The Era of Gravitational Wave Astonromy, IAP *Paris, June 2017*

Gravitational waves from compact binaries in scalar-tensor gravity to 2PN order

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Outline

- Scalar-Tensor Gravity
- Motivation
- To-date
- Landau-Lifshitz Formalism
- Direct Integration of Relaxed Einstein Equations (DIRE)
- To-do

Scalar-Tensor Gravity

- Alternate theory of gravity (ATG)
  - Alternative to Newtonian:

    Nordström: $g_{\mu\nu} = \Phi \eta_{\mu\nu},$ General Relativity: $8\pi G T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}.$
  - Alternative to GR:
    - Supported by same experimental evidence: Whitehead, Scalar-Tensor.
    - Seeks to explain recent observations / issues: (super)String Theory, f(R), etc.
- Variable gravitational "constant" $\rightarrow$ Scalar field(s),

$$T^{\alpha\beta}{}_{,\beta} = 0$$

$$S_{GR} = \frac{1}{16\pi G} \int R\sqrt{-g} d^4x + \int \mathcal{L}\left(m, g_{\alpha\beta}\right) \sqrt{g} d^4x,$$

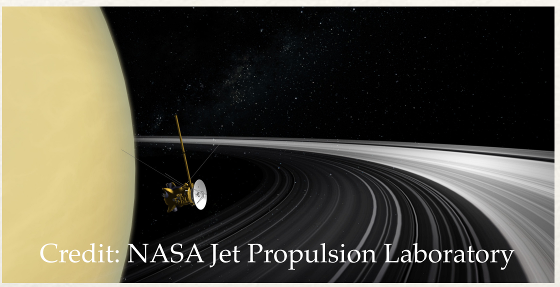
$$S_{ST} = \frac{1}{16\pi} \int \left[\phi R - \frac{1}{\phi} \omega\left(\phi\right) g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi\right] \sqrt{-g} d^4x + \int \mathcal{L}\left(m, g_{\alpha\beta}\right) \sqrt{g} d^4x.$$

- Obeys Einstein's Equivalence principle
- Violates the Strong Equivalence Principle

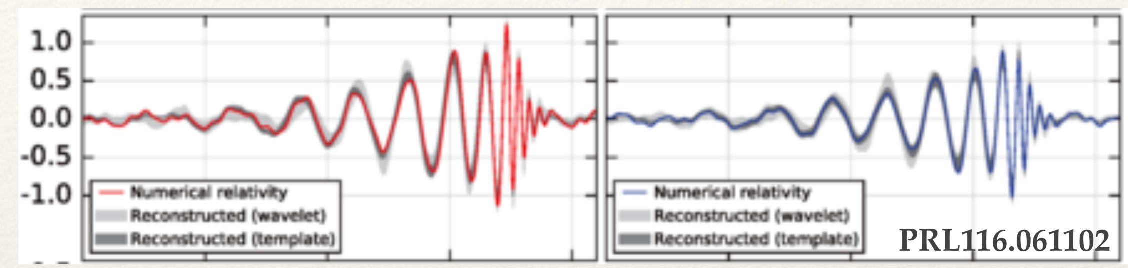
Internal Structure effects motion and GW emission

Motivation

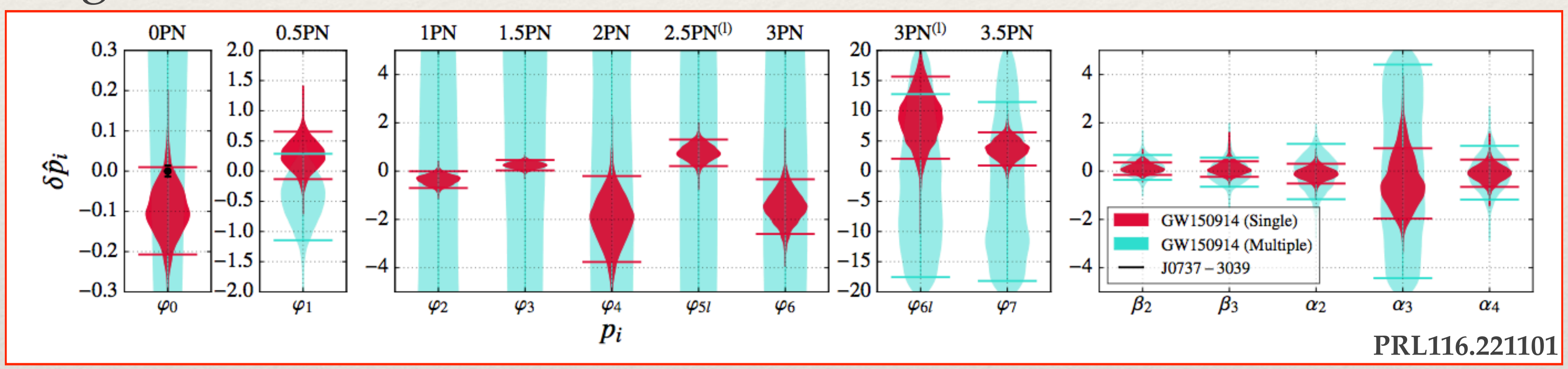
- Seek to verify / constrain / discard ATG's



- New tool → Gravitational Wave Astronomy



- Current Testing



- Why Scalar Tensor?
  - One of the simplest variations of GR
  - Encapsulates some of F(R) and Superstring theories
- Warning
  - Hawking Theorem - GR and ST black hole binaries indistinguishable
  - General argument for small differences expected at every PN order :(

To date …

- Necessary Ingredients:

3PN EOM

  - EOM (Mirshekeri & Will 2013: 2.5PN)
  - Tensor gravitational waves and tensorial energy flux 2PN (Lang 2014: 2PN)

2.5PN Ψ

  - Scalar gravitational waves and scalar energy flux (Lang 2015: 1.5PN and 1PN respectively)
  - Ready to use waveforms (Sennett, Marsat, Buonanno 2016: incomplete 2PN)

- Culprit: Non-vanishing scalar dipole moment:

$\Psi = \Psi_{-1/2} + \Psi_0 + \Psi_{1/2} + \Psi_1 + \Psi_{3/2},$ $\Rightarrow$We require $\Psi_{n+1/2}$ for $\dot{E}_n$,

$\dot{E}_S \propto \dot{\Psi}^2 \Rightarrow \dot{E} = \dot{E}_{-1} + \dot{E}_0 + \dot{E}_{1/2} + \dot{E}_1.$ $\Rightarrow$ We require $\mathrm{EOM}_{(n+1)}$ for $\Psi_{n+1/2}$.

Landau-Lifshitz Formalism

- Field equations:

$$G_{\alpha\beta} = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega(\phi)}{\phi^2}\left(\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2} g_{\alpha\beta}\phi_{,\gamma}\phi^{,\gamma}\right) + \frac{1}{\phi}\left(\phi_{,\alpha\beta} - g_{\alpha\beta}\Box_g\phi\right),$$

$$\Box_g \phi = \frac{1}{3+2\omega(\phi)}\left(8\pi T - 16\pi\phi\frac{\partial T}{\partial\phi} - \frac{d\omega}{d\phi}\phi_{,\gamma}\phi^{,\gamma}\right).$$

$$\psi = \frac{\phi}{\phi_0}.$$

- Wave equations:

$$\Box_\eta \tilde{h}^{\alpha\beta} = -16\pi\tau^{\alpha\beta}, \qquad \Box_\eta\psi = -8\pi\tau_s.$$

$$\tau^{\alpha\beta}{}_{,\beta} = 0 \qquad 16\pi\tau^{\alpha\beta} = 16\pi(-g)\frac{\psi}{\phi_0}T^{\alpha\beta} + \Lambda^{\alpha\beta} + \Lambda^{\alpha\beta}_S,$$

$$\tau_s = -\frac{1}{3+2\omega}\sqrt{-g}\frac{\psi}{\phi_0}\left(T - 2\psi\frac{\partial T}{\partial\psi}\right) - \frac{1}{8\pi}\tilde{h}^{\mu\nu}\psi_{,\mu\nu} + \frac{1}{16\pi}\frac{d}{d\psi}\left[\ln\left(\frac{3+2\omega}{\psi^2}\right)\right]\psi_{,\mu}\psi_{,\nu}\tilde{\mathbf{g}}^{\mu\nu}$$

- Integration of wave equations via flat Green function:

$$\tilde{h}^{\alpha\beta}(t,x) = 4\int \frac{\tau^{\alpha\beta}(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|}d^4x', \quad \psi(t,x) = 2\int\frac{\tau_s(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|}d^4x'.$$

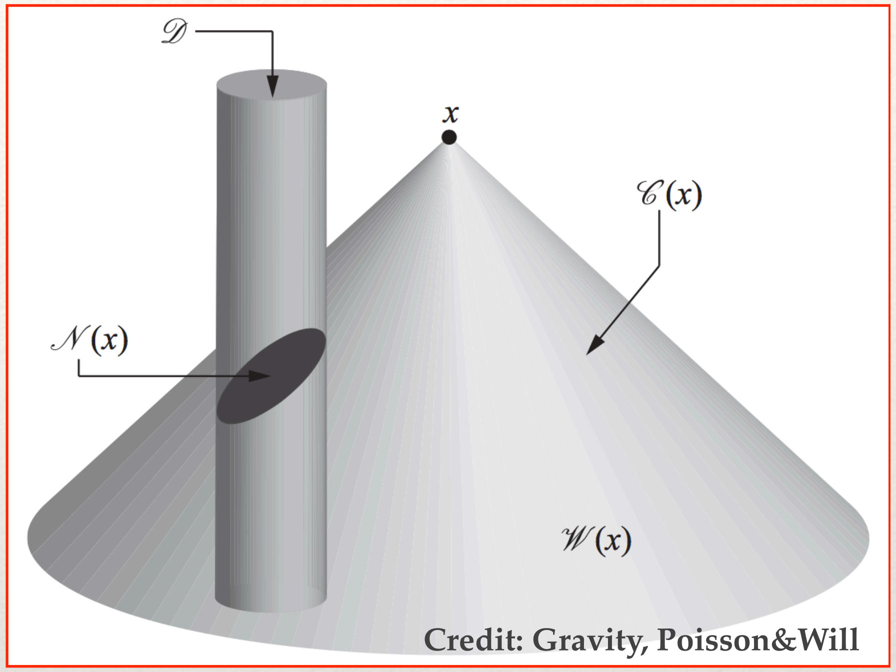
Direct Integration of Relaxed Einstein Equations

- Zones: Near zone, wave zone

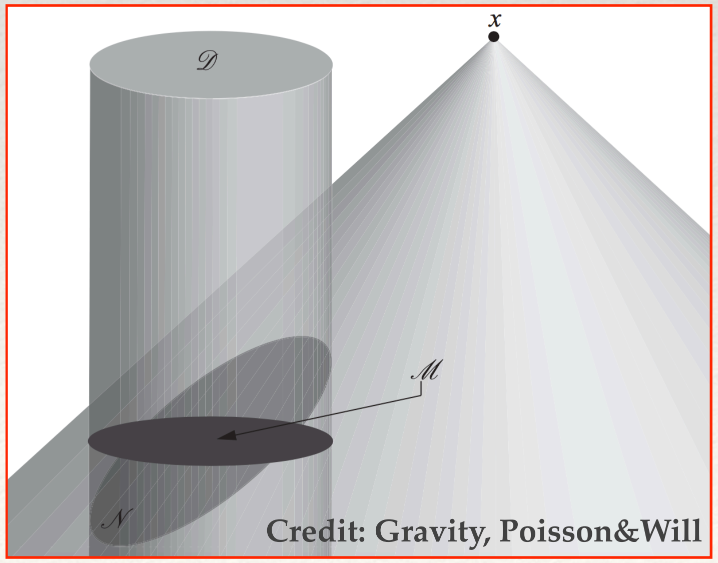
$$\Psi = \Psi_{\mathcal{N}} + \Psi_{\mathcal{W}}, \quad h^{\alpha\beta} = h^{\alpha\beta}_{\mathcal{N}} + h^{\alpha\beta}_{\mathcal{W}}$$

$$\tilde{h}^{\alpha\beta}_{(\mathcal{N}/\mathcal{W})}(t,x) = 4\int_{\mathcal{N}/\mathcal{W}} \frac{\tau^{\alpha\beta}(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|} d^4x',$$

$$\psi_{(\mathcal{N}/\mathcal{W})}(t,x) = 2\int_{\mathcal{N}/\mathcal{W}} \frac{\tau_s(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|} d^4x'.$$



- Near zone field, wave zone field point



$$\psi_{\mathcal{N}}/h^{\alpha\beta}_{\mathcal{N}} = \int_{\mathcal{N}}\int \frac{\tau^A(t-|x-x'|,y)}{|x-x'|}\delta(y-x')d^3y\,d^3x', \quad \Rightarrow x' \, small$$

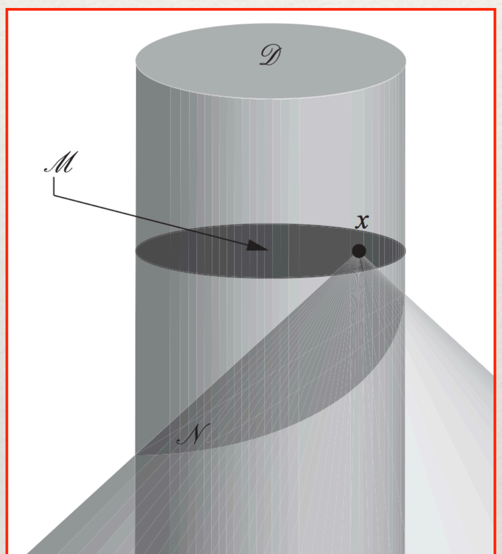
$$= \sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\partial_L\left[\frac{1}{r}\int_{\mathcal{M}}\tau^A(\underbrace{t-|x|}_{\tau},x')x'^L d^3x'\right],$$

Far away wave zone

$$= \frac{1}{r}\sum_{l=0}^{\infty}\frac{1}{l!}n_L\left(\frac{d}{d\tau}\right)^l\int_{\mathcal{M}}\tau^A(\tau,x')x'^L d^3x' + \mathcal{O}\left(r^{-2}\right).$$

Direct Integration of Relaxed Einstein Equations

- Near zone field, near zone field point



Credit: Gravity, Poisson&Will

$\Rightarrow |x - x'| small$

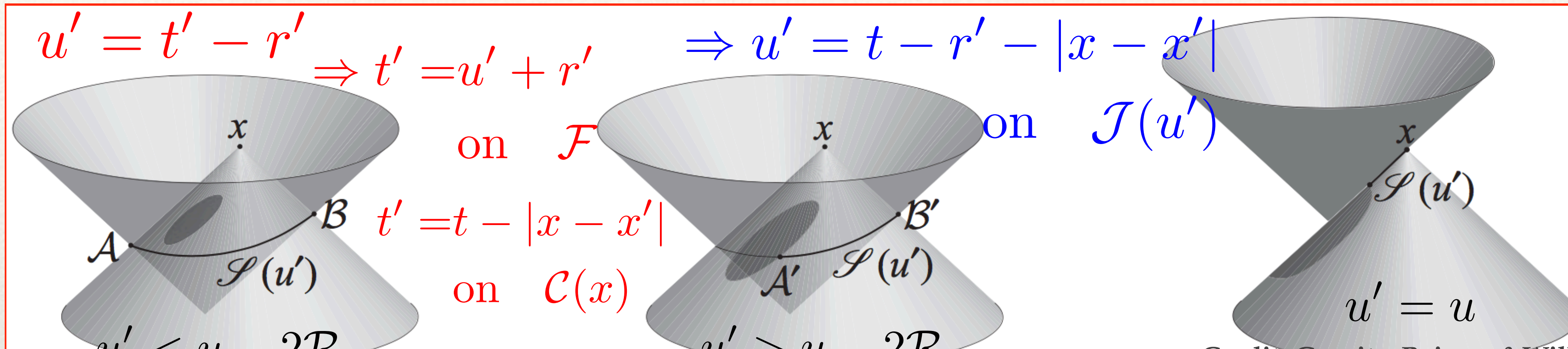
$$\psi_{\mathcal{N}}/h^{\alpha\beta}_{\mathcal{N}} = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left(\frac{\partial}{\partial t}\right)^l \times \int_{\mathcal{M}} \tau^A(t, x') |x - x'|^{l-1} d^3x'$$

- Far zone field

$u' = t' - r'$ $\Rightarrow t' = u' + r'$ on $\mathcal{F}$

$t' = t - |x - x'|$ on $\mathcal{C}(x)$

$\Rightarrow u' = t - r' - |x - x'|$ on $\mathcal{J}(u')$



Credit: Gravity, Poisson&Will

$$\psi_{\mathcal{W}}/h^{\alpha\beta}_{\mathcal{W}} = \int_{-\infty}^{u} du' \oint_{\mathcal{J}(u')} \frac{\tau^A(u' + r', x')}{t - u' - n' \cdot x} r'(u', \theta', \phi')^2 d\Omega', \quad \tau^A(x') = \frac{1}{4\pi} \frac{f^A(\tau')}{r'^n} n'^{<L>}$$

$$= \frac{1}{2} n^{<L>} \int_{-\infty}^{u} du' f(u') \int \frac{P_l(\xi)}{r r'^{(n-1)}} dr'$$

$n = e_z$

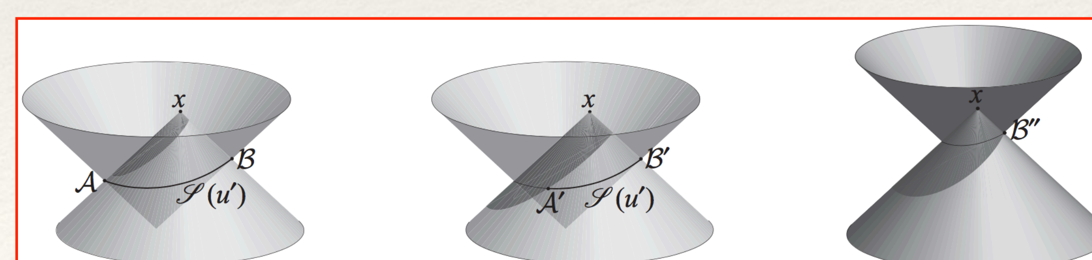
$$A(s,r) = \int_{\mathcal{R}}^{r+s} \frac{P_l(\cos\theta')}{r'^{(n-1)}} dr', \quad B(s,r) = \int_{s}^{r+s} \frac{P_l(\cos\theta')}{r'^{(n-1)}} dr', \quad s = \frac{1}{2}(u - u')$$

$$\psi_{\mathcal{W}}/h^{\alpha\beta}_{\mathcal{W}} = \frac{n^{<L>}}{r} \left[\int_{\cancel{0}}^{\mathcal{R}} ds f^A(\tau - 2s) A(s,r) + \int_{\mathcal{R}}^{\infty} ds f^A(\tau - 2s) B(s,r) \right]$$

$\mathcal{R} - r$

Far zone field pt

Near zone field pt



Direct Integration of Relaxed Einstein Equations

- Solve simultaneously for source terms $(\tau^{\alpha\beta}, \tau_s)$ and fields $(g^{\alpha\beta}, \Psi)$

3PN

Flat metric
$g^{\alpha\beta} \approx \eta^{\alpha\beta}$

Calculate 0PN source term

$$\tau^{00} = \frac{\sigma}{\phi_0} + \mathcal{O}(\epsilon\rho)\,,$$
$$\tau^{0a} = \tau^{ab} = \tau^{aa} = \mathcal{O}(\epsilon\rho)\,,$$
$$\tau_s = G\zeta\sigma_s + \mathcal{O}(\epsilon\rho)\,.$$

Add perturbation to fields

$$g^{\alpha\beta}_{(n+1/2)} = g^{\alpha\beta}_{(n)} + \epsilon^{n+1/2} h^{\alpha\beta}_{(n+1/2)}$$
$$\Psi_{s(n+1/2)} = \Psi_{s(n)} + \epsilon^{n+1/2} \Psi_{s(n+1/2)}$$

Calculate next +0.5PN source terms
$(\tau^{\alpha\beta}_{(n+1/2)}, \tau_{s(n+1/2)})$

Solve for next +0.5PN field terms
$(g^{\alpha\beta}_{(n+1/2)}, \Psi_{s(n+1/2)})$

- Stress energy tensor

$$T^{\alpha\beta} = \rho * (-g)^{-1/2} u^\alpha u^\beta (u^0)^{-1}, \quad \rho* = \sum_A m_A \delta^3(x - x_A),$$

$$\Rightarrow T^{\alpha\beta} = \rho * (-g)^{-1/2} v^\alpha v^\beta u^0 \left[1 + S(s;\Psi)\right]$$

$$\sigma \equiv T^{00} + T^{ii},$$
$$\sigma^a \equiv T^{0a},$$
$$\sigma^{ab} \equiv T^{ab},$$
$$\sigma_s \equiv -T + 2\psi \frac{\partial T}{\partial \psi}.$$

$$\sigma = \rho * (-g)^{-1/2} u^0 (1 + v^2) \left[1 + S(s;\Psi)\right], \quad \sigma^a = \rho * (-g)^{-1/2} u^0 v^a \left[1 + S(s;\Psi)\right],$$
$$\sigma^{ab} = \rho * (-g)^{-1/2} u^0 v^a v^b \left[1 + S(s;\Psi)\right], \quad \sigma_s = \rho * (-g)^{-1/2} (u^0)^{-1} \left[1 - 2s + S_s(s;\Psi)\right].$$

$$\begin{aligned} S(s;\Psi) =& \epsilon\Psi s + \frac{1}{4}\epsilon^2\Psi^2(2a_s - s) + \frac{1}{24}\epsilon^3\Psi^3(-6a_s + 3s + 4b_s) \\ &+ \frac{1}{192}\epsilon^4\Psi^4(30a_s - 15s - 20b_s + 24c_s) + \mathcal{O}(\epsilon^5)\,, \\ S_s(s;\Psi) =& -2\epsilon(\Psi a_s) - \epsilon^2\Psi^2 b_s - \epsilon^3\Psi^3 c_s + \mathcal{O}(\epsilon^4) \end{aligned}$$

$$a_s \equiv s^2 + s' - \frac{1}{2}s,$$
$$b_s \equiv a_s' - a_s - s a_s,$$
$$c_s \equiv \frac{1}{3}(b_s' - 2b_s - s b_s)\,.$$

$$s_A \equiv \left(\frac{d\ln M_A(\phi)}{d\ln\phi}\right)_0$$

In progress …

Use source terms to calculate multipole moments

$$\mathcal{I}^{Q}_{(s)} \equiv \int \tau_{(s)} x^{Q} d^3x$$

$G_1, G_{1s}, G_2, G_{2s}, G_3, G_{3s}, G_4, G_6, G_{6s}, v.G_7, H, H_s, H^s{}_s, H^s, U^3, U_s{}^3, U_s U^2, v^2 U^2, v^2 U_s{}^2, v^a v^b P_{2ab}, v^a v^b P_{2sab}, V^2, v^2 v.V, v.V_2, \overset{(4)}{Y}, \overset{(4)}{Y}_s, v^2 \ddot{\chi}, \ddot{\chi}_1, \ddot{\chi}_2, v.\ddot{\chi}, v^2 \ddot{\chi}_s, \ddot{\chi}^s{}_2, \ddot{\chi}^s{}_{2s}, \Sigma_s(\ddot{\chi}),$
$\Sigma_s(\ddot{\chi}_s), \Sigma_s(\Phi_1), \Sigma_s(\Phi^s{}_2), \Sigma_s(\Phi^s{}_{2s}), \Sigma(\ddot{\chi}), \Sigma(\Phi_1), \Sigma(\Phi^s{}_{2s}), v^2\Phi_1, v^a v^b \Phi_{1ab}, v^2\Phi_2, v.\Phi_2, P(\dot{U}_s \nabla U_s.v), v^6, v.V^s{}_{2s}, \ddot{\chi}^s{}_1, \ddot{\chi}(U_s a_s), \Sigma_a{}^a(U), \Sigma(a_s \Sigma(U_s a_s)), \Sigma_s(U^2), \Sigma_s(U_s{}^2), \Sigma_s(U_s v^2),$
$\Sigma_s(U U_s), \Sigma_s(U v^2), \Sigma_s(v^4), \Sigma_s(v.V), \Sigma_s(\Sigma(U_s a_s)), \Sigma_s(\Phi_2), \Sigma_s(\Phi^s{}_1), \Sigma(U_s{}^2), \Sigma(U^2), \Sigma(U_s{}^2 a_s), \Sigma(U_s{}^2 b_s), v^2\Sigma(U_s a_s), \Sigma(U_s v^2 a_s), \Sigma(U U_s a_s), \Sigma(U v^2), v.\Sigma(v^2 v), \Sigma(v^4), \Sigma(v.V), \Sigma(\ddot{\chi}_s a_s),$
$\Sigma(\Phi_2), \Sigma(\Phi^s{}_1 a_s), \Sigma(\Phi^s{}_2 a_s), \Sigma(\Phi^s{}_{2s} a_s), v^2\Phi^s{}_1, v^2\Phi^s{}_2, v^2\Phi^s{}_2, v^2\Phi^s{}_{2s}, U U_s{}^2, v^2 U U_s, v.V U, U\ddot{\chi}, U\ddot{\chi}_s, U\Phi_1, U\Phi_2, v^4 U, U\Sigma(U_s a_s), U\Phi^s{}_1, U\Phi^s{}_2, U\Phi^s{}_{2s}, U_s v.V, U_s\ddot{\chi}, U_s\ddot{\chi}_s,$
$U_s\Phi_1, U_s\Phi_2, v^4 U_s, U_s\Sigma(U_s a_s), U_s\Phi^s{}_1, U_s\Phi^s{}_2, U_s\Phi^s{}_{2s}.$

ρ^* coefficients

$a_s G_{2s}, a_s G_{3s}, a_s G_{6s}, a_s H^s{}_s, a_s H^s, c_s U_s{}^3, b_s U_s{}^3, a_s U_s{}^3, a_s U_s U^2, b_s v^2 U_s{}^2, a_s v^2 U_s, a_s \overset{(4)}{Y}_s, a_s v^2\ddot{\chi}_s, a_s\ddot{\chi}^s{}_2, a_s\ddot{\chi}^s{}_{2s}, a_s\Sigma_s(\ddot{\chi}), a_s\Sigma_s(\ddot{\chi}_s), a_s\Sigma_s(\Phi_1), a_s\Sigma_s(\Phi^s{}_2), a_s\Sigma_s(\Phi^s{}_{2s}), a_s\ddot{\chi}^s{}_1,$
$a_s\ddot{\chi}(U_s a_s), a_s\Sigma(a_s\Sigma(a_s U_s)), a_s\Sigma_s(U^2), a_s\Sigma_s(U_s{}^2), a_s\Sigma_s(U_s v^s), a_s\Sigma_s(U U_s), a_s\Sigma_s(Uv^2), a_s\Sigma_s(v^4), a_s\Sigma_s(v.V), a_s\Sigma_s(\Sigma(U_s a_s)), a_s\Sigma_s(\Phi_2), a_s\Sigma_s(\Phi^s{}_1), a_s\Sigma(U_s{}^2 a_s), a_s\Sigma(U_s{}^2 a_s),$
$a_s\Sigma(U_s{}^2 b_s), a_s v^2\Sigma(U a_s), a_s\Sigma(U_s v^2 a_s), a_s\Sigma(U U_s a_s), a_s\Sigma(\ddot{\chi}_s a_s), a_s\Sigma(\Phi^s{}_1 a_s), a_s\Sigma(\Phi^s{}_2 a_s), a_s\Sigma(\Phi^s{}_{2s} a_s), v^2\Phi^s{}_1, a_s v^2\Phi^s{}_2, a_s v^2\Phi^s{}_2, a_s v^2\Phi^s{}_{2s}, b_s U U_s{}^2, a_s U U_s{}^2, a_s v^2 U U_s,$
$a_s U\ddot{\chi}_s, a_s U\Sigma(U_s a_s), a_s U\Phi^s{}_1, a_s U\Phi^s{}_2, a_s U\Phi^s{}_{2s}, a_s v.V U_s, a_s U_s\ddot{\chi}, b_s U_s\ddot{\chi}_s, a_s U_s\ddot{\chi}_s, a_s U_s\Phi_1, a_s U_s\Phi_2, b_s U_s\Sigma(U_s a_s), b_s U_s\Phi^s{}_1, a_s v^4 U_s, a_s U_s\Sigma(U_s a_s), a_s U_s\Phi^s{}_1, b_s U_s\Phi^s{}_2,$
$a_s U_s\Phi^s{}_2, b_s U_s\Phi^s{}_{2s}, a_s U_s\Phi^s{}_{2s}.$

$\tau_{s,F1}$

$U\ddot{U}_s{}^2, \dot{U}_s\overset{(3)}{\chi}_s, \dot{U}_s\dot{\Sigma}(U_s a_s), \dot{U}_s\dot{\Phi}^s{}_1, \dot{U}_s\dot{\Phi}^s{}_2, \dot{U}_s\dot{\Phi}^s{}_{2s}, \dot{U}_s V.\nabla U_s, U^2(\nabla U_s)^2, U_s{}^2(\nabla U_s)^2, \Phi_2(\nabla U_s)^2, \Sigma(U_s a_s)(\nabla U_s)^2, \Phi^s{}_1(\nabla U_s)^2, \nabla\Sigma_s(U^2).\nabla U_s, \nabla\Sigma_s(U_s{}^2).\nabla U_s, \nabla\Sigma_s(U_s v^2).\nabla U_s,$
$\nabla\Sigma_s(U U_s).\nabla U_s, \nabla\Sigma_s(U v^2).\nabla U_s, \nabla\Sigma_s(\Sigma(U_s a_s)).\nabla U_s, \nabla\Sigma_s(\Phi_1).\nabla U_s, \nabla\Sigma_s(\Phi_2).\nabla U_s, \nabla\Sigma_s(\Phi_{2s}).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_1).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_2).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_{2s}).\nabla U_s, \nabla\Sigma(U_s{}^2 a_s).\nabla U_s, \nabla\Sigma(U_s U a_s).\nabla U_s,$
$\nabla\Sigma(a_s\Sigma(U_s a_s)).\nabla U_s, \nabla\Sigma_s(v^4).\nabla U_s, \nabla\Sigma_s(v.V).\nabla U_s, \nabla\Sigma_s(\ddot{\chi}).\nabla U_s, \nabla\Sigma_s(\ddot{\chi}_s).\nabla U_s, \nabla\Sigma(U_s{}^2 b_s).\nabla U_s, \nabla\Sigma(U_s v^2 a_s).\nabla U_s, \nabla\Sigma(\ddot{\chi}_s a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_1 a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_2 a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_{2s} a_s).\nabla U_s,$

$\nabla G_{2s}.\nabla U_s, \nabla G_{3s}.\nabla U_s, \nabla G_{6s}.\nabla U_s, \nabla H^s{}_s.\nabla U_s, \nabla H^s.\nabla U_s, \ddot{\chi}_s(\nabla U_s)^2, (\nabla\ddot{\chi}_s)^2, \nabla\Sigma(U_s a_s).\nabla\ddot{\chi}_s, \nabla\Phi^s{}_1.\nabla\ddot{\chi}_s, \nabla\Phi^s{}_2.\nabla\ddot{\chi}_s, \nabla\Phi^s{}_{2s}.\nabla\ddot{\chi}_s, \nabla U_s.\nabla\overset{(4)}{Y}_s, \nabla U_s.\nabla\ddot{\chi}^s{}_2, \nabla U_s.\nabla\ddot{\chi}^s{}_{2s},$
$\nabla U_s.\nabla\ddot{\chi}^s{}_1, \nabla U_s.\nabla\ddot{\chi}(U_s a_s), (\nabla\Sigma(U_s a_s))^2, \nabla\Phi^s{}_1.\nabla\Sigma(U_s a_s), (\nabla\Phi^s{}_1)^2, \nabla\Phi^s{}_2.\nabla\Phi^s{}_{2s}, \nabla\Phi^s{}_2.\nabla\Sigma(U_s a_s), \nabla\Phi^s{}_2.\nabla\Phi^s{}_1, (\nabla\Phi^s{}_2)^2, \nabla\Phi^s{}_{2s}.\nabla\Sigma(U_s a_s), \nabla\Phi^s{}_{2s}.\nabla\Phi^s{}_1, (\nabla\Phi^s{}_{2s})^2, P_2{}^{ab}\nabla_a U_s\nabla_b U_s,$
$P_{2s}{}^{ab}\nabla_a U_s\nabla_b U_s, \Phi_1{}^{ab}\nabla_a U_s\nabla_b U_s, \Phi^s{}_2(\nabla U_s)^2, \Phi^s{}_{2s}(\nabla U_s)^2, U_s\ddot{U}_s{}^2, U_s\nabla U_s.\nabla\Sigma(U_s a_s), U_s\nabla U_s.\nabla\Phi^s{}_1, U_s\nabla U_s.\nabla\Phi^s{}_2, U_s\nabla U_s.\nabla\Phi^s{}_{2s}, U_s\nabla U_s.\nabla\ddot{\chi}_s$

$\tau_{s,F2}$

$\rho_s G_1, \rho_s G_4, \rho_s G_5, \rho_s U^3, \rho_s U_s{}^3, \rho_s V^2, \rho_s\ddot{\chi}_2, \rho_s\ddot{\chi}^s{}_{2s}, \rho_s\Sigma_a(V^a), \rho_s\Sigma_s(\ddot{\chi}_s), \rho_s\Sigma_s(\Phi^s{}_2), \rho_s\Sigma_s(\Phi^s{}_{2s}), \rho_s\Sigma_s(U_s{}^2), \rho_s\Sigma_s(U_s U), \rho_s\Sigma_s(U_s v^2), \rho_s\Sigma_s(\Sigma(U_s a_s)), \rho_s\Sigma_s(\Phi^s{}_1), \rho_s\Sigma(U_s U), \rho_s\Sigma(\ddot{\chi}),$
$\rho_s\Sigma(\Phi_{2s}), \rho_s\Sigma(U^2), \rho_s\Sigma(U_s{}^2 a_s), \rho_s\Sigma(U v^2), \rho_s\Sigma(\Phi_1), \rho_s\Sigma(\Phi_2), \rho_s\Sigma(\Phi_{2s}), \rho_s\Sigma(\Phi^s{}_{2s}), \rho_s U U_s{}^2, \rho_s U\ddot{\chi}, \rho_s U\Phi_2, \rho_s U\Phi^s{}_{2s}, \rho_s U\Phi_{2s}, \rho_s U\Phi_1, \rho_s U^2 U_s, \rho_s U_s\ddot{\chi}_s, \rho_s U_s\Phi_2, \rho_s U_s\Phi^s{}_2,$
$\rho_s U_s\Phi^s{}_{2s}, \ddot{U}_s\ddot{\chi}, \ddot{U}_s\Phi_1, G_7.\nabla\dot{U}_s, P(\dot{U}_s\nabla U_s).\nabla\dot{U}_s, V_2.\nabla\dot{U}_s, V^s{}_{2s}.\nabla\dot{U}_s, \ddot{\chi}.\nabla\dot{U}_s, \Sigma(v^2 v).\nabla\dot{U}_s, \Phi_2.\nabla\dot{U}_s, V.\nabla\dot{\Sigma}(U_s a_s), V.\nabla\dot{\Phi}^s{}_1, V.\nabla\dot{\Phi}^s{}_2, V.\nabla\dot{\Phi}^s{}_{2s}, V.\nabla\dot{U}_s\dot{U}_s, V.\nabla\overset{(3)}{\chi}_s, P_{3s}{}^{ab}\nabla_a\nabla_b U_s,$
$P^S{}_{s3s}{}^{ab}\nabla_a\nabla_b U_s, P^S{}_{s3}{}^{(ab)}\nabla_a\nabla_b U_s, P(\nabla^{(a}U V^{b)})\nabla_a\nabla_b U_s, P(\nabla^{(a}U\nabla^{b)}\ddot{\chi})\nabla_a\nabla_b U_s, P(\nabla^{(a}U_s\nabla^{b)}\ddot{\chi}_s)\nabla_a\nabla_b U_s, S_2{}^{ab}\nabla_a\nabla_b U_s, S_{2s}{}^{ab}\nabla_a\nabla_b U_s, \ddot{\chi}^{ab}\nabla_a\nabla_b U_s, \Sigma^{ab}(U)\nabla_a\nabla_b U_s, \Sigma^{ab}(U_s)\nabla_a\nabla_b U_s,$
$P_3{}^{(ab)}\nabla_a\nabla_b U_s, P_{3s}{}^{(ab)}\nabla_a\nabla_b U_s, P_{s3s}{}^{(ab)}\nabla_a\nabla_b U_s, P^S{}_{s3}{}^{(ab)}\nabla_a\nabla_b U_s, P^S{}_{s3s}{}^{(ab)}\nabla_a\nabla_b U_s, P(\nabla^{(a}U_s\nabla^{b)}\Sigma(U_s a_s))\nabla_a\nabla_b U_s, P(\nabla^{(a}U_s\nabla^{b)}\Phi^s{}_1)\nabla_a\nabla_b U_s, P(\nabla^{(a}U\nabla^{b)}\Phi_1)\nabla_a\nabla_b U_s,$
$\Sigma_s(U_s v^a v^b)\nabla_a\nabla_b U_s, \Sigma(U_s v^a v^b)\nabla_a\nabla_b U_s, \Sigma(U v^a v^b)\nabla_a\nabla_b U_s, \Sigma(v^2 v^a v^b)\nabla_a\nabla_b U_s, \Phi_1{}^{ab}\nabla_a\nabla_b\Sigma(U_s a_s), \Phi_1{}^{ab}\nabla_a\nabla_b\Phi^s{}_1, \Phi_1{}^{ab}\nabla_a\nabla_b\Phi^s{}_2, \Phi_1{}^{ab}\nabla_a\nabla_b\Phi^s{}_{2s}, P_2{}^{ab}\nabla_a\nabla_b\Sigma(U_s a_s),$
$P_2{}^{ab}\nabla_a\nabla_b\Phi^s{}_1, P_2{}^{ab}\nabla_a\nabla_b\Phi^s{}_2, P_2{}^{ab}\nabla_a\nabla_b\Phi^s{}_{2s}, P_{2s}{}^{ab}\nabla_a\nabla_b\Sigma(U_s a_s), P_{2s}{}^{ab}\nabla_a\nabla_b\Phi^s{}_1, P_{2s}{}^{ab}\nabla_a\nabla_b\Phi^s{}_2, P_{2s}{}^{ab}\nabla_a\nabla_b\Phi^s{}_{2s}, \Phi_1{}^{ab}\nabla_a\nabla_b\ddot{\chi}_s, P_2{}^{ab}\nabla_a\nabla_b\ddot{\chi}_s, P_{2s}{}^{ab}\nabla_a\nabla_b\ddot{\chi}_s, U\overset{(4)}{\chi}_s, U U_s\ddot{U}_s,$
$U\ddot{\Sigma}(U_s a_s), U\ddot{\Phi}^s{}_1, U\ddot{\Phi}^s{}_2, U\ddot{\Phi}^s{}_{2s}, U\dot{U}_s{}^2, U\nabla\dot{U}_s.V, U^2\ddot{U}_s, U^2(\nabla U_s)^2, \Phi_2\ddot{U}_s, \Phi_2(\nabla U_s)^2, \Phi^s{}_{2s}\ddot{U}_s, \Phi^s{}_{2s}(\nabla U_s)^2, U_s{}^2(\nabla U_s)^2, P_2{}^{ab}\nabla_a U_s\nabla_b U_s, P_{2s}{}^{ab}\nabla_a U_s\nabla_b U_s, \Phi_1{}^{ab}\nabla_a U_s\nabla_b U_s,$
$P(V^{[c,a]}V_{[c}{}^{,b]})\nabla_a\nabla_b U_s, U_s V.\nabla\dot{U}_s, P_2{}^{ab}\nabla_a\nabla_b U_s, P_{2s}{}^{ab}\nabla_a\nabla_b U_s, \Phi_1{}^{ab}\nabla_a\nabla_b U_s$

$\rho U^2 U_s a_s, \rho_s U^2 v^2, \rho_s U^3, \rho_s U^2 U_s, \rho\Phi_2 U_s a_s, \rho_s\Phi_2 v^2, \rho_s\Phi_2 U, \rho_s\Phi_2 U_s, \rho\Phi^s{}_{2s} U_s a_s, \rho_s\Phi^s{}_{2s} v^2, \rho_s\Phi^s{}_{2s} U, \rho_s\Phi^s{}_{2s} U_s, \rho U_s{}^3 a_s, \rho_s U_s{}^2 v^2, \rho_s U_s{}^2 U, \rho_s U_s{}^3, \rho_s U_s\Sigma(U_s a_s),$
$\rho_s U_s\Phi^s{}_1$

- Terms like $\dddot{\mathcal{I}}$ require 3PN equations of motion

Tricky in Will, Wiseman &Pati

Solution: Laura Bernard!

To-do

- 3 PN EOM (Laura Bernard et al.)
- Multipole moments (us)

→ Scalar Waves

↓

Energy Flux

←

Feed into ready to use waveforms (Sennett et al.)

↓

2PN scalar tensor templates for LIGO data analysis