Quantum mechanics and large-scale CMB anomalies

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A. Valentini, “Inflationary cosmology as a probe of primordial quantum mechanics”, Phys. Rev. D **82**, 063513 (2010).

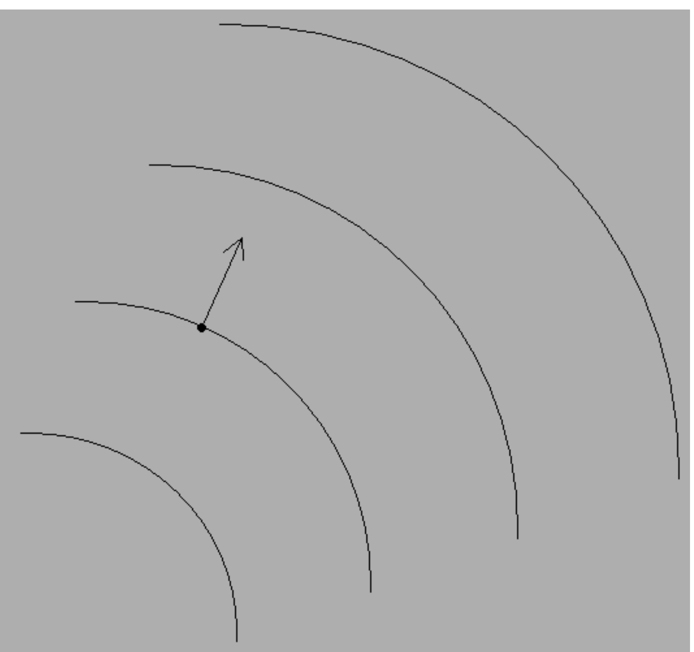
S. Colin and A. Valentini, “Mechanism for the suppression of quantum noise at large scales on expanding space”, Phys. Rev. D **88**, 103515 (2013).

S. Colin and A. Valentini, “Primordial quantum nonequilibrium and large-scale cosmic anomalies”, arXiv:1407.8262 [astro-ph.CO].

De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2\Psi + V\Psi$$

$$(\ \Psi = |\Psi|\, e^{iS}\ ) \qquad m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$



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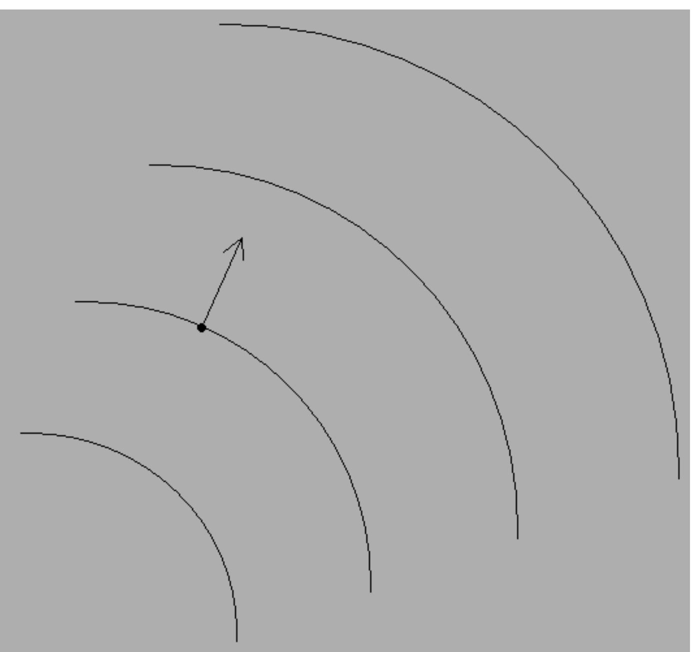
(cf. WKB, but for *any* wave function)



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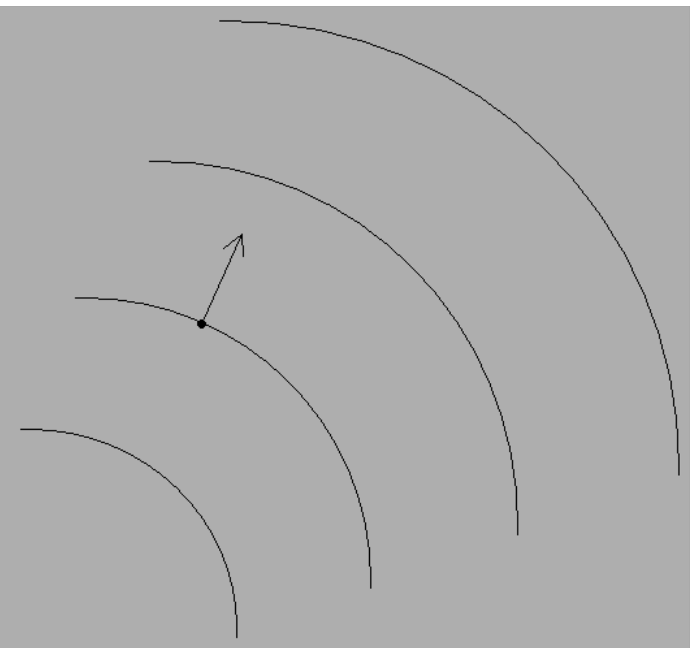

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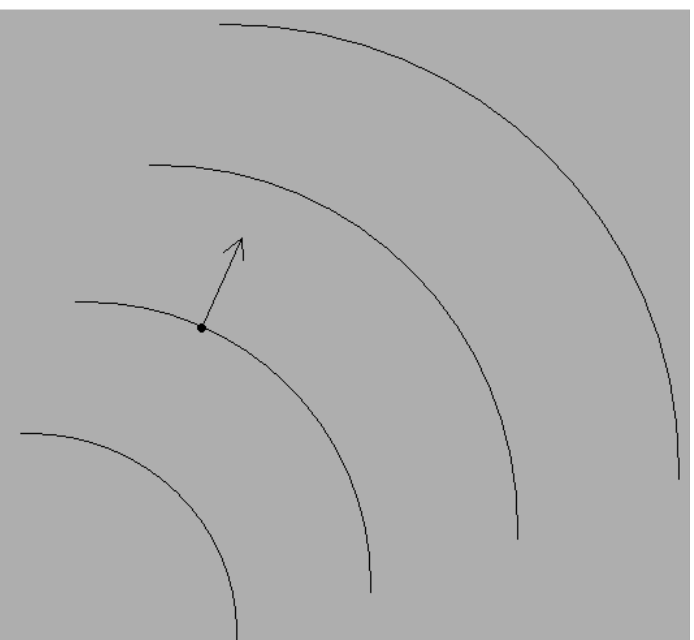
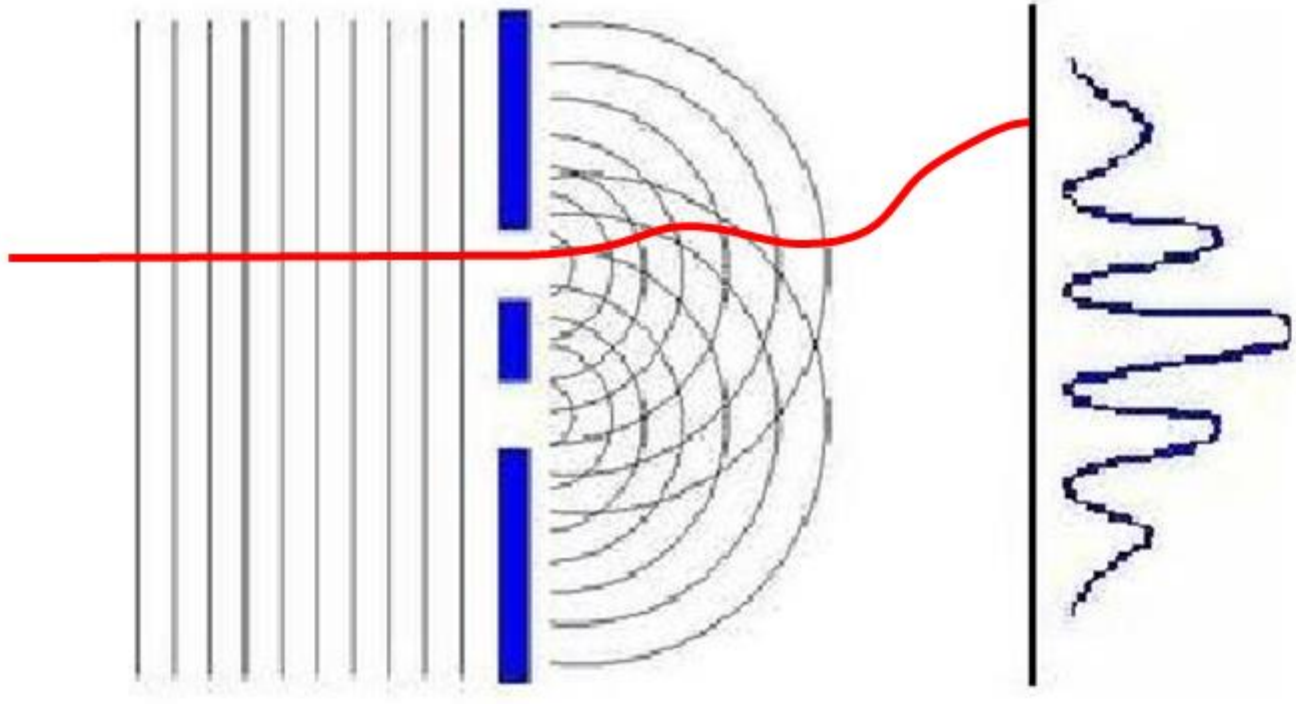


(Generalise: configuration $q(t)$)

Get QM if *assume* initial Born-rule distribution, $P = |\Psi|^2$ (preserved in time by the dynamics)

(shown fully by Bohm in 1952)

Example of one particle

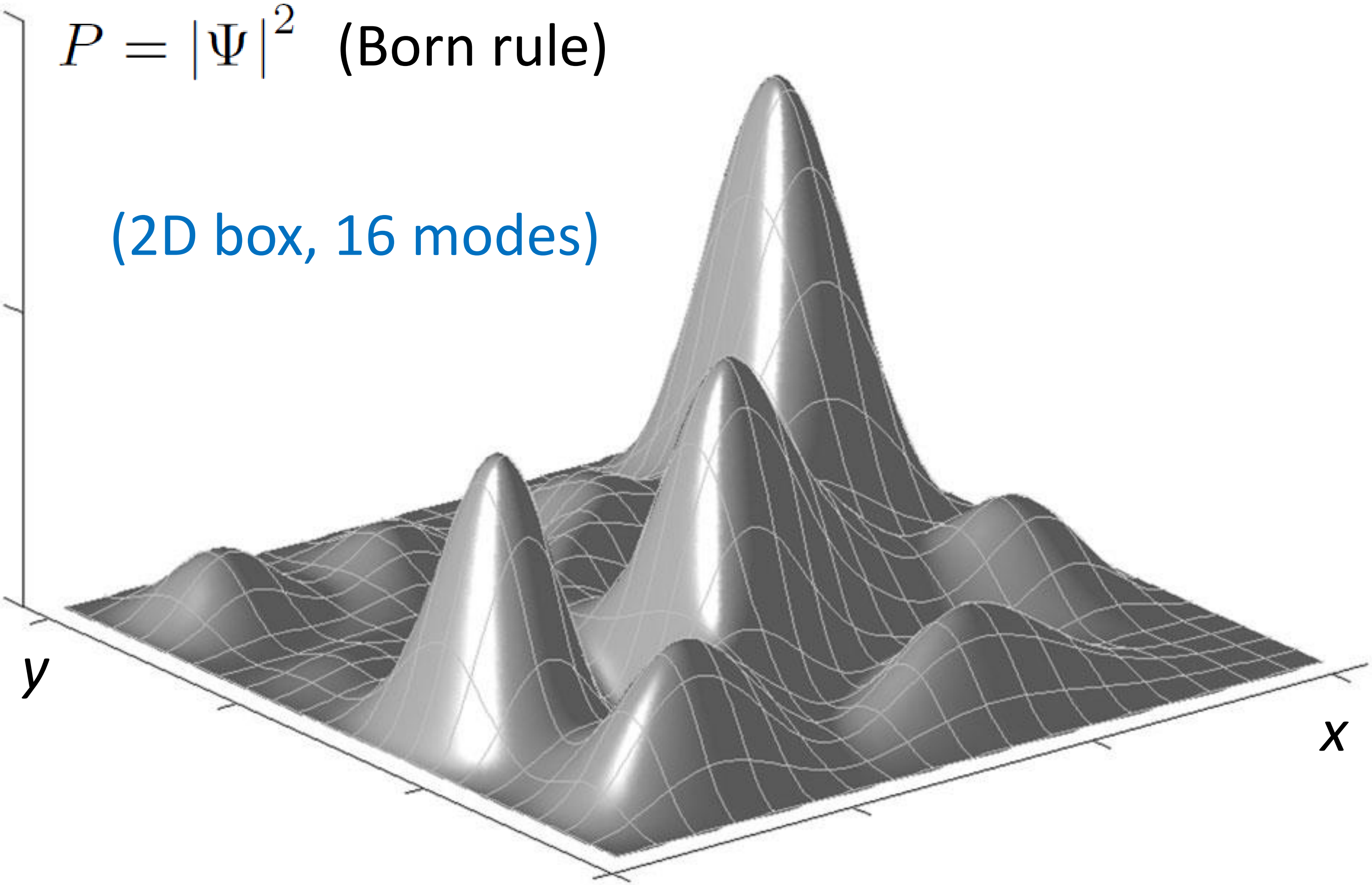


In agreement with experiment
if assume initial $P = |\Psi|^2$

Disagrees with experiment for initial $P \neq |\Psi|^2$

Quantum theory = special case of a wider physics

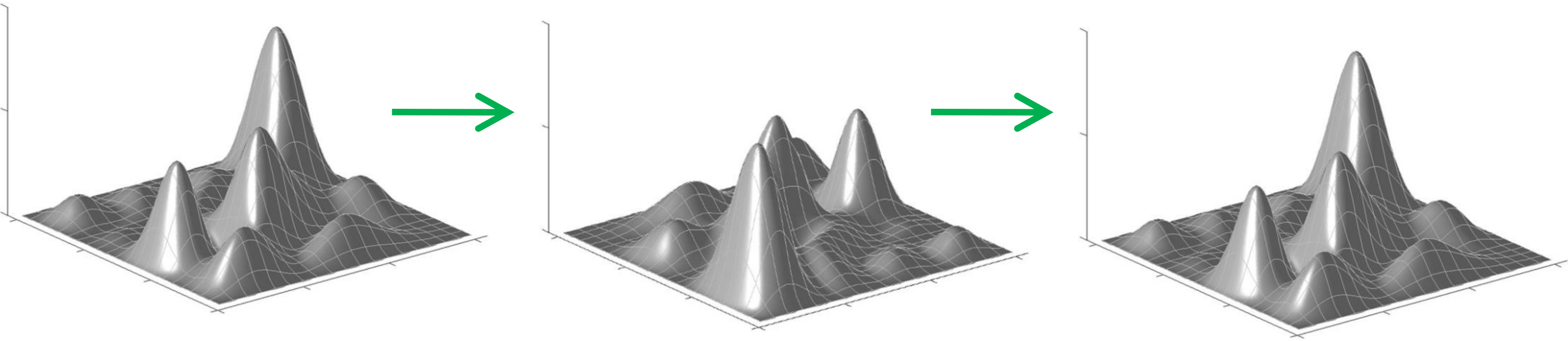
BUT: *experimentally* quantum dof's are always found to have the "quantum equilibrium" distribution:



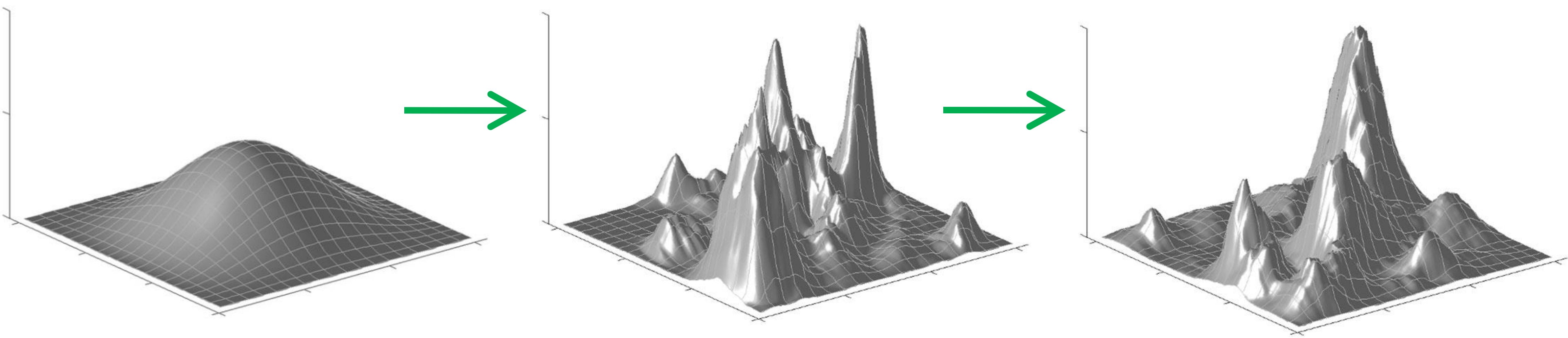
Why?

Relaxation to quantum equilibrium

Equilibrium ($P = |\Psi|^2$) changes with time

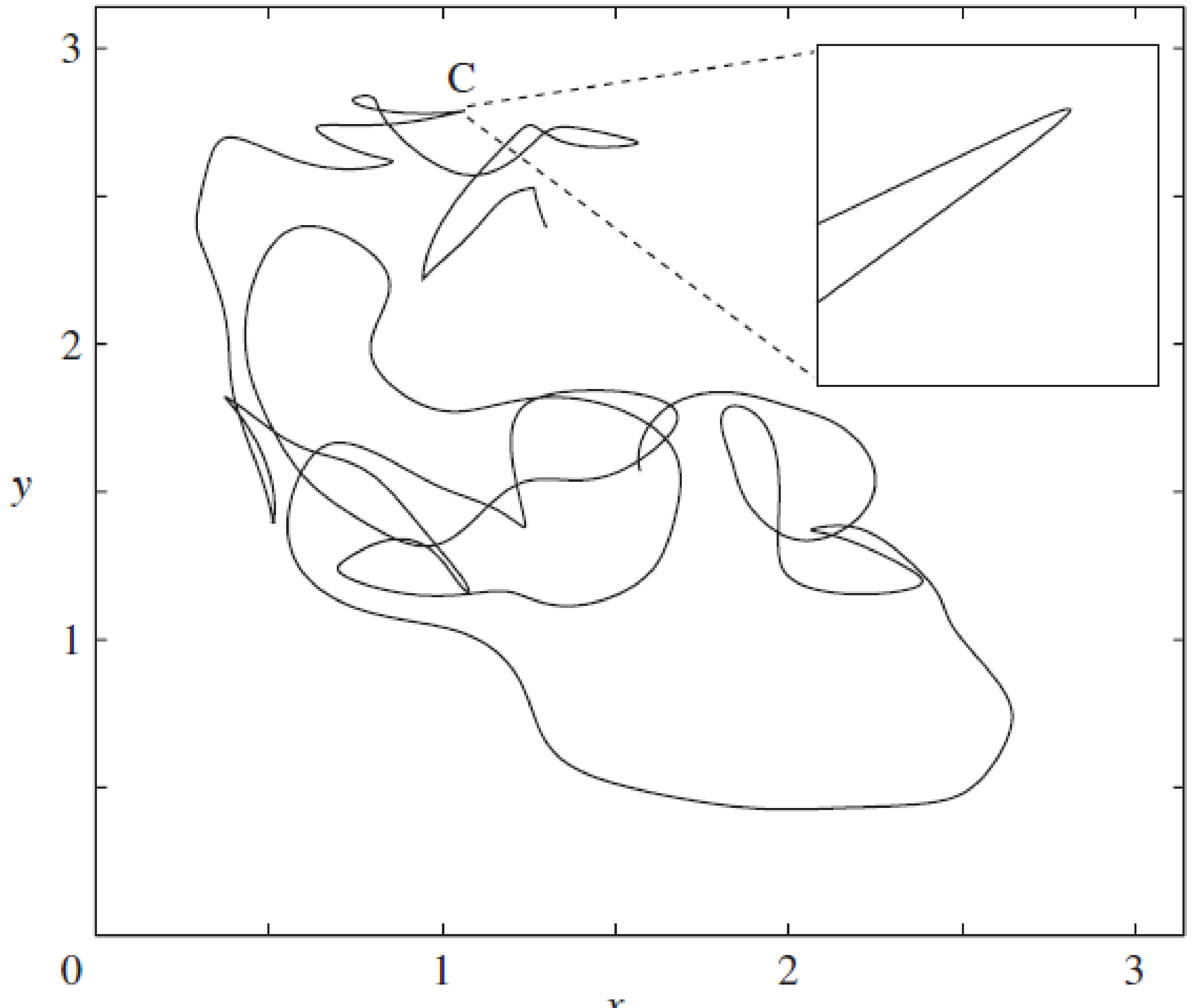


Non-equilibrium ($P \neq |\Psi|^2$) relaxes to equilibrium



(Valentini and Westman, Proc. Roy. Soc. A 2005)

Superposed energies give rapidly-varying velocity fields



Trajectories are erratic and tend to explore the region

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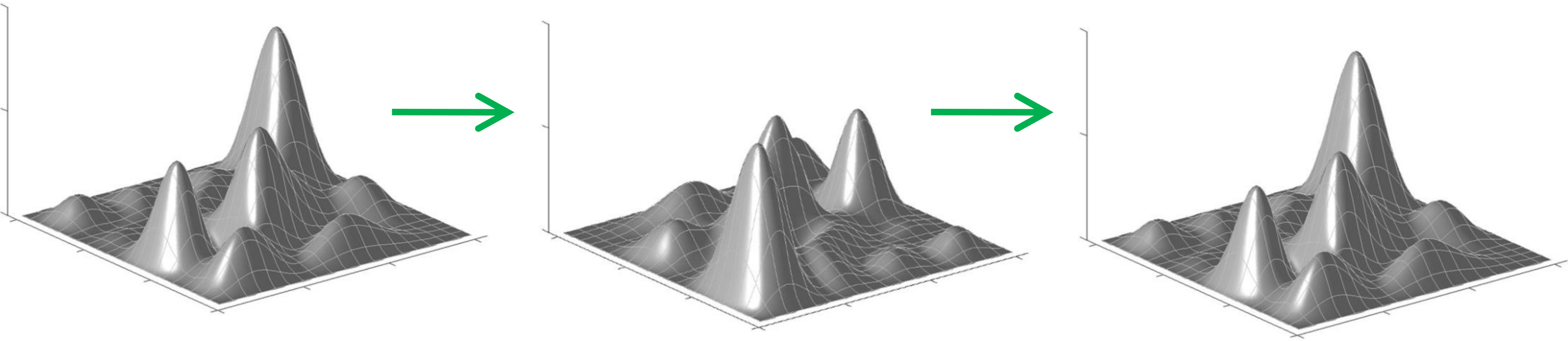

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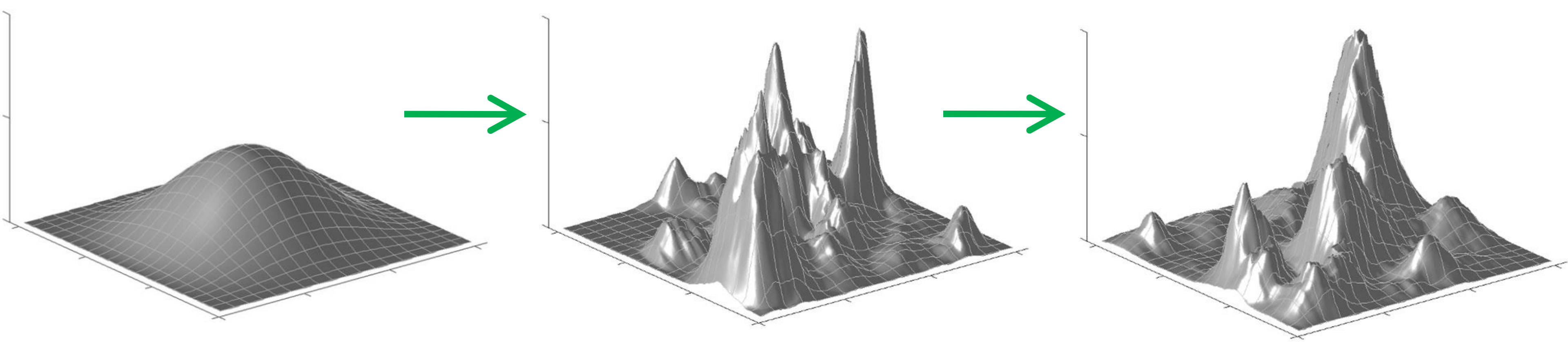


(Valentini and Westman, Proc. Roy. Soc. A 2005)

Quantify relaxation with a coarse-grained *H*-function

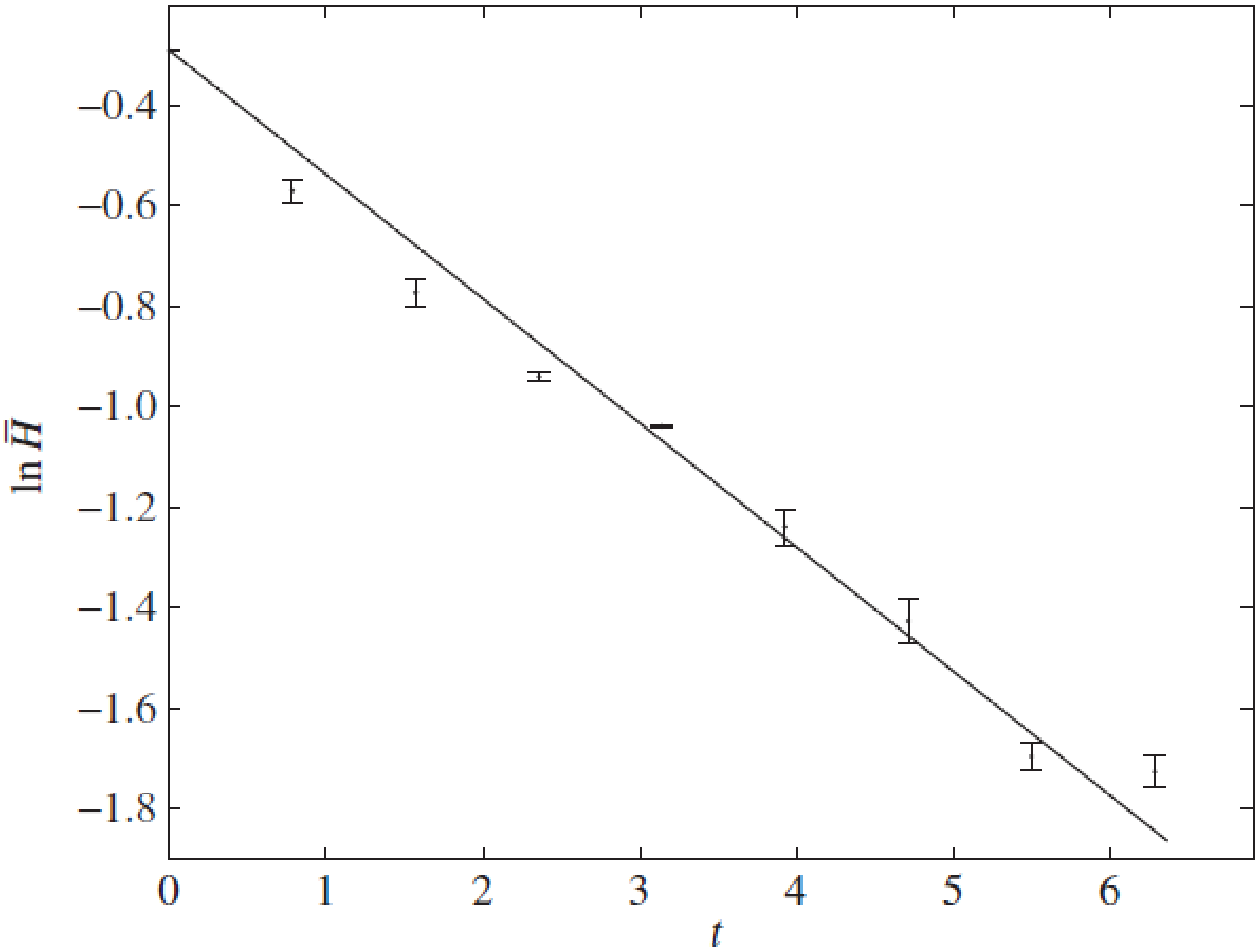
$$\bar{H} = \int \mathrm{d}q\, \bar{\rho} \ln(\bar{\rho}/\overline{|\psi|^2}),$$ (minus the relative entropy)

Obeys the *H*-theorem (Valentini 1991, 1992)

$$\bar{H}(t) \leqslant \bar{H}(0)$$ (cf. classical analogue)

assuming no initial fine-grained structure in ρ and $|\psi|^2$

Simulations show *exponential decay* of *H*-function



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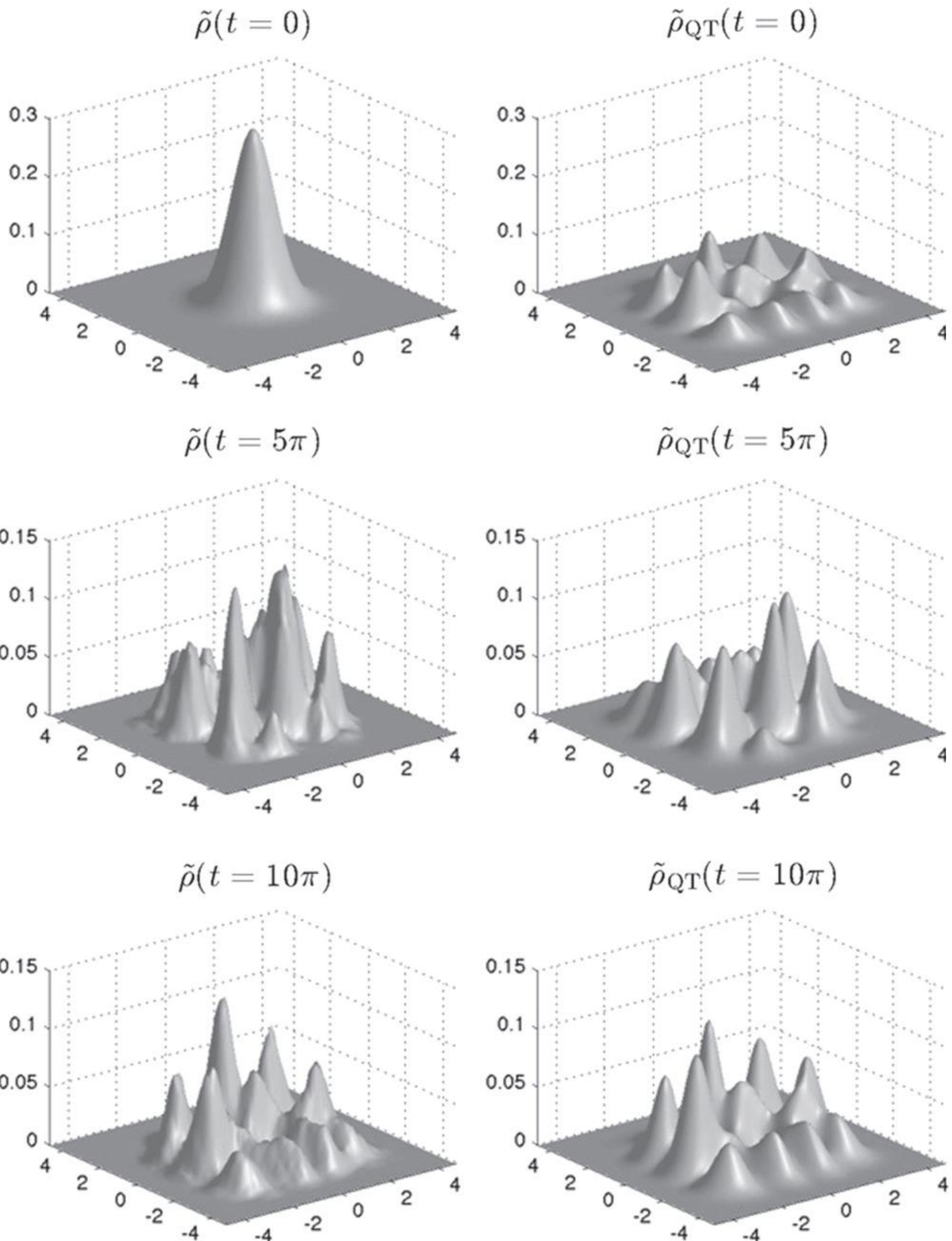
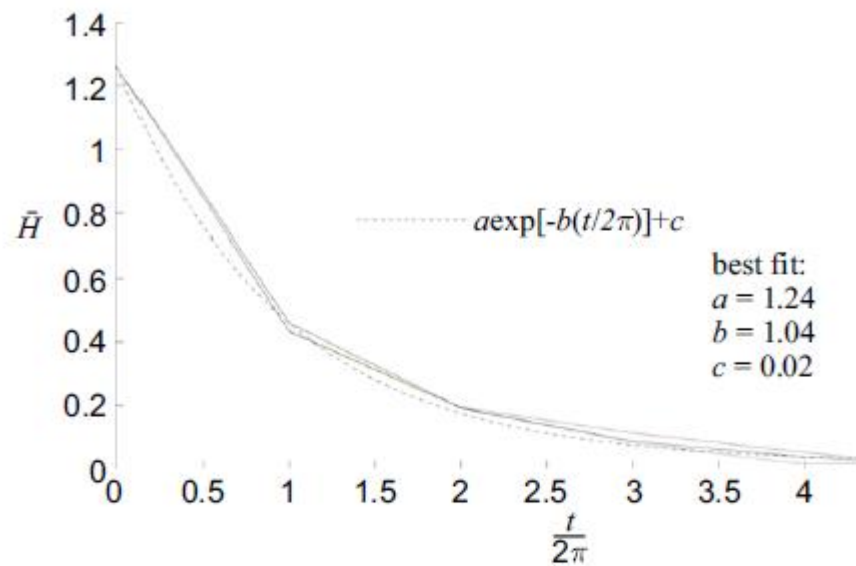
Confirmed and extended by many independent simulations



2D oscillator, 25 modes in superposition (Abraham, Colin and Valentini, J. Phys. A 2014)



Relaxation is faster for larger numbers of modes.

Very crudely: timescale is of order the timescale for wave function evolution.

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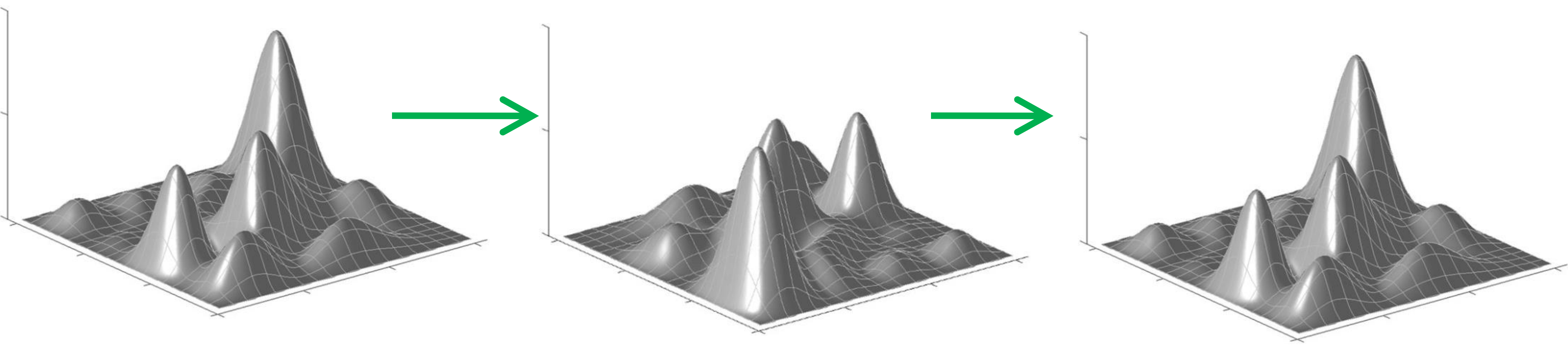
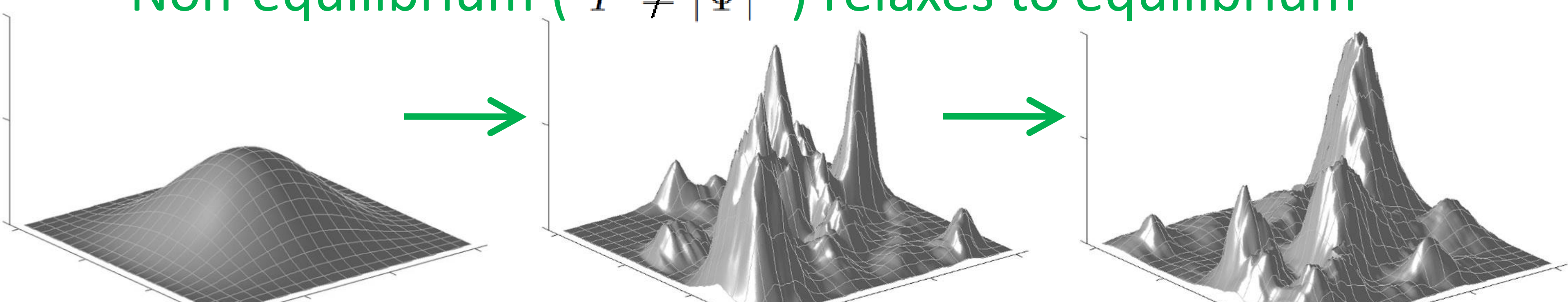
Equilibrium ($P = |\Psi|^2$) changes with time



Non-equilibrium ($P \neq |\Psi|^2$) relaxes to equilibrium



When did relaxation to equilibrium happen?

Presumably, a long time ago, in the very early universe, soon after the big bang.

Quantum noise is a relic of the big bang

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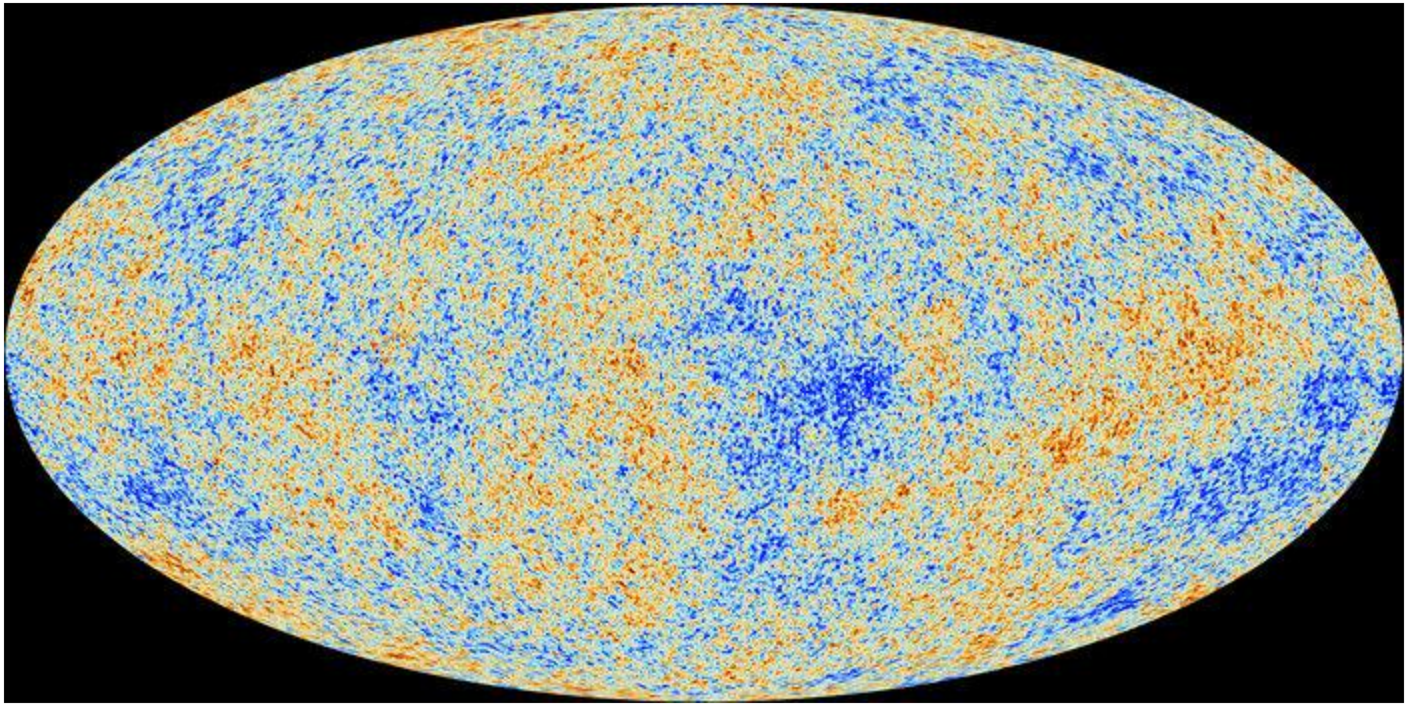
CMB anisotropies are ultimately generated by early quantum noise (inflationary vacuum)



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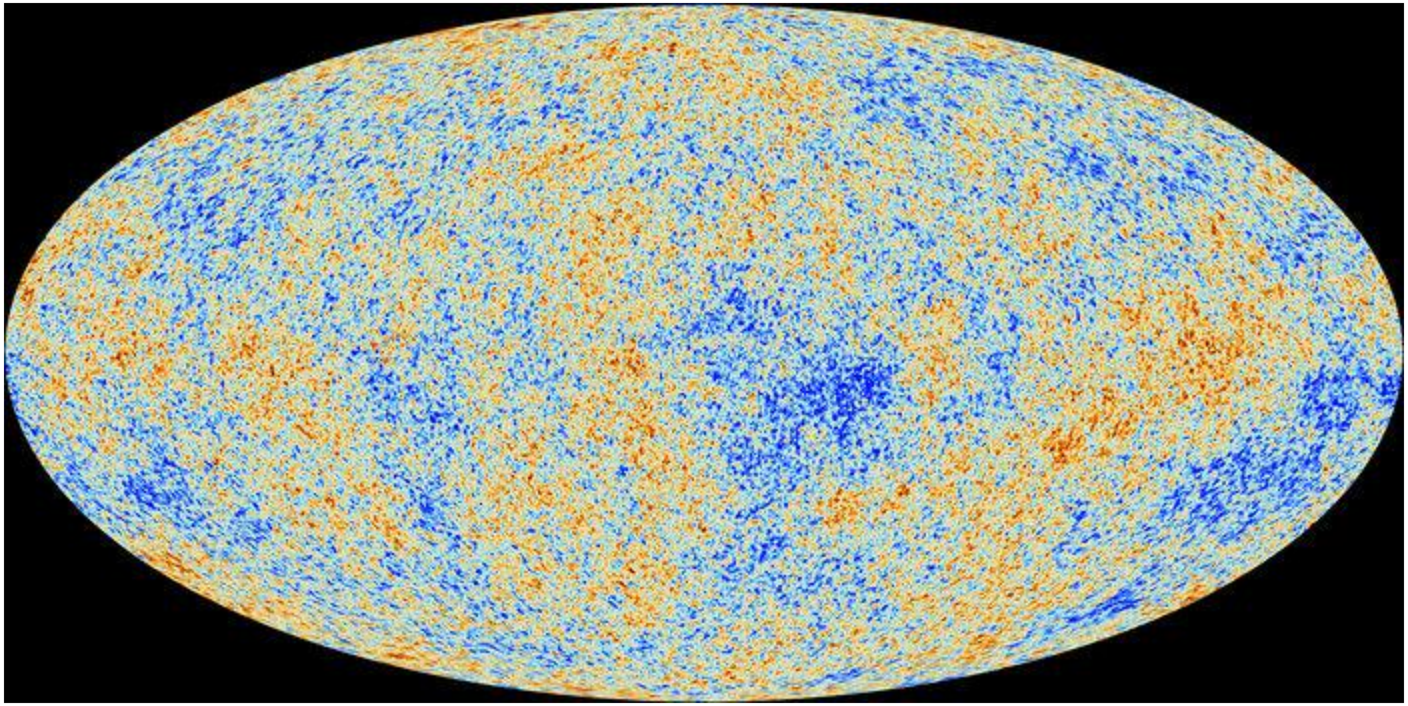

Can test early Born rule by measuring the CMB

System with configuration $q(t)$ and wave function(al) $\psi(q,t)$

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

$$\frac{dq}{dt} = \frac{j}{|\psi|^2}$$

These equations define a *pilot-wave dynamics* for any system whose Hamiltonian $\hat{H}$ is given by a differential operator (Struyve and Valentini 2009)

where $j = j\left[\psi\right] = j(q,t)$ is the Schrödinger current

[Requires an underlying preferred foliation with time function *t*.
Valid in any globally-hyperbolic spacetime (Valentini 2004)]

By construction $\rho(q,t)$ will obey

$$\frac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0 \qquad \frac{dq}{dt} = v$$

and $\rho(q,t) = |\psi(q,t)|^2$ is preserved in time (Born rule).

Pilot-wave field theory on expanding space

Flat metric $d\tau^2 = dt^2 - a^2 d\mathbf{x}^2$ (scale factor $a = a(t)$)

free (minimally-coupled) massless scalar field ϕ

Hamiltonian density $\mathcal{H} = \frac{1}{2}\frac{\pi^2}{a^3} + \frac{1}{2}a(\nabla\phi)^2$

Fourier components $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + iq_{\mathbf{k}2})$

Hamiltonian $H = \int d^3\mathbf{x}\,\mathcal{H}$ becomes $H = \sum_{\mathbf{k}r} H_{\mathbf{k}r}$

with $H_{\mathbf{k}r} = \frac{1}{2a^3}\pi_{\mathbf{k}r}^2 + \frac{1}{2}ak^2q_{\mathbf{k}r}^2$

Schrödinger equation for $\Psi = \Psi[q_{\mathbf{k}r}, t]$ is

$$i\frac{\partial \Psi}{\partial t} = \sum_{\mathbf{k}r} \left(-\frac{1}{2a^3} \frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{1}{2} a k^2 q_{\mathbf{k}r}^2 \right) \Psi$$

and the de Broglie velocities

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}}$$

initial distribution $P[q_{\mathbf{k}r}, t_i]$,

time evolution $P[q_{\mathbf{k}r}, t]$ will be determined by

$$\frac{\partial P}{\partial t} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}} \left(P \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}} \right) = 0$$

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decoupled mode $\mathbf{k}$ $\quad \Psi = \psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)\varkappa$

drop index **k**, wave function $\quad \psi = \psi(q_1, q_2, t)$

initial distribution $\quad \rho(q_1, q_2, t_i)$

THE MODEL (one mode)

$$i\frac{\partial\psi}{\partial t} = \sum_{r=1,\ 2}\left(-\frac{1}{2m}\partial_r^2 + \frac{1}{2}m\omega^2 q_r^2\right)\psi$$

$$\dot{q}_r = \frac{1}{m}\,\mathrm{Im}\,\frac{\partial_r\psi}{\psi}$$ [= (1/*m*)grad *S*]

$$\frac{\partial\rho}{\partial t} + \sum_{r=1,\ 2}\partial_r\left(\rho\frac{1}{m}\,\mathrm{Im}\,\frac{\partial_r\psi}{\psi}\right) = 0$$

$$m = a^3\ , \quad \omega = k/a$$

STRATEGY

Apply to a pre-inflationary era (rad.-dom. $a \propto t^{1/2}$).

Derive large-scale "squeezing" of the Born rule for a spectator scalar field (suppression of relaxation at long wavelengths).

Assume that similar "squeezing" of the Born rule is imprinted on the inflationary spectrum (pending a model of the transition, future work).

NB: no relaxation during inflation itself, the Bunch-Davies dynamics is too simple (Valentini, Phys. Rev. D 2010)

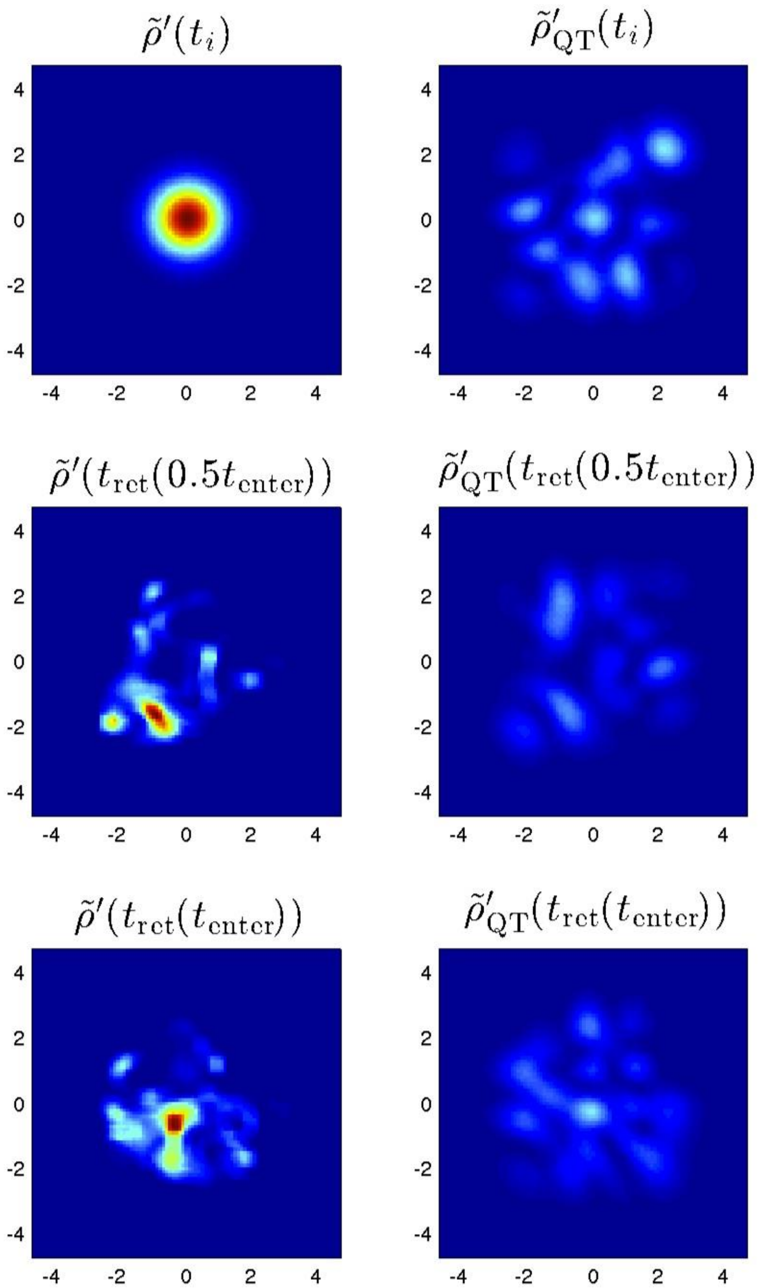
Suppression of quantum noise at super-Hubble wavelengths
(Colin and Valentini, Phys. Rev. D 2013)

Superposition of M=25 energy states, random initial phases

$$\psi(q_1, q_2, t_i) = \frac{1}{\sqrt{M}} \sum_{n_1=0}^{\sqrt{M}-1} \sum_{n_2=0}^{\sqrt{M}-1} e^{i\theta_{n_1 n_2}} \Phi_{n_1}(q_1) \Phi_{n_2}(q_2)$$

Initial non-equilibrium = a 'ground-state' Gaussian

Mode begins outside Hubble radius, evolve until time t_{enter}



expanding space

We are simply evolving this equation

$$\frac{\partial \rho}{\partial t} + \sum_{r=1,\ 2} \partial_r \left(\rho \frac{1}{m} \operatorname{Im} \frac{\partial_r \psi}{\psi} \right) = 0$$

forwards in time.

Right column: equilibrium initial conditions

$$\rho(q_1, q_2, t_i) = |\psi(q_1, q_2, t_i)|^2$$

Left column: nonequilibrium initial conditions

$$\rho(q_1, q_2, t_i) \neq |\psi(q_1, q_2, t_i)|^2$$

(assume subquantum width)

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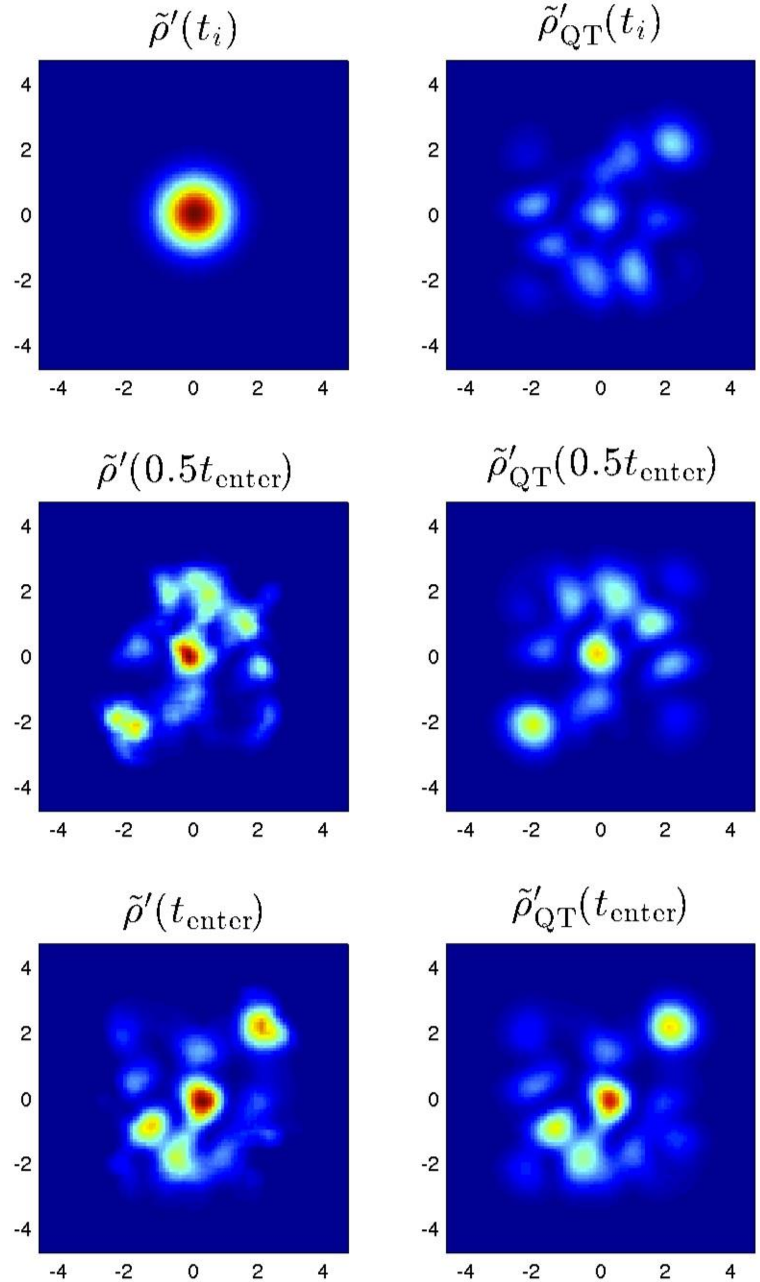
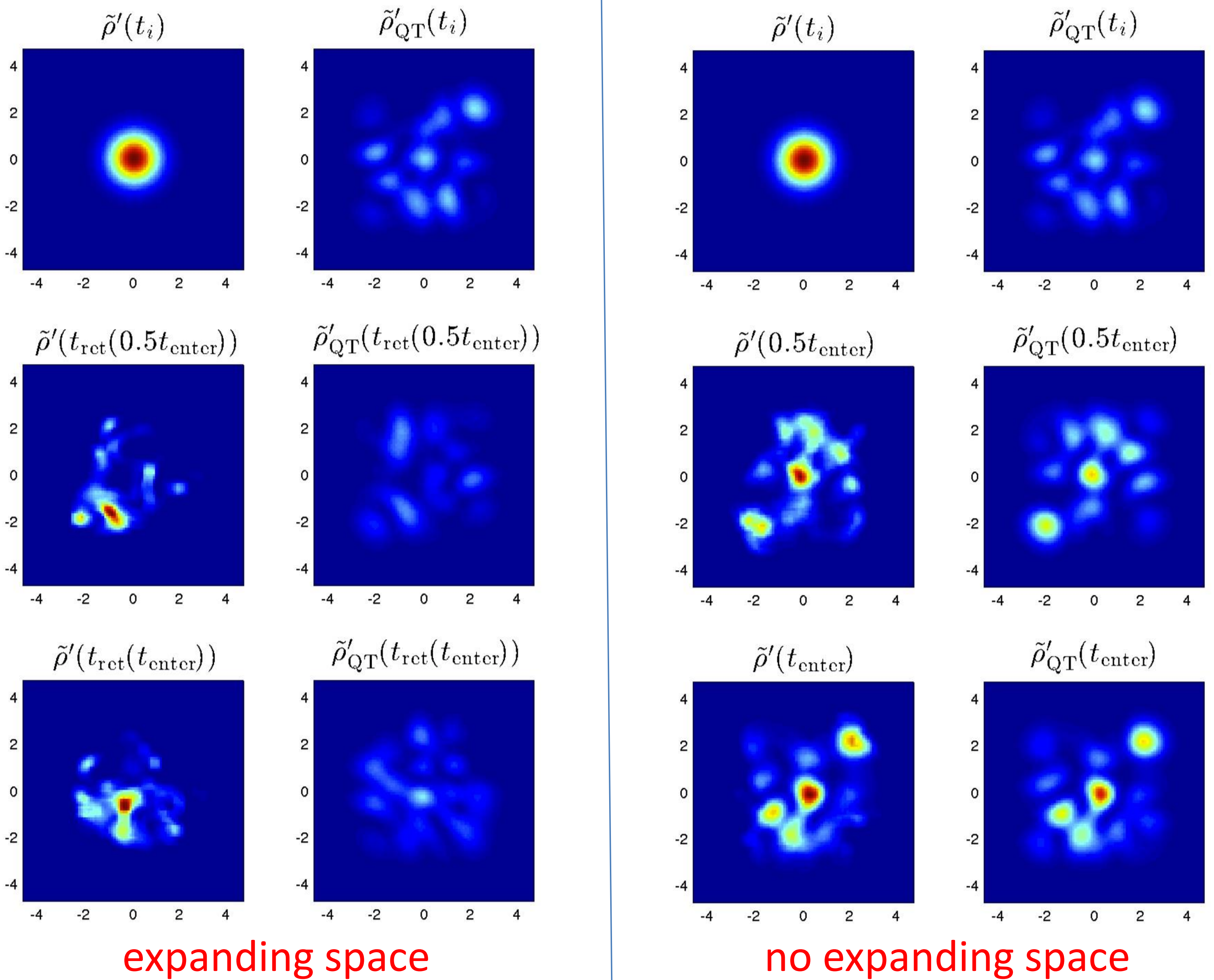


no expanding space


$\tilde{\rho}'(t_i)$
$\tilde{\rho}'_{\mathrm{QT}}(t_i)$
$\tilde{\rho}'(t_{\mathrm{ret}}(0.5t_{\mathrm{enter}}))$
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no expanding space

Write

$$\langle |\phi_{\mathbf{k}}|^2 \rangle = \langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}} \, \xi(k)$$

The function $\xi(k)$ measures the *power deficit* at the end of pre-inflation ("squeezed" Born rule)

Write

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The function $\xi(k)$ measures the *power deficit* at the end of pre-inflation ("squeezed" Born rule)

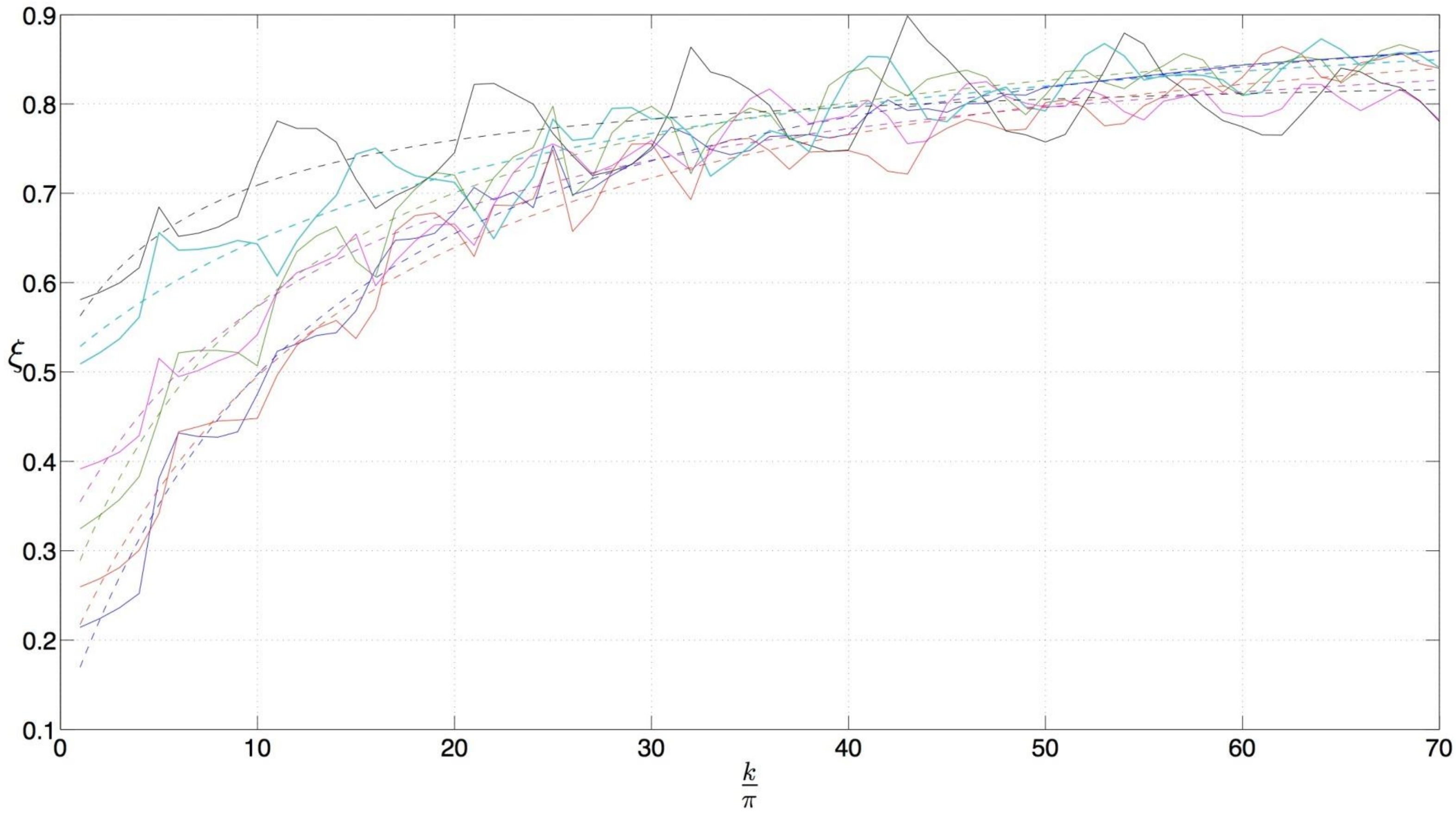
Expect $\xi(k)$ to be smaller (< 1) for smaller *k* (i.e. for longer wavelengths, less relaxation).

Expect $\xi(k)$ to approach 1 for large *k* (i.e. for shorter wavelengths, more relaxation)

Repeat the above simulation for varying *k* , plot the results as a function of *k*

(S. Colin and A. Valentini, arXiv:1407.8262)

Results for M = 4, 6, 9, 12, 16, 25 modes (fixed time interval)



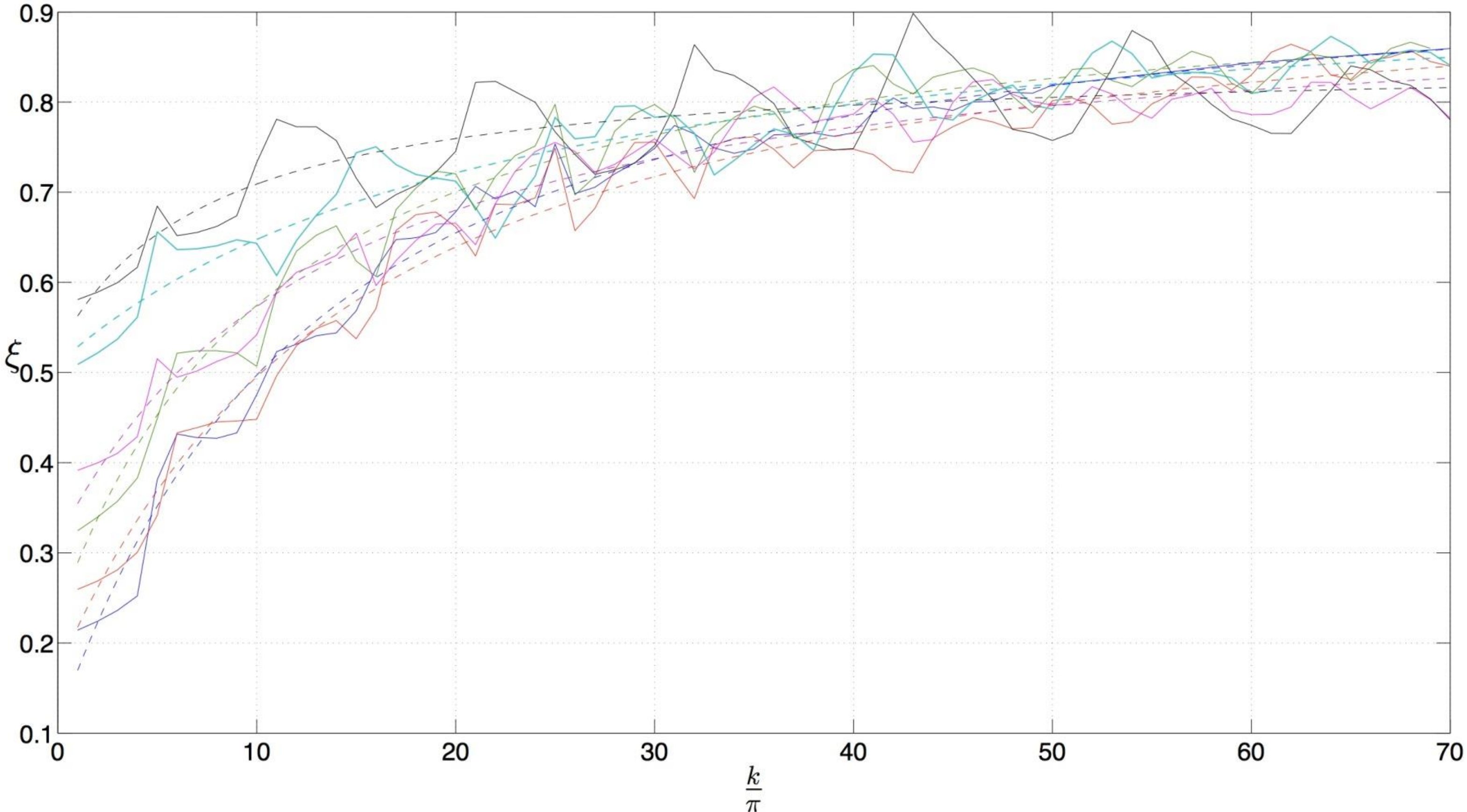
$$\xi(k) = \tan^{-1}(c_1(k/\pi) + c_2) - (\pi/2) + c_3$$

c_1, c_2 and c_3 are free parameters

First approximation: ignore oscillations in $\xi(k)$

We have derived a "squeezed Born rule" for a spectator scalar field at the end of a pre-inflationary era.

Assume a similar correction to the Born rule in the Bunch-Davies vacuum (pending model of transition), with the Born rule "squeezed" by the same factor $\xi(k)$.



Predicted *shape* for the CMB power deficit

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}$$ (ϕ is now the inflaton perturbation)

$$\left\langle |\phi_{\mathbf{k}}|^2 \right\rangle = \left\langle |\phi_{\mathbf{k}}|^2 \right\rangle_{\mathrm{QT}} \xi(k)$$

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)\xi(k)$$

$$\xi(k) = \tan^{-1}(c_1(k/\pi) + c_2) - (\pi/2) + c_3$$

(S. Colin and A. Valentini, arXiv:1407.8262)

In effect we have a two-parameter model

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)\xi(k)$$

$$\xi(k) = \tan^{-1}(c_1(k/\pi) + c_2) - (\pi/2) + c_3$$

where c_1,c_2,c_3 depend on the *number of modes* and the *time interval* (in the pre-inflationary phase).

Current work (with P. Peter and S. Vitenti):

-- using COSMOMC to explore the parameter space

-- preliminary fair fit but no conclusions yet about likelihood or significance

STATISTICAL ANISOTROPY

Breaking the Born rule in the Bunch-Davies vacuum will generically *break statistical isotropy*:

-- "squeezing" factor ξ can depend on the direction of the wave vector **k**
(Colin and Valentini 2013)

-- anomalous phases of $\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}} (q_{\mathbf{k}1} + i q_{\mathbf{k}2})$

(Valentini 2010, Colin and Valentini 2014)

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Therefore we expect:

-- isotropy at short wavelengths (equilibrium)

-- anisotropy at long wavelengths (nonequilibrium)

NOTES ON OUR PREDICTIONS

-- Cannot predict lengthscale at which power deficit

$$\xi(k) = \tan^{-1}(c_1(k/\pi) + c_2) - (\pi/2) + c_3$$

will set in, since measured c_1 will be rescaled by inflationary expansion (depends on unknown number of e-folds)

-- But: we *can* predict that anomalous phases/anisotropies are expected at comparable (slightly larger) lengthscales (S. Colin and A. Valentini, arXiv:1407.8262)

-- Superficial resemblance to data:

power deficit for $l \lesssim 40$,

anisotropy for $l \lesssim 10$

Planck 2013 results. XXIII. Isotropy and statistics of the CMB

pected. However, it should be clear that the evidence for some of the large-angular scale anomalies is significant indeed, yet few physically compelling models have been proposed to account for them, and none so far that provide a common origin. The dipole

We have proposed a mechanism for a common origin

All of our results come simply from the standard quantum-mechanical equation

$$\frac{\partial \rho}{\partial t} + \sum_{r=1,\ 2} \partial_r \left(\rho \frac{1}{m} \operatorname{Im} \frac{\partial_r \psi}{\psi} \right) = 0$$

The only change is in the initial conditions.

We assume that at the initial time the width of $\rho(q_1, q_2, t_i)$ *is smaller than the width of* $|\psi(q_1, q_2, t_i)|^2$

This (mathematically) tiny change might provide a common origin for the observed large-scale CMB anomalies.

SUMMARY

1. De Broglie-Bohm formulation of quantum theory: allows non-Born rule probabilities ($P \neq |\Psi|^2$)

2. Relaxation to "equilibrium", $\bar{P} \to \overline{|\Psi|^2}$ (cf. thermal)

3. Expanding space, relaxation is suppressed at long wavelengths; expect $P \neq |\Psi|^2$ on large scales

4. Single mechanism for both power deficit and statistical anisotropy in low-*l* region (CMB)

5. Inverse-tangent prediction for $\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)\xi(k)$; comparison with data (in progress)