Implications of the No-Boundary Proposal for Ekpyrotic and Cyclic Cosmologies

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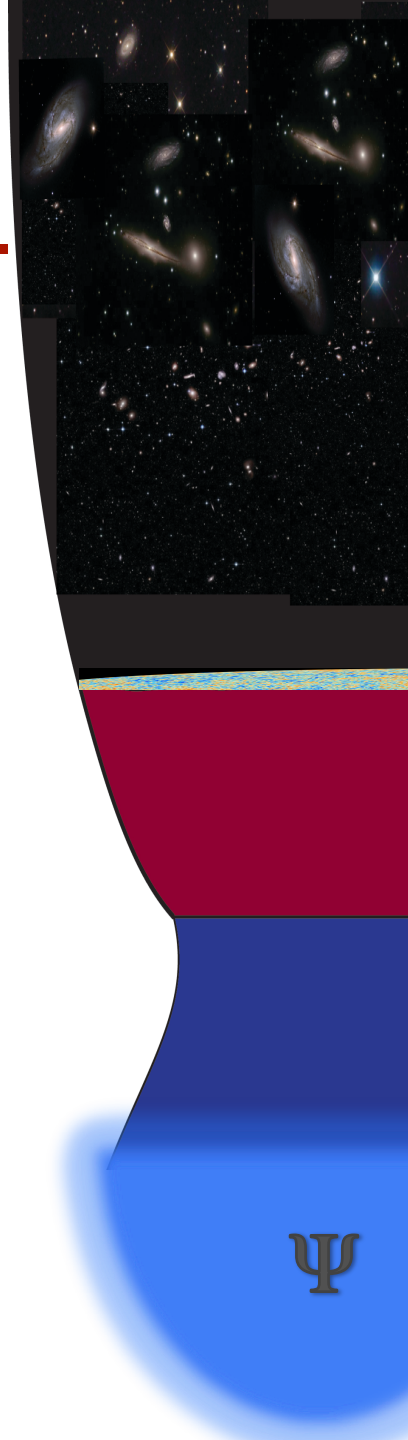
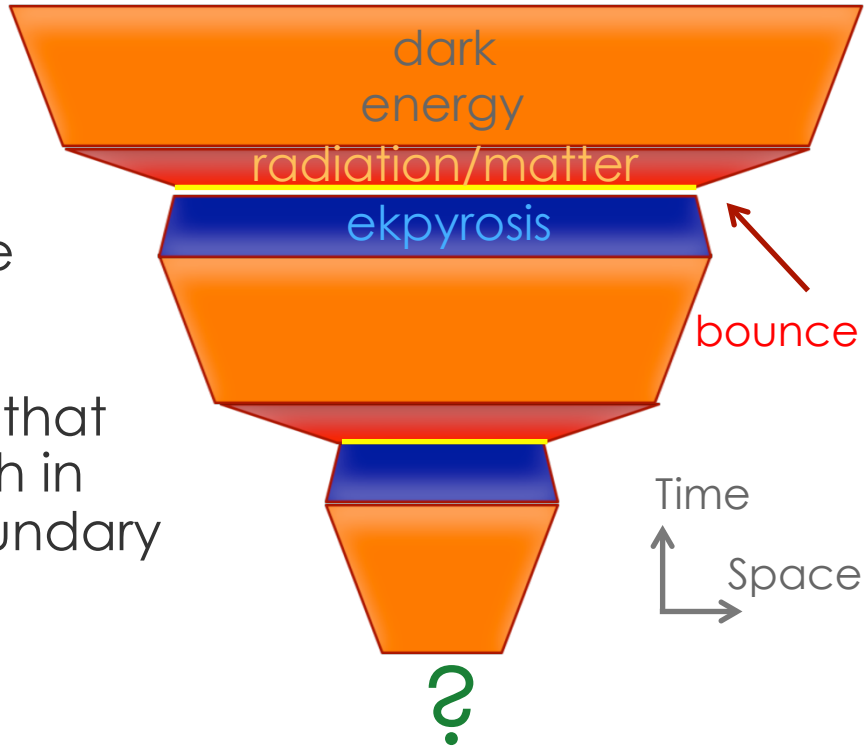
[work done with Lorenzo Battarra]



Initial Conditions

- Asking what came before? inevitably leads to the question of how the universe "started"
- Singularity theorems of Hawking & Penrose
- Inflation (& eternal inflation): theorem of Borde, Guth & Vilenkin shows again that classically a singularity is unavoidable
- Cyclic universe: grows from cycle to cycle, hence, if finite, will have been in quantum regime at a finite time in the past
- Perhaps there are cyclic universes that do not need a beginning, although in that case one may still need a boundary condition at minus infinity



- Even though inflation & ekpyrosis are dynamical attractors, we know that many features/ predictions depend on initial conditions
- If several alternative cosmological histories are allowed by a theory, initial conditions will determine which is the relevant one
- More generally, for any early universe model, a big question remains unanswered: given quantum theoretical laws, how did a classical universe emerge?

This talk:

- Can Semi-Classical Quantum Gravity, together with the No-Boundary Proposal, address these open issues?

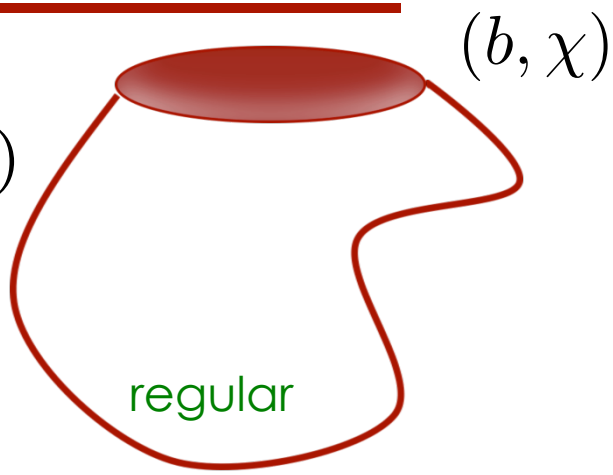
Note:

- Many mathematical footnotes are attached to the no-boundary proposal: definition of path integral, Wick rotation, non-renormalizability of ordinary gravity, difficulties of going beyond the mini-superspace simplification,...
- Here we will find that the configurations of greatest interest involve small curvatures -> this suggests that we may nevertheless be able to trust the semi-classical approximation that is used throughout

Review of the No-Boundary Proposal

$$\Psi(b,\chi) = \int_{\mathcal{C}} \mathcal{D}a\mathcal{D}\phi e^{-S_E(a,\phi)}$$

$$\approx e^{-S_{E,ext}(b,\chi)}$$



- The wavefunction is given by a path integral over all possible four-geometries that are regular in the past (i.e. the possible paths are *restricted*)
- Hartle-Hawking b.c.: the universe is *finite and self-contained*
- No-Boundary Proposal is *supported by AdS/CFT*
- Saddle point approximation: the geometries that are an extremum of the action with the required boundary conditions are typically *complex – "fuzzy" instantons*

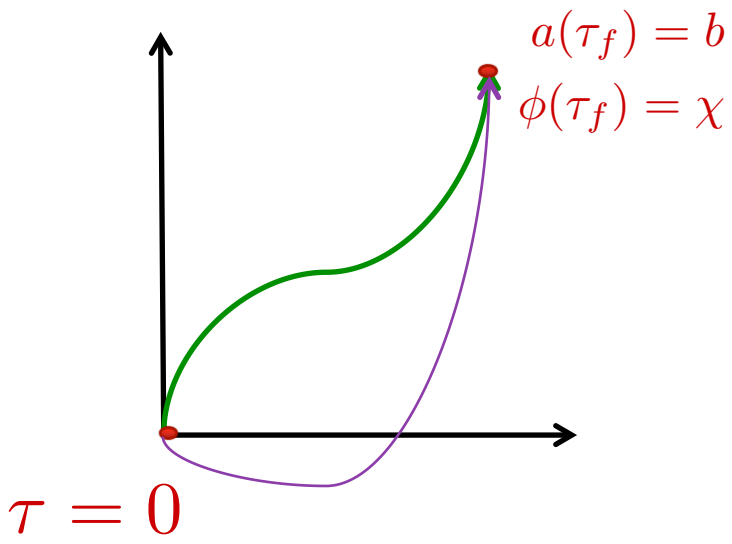
[Hartle, Hawking & Hertog]

- The metric is taken to be of (complexified) FLRW type with a fixed 3-sphere spatial geometry

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3^2$$

- In this minisuperspace approximation, the action then becomes

$$S_E = 6\pi^2 \int d\tau \left(-a a'^2 - a + \frac{a}{3} \left(\frac{1}{2} \phi'^2 + V \right) \right)$$



Complex geometries -> the above integral is a *contour integral* in the complex τ plane

At τ=0, no-boundary conditions are imposed:

$$a = 0, \quad a' = 1$$

$$\phi = \phi^R_{SP} + i\, \phi^I_{SP}, \quad \phi' = 0$$

[Hartle & Hawking]

Probabilities

$$S_E = 6\pi^2 \int d\tau \left(-a a'^2 - a + \frac{a}{3} \left(\frac{1}{2} \phi'^2 + V \right) \right)$$

- A standard Lorentzian history corresponds to evolving in the imaginary τ direction ($d\tau = i dt$), with a and ϕ being (approximately) real

$$\Psi^* \Psi \sim e^{-2Re(S_E)}$$

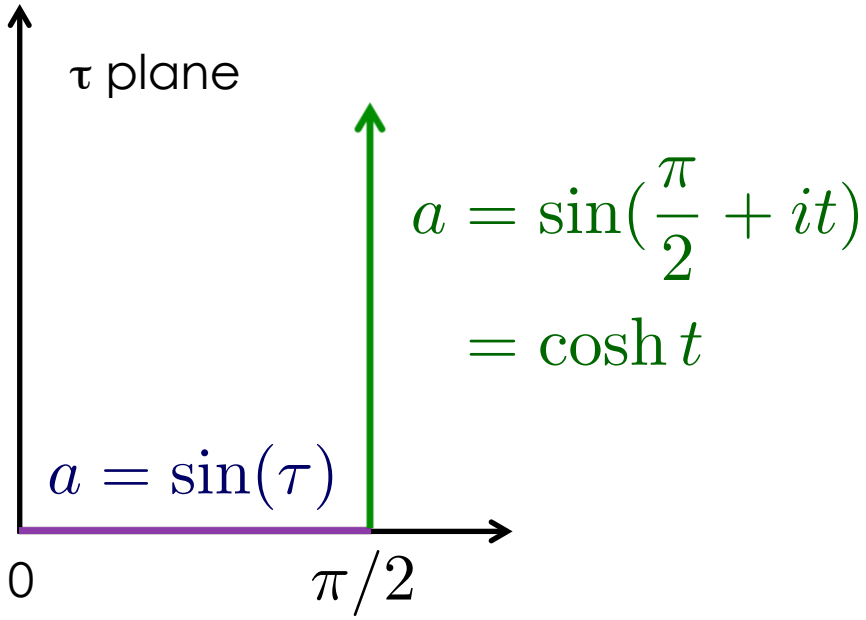
- When (and only when) the universe becomes classical, the real part of S_E stops evolving and we get a meaningful notion of (relative) probability

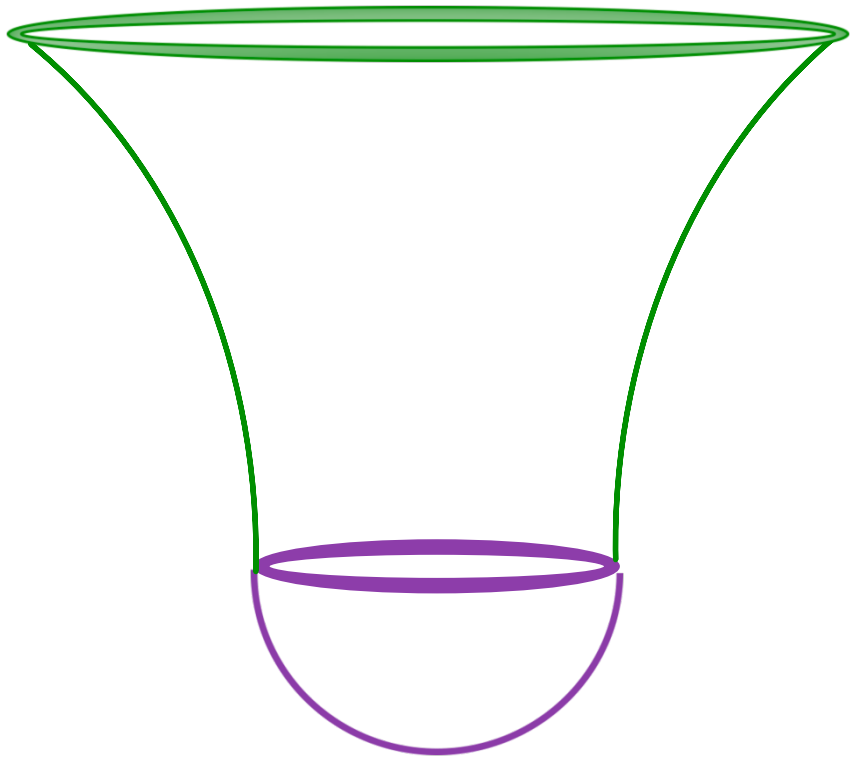
[Hartle & Hawking]

Hawking's Prototype Instanton: Pure de Sitter

- Here there is no scalar field, only a cosmological constant $\Lambda = 3\,H^2$
- Probability

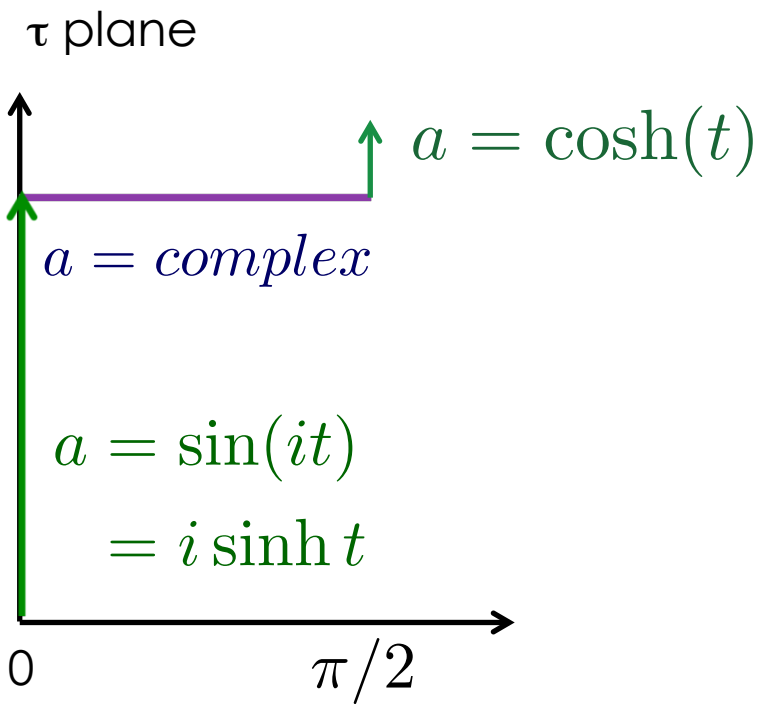
$$e^{-2Re(S)} = e^{\frac{24\pi^2}{H^2}}$$

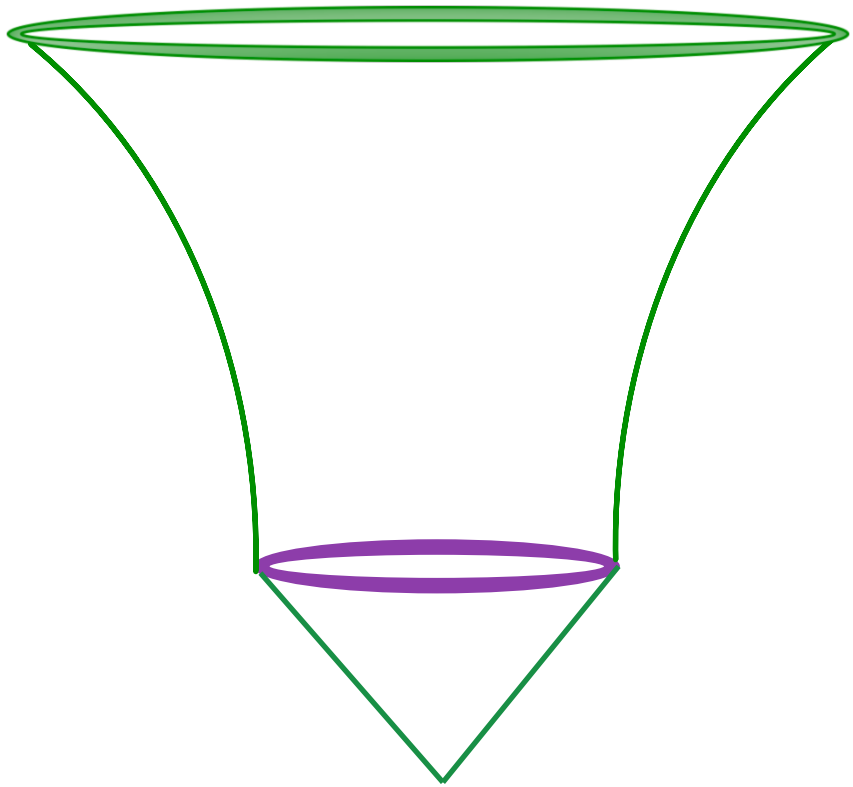




Alternative Representation: AdS Contour

- By running the contour up the y-axis first, we obtain a (wrong-signature) Euclidean AdS representation (and a possible connection with AdS/CFT)

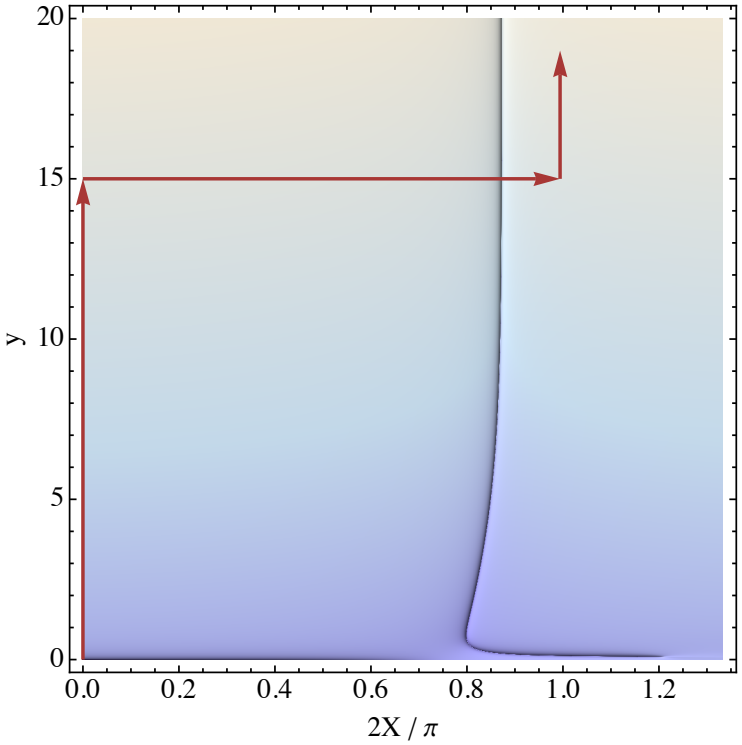




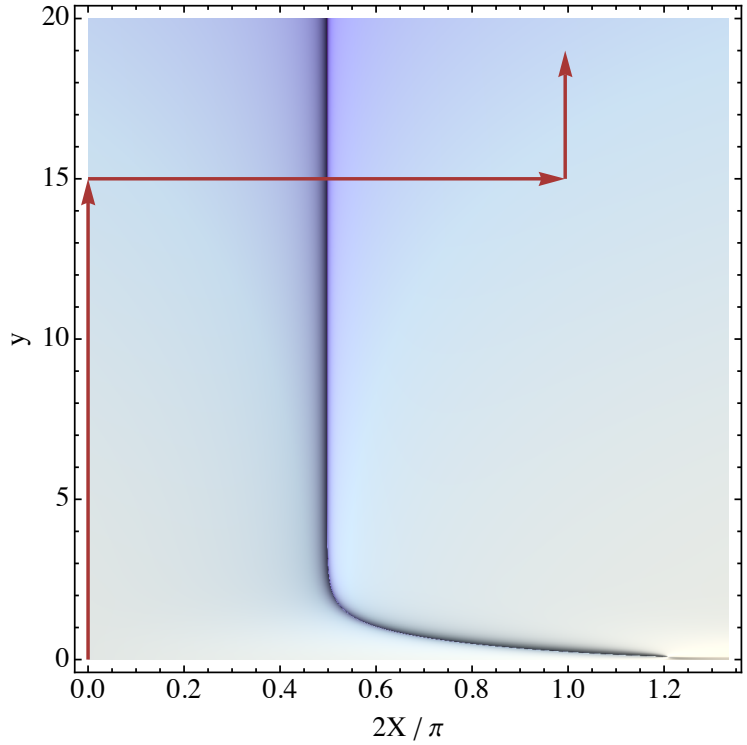
Hartle & Hertog

No-Boundary "Fuzzy" Instantons – including the scalar

- For an *arbitrary* value of ϕ at τ=0, the lines where a is real do not match up with those where ϕ is real, i.e. we do *not* get a classical universe



Dark line depicts locus of real scale factor a

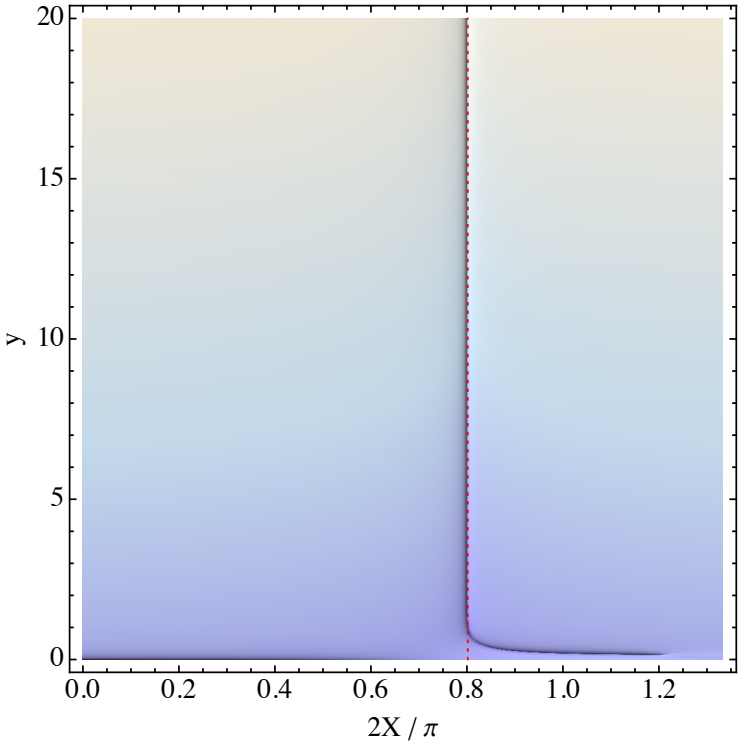


Dark = real scalar field

[Battarra & JLL]

No-Boundary "Fuzzy" Instantons – including the scalar

- We must tune the *imaginary part* of ϕ at the origin $\tau=0$ in order for the lines of real a and real ϕ to match up *at late times*



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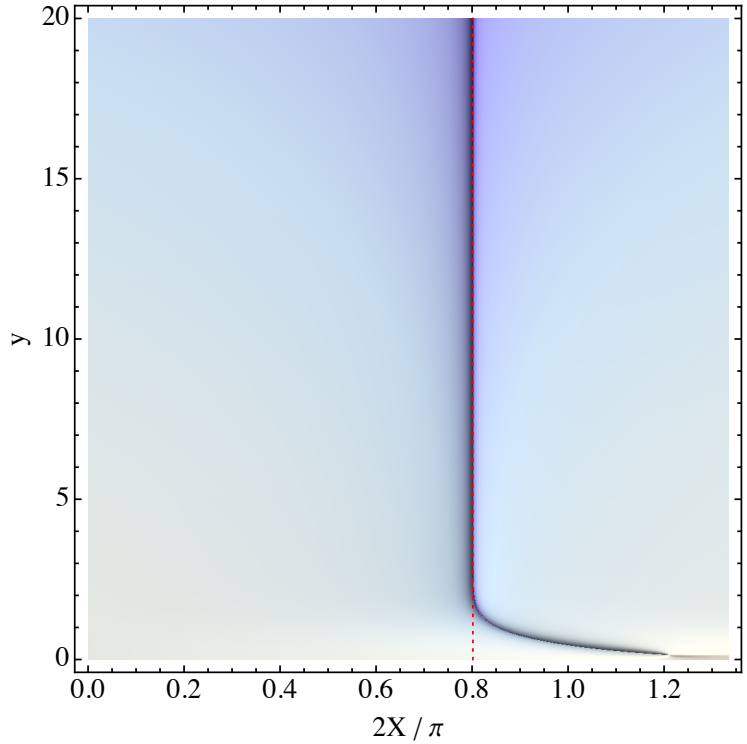


Dark = real scalar field ϕ

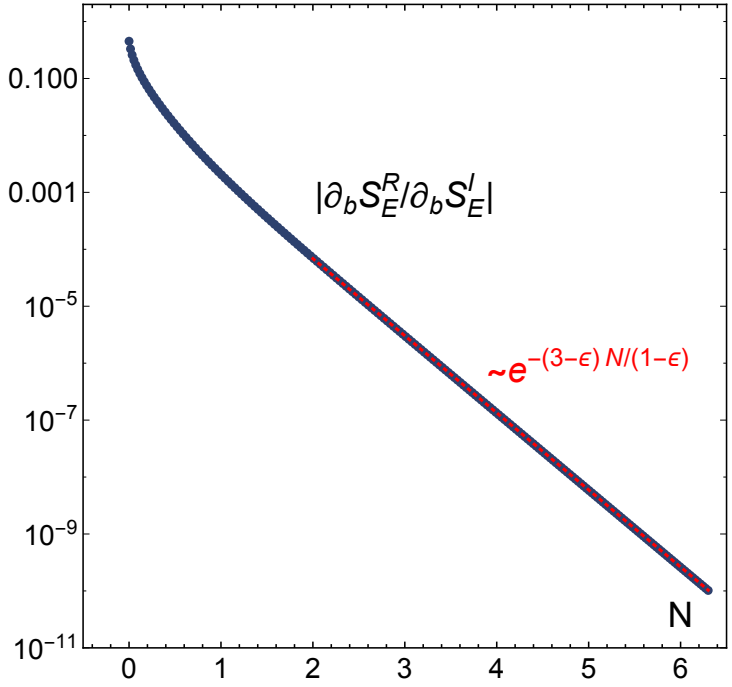
- The inflationary attractor makes this possible

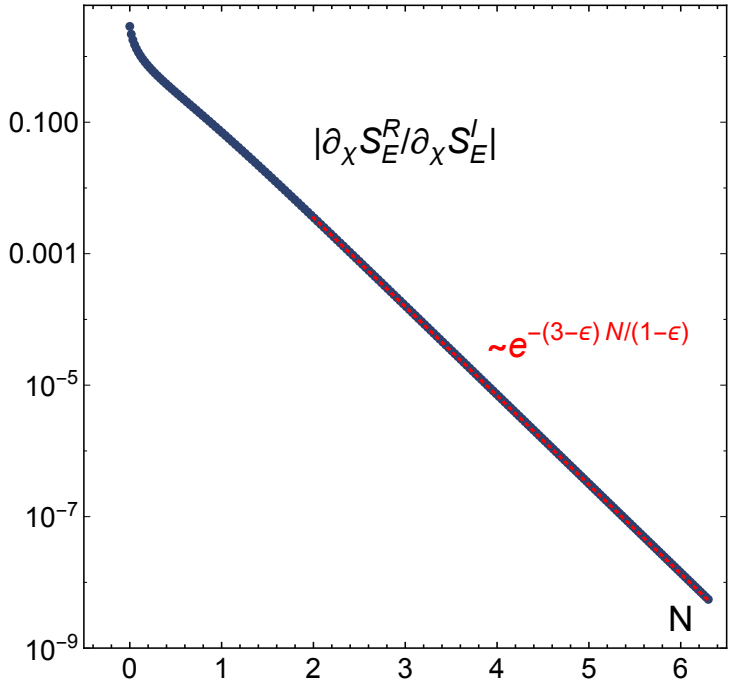
[Hartle, Hawking & Hertog]

WKB Classicality – Inflation (ε constant)

- As the inflationary phase proceeds, the wavefunction of the universe $\Psi \sim e^{-S_E}$ becomes increasingly classical, in the sense that its phase varies rapidly compared to the amplitude – WKB conditions:

$$|\partial_b S_E^R \,/\, \partial_b S_E^I| \ll 1, \qquad |\partial_\chi S_E^R \,/\, \partial_\chi S_E^I| \ll 1$$





[JLL]

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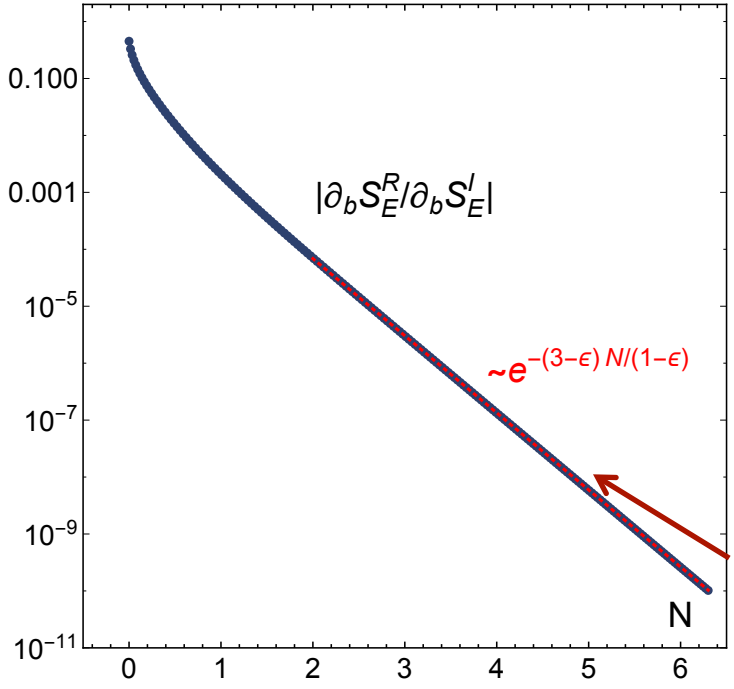
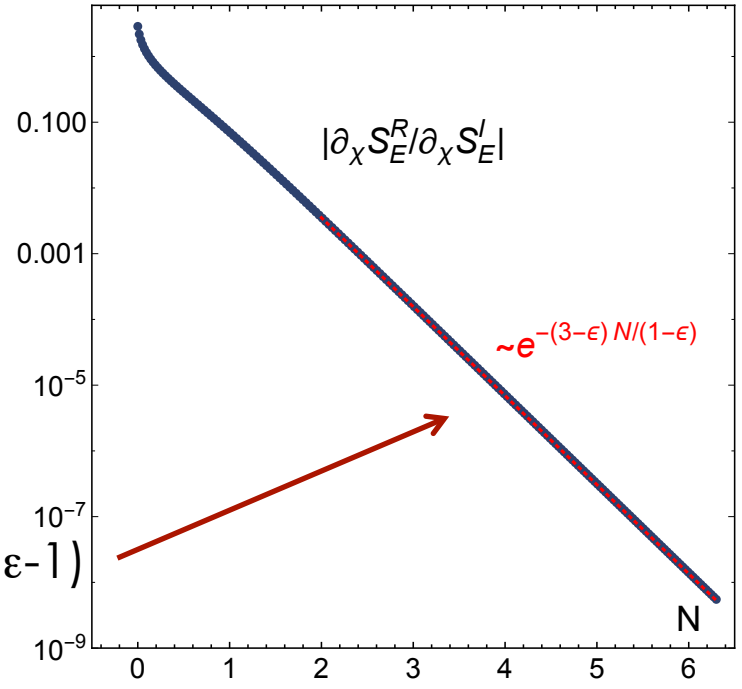


$e^{-(\varepsilon-3)N/(\varepsilon-1)}$

$\sim e^{-3N}$



[JLL]

Inflationary instantons - Comments

- It has been claimed that inflation is necessary in order to explain the classicality of the universe

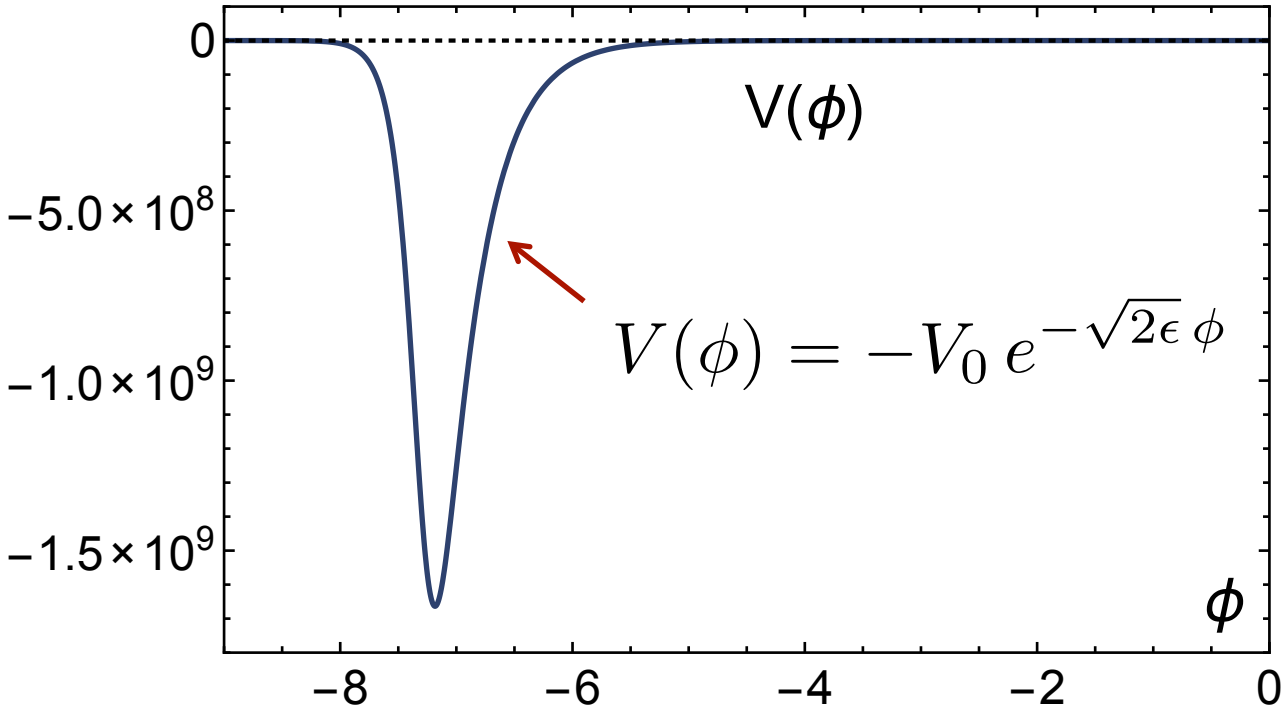
[Hartle, Hawking & Hertog]

- Relative probabilities:

$$\Psi \sim e^{-\frac{\mathcal{O}(10^2)}{V(\phi^R_{SP})}}$$

Low values of the potential are preferred

Ekpyrotic Instantons

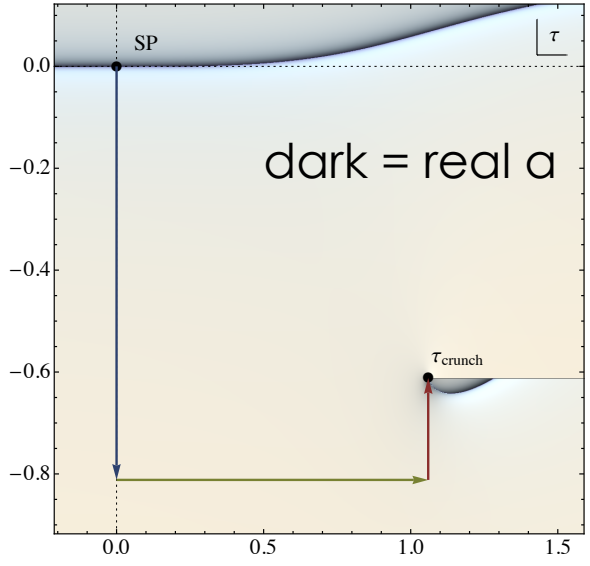


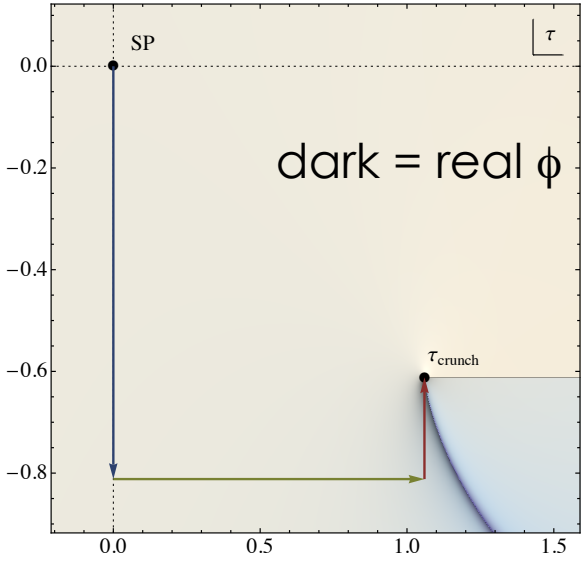
- Can one make sense of the no-boundary proposal when the potential is negative?
- How can a contracting universe emerge from nothing?

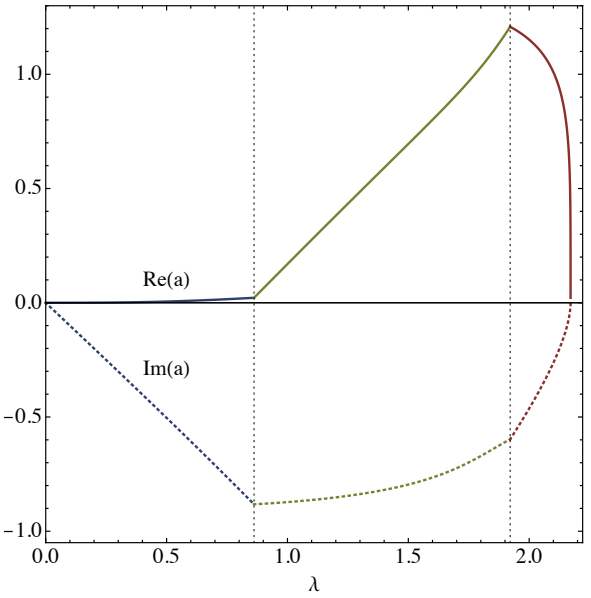
[Battarra & JLL]

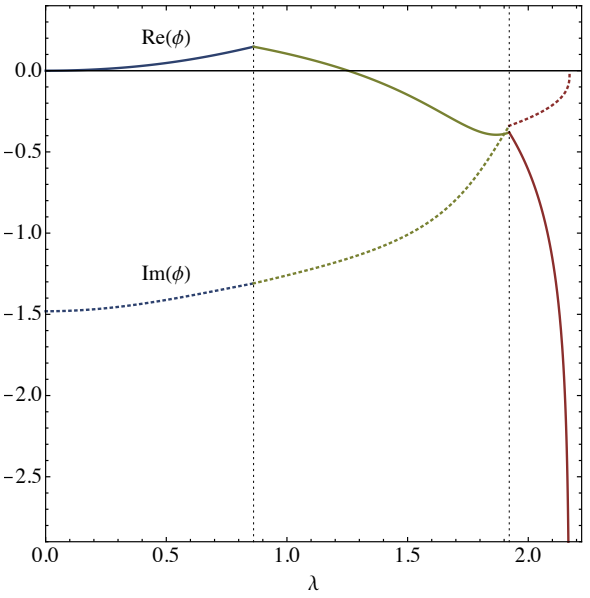
Ekpyrotic Instantons

- Require a different contour
- No bounce dynamics included (yet), hence these instantons end in a crunch
- Classicality is reached during the ekpyrotic fast-roll









[Battarra & JLL]

Ekpyrotic Instantons - Shape

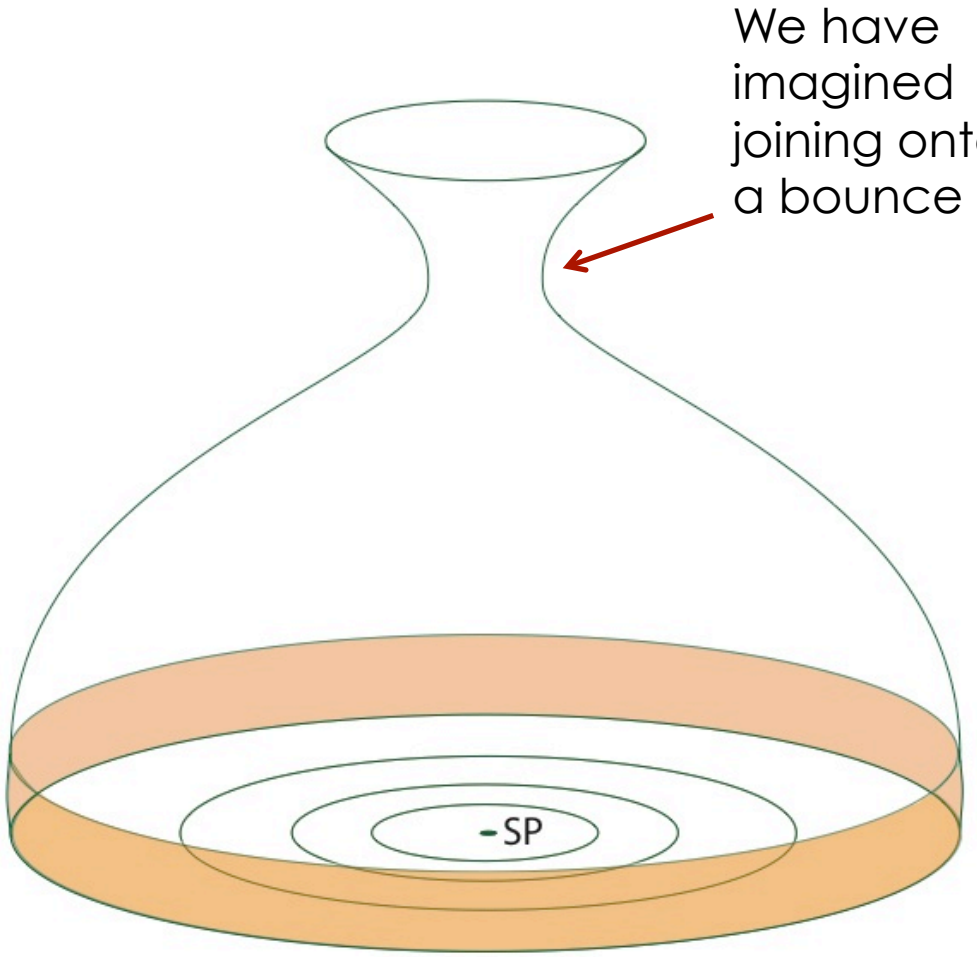
- Along this contour, the shape is as follows:

Bottom: portion of Euclidean space

Middle: fully complex

Top: increasingly classical contracting universe

- Thus a contracting Lorentzian universe can emerge from nothing as a region of Euclidean space gets created first

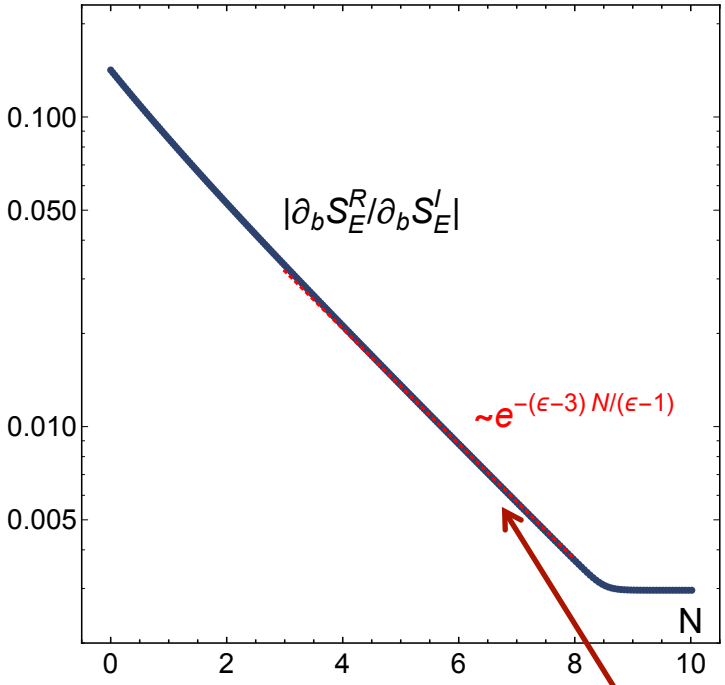


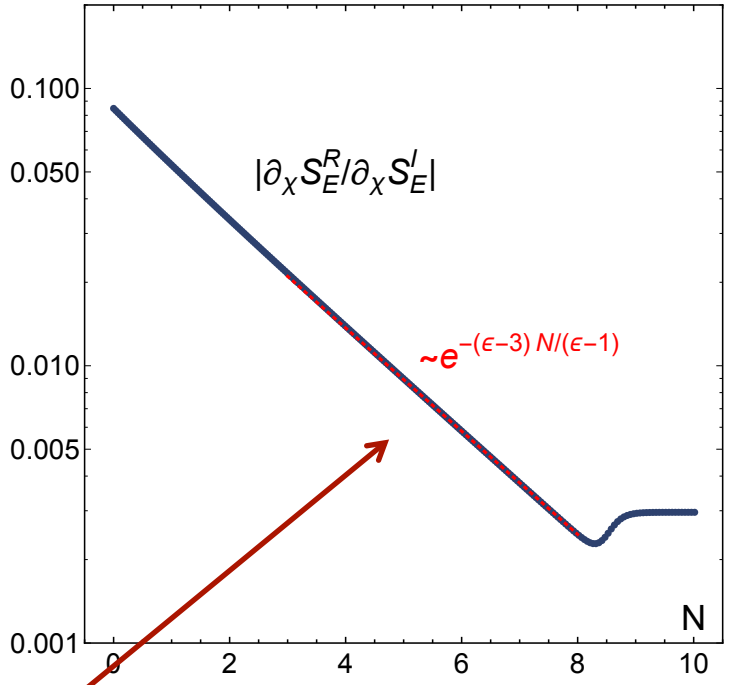
[Battarra & JLL]

WKB Classicality - Ekpyrosis

- In this case also, the wavefunction becomes increasingly classical in a WKB sense

$$|\partial_b S_E^R / \partial_b S_E^I| \ll 1, \qquad |\partial_\chi S_E^R / \partial_\chi S_E^I| \ll 1$$





$e^{-(\varepsilon-3)N/(\varepsilon-1)}$

$\sim e^{-N}$

[Battarra & JLL]

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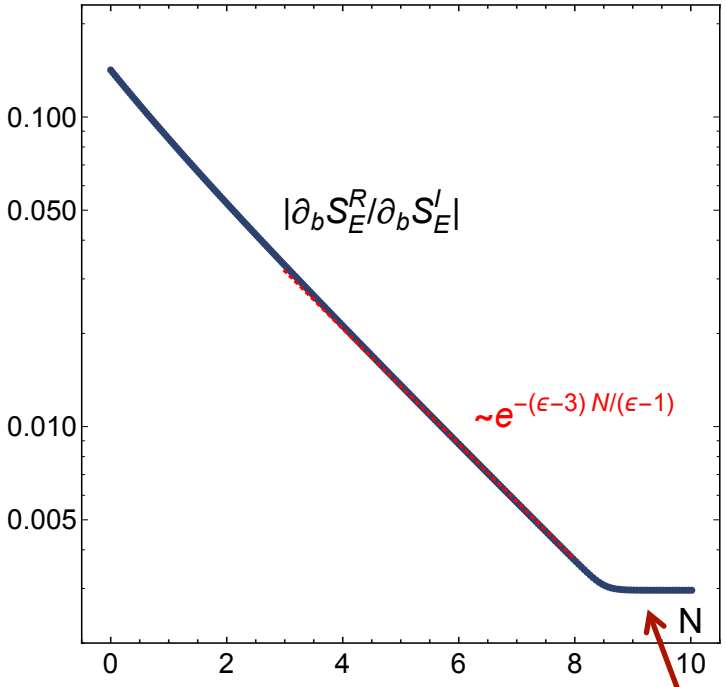
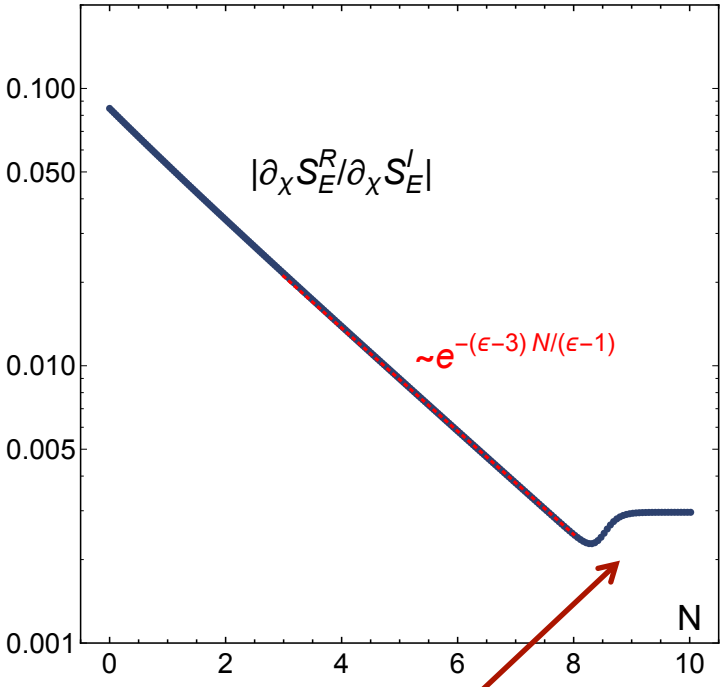


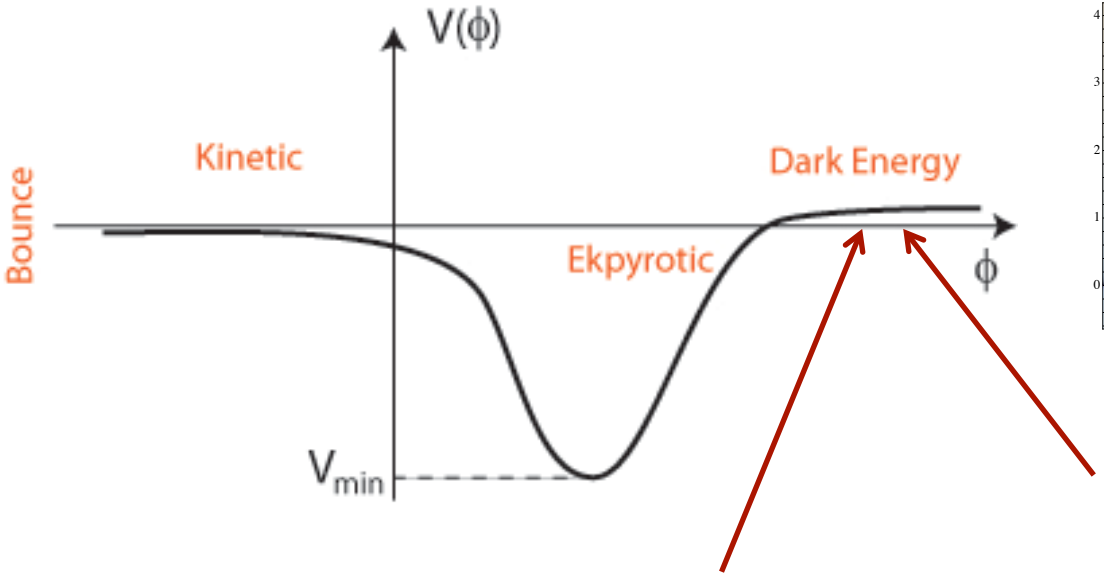


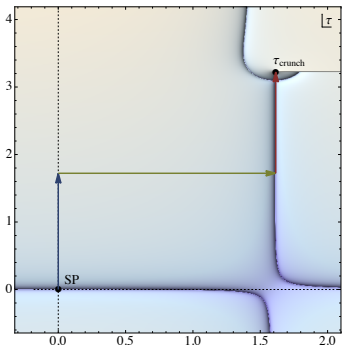
Classicality is preserved during the subsequent kinetic phase

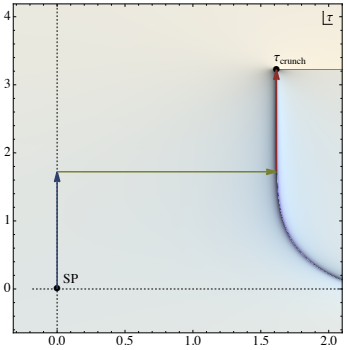
[JLL]

Cyclic Potential

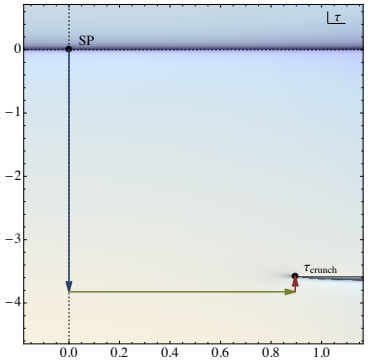
- On dark energy plateau, inflationary and ekpyrotic instantons coexist

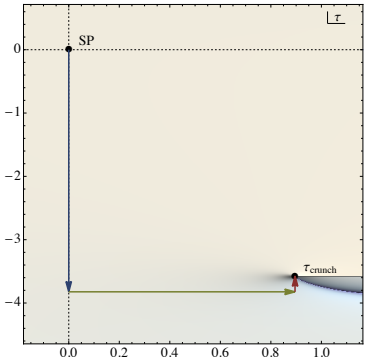






Inflationary instanton
Classicality reached during phase of exponential expansion

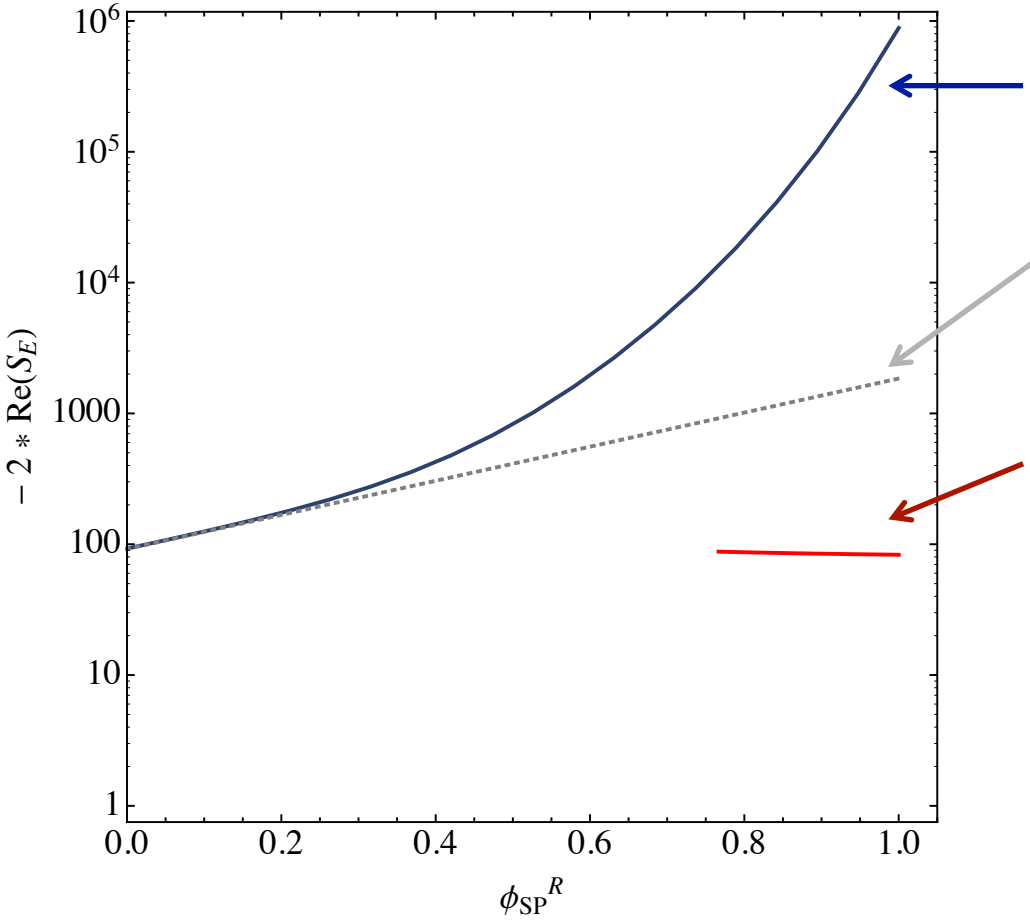




Ekpyrotic instanton
Classicality reached during subsequent ekpyrotic rolling-down

Cyclic Potential - Probabilities

- Relative probability of inflationary vs. cyclic instantons on dark energy plateau



Ekpyrotic instantons

Expected scaling assuming a pure exponential potential

Inflationary instantons

The relative probability for ekpyrotic instantons on the dark energy plateau is high – why?

Conclusions

- Inflation and ekpyrosis are the only two theories known that can render the universe classical, starting from a quantum state
- In both cases classicality is reached as a power-law in the scale factor of the universe

$$WKB \propto e^{-\frac{\epsilon-3}{\epsilon-1}N}$$

- In a potential energy landscape the relative probability of the various classical histories is approx. given by a simple formula

$$\Psi^{\star}\Psi \propto e^{\frac{1}{|V(\phi^{R}_{SP})|}}$$

- This implies that ekpyrotic histories are vastly preferred
- Open questions:
  - Can one can add a bounce? Is classicality preserved across the bounce?
  - How can one incorporate tunneling events?

Volume Weighting and Eternal Inflation

- It has been argued that one should weight by physical volume in order to obtain the relevant probabilities:

$$P \propto e^{-2Re(S_E)+3N}$$

[Hartle, Hawking & Hertog]

- It is then easy to determine the minimum of the probability distribution:

$$\begin{aligned}(-2Re(S)+3N)_{,\phi} &= \left(\frac{24\pi^2}{V}+3\int\frac{V}{V_{,\phi}}d\phi\right)_{,\phi} \\ &= -\frac{24\pi^2 V_{,\phi}}{V^2}+3\frac{V}{V_{,\phi}} \\ &= 0 \\ \rightarrow \quad \frac{V_{,\phi}^2}{V^3} &= \frac{1}{8\pi^2}\end{aligned}$$

- Thus we have the lowest probability at the onset of eternal inflation!

[Battarra & JLL]