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Model predictions with ASPIC Schwarz Terrero-Escalante classification

Data analysis in model space

Using the slow-roll approximation as a proxy Bayesian model comparison Jeffreys' scale Speeding up evidence calculation Accuracy of ASPIC + effective likelihood Bayes factor for hundred of models And the winners are... Narrowing down the simplest with complexity

Planck constraints on reheating

Posteriors on the reheating parameter Prior-to-posterior width ratio Reheating constraints versus evidence

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C	CR
	J. Martin, CR and V. Vennin

J. Martin, CR, R. Trotta and V. Vennin arXiv:1312.3529, arXiv:1405.7272 0

arXiv:1312.2347 arXiv:1303.3787, arXiv:1410.7958



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- Favour minimal assumption scenarios
 - Flatness ($\Omega_{\rm K} = 0$) $\Omega_{\rm K} = 1 - \Omega_{\rm dm} - \Omega_{\rm b} - \Omega_{\Lambda} = 0.00^{+0.0066}_{-0.0067}$ (PLANCK+WP+BAO)
 - ♦ Adiabatic initial conditions: isocurvature modes are constrained
 ∀X $P_X(k) = P(k)$
 - Quasi scale invariance of the scalar modes

$$k^{3}P(k) = A\left(\frac{k}{k_{*}}\right)^{n_{\rm S}-1} \Rightarrow n_{\rm S} = 0.9619 \pm 0.0073$$

Gaussianity of the CMB anisotropies $f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm eq} = -42 \pm 75, \quad f_{\rm NL}^{\rm ortho} = -25 \pm 39$



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- Flatness $(\Omega_{\rm K} = 0)$
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 - ✦ Adiabatic initial conditions: isocurvature modes are constrained
 ∀X · P_X(k) = P(k)
 - Quasi scale invariance of the scalar modes
 - $k^{3}P(k) = A\left(\frac{k}{k_{*}}\right)^{n_{\rm S}-1} \Rightarrow n_{\rm S} = 0.9619 \pm 0.0073$
 - Gaussiance of the CMB anisotropies

 $f_{\rm TL}^{\rm lo} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm eq} = -42 \pm 75, \quad f_{\rm NL}^{\rm ortho} = -25 \pm 39$

- This is also called: single-field slow-roll inflation
 - Makes extra-predictions: $f_{\rm NL}^{\rm loc} = \mathcal{O}(n_{\rm s} 1)$ and $\exists r > 0$



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Basic theoretical assumptions

• Dynamics given by $(\kappa^2 = 1/M_{\rm P}^2)$

 $S = \int dx^4 \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \mathcal{L}(\phi) \right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$

- Can be used to describe:
 - Minimally coupled scalar field to General Relativity
 - Scalar-tensor theory of gravitation in the Einstein frame the graviton' scalar partner is also the inflaton (HI, RPI1,...)
- Everything can be consistently solved in the slow-roll approximation
 - Background evolution $\phi(N)$ where $N \equiv \ln a$
 - Linear perturbations for the field-metric system $\zeta(t, \boldsymbol{x})$, $\delta \phi(t, \boldsymbol{x})$
 - Slow-roll = expansion in terms of the Hubble flow functions [Schwarz 01]

$$\epsilon_0 = \frac{H_{\text{ini}}}{H}, \quad \epsilon_{i+1} = \frac{\ln |\epsilon_i|}{\mathrm{d}N}$$
 measure deviations from de-Sitter



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Decoupling field and space-time evolution

Friedmann-Lemaître equations in e-fold time (with $M_{_{
m P}}^2=1)$

$$\begin{pmatrix} H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V \right) \\ \frac{\ddot{a}}{a} = -\frac{1}{3} \left(\dot{\phi}^2 - V \right) \end{pmatrix} \Rightarrow \begin{cases} H^2 = \frac{V}{3 - \frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}N} \right)^2} \\ -\frac{\mathrm{d}\ln H}{\mathrm{d}N} = \frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}N} \right)^2 \end{cases} \Leftrightarrow \begin{cases} H^2 = \frac{V}{3 - \epsilon_1} \\ \epsilon_1 = \frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}N} \right)^2 \end{cases}$$

• Klein-Gordon equation in e-folds: relativistic kinematics with friction

$$\frac{1}{3-\epsilon_1}\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi} \quad \Leftrightarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{3-\epsilon_1}{3-\epsilon_1+\frac{\epsilon_2}{2}}\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

Slow-roll approximation: all $\epsilon_i = \mathcal{O}(\epsilon)$ and $\epsilon_1 < 1$ is the definition of inflation ($\ddot{a} > 0$)

The trajectory can be solved for N

$$N - N_{\text{end}} \simeq \int_{\phi}^{\phi_{\text{end}}} \frac{V(\psi)}{V'(\psi)} \,\mathrm{d}\psi$$



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The end of inflation and after

Accelerated expansion stops for $\epsilon_1 > 1$ ($\ddot{a} < 0$) at $N = N_{
m end}$

- Naturally happens during field evolution (graceful exit) at $\phi = \phi_{end}$ $\epsilon_1(\phi_{end}) = 1$
- Or, there is another mechanism ending inflation (tachyonic instability) and ϕ_{end} is a model parameter that has to be specified
- The reheating stage: everything after N_{end} till radiation domination
 - $\blacktriangleright \quad \mathsf{Basic \ picture} \longrightarrow$
 - But in reality a very complicated process, microphysics dependent
 - Reheating duration is unknown:

 $\Delta N_{\rm reh} \equiv N_{\rm reh} - N_{\rm end}$





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Redshift at which reheating ends

Denoting $N = N_{reh}$ the end of reheating = beginning of radiation era

• If thermalized, and no extra entropy production: $a_{reh}^3 s_{reh} = a_0^3 s_0$

$$\begin{cases} s_{\rm reh} = q_{\rm reh} \frac{2\pi^2}{45} T_{\rm reh}^3 \\ \rho_{\rm reh} = g_{\rm reh} \frac{\pi^2}{30} T_{\rm reh}^4 \end{cases} \Rightarrow \qquad \frac{a_0}{a_{\rm reh}} = \left(\frac{q_{\rm reh}^{1/3} g_0^{1/4}}{q_0^{1/3} g_{\rm reh}^{1/4}}\right) \frac{\rho_{\rm reh}^{1/4}}{\rho_{\gamma}} \\ \sigma_{\gamma} = \frac{\pi^2}{30} T_{\rm reh}^4 \qquad \sigma_{\gamma} = \left(\frac{\rho_{\rm reh}}{\tilde{\rho}_{\gamma}}\right)^{1/4} \end{cases}$$

Depends on
$$ho_{
m reh}$$
 and $\widetilde
ho_\gamma\equiv {\cal Q}_{
m reh}
ho_\gamma$

- Energy density of radiation today: $\rho_{\gamma} = 3 \frac{H_0^2}{M_p^2} \Omega_{\rm rad}$
- Change in the number of entropy and energy relativistic degrees of freedom (small effect compared to $\rho_{\rm reh}/\rho_{\gamma}$)

$$\mathcal{Q}_{\mathrm{reh}} \equiv rac{g_{\mathrm{reh}}}{g_0} \left(rac{q_0}{q_{\mathrm{reh}}}
ight)^{1/4}$$



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Redshift at which inflation ends

Depends on the redshift of reheating

$$1 + z_{\text{end}} = \frac{a_0}{a_{\text{end}}} = \frac{a_{\text{reh}}}{a_{\text{end}}} (1 + z_{\text{reh}}) = \frac{a_{\text{reh}}}{a_{\text{end}}} \left(\frac{\rho_{\text{reh}}}{\tilde{\rho}_{\gamma}}\right)^{1/4} = \frac{1}{R_{\text{rad}}} \left(\frac{\rho_{\text{end}}}{\tilde{\rho}_{\gamma}}\right)^{1/4}$$

- The reheating parameter $R_{\rm rad} \equiv \frac{a_{\rm end}}{a_{\rm reh}} \left(\frac{\rho_{\rm end}}{\rho_{\rm reh}}\right)^{1/4}$
- Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{rad} = 1$
- $R_{
 m rad}$ can be expressed in terms of $(
 ho_{
 m reh}, \overline{w}_{
 m reh})$ or $(\Delta N_{
 m reh}, \overline{w}_{
 m reh})$

$$\ln R_{\rm rad} = \frac{\Delta N_{\rm reh}}{4} (3\overline{w}_{\rm reh} - 1) = \frac{1 - 3\overline{w}_{\rm reh}}{12(1 + \overline{w}_{\rm reh})} \ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right)$$

where
$$\overline{w}_{\rm reh} \equiv \frac{1}{\Delta N_{\rm reh}} \int_{N_{\rm end}}^{N_{\rm reh}} \frac{P(N)}{\rho(N)} dN$$

• A fixed inflationary parameters, $z_{\rm end}$ can still be affected by $R_{\rm rad}$



Reheating effects on inflationary observables

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• Model testing: reheating effects must be included!



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Inflationary perturbations in slow-roll

Equations of motion for the linear perturbations

$$\mu_{\mathbf{T}} \equiv ah \mu_{\mathbf{S}} \equiv a\sqrt{2}\phi_{,N}\boldsymbol{\zeta} \right\} \Rightarrow \mu_{\mathbf{TS}}'' + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathbf{TS}} = 0$$

Can be consistently solved using slow-roll and pivot expansion [Stewart:1993,

Gong:2001, Schwarz:2001, Leach:2002, Martin:2002, Habib:2002, Casadio:2005, Lorenz:2008, Martin:2013, Beltran:2013]

$$\begin{aligned} \mathcal{P}_{\zeta} &= \frac{H_{*}^{2}}{8\pi^{2}M_{\mathrm{P}}^{2}\epsilon_{1*}} \left\{ 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^{2}}{2} - 3 + 2C + 2C^{2}\right)\epsilon_{1*}^{2} + \left(\frac{7\pi^{2}}{12} - 6 - C + C^{2}\right)\epsilon_{1*}\epsilon_{2*} \\ &+ \left(\frac{\pi^{2}}{8} - 1 + \frac{C^{2}}{2}\right)\epsilon_{2*}^{2} + \left(\frac{\pi^{2}}{24} - \frac{C^{2}}{2}\right)\epsilon_{2*}\epsilon_{3*} \\ &+ \left[-2\epsilon_{1*} - \epsilon_{2*} + (2 + 4C)\epsilon_{1*}^{2} + (-1 + 2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^{2} - C\epsilon_{2*}\epsilon_{3*} \right] \ln\left(\frac{k}{k_{*}}\right) \\ &+ \left[2\epsilon_{1*}^{2} + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^{2} - \frac{1}{2}\epsilon_{2*}\epsilon_{3*} \right] \ln^{2}\left(\frac{k}{k_{*}}\right) \right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{P}_{h} &= \frac{2H_{*}^{2}}{\pi^{2}M_{\mathrm{P}}^{2}} \left\{ 1 - 2(1 + C)\epsilon_{1*} + \left[-3 + \frac{\pi^{2}}{2} + 2C + 2C^{2} \right]\epsilon_{1*}^{2} + \left[-2 + \frac{\pi^{2}}{12} - 2C - C^{2} \right]\epsilon_{1*}\epsilon_{2*} \\ &+ \left[-2\epsilon_{1*} + (2 + 4C)\epsilon_{1*}^{2} + (-2 - 2C)\epsilon_{1*}\epsilon_{2*} \right] \ln\left(\frac{k}{k_{*}}\right) + \left(2\epsilon_{1*}^{2} - \epsilon_{1*}\epsilon_{1*}\right) \ln^{2}\left(\frac{k}{k_{*}}\right) \right\} \end{aligned}$$

$$\begin{aligned} \mathbf{Notice that:} \quad H_{*} \equiv H(N_{*}) \text{ and } \epsilon_{i*} \equiv \epsilon_{i}(N_{*}) \text{ with } k_{*}\eta(N_{*}) = -1 \end{aligned}$$



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Solving for the time of pivot crossing

To make inflationary predictions, one has to solve $k_*\eta_*=-1$

$$\frac{k_*}{a_0} = \frac{a(N_*)}{a_0} H_* = e^{N_* - N_{\text{end}}} \frac{a_{\text{end}}}{a_0} H_* = \frac{e^{\Delta N_*} H_*}{1 + z_{\text{end}}} = e^{\Delta N_*} R_{\text{rad}} \left(\frac{\rho_{\text{end}}}{\tilde{\rho}_{\gamma}}\right)^{-\frac{1}{4}} H_*$$

- Defining $N_0 \equiv \ln\left(\frac{k_*}{a_0}\frac{1}{\tilde{\rho}_{\gamma}^{1/4}}\right)$ (number of e-folds of deceleration)
 - This is a non-trivial integral equation that depends on: model + how inflation ends + reheating + data

$$-\left[\int_{\phi_{\text{end}}}^{\phi_{*}} \frac{V(\psi)}{V'(\psi)} d\psi\right] = \ln R_{\text{rad}} - N_{0} + \frac{1}{4} \ln(8\pi^{2}P_{*}) \\ -\frac{1}{4} \ln \left\{\frac{9}{\epsilon_{1}(\phi_{*})[3 - \epsilon_{1}(\phi_{\text{end}})]} \frac{V(\phi_{\text{end}})}{V(\phi_{*})}\right\}$$

• Arbitrarily fixing ΔN_* (or ϕ_*) = postulating a generally wrong solution to this trivial equation!



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The optimal reheating parameter

Defining the rescaled reheating parameter (astro-ph/0605367)

$$\ln \mathbf{R_{reh}} \equiv \ln R_{rad} + \frac{1}{4} \ln \rho_{end}$$

+ Within a given model, one-to-one correspondance between $R_{\rm rad}$ and $R_{\rm reh}$

"'Magic" cancellation in the reheating equation (also valid out of slow-roll)

$$-\left[\int_{\phi_{\text{end}}}^{\phi_*} \frac{V(\psi)}{V'(\psi)} \mathrm{d}\psi\right] = \ln \mathbf{R}_{\text{reh}} - N_0 - \frac{1}{2} \ln \left[\frac{9}{3 - \epsilon_1(\phi_{\text{end}})} \frac{V(\phi_{\text{end}})}{V(\phi_*)}\right]$$

Using R_{reh} avoids correlations with P_{*} in performing data analysis
 Assuming -1/3 < w
 ⁻_{reh} < 1 and ρ_{nuc} ≡ (10 MeV)⁴ < ρ_{reh} < ρ_{end}

$$-46 < \ln \frac{R_{\rm reh}}{R_{\rm reh}} < 15 + \frac{1}{3} \ln \rho_{\rm end}$$



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Planck 2013 constraints on slow-roll

From the slow-roll expanded expression of $\mathcal{P}_{\zeta}(k)$ and $\mathcal{P}_{h}(k)$

• Constraints on ϵ_{i*} and P_* (or H^2_*/ϵ_{1*})





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- Can only be done from the input of $R_{
 m reh}$, or $R_{
 m rad}$, or $(\overline{w}_{
 m reh},
 ho_{
 m reh})$
 - One can scan various reheating histories: ΔN_* is not arbitrary!
 - Example: LFI₂ with $\overline{w}_{reh} = 0$ and $\rho_{nuc} < \rho_{reh} < \rho_{end}$





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- In the abscence of any information on the reheating, one should use $R_{\rm reh}$ (or $R_{\rm rad}$)
- Same example: LFI₂ without assuming $\overline{w}_{\rm reh} = 0$





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With J. Martin and V. Vennin



http://arxiv.org/abs/1303.3787 http://cp3.irmp.ucl.ac.be/~ringeval/aspic.html

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- Quasi-exhaustive analysis to derive reheating consistent observable predictions for all slow-roll single-field inflationary models
- Comes with a public code (ASPIC)
- Currently supports more than 50 motivated classes of potential

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)$
RCHI	1	1	$M^4 \left(1 - 2e^{-\sqrt{2/3}\phi/M_{Pl}} + \frac{A_I}{16\pi^2}\frac{\phi}{\sqrt{6}M_{Pl}}\right)$
LFI	1	1	$M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^p$
MLFI	1	1	$M^4 \frac{\phi^2}{M_{Pl}^2} \left[1 + \alpha \frac{\phi^2}{M_{Pl}^2}\right]$
RCMI	1	1	$M^4 \left(\frac{\phi}{M_{\text{Pl}}}\right)^2 \left[1 - 2\alpha \frac{\phi^2}{M_{\text{Pl}}^2} \ln \left(\frac{\phi}{M_{\text{Pl}}}\right)\right]$
RCQI	1	1	$M^4 \left(\frac{\phi}{M_{\text{Pl}}}\right)^4 \left[1 - \alpha \ln \left(\frac{\phi}{M_{\text{Pl}}}\right)\right]$
NI	1	1	$M^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4 (1 - e^{-q\phi/M_{Pl}})$
PLI	1	1	$M^4 e^{-\alpha \phi/M_{\rm Pl}}$
KMII	1	2	$M^4 \left(1 - \alpha \frac{\phi}{M_{\rm Pl}} e^{-\phi/M_{\rm Pl}}\right)$
HF1I	1	1	$M^4 \left(1 + A_1 \frac{\phi}{M_{\rm Pl}}\right)^2 \left[1 - \frac{2}{3} \left(\frac{A_1}{1 + A_1 \phi/M_{\rm Pl}}\right)^2\right]$
CWI	1	1	$M^4 \left[1 + \alpha \left(\frac{\phi}{Q}\right)^4 \ln \left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4 \left[1 + \alpha \ln \left(\frac{\phi}{M_{\text{Pl}}} \right) \right]$
RpI	1	3	$M^4 e^{-2\sqrt{2/3}\phi/M_{\rm Pl}} \left e^{\sqrt{2/3}\phi/M_{\rm Pl}} - 1 \right ^{2p/(2p-1)}$
DWI	1	1	$M^4 \left[\left(\frac{\phi}{\phi_0} \right)^2 - 1 \right]^2$
MHI	1	1	$M^4 \left[1 - \operatorname{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4 \frac{(\phi/M_{\rm Pl})^2}{\alpha + (\phi/M_{\rm Pl})^2}$
MSSMI	1	1	$M^4 \left[\left(\frac{\phi}{\phi_0} \right)^2 - \frac{2}{3} \left(\frac{\phi}{\phi_0} \right)^6 + \frac{1}{5} \left(\frac{\phi}{\phi_0} \right)^{10} \right]$
RIPI	1	1	$M^4 \left[\left(\frac{\phi}{\phi_0} \right)^2 - \frac{4}{3} \left(\frac{\phi}{\phi_0} \right)^3 + \frac{1}{2} \left(\frac{\phi}{\phi_0} \right)^4 \right]$
AI	1	1	$M^4 \left[1 - \frac{2}{\pi} \arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4 \left[3 - (3 + \alpha^2) \tanh^2 \left(\frac{\alpha}{\sqrt{2}} \frac{\phi}{M_{\text{Pl}}}\right)\right]$
CNBI	1	1	$M^4 \left[(3 - \alpha^2) \tan^2 \left(\frac{\alpha}{\sqrt{2}} \frac{\phi}{M_{Pl}} \right) - 3 \right]$
OSTI	1	1	$-M^4 \left(\frac{\phi}{\phi_0}\right)^2 \ln \left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4 \ln \left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$

II	2	1	$M^4 \left(\frac{\phi - \phi_0}{M_{Pl}}\right)^{-\beta} - M^4 \frac{\beta^2}{6} \left(\frac{\phi - \phi_0}{M_{Pl}}\right)^{-\beta-2}$
KMIII	2	1	$M^4 \left[1 - \alpha \frac{\phi}{M_{\text{Pl}}} \exp \left(-\beta \frac{\phi}{M_{\text{Pl}}} \right) \right]$
LMI	2	2	$M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^{\alpha} \exp\left[-\beta(\phi/M_{\rm Pl})^{\gamma}\right]$
TWI	2	1	$M^4 \left[1 - A \left(\frac{\phi}{\phi_0} \right)^2 e^{-\phi/\phi_0} \right]$
GMSSMI	2	2	$M^{4}\left[\left(\frac{\phi}{\phi_{0}}\right)^{2} - \frac{2}{3}\alpha\left(\frac{\phi}{\phi_{0}}\right)^{6} + \frac{\alpha}{5}\left(\frac{\phi}{\phi_{0}}\right)^{10}\right]$
GRIPI	2	2	$M^4 \left[\left(\frac{\phi}{\phi_0} \right)^2 - \frac{4}{3} \alpha \left(\frac{\phi}{\phi_0} \right)^3 + \frac{\alpha}{2} \left(\frac{\phi}{\phi_0} \right)^4 \right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\text{Pl}}}} + e^{\sqrt{6}\gamma\frac{\phi}{M_{\text{Pl}}}}\right)$
TI	2	3	$M^4 \left(1 + \cos\frac{\phi}{\mu} + \alpha \sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4 \exp_{1-\beta} \left(-\lambda \frac{\phi}{M_{\text{Pl}}}\right)$
PSNI	2	1	$M^4 \left[1 + \alpha \ln \left(\cos \frac{\phi}{f} \right) \right]$
NCKI	2	2	$M^4 \left[1 + \alpha \ln \left(\frac{\phi}{M_{\text{Pl}}}\right) + \beta \left(\frac{\phi}{M_{\text{Pl}}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1 - \alpha \frac{\phi}{M_{\text{Pl}}}\right)^2}$
OI	2	1	$M^4 \left(\frac{\phi}{\phi_0}\right)^4 \left[\left(\ln \frac{\phi}{\phi_0}\right)^2 - \alpha \right]$
CNCI	2	1	$M^4\left[(3 + \alpha^2) \operatorname{coth}^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\text{Pl}}}\right) - 3\right]$
SBI	2	2	$M^{4}\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{4}\right\}$
SSBI	2	6	$M^4 \left[1 + \alpha \left(\frac{\phi}{M_{Pl}}\right)^2 + \beta \left(\frac{\phi}{M_{Pl}}\right)^4\right]$
IMI	2	1	$M^4 \left(\frac{\phi}{M_{\rm Pl}}\right)^{-p}$
BI	2	2	$M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^{-p}\right]$
RMI	3	4	$M^{4}\left[1 - \frac{c}{2}\left(-\frac{1}{2} + \ln \frac{\phi}{\phi_{0}}\right)\frac{\phi^{2}}{M_{Pl}^{2}}\right]$
VHI	3	1	$M^4 \left[1 + \left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4 \left[1 + \left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4 \left(\frac{\phi}{M_{\text{Pl}}}\right)^p \left[1 + \alpha \left(\frac{\phi}{M_{\text{Pl}}}\right)^q\right]$
LPI	3	3	$M^4 \left(\frac{\phi}{\phi_0}\right)^p \left(\ln \frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta \cos \left[\alpha \left(\frac{\phi - \phi_0}{M_{Pl}}\right)\right]\right\}^2}$



ASPIC example program with LFI

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♦ ASPIC example program with LFI

♦ ASPIC and alternative ∩ parameterizations

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print *, 'xend= xstar= DeltaN= ', xend, xstar, DeltaN eps(1) = lfi_epsilon_one(xstar,p) eps(2) = lfi_epsilon_two(xstar,p) eps(3) = lfi_epsilon_three(xstar,p)

n=2

lnR = 0. kp

ns = scalar_spectral_index(eps) r = tensor_to_scalar_ratio(eps)

print *, 'ns=r=',ns,r

read(*,*)

program toy

use infprec, only : kp

real(kp), dimension(3) :: eps

real(kp) :: ErehGeV, wreh,lnRhoReh

xstar = lfi_x_rreh(p,lnR,DeltaN)

real(kp) :: p, xstar, xend real(kp) :: ns, r

radiation-like reheating

xend = lfi x endinf(p)

implicit none real(kp) :: lnR

real(kp) :: DeltaN

use |fisr, only : lfi_epsilon_one, lfi_epsilon_two use Ifisr, only : Ifi epsilon three, Ifi x endinf

use sflow, only : scalar_spectral_index, tensor_to_scalar_ratio
use cosmopar, only : lnMpinGeV, PowerAmpScalar

use lfireheat, only : lfi_x_rreh, lfi_x_star

matter like reheating at Ereh=10^8 GeV ErehGeV = 1e8wreh = 0

lnRhoReh = 4._kp*(log(ErehGev)-lnMpinGev)

xstar = lfi_x_star(p,wreh,lnRhoReh,PowerAmpScalar,DeltaN)

print *, 'xend= xstar= DeltaN= ', xend, xstar, DeltaN

eps(1) = lfi_epsilon_one(xstar,p) eps(2) = lfi_epsilon_two(xstar,p) eps(3) = lfi_epsilon_three(xstar,p)

ns = scalar_spectral_index(eps) r = tensor_to_scalar_ratio(eps)

print *. 'ns=r='.ns.r

end program toy



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Comparison with

ASPIC and alternative parameterizations

Postulating an evolution for $w(N) = \frac{P(N)}{\rho(N)} = \frac{2}{3}\epsilon_1(N) - 1 \Leftrightarrow$

$$\frac{d\phi}{dN} = \pm\sqrt{3} (1+w)^{1/2} \\ \frac{d\ln V}{dN} = -3 (1+w) + \frac{d\ln(1-w)}{dN} \Rightarrow \begin{cases} \phi = \phi_{\text{end}} \mp \sqrt{3} \int (1+w)^{1/2} dN \\ V \propto (1-w) e^{-3 \int (1+w) dN} \end{cases}$$

- Strictly equivalent to specify $V(\phi)$ up to the normalisation M^4 M^4 , ΔN_* are obtained from $P_* + R_{\rm reh} + {\rm solving} w(N_{\rm end}) = 1/3$
- Expanding $n_{\rm s}(N)$ and r(N) around $N_* \Leftrightarrow {\sf choosing} \ V(\phi)$ around ϕ_*

$$n_{\rm S} = 1 - 2\epsilon_1 - \epsilon_2 + \mathcal{O}(\epsilon^2)$$

$$r = 16\epsilon_1 + \mathcal{O}(\epsilon^2) \Rightarrow \begin{cases} \frac{\mathrm{d}\phi}{\mathrm{d}N} \simeq \pm \frac{r^{1/2}}{\sqrt{8}} \\ \frac{\mathrm{d}\ln V}{\mathrm{d}N} \simeq -\frac{r}{8} \left(1 + \frac{1 - n_{\rm S} - r/8}{6 - r/8}\right) \end{cases}$$

• But M^4 , ΔN_* have to be postulated, reheating consistency lost

A given parameterization = 1 model in ASPIC

Comparison with observations

- Planck 2013 constraints
 on-slow-roll
- Comparison with model predictions
- * Most generic reheating parametrization
- Encyclopædia
 Inflationaris
- Purpose
- ♦ ASPIC example
- program with ${\rm LFI}$
- * ASPIC and alternative parameterizations

Model predictions with ASPIC

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 classification

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Data analysis in model space
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Model predictions with ASPIC

For all Encyclopædia Inflationaris models

potential parameters + reheating $\longrightarrow \epsilon_{i*} \longrightarrow n_{s}$, r, α_{s} ... (with consistency relations)

• Easy to check for which reheating history a model is compatible

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✤ Most generic rebeating

Schwarz Terrero-Escalante classification

Based on the relative energy evolution at the pivot scale (ϕ_*)

In terms of slow-roll parameters

ST1: $\epsilon_{2*} > 2\epsilon_{1*}$, ST2: $0 < \epsilon_{2*} < 2\epsilon_{1*}$, ST3: $\epsilon_{2*} < 0$

Comparison with observations

Data analysis in model space

* Using the slow-roll approximation as a proxy

Bayesian modelcomparison

✤ Jeffreys' scale ^O

Speeding up evidence calculation

Accuracy of ASPIC + effective likelihood

✤ Bayes factor for mundred
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Using the slow-roll approximation as a proxy

To constrain the fundamental inflationary parameters: $oldsymbol{ heta}_{ ext{inf}}$

$$(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}) \longrightarrow \mathsf{ASPIC} \longrightarrow \boldsymbol{\epsilon_{i*}} \longrightarrow \begin{cases} \mathcal{P}_{\zeta}(k) \\ \mathcal{P}_{h}(k) \end{cases} \longrightarrow \mathsf{CAMB} \longleftrightarrow \mathrm{CMB} \mathrm{data} \end{cases}$$

Example: Planck 2013 data analysis with LFI

• Confidence intervals are on the relevant parameters (95% CL)

$$p < 2.3, \qquad -37 < \ln R_{\rm reh} < 6$$

Comparison with observations

Data analysis in model space

- Substant Using the slow-roll approximation as a proxy
- Bayesian model comparison
- ✤ Jeffreys' scale
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- ♦ Accuracy of ASPIC + effective likelihood ○
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Bayesian model comparison

- Bayesian evidence
 - + For each model \mathcal{M} , marginalisation over all parameters

$$\mathcal{E}(D|\mathcal{M}) = \int \mathrm{d}\boldsymbol{\theta} \mathcal{L}(\boldsymbol{\theta}) \pi(\boldsymbol{\theta}|\mathcal{M})$$

 \blacklozenge Gives the posterior probability of ${\mathcal M}$ to explain the data D

$$p(\mathcal{M}|D) = \frac{\mathcal{E}(D|\mathcal{M})\pi(\mathcal{M})}{p(D)} \qquad \text{where} \qquad p(D) = \sum_{i} \mathcal{E}(\mathcal{M}_{i}|D)\pi(\mathcal{M}_{i})$$

Bayes' factor

 \blacklozenge Gives the posterior odds between $\mathcal M$ and a reference model $\mathcal M_0$

$$\frac{p(\mathcal{M}|D)}{p(\mathcal{M}_0|D)} = \mathbf{B} \frac{\pi(\mathcal{M})}{\pi(\mathcal{M}_0)} \Rightarrow \mathbf{B} = \frac{\mathcal{E}(D|\mathcal{M})}{\mathcal{E}(D|\mathcal{M}_0)}$$

Measure of how much the prior information has been updated

Jeffreys' scale

Strength of evidence of ${\mathcal M}$ compared to ${\mathcal M}_0$

 ASPIC allows to fastly do that for all the *Encyclopædia Inflationaris* models

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Speeding up evidence calculation

- Marginalisation over all parameters is numerically challenging!
- Effective likelihood for slow-roll inflation
 - Requires only one complete data analysis to get

$$\mathcal{L}_{\text{eff}}(D|P_*, \epsilon_{i*}) = \int p(D|\boldsymbol{\theta}_{\text{cosmo}}, P_*, \epsilon_{i*}) \pi(\boldsymbol{\theta}_{\text{cosmo}}) d\boldsymbol{\theta}_{\text{cosmo}}$$

- Use machine-learning algorithm to fit its multidimensional shape
- + For each model \mathcal{M} and their parameters $\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}$

 $p(\boldsymbol{\theta}_{\inf}, R_{\operatorname{reh}} | D, \mathcal{M}) = \frac{\mathcal{L}_{\operatorname{eff}}[D | P_*(\boldsymbol{\theta}_{\inf}, R_{\operatorname{reh}}), \epsilon_{i*}(\boldsymbol{\theta}_{\inf}, R_{\operatorname{reh}})] \pi(\boldsymbol{\theta}_{\inf}, R_{\operatorname{reh}} | \mathcal{M})}{p(D | \mathcal{M})}$

- All posteriors and evidences can be obtained by integrating $\mathcal{L}_{\mathrm{eff}}$
- In practice: ASPIC + MultiNest + $\mathcal{L}_{
 m eff}$ = 1 hour per model

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Accuracy of ASPIC + effective likelihood

First order quantities marginalized over second order

Bayes factor for hundred of models

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Data analysis in model	BI _{nb}
	BI _{ls}
space	BI_{2s}
	$\mathrm{BI}_{3\mathrm{s}}$
Using the slow-roll	$\mathrm{BI}_{4\mathrm{s}}$
approximation as a proxy	BI_{58}
	BI_{6i}
Bavesian model	BI
•	, Di _{stg}
comparison	ĊWI _f
A laffana a' anala	DWI
* Jettreys scare	ESI
Speeding up ovidence	ESI
* Speeding up evidence	ESI
calculation	$ESI_{\sqrt{2}}$
	$\mathrm{ESI}_{\sqrt{2/3}}$
* Accuracy of ASPIC +	GMSSMI _{ep}
effective likeliho	GMSSMI _{opA}
	GRIPI
Bayes factor for hundred	GRIPI
of models	GRIPI _{onA}
	$GRIPI_{opB}$
And the winners are	н
	KKLTI
Narrowing down the	KKLTI _s
simplest with complexity	KKLTI _{stg}
simplest with complexity	KMII KAUI
	KMII.
Planck constraints on	LI LI
reheating	LI _{a>0}
Teneating	MHI
	MHI
Perspective Q	MHI _s
	MSSMI _o
	MSSMI _p
	NCKI _{β<0}
	BCHI BCHI
	RCHI
	RGI
	$RGI_{1/16}$
	RGI
	RGIs
	RIPI _{sugra}
	RIPIo

-5

-2.5 -1.1 1.1 2.5

Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{HI})$

	RMI_1			GMLFI _{1,3}			TI^e	
	RMI_{11}			GMLFI _{2,1}			$TI^{6}_{q < 1/2}$	
	RMI_2			MLFI	1 1		$TI_{q > 1/2}^{6}$: :
	RMI_{21}	1	: · · · · · · · · · · · · · · · · · · ·	GMLFI _{2,3}	1		TWI_{ϕ_0}	1 1
	SBI		GM	$LFL_{2/3,1/3}$	1		$TWI_{\phi_0}^r$	1 1
	$SBI_{\alpha_{min}}$: : :	ĢM	$LFL_{2/3,4/3}$: :	:	: TWI	1 1
	SFI	1	: : : : : : : : : : : : : : : : : : : :	GMLFI _{3,1}	1		TWI ^r	1 1
	SFI_1	1	: : : : : : : : : : : : : : : : : : :	GMLFI _{3,2}	1	:	Π_{β}	1 1
	SFI ₂		: : : : : : : : : : : : : : : : : : :	GMLFI _{3,3}	: : : · · ·		$:$ $:$ Π_f	1 1
	SFI ₂₁		1	LFI	1		Π_{λ}	1 1
	SFI ₃		1 I I I I I I I I I I I I I I I I I I I	LFI_1			PLI	1 1
	SFI ₃₁			LFI_2	1		PLI _p	1 1
	SFI_{3s}			$LFI_{2/3}$	1		$BSUSYBI_{f}$	
	SFI_4		: : :	LFI_3			BSUSYBI	
	SFI_{41}		: : : :	LFI_4	1		CNCI	
	SFI_{48}			LPI1			CNDI	
	SFI_1			$LPI1_{4,1}$			CSI	
	SFI_s			$LPI1_{4,2}$			DSI	
	SSB12			$LPI1_{4,3}$			DSI_2	
	$SSB12_f$			$NCKI_{\beta>0}$			DSI_0	
	SSB13			NI			IMI	
	$SSBI3_{f}$	1.1.1		IO :	1		Į MI1	: :
	S\$BI4			PSNI _{epA}			ĮMI2	
	SSBI4 _f	1	: :	$PSNI_{epB}$	1 1		1MI3	1 1
	SSB15		1 1	$PSNI_{epC}$	1		1MI4	1 1
	SSB15 _f	1	1 1	PSNI _{ft1}	1		İMI5	1 1
	WRI _g		1	PSNI _{ft2}	1		İ İMI6	1 1
	WRI ₀	1	1	PSNI _{ft3}	1		ŔMI ₃	1 1
	BEI		1	PSNI _{ft4}	1		RMI31	
	CNAI	: : :	: : : :	PSNI	: :	:	ŔMI4	1 1
	HF1I		1	PSNI _{oB}	1		RMI_{41}	
	$LI_{\alpha < 0}$	1	1	PSNLC	: : · ·		VHI	
	LMI1 _o		1	RCMI	1		VHI1	1 1
	LMI1 _p			RCQI	1.1.1		$VHI_{1/2}$	
	LMI2 _o		: : :	SSB11	: : :		VHI ₂	1 1
	LMI2 _p		: : : :	SSBI1			VHI ₃	
	LPI2 ₂			SSB16			VHI_4	
	LPI2 ₄			SSBI6 _f			$VHI_{n \leq 1}$	
	LPI2 ₆			$TI_{1/2}$			CNBI	
	LPI3 ₂			$TI_{\alpha>1/2}^{ft+}$			GMSSMI	
	LPI3 ₄			$TI_{\alpha>1/2}^{ft}$			GMSSMI	
	L'PI3 ₆	1	1 1	$TI_{\alpha > 1/2}^{ft-}$	1		ĢMSSMI _{omA}	1 1
	RPI1			$TI_{\alpha < 1/2}^{ft +}$: :		GMSSMI _{omB}	
	RPI2			$TI_{\sigma < 1/2}^{ft}$	1		GRIPL	
	RPI3		1	$TI_{d\leq 1/2}^{ft-}$	1		GRIPI	
	GMLFI	1		$\dot{T}I_{ft+}$	1		GRIPI	
	GMLFI _{1.1}	1	1	TI_{ft}	1		GRIPI	
	GMLFI _{1.2}		1	TI_{t-}	1	:	GRIPI	
_	5 -25 -11	11 25	-5 -25		1 2 5	_	25 .1 1	11 25
	J -Z.J -L.L	1.1 L.J	-5 -2.5	1			/ <u>-</u> Z.J -I.I	I.I Z.J

J.Martin, C.Ringeval, R.Trotta, V.Vennin ASPIC project

Bayes factor for hundred of models

WMAP7, arXiv:1009.4157

RGI.

RIPI

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RIPL

-2.5 -1.1 1.1 2.5

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Data analysis in model	AI	1 :
	$\mathrm{BI}_{\mathrm{ph}}$	1
space	Bl _{ls}	
	Bl _{2s}	
Using the slow-roll		
approximation as a proxy	BI ₅₅	
	BI_{0s}	
Bayesian model	BI_{s}	1 :
comparison o	$\mathrm{BI}_{\mathrm{stg}}$	
✤ Jeffreys' scate	DWI	
	ESI	
Speeding up evidence	ESI	1
calculation	ESI.	
	ESI/2 ESI	
♦ Accuracy of ASPIC +	GMSSMI _{en}	
effective likelihood	GMSSMI _{opA}	
	$GMSSMI_{opB}$	
Bayes factor for hundred	GRIPI _{sugra}	:
of models	$\mathrm{GRIPI}_{\mathrm{ep}}$	
or models	GRIPI _{opA}	
And the winners are	GrifiopB	
	KKLTI	
Narrowing down the	KKLTI,	
simplest with complexity	KKLTI _{stg}	1
Simplest with complexity	КМП	
	KMIII	1
Planck constraints on	KMH _{V>0}	
reheating	Ш	
	MHI	1
	MHI ₁	
Perspective	MHI _s	1
	MSSMI _o	
	MSSMIp	
	NCKIβ<0 ØCTI	
	RCHI BCHI	
	RCHI	
	RGI	
	RGI _{1/16}	:
	RGI,	1 0

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Bayes factor for hundred of models

Planck 2013, arXiv:1303.5082

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Bayes factor for hundred of models

Planck 2013, arXiv:1312.3529

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- approximation as a proxy Bayesian model comparison \cap ✤ Jeffreys' scate Speeding up evidence calculation ♦ Accuracy of ASPIC + effective likeliho Bayes factor for hundred
- of models
- ♦ And the winners are. . . ✤ Narrowing down the simplest with complexity
- Planck constraints on reheating

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Schwarz-Terrero-Escalante Classification: 2

Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{HI})$

GMLFL

GMLEL

GMLFL.

GMLEL

LPII

LPI1

PSNL.

CMLE

MLEI

GMLFL_{2/2}

PSNL.'.

PSNL.

PSNI_{e2}

PSNL A

PSNI_{oC}

-2.5 -1.1

-5

PSNI₆₃

PSNL

P\$NI_{ep}

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Bayes factor for hundred of models

LPI2.

GMLFI

Planck 2013, arXiv:1312.3529

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Comparison with observations

Bayesian Evidences $\ln({\cal E}/{\cal E}_{\rm HI})$ and $\ln({\cal L}_{\rm max}/{\cal E}_{\rm HI})$

GMLEL

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Displayed Evidences: 194

Comparison with Oobservations

Data analysis in model space

- Using the slow-roll approximation as a proxy
 Bayesian model
- comparison ♦ Jeffreys' scale
- Speeding up evidence calculation
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\clubsuit And the winners are. . .

Narrowing down the simplest with complexity

Planck constraints on reheating

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And the winners are...

- From non-commital priors: $\pi(\mathcal{M}) = 1/N_{\text{model}}$
- Posterior-to-prior ratio: Planck 2013

• Some numbers

- ◆ 52 models are in the inconclusive region "Some Good": AI, BI, ESI, HI, KKLTI, KMII, KMIII, LI, MHI, PSNI, RGI, SBI, SFI, SSBI2, TWI
- ♦ 66 models are strongly disfavoured (some "Bad" others "Ugly")

Comparison with observations

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Narrowing down the simplest with complexity

Bayesian complexity \simeq the number of constrained parameters

 $C = \langle -2 \ln \mathcal{L} \rangle + 2 \ln \mathcal{L}_{\max} \quad \Rightarrow \quad N_{\text{unconstrained}} = N_{\text{param}} - C$

• Planck 2013

arXiv:1312.3529 For the most probable and simplest scenarios \longrightarrow Displayed Models: 66/193

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Planck constraints on reheating

 Posteriors on the reheating parameter

 Prior-to-posterior width ratio
 Reheating constraints versus evidence

Planck constraints on reheating

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Posteriors on the reheating parameter

 Prior-to-posterior width ratio
 Reheating constraints versus evidence

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Posteriors on the reheating parameter

- For each model, we use the most generic parameterization: $R_{
 m reh}$
 - Prior choice: Jeffreys' on $R_{\rm reh} \Leftrightarrow$ flat on $\ln R_{\rm reh}$ with:

$$-46 < \ln \frac{R_{\rm reh}}{R_{\rm reh}} < 15 + \frac{1}{3} \ln \rho_{\rm end}$$

Planck 2013 data put non-trivial constraints on many models

Examples: LI with
$$V(\phi) = M^4 (1 + \alpha \ln \phi)$$

prior

posterior

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 Prior-to-posterior width ratio
 Reheating constraints versus evidence

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Posteriors on the reheating parameter

- For each model, we use the most generic parameterization: $R_{
 m reh}$
 - Prior choice: Jeffreys' on $R_{\rm reh} \Leftrightarrow$ flat on $\ln R_{\rm reh}$ with:

$$-46 < \ln \frac{R_{\rm reh}}{R_{\rm reh}} < 15 + \frac{1}{3} \ln \rho_{\rm end}$$

- Planck 2013 data put non-trivial constraints on many models
- Examples: SBI with $V(\phi) = M^4 \left[1 + \phi^4 \left(-\alpha + \beta \ln \phi \right) \right]$

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Prior-to-posterior width ratio

Reheating is constrained \Leftrightarrow posterior of $\ln R_{\rm reh}$ is peaked

- The most probable value of $R_{\rm reh}$ is model-dependent
- We introduce the ratio between the prior and posterior standard deviation of $\ln R_{\rm reh}$

$$\frac{\Delta \pi_{\ln R_{\rm reh}}}{\Delta \mathcal{P}_{\ln R_{\rm reh}}}\Big|_{\mathcal{M}} = \sqrt{\frac{\int \left(\ln R_{\rm reh} - \langle \ln R_{\rm reh} \rangle_{\pi}\right)^2 \pi \left(\ln R_{\rm reh} | \mathcal{M} \right) \mathrm{d} \ln R_{\rm reh}}{\int \left(\ln R_{\rm reh} - \langle \ln R_{\rm reh} \rangle_p\right)^2 p \left(\ln R_{\rm reh} | D, \mathcal{M} \right) \mathrm{d} \ln R_{\rm reh}}}$$

- Disfavoured models exhibit larger values for $\Delta \pi_{\ln R_{\rm reh}} / \Delta \mathcal{P}_{\ln R_{\rm reh}}$
 - In the space of models, a fair estimate of the Planck's constraining power on reheating is

$$\left\langle \frac{\Delta \pi_{\ln R_{\rm reh}}}{\Delta \mathcal{P}_{\ln R_{\rm reh}}} \right\rangle \equiv \sum_{\mathcal{M}_i} p(\mathcal{M}_i | D) \left. \frac{\Delta \pi_{\ln R_{\rm reh}}}{\Delta \mathcal{P}_{\ln R_{\rm reh}}} \right|_{\mathcal{M}_i}$$

• For Planck 2013: $\left\langle \frac{\Delta \pi_{\ln R_{\rm reh}}}{\Delta \mathcal{P}_{\ln R_{\rm reh}}} \right\rangle \simeq 1.66 \implies$ prior cut by 40%

Reheating constraints versus evidence

No assumption on reheating (= using $R_{
m reh}$)

Displayed Models: 170/193

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Reheating constraints versus evidence

Assuming the equation of state $\overline{w}_{\rm reh}$ to be fixed

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Novel and efficient approach applicable to any cosmological data set

- Reheating is included and already constrained by Planck 2013
- Provides new insights in the most difficult to disambiguate situation: slow-roll inflation

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Novel and efficient approach applicable to any cosmological data set

- Reheating is included and already constrained by Planck 2013
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- After Planck 2014?

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 - Reheating is included and already constrained by Planck 2013
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- After Planck 2014?
 - ♦ Future CMB missions: See V. Vennin's talk

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Novel and efficient approach applicable to any cosmological data set

- Reheating is included and already constrained by Planck 2013
- Provides new insights in the most difficult to disambiguate situation: slow-roll inflation
- After Planck 2014?
 - Future CMB missions: See V. Vennin's talk
 - ♦ Galaxy surveys: Euclid

From Basse et al., arXiv:1409.3469

