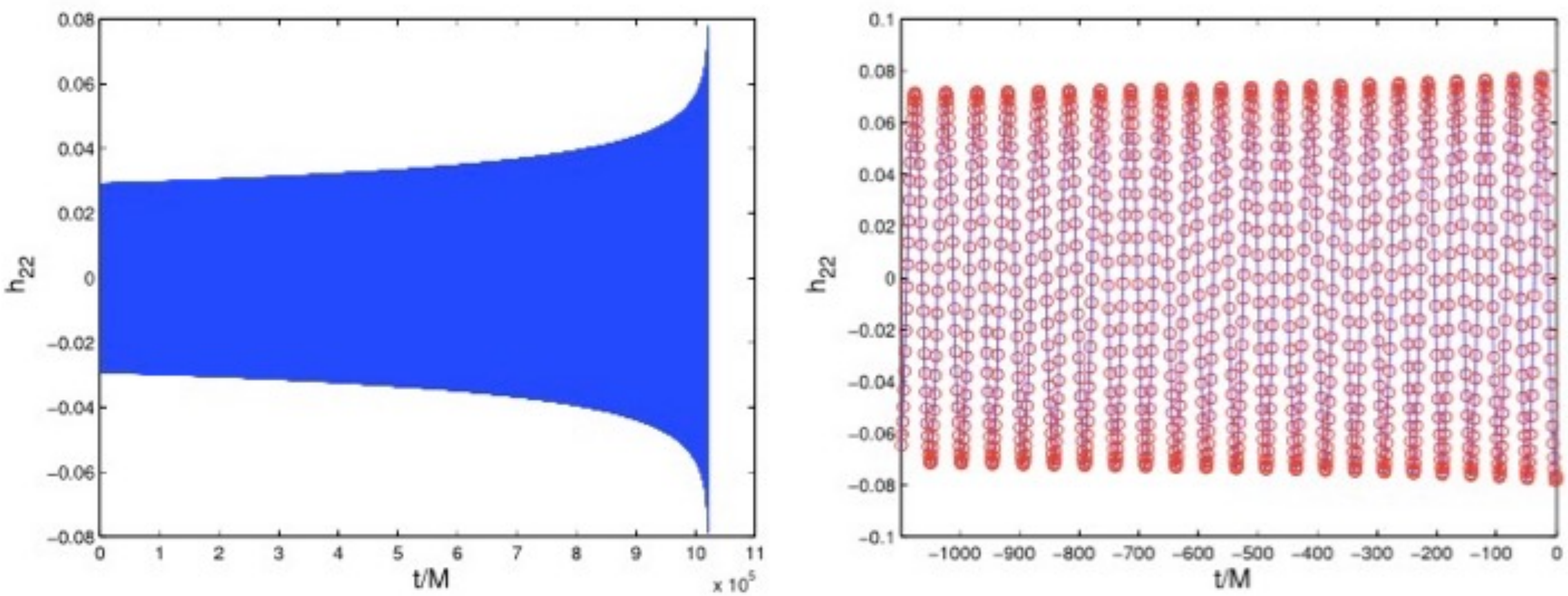


Figure 5. Orbital evolution of the EMRIs (mass ratio $1/1000$) for $a = 0$; the right panel shows the details of the orbit evolution at the final time.

Han CQG 2016



Our Fully calibrated version1: shortcoming

- Each group of polynomials only works in one special case;
- For getting the polynomials, one firstly must generate some numerical reference data;
- Polynomials do not have physics inside.
- Inspiring us to use factorized forms in EOB frame

Version2: recalibrated waveforms in EOB frame

$$\rho_{lm} = 1 + a_1 v^2 + b_1 q v^3 + (a_2 + b_2 q^2) v^4 + b_3 q v^5 + [b_4 q^2 + a_3 + a_4 \mathrm{eulerlog}(mv)] v^6$$
$$[b_5 q + b_6 q^3] v^7 + [b_7 q^2 + b_8 q^4 + a_5 + a_6 \mathrm{eulerlog}(mv)] v^8 + (b_9 q + b_{10} q^3) v^9 +$$
$$[a_7 + a_8 \mathrm{eulerlog}(mv)] v^{10} + [a_9 + a_{10} \mathrm{eulerlog}(mv)] v^{12}$$

for l = m

$$\rho_{lm} = 1 + b_1 q v + (a_1 + b_2 q^2) v^2 + (b_3 q + b_4 q^3) v^3 + (a_2 + b_5 q^2 + b_6 q^4) v^4 +$$
$$(b_7 q + b_8 q^3 + b_9 q^5) v^5 + [a_3 + a_4 \mathrm{eulerlog}(mv) + b_{10} q^2 + b_{11} q^4 + b_{12} q^6] v^6 +$$
$$[b_{13} q + b_{14} \mathrm{eulerlog}(mv) q + b_{15} q^3 + b_{16} q^5 + b_{17} q^7] v^7 + [a_5 + a_6 \mathrm{eulerlog}(mv)] v^8 +$$
$$[a_7 + a_8 \mathrm{eulerlog}(mv)] v^{10} + [a_9 + a_{10} \mathrm{eulerlog}(mv)] v^{12}$$

for l =/ m

$$a_{11}^{\mathrm{Hor,S}} = p_1 q + p_2 q^2 + p_3 q^3 + p_4 q^4 + p_5 q^5 + p_6 q^6$$
$$a_{12}^{\mathrm{Hor,S}} = p_7 q + p_8 q^2 + p_9 q^3 + p_{10} q^4 + p_{11} q^5 + p_{12} q^6 + p_{13} q^7$$

for horizon absorption

Version2: recalibrated waveforms in EOB frame

We use a least square method to find the global coefficients with the highly accurate Teukolsky-based data which q=0.9~0,-0.3,-0.5,-0.7,-0.9

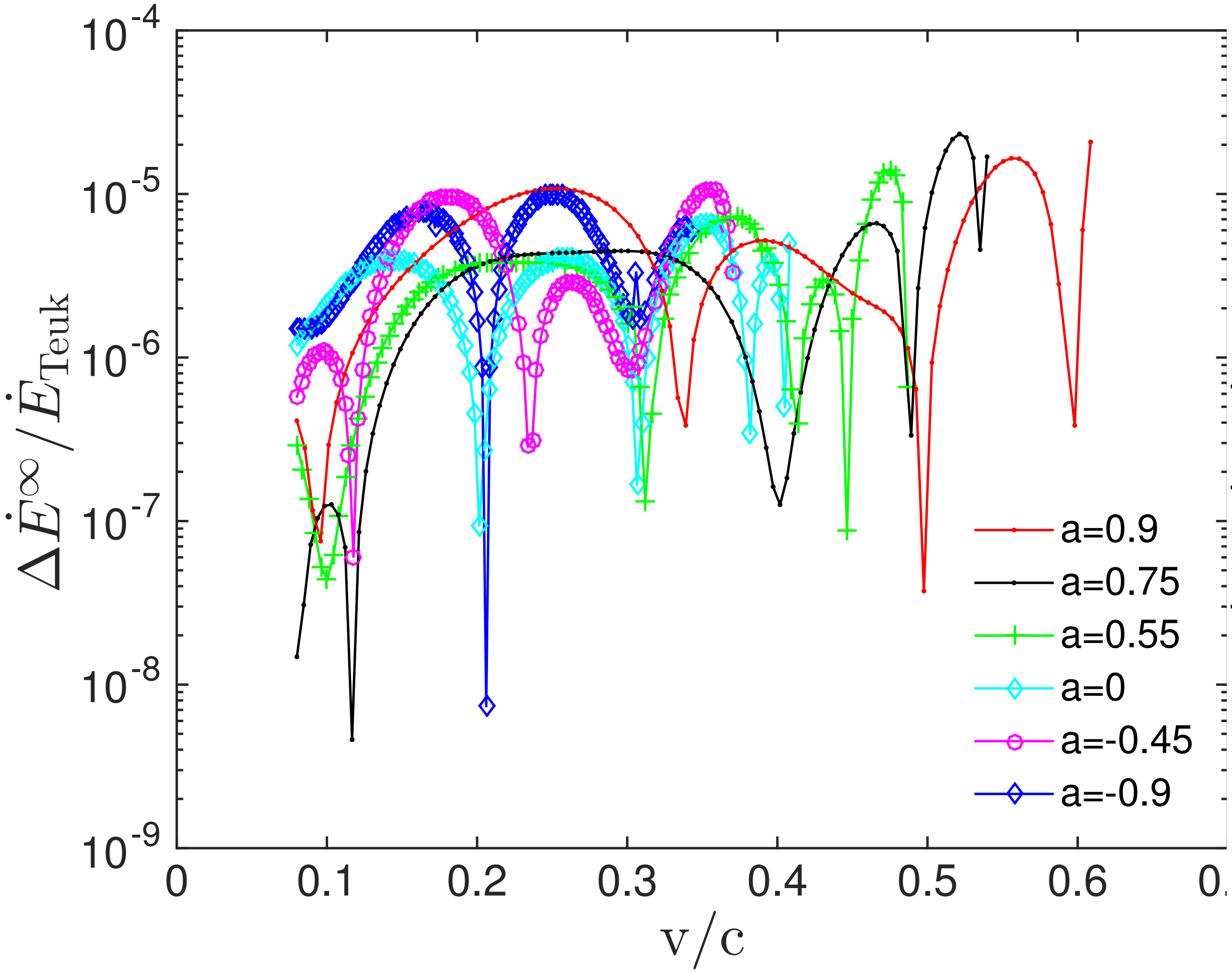
	$l=2, m=2$	$l=3, m=3$	$l=4, m=4$	$l=5, m=5$	$l=6, m=6$	
a_1	-1.023975805956E+00	-1.166721079966E+00	-1.2227931618E+00	-1.2487811477E+00	-1.2619660215E+00	-1.
a_2	-1.773785209399E+00	-1.632220902128E+00	-1.5488861713E+00	-1.5137494102E+00	-1.4927232619E+00	-1.
a_3	3.318171742361E+01	1.485007940230E+01	1.1208215801E+01	8.7424413748E+00	6.5654511682E+00	4.
a_4	3.953835298796E+01	1.245654119994E+01	1.2709894830E+01	1.1883585053E+01	1.1939038028E+01	1.
a_5	-1.104050524831E+03	-6.678847665787E+02	-8.6428660908E+02	-9.5407467730E+02	-1.0672751384E+03	-1.
a_6	1.765670955233E+03	6.364340267479E+02	6.4805577773E+02	6.1182473525E+02	6.1235129084E+02	6.
a_7	-1.584521768582E+04	-7.018461656809E+03	-8.2330639578E+03	-8.5105354171E+03	-9.1262632809E+03	-9.
a_8	1.007746475744E+04	3.520695972919E+03	3.6160554839E+03	3.4023945061E+03	3.4000165921E+03	3.
a_9	-1.943757060254E+04	-7.516377182961E+03	-8.4638217440E+03	-8.4396976216E+03	-8.8310465543E+03	-9.
a_{10}	6.701134943008E+03	2.262034338117E+03	2.3482970193E+03	2.2049397140E+03	2.2024481099E+03	2.
b_1	-6.733755278411E-01	-6.696972546143E-01	-6.6915298811E-01	-6.6901906546E-01	-6.6904049613E-01	-6
b_2	5.843602324003E-01	5.360214944791E-01	5.2813489804E-01	5.2594458075E-01	5.2571830732E-01	5.
b_3	-1.662151538548E+00	-1.308471947237E+00	-1.2252358713E+00	-1.2090008750E+00	-1.2146809603E+00	-1.
b_4	-1.061146392648E+00	-4.975131465872E-01	-4.5878852649E-01	-4.6545217148E-01	-4.8516130249E-01	-5
b_5	3.827859278692E+00	2.543651584540E+00	2.4583522998E+00	2.4535277945E+00	2.4730693730E+00	2.
b_6	-7.756293634998E-01	-3.514346900192E-01	-1.8135025452E-01	-1.4988716272E-01	-1.4384902166E-01	-1

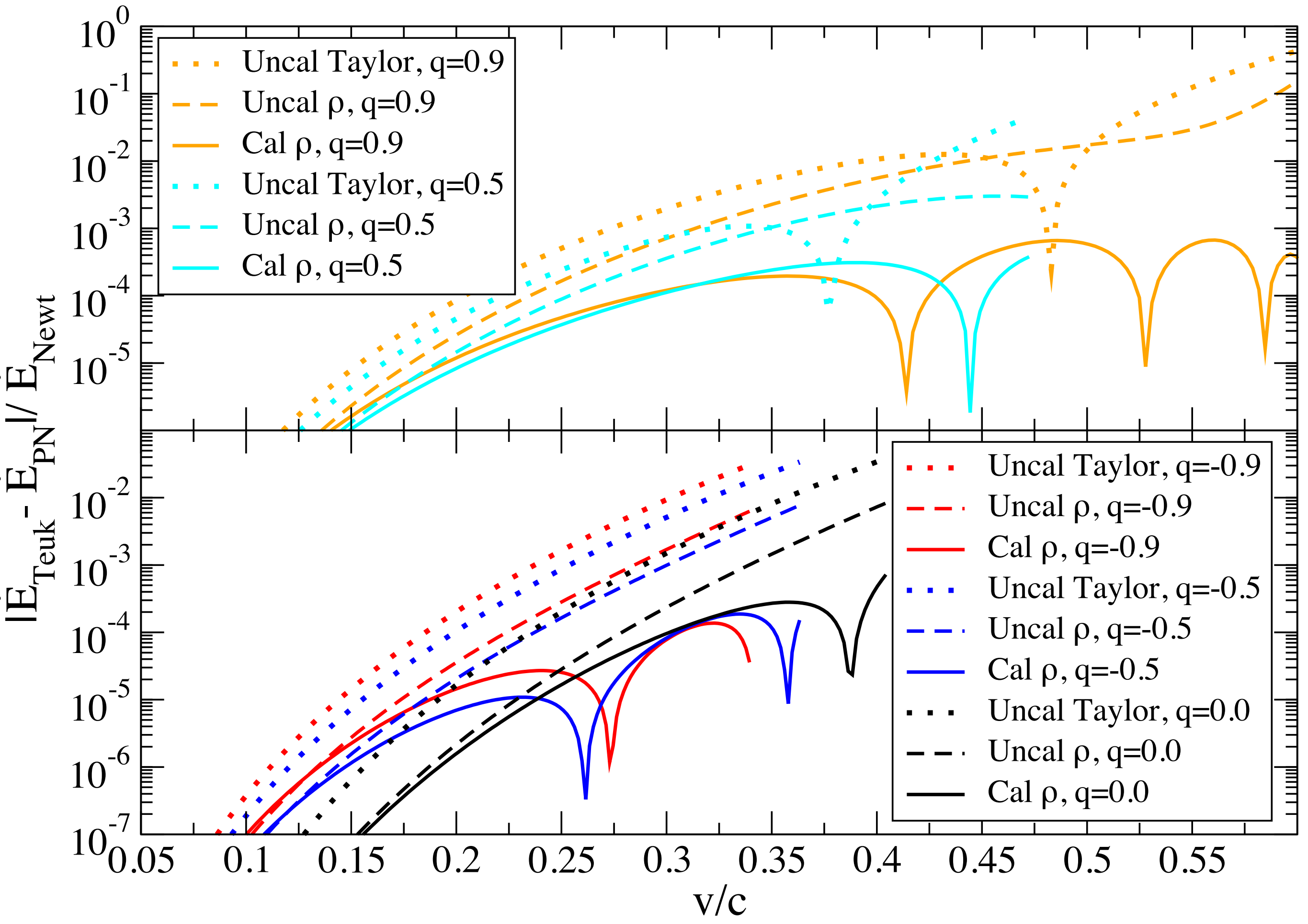
Version2: recalibrated waveforms in EOB frame

TABLE III. Total coefficients for $m \neq l$

	$l=5, m=4$	$l=5, m=3$	$l=6, m=5$	$l=6, m=4$
a_1	-1.2781593391E+00	-9.6137951691E-01	-1.2846696264E+00	-1.0236865411E+00
a_2	-9.7034478849E-01	-6.7312218658E-01	-1.0280853586E+00	-8.0181417330E-01
a_3	2.8215508168E+00	6.6046438604E+00	1.4723471598E+00	1.3317579430E+01
a_4	1.2611970259E+01	-2.3057664622E+01	1.2336196550E+01	-1.9494642667E+01
a_5	-5.2596249376E+02	8.0535017960E+02	-6.2959344624E+02	8.2565169087E+02
a_6	3.0038340199E+02	-7.0899298915E+02	3.3177604825E+02	-5.8059965421E+02
a_7	-1.7661904159E+03	6.8217825869E+03	-2.5543377525E+03	6.3637960489E+03
a_8	6.0774411335E+02	-3.2601890463E+03	8.8256123155E+02	-2.6727578200E+03
a_9	-2.4280639567E+02	5.8657539419E+03	-1.0359578359E+03	5.2005709295E+03
a_{10}	3.2316814336E+01	-1.7108913454E+03	2.3941084370E+02	-1.3988354619E+03
b_1	-2.4005464853E-01	8.6374961193E-06	-1.9451648765E-01	-9.2761151062E-06
b_2	-1.1607843889E-01	1.2299226986E-04	-9.5157480501E-02	2.0275436115E-04
b_3	3.2207333467E-01	-1.1811709880E+00	1.6169577622E-01	-1.0374290314E+00
b_4	-8.2537773600E-02	5.7012441031E-03	-6.6960649581E-02	5.2750953889E-03
b_5	3.3019097935E-01	5.8255283839E-01	3.7135374972E-01	5.2694468496E-01
b_6	-7.1149022817E-02	1.6974336944E-02	-5.8801917957E-02	1.3846828797E-02

Version2: recalibrated waveforms in EOB frame

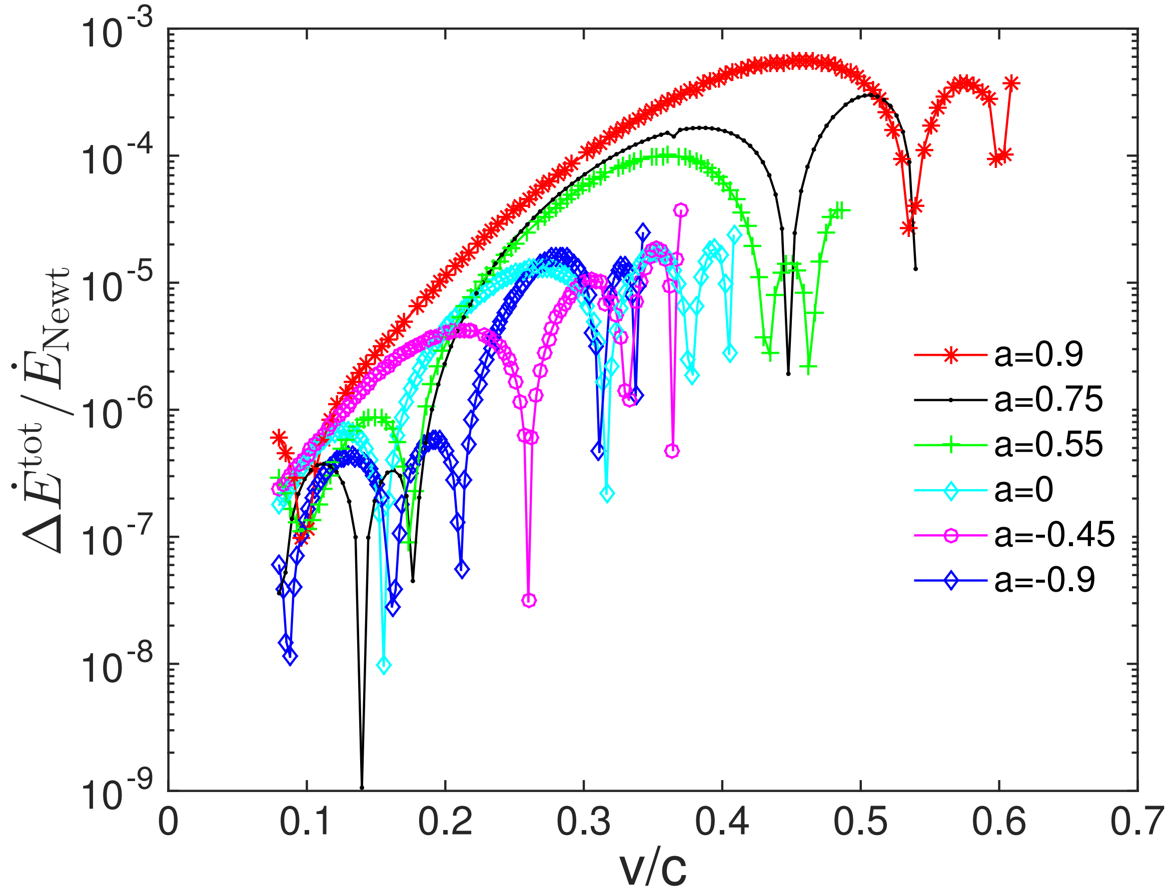


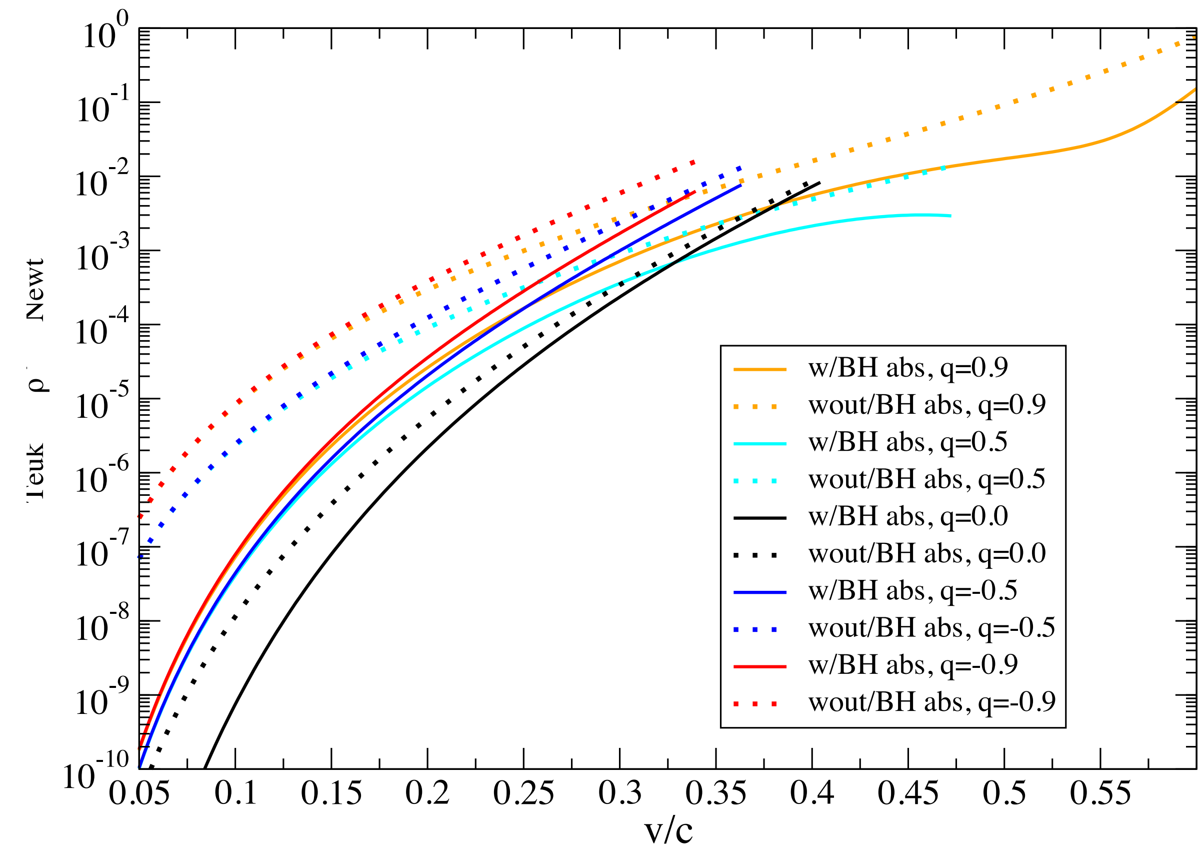


Cheng & Han, 2017,submitted to PRD, arXiv:1706.03884

Version2: recalibrated waveforms in EOB frame

Including fluxes to horizon

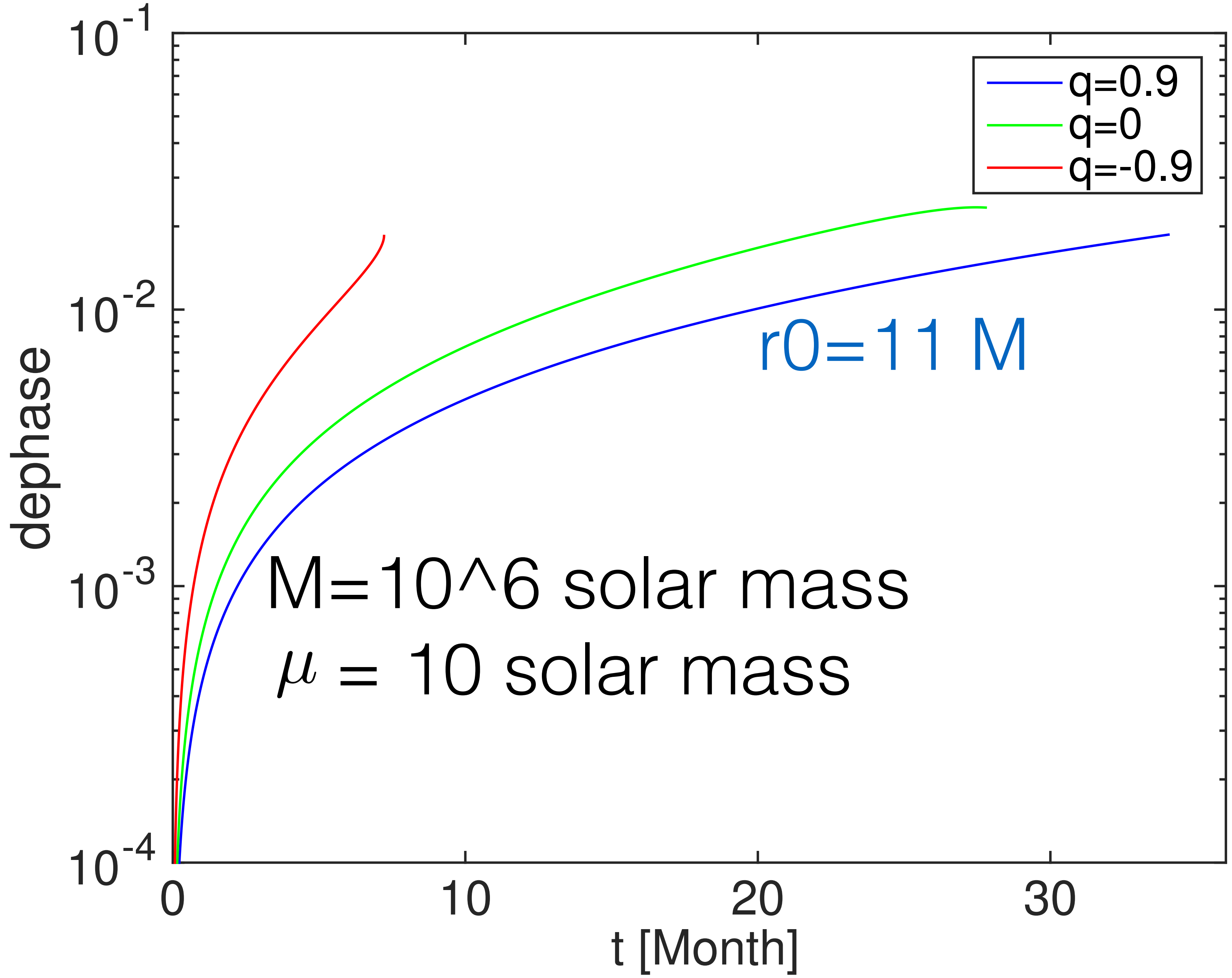




Cheng & Han, 2017,submitted to PRD, arXiv:1706.03884

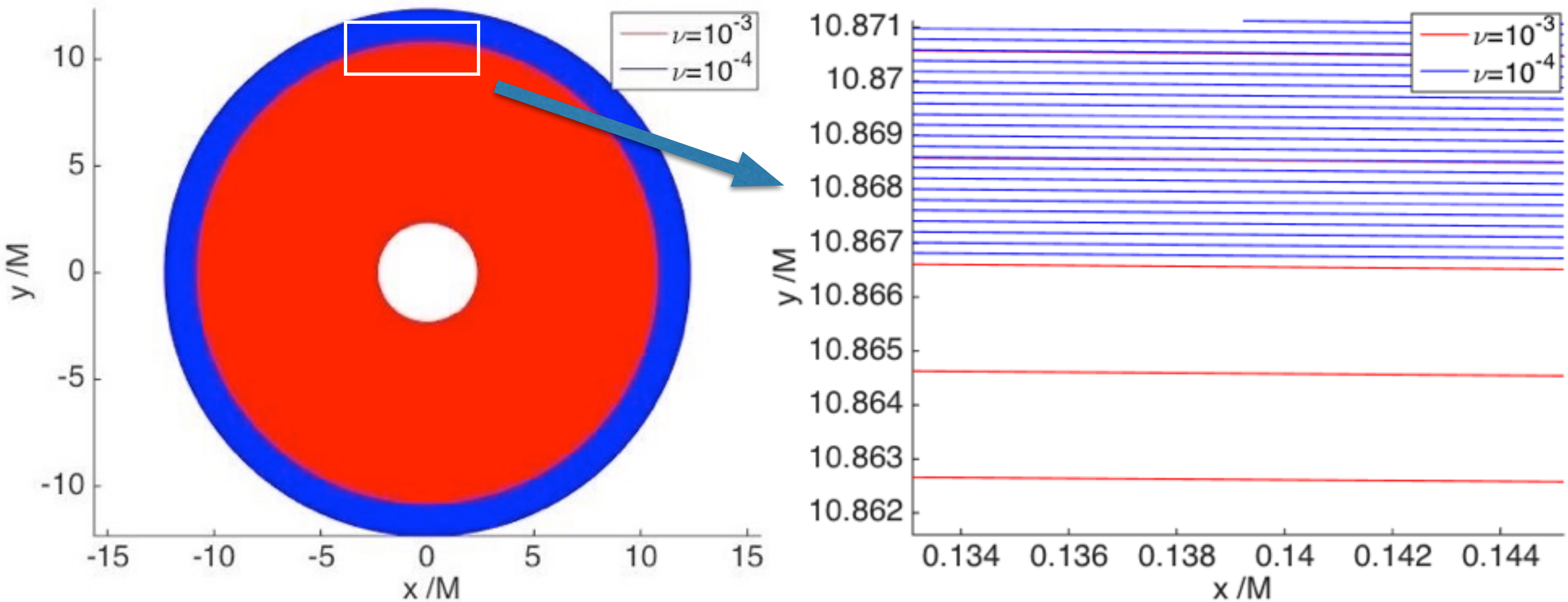
Version2: recalibrated waveforms in EOB frame

Dephasing between recalibrated model and Teukolsky-based waveforms



Version2: recalibrated waveforms in EOB frame

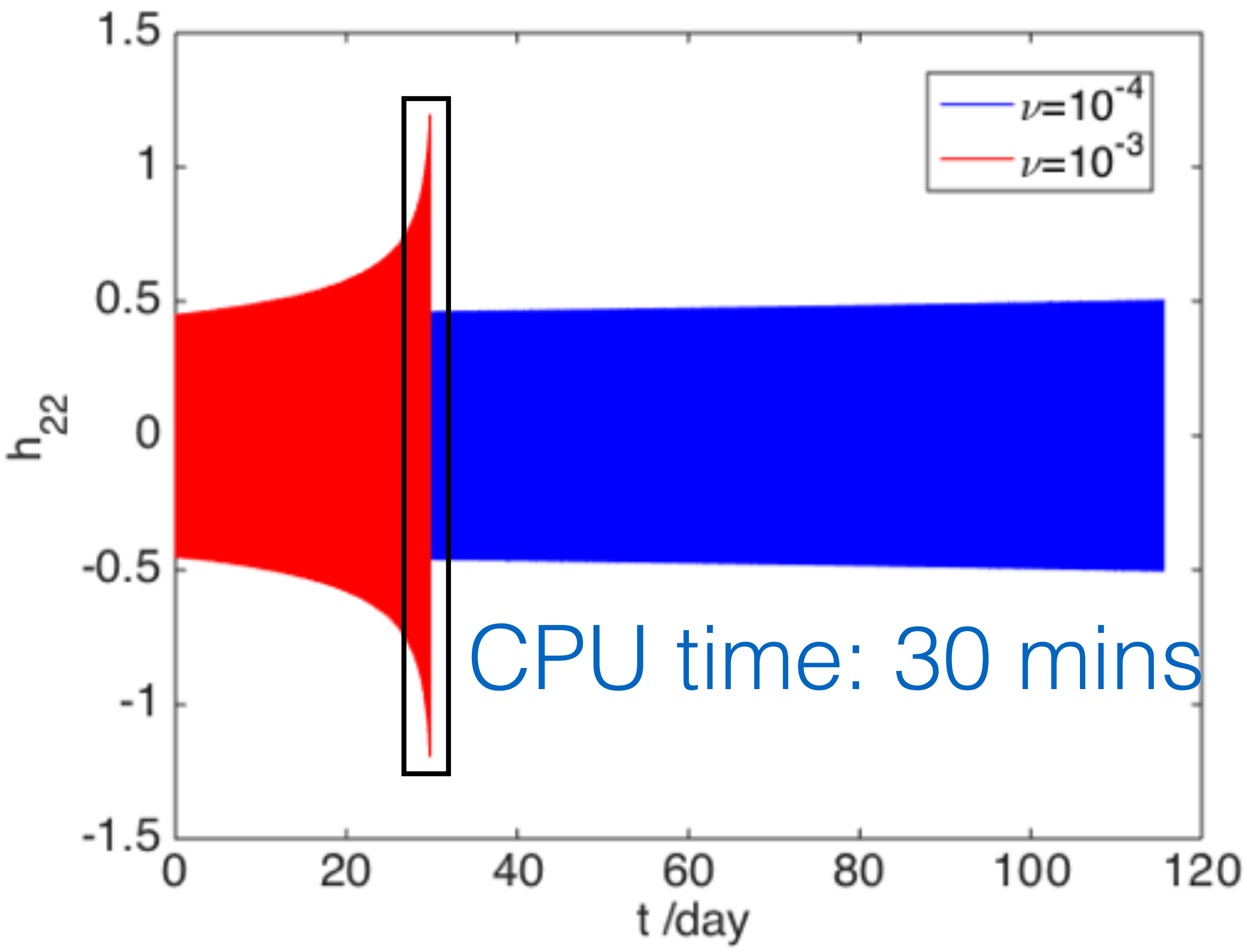
Evolution

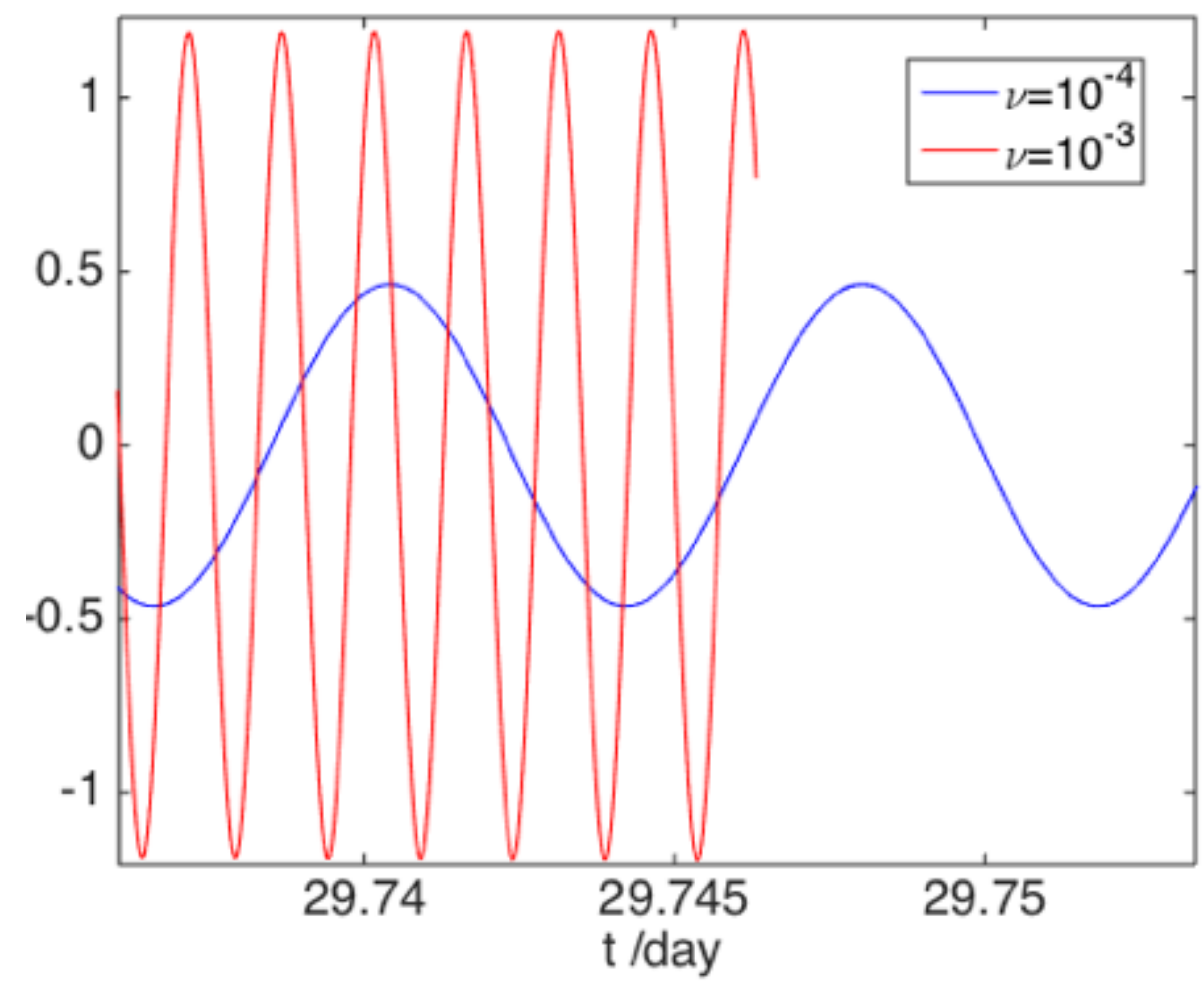


q=0.9, M=10^5 solar mass, r0=11M

Version2: recalibrated waveforms in EOB frame

Waveforms





Conclusions

- The recalibrated formalism looks ugly but works
- Only works for EMRIs
- Published all the coefficients and will publish our codes;
- It is an effort to construct the waveform templates for LISA et. al. in the EOB formalisms.
- recalibrating mass-ratios dependent terms;

Thank you!