Inflationary models after Planck

Christophe Ringeval
Centre for Cosmology, Particle Physics and Phenomenology
Institute of Mathematics and Physics
Louvain University, Belgium

Paris, 16/12/2014

Outline

CR	arXiv:1312.2347
J. Martin, CR and V. Vennin	arXiv:1303.3787, arXiv:1410.7958
J. Martin, CR, R. Trotta and V. Vennin	arXiv:1312.3529, arXiv:1405.7272

Planck 2013 results for inflation summarized

- Favour minimal assumption scenarios
  - Flatness ($\Omega_K = 0$)

$$\Omega_K = 1 - \Omega_{\mathrm{dm}} - \Omega_{\mathrm{b}} - \Omega_\Lambda = 0.00^{+0.0066}_{-0.0067} \quad \texttt{(PLANCK+WP+BAO)}$$

  - Adiabatic initial conditions: isocurvature modes are constrained

$$\forall X \quad P_X(k) = P(k)$$

  - Quasi scale invariance of the scalar modes

$$k^3 P(k) = A \left(\frac{k}{k_*}\right)^{n_S - 1} \Rightarrow n_S = 0.9619 \pm 0.0073$$

  - Gaussianity of the CMB anisotropies

$$f_{NL}^{\mathrm{loc}} = 2.7 \pm 5.8, \quad f_{NL}^{\mathrm{eq}} = -42 \pm 75, \quad f_{NL}^{\mathrm{ortho}} = -25 \pm 39$$

Planck 2013 results for inflation summarized

- Favour minimal assumption scenarios
  - Flatness ($\Omega_{\rm K} = 0$)

$$\Omega_{\rm K} = 1 - \Omega_{\rm dm} - \Omega_{\rm b} - \Omega_{\Lambda} = 0.00^{+0.0066}_{-0.0067} \quad \texttt{(PLANCK+WP+BAO)}$$

  - Adiabatic initial conditions: isocurvature modes are constrained

$$\forall X \quad P_{\rm X}(k) = P(k)$$

  - Quasi scale invariance of the scalar modes

$$k^3 P(k) = A\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} \Rightarrow n_{\rm S} = 0.9619 \pm 0.0073$$

  - Gaussianity of the CMB anisotropies

$$f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm eq} = -42 \pm 75, \quad f_{\rm NL}^{\rm ortho} = -25 \pm 39$$

PLANCK 2014 certified!

Planck 2013 results for inflation summarized

PLANCK 2014 certified!

- Favour minimal assumption scenarios
  - Flatness ($\Omega_{\mathrm{K}} = 0$)

$$\Omega_{\mathrm{K}} = 1 - \Omega_{\mathrm{dm}} - \Omega_{\mathrm{b}} - \Omega_{\Lambda} = 0.00^{+0.0066}_{-0.0067} \quad \texttt{(PLANCK+WP+BAO)}$$

  - Adiabatic initial conditions: isocurvature modes are constrained

$$\forall X \quad P_{\mathrm{X}}(k) = P(k)$$

  - Quasi scale invariance of the scalar modes

$$k^3 P(k) = A\left(\frac{k}{k_*}\right)^{n_{\mathrm{S}}-1} \Rightarrow n_{\mathrm{S}} = 0.9619 \pm 0.0073$$

  - Gaussianity of the CMB anisotropies

$$f_{\mathrm{NL}}^{\mathrm{loc}} = 2.7 \pm 5.8, \quad f_{\mathrm{NL}}^{\mathrm{eq}} = -42 \pm 75, \quad f_{\mathrm{NL}}^{\mathrm{ortho}} = -25 \pm 39$$

- This is also called: single-field slow-roll inflation
  - Makes extra-predictions: $f_{\mathrm{NL}}^{\mathrm{loc}} = \mathcal{O}(n_{\mathrm{S}} - 1)$ and $\exists r > 0$

Basic theoretical assumptions

- Dynamics given by $(\kappa^2 = 1/M_{\mathrm{P}}^2)$

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2\kappa^2}R + \mathcal{L}(\phi)\right] \quad \text{with} \quad \mathcal{L}(\phi) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

- Can be used to describe:
  - Minimally coupled scalar field to General Relativity
  - Scalar-tensor theory of gravitation in the Einstein frame

    the graviton' scalar partner is also the inflaton (HI, RPI1,...)

- Everything can be consistently solved in the slow-roll approximation
  - Background evolution $\phi(N)$ where $N \equiv \ln a$
  - Linear perturbations for the field-metric system $\zeta(t, \boldsymbol{x})$, $\delta\phi(t, \boldsymbol{x})$

- Slow-roll = expansion in terms of the Hubble flow functions [Schwarz 01]

$$\epsilon_0 = \frac{H_{\mathrm{ini}}}{H}, \qquad \epsilon_{i+1} = \frac{\ln|\epsilon_i|}{\mathrm{d}N} \qquad \text{measure deviations from de-Sitter}$$

Decoupling field and space-time evolution

- Friedmann-Lemaître equations in e-fold time (with $M_{\mathrm{P}}^2 = 1$)

$$\begin{cases} H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) \\ \frac{\ddot{a}}{a} = -\frac{1}{3}\left(\dot{\phi}^2 - V\right) \end{cases} \Rightarrow \begin{cases} H^2 = \dfrac{V}{3 - \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2} \\ -\frac{\mathrm{d}\ln H}{\mathrm{d}N} = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases} \Leftrightarrow \begin{cases} H^2 = \dfrac{V}{3 - \epsilon_1} \\ \epsilon_1 = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2 \end{cases}$$

- Klein-Gordon equation in e-folds: relativistic kinematics with friction

$$\frac{1}{3 - \epsilon_1}\frac{\mathrm{d}^2\phi}{\mathrm{d}N^2} + \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{\mathrm{d}\ln V}{\mathrm{d}\phi} \quad \Leftrightarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{3 - \epsilon_1}{3 - \epsilon_1 + \frac{\epsilon_2}{2}}\frac{\mathrm{d}\ln V}{\mathrm{d}\phi}$$

- Slow-roll approximation: all $\epsilon_i = \mathcal{O}(\epsilon)$ and $\epsilon_1 < 1$ is the definition of inflation ($\ddot{a} > 0$)
  - ✦ The trajectory can be solved for N

$$N - N_{\mathrm{end}} \simeq \int_{\phi}^{\phi_{\mathrm{end}}} \frac{V(\psi)}{V'(\psi)}\,\mathrm{d}\psi$$

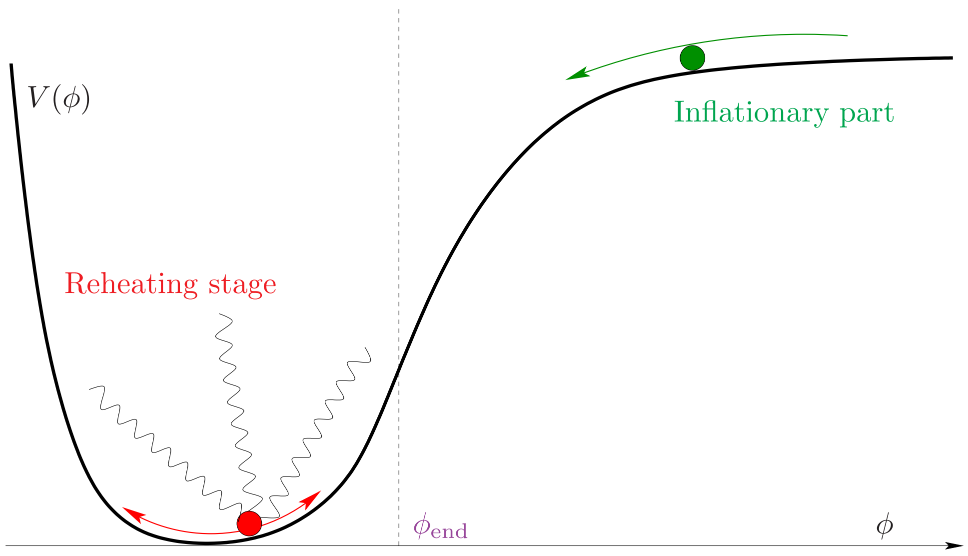
The end of inflation and after

- Accelerated expansion stops for $\epsilon_1 > 1$ ($\ddot{a} < 0$) at $N = N_{\mathrm{end}}$
  - Naturally happens during field evolution (graceful exit) at $\phi = \phi_{\mathrm{end}}$

$$\epsilon_1(\phi_{\mathrm{end}}) = 1$$

  - Or, there is another mechanism ending inflation (tachyonic instability) and ϕ_{end} is a model parameter that has to be specified
- The reheating stage: everything after N_{end} till radiation domination
  - Basic picture $\longrightarrow$
  - But in reality a very complicated process, microphysics dependent
  - Reheating duration is unknown:

$$\Delta N_{\mathrm{reh}} \equiv N_{\mathrm{reh}} - N_{\mathrm{end}}$$



Redshift at which reheating ends

- Denoting $N = N_{\mathrm{reh}}$ the end of reheating = beginning of radiation era
  - If thermalized, and no extra entropy production: $a_{\mathrm{reh}}^3 s_{\mathrm{reh}} = a_0^3 s_0$

$$\begin{cases} s_{\mathrm{reh}} = q_{\mathrm{reh}} \dfrac{2\pi^2}{45} T_{\mathrm{reh}}^3 \\ \rho_{\mathrm{reh}} = g_{\mathrm{reh}} \dfrac{\pi^2}{30} T_{\mathrm{reh}}^4 \end{cases} \Rightarrow \quad \frac{a_0}{a_{\mathrm{reh}}} = \left(\frac{q_{\mathrm{reh}}^{1/3} g_0^{1/4}}{q_0^{1/3} g_{\mathrm{reh}}^{1/4}} \right) \frac{\rho_{\mathrm{reh}}^{1/4}}{\rho_\gamma^{1/4}}$$

$$\text{or} \quad 1 + z_{\mathrm{reh}} = \left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma} \right)^{1/4}$$

- Depends on ρ_{reh} and $\tilde{\rho}_\gamma \equiv \mathcal{Q}_{\mathrm{reh}} \rho_\gamma$
  - Energy density of radiation today: $\rho_\gamma = 3 \dfrac{H_0^2}{M_{\mathrm{P}}^2} \Omega_{\mathrm{rad}}$
  - Change in the number of entropy and energy relativistic degrees of freedom (small effect compared to $\rho_{\mathrm{reh}}/\rho_\gamma$)

$$\mathcal{Q}_{\mathrm{reh}} \equiv \frac{g_{\mathrm{reh}}}{g_0} \left(\frac{q_0}{q_{\mathrm{reh}}} \right)^{1/4}$$

Redshift at which inflation ends

- Depends on the redshift of reheating

$$1+z_{\mathrm{end}} = \frac{a_0}{a_{\mathrm{end}}} = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}(1+z_{\mathrm{reh}}) = \frac{a_{\mathrm{reh}}}{a_{\mathrm{end}}}\left(\frac{\rho_{\mathrm{reh}}}{\tilde{\rho}_\gamma}\right)^{1/4} = \frac{1}{R_{\mathrm{rad}}}\left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{1/4}$$

  - ✦ The reheating parameter $R_{\mathrm{rad}} \equiv \frac{a_{\mathrm{end}}}{a_{\mathrm{reh}}}\left(\frac{\rho_{\mathrm{end}}}{\rho_{\mathrm{reh}}}\right)^{1/4}$
  - ✦ Encodes any observable deviations from a radiation-like or instantaneous reheating $R_{\mathrm{rad}} = 1$

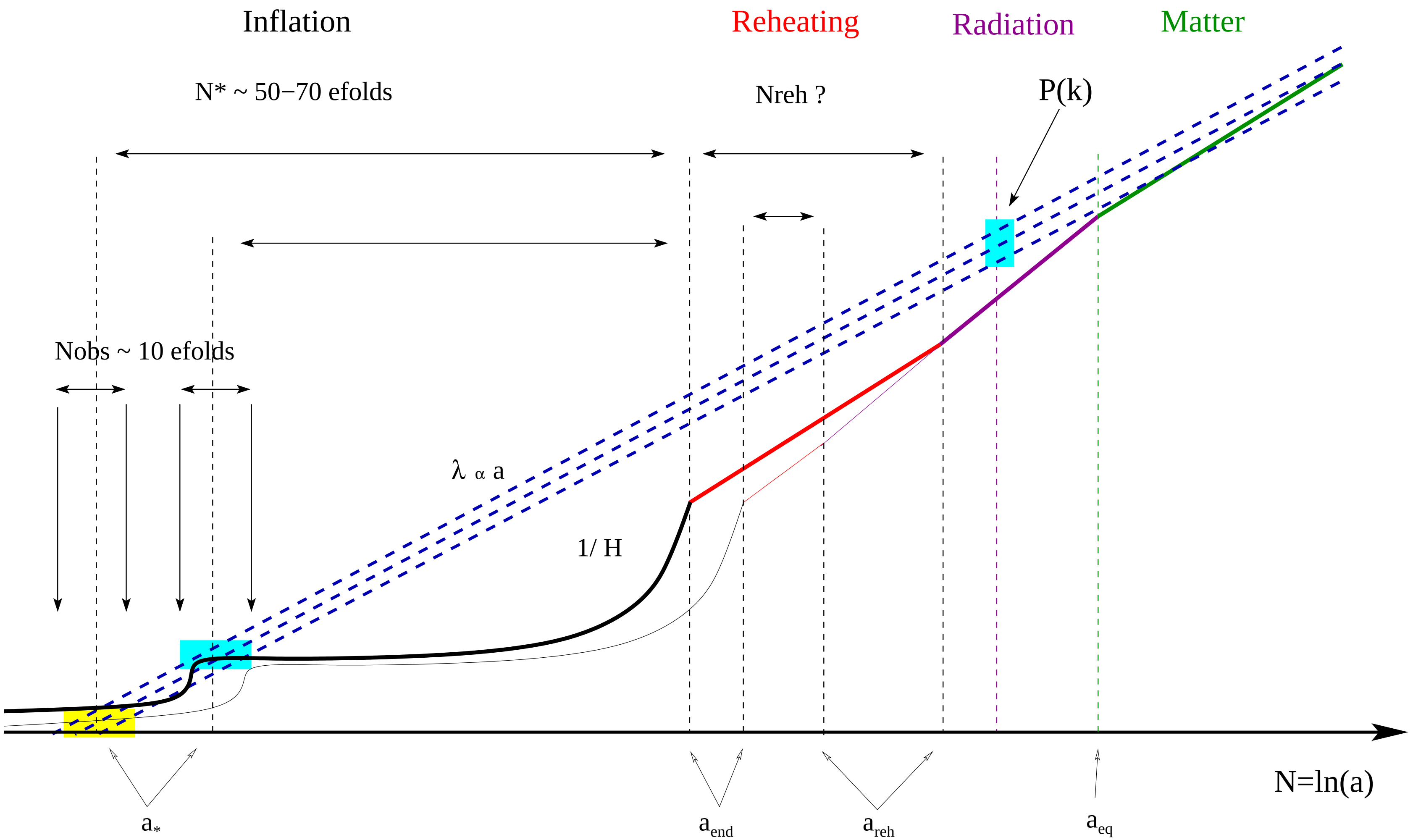
- R_{rad} can be expressed in terms of $(\rho_{\mathrm{reh}}, \overline{w}_{\mathrm{reh}})$ or $(\Delta N_{\mathrm{reh}}, \overline{w}_{\mathrm{reh}})$

$$\ln R_{\mathrm{rad}} = \frac{\Delta N_{\mathrm{reh}}}{4}(3\overline{w}_{\mathrm{reh}} - 1) = \frac{1-3\overline{w}_{\mathrm{reh}}}{12(1+\overline{w}_{\mathrm{reh}})} \ln\left(\frac{\rho_{\mathrm{reh}}}{\rho_{\mathrm{end}}}\right)$$

where $\overline{w}_{\mathrm{reh}} \equiv \frac{1}{\Delta N_{\mathrm{reh}}}\int_{N_{\mathrm{end}}}^{N_{\mathrm{reh}}} \frac{P(N)}{\rho(N)} \mathrm{d}N$

- A fixed inflationary parameters, z_{end} can still be affected by R_{rad}

Reheating effects on inflationary observables



- Model testing: reheating effects must be included!

Inflationary perturbations in slow-roll

- Equations of motion for the linear perturbations

$$\left.\begin{aligned}\mu_{\mathrm{T}} &\equiv a h\\ \mu_{\mathrm{S}} &\equiv a\sqrt{2}\phi_{,N}\zeta\end{aligned}\right\} \Rightarrow \mu_{\mathrm{TS}}'' + \left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]\mu_{\mathrm{TS}} = 0$$

- Can be consistently solved using slow-roll and pivot expansion [Stewart:1993, Gong:2001, Schwarz:2001, Leach:2002, Martin:2002, Habib:2002, Casadio:2005, Lorenz:2008, Martin:2013, Beltran:2013]

$$\begin{aligned}\mathcal{P}_\zeta = \frac{H_*^2}{8\pi^2 M_{\mathrm{P}}^2\epsilon_{1*}}\Bigg\{ & 1 - 2(1+C)\epsilon_{1*} - C\epsilon_{2*} + \left(\frac{\pi^2}{2} - 3 + 2C + 2C^2\right)\epsilon_{1*}^2 + \left(\frac{7\pi^2}{12} - 6 - C + C^2\right)\epsilon_{1*}\epsilon_{2*}\\ & + \left(\frac{\pi^2}{8} - 1 + \frac{C^2}{2}\right)\epsilon_{2*}^2 + \left(\frac{\pi^2}{24} - \frac{C^2}{2}\right)\epsilon_{2*}\epsilon_{3*}\\ & + \left[-2\epsilon_{1*} - \epsilon_{2*} + (2+4C)\epsilon_{1*}^2 + (-1+2C)\epsilon_{1*}\epsilon_{2*} + C\epsilon_{2*}^2 - C\epsilon_{2*}\epsilon_{3*}\right]\ln\left(\frac{k}{k_*}\right)\\ & + \left[2\epsilon_{1*}^2 + \epsilon_{1*}\epsilon_{2*} + \frac{1}{2}\epsilon_{2*}^2 - \frac{1}{2}\epsilon_{2*}\epsilon_{3*}\right]\ln^2\left(\frac{k}{k_*}\right)\Bigg\},\\ \mathcal{P}_h = \frac{2H_*^2}{\pi^2 M_{\mathrm{P}}^2}\Bigg\{ & 1 - 2(1+C)\epsilon_{1*} + \left[-3 + \frac{\pi^2}{2} + 2C + 2C^2\right]\epsilon_{1*}^2 + \left[-2 + \frac{\pi^2}{12} - 2C - C^2\right]\epsilon_{1*}\epsilon_{2*}\\ & + \left[-2\epsilon_{1*} + (2+4C)\epsilon_{1*}^2 + (-2-2C)\epsilon_{1*}\epsilon_{2*}\right]\ln\left(\frac{k}{k_*}\right) + \left(2\epsilon_{1*}^2 - \epsilon_{1*}\epsilon_{1*}\right)\ln^2\left(\frac{k}{k_*}\right)\Bigg\}\end{aligned}$$

- Notice that: $H_* \equiv H(N_*)$ and $\epsilon_{i*} \equiv \epsilon_i(N_*)$ with $k_*\eta(N_*) = -1$

Solving for the time of pivot crossing

- To make inflationary predictions, one has to solve $k_*\eta_* = -1$

$$\frac{k_*}{a_0} = \frac{a(N_*)}{a_0} H_* = e^{N_*-N_{\mathrm{end}}} \frac{a_{\mathrm{end}}}{a_0} H_* = \frac{e^{\Delta N_*} H_*}{1+z_{\mathrm{end}}} = e^{\Delta N_*} R_{\mathrm{rad}} \left(\frac{\rho_{\mathrm{end}}}{\tilde{\rho}_\gamma}\right)^{-\frac{1}{4}} H_*$$

- Defining $N_0 \equiv \ln\left(\frac{k_*}{a_0}\frac{1}{\tilde{\rho}_\gamma^{1/4}}\right)$ (number of e-folds of deceleration)
  - ✦ This is a non-trivial integral equation that depends on: model + how inflation ends + reheating + data

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*} \frac{V(\psi)}{V'(\psi)} \mathrm{d}\psi\right] = \ln R_{\mathrm{rad}} - N_0 + \frac{1}{4}\ln(8\pi^2 P_*) - \frac{1}{4}\ln\left\{\frac{9}{\epsilon_1(\phi_*)[3-\epsilon_1(\phi_{\mathrm{end}})]}\frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right\}$$

  - ✦ Arbitrarily fixing ΔN_* (or ϕ_*) = postulating a generally wrong solution to this trivial equation!

The optimal reheating parameter

- Defining the rescaled reheating parameter (astro-ph/0605367)

$$\ln R_{\mathrm{reh}} \equiv \ln R_{\mathrm{rad}} + \frac{1}{4} \ln \rho_{\mathrm{end}}$$

  - Within a given model, one-to-one correspondance between R_{rad} and R_{reh}

- "Magic" cancellation in the reheating equation (also valid out of slow-roll)

$$-\left[\int_{\phi_{\mathrm{end}}}^{\phi_*} \frac{V(\psi)}{V'(\psi)} \mathrm{d}\psi\right] = \ln R_{\mathrm{reh}} - N_0 - \frac{1}{2} \ln \left[\frac{9}{3 - \epsilon_1(\phi_{\mathrm{end}})} \frac{V(\phi_{\mathrm{end}})}{V(\phi_*)}\right]$$

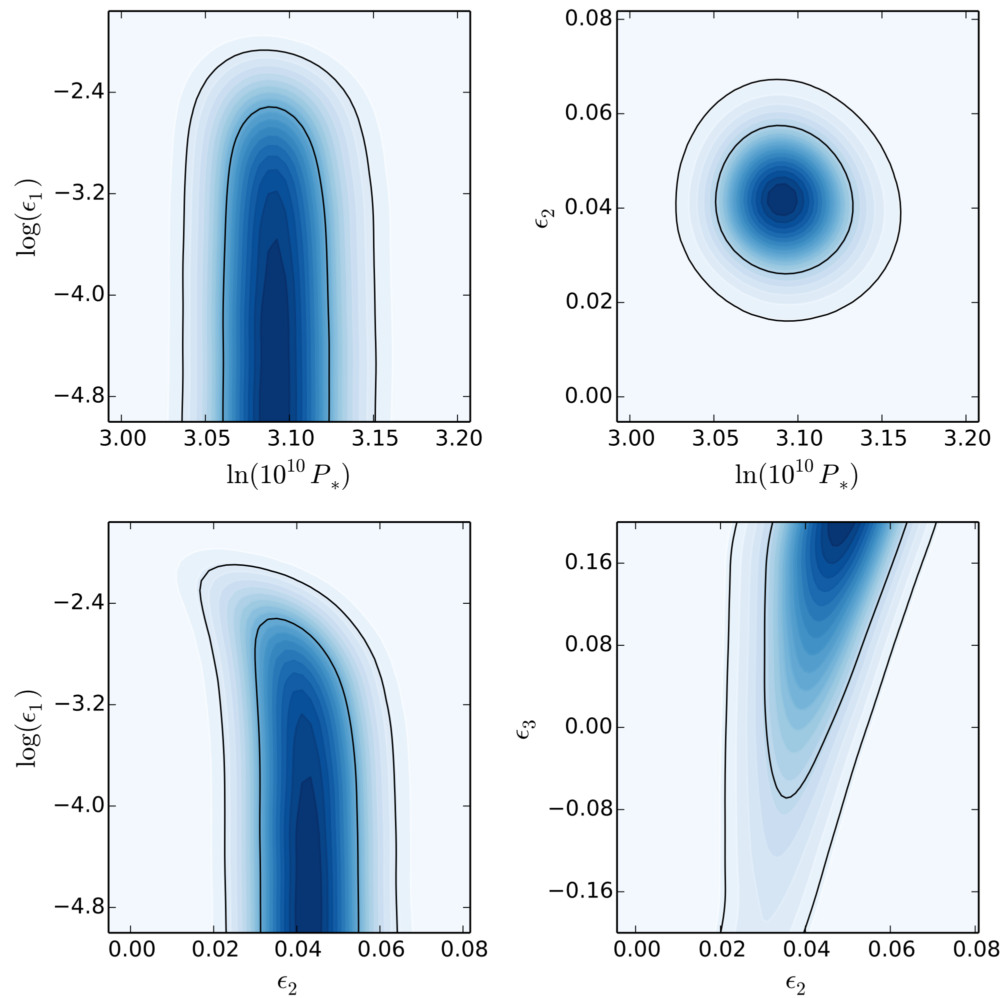
- Using R_{reh} avoids correlations with P_* in performing data analysis
- Assuming $-1/3 < \overline{w}_{\mathrm{reh}} < 1$ and $\rho_{\mathrm{nuc}} \equiv (10\,\mathrm{MeV})^4 < \rho_{\mathrm{reh}} < \rho_{\mathrm{end}}$

$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3} \ln \rho_{\mathrm{end}}$$

Comparison with observations

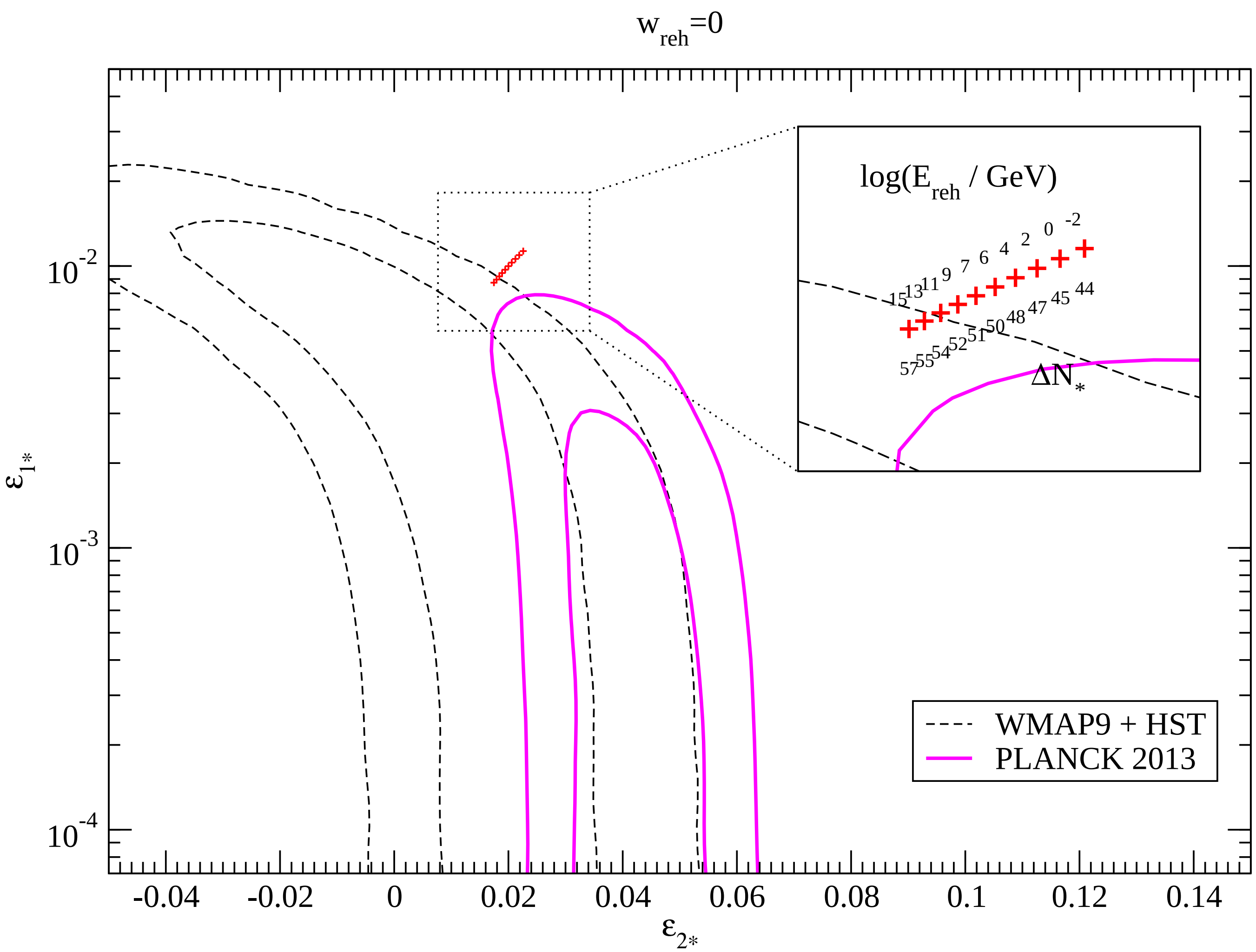
Planck 2013 constraints on slow-roll

- From the slow-roll expanded expression of $\mathcal{P}_\zeta(k)$ and $\mathcal{P}_h(k)$
  - ✦ Constraints on ϵ_{i*} and P_* (or H_*^2/ϵ_{1*})



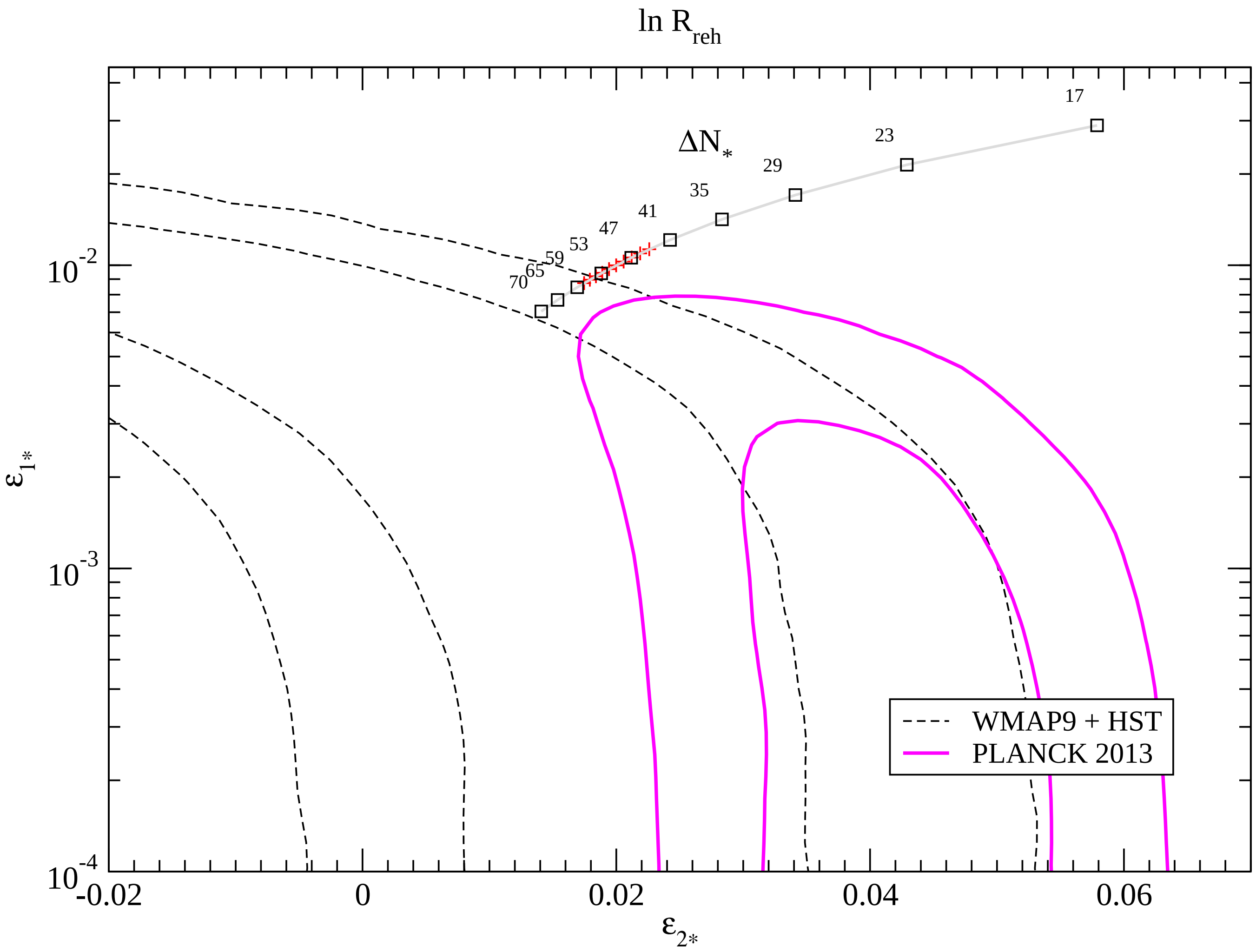
Comparison with model predictions

- Can only be done from the input of R_{reh}, or R_{rad}, or $(\overline{w}_{\mathrm{reh}}, \rho_{\mathrm{reh}})$
  - One can scan various reheating histories: ΔN_* is not arbitrary!
  - Example: LFI_2 with $\overline{w}_{\mathrm{reh}} = 0$ and $\rho_{\mathrm{nuc}} < \rho_{\mathrm{reh}} < \rho_{\mathrm{end}}$



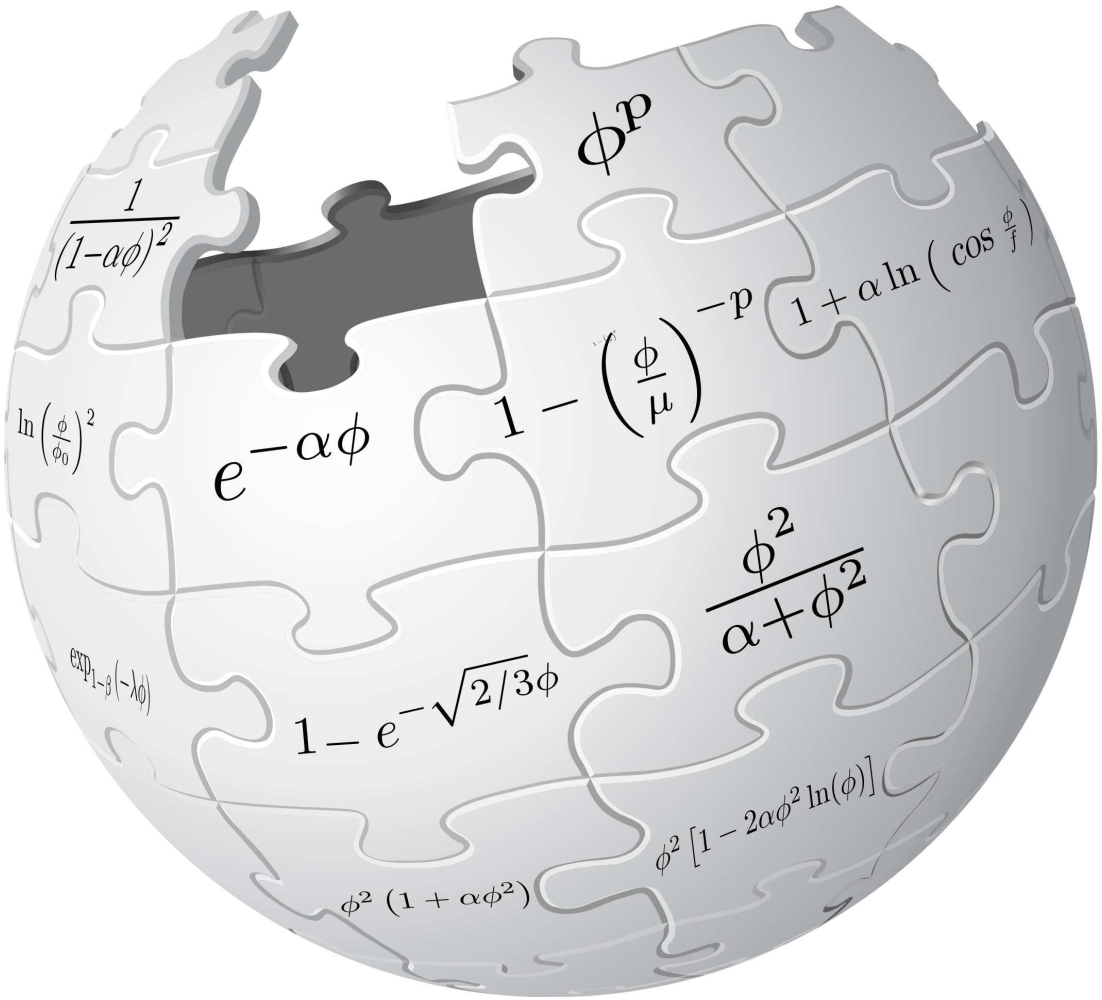
Most generic reheating parametrization

- In the absence of any information on the reheating, one should use R_{reh} (or R_{rad})
- Same example: LFI_2 without assuming $\overline{w}_{\mathrm{reh}} = 0$



Encyclopædia Inflationaris

- With J. Martin and V. Vennin



http://arxiv.org/abs/1303.3787
http://cp3.irmp.ucl.ac.be/~ringeval/aspic.html

Purpose

- Quasi-exhaustive analysis to derive reheating consistent observable predictions for all slow-roll single-field inflationary models
- Comes with a public code (`ASPIC`)
- Currently supports more than 50 motivated classes of potential

Name	Parameters	Sub-models	$V(\phi)$
HI	0	1	$M^4\left(1-e^{-\sqrt{2/3}\phi/M_{\mathrm{Pl}}}\right)$
RCHI	1	1	$M^4\left(1-2e^{-\sqrt{2/3}\phi/M_{\mathrm{Pl}}}+\frac{A_{\mathrm{I}}}{16\pi^2}\frac{\phi}{\sqrt{6}M_{\mathrm{Pl}}}\right)$
LFI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p$
MLFI	1	1	$M^4\frac{\phi^2}{M_{\mathrm{Pl}}^2}\left[1+\alpha\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right]$
RCMI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\left[1-2\alpha\frac{\phi^2}{M_{\mathrm{Pl}}^2}\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
RCQI	1	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\left[1-\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
NI	1	1	$M^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$
ESI	1	1	$M^4\left(1-e^{-q\phi/M_{\mathrm{Pl}}}\right)$
PLI	1	1	$M^4e^{-\alpha\phi/M_{\mathrm{Pl}}}$
KMII	1	2	$M^4\left(1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}e^{-\phi/M_{\mathrm{Pl}}}\right)$
HF1I	1	1	$M^4\left(1+A_1\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\left[1-\frac{2}{3}\left(\frac{A_1}{1+A_1\phi/M_{\mathrm{Pl}}}\right)^2\right]$
CWI	1	1	$M^4\left[1+\alpha\left(\frac{\phi}{Q}\right)^4\ln\left(\frac{\phi}{Q}\right)\right]$
LI	1	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
RpI	1	3	$M^4e^{-2\sqrt{2/3}\phi/M_{\mathrm{Pl}}}\left\|e^{\sqrt{2/3}\phi/M_{\mathrm{Pl}}}-1\right\|^{2p/(2p-1)}$
DWI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-1\right]^2$
MHI	1	1	$M^4\left[1-\mathrm{sech}\left(\frac{\phi}{\mu}\right)\right]$
RGI	1	1	$M^4\frac{(\phi/M_{\mathrm{Pl}})^2}{\alpha+(\phi/M_{\mathrm{Pl}})^2}$
MSSMI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\left(\frac{\phi}{\phi_0}\right)^6+\frac{1}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
RIPI	1	1	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^3+\frac{1}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
AI	1	1	$M^4\left[1-\frac{2}{\pi}\arctan\left(\frac{\phi}{\mu}\right)\right]$
CNAI	1	1	$M^4\left[3-\left(3+\alpha^2\right)\tanh^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
CNBI	1	1	$M^4\left[\left(3-\alpha^2\right)\tan^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)-3\right]$
OSTI	1	1	$-M^4\left(\frac{\phi}{\phi_0}\right)^2\ln\left[\left(\frac{\phi}{\phi_0}\right)^2\right]$
WRI	1	1	$M^4\ln\left(\frac{\phi}{\phi_0}\right)^2$
SFI	2	1	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^p\right]$

– 15 –

II	2	1	$M^4\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)^{-\beta}-M^4\frac{\beta^2}{6}\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)^{-\beta-2}$
KMIII	2	1	$M^4\left[1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}\exp\left(-\beta\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]$
LMI	2	2	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{\alpha}\exp\left[-\beta(\phi/M_{\mathrm{Pl}})^{\gamma}\right]$
TWI	2	1	$M^4\left[1-A\left(\frac{\phi}{\phi_0}\right)^2e^{-\phi/\phi_0}\right]$
GMSSMI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{2}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^6+\frac{\alpha}{5}\left(\frac{\phi}{\phi_0}\right)^{10}\right]$
GRIPI	2	2	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2-\frac{4}{3}\alpha\left(\frac{\phi}{\phi_0}\right)^3+\frac{\alpha}{2}\left(\frac{\phi}{\phi_0}\right)^4\right]$
BSUSYBI	2	1	$M^4\left(e^{\sqrt{6}\frac{\phi}{M_{\mathrm{Pl}}}}+e^{\sqrt{6}\gamma\frac{\phi}{M_{\mathrm{Pl}}}}\right)$
TI	2	3	$M^4\left(1+\cos\frac{\phi}{\mu}+\alpha\sin^2\frac{\phi}{\mu}\right)$
BEI	2	1	$M^4\exp_{1-\beta}\left(-\lambda\frac{\phi}{M_{\mathrm{Pl}}}\right)$
PSNI	2	1	$M^4\left[1+\alpha\ln\left(\cos\frac{\phi}{f}\right)\right]$
NCKI	2	2	$M^4\left[1+\alpha\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)+\beta\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2\right]$
CSI	2	1	$\frac{M^4}{\left(1-\alpha\frac{\phi}{M_{\mathrm{Pl}}}\right)^2}$
OI	2	1	$M^4\left(\frac{\phi}{\phi_0}\right)^4\left[\left(\ln\frac{\phi}{\phi_0}\right)^2-\alpha\right]$
CNCI	2	1	$M^4\left[\left(3+\alpha^2\right)\coth^2\left(\frac{\alpha}{\sqrt{2}}\frac{\phi}{M_{\mathrm{Pl}}}\right)-3\right]$
SBI	2	2	$M^4\left\{1+\left[-\alpha+\beta\ln\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\right\}$
SSBI	2	6	$M^4\left[1+\alpha\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^2+\beta\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^4\right]$
IMI	2	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^{-p}$
BI	2	2	$M^4\left[1-\left(\frac{\phi}{\mu}\right)^{-p}\right]$
RMI	3	4	$M^4\left[1-\frac{c}{2}\left(-\frac{1}{2}+\ln\frac{\phi}{\phi_0}\right)\frac{\phi^2}{M_{\mathrm{Pl}}^2}\right]$
VHI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^p\right]$
DSI	3	1	$M^4\left[1+\left(\frac{\phi}{\mu}\right)^{-p}\right]$
GMLFI	3	1	$M^4\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^p\left[1+\alpha\left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^q\right]$
LPI	3	3	$M^4\left(\frac{\phi}{\phi_0}\right)^p\left(\ln\frac{\phi}{\phi_0}\right)^q$
CNDI	3	3	$\frac{M^4}{\left\{1+\beta\cos\left[\alpha\left(\frac{\phi-\phi_0}{M_{\mathrm{Pl}}}\right)\right]\right\}^2}$

– 16 –

ASPIC example program with LFI

```
program toy
  use infprec, only : kp
  use lfisr, only : lfi_epsilon_one, lfi_epsilon_two
  use lfisr, only : lfi_epsilon_three, lfi_x_endinf
  use lfireheat, only : lfi_x_rreh, lfi_x_star
  use srflow, only : scalar_spectral_index, tensor_to_scalar_ratio
  use cosmopar, only : lnMpinGeV, PowerAmpScalar
  implicit none

  real(kp) :: lnR
  real(kp), dimension(3) :: eps

  real(kp) :: DeltaN
  real(kp) :: p, xstar, xend
  real(kp) :: ns, r

  real(kp) :: ErehGeV, wreh,lnRhoReh

  p=2

!radiation-like reheating
  lnR = 0._kp

  xend = lfi_x_endinf(p)
  xstar = lfi_x_rreh(p,lnR,DeltaN)

  print *,'xend= xstar= DeltaN= ',xend,xstar,DeltaN

  eps(1) = lfi_epsilon_one(xstar,p)
  eps(2) = lfi_epsilon_two(xstar,p)
  eps(3) = lfi_epsilon_three(xstar,p)

  ns = scalar_spectral_index(eps)
  r = tensor_to_scalar_ratio(eps)

  print *,'ns= r= ',ns,r

  read(*,*)

!matter like reheating at Ereh=10^8 GeV
  ErehGeV = 1e8
  wreh = 0

  lnRhoReh = 4._kp*(log(ErehGev)-lnMpinGev)

  xstar = lfi_x_star(p,wreh,lnRhoReh,PowerAmpScalar,DeltaN)

  print *,'xend= xstar= DeltaN= ',xend,xstar,DeltaN

  eps(1) = lfi_epsilon_one(xstar,p)
  eps(2) = lfi_epsilon_two(xstar,p)
  eps(3) = lfi_epsilon_three(xstar,p)

  ns = scalar_spectral_index(eps)
  r = tensor_to_scalar_ratio(eps)

  print *,'ns= r= ',ns,r

end program toy
```

```
FC=gfortran

FCFLAGS=-g
LFLAGS=-L/home/chris/usr/lib -laspic

INCLUDE=-I/home/chris/usr/include/aspic

default: toy

%.o: %.f90
	$(FC) $(FCFLAGS) $(INCLUDE) -c $<

toy: toy.o
	$(FC) $(FCFLAGS) toy.o -o $@ $(LFLAGS)


clean:
	rm toy *.o *.mod
```

ASPIC and alternative parameterizations

- Postulating an evolution for $w(N) = \dfrac{P(N)}{\rho(N)} = \dfrac{2}{3}\epsilon_1(N) - 1 \Leftrightarrow$

$$\begin{cases} \dfrac{\mathrm{d}\phi}{\mathrm{d}N} = \pm\sqrt{3}\,(1+w)^{1/2} \\ \dfrac{\mathrm{d}\ln V}{\mathrm{d}N} = -3\,(1+w) + \dfrac{\mathrm{d}\ln(1-w)}{\mathrm{d}N} \end{cases} \Rightarrow \begin{cases} \phi = \phi_{\mathrm{end}} \mp \sqrt{3}\displaystyle\int (1+w)^{1/2}\mathrm{d}N \\ V \propto (1-w)\, e^{-3\int(1+w)\mathrm{d}N} \end{cases}$$

  - ✦ Strictly equivalent to specify $V(\phi)$ up to the normalisation M^4
    M^4, ΔN_* are obtained from $P_* + R_{\mathrm{reh}} +$ solving $w(N_{\mathrm{end}}) = 1/3$

- Expanding $n_{\mathrm{S}}(N)$ and $r(N)$ around $N_* \Leftrightarrow$ choosing $V(\phi)$ around ϕ_*

$$\begin{aligned} n_{\mathrm{S}} &= 1 - 2\epsilon_1 - \epsilon_2 + \mathcal{O}(\epsilon^2) \\ r &= 16\epsilon_1 + \mathcal{O}(\epsilon^2) \end{aligned} \Rightarrow \begin{cases} \dfrac{\mathrm{d}\phi}{\mathrm{d}N} \simeq \pm\dfrac{r^{1/2}}{\sqrt{8}} \\ \dfrac{\mathrm{d}\ln V}{\mathrm{d}N} \simeq -\dfrac{r}{8}\left(1 + \dfrac{1 - n_{\mathrm{S}} - r/8}{6 - r/8}\right) \end{cases}$$

  - ✦ But M^4, ΔN_* have to be postulated, reheating consistency lost

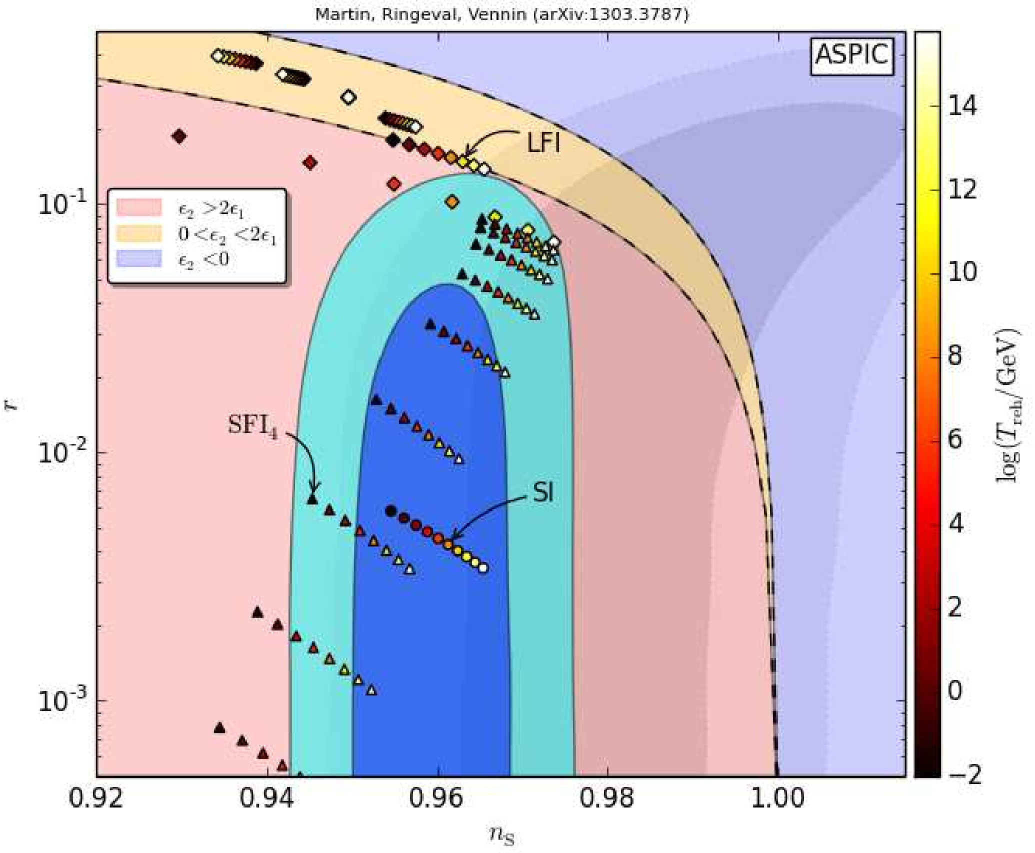
- A given parameterization = 1 model in ASPIC

Model predictions with ASPIC

- For all *Encyclopædia Inflationaris* models

$$\text{potential parameters} + \text{reheating} \longrightarrow \epsilon_{i*} \longrightarrow n_S, r, \alpha_S \ldots \text{(with consistency relations)}$$

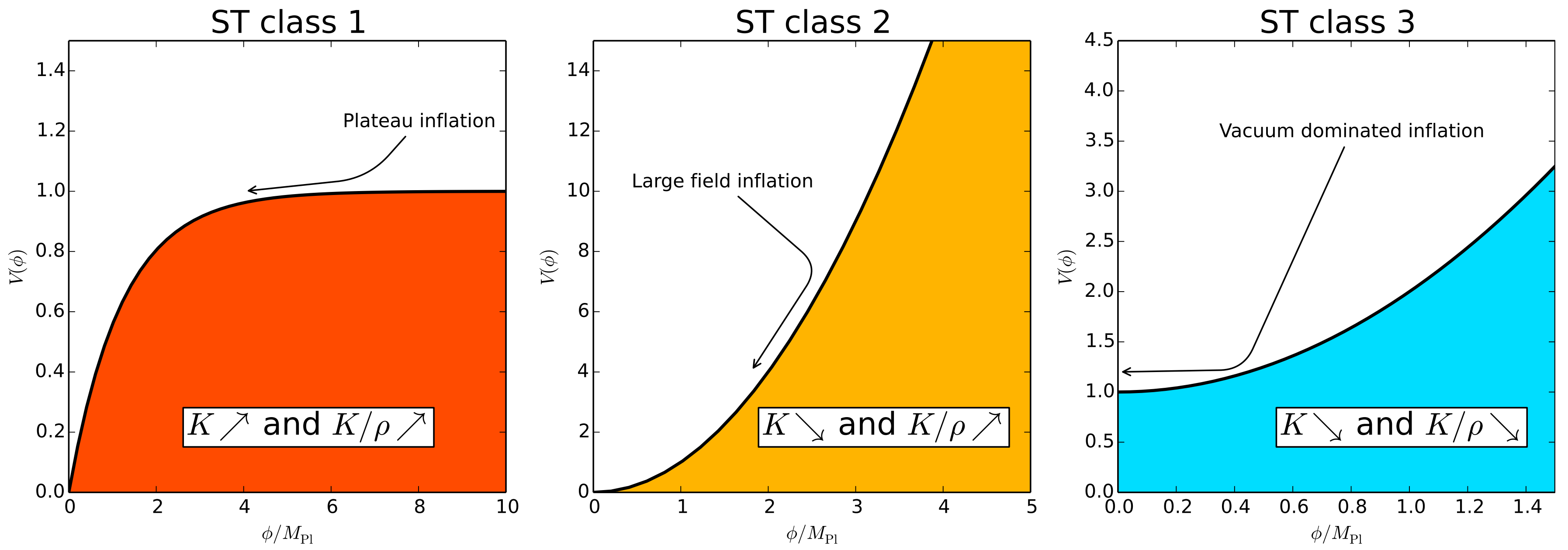
- Easy to check for which reheating history a model is compatible



Schwarz Terrero-Escalante classification

- Based on the relative energy evolution at the pivot scale (ϕ_*)

$$K = \frac{1}{2}\dot{\phi}^2 \qquad \rho = K + V \qquad P = K - V \simeq -\rho$$



- In terms of slow-roll parameters

$$\mathrm{ST1:}\ \epsilon_{2*} > 2\epsilon_{1*}, \quad \mathrm{ST2:}\ 0 < \epsilon_{2*} < 2\epsilon_{1*}, \quad \mathrm{ST3:}\ \epsilon_{2*} < 0$$

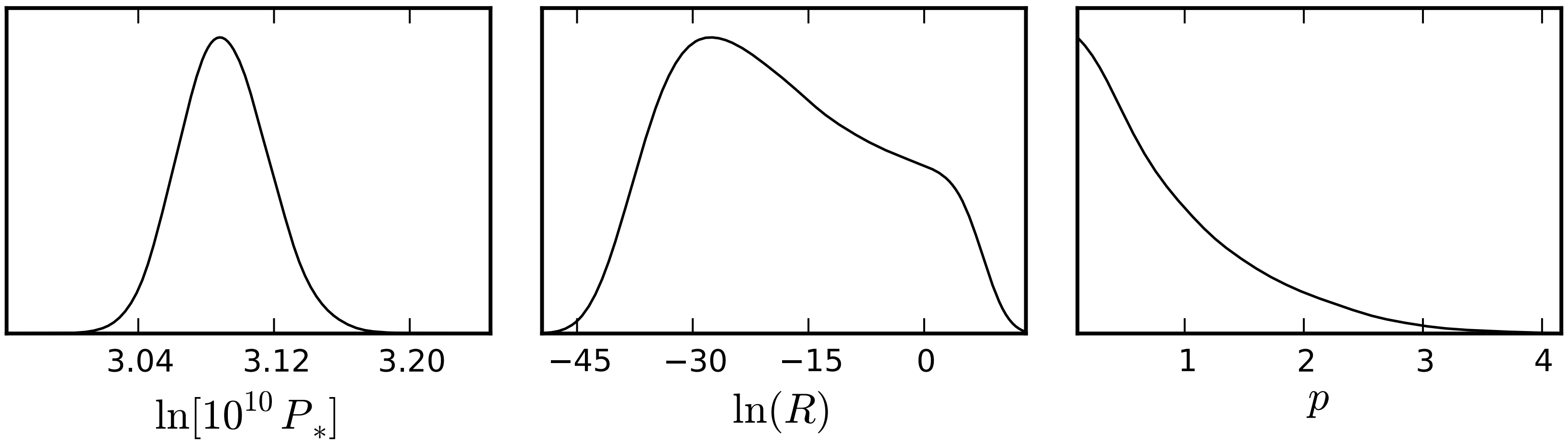
Data analysis in model space

Using the slow-roll approximation as a proxy

- To constrain the fundamental inflationary parameters: $\boldsymbol{\theta}_{\mathrm{inf}}$

$$(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}) \longrightarrow \mathtt{ASPIC} \longrightarrow \epsilon_{i*} \longrightarrow \begin{cases} \mathcal{P}_\zeta(k) \\ \mathcal{P}_h(k) \end{cases} \longrightarrow \mathtt{CAMB} \longleftrightarrow \text{CMB data}$$

- Example: Planck 2013 data analysis with LFI



- Confidence intervals are on the relevant parameters (95% CL)

$$p < 2.3, \qquad -37 < \ln R_{\mathrm{reh}} < 6$$

Bayesian model comparison

- Bayesian evidence
  - For each model $\mathcal{M}$, marginalisation over all parameters

$$\mathcal{E}(D|\mathcal{M}) = \int \mathrm{d}\boldsymbol{\theta}\mathcal{L}(\boldsymbol{\theta})\pi(\boldsymbol{\theta}|\mathcal{M})$$

  - Gives the posterior probability of $\mathcal{M}$ to explain the data D

$$p(\mathcal{M}|D) = \frac{\mathcal{E}(D|\mathcal{M})\pi(\mathcal{M})}{p(D)} \qquad \textit{where} \qquad p(D) = \sum_i \mathcal{E}(\mathcal{M}_i|D)\pi(\mathcal{M}_i)$$

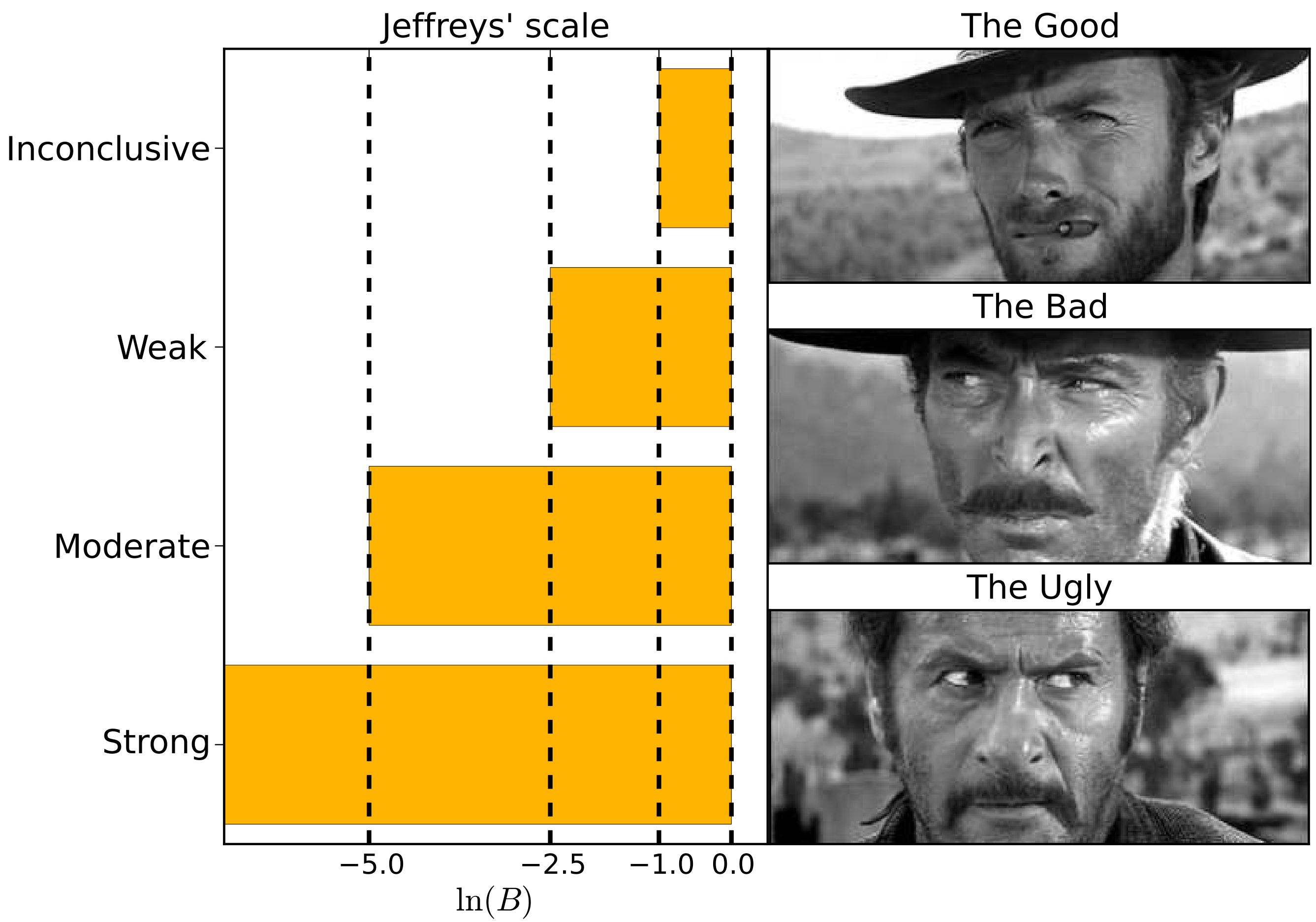
- Bayes' factor
  - Gives the posterior odds between $\mathcal{M}$ and a reference model $\mathcal{M}_0$

$$\frac{p(\mathcal{M}|D)}{p(\mathcal{M}_0|D)} = B\frac{\pi(\mathcal{M})}{\pi(\mathcal{M}_0)} \Rightarrow B = \frac{\mathcal{E}(D|\mathcal{M})}{\mathcal{E}(D|\mathcal{M}_0)}$$

  - Measure of how much the prior information has been updated

Jeffreys' scale

- Strength of evidence of $\mathcal{M}$ compared to $\mathcal{M}_0$



- ASPIC allows to fastly do that for all the *Encyclopædia Inflationaris* models

Speeding up evidence calculation

- Marginalisation over all parameters is numerically challenging!
- Effective likelihood for slow-roll inflation
  - Requires only one complete data analysis to get

$$\mathcal{L}_{\mathrm{eff}}(D|P_*, \epsilon_{i*}) = \int p(D|\boldsymbol{\theta}_{\mathrm{cosmo}}, P_*, \epsilon_{i*})\pi(\boldsymbol{\theta}_{\mathrm{cosmo}})\mathrm{d}\boldsymbol{\theta}_{\mathrm{cosmo}}$$

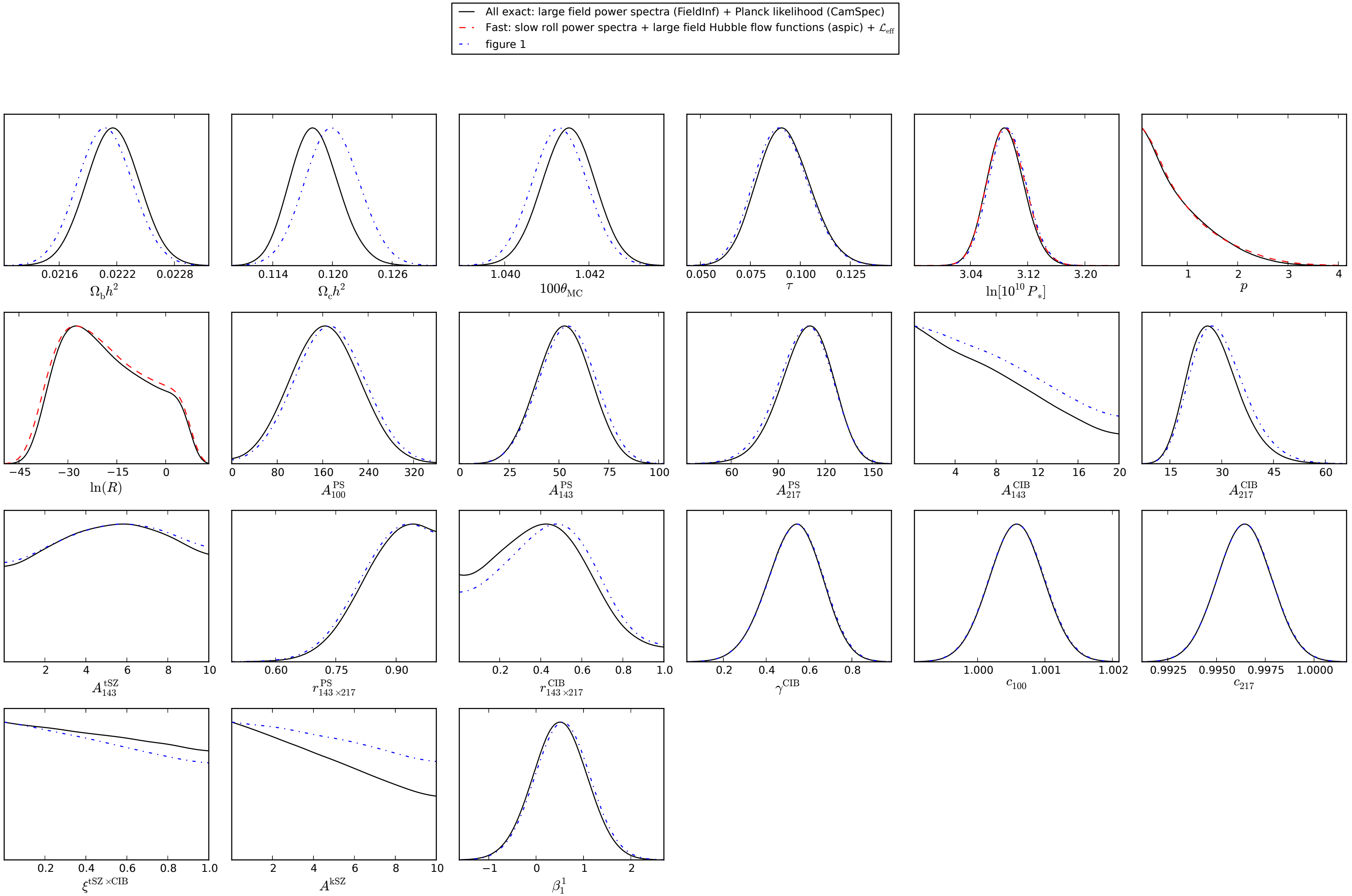
  - Use machine-learning algorithm to fit its multidimensional shape
  - For each model $\mathcal{M}$ and their parameters $\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}$

$$p(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}|D, \mathcal{M}) = \frac{\mathcal{L}_{\mathrm{eff}}[D|P_*(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}), \epsilon_{i*}(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}})]\pi(\boldsymbol{\theta}_{\mathrm{inf}}, R_{\mathrm{reh}}|\mathcal{M})}{p(D|\mathcal{M})}$$

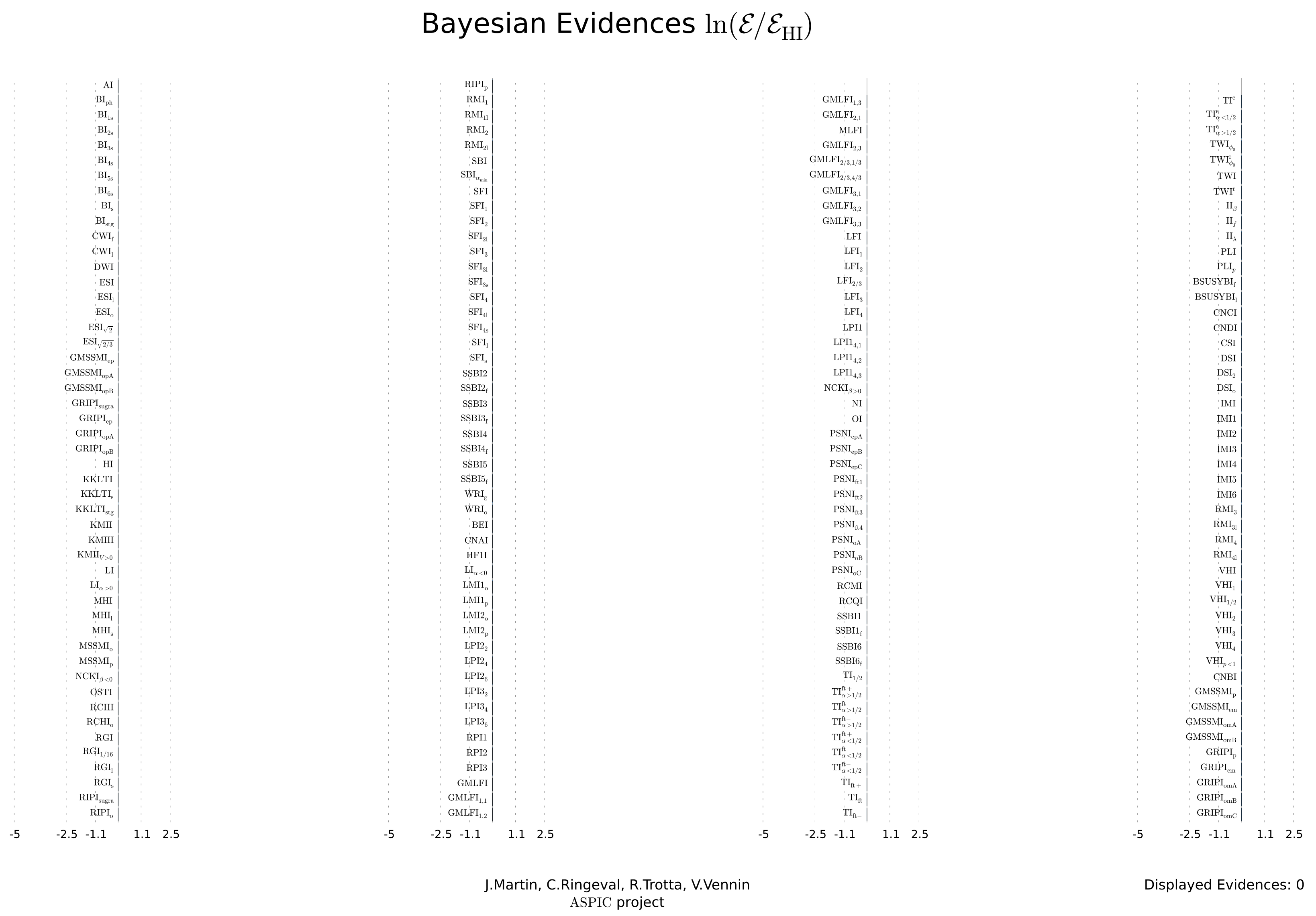
- All posteriors and evidences can be obtained by integrating $\mathcal{L}_{\mathrm{eff}}$
- In practice: ASPIC + MultiNest + $\mathcal{L}_{\mathrm{eff}}$ = 1 hour per model

Accuracy of ASPIC + effective likelihood

- First order quantities marginalized over second order

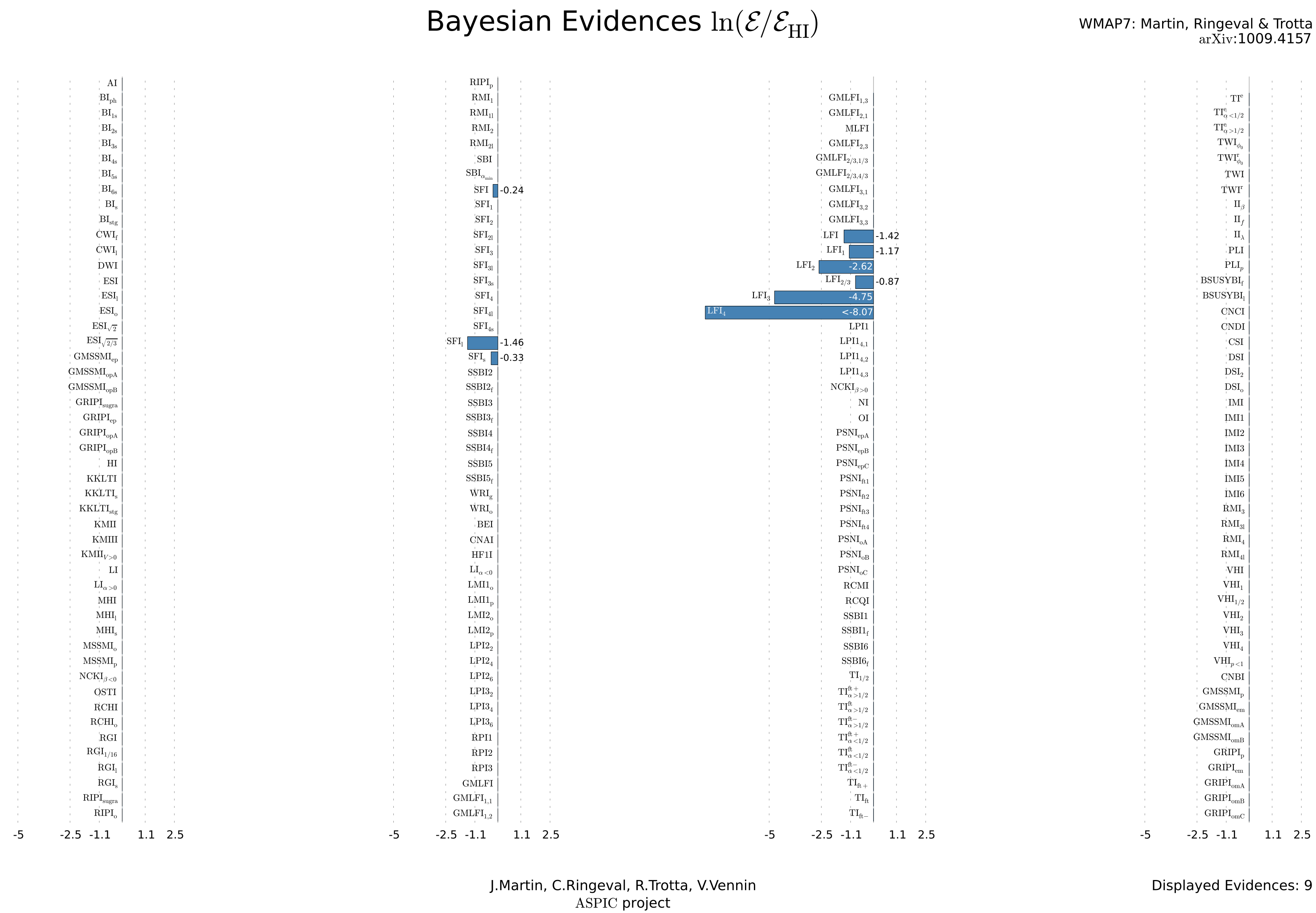


Bayes factor for hundred of models



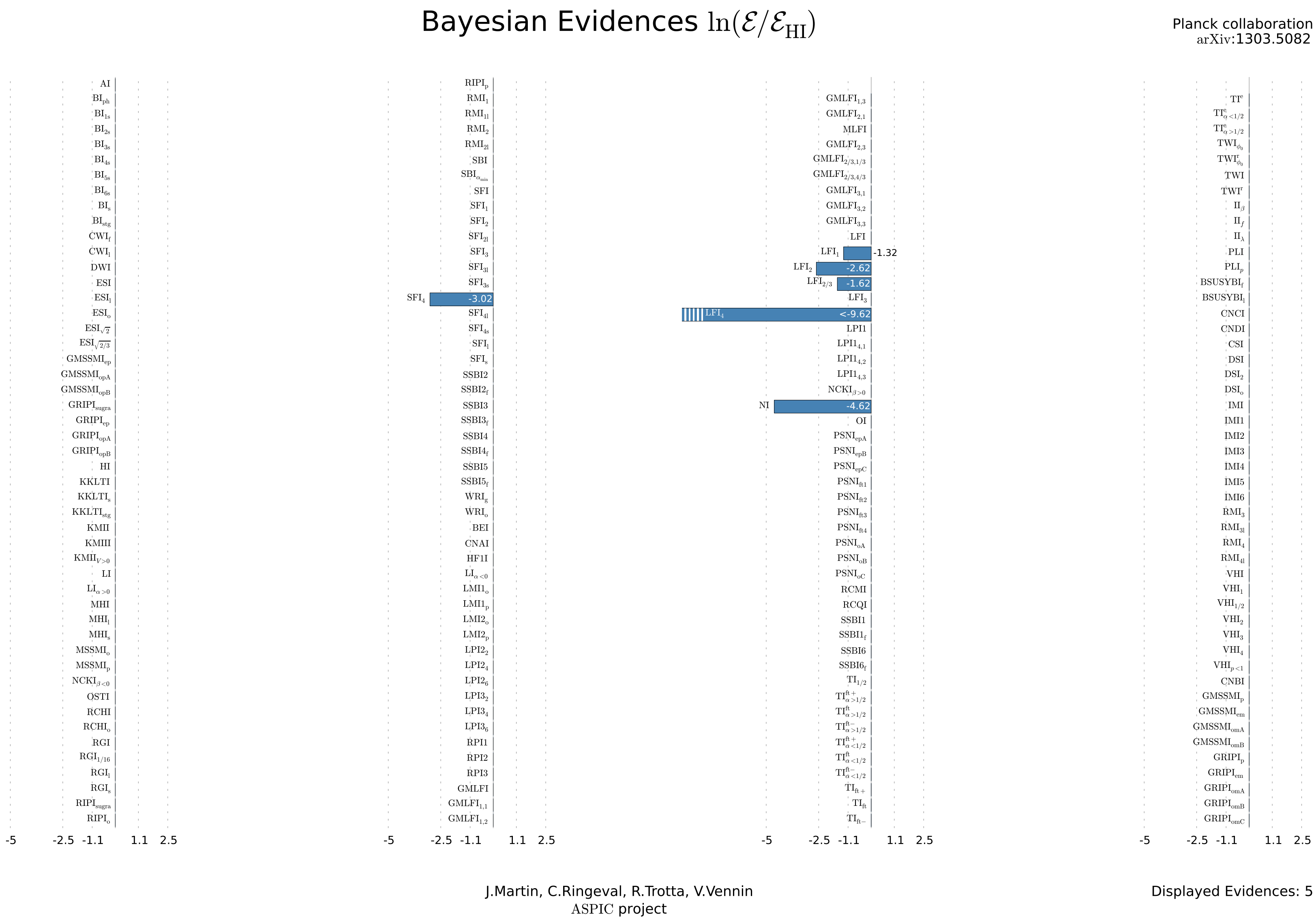
Bayes factor for hundred of models

WMAP7, arXiv:1009.4157



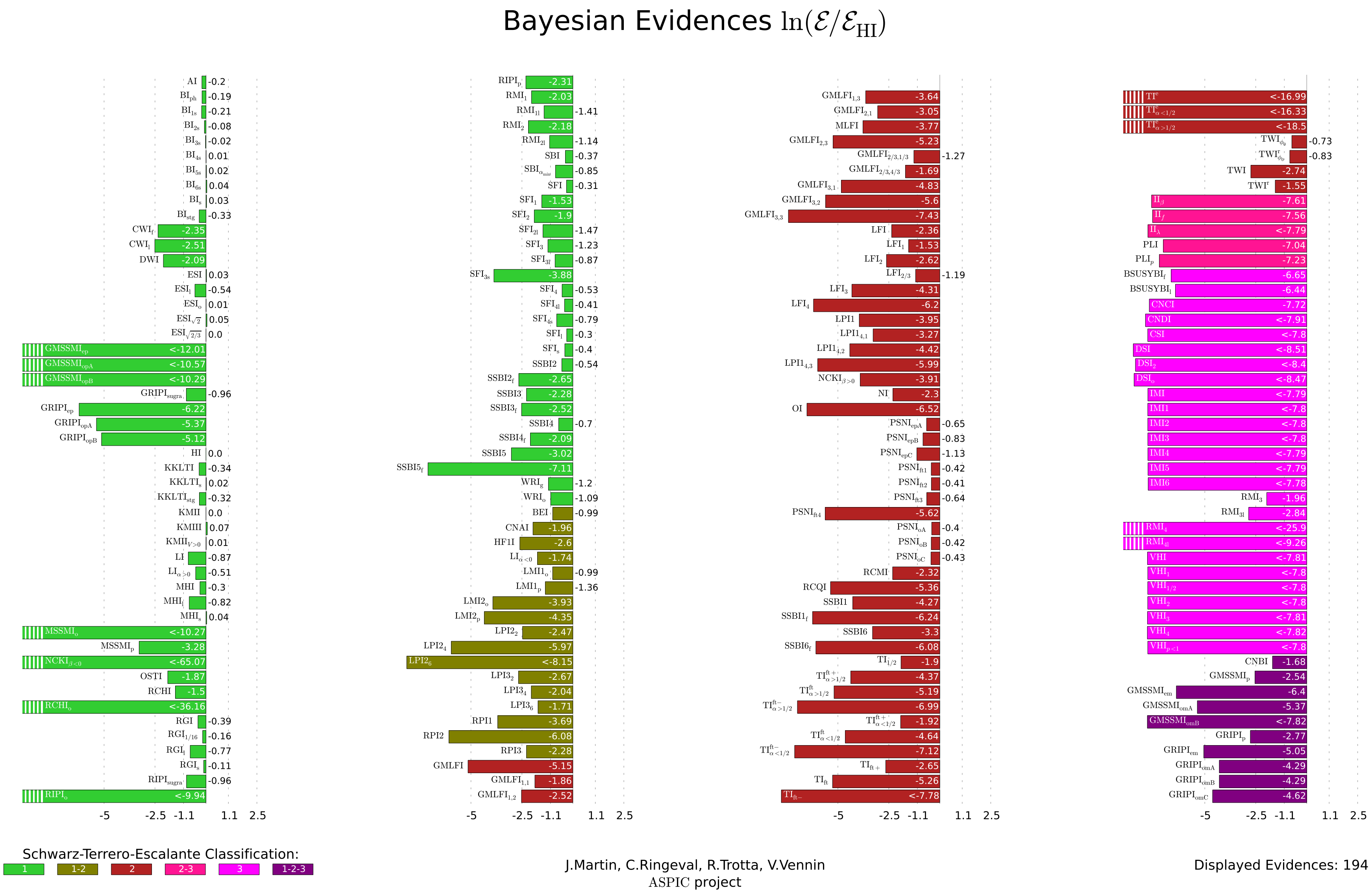
Bayes factor for hundred of models

Planck 2013, arXiv:1303.5082



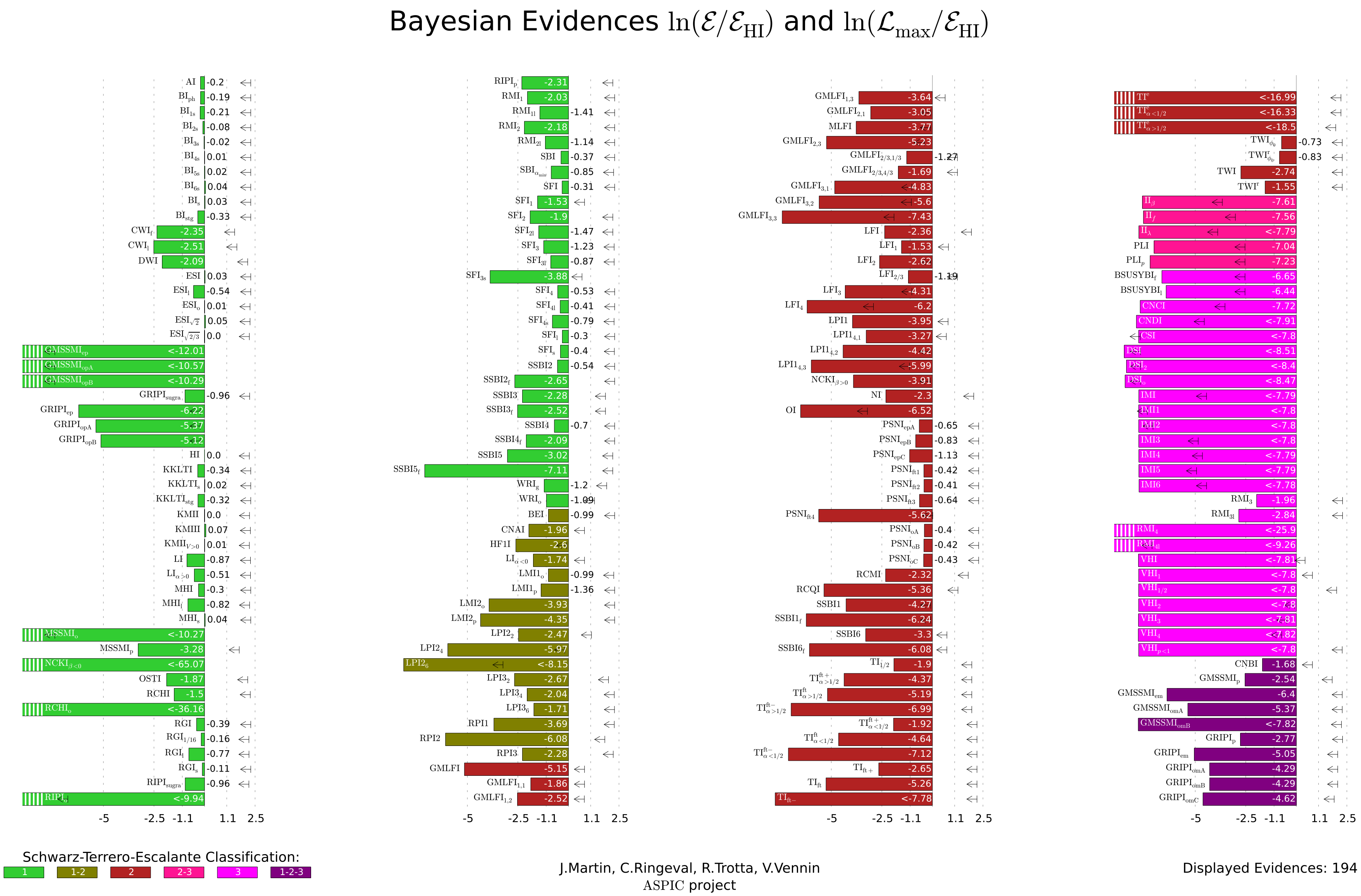
Bayes factor for hundred of models

Planck 2013, arXiv:1312.3529



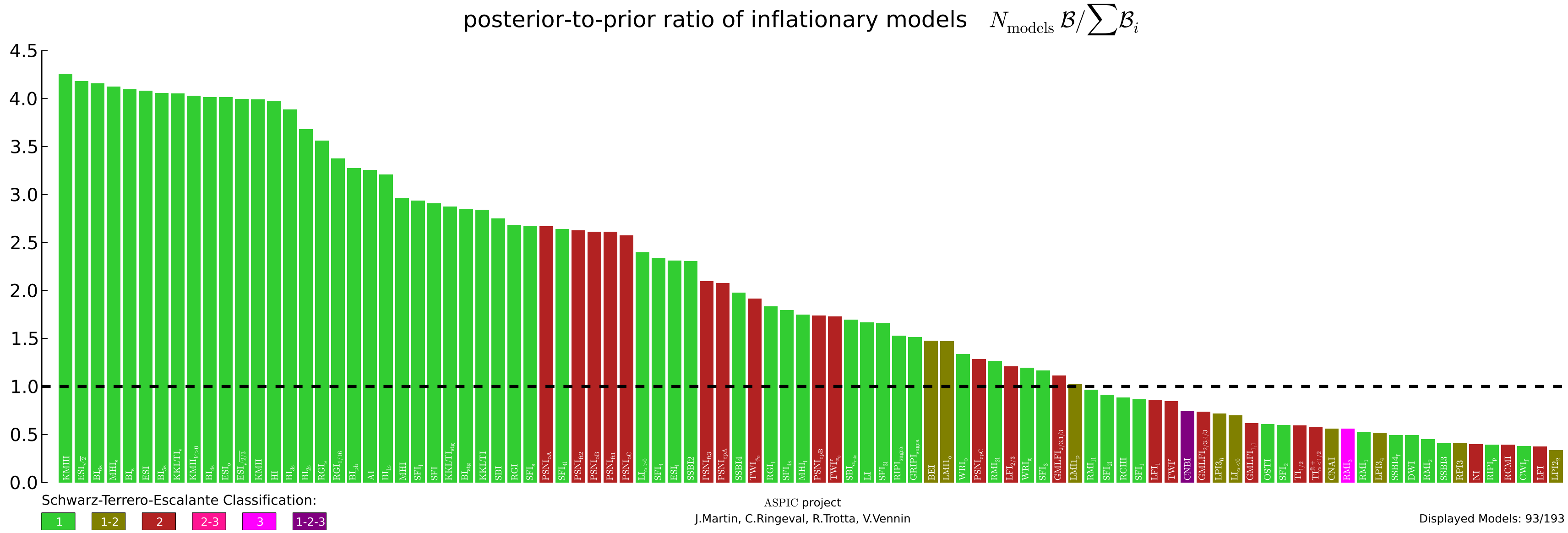
Bayes factor for hundred of models

Planck 2013, arXiv:1312.3529



And the winners are. . .

- From non-commital priors: $\pi(\mathcal{M}) = 1/N_{\rm model}$
- Posterior-to-prior ratio: Planck 2013



- Some numbers
  - ✦ 52 models are in the inconclusive region "Some Good": AI, BI, ESI, HI, KKLTI, KMII, KMIII, LI, MHI, PSNI, RGI, SBI, SFI, SSBI2, TWI
  - ✦ 66 models are strongly disfavoured (some "Bad" others "Ugly")

Narrowing down the simplest with complexity

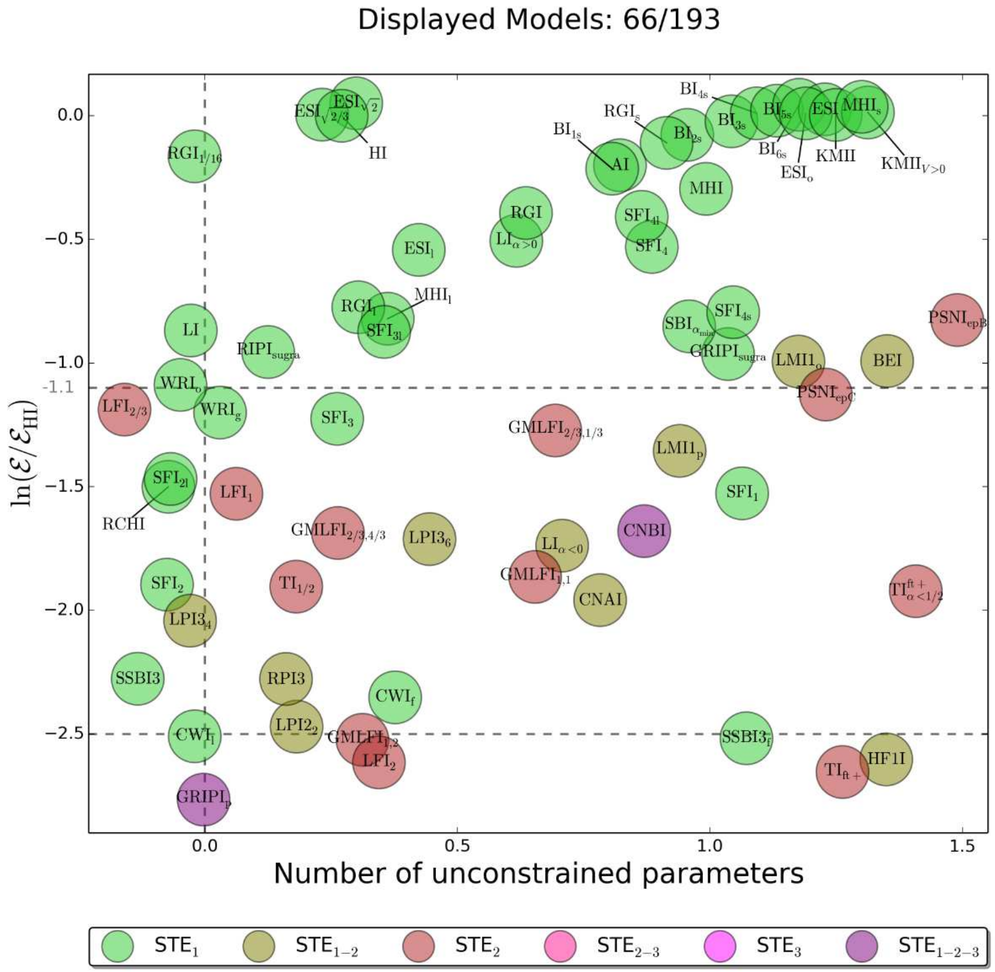
- Bayesian complexity $\simeq$ the number of constrained parameters

$$\mathcal{C} = \langle -2\ln\mathcal{L}\rangle + 2\ln\mathcal{L}_{\max} \quad \Rightarrow \quad N_{\text{unconstrained}} = N_{\text{param}} - \mathcal{C}$$

- Planck 2013

  arXiv:1312.3529

  For the most probable and simplest scenarios $\longrightarrow$



Planck constraints on reheating

Posteriors on the reheating parameter

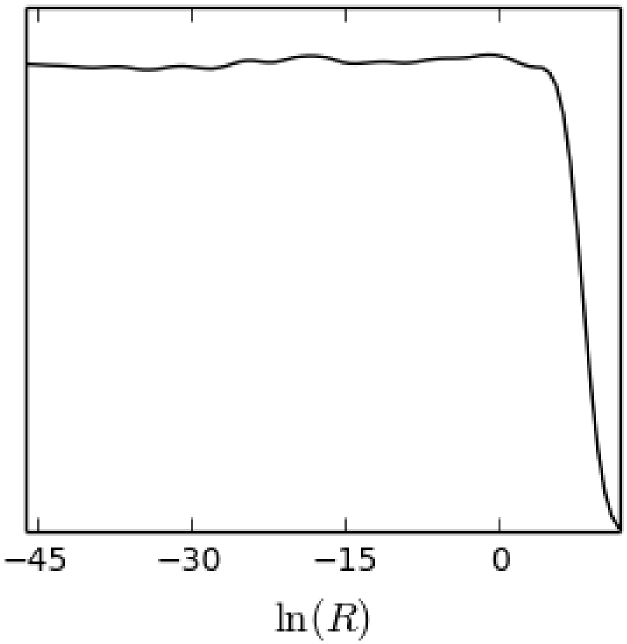
- For each model, we use the most generic parameterization: $R_{\rm reh}$
  - ✦ Prior choice: Jeffreys' on $R_{\rm reh} \Leftrightarrow$ flat on $\ln R_{\rm reh}$ with:

$$-46 < \ln R_{\rm reh} < 15 + \frac{1}{3}\ln \rho_{\rm end}$$

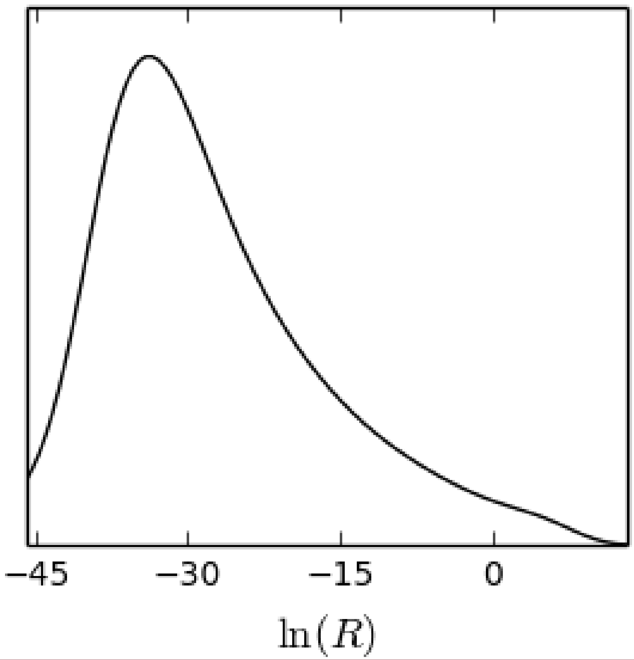
  - ✦ Planck 2013 data put non-trivial constraints on many models

- Examples: LI with $V(\phi) = M^4 (1 + \alpha \ln \phi)$

prior



posterior



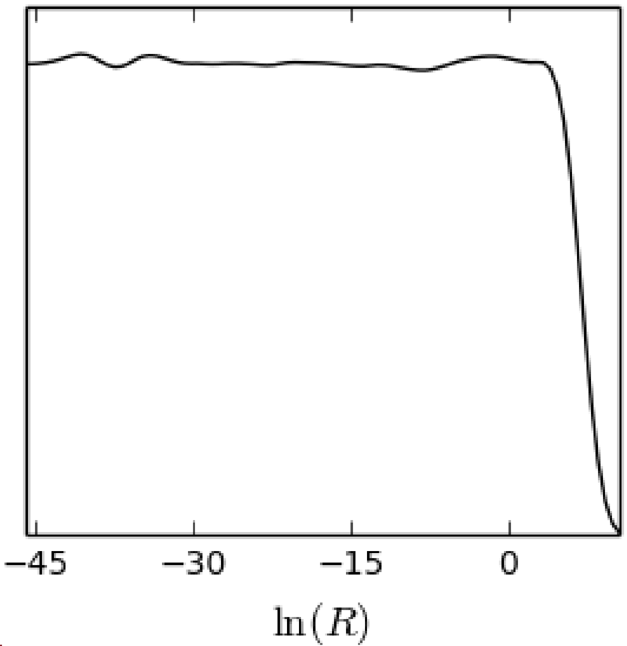
Posteriors on the reheating parameter

- For each model, we use the most generic parameterization: R_{reh}
  - Prior choice: Jeffreys' on $R_{\mathrm{reh}} \Leftrightarrow$ flat on $\ln R_{\mathrm{reh}}$ with:

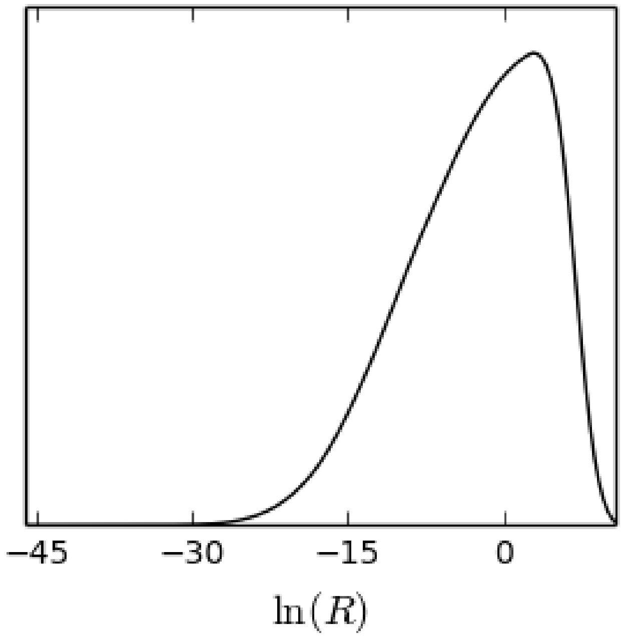
$$-46 < \ln R_{\mathrm{reh}} < 15 + \frac{1}{3}\ln \rho_{\mathrm{end}}$$

  - Planck 2013 data put non-trivial constraints on many models
- Examples: SBI with $V(\phi) = M^4\left[1 + \phi^4\left(-\alpha + \beta \ln \phi\right)\right]$

prior



posterior



Prior-to-posterior width ratio

- Reheating is constrained $\Leftrightarrow$ posterior of $\ln R_{\mathrm{reh}}$ is peaked
  - The most probable value of R_{reh} is model-dependent
  - We introduce the ratio between the prior and posterior standard deviation of $\ln R_{\mathrm{reh}}$

$$\left.\frac{\Delta\pi_{\ln R_{\mathrm{reh}}}}{\Delta\mathcal{P}_{\ln R_{\mathrm{reh}}}}\right|_{\mathcal{M}} = \sqrt{\frac{\int \left(\ln R_{\mathrm{reh}} - \langle \ln R_{\mathrm{reh}}\rangle_\pi\right)^2 \pi(\ln R_{\mathrm{reh}}|\mathcal{M})\mathrm{d}\ln R_{\mathrm{reh}}}{\int \left(\ln R_{\mathrm{reh}} - \langle \ln R_{\mathrm{reh}}\rangle_p\right)^2 p(\ln R_{\mathrm{reh}}|D,\mathcal{M})\mathrm{d}\ln R_{\mathrm{reh}}}}$$

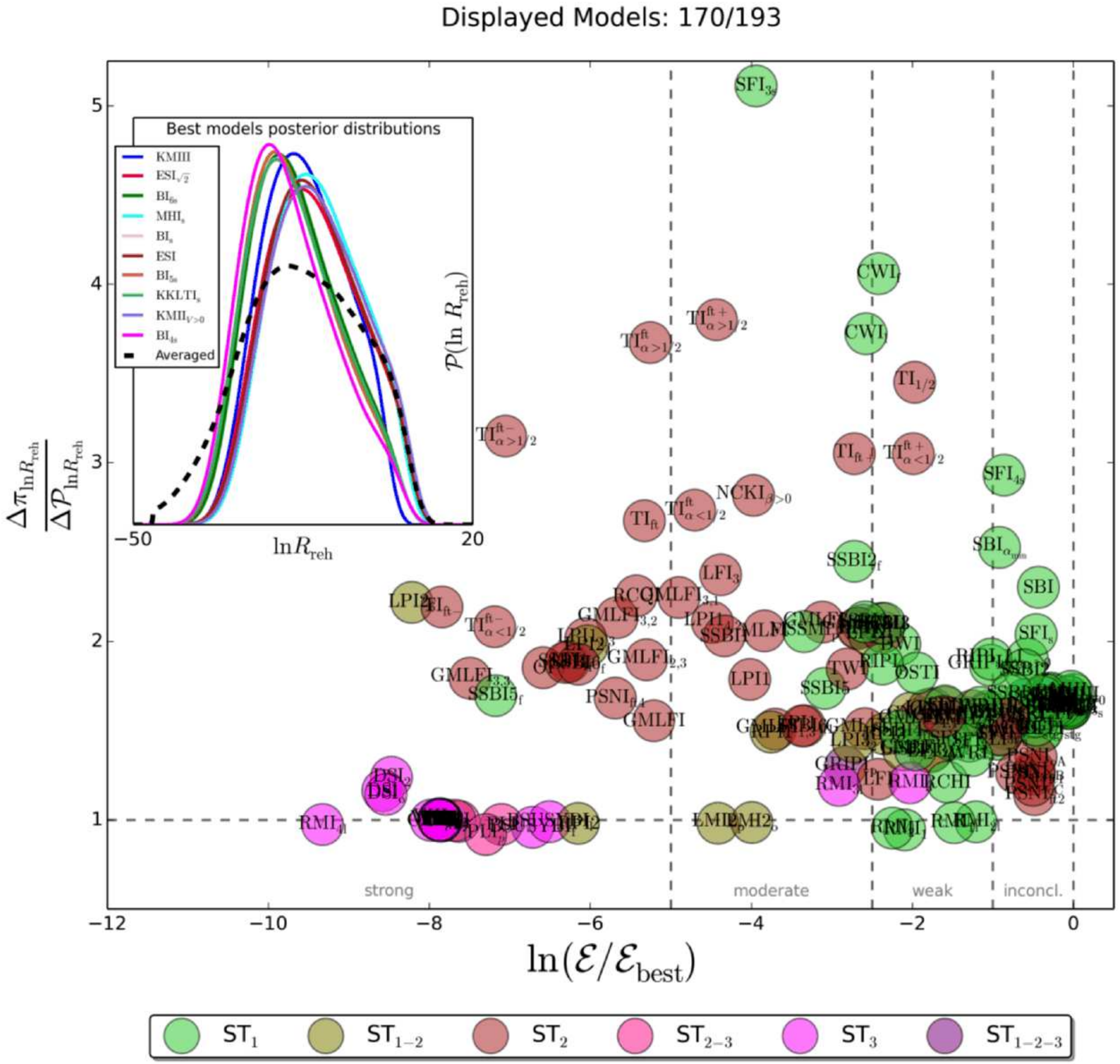
- Disfavoured models exhibit larger values for $\Delta\pi_{\ln R_{\mathrm{reh}}}/\Delta\mathcal{P}_{\ln R_{\mathrm{reh}}}$
  - In the space of models, a fair estimate of the Planck's constraining power on reheating is

$$\left\langle \frac{\Delta\pi_{\ln R_{\mathrm{reh}}}}{\Delta\mathcal{P}_{\ln R_{\mathrm{reh}}}} \right\rangle \equiv \sum_{\mathcal{M}_i} p(\mathcal{M}_i|D) \left.\frac{\Delta\pi_{\ln R_{\mathrm{reh}}}}{\Delta\mathcal{P}_{\ln R_{\mathrm{reh}}}}\right|_{\mathcal{M}_i}$$

- For Planck 2013: $\left\langle \frac{\Delta\pi_{\ln R_{\mathrm{reh}}}}{\Delta\mathcal{P}_{\ln R_{\mathrm{reh}}}} \right\rangle \simeq 1.66 \implies$ prior cut by 40%

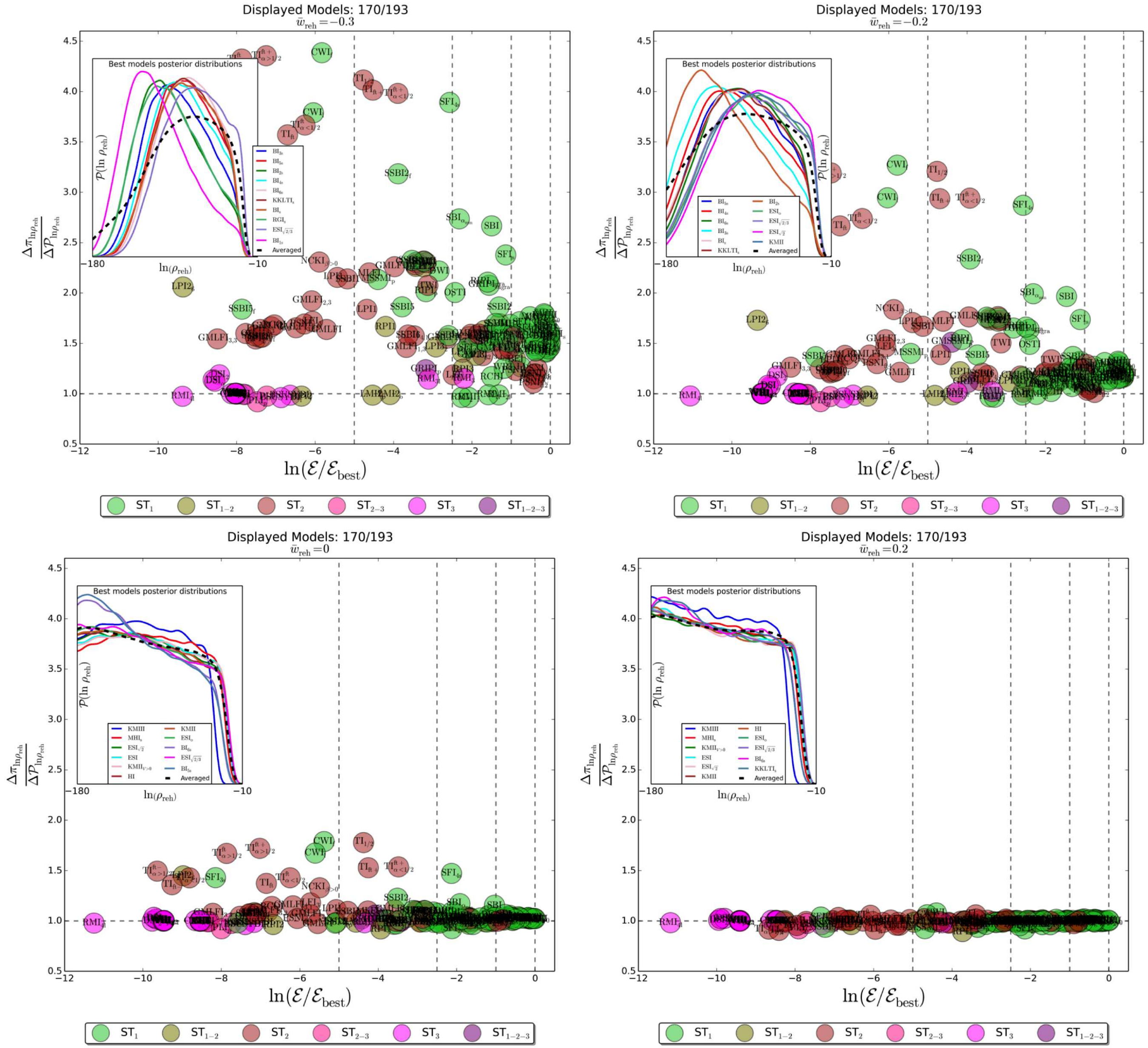
Reheating constraints versus evidence

- No assumption on reheating (= using $R_{\rm reh}$)



Reheating constraints versus evidence

- Assuming the equation of state $\overline{w}_{\mathrm{reh}}$ to be fixed



Perspective

- Novel and efficient approach applicable to any cosmological data set
  - Reheating is included and already constrained by Planck 2013
  - Provides new insights in the most difficult to disambiguate situation: slow-roll inflation

Perspective

- Novel and efficient approach applicable to any cosmological data set
  - Reheating is included and already constrained by Planck 2013
  - Provides new insights in the most difficult to disambiguate situation: slow-roll inflation
- After Planck 2014?

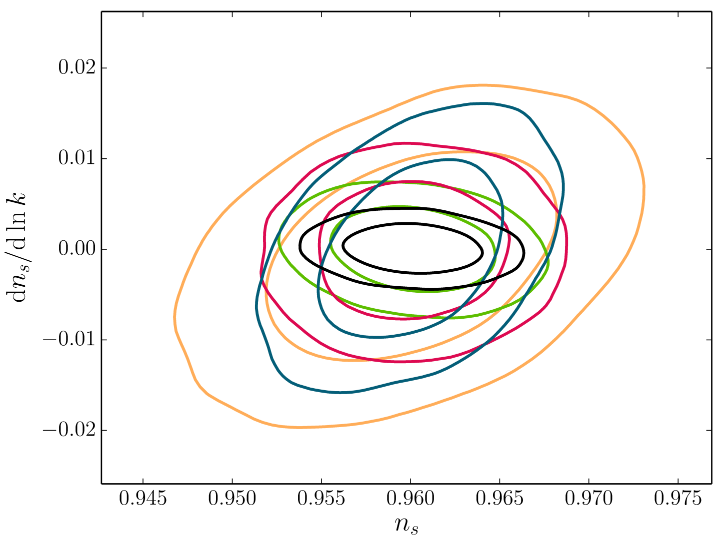
Perspective

- Novel and efficient approach applicable to any cosmological data set
  - Reheating is included and already constrained by Planck 2013
  - Provides new insights in the most difficult to disambiguate situation: slow-roll inflation
- After Planck 2014?
  - Future CMB missions: See V. Vennin's talk

Perspective

- Novel and efficient approach applicable to any cosmological data set
  - Reheating is included and already constrained by Planck 2013
  - Provides new insights in the most difficult to disambiguate situation: slow-roll inflation
- After Planck 2014?
  - Future CMB missions: See V. Vennin's talk
  - Galaxy surveys: Euclid

From Basse et al., `arXiv:1409.3469`



Courtesy of S. Clesse (in prep.)