Effective field theory for inflation

And to what extent can we push EFT in UV?

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)
CQG (2013), Phys. Rev. D (gr-qc/1408.620)

Einstein's GR is well behaved in IR, but in UV there are corrections which are essential for cosmology

Many have contributed

Born, Enfeld, Utiyama, Efimov, Pias, Tseytlin, Siegel,

Grisaru, Biswas, Krasnov, Ashtekar, Nicolai, Anselmi,

DeWitt, Desser, Stelle, Witten, Sen, Zwiebach,

Kostelecky, Samuel, Frampton, Okada, Olson, Freund,

Tomboulis, Talaganis, Khoury, Modesto, Bravisnky,

Koivisto, Cline, Barnaby, Kamran, Woodard, Vernov,

Kapusta, Daffayet, Arefeva, Dvali, Arkani-Hamed,

Koshelev, Conroy, Craps, Sagnotti, Mainheim, Rubakov,

Wetterich …

Inflation + Curvaton

There are many fundamental assumptions we make, which we do not justify ...

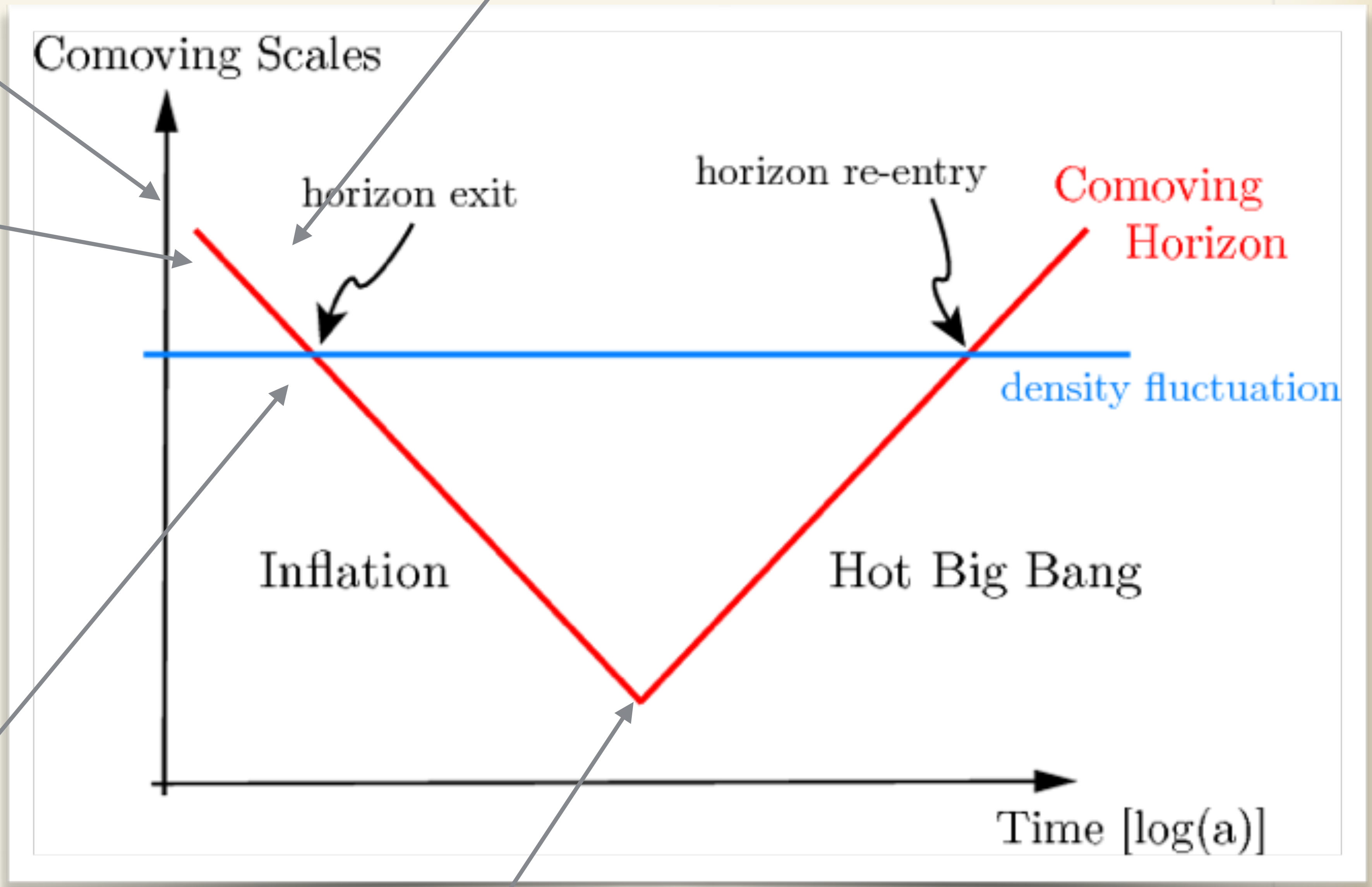
Questions & Assumptions:

within my causal patch

Why Inflationary patch is homogeneous & isotropic ?

Is there a dynamical attractor solution?

Whether EFT is valid close to the cut-off too ?



Assuming Validity of EFT produces data, i.e. slow roll inflation

How do we produce the SM dof ?

Our key to understand the Early Universe Physics

Only 3 models which can possibly connect to phenomenology

Starobinsky

Needs UV completion
+
Need to specify the visible sector

Higgs

Inlaton VEV above the cut-off
+
EFT is invalid
+
Needs UV completion in gravity ==> Gravity should be scale invariant

MSSM type inflation

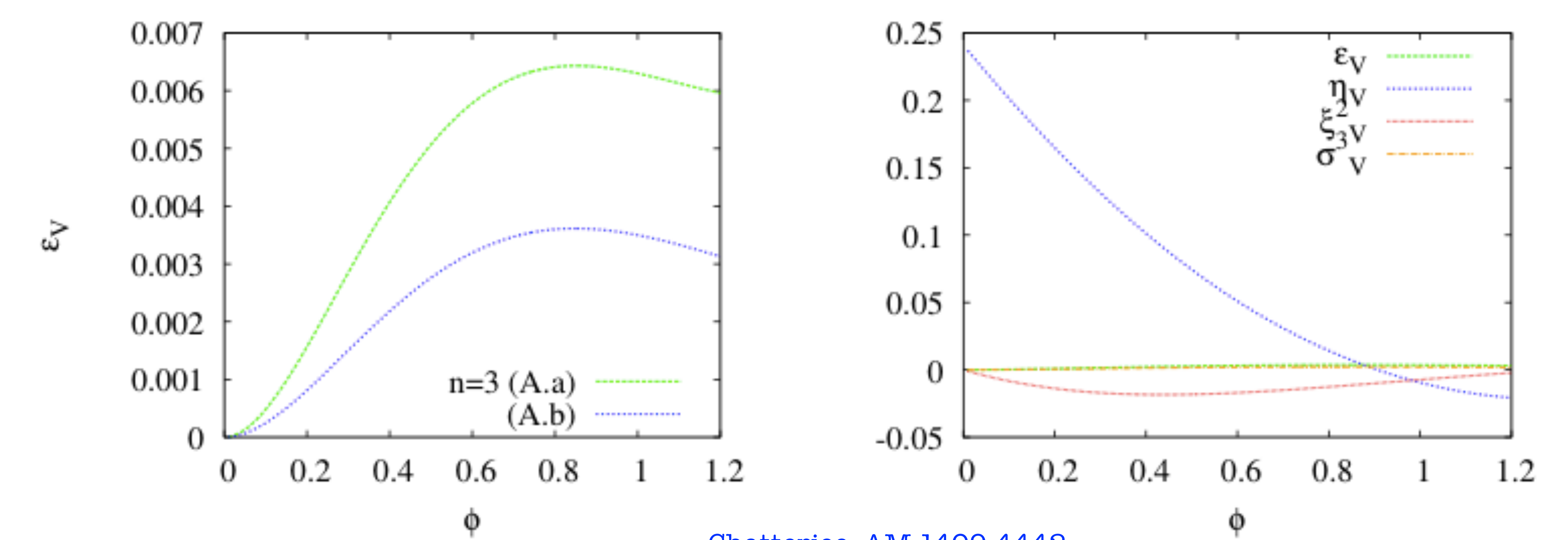
Sub-Plankian inflation
+
EFT is under control
+
Requires SUSY
+
Low scale SUSY requires fine tuning
+
High scale SUSY or SUGRA corrections reduce fine tuning

I am biased, because I am alive, and I am made up of SM dof.

Largest 'r' for sub-Planckian Inflation?

$$V = V_0 + A\phi^2 - B\phi^n + C\phi^{2n-2}$$

$$V(\phi) \approx V_0 + V'(\phi-\phi_0) + V''(\phi-\phi_0)^2 + V'''(\phi-\phi_0)^3 + V''''(\phi-\phi_0)^4 + \cdots$$



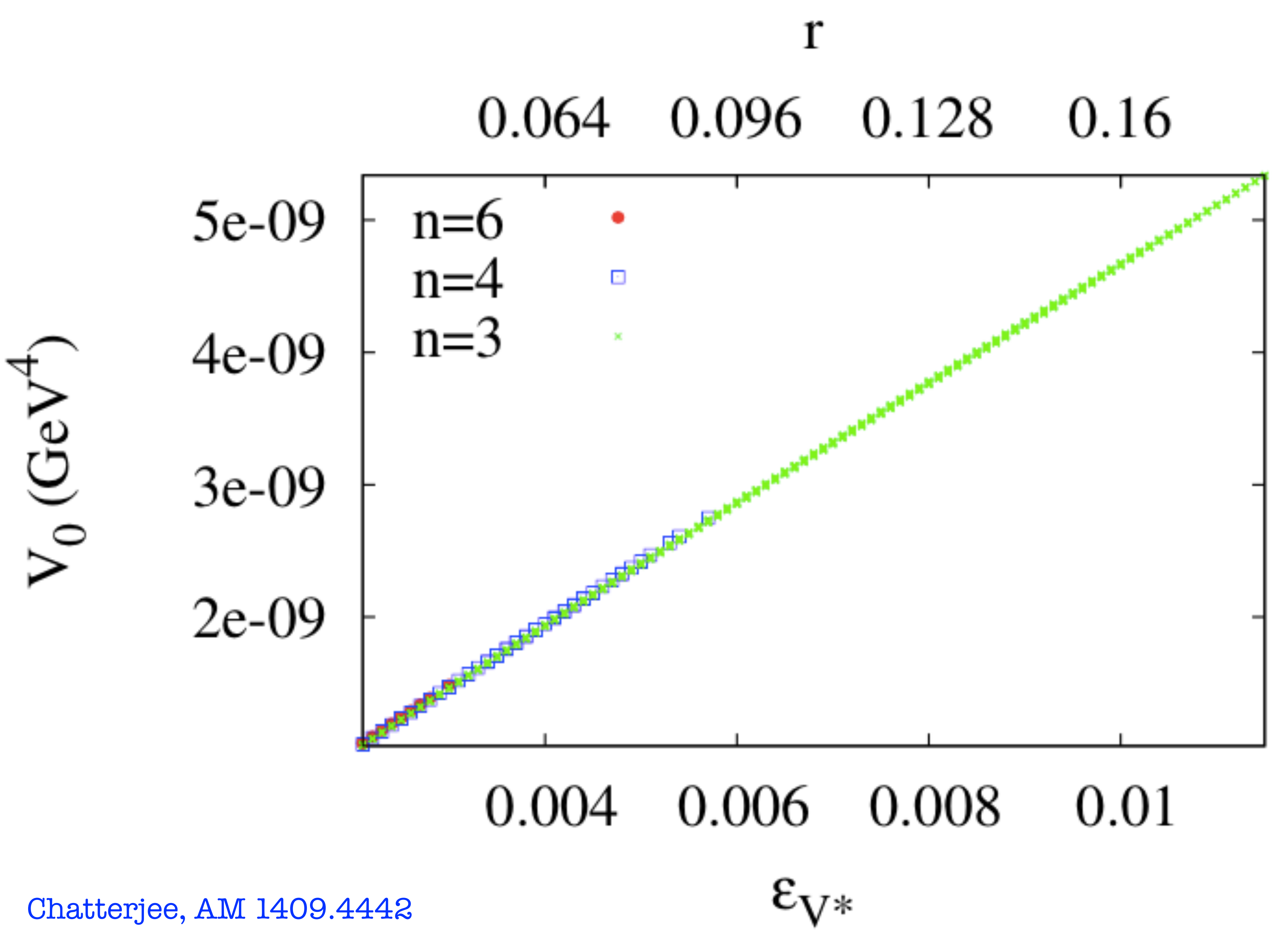
Chatterjee, AM 1409.4442

Slow roll parameter evolves non-monotonically

Inflection point inflation

Largest r for sub-Planckian Inflation?

$$V = V_0 + A\phi^2 - B\phi^n + C\phi^{2n-2}$$



Chatterjee, AM 1409.4442

$$n = 3$$

$$\phi = \frac{\widetilde{N} + H_u + \widetilde{L}}{\sqrt{3}}$$

$$n = 4$$

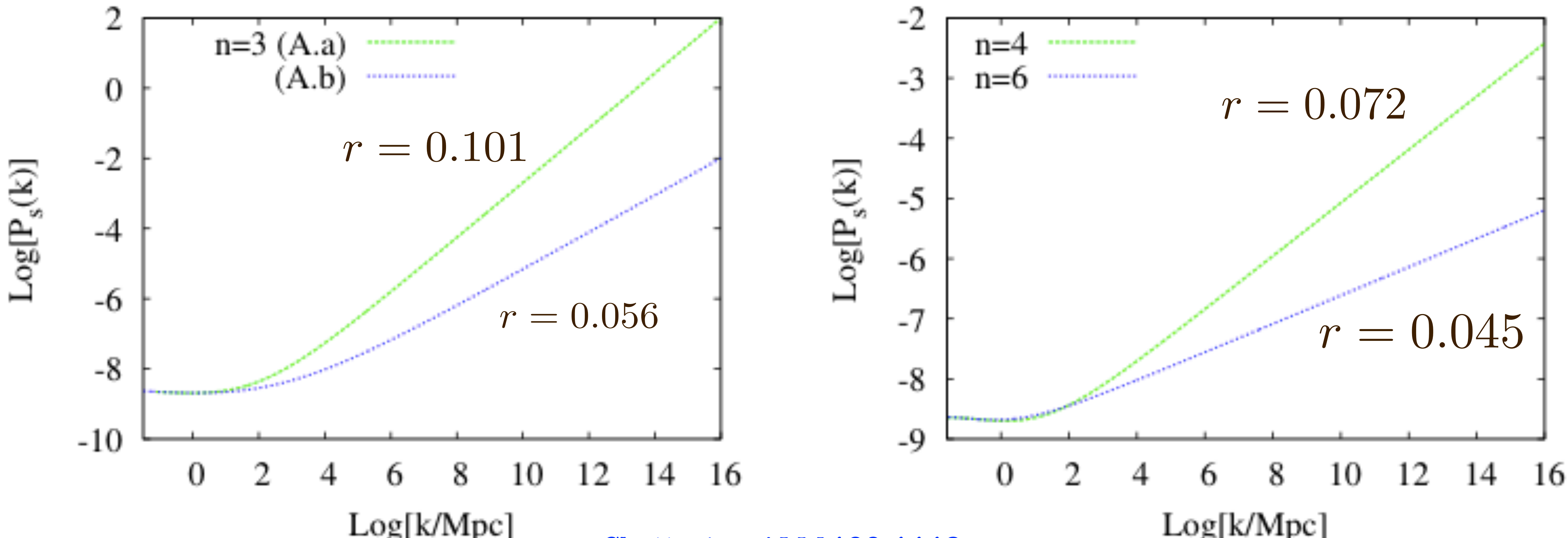
$$\phi = \frac{H_u + H_d}{\sqrt{2}}$$

$$n = 6$$

$$\phi = \frac{\widetilde{u} + \widetilde{d} + \widetilde{d}}{\sqrt{3}}$$

$$\phi = \frac{\widetilde{L} + \widetilde{L} + \widetilde{e}}{\sqrt{3}}$$

Largest r for sub-Planckian Inflation?

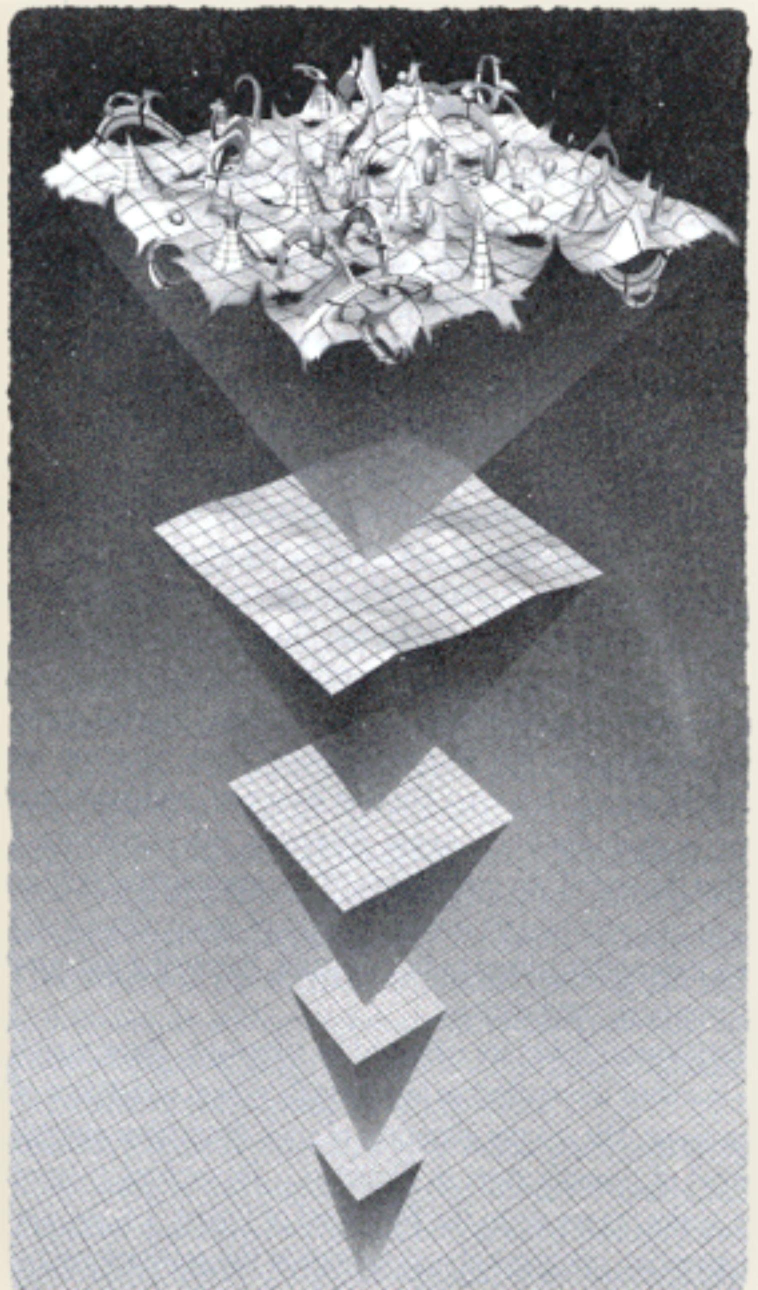


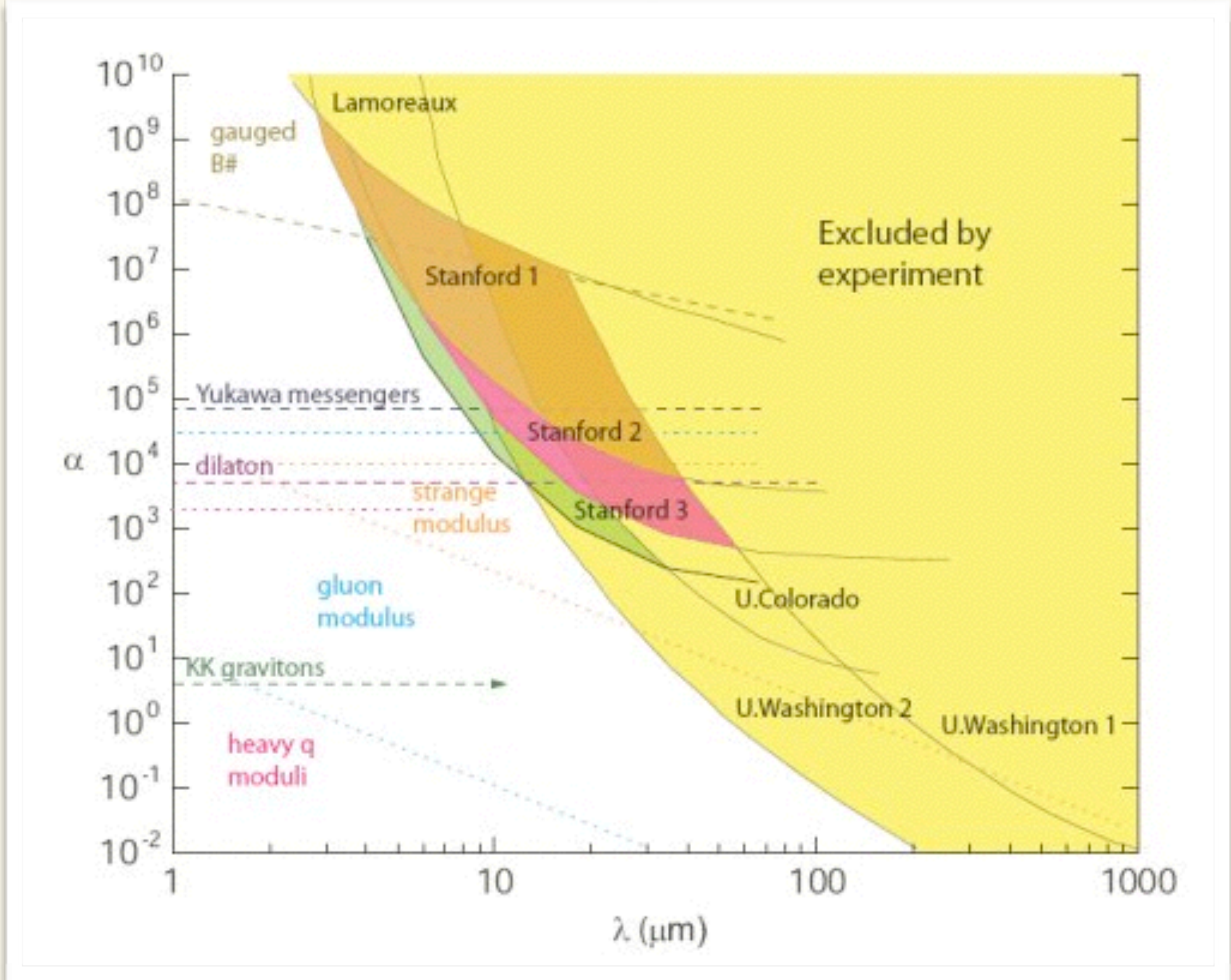
Chatterjee, AM 1409.4442

	n	V_0	A	B	C	r
(A.a)	3	3.010×10^{-9}	5.114×10^{-10}	3.126×10^{-10}	7.075×10^{-11}	0.101
(A.b)	3	1.710×10^{-9}	2.061×10^{-10}	1.165×10^{-10}	2.248×10^{-11}	0.056
(B)	4	2.184×10^{-9}	2.238×10^{-10}	8.301×10^{-11}	1.778×10^{-11}	0.072
(C)	6	1.388×10^{-9}	7.884×10^{-11}	1.096×10^{-11}	1.712×10^{-12}	0.045

It would be hard to make: r> 0.1 for sub-Planckian

In UV there are many scales





To what extent Einstein's Gravity is a good description of nature ?

4th Derivative Gravity & Power Counting renormalisability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k\, R + a\, R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}\, (b + a) R^2 \right]$$

$$D \propto \frac{1}{k^4 + A k^2} = \frac{1}{A} \left(\frac{1}{k^2} - \frac{1}{k^2 + A} \right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m"(No-Tachyon)

$$S = \int d^4x \; \phi\Box(\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\Box e^{-\Box}\phi = 0$$

No extra states other than the original dof.

Higher Derivative Action around Minkowski

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}\,[R_{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + \cdots]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad R \sim \mathcal{O}(h)$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Unknown Infinite Functions of Covariant Derivatives

Redundancies

$$
\begin{aligned}
S_q \;=\; \int d^4x\sqrt{-g}[& RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R^\nu_\mu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
+ & R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ & R^\rho_\lambda F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ & R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
+ & R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
\end{aligned}
$$

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$\Delta\mathcal{L} = \sqrt{-g}\left(\alpha R^2 + \beta R^2_{\mu\nu} + \gamma R^2_{\alpha\beta\mu\nu}\right)$$

$$\int \mathrm{d}^4x\sqrt{-g}(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\alpha\beta})$$

Gauss-Bonet Gravity

$$= \int d^4x \sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x\Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h^\sigma_\mu b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$

$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$

$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$

$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$

$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$

$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]})$$

$$R_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu})$$

$$R = \partial_\nu\partial_\mu h^{\mu\nu} - \Box h$$

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

$\mathcal{F}_3(\Box)$ is redundant around Minkowski

Impose Three Conditions

M_p

General Covariance

M $$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \cdots \right)$$

Smooth IR limit

Tree-level Unitarity

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

As a starting point these 3 conditions are sufficient

Fixing the Action up to quadratic in curvature around Minkowski, one can go beyond quadratic also

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$\Pi = \frac{1}{a(k^2)}\left[\frac{P^2}{k^2} - \frac{P^0}{2k^2}\right] = \frac{1}{a(k^2)} \times \text{Propagator of GR}$$

Should be analytic function and should not contain any extra pole

"Entire Function" e.g.

$$a(\Box) = e^{\Box/M^2}$$

Simplified action (dream will be to push for renormalisability, but it is very challenging ...)

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

↓

$$S = \int d^4x\, \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$

Applications

Black Hole Singularity, i.e. Schwarzschild Type

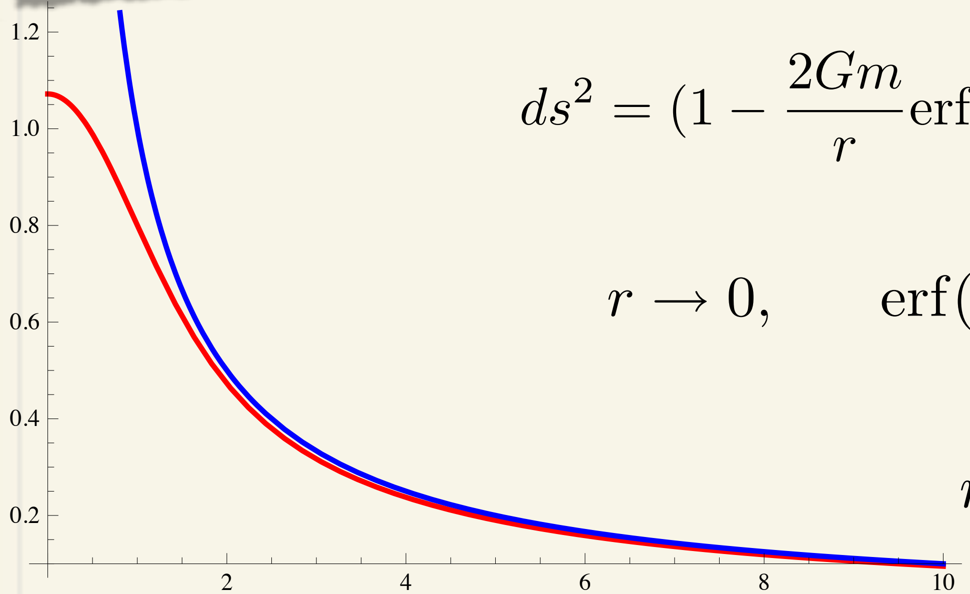
Biswas, Gerwick, Koivisto, AM, PRL (2012)
(gr-qc/1110.5249)

Cosmological Singularity, i.e. Big Bang Type

Biswas, AM, Siegel, JCAP (hep-th/0508194),
Brandenberger, Biswas, AM, Siegel, JCAP (hep-th/0610274)
Biswas, AM, Koivisto, JCAP (1005.0590)

Ameliorating the blackhole & cosmological singularity

$$S=\int d^4x\,\sqrt{-g}\left[\frac{R}{2}+R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R-2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}}-1}{\Box}\right]R^{\mu\nu}\right]$$



$$ds^2=(1-\frac{2Gm}{r}\mathrm{erf}\,(rM/2))dt^2-\frac{dr^2}{(1-\frac{2Gm}{r}\mathrm{erf}\,(rM/2))}$$

Valid for $mM \leq M_p^2$

$$r\to 0,\qquad \mathrm{erf}(r)\to r \qquad \Phi(r)\to \text{const.}$$

$$r\to\infty,\qquad \mathrm{erf}(r)\to 1 \quad \Phi(r)\to\frac{1}{r}$$

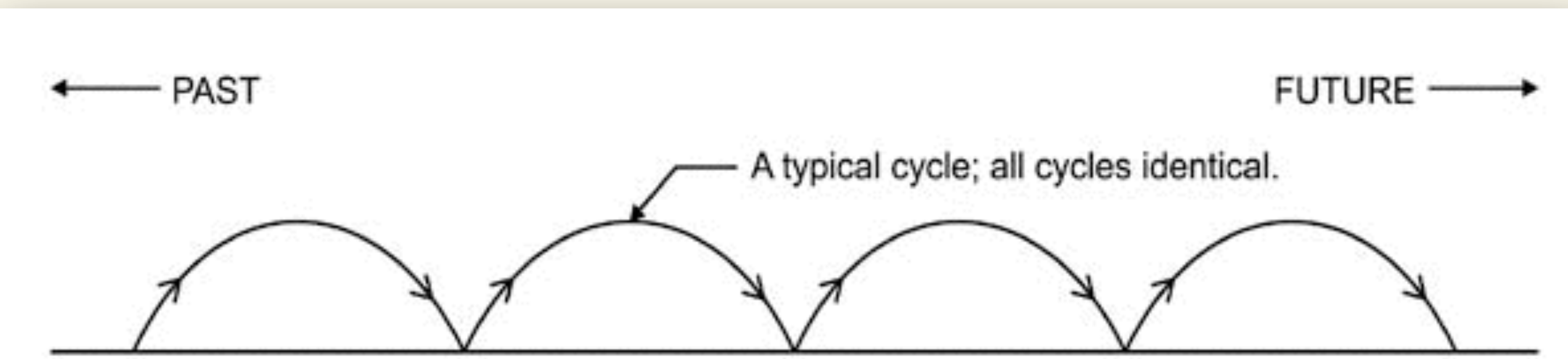
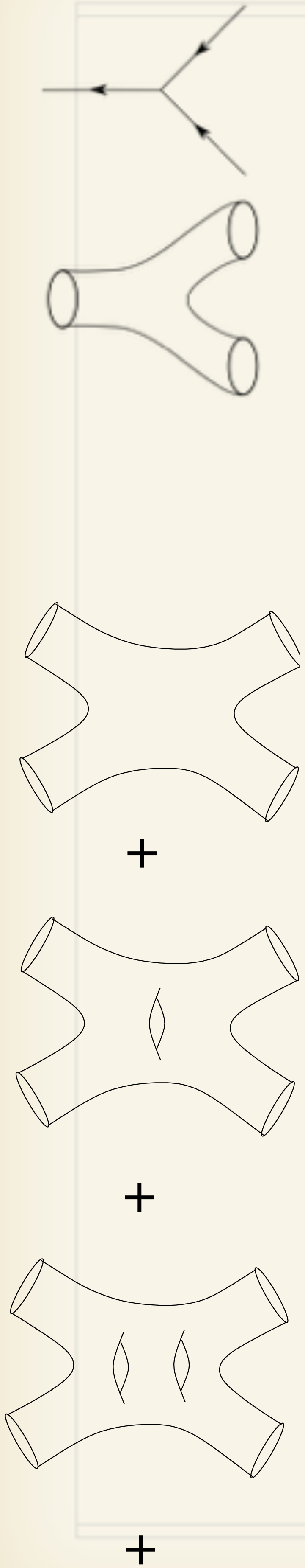


Fig 02

$$h\sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t)\ \text{with}\ A\ll 1$$

Biswas, Gerwick, Koivisto, AM, PRL (2012)

Connection to string theory



- **String Theory Introduces 2 Parameters**

$$\kappa^2 \approx g_s^2(\alpha')^{12}$$

- **DBI action ameliorates the Point like Singularity of Coulomb Solution**

$$S = -T_p \int d^{p+1}\zeta \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab})}$$

- **DBI Action Provides a Description of Open Strings to All Orders in α' at One-Loop**

Challenge for String Theorists:

To Construct a similar Action for Closed Strings with All Orders in α'

Remnants of stringy Gravity

$\mathrm{M_p}$

$\mathrm{m_W}$

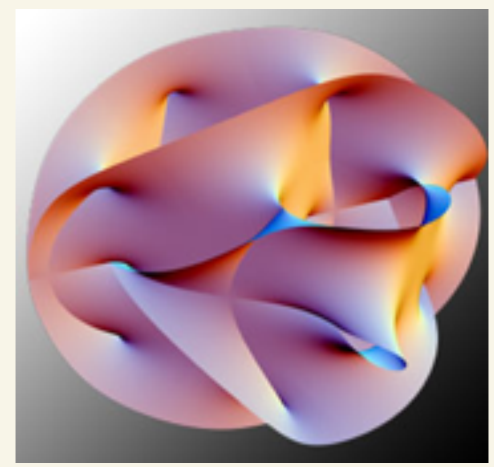
$\mathrm{m_s}$

$\mathrm{m_{KK}}$

$$\mathcal{L}^{10\mathrm{d}} \sim R + R^4 + \cdots \qquad \kappa^2 = g_s^2(\alpha')^4$$

Perturbative string theory has α' & g_s corrections

For all orders : String field theory



$$\mathcal{L}^{4\mathrm{d}} \sim R + \sum_i c_i R \left(\frac{\Box}{m_{kk}}\right)^i R + \cdots$$

$1-$ loop in g_s all orders in α'

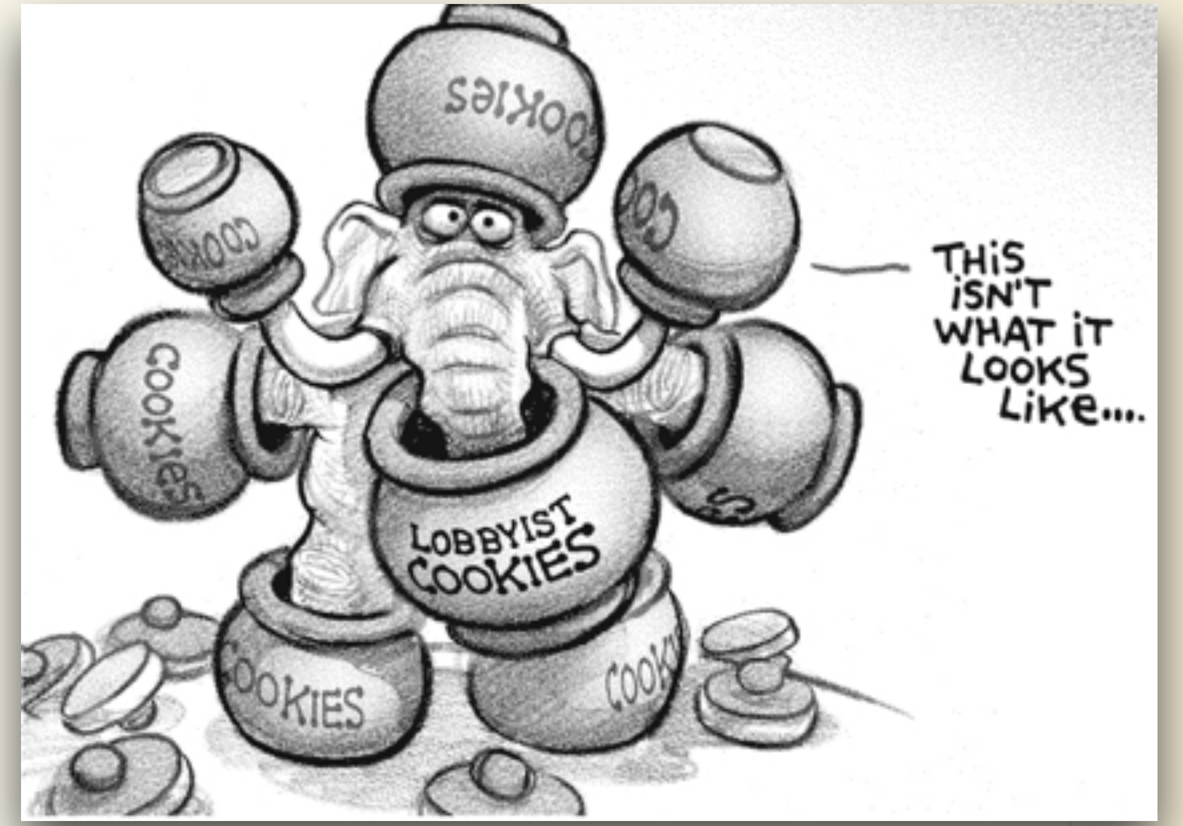
Witten (1998) , Tseytlin (1995), Zwiebach (2000), Sigel (1998, 2003), ...

So, if tensor to scalar ratio is large, and if we believe in string theory...

M_p

m_{KK}

The spectrum is squeezed, we are fighting for factors of 2pi



Inflation becomes even more interesting : pose new challenges

Not only have to worry about the matter sector, such as fluxes, string axions, non-perturbative gaugino condensations, etc.

During Inflation we may have a small curvature, but we do not know is where the scale of new physics

Inflation below KK mass

$$H_{inf} \ll m_{KK} \ll m_s \ll M_p$$

$$V(\phi) \ll m_{KK}^4$$

$\mathrm{M_p}$

$\mathrm{m_W}$

$$m_{\rm KK} < m_{\rm s} \Longrightarrow \mathcal{V}_s > (2\pi)^6$$

$\mathrm{m_s}$

$$m_{\rm s} < M_{\rm p} \Longrightarrow \mathcal{V}_s > (2\pi)^6 \times \pi g_s^2$$

$$\mathrm{m_{KK}} \sim \frac{2\pi}{\mathcal{V}_s^{1/6}} m_{\rm s}$$

$$r \ll \frac{(2.4 \times 10^8 g_s^4)}{\mathcal{V}_s^{8/3}/(2\pi)^{16}}$$

$$V(\phi)^{1/4} \sim \left(\frac{r}{0.1}\right)^{1/4} \times (8 \times 10^{-3}) M_{\rm p}$$

AM, Pramod Shukla 1411.4636

Loop quantum gravity

or

Causal set approach

↓

Wilson loops

↓

Non-local objects

Whatever I have spoken here perhaps there is a neat connection to loop quantum gravity, but I need to understand it better

In a time dependent background and in case of large ‘r’

Non-perturbative corrections arising from gravity becomes a mammoth …

We need to take these gravitational corrections especially for Starobinsky type and Higgs inflation, or high scale inflation

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Graviton Propagator

$$a(\Box)\Box h_{\mu\nu} \quad + \quad b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h)$$
$$+\eta_{\mu\nu}d(\Box)\Box h \quad + \quad \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}$$

Bianchi Identity

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

Biswas, Koivisto, AM
1302.0532

$$\Pi^{-1\lambda\sigma}_{\mu\nu}h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a-3c)k^2}$$

Covariant Modification of a Graviton Propagator : Only 1 Entire Function

UV → IR

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad = \frac{1}{a}\left[\frac{P^2}{k^2} - \frac{P_s^0}{2k^2}\right]$$

Demand: $a(k^2) = c(k^2)$

Recovers GR $\lim_{k^2\to 0} \Pi^{\mu\nu}{}_{\lambda\sigma} = (P^2/k^2) - (P_s^0/2k^2)$

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

ONLY 1 Non-Singular, Analytic functions at k=0, is required to Ameliorate the UV property of GR

'a' should be an Entire Function & cannot contain non-local operators, such as $a(\Box) \sim 1/\Box$

Ghost Free Gravity

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

Entire Function

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} = \frac{1}{a}\left[\frac{P^2}{k^2} - \frac{P_s^0}{2k^2}\right]$$

$$a(\Box) = c(\Box) = e^{-\Box/M^2}$$

Some function of k which falls faster than$1/k^2$

$$a(\Box) = e^{-\frac{\Box}{M^2}} \text{ and } \mathcal{F}_3 = 0 \Rightarrow \mathcal{F}_1(\Box) = \frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} = -\frac{\mathcal{F}_2(\Box)}{2}$$

Spin projectors

Let us introduce

$$\begin{aligned}
\mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho})-\frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\\
\mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho})\,,\\
\mathcal{P}^0_s &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\quad \mathcal{P}^0_w=\omega_{\mu\nu}\omega_{\rho\sigma}\,,\\
\mathcal{P}^0_{sw} &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}\,,\quad \mathcal{P}^0_{ws}=\frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}\,,
\end{aligned}\tag{16}$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\qquad \omega_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}.$$

Note that the operators $\mathcal{P}^i$ are in fact 4-rank tensors, $\mathcal{P}^i_{\mu\nu\rho\sigma}$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2,\mathcal{P}^1,\mathcal{P}^0_s,\mathcal{P}^0_w\}$, form a complete set of projection operators:

$$\mathcal{P}^i_a\mathcal{P}^j_b=\delta^{ij}\delta_{ab}\mathcal{P}^i_a \quad\text{and}\quad \mathcal{P}^2+\mathcal{P}^1+\mathcal{P}^0_s+\mathcal{P}^0_w=1\,,\tag{17}$$

$$\mathcal{P}^0_{ij}\mathcal{P}^0_k=\delta_{jk}\mathcal{P}^0_{ij}\,,\quad \mathcal{P}^0_{ij}\mathcal{P}^0_{kl}=\delta_{il}\delta_{jk}\mathcal{P}^0_k\,,\quad \mathcal{P}^0_k\mathcal{P}^0_{ij}=\delta_{ik}\mathcal{P}^0_{ij}\,,$$

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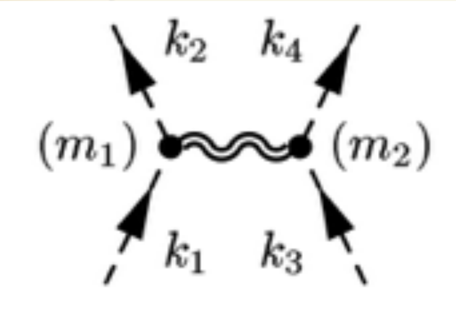
Newtonian Potential



Linearized Solution

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\begin{aligned}(a(\Box) - 3c(\Box))\Box h + (4c(\Box) - 2a(\Box) + f(\Box))\partial_\mu\partial_\nu h^{\mu\nu} &= \kappa\rho \\ a(\Box)\Box h_{00} + c(\Box)\Box h - c(\Box)\partial_\mu\partial_\nu h^{\mu\nu} &= -\kappa\rho\end{aligned}$$

For $f = 0$ and $a(\Box) = c(\Box)$

$$4a(\nabla^2)\nabla^2\Phi = 4a(\nabla^2)\nabla^2\Psi = \kappa\rho = \kappa m\delta^3(\vec{r})$$

$a(\Box) = e^{-\Box/M^2}$ **Varying slowly with time** $\Box \longrightarrow \nabla^2$

$$\Phi(r) \sim \kappa m \int \frac{dp}{p} e^{-p^2/M^2} \sin(pr) = \kappa \frac{m\pi}{4\pi^2 r} \mathrm{erf}\left(\frac{rM}{2}\right) = \frac{Gm}{r}\mathrm{erf}\left(\frac{rM}{2}\right) = \frac{m}{4\pi M_p^2 r}\mathrm{erf}\left(\frac{rM}{2}\right)$$

$$mM \ll M_p^2 \implies m \ll M_p$$

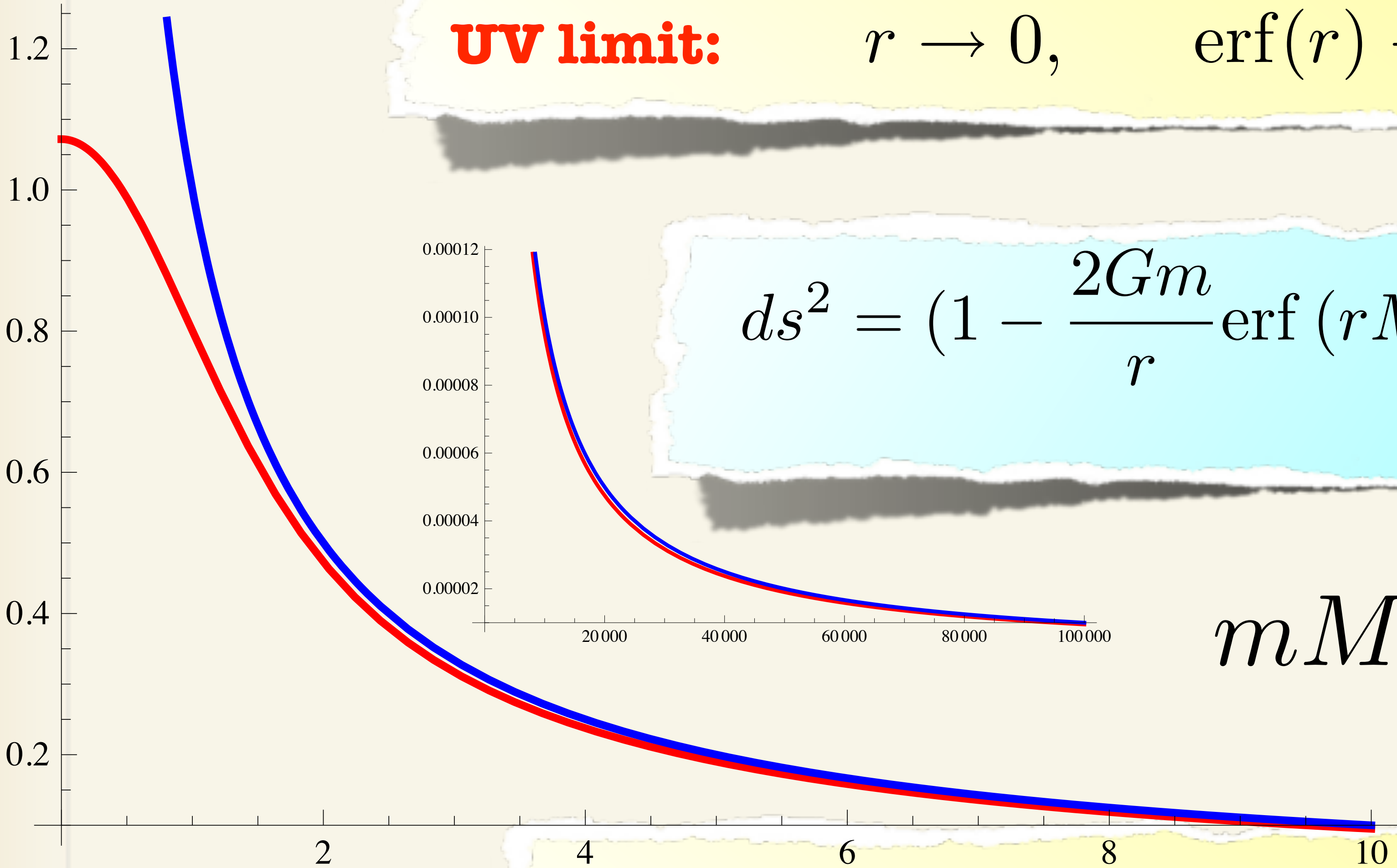
Biswas, Gerwick, Koivisto, AM, PRL, (gr-qc/1110.5249)

Non Singular Solution

UV limit: $r \to 0, \quad \mathrm{erf}(r) \to r \qquad \Phi(r) \to \mathrm{const.}$

$$ds^2 = \left(1 - \frac{2Gm}{r}\mathrm{erf}\left(rM/2\right)\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\mathrm{erf}\left(rM/2\right)\right)}$$

$$mM \ll M_p^2 \implies m \ll M_p$$



IR limit: $r \to \infty, \quad \mathrm{erf}(r) \to 1 \qquad \Phi(r) \to \frac{1}{r}$

No Singularity → No Horizon

Non-Singular Bouncing Solution

$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

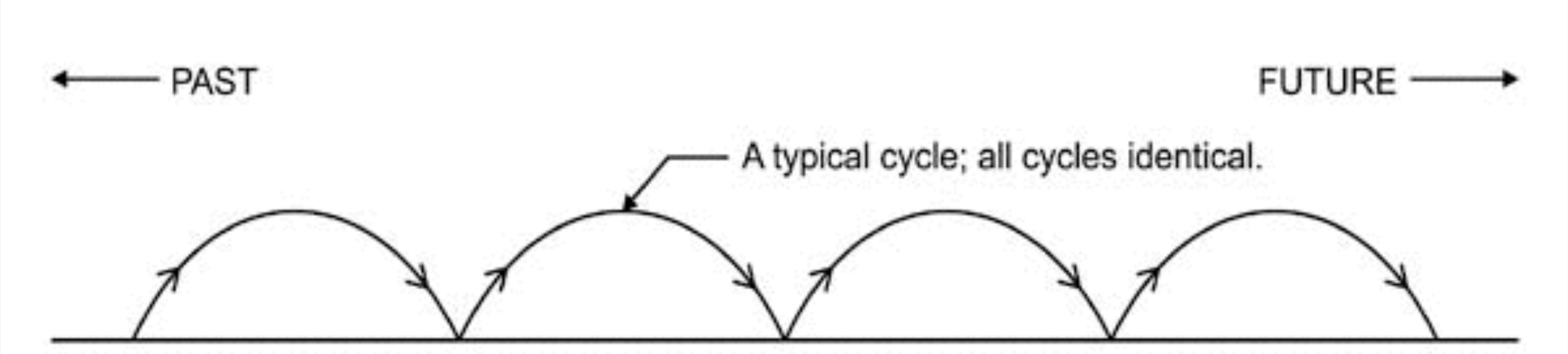


Fig 02

$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \text{ with } A \ll 1$$

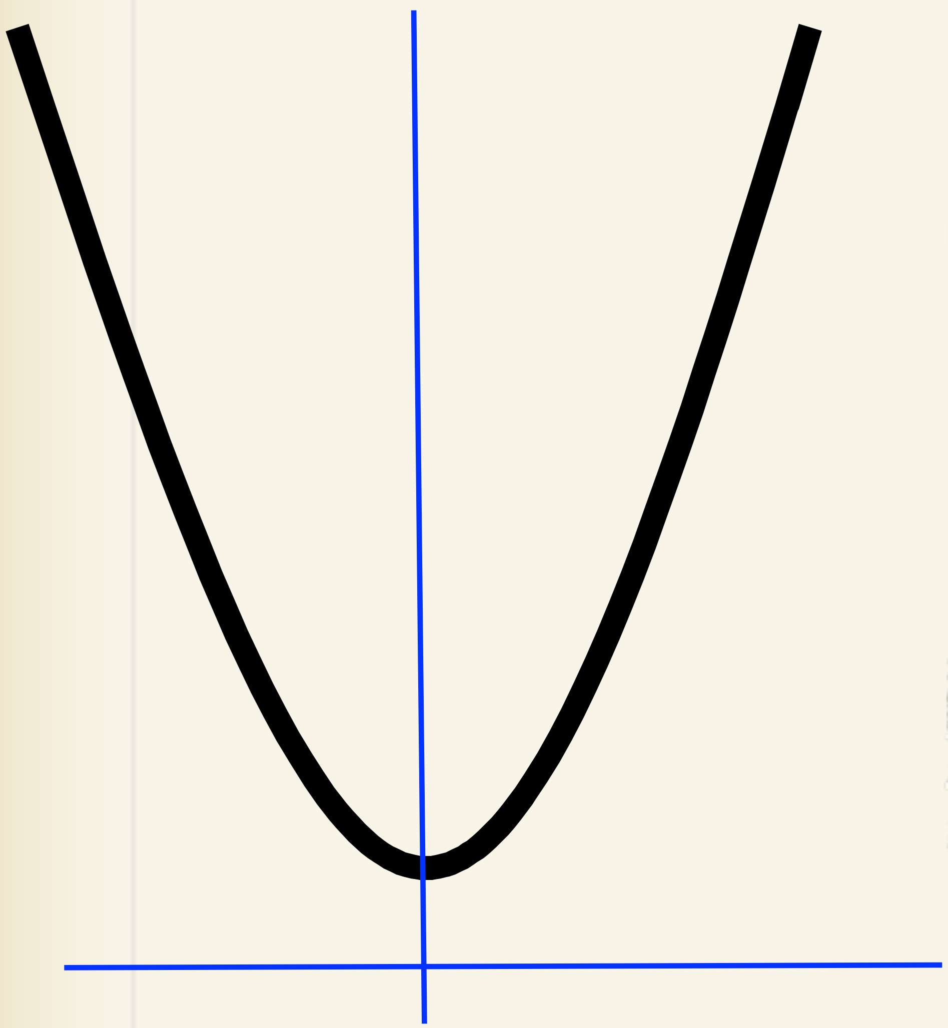
Non- Singular Bouncing, Homogeneous & Isotropic Universe

Such a solution is not possible in GR

Biswas, Gerwick, Koivisto, AM, Phys. Rev. Lett. (gr-qc/1110.5249)

Full Non-Singular Solution

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box}\right]R^{\mu\nu}\right]$$



$$\Box R = r_1 R + r_2$$

$$\Lambda = -\frac{r_2 M_P^2}{4r_1}$$

$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}t\right)$$

$$\ddot{a} > 0 \Longrightarrow \Lambda > 0$$

Does Not Contribute to Dynamics But to Perturbations

Biswas, AM, Siegel, JCAP (hep-th/0508194)

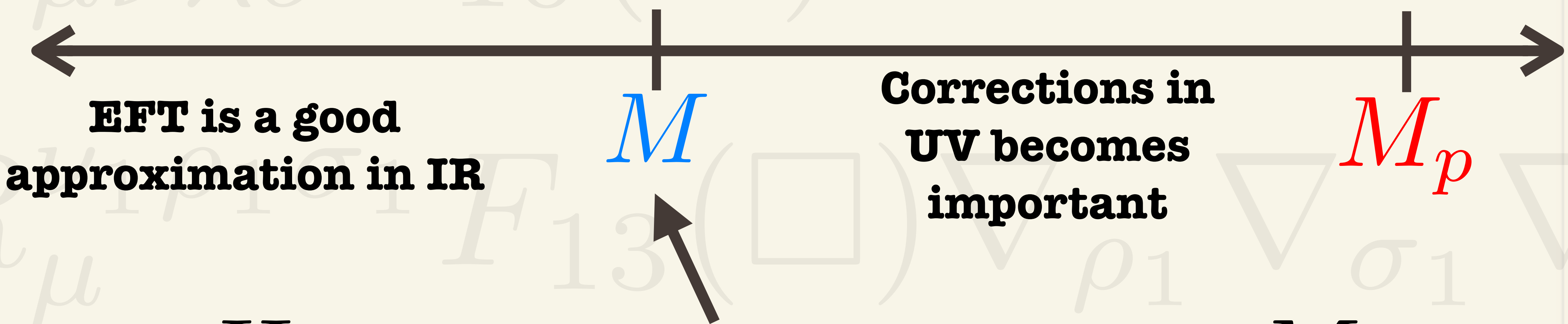
Biswas, Koivisto, AM, JCAP (hep-th/1005.0590)

Conclusions

- **We have constructed a Ghost Free & Singularity Free Theory of Gravity**
- **If we can show higher loops are finite then it is a great news --** this is what we are working now
- **But, studying singularity theorems, positive energy theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems can be studied in this framework**
- **Holography is no longer a property of UV, becomes part of an IR effect. The area law of gravitational entropy will no longer hold true in UV.**

Implications for Cosmic Inflation

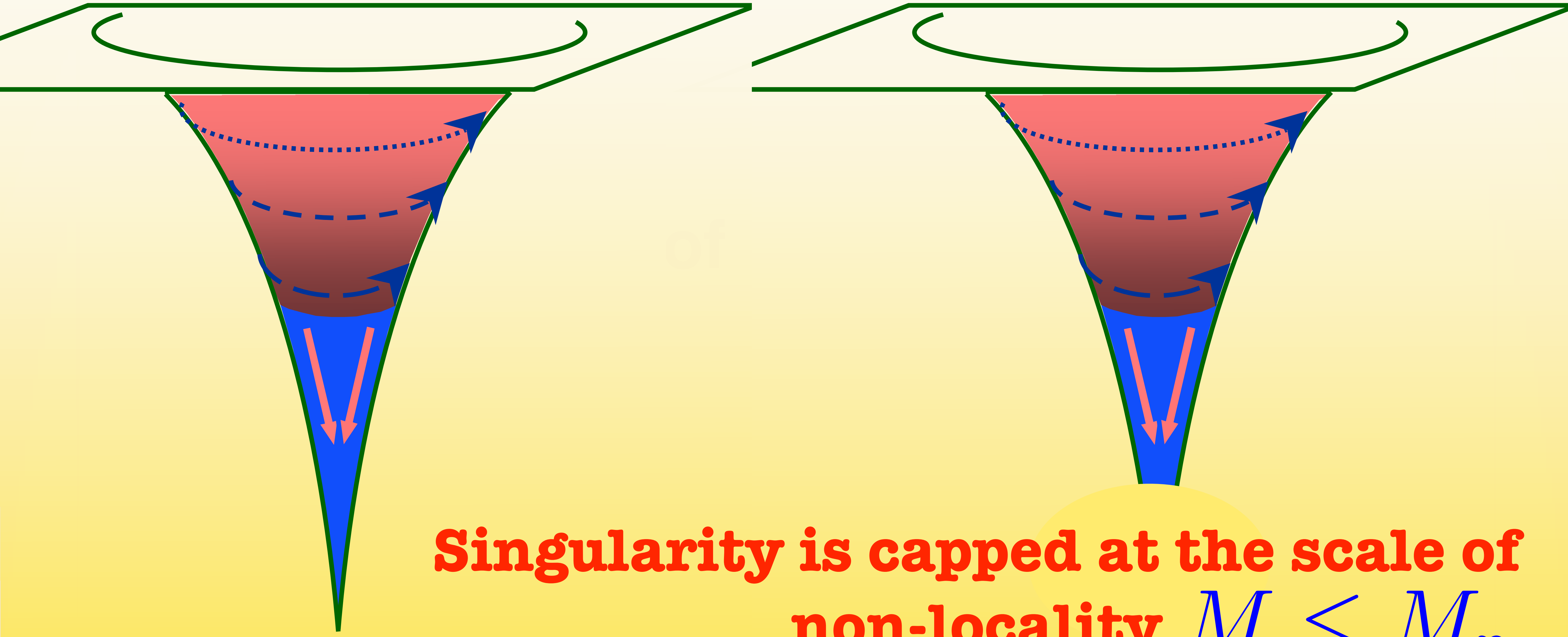
This could be the KK -scale



$$H_{inf} \ll m_{KK} \ll m_s \ll M_p$$

$$\mathcal{L} \sim \frac{1}{2}\phi e^{\frac{\Box + m^2}{M^2}}(\Box + m^2)\phi + V(\phi)$$

Where would you expect the modifications?



Singularity is capped at the scale of non-locality $M \leq M_p$

Revisiting Hawking-Penrose Singularity Theorems

$$\theta = \nabla_a N^a \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{ab}N^a N^b$$

General Relativity

$$R_{ab}N^a N^b = 8\pi T_{ab}N^a N^b \geq 0$$

$$\frac{d\theta}{d\tau} \leq 0 \qquad \rho + p \geq 0$$

Non-local extension of GR

$$R_{ab}N^a N^b \leq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

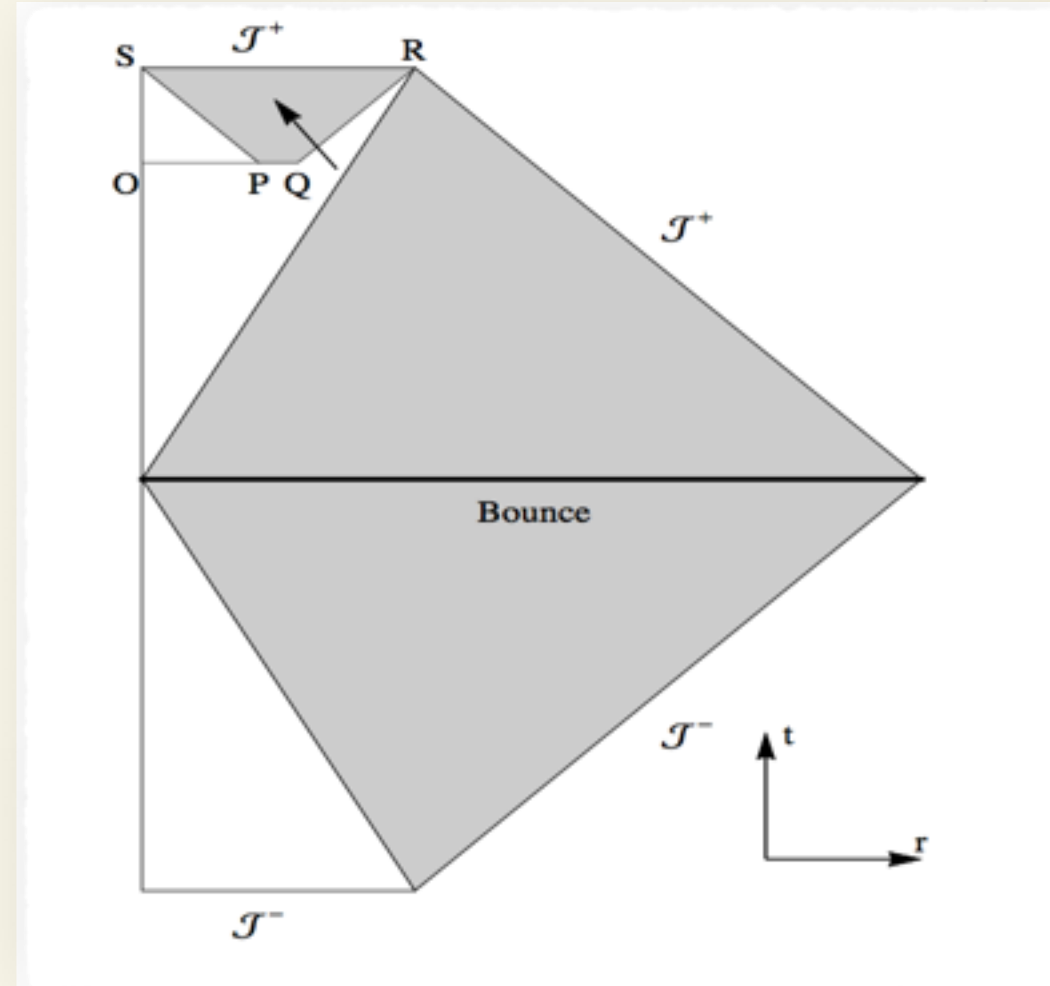
$$R_{ab}N^a N^b \neq 8\pi T_{ab}N^a N^b$$

Conroy, Koshlev, AM,
(gr-qc/1408.6205)

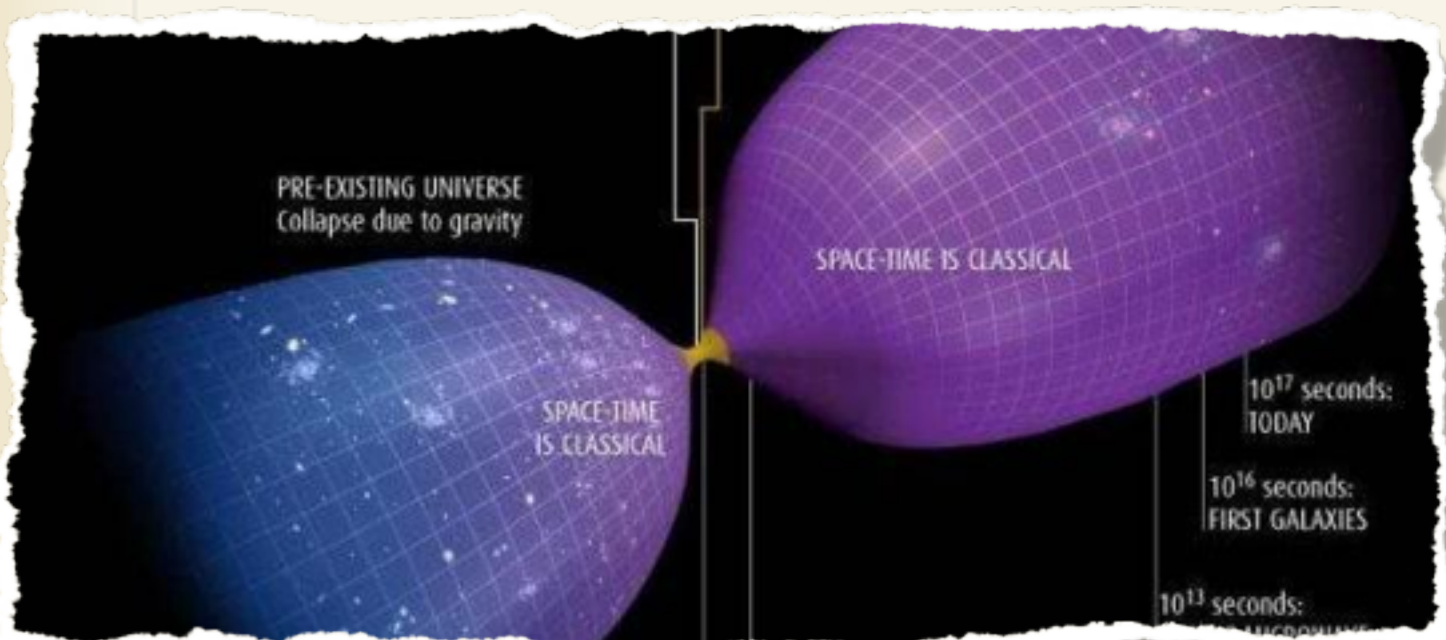
Revisiting Singularity Theorems

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{R\mathcal{F}(\Box)R}{2}\right)$$

$$R_{\mu\nu}k^\mu k^\nu = (k^0)^2\frac{(\rho+p) + 2\partial_t^2(\mathcal{F}(\Box)R)}{M_p^2 + 2\mathcal{F}(\Box)R}$$



$$R_{\mu\nu}k^\mu k^\nu \leq 0, \qquad T_{\mu\nu}k^\mu k^\nu \geq 0 \rightarrow (\rho + p \geq 0)$$



$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

Conroy, Koshlev, AM,
(gr-qc/1408.6205)