4.5 Post-Newtonian order gravitational radiation

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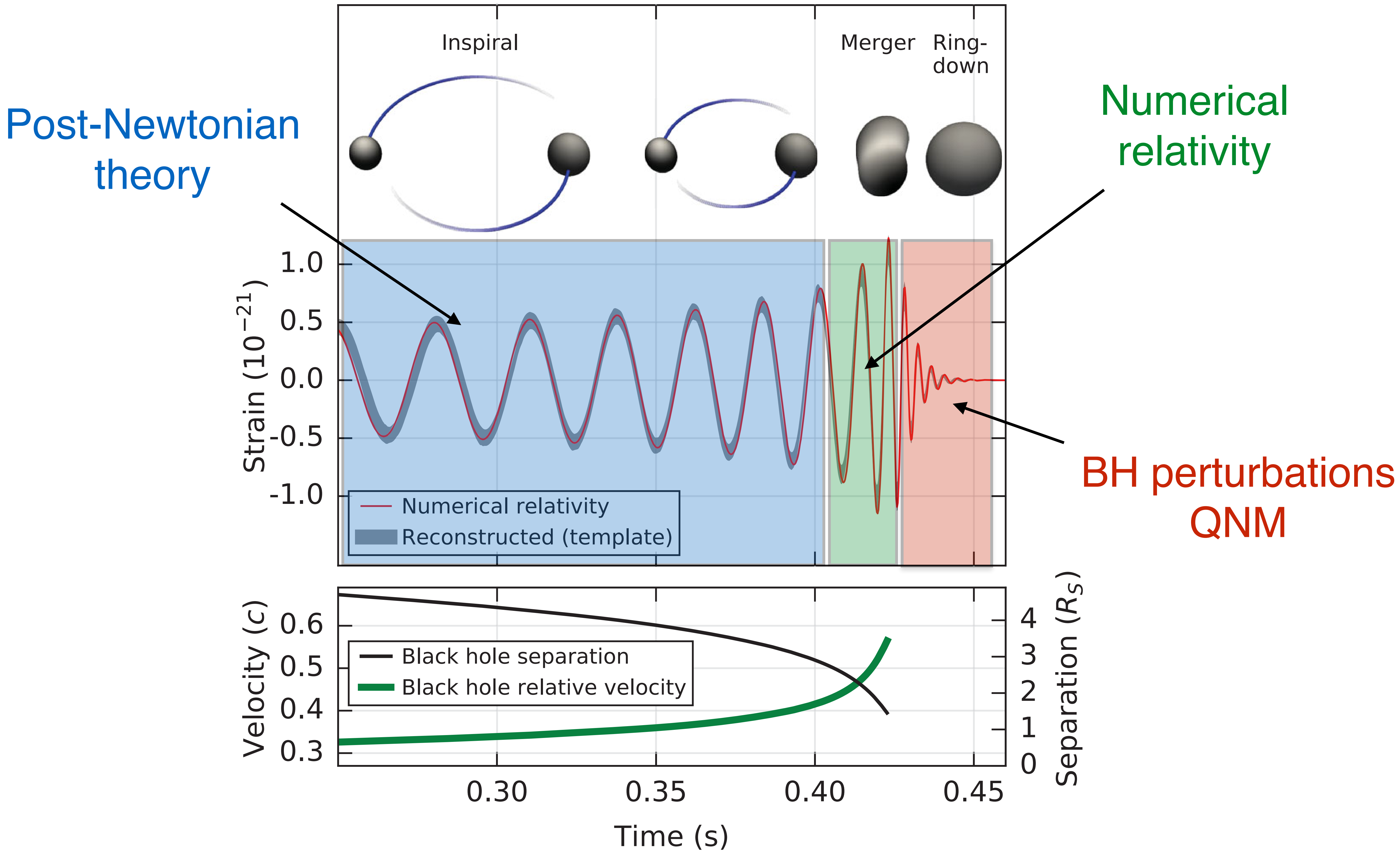
I. INTRODUCTION



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Post-Newtonian theory

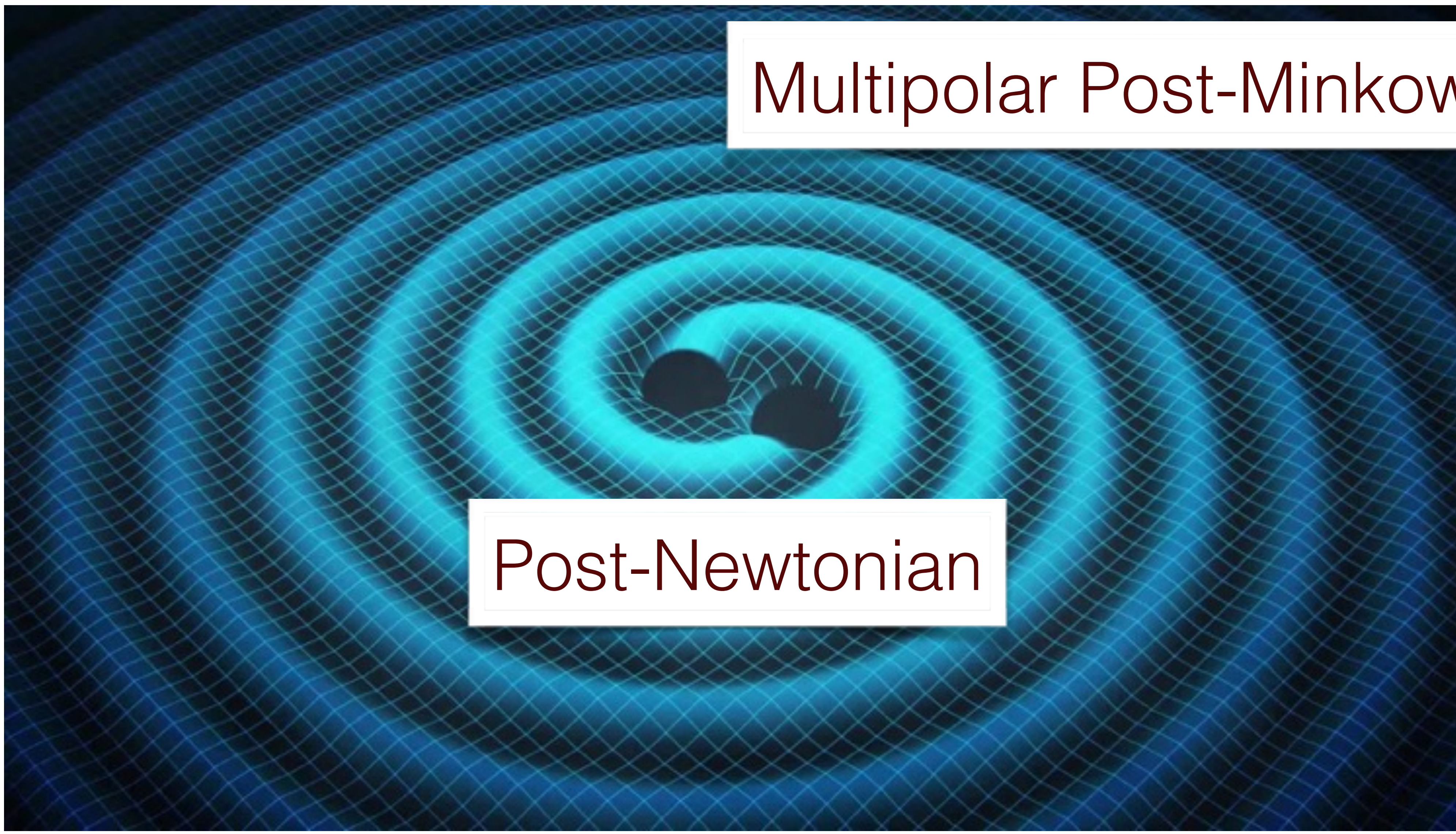
- Perturbative expansion of relativistic effects

- 1 PN $\longrightarrow \left(\frac{v}{c}\right)^2$

- More and more difficulties appear as we go to higher orders

Blanchet-Damour-Iyer formalism

Blanchet-Damour-Iyer formalism



[Matching equation Blanchet 1998]

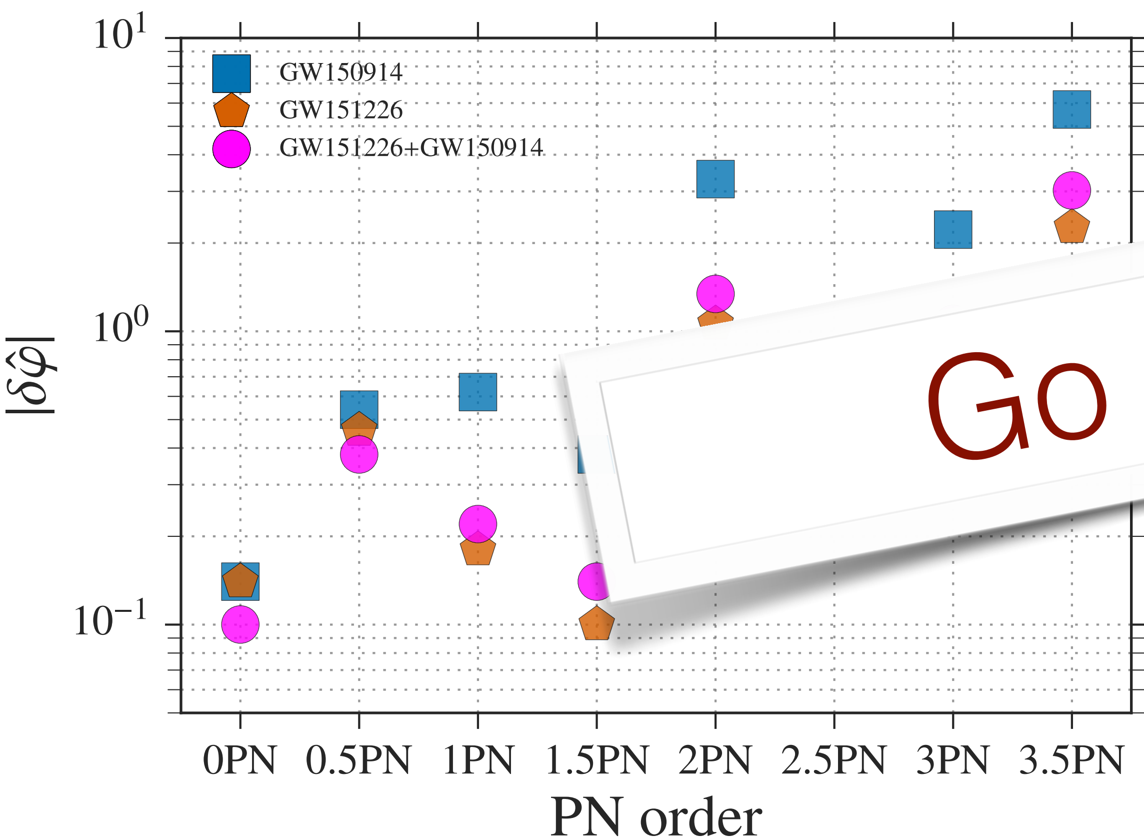
figure: www.virgo-gw.eu

Once the **equations of motions** and the **flux** is known at n-PN

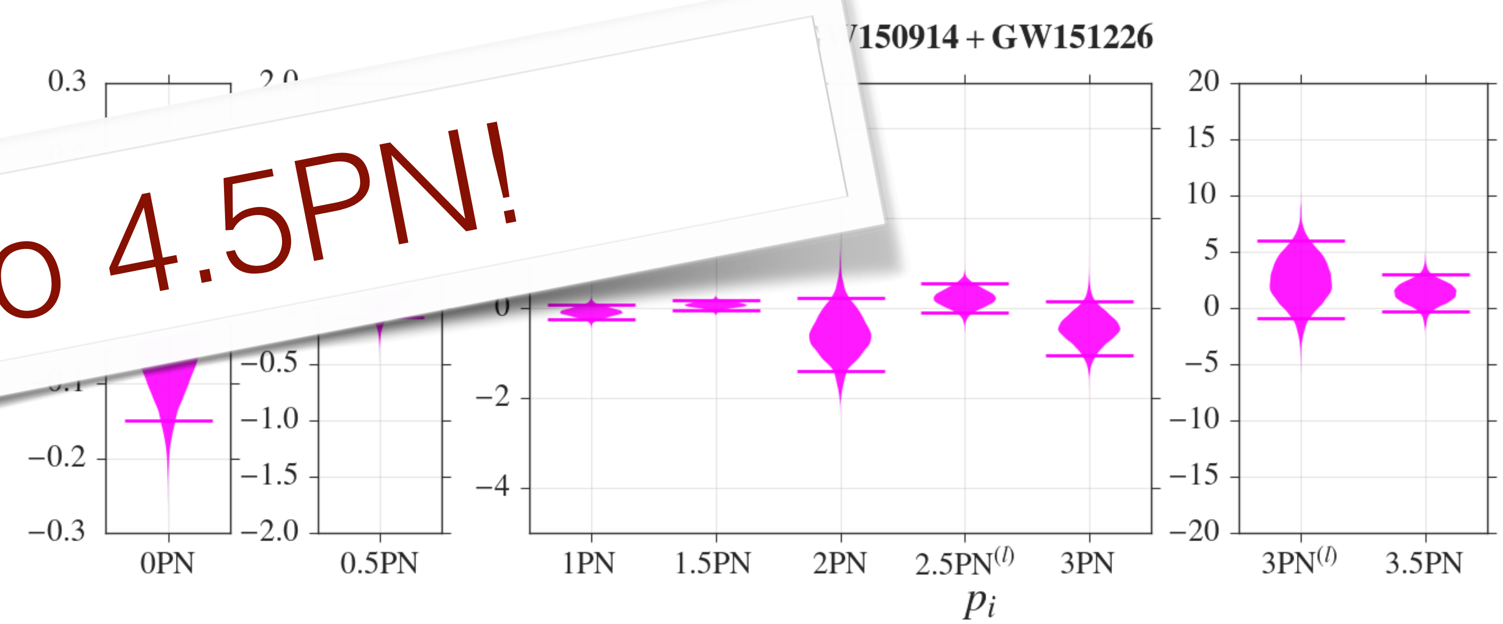
We consider consecutive circular orbits of orbital frequencies Ω , energy E_n and emitting a flux $\mathcal{F}_n$

$$x = \left(\frac{Gm\Omega}{c^3}\right)^{2/3}$$

$$\frac{\mathrm{d}E_n(x)}{\mathrm{d}t} = \mathcal{F}_n(x) \quad \Rightarrow \quad \phi_{\mathrm{n}} = f_{\mathrm{n}}(x)$$



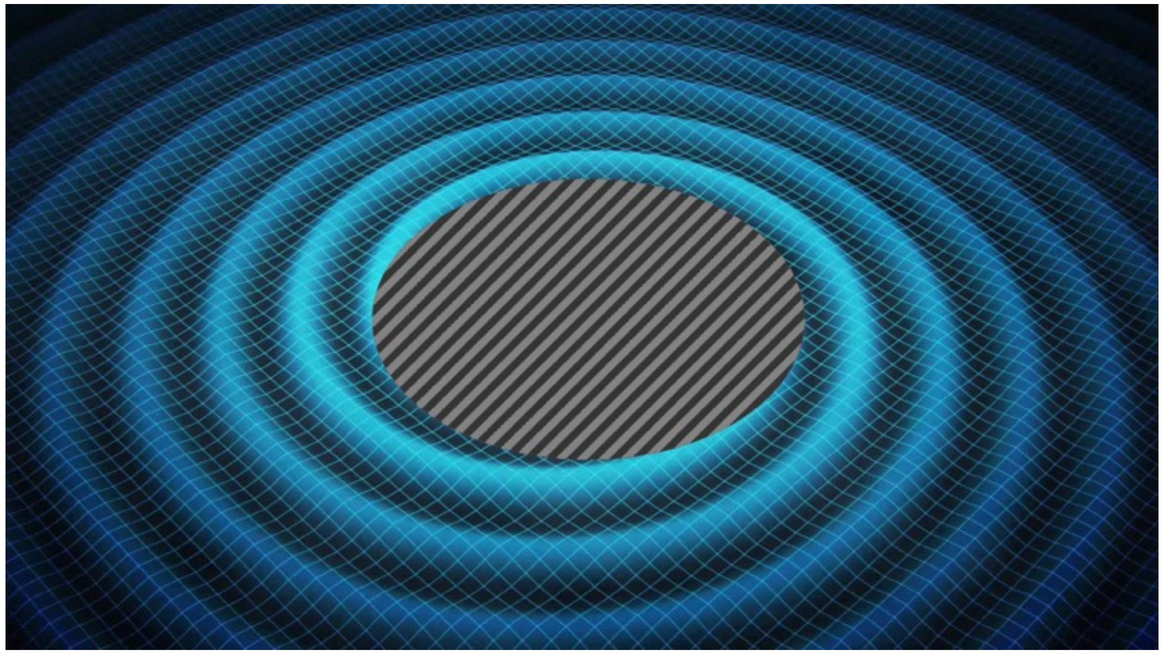
Go to 4.5PN!



LIGO Scientific and Virgo collaboration arxiv:1606.04856

II. The multipolar post-Minkowskian (MPM) algorithm

The MPM algorithm



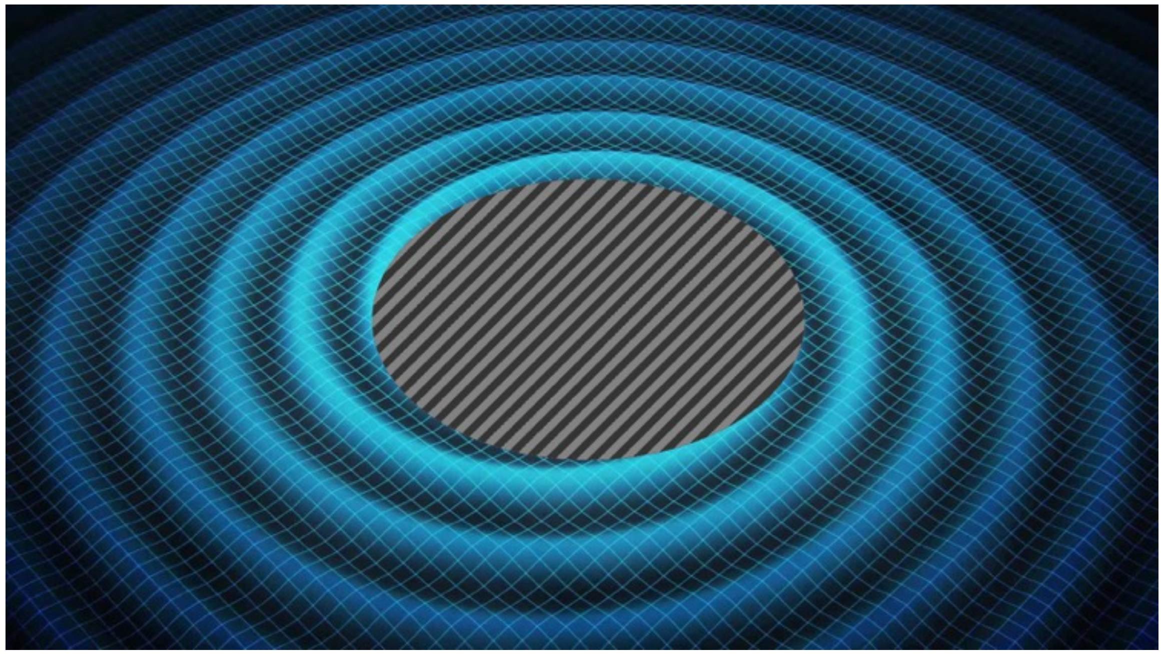
$$G_{\mu\nu}(g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}) = 0$$

$$h^{\mu\nu} \equiv \sqrt{-g}g^{\mu\nu} - \eta^{\mu\nu} = \mathcal{G}h^{\mu\nu}_{(1)} + \mathcal{G}^2 h^{\mu\nu}_{(2)} + \dots$$

$$\begin{cases} \Box h^{\mu\nu}_{(i)} = \Lambda^{\mu\nu}_{(i)}(h^{\alpha\beta}_{(1)}, \dots, h^{\alpha\beta}_{(i-1)}) \\ \partial^\mu h_{(i)\mu\nu} = 0 \end{cases}$$

[harmonic coordinates]

The MPM algorithm



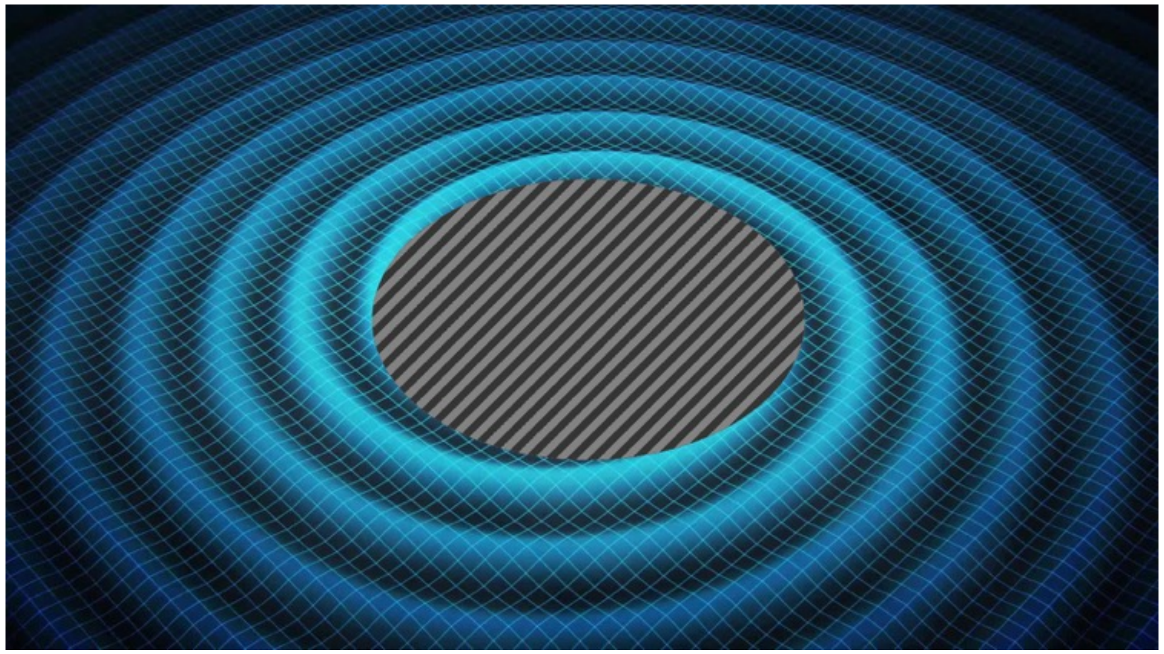
$$\begin{cases} \Box h^{\mu\nu}_{(i)} = \Lambda^{\mu\nu}_{(i)}(h^{\alpha\beta}_{(1)}, \dots, h^{\alpha\beta}_{(i-1)}) \\ \partial^\mu h_{(i)\mu\nu} = 0 \end{cases}$$

$$h^{\mu\nu}_{(1)} \sim \sum_{l\geq 0} \partial_{i_1,\dots,i_l} \left(\frac{M_{i_1\dots i_l}(t-r)}{r} \right) + \sum_{l\geq 2} \partial_{i_1,\dots,i_l} \left(\frac{S_{i_1\dots i_l}(t-r)}{r} \right)$$

$$= h^1_M + h^1_{M_{ij}} + h^1_{M_{ijk}} + \dots + h^1_{S_{ij}} + h^1_{S_{ijk}} + \dots$$ [Thorne 80]

$$h^{\mu\nu}_{(2)} = h^{\mu\nu}_{(2)M\times M} + h^{\mu\nu}_{(2)M\times M_{ij}} + h^{\mu\nu}_{(2)M_{ij}\times M_{ij}} + \dots$$

First issue: regularization



$$\begin{cases} \Box h^{\mu\nu}_{(i)} = \Lambda^{\mu\nu}_{(i)}(h^{\alpha\beta}_{(1)}, \ldots, h^{\alpha\beta}_{(i-1)}) \\ \partial^\mu h_{(i)\mu\nu} = 0 \end{cases}$$

$$\Box^{-1}\Lambda(x,t) = \int \mathrm{d}^3x' \frac{\Lambda(x', t - \mid x - x' \mid)}{\mid x - x' \mid}$$

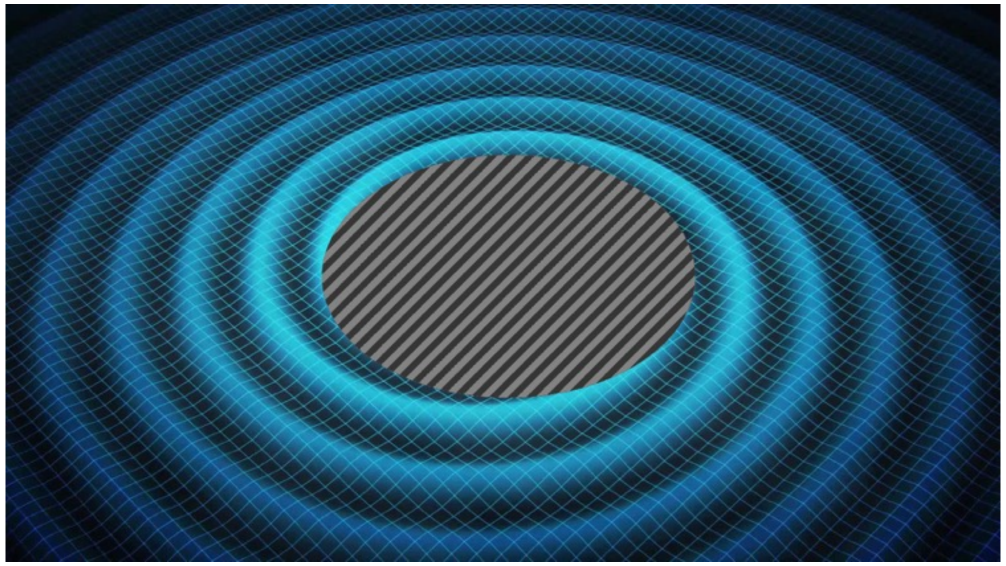
Issue: $\Lambda \sim_{r\to 0} \frac{1}{r^k},\ k \geq 3$

$$\Rightarrow \quad \mathrm{FP}_{B=0}\Box^{-1}\left[\left(\frac{r}{r_0}\right)^B \Lambda\right]$$

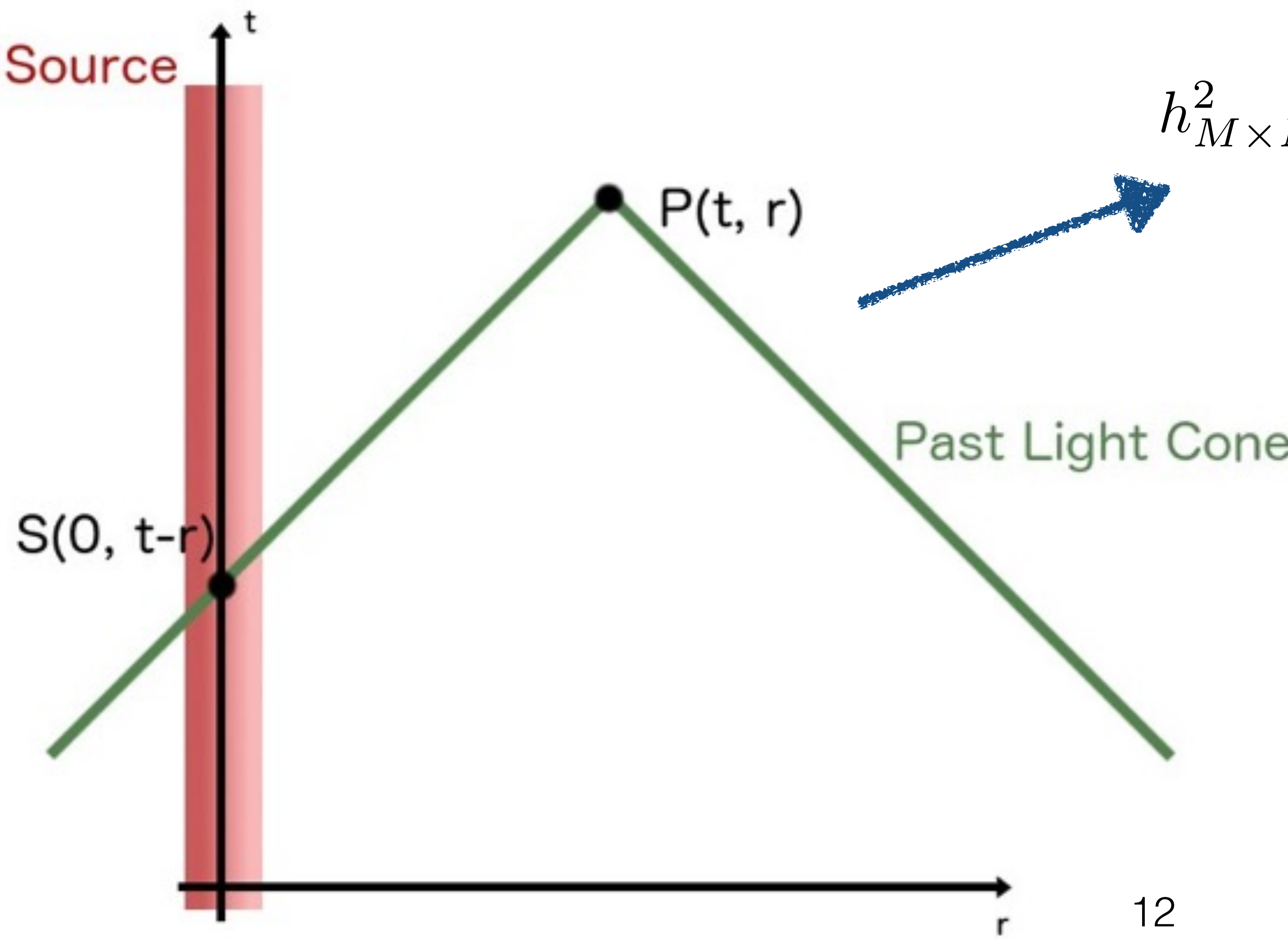
[Analytic continuation in $B \in \mathbb{C}$]

[Finite Part when $B \to 0$]

Second issue: tails



$$\begin{cases} \Box h^{\mu\nu}_{(i)} = \Lambda^{\mu\nu}_{(i)}(h^{\alpha\beta}_{(1)}, \ldots, h^{\alpha\beta}_{(i-1)}) \\ \partial^\mu h_{(i)\mu\nu} = 0 \end{cases}$$



$$h^2_{M\times M_{ij}}(t,r) \sim M \int_0^\infty \mathrm{d}\tau M_{ij}(t-r-\tau)\mathcal{Q}(\tau)$$

4.5PN project

Ultimate Goal: compute the flux up to 4.5PN

Done so far: compute all the 4.5PN contributions of the tails:

$$h^{\mu\nu}_{(2)M\times M_{ij}},\ h^{\mu\nu}_{(3)M\times M\times M_{ij}},\ h^{\mu\nu}_{(4)M\times M\times M\times M_{ij}}$$

$$h^{\mu\nu}_{(2)M\times M_{ijk}},\ h^{\mu\nu}_{(3)M\times M\times M_{ijk}}$$

$$h^{\mu\nu}_{(2)M\times M_{ijkl}}\ ,\ \ldots$$

[TM Blanchet Faye (2016) 1607.07601]

- Required new analytical formulae

$$\mathrm{FP}_{B=0}\Box^{-1}\left[\hat{n}_L\left(\frac{r}{r_0}\right)^B r^{-k}\int_1^\infty \mathrm{d}y Q_m(y)F(t-ry)\right]$$
$$= -\hat{n}_L\int_1^\infty \mathrm{d}s F^{(k-2)}(t-rs)\left(Q_l(s)\int_1^s \mathrm{d}y Q_m^{(-k+2)}(y)P_l(y) + P_l(s)\int_s^\infty \mathrm{d}y Q_m^{(-k+2)}(y)Q_l(y)\right)$$

- Implementing the algorithm into Mathematica

[xAct - xTensor]

Going to radiative coordinate

$$h^{00}_{M^3\times M_{ij}} = \frac{M^3 \hat{n}_{ab}}{r}\int_0^{+\infty} \mathrm{d}\tau\, M^{(6)}_{ab}\left\{-\frac{8}{3}\ln^3\left(\frac{\tau}{2r}\right) + \frac{148}{21}\ln^2\left(\frac{\tau}{2r}\right) + \frac{232}{21}\ln\left(\frac{r}{r_0}\right)\ln\left(\frac{\tau}{2r}\right) + \frac{1016}{2205}\ln\left(\frac{\tau}{2r}\right) + \frac{104}{15}\ln\left(\frac{r}{r_0}\right) + \frac{16489}{1575} - \frac{232\pi^2}{63}\right\} + \mathcal{O}\left(\frac{1}{r^{2-\epsilon}}\right)$$

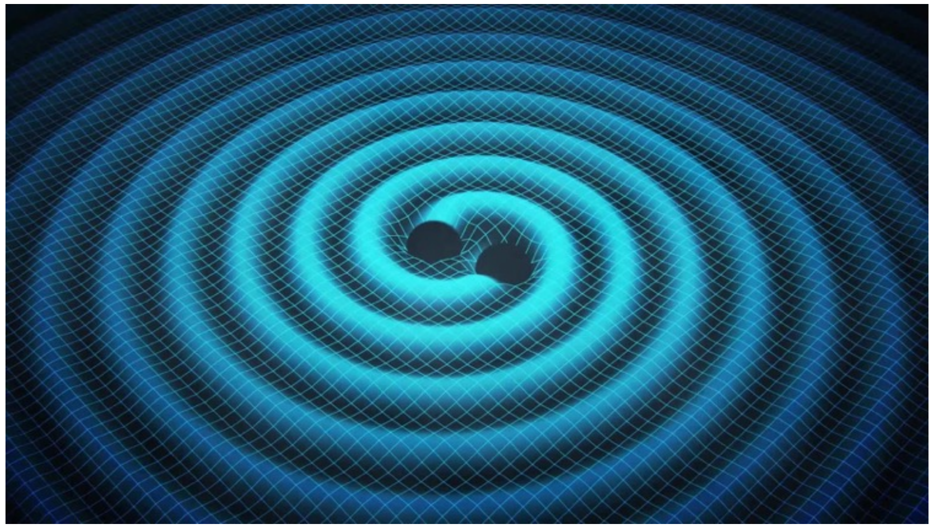
$$X^\mu = x^\mu + \xi^\mu(x) \qquad \begin{cases} \xi^0 = -2M\ln\left(\frac{r}{b_0}\right) \\ \xi^i = 0 \end{cases}$$

$$H^{00}_{M^3\times M_{ij}} = \frac{M^3 \hat{N}_{ab}}{R}\int_0^{+\infty} \mathrm{d}\tau\, M^{(6)}_{ab}\left\{-\frac{8}{3}\ln^3\left(\frac{\tau}{2b_0}\right) - 4\ln^2\left(\frac{\tau}{2b_0}\right) + \frac{232}{21}\ln\left(\frac{\tau}{2b_0}\right)\ln\left(\frac{\tau}{2r_0}\right) - \frac{14272}{2205}\ln\left(\frac{\tau}{2b_0}\right) + \frac{104}{15}\ln\left(\frac{\tau}{2r_0}\right) + \frac{16489}{1575} - \frac{232\pi^2}{63}\right\} + \mathcal{O}\left(\frac{1}{R^2}\right)$$

We can compute the flux directly from $H^{\mu\nu}$

Matching equation to compute the multipole moments

[e.g. Blanchet 1998]



$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

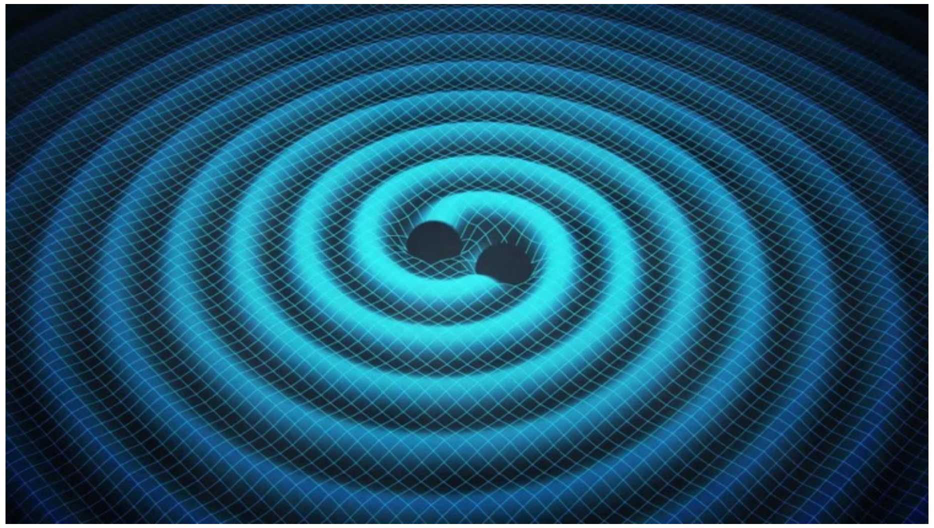
$$\gamma = \frac{Gm}{rc^2} \qquad \text{(circular orbits)}$$

Tails

$$\mathcal{F}_{\text{tails}} = \frac{32c^5}{5G}\nu^2\gamma^5\Bigg\{4\pi\gamma^{3/2} + \ldots + \left(\frac{9997778801}{106444800} - \frac{6848}{105}\ln\left(\frac{r}{r_0}\right) + \left[-\frac{8058312817}{2661120} + \frac{287}{32}\pi^2 + \frac{572}{3}\ln\left(\frac{r}{r'_0}\right)\right]\nu - \frac{12433367}{13824}\nu^2 - \frac{1026257}{266112}\nu^3\right)\pi\gamma^{9/2} + \mathcal{O}\left(\frac{1}{c^{11}}\right)\Bigg\},$$

Tails-of-tails-of-tails and Tails-of-tails × tails

$$\mathcal{F}_{\text{T}-\text{T}-\text{T and T}-\text{T}*\text{T}} = \frac{32c^5}{5G}\nu^2\gamma^5\Bigg\{\left(-\frac{467044}{3675} - \frac{3424}{105}\ln(16\gamma) + \frac{6848}{105}\ln\left(\frac{r}{r_0}\right) - \frac{6848}{105}\gamma_{\text{E}}\right)\pi\gamma^{9/2} + \mathcal{O}\left(\frac{1}{c^{11}}\right)\Bigg\}.$$



$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$x = \left(\frac{G m \Omega}{c^3}\right)^{2/3} = \mathcal{O}\left(\frac{1}{c^2}\right)$$

$$\begin{aligned}
\mathcal{F}_{\rm total} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right) x^2 \\
& + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E \right. \\
& \left. - \frac{856}{105}\ln(16x) + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \right] x^3 \\
& + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + (\text{unknown coefficients}) x^4 \\
& + \left(\frac{265978667519}{745113600} - \frac{6848}{105}\gamma_{\rm E} - \frac{3424}{105}\ln(16x) + \left[\frac{2062241}{22176} + \frac{41}{12}\pi^2\right]\nu \right. \\
& \left. - \frac{133112905}{290304}\nu^2 - \frac{3719141}{38016}\nu^3\right)\pi x^{9/2} + \mathcal{O}(x^5) \Bigg\}.
\end{aligned}$$

[Tanaka et al. gr-qc/9701050]

arxiv:1607.07601

[See also Messina&Nagar 2017]

Conclusion

- Goal: reach 4.5PN Flux in order the have the 4.5PN phase
- Done so far: Tail effects up to 4.5PN, and the 4.5PN term in the flux of circular orbits.
- What's next: Computing M_{ij} at 4PN and S_{ij} at 3PN.

Ultimately, 4.5PN phase and polarization

Thank you !