Universal Constraints on Axions from Inflation

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Based on arXiv:1409.5799 w. R. Ferreira

Quick Outline

★ Axions in inflation and gauge field production

★ Universal constraints on axions

★ Consequences for

- Synthetic tensor modes and the scale of inflation
- Natural inflation
- The axion as a curvaton

Axions in inflation

Axions are a popular ingredient in inflationary model building

- Flatness of potential protected by weakly broken shift symmetry

- Axion pNGB of broken shift symmetry

$$V(\sigma) \supset \Lambda^4 \cos(\sigma/f)$$

- Axions will generically couple to gauge fields with a coupling of the form

$$\mathcal{L}_{\text{int}} = -\frac{\alpha\sigma}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

The interaction term

$$\mathcal{L}_{\rm int} = -\frac{\alpha\sigma}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

is a total derivative if the axion is time-independent

- All dynamical effects are suppressed by $\dot{\sigma}$
- If axion is in slow-roll there will be non-trivial effects

E.o.m. for gauge field becomes

$$A_\pm(\tau,k)'' + \left(k^2 \pm \frac{2k\xi}{\tau}\right) A_\pm(\tau,k) = 0, \quad \xi \equiv \frac{\alpha\dot{\sigma}}{2fH} = \frac{\alpha}{f}\sqrt{\frac{\epsilon_\sigma}{2}}$$

For a narrow interval around horizon crossing

$$(8\xi)^{-1} \lesssim \; -k\tau \lesssim 2\xi$$

one helicity mode is exponentially enhanced giving

$$A(\tau,k) \simeq \left(\frac{-\tau}{2^3 k\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}} \quad \text{and} \quad A'(\tau,k) \simeq \left(\frac{-k\xi}{2\tau}\right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}}$$

[Anber, Sorbo 09; Peloso Barnaby 10]

➡ Back reaction constraint, but strongest constraint comes from perturbations

Perturbations of axions during inflation

Consider the effect of perturbations of the axion

$$\mathcal{L}_{\delta\sigma AA} = -\frac{\alpha\delta\sigma}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

If the axion is the inflaton, then

$$\zeta = -\frac{H}{\dot{\sigma}}\delta\sigma$$

and

$$\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu} \quad , \qquad \xi = \alpha\dot{\sigma}/(fH)$$

The coupling

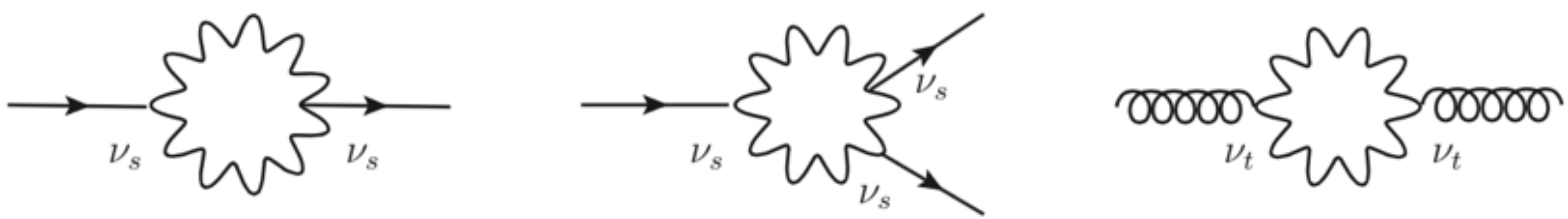
$$\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

can lead to large:

- Corrections to power spectrum
- Non-Gaussianity
- Tensor modes

[Barnaby, Peloso 2011; Barnaby, Pajer, Peloso 2011; Meerburg, Pajer 2012; Cook, Sorbo 2013; Linde et al. 2013]

from exponentially enhanced gauge fields in loops



➡ NG gives strongest constraint

$$\xi \lesssim 3$$

➡ Corrections to tensor spectrum small

Axions different from the inflaton

It was suggested that if the axion is not identified with the inflaton, then coupling

$$\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

is absent, since we no longer have

$$\zeta = -\frac{H}{\dot{\sigma}}\delta\sigma$$

➡ Large tensor modes from gauge field production?

➡ Large observable *r* not related to the scale of inflation?

On the other hand, the axion field fluctuation is not generally gauge invariant

➡ Write the axion perturbation in terms of gauge invariant variables (in flat gauge)

$$\mathcal{R} = H\frac{\delta\phi}{\dot{\phi}} \quad \text{and} \quad \mathcal{S}_{\sigma\phi} = H\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\phi}{\dot{\phi}}\right) \qquad (\mathcal{R} \approx -\zeta)$$

Which we can invert

$$\delta\sigma = \frac{\dot{\sigma}}{H}\left(\mathcal{S}_{\sigma\phi} + \mathcal{R}\right)$$

➡ $\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu}$ $\qquad$ $\mathcal{L}_{\mathcal{S}AA} = -\frac{\xi}{4}\mathcal{S}_{\sigma\phi}F_{\mu\nu}\tilde{F}^{\mu\nu}$

[see arXiv:1409.5799]

Universality of gravity

Even if the axion is not the inflaton, the coupling

$$\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

will be there!

➡ A curvature perturbation at horizon crossing is created from exponentially enhanced gauge field of the same size in terms of ξ regardless whether the axion is the inflation

➡ If the curvature perturbation is frozen on super-horizon scales this leads to strong constraints even if the axion is not the inflaton

[see arXiv:1409.5799]

Super-horizon evolution

Two cases to consider:

1. The axion decays after inflation or not at all

2. The axion decays during inflation

1. The axion decays after inflation or not at all

- The curvature perturbation will remain constant during inflation
- It will not change after inflation if it decays quickly into radiation
- If it doesn't, the field will become energetically more relevant which will only increase the curvature perturbation
- If the field instead decays into cold dark matter then isocurvature perturbations are generated in addition.

➡ We can assume that the curvature perturbation remains at least constant at late times.

2. The axion decays during inflation

In this case the curvature perturbation at the end of inflation will be given only by the inflaton perturbation

$$\zeta = \zeta_\phi = -\frac{H}{\dot{\phi}}\delta\phi$$

However, energy conservation gives us

$$\zeta_\phi' = -\left(\frac{\dot{\sigma}}{\dot{\phi}}\right)^2 \zeta_\sigma'$$

If the axion stays around just a little bit of time, it will source perturbations in the inflaton fluid due to gravitational interactions!

Implications for synthetic tensor modes

Assuming tensor modes are induced by loops of exponentially enhanced gauge fields and *r* can not be related to the scale of inflation

[Barnaby, Moxon,Namba,Peloso, Shiu 12]
(see also Senatore, Silverstein, Zaldarriaga 11)

1. The axion decays after inflation or not at all

In this case the universal coupling

$$\mathcal{L}_{\zeta AA} = \frac{\xi}{4}\zeta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

implies that too large curvature perturbations are induced unless

$$\boxed{r \lesssim 0.01\epsilon^2 (f_{\rm NL}^{\rm eq})^{2/3} \simeq 0.2\epsilon^2}$$

[see arXiv:1409.5799]
(see also related work MSSZ 14)

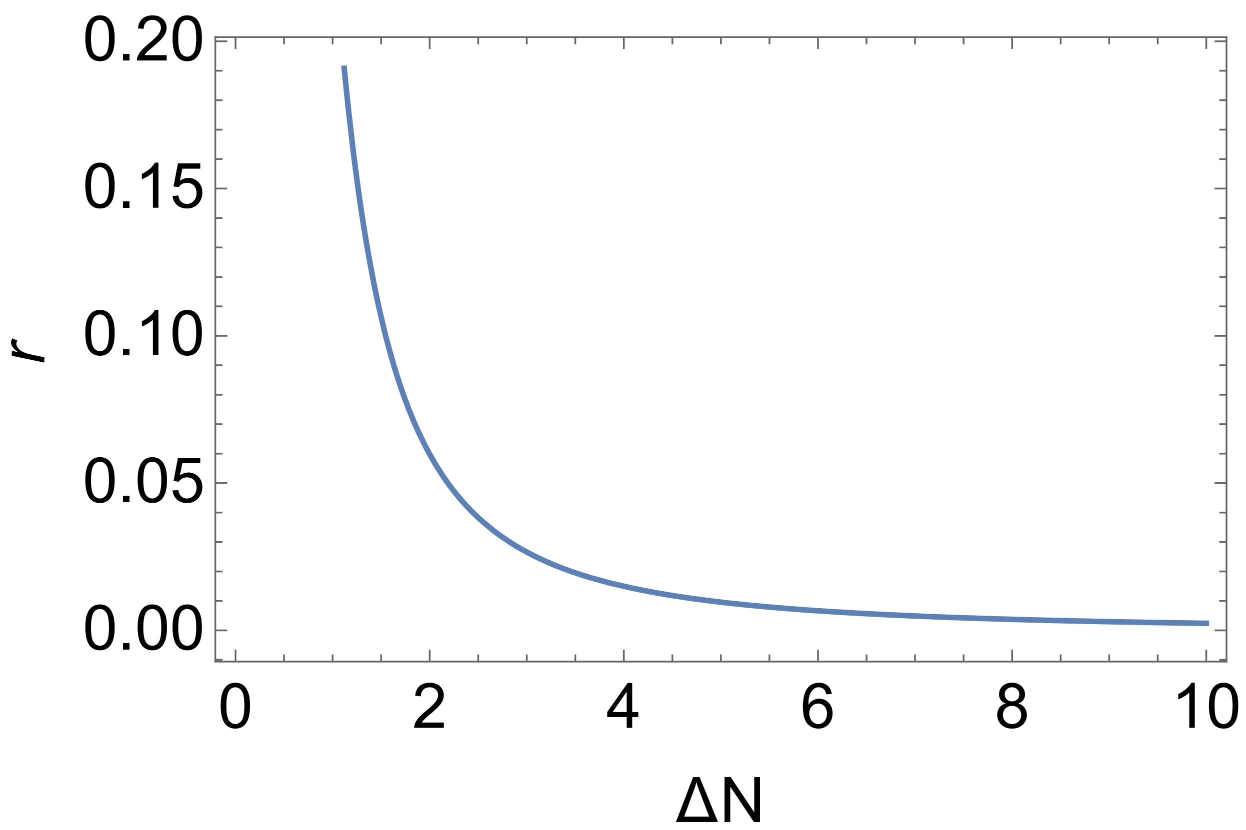
2. The axion decays during inflation

In this case the energy conservation

$$\zeta'_\phi = -\left(\frac{\dot{\sigma}}{\dot{\phi}}\right)^2 \zeta'_\sigma$$

implies

$$r \lesssim 0.01 \frac{(f_{NL}^{\rm eq})^{2/3}}{\Delta N^2} \simeq \frac{0.2}{\Delta N^2}$$



[see arXiv:1409.5799]

Axion decay constants and Natural Inflation

The non-Gaussianity constraint

$$\xi_i \lesssim 3 \quad \Rightarrow \quad f_i \gtrsim 5 \times 10^{-2} \sqrt{\epsilon_i}\, M_p$$

has to be satisfied by all axions around during inflation

[see arXiv:1409.5799]

➡ Non-trivial constraints for realizations of natural inflation where several/many axions with small *f*'s combine to form an effective inflaton direction

[Kim, Nilles, Peloso 04; Dimopoulos,McGreevy,Wacker 05,…]

Axion as a curvaton

Consider the case where the axion is a curvaton with potenital

[Enqvist, MSS 01; Wands, Lyth 01; Moroi, Takahashi 01]

$$V(\sigma) = \Lambda^4 \left[1 - \cos(\sigma/f_1)\right]$$

The spectral index and NG are

$$n_\sigma - 1 = -2\epsilon + \frac{2V''(\sigma)}{3H^2} \qquad |f_{NL}| \approx 5/4$$

If the curvaton is coupled to a second gauge group the universal non-gaussianity constraint

$$\xi_2 \lesssim 3$$

translates into

$$f_2 \gtrsim 0.01 \frac{m_\sigma^2}{H^2} f_1$$

[see arXiv:1409.5799]

More for discussion:

It appears reasonable to stress test the robustness of our assumptions before canonicalizing them

one lesson from the curvaton is that constraining $|f_{NL}| < 1$ appears like a good benchmark test of the minimal single field paradigm

More for (pseudo-scientific) discussion:

As a Scandinavian I too have a bias for minimalism in architecture

But minimalism is not always preferred

Danish minimal architecture from 60's

Versailles

There might be reasons that you would want to live in a more complex place than the apparent minimal required for your existence

Conclusions

- Coupling of curvature perturbation to gauge fields takes a universal form in presence of axions
- Large r from synthetic tensor modes only possible if ΔN is fine-tuned to be less than 2
- The decay rate is bounded from below
- Axion as a curvaton is fine, but if coupled to many gauge groups there are additional constraints