Inflation and Quantum Theory

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Contents

The superposition principle

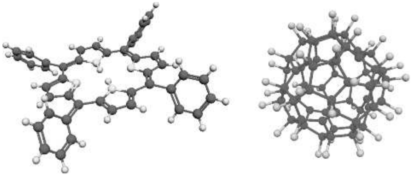
- Let Ψ_1 and Ψ_2 be physical states. Then, $\alpha\Psi_1 + \beta\Psi_2$ is again a physical state.
  For more than one degree of freedom, this leads to an entanglement between systems.
- Linearity of the Schrödinger equation: the sum of two solutions is again a solution.

Classical states only form a tiny subset of all possible states.

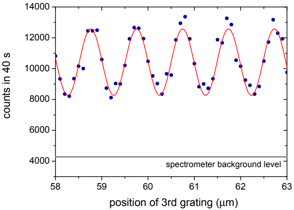
Erwin Schrödinger 1935:

I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. . . . Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*, even though they may be entirely separated . . .

A particular example (Vienna experiment)



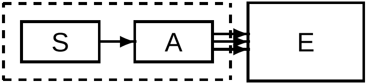
tetraphenylporphyrin ($C_{44}H_{30}N_4$) (left) and fluorofullerene $C_{60}F_{48}$ (right)



Interference pattern of tetraphenylporphyrin

L. Hackermüller *et al*., *Phys. Rev. Lett.* **91** (2003) 090408

Decoherence



The superposition principle leads to

$$\left(\sum_n c_n|n\rangle|\Phi_n\rangle\right)|E_0\rangle \quad \xrightarrow{t} \quad \sum_n c_n|n\rangle|\Phi_n\rangle|E_n\rangle$$

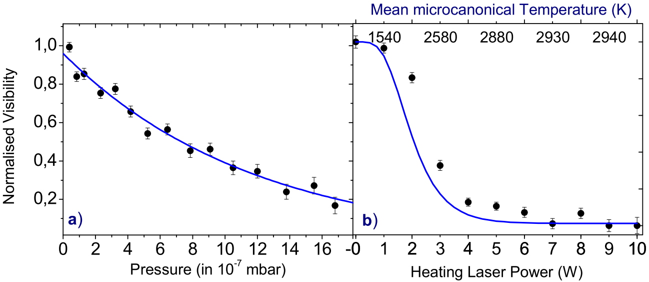
All *local* observations follow from the reduced density matrix for system plus apparatus:

$$\rho_{\mathrm{SA}} \approx \sum_n |c_n|^2|n\rangle\langle n| \otimes |\Phi_n\rangle\langle\Phi_n| \qquad \text{if} \qquad \langle E_n|E_m\rangle \approx \delta_{nm}$$

The interferences exist, but they are not ***there****.*

Decoherence: Emergence of classical properties through the unavoidable, ubiquitous, and irreversible interaction with the environment (Zeh 1970)

Experimental test of decoherence



Left: Decoherence through particle collisions.

Right: Decoherence through heating of fullerenes.

Figure credit: M. Arndt and K. Hornberger, `arXiv:0903.1614v1`

Interpretation

- Universality of unitary quantum theory: *Everett* or *many-worlds* interpretation
- Unitary quantum theory *plus* classical configurations: *de Broglie–Bohm theory*
- Unitarity-violating modifications: collapse models
- Pragmatic approaches such as the *Copenhagen interpretation*

Here, I will restrict myself to the first option.[1]

[1] For other approaches, cf. the talks by S. Das, P. Peter, N. Pinto-Neto, . . .

Quantum cosmology

Gell-Mann and Hartle 1990:

Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

Bryce DeWitt 1967:

Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

Particular approaches

- Quantum general relativity
  - Covariant approaches (dynamical triangulation, . . .)
  - Canonical approaches (quantum geometrodynamics, loop quantum gravity, . . .)
- String theory

Here: Wheeler–DeWitt equation for Friedmann–Lemaître universe plus scalar field plus small fluctuations of metric and field

$$\boxed{H_{\rm tot}\Psi = 0}$$

Semiclassical approximation

Expansion of the Wheeler–DeWitt equation in powers of m_{P}^{-2}

- *First order*: Hamilton–Jacobi equation for general relativity
- *Second order*: Quantum theory in curved spacetime; wave function is of the form

  $$\Psi \propto \exp(\mathrm{i} m_{\mathrm{P}}^2 S_0[\text{gravity}])\psi[\text{gravity}, \text{matter}]$$

  and obeys a Schrödinger equation with time defined from S_0.
- *Third order*: Quantum gravitational corrections $\propto m_{\mathrm{P}}^{-2}$.

Decoherence in quantum cosmology

How do the superpositions of different geometries decohere?

- *System*: global degrees of freedom (scale factor, inflaton, ...)
- *Environment*: small density fluctuations, gravitational waves, ...

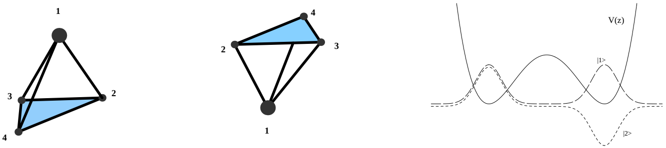
(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) (*system*) experiences decoherence by gravitons (*environment*) according to

$$\rho_0(a,a') \to \rho_0(a,a')\exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right)\ ,\ C>0$$

The Universe assumes classical properties at the beginning of inflation (Barvinsky, Kamenshchik, C.K. 1999)

Superpositions of states containing $\exp(\mathrm{i}m_{\mathrm{P}}^2 S_0[\mathrm{gravity}])$ with their complex conjugate experience decoherence through interaction with (for example) weak gravitational waves



Analogy: Molecular structure from decoherence by scattering events

Example for the decoherence factor (C.K. 1992):

$$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$$

This justifies the *semiclassical approximation* to quantum gravity.

Predictions in quantum cosmology

Anthropic interpretation We find ourselves in a decohered branch of the wave function that is suitable for life (*landscape* picture)

Peak in the wave function If the wave function is peaked around particular values of $a, \phi, \ldots$, this corresponds to the prediction that these values occur with high probability; if the wave function vanishes, the corresponding values are not allowed (relevant e.g. for singularity avoidance).

Semiclassical interpretation The wave function can only be interpreted in the semiclassical regime, where an approximate *WKB time* emerges.

Interpret heuristically a sharp peak in the wave function as a prediction: Inflation then occurs "naturally" if Ψ has a peak at a sufficiently large value of the inflaton field ϕ.

Inflation from quantum gravity

Which wave function, if any, does predict the occurrence of inflation?

- No-boundary condition: since $\psi_{\text{no-boundary}} \sim \exp\left(\frac{1}{3V(\phi)}\right)$, it favours *small* values of ϕ unsuitable for inflation
- Tunnelling condition: since $\psi_{\text{tunnel}} \sim \exp\left(-\frac{1}{3V(\phi)}\right)$, it favours *large* values of ϕ potentially suitable for inflation

Beyond tree-level approximation?

- This result also holds at the one-loop order (Barvinsky and Kamenshchik 1990)
- In this way, Higgs inflation[2] and natural inflation[3] can be predicted from the tunnelling wave function

[2] Barvinsky, Kamenshchik, Steinwachs, C.K. (2010)
[3] Calcagni, Steinwachs, C.K. (2014)

Hierarchy of classicality

Full quantum state of gravity plus matter

$\Downarrow$

Classicality of ‘background’ gravitational field

$\Downarrow$

Classicality of other ‘background’ fields

$\Downarrow$

Classicality of primordial fluctuations

Primordial fluctuations

What are they?

The primordial fluctuations in cosmology are small perturbations of the metric and the inflaton field; they can be described by a massless scalar field ϕ in a (here, flat) Friedmann–Lemaître universe.

One has tensor modes (gravitons)[4] and scalar modes.

Fourier transform: $y_k := a\phi_k = y^*_{-k}$

In the following (for simplicity): one real mode y

[4] If the original interpretation of the BICEP2 observations were correct, this would be the first empirical test of quantum gravity.

Quantum description of the modes

Schrödinger equation per mode:

$$\mathrm{i}\hbar \frac{\partial \psi(y,t)}{\partial t} = \hat{H}\psi(y,t)$$

Hamiltonian per mode:

$$\hat{H} = \frac{1}{2}\left(p^2 + k^2 y^2 + \underbrace{\frac{2\dot{a}}{a} yp}_{\to \text{squeezing}} \right)$$

Initial condition given by ground state:

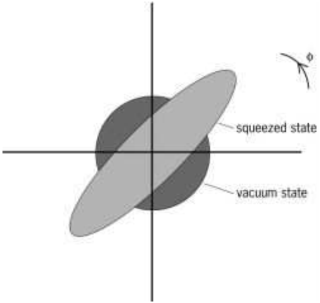
$$\psi(y,t) = \left(\frac{2\Omega_{\mathrm{R}}(t)}{\pi}\right)^{1/4} \exp\left(-\Omega(t) y^2\right) \ , \quad \Omega := \Omega_{\mathrm{R}} + \mathrm{i}\Omega_{\mathrm{I}}$$

The dynamics generates a squeezing of the quantum state:[5]

$$\Omega_{\mathrm{R}} = \frac{k}{\cosh 2r + \cos 2\varphi \sinh 2r} \,, \qquad \Omega_{\mathrm{I}} = -\Omega_{\mathrm{R}} \sin 2\varphi \sinh 2r \,,$$

with $\Omega_{\mathrm{R}} = k$ and $\Omega_{\mathrm{I}} = 0$ for the initial state.

r: squeezing parameter
φ: squeezing angle



[5]Grishchuk and Sidorov (1989)

Pure exponential inflation

The exponential evolution

$$a(t) = a_0 \exp(H_{\rm I} t)$$

for the scale factor leads to

$$\sinh r = \frac{aH_{\rm I}}{2k} \ , \quad \cos 2\varphi = \tanh r \ .$$

(largest cosmological scales: $r \approx 120$, $\varphi \approx 0$)

Quantum-to-classical transition and entropy

- Primordial fluctuations (if treated as isolated): For $r \to \infty$ we have a semiclassical quantum state; expectation values are indistinguishable from classical stochastic mean values.[6]
- Highly squeezed states are extremely sensitive to interactions – decoherence of primordial fluctuations[7]

Simplest case of ideal interaction (only entanglement):

$$\rho_0(y, y') \longrightarrow \rho_\xi(y, y') = \rho_0(y, y') \exp\left(-\frac{\xi}{2}(y - y')^2\right)$$

ξ: Phenomenological parameter; contains details of interaction

[6] Polarski and Starobinsky (1996)
[7] Polarski, Starobinsky, C.K. (1998)

Realistic situation (decoherence condition):

$$\xi \gg \Omega_{\mathrm{R}} \approx k\mathrm{e}^{-2r}$$

Decoherence time scale for inflation: $t_{\mathrm{d}} \sim H_{\mathrm{I}}^{-1}$
Analogy to chaotic systems: H_{I} corresponds to the Lyapunov parameter

Axes of the Wigner ellipse for large r:

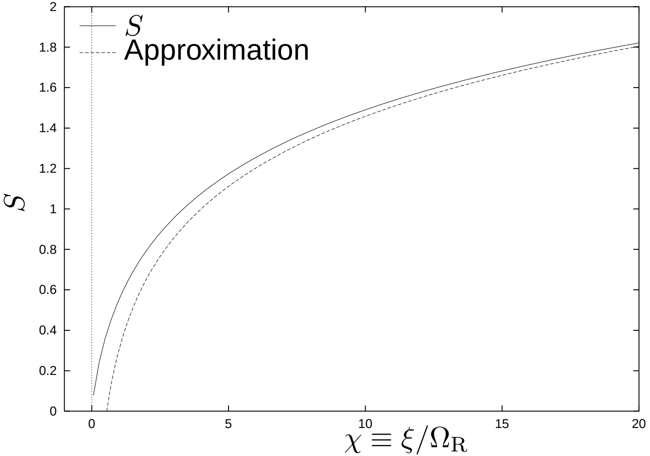
$$\alpha \approx \mathrm{e}^{r} \; , \quad \beta \approx \sqrt{\frac{\xi}{k}} \gg \mathrm{e}^{-r}$$

Correlation condition:

$$\frac{\xi}{k} \ll \mathrm{e}^{2r}$$

Entropy

$$S = -\mathrm{tr}(\rho_\xi \ln \rho_\xi)$$



$$\boxed{S < r = S_{\max}/2}$$

Decoherence time and pointer states

Reconsider the master equation for localization:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\frac{\mathrm{i}}{2m\hbar}[\hat{p}^2, \hat{\rho}] - \Lambda[\hat{x}, [\hat{x}, \hat{\rho}(t)]]$$

- Wigner function becomes positive after a certain time t_d, independent of the initial state (Diósi and C.K. 2002)
- The reduced density operator $\hat{\rho}$ can for $t > t_\mathrm{d}$ be decomposed in the form

  $$\hat{\rho} = \int \mathrm{d}\Gamma \; P(\Gamma, t)|\Gamma\rangle\langle\Gamma| \; , \quad P(\Gamma, t) \geq 0 \; ,$$

  where $|\Gamma\rangle$ denotes a set of Gaussian states (*op. cit.*).
- Generalization to quadratic Hamiltonians and general Lindblad equation[8] and Non-Markovian situations[9]

[8] Brodier and Ozoro de Almeida (2004)

[9] Eisert (2004)

General master equation:

$$\frac{\partial \hat{\rho}}{\partial t} = -\mathrm{i}[\hat{H}, \hat{\rho}] + \hat{L}\hat{\rho}\hat{L}^\dagger - \frac{1}{2}\hat{L}^\dagger\hat{L}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{L}^\dagger\hat{L}$$

Write Lindblad operator in the form

$$L = (\mathbf{l}' + \mathrm{i}\mathbf{l}'') \cdot \begin{pmatrix} p \\ y \end{pmatrix} \ ,$$

where the real components of the ‘vectors’

$$\mathbf{l}' = \begin{pmatrix} \lambda' \\ \mu' \end{pmatrix} \quad \text{and} \quad \mathbf{l}'' = \begin{pmatrix} \lambda'' \\ \mu'' \end{pmatrix}$$

contain all the information on the interaction of the system with its environment

Results

- Pointer basis is given by the field-amplitude basis
- For modes outside the Hubble radius during inflation:

$$t_{\rm d} \sim H_{\rm I}^{-1} \ln \frac{H_{\rm I}^{-1}|\lambda' \mu'|}{a_0}$$

(as seen also in concrete models)

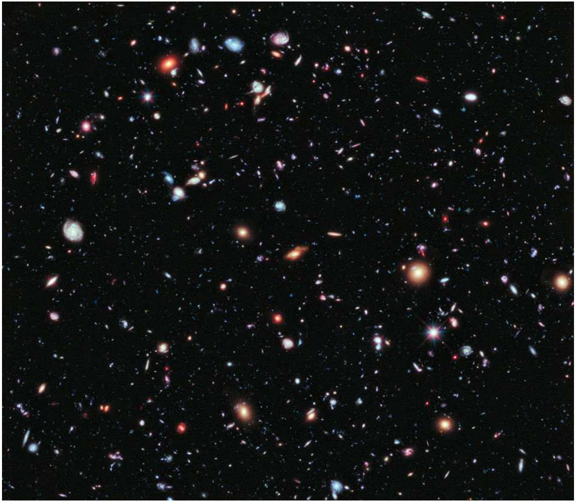
- Outside but radiation dominance:

$$t_{\rm d} \sim \frac{H_{\rm I} a_{\rm e}^2}{2(\lambda' \mu')^2}$$

- Inside Hubble radius:
  $t_{\rm d}$ dominated by dissipation

(Lohmar, Polarski, Starobinsky, C.K. 2007)

Origin of structure from quantum fluctuations



Analogy in nuclear physics: deformed nuclei (Zeh 1967)