The curvaton scenario: Pre and “post” Planck

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CB, Cortes & Liddle 2014 + numerous older papers + 2 works in progress

We have an open postdoc position in cosmology, deadline January 2nd

IAP: The primordial universe after Planck; 16th December 2014 - 20 min

The curvaton scenario

- An alternative model to single-field inflation for the origin of structures
- The inflaton drives inflation while the curvaton generates curvature perturbations (hence the name)
- This "liberates" the inflaton, at the expense of making inflation less predictive
- We now have two light degrees of freedom during inflation, sensitive to two potentials and initial conditions.

- The curvaton is a light field which
  1. has a subdominant energy density during inflation
  2. Is long lived (compared to the inflaton)
  3. Generates the primordial curvature perturbation

- We will often drop assumption 3, and consider the mixed inflaton-curvaton scenario

Curvaton phenomenology

- Adding one extra field allows for interesting new phenomenology which single-field inflation cannot generate

  1. Large local non-Gaussianity
  2. Isocurvature perturbations

Observations don't (currently) require a second field, but high energy theories might

A brief history: The trilogy from 2001: Enqvist and Sloth, Lyth and Wands (who created the name and got ~900 citations), Moroi and Takahashi.

Plus two related older papers, Linde and Mukhanov (1996), Mollerach (1990)

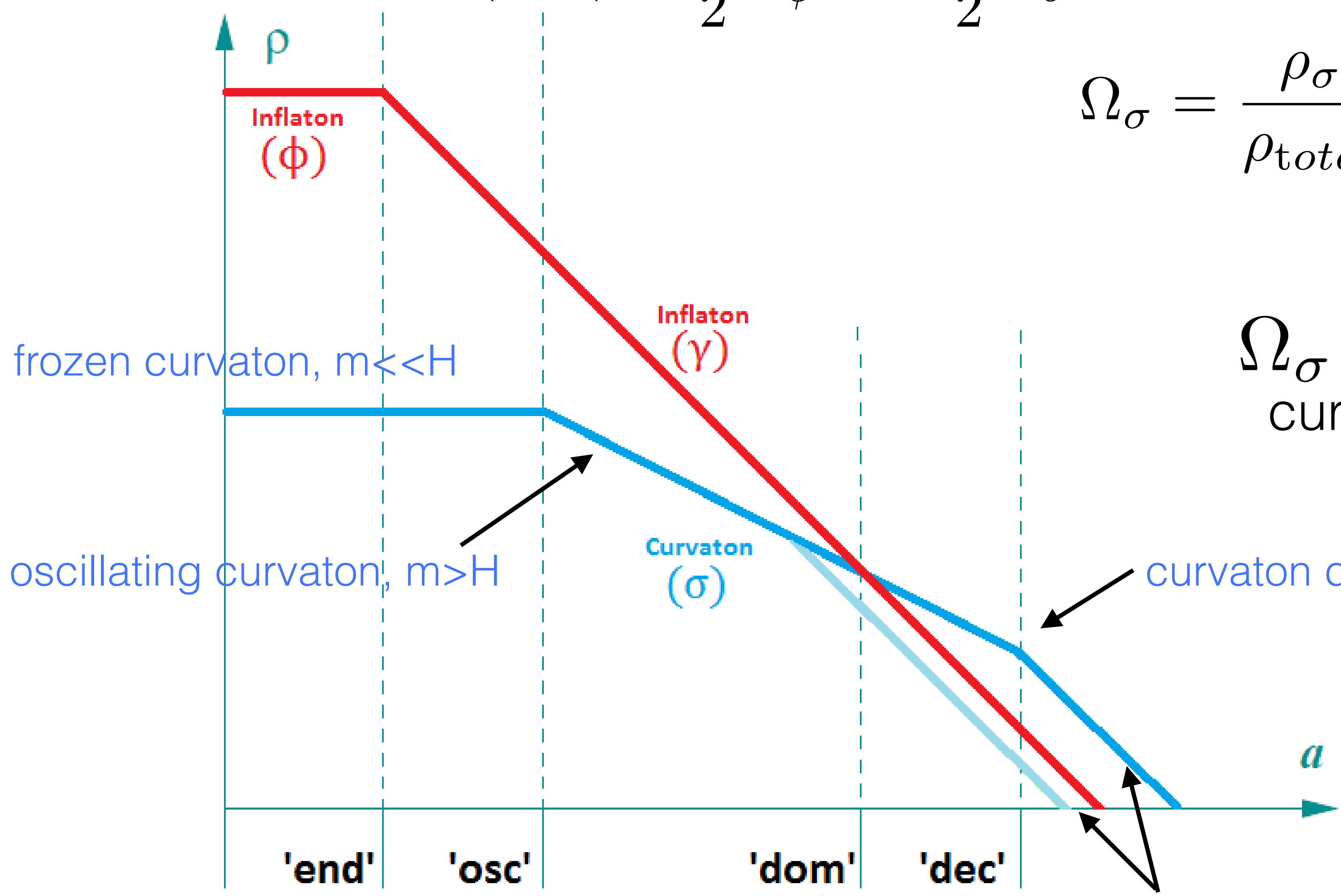
Curvaton (σ) background evolution:

Log of scale factor versus log of energy density

$$V(\phi, \sigma) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\sigma^2 \sigma^2$$

$$\Omega_\sigma = \frac{\rho_\sigma}{\rho_{total}}$$ measured at the curvaton decay time

$\Omega_\sigma = 1$ is an attractor if the curvaton decays late enough



curvaton decays into radiation

The curvaton may decay before or after it becomes dominant

The longer the curvaton lives, the larger its relative energy density becomes, as measured by r_{dec}

What non-Gaussianity does the (quadratic) curvaton predict?

- The curvature perturbation is approximately $\zeta \simeq \Omega_\sigma \frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \Omega_\sigma \left(\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2 \right)$
- Local non-Gaussianity is generated: $f_{NL} \sim 1/\Omega_\sigma$
- The Planck constraint $f_{NL}<10$, tells us $\Omega_\sigma>0.1$. A priori, $\Omega_\sigma \sim 10^{-5}$ (and $f_{NL} \sim 10^5$) was possible.
- If the curvaton dominates before decay, $\Omega_\sigma=1$ and $f_{NL}=-5/4$
- In terms of a linear scale on $-5/4<f_{NL}<10^5$ - 99.99% has already been ruled out
- In terms of a linear scale on $10^{-5}<\Omega_\sigma<1$ - 10% has been ruled out
- A strongly subdominant curvaton is totally ruled out, so the dominant curvaton case becomes our "prediction". Detecting $f_{NL}=3$ or 7 seems unlikely, although it is compatible with the model

Isocurvature perturbations

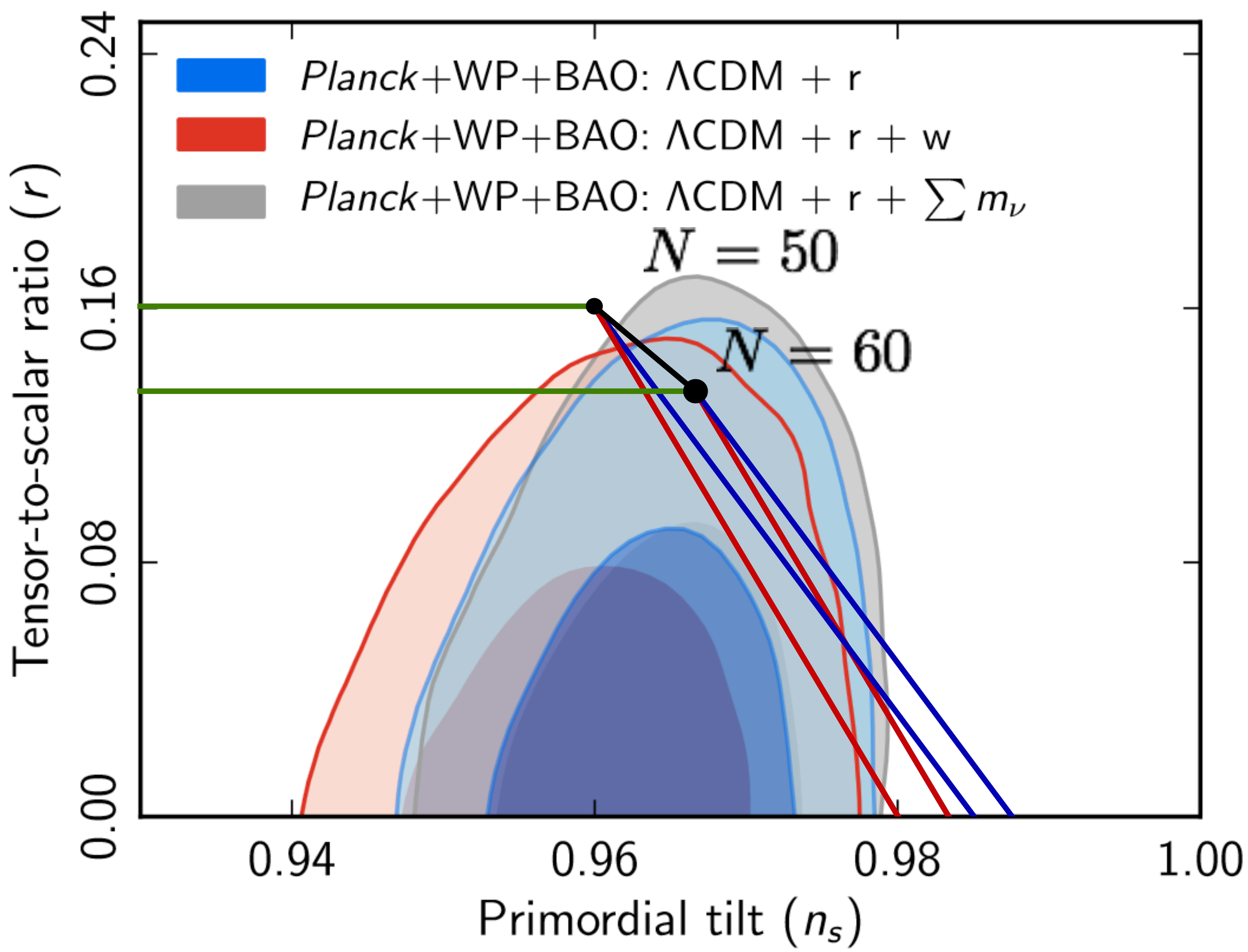
- Cosmological perturbations may be of two classes, adiabatic or isocurvature - Bartjan's talk today
- Adiabatic perturbations mean that locally all parts of the universe look the same, so e.g. the ratio of photons to baryons to CDM is the same everywhere
- The curvaton can generate isocurvature perturbations (most multi-field models can, single-field models never can), but if the universe thermalises after curvaton decay then none will survive.
- Theorists are not really able to interpret the 1% level isocurvature constraints in terms of early universe models, the thermal history of the universe prior to BBN is poorly understood

The simplest curvaton scenario

$$V(\phi, \sigma) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\sigma^2\sigma^2$$

- Parameter constraints were originally made by Bartolo and Liddle (2002), the data allowed so much freedom they restricted the model to i) the Gaussian case ii) negligible inflaton perturbations
- CB, Cortes and Liddle (2014) revisited the model with Planck data. Even dropping those two assumptions we find the model is close to being ruled out. Observational data has improved a lot.
- We also allow the inflating curvaton scenario, in which the curvaton drives a second period of inflation. Applies when sigma$_*$>M_{Pl}.

Curvaton post Planck1



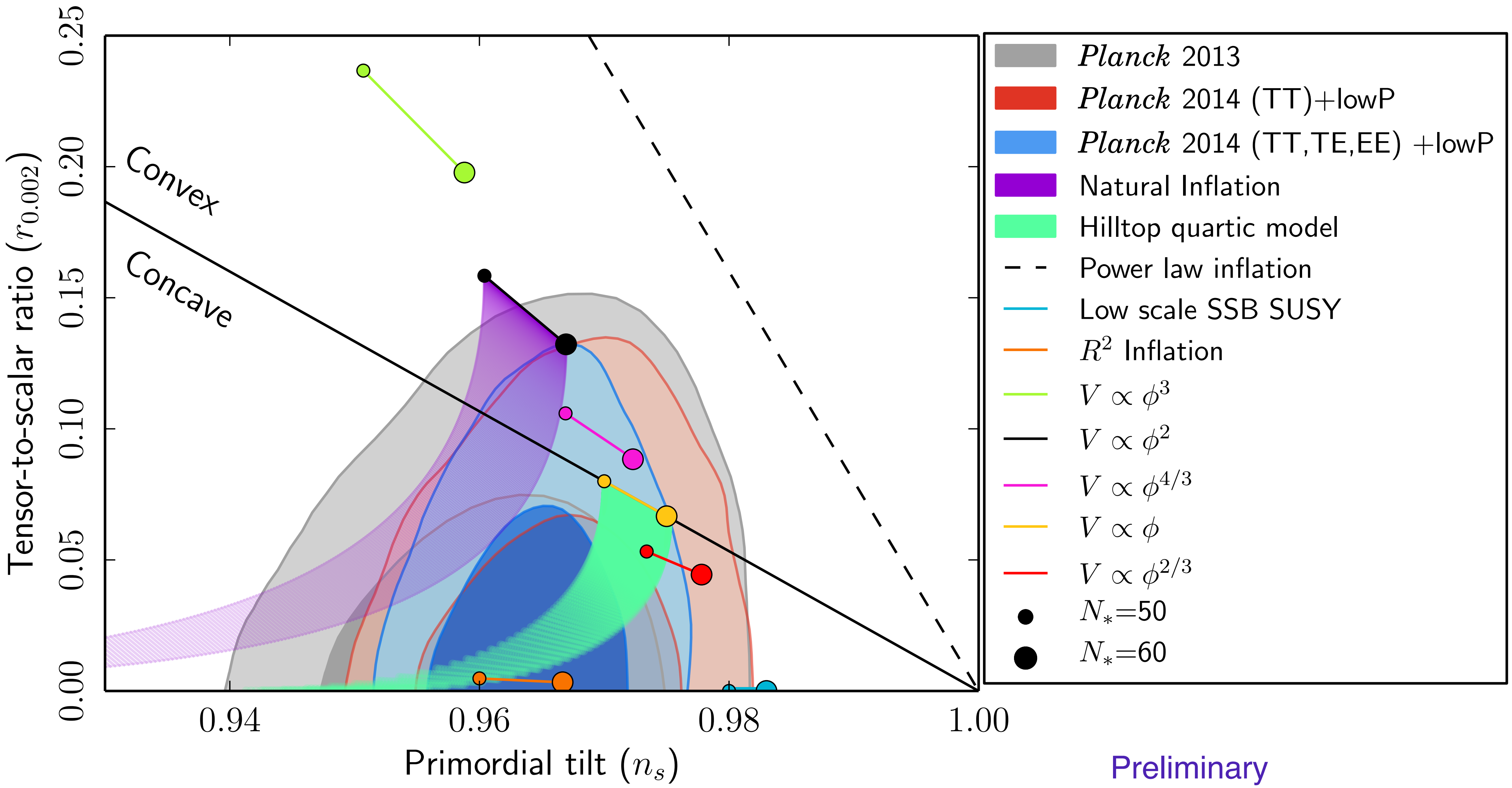
The uncertainty in matching the Planck pivot scale to N is significant. We don't know the expansion history of the universe between inflation and BBN. Smaller values of N are possible, which the data prefers.

Red lines are for a negligible curvaton mass

Blue lines have m_sigma=m_phi/2 (it is hard to make the curvaton heavier, and a bluer spectrum results)

Green lines are the inflating curvaton regime, where it drives a second period of inflation

Post Planck2?



From Finelli's talk in Ferrara Dec 14.

Only relevant change if the E mode polarisation is included, then r=0 and n_s=0.98 is more than 2 sigma ruled out. This further rules out the two field scenario.

We need to wait for the joint BICEP/Planck analysis to see what happens to the tensor constraint. Will the quadratic single field model survive?

Planck data alone may be close to ruling out a two field model

$$V(\phi, \sigma) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\sigma^2 \sigma^2$$

The simplest curvaton model, with 5 free parameters no longer provides an excellent match to the data **anywhere in its (large) parameter space**.

This model has five free parameters, all of which can vary by many orders of magnitude. But there are still only specific areas of the ns vs r plot which the model can fill. It is not the non-Gaussianity constraints which is putting the curvaton model under pressure.

The five free parameters are:

The two mass parameters

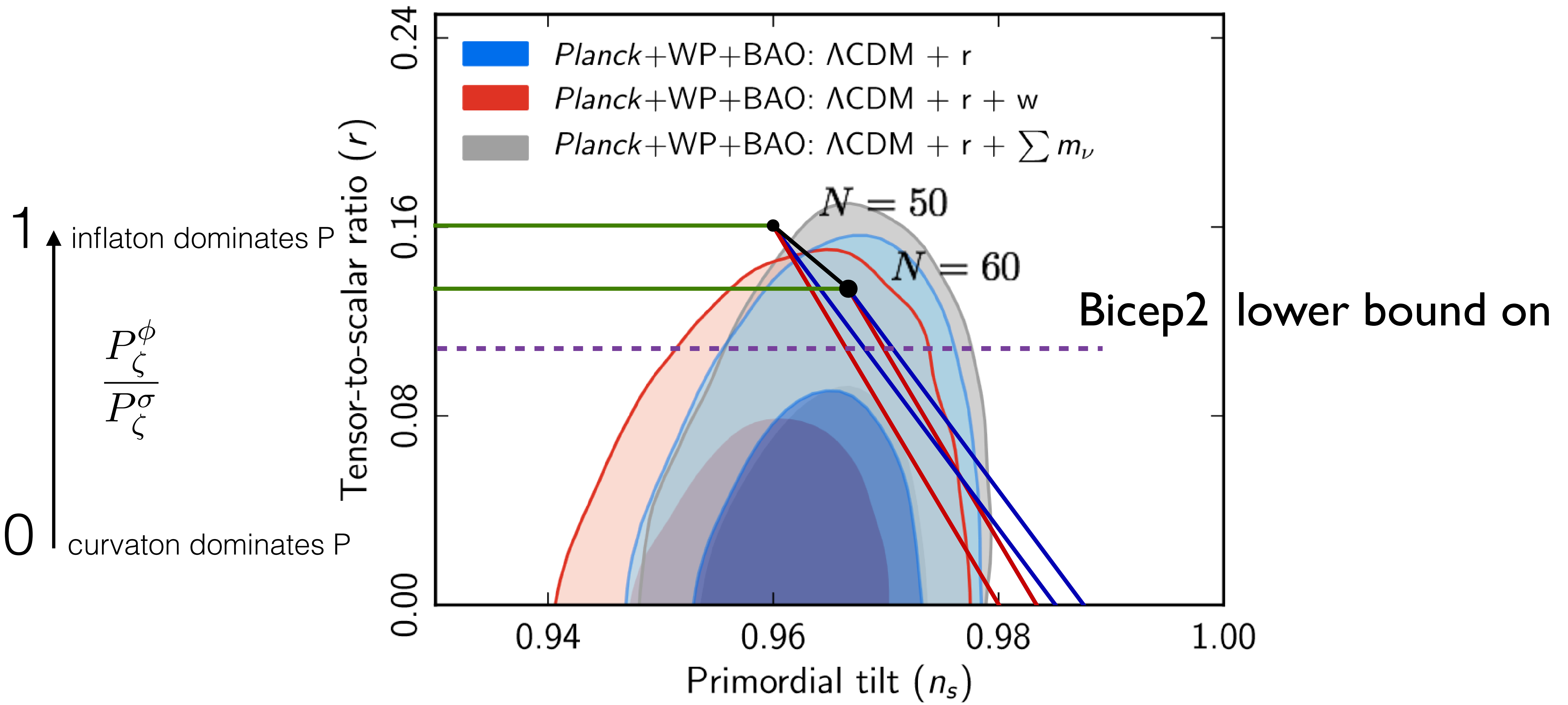
The decay rates of each field

The initial value of the curvaton field (the initial inflaton VEV is determined by the thermal history via N)

The pure curvaton scenario makes an additional prediction that the tensors are negligible

A Bayesian model comparison is in progress, results soon after we have Planck data

Curvaton post Planck1 and BICEP2



Red lines are for negligible curvaton mass, blue lines have m_sigma=m_phi/2. Green lines are the inflating curvaton regime, where it drives a second period of inflation.

BICEP2 adds a lower bound on the tensor to scalar ratio, which requires that the inflaton perturbations contribute at least 50% of the total curvature perturbation. If confirmed, this rules out the original curvaton scenario, in which the inflaton perturbations and hence r are negligible.

A difficult time for curvaton fans?

- If confirmed with Planck, the BICEP2 detection of large tensor modes has ruled out the original curvaton scenario

- Mixed scenarios in which both the inflaton and curvaton contribute to the primordial curvature perturbation can never be ruled out by a detection of tensors

- We can (or could) take a positive view, either a large negative running of the curvaton (Sloth 2014) or anti-correlated isocurvature modes (Kawasaki & Yokoyama 2014) as means to suppress the large scale power and alleviate possible Planck/BICEP tension

- Planck did significantly improve the constraints on both local non-Gaussianity and isocurvature perturbations, but there was no detection of either. This makes the curvaton phenomenology less interesting.

- However, the curvaton does not in any way require the existence of isocurvature perturbations today, and a natural limit of non-Gaussianity is local f_{NL}=-5/4. So Planck data does not come close to ruling it out all curvaton scenarios.

- Planck data alone puts pressure on the simplest curvaton scenario

Model independent curvaton statements

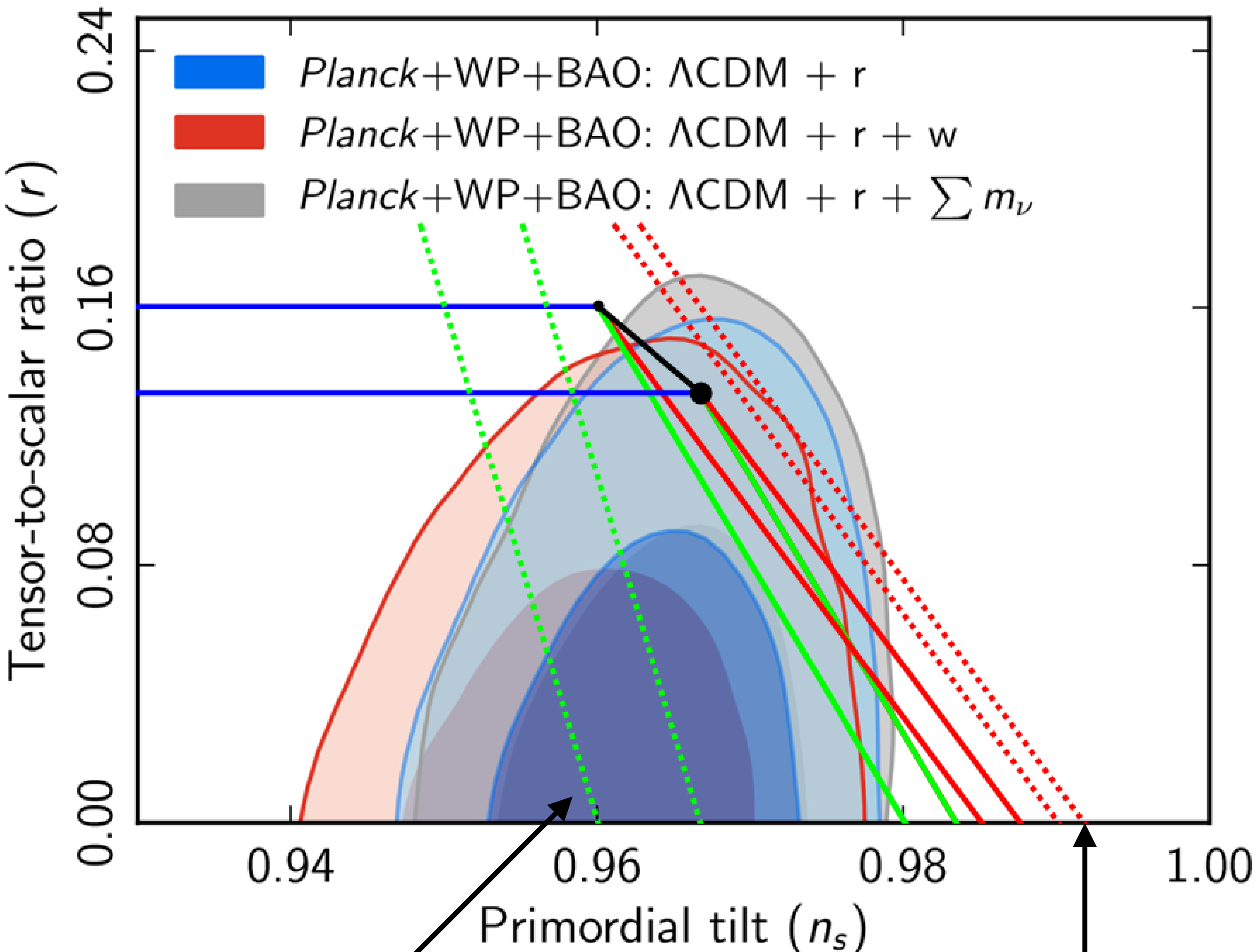
- The pure curvaton scenario has a suppressed tensor spectrum, a detection of r can force us into the mixed inflaton-curvaton scenario
- By tuning the inflaton potential, any value of n_s and r can be achieved with a quadratic curvaton
- A detection of (local) $f_{NL}<-5/4$ would rule out all quadratic curvaton models (but not non-qadratic curvaton potentials)
- A constraint $|f_{NL}|<1$ would be a very strong hint against all curvaton scenarios, independently of the potential of either field (even independently of the number of curvaton and inflaton fields)

Filling the n_s-r plane

$$V = \lambda\phi^4 + \frac{1}{2}m^2\sigma^2$$

For this simple potential, the mixed inflaton-curvaton scenario can lie anywhere between the outer dashed green and red lines

Credit: Robert Hardwick (MSc student)



$$N = 50 \ and \ \eta_\sigma \simeq 0$$

$$N = 60 \ and \ \eta_\sigma \simeq \frac{1}{4}\eta_\phi$$

The early universe is very poorly constrained

- The curvaton scenario really is different from single-field inflation
- During inflation we have a second, perturbed degree of freedom
- From the end of inflation until after the curvaton decays, the universe behaves very differently. Both at the homogeneous and the perturbed level.
- What was the background equation of state during baryogenesis? Did isocurvature perturbations exist? Are the perturbations on these small scales Gaussian? We have no idea.
- Because the perturbations are so tiny, f_{NL}=-5/4 is a small correction, except when the amplitude of perturbations is large. For small scale perturbations where power spectrum bounds are very weak, this value has a huge effect. Example: Primordial black hole formation rates - S. Young & CB 2013

We lack guidance - see Cliff's talk

Wilsonville

Occamopolis
≠ Bayes cité

- Model building is important: for model comparison we need to know the priors (or make an arbitrary choice)
- Models are incomplete if they do not specify reheating/thermal history, observations are now sensitive enough to care, e.g. whether N=50 or 60 - Ringeval's talk
- The data is not good enough to distinguish between many classes of models, in some cases it will never ever be (two models can predict exactly the same CMB spectrum)

- Top down and bottom up approaches are needed

Why are the curvaton and inflaton scenarios so hard to distinguish?

- The models were not made to look similar by fine tuning
- A simple non-linear transformation, x -> $x+x^2$, generates order unity non-Gaussianity, i.e. f_{NL}~1
- This is the same level as the secondary non-Gaussianity's present for all models, e.g. the ISW-lensing bispectrum detected by Planck
- These two types of non-Gaussianity are distinguishable
- f_{NL}=-5/4 is hard to detect only because the perturbations are tiny. It is a 0.001% effect
- But this might be the only way to distinguish the two scenarios

Related models

- Several other models predict essentially identical phenomenology (local non-Gaussianity, isocurvature perturbations and suppressed tensor perturbations)
- For example
    1. Modulated reheating (the efficiency of reheating is a function of position)
    2. Inhomogeneous end of inflation (inflation ends later in some positions)
    3. Models with a subdominant field curving the trajectory during inflation
- This is not a coincidence, all models are tracking the conversion of an initial isocurvature perturbation (corresponding to a light and subdominant field) into the adiabatic perturbation after inflation
- The models are physically different, and detailed predictions for the simplest realisations do vary
- For various classes of models, f_{NL}~1 is a natural target (including the curvaton with any potential) - c.f. r~$(n_s$-1$)^2$ - Linde's talk
- In particular, multifield models in which all fields decay independently and at the same time "typically" predict f_{NL}~0 and are probably indistinguishable from single-field models

Conclusions

- If a detection of (large) r is made, the original curvaton scenario in which the inflaton perturbations can be neglected is ruled out

- Ignoring BICEP2, Planck alone has put pressure on the simplest curvaton scenario (quadratic inflaton and curvaton potentials), due to a combination of the spectral index and r.

- The above is true even if we allow an arbitrary proportion of the perturbations to come from the inflaton (we also allow the curvaton to drive a second period of inflation). The data is good enough to start ruling out two-field scenarios

- Non-Gaussianity constrains the curvaton to not be too subdominant, but are a long way from testing the f_{NL}=-5/4 limit. If non-G is detected, we could learn a lot.

- Without a detection of local non-Gaussianity or isocurvature perturbations we will never need a curvaton type mechanism, but this does not imply the curvaton didn't exist. How should we proceed?

Curvaton evolution

$$V = \frac{1}{2} m_\sigma^2 \sigma^2$$

- For simplicity, we initially assume a quadratic potential for the curvaton, most papers in the literature do so

$$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0,$$
$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V_{,\sigma\sigma}\delta\sigma = 0.$$

- Just for a quadratic potential, the two evolution equations are the same. This implies that the ratio of the two solutions is constant in time. The second equation neglects back reaction from gravity, accurate as long as its energy is subdominant

Curvaton density perturbations

- The curvaton density perturbation $$\frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma+\delta\sigma)-V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$$
- The truncation at second order follows if we assume a quadratic curvaton potential, deviations can be tested/constrained by g_{NL}
- The curvaton perturbation are Gaussian, the above form of non-Gaussianity, Gaussian + Gaussian squared is known as the local form of non-Gaussianity
- The above formula matches the local model of non-Gaussianity, and if the above was the final result for zeta we would have $f_{NL}\sim 1$
- Gravity is non-linear, so further non-Gaussianities will be generated in all models, this also generates $f_{NL}\sim 1$, but with a different shape which can be observationally distinguished
- However, we should consider that the curvaton is not the only component of the universe

$$\zeta = \Omega_\sigma \zeta_\sigma \qquad \Omega_\sigma = \rho_\sigma/\rho_{tot}$$

$$f_{\rm NL} \propto \zeta^{(2)}/\zeta^{(1)2} \propto 1/\Omega_\sigma$$

Corrections to f_{NL}

- The basic result is correct, the less efficient the transfer from the curvaton perturbation to total curvature perturbation, the larger the non-Gaussianity becomes. This holds quite generally

- The "full" result is

$$f_{NL} = \frac{5}{4r_{\rm dec}} - \frac{5}{3} - \frac{5}{6}r_{\rm dec} \qquad r_{\rm dec} \equiv \left.\frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma}\right|_{\rm decay}$$

- If f_{NL} is large, $f_{\rm NL} \propto \frac{1}{r_{\rm dec}} \propto \frac{1}{\Omega_\sigma}$

- The Planck constraint, $f_{NL}<10$, tells us $r_{dec}>0.1$. A priori, $r_{dec}\sim10^{-5}$ (and $f_{NL}\sim10^5$) was possible.

- If the curvaton dominates before it decays $f_{NL}=-5/4$

What does the curvaton predict?

- The Planck constraint $f_{NL}<10$, tells us $\Omega>0.1$. A priori, $\Omega r_{dec} \sim 10^{-5}$ (and $f_{NL} \sim 10^5$) was possible. If the curvaton dominates before decay $f_{NL}=-5/4$

- In terms of a linear scale on $-5/4<f_{NL}<10^5$ - 99.99% has already been ruled out

- In terms of a linear scale on $10^{-5}<r_{dec}<1$ - 10% has been ruled out

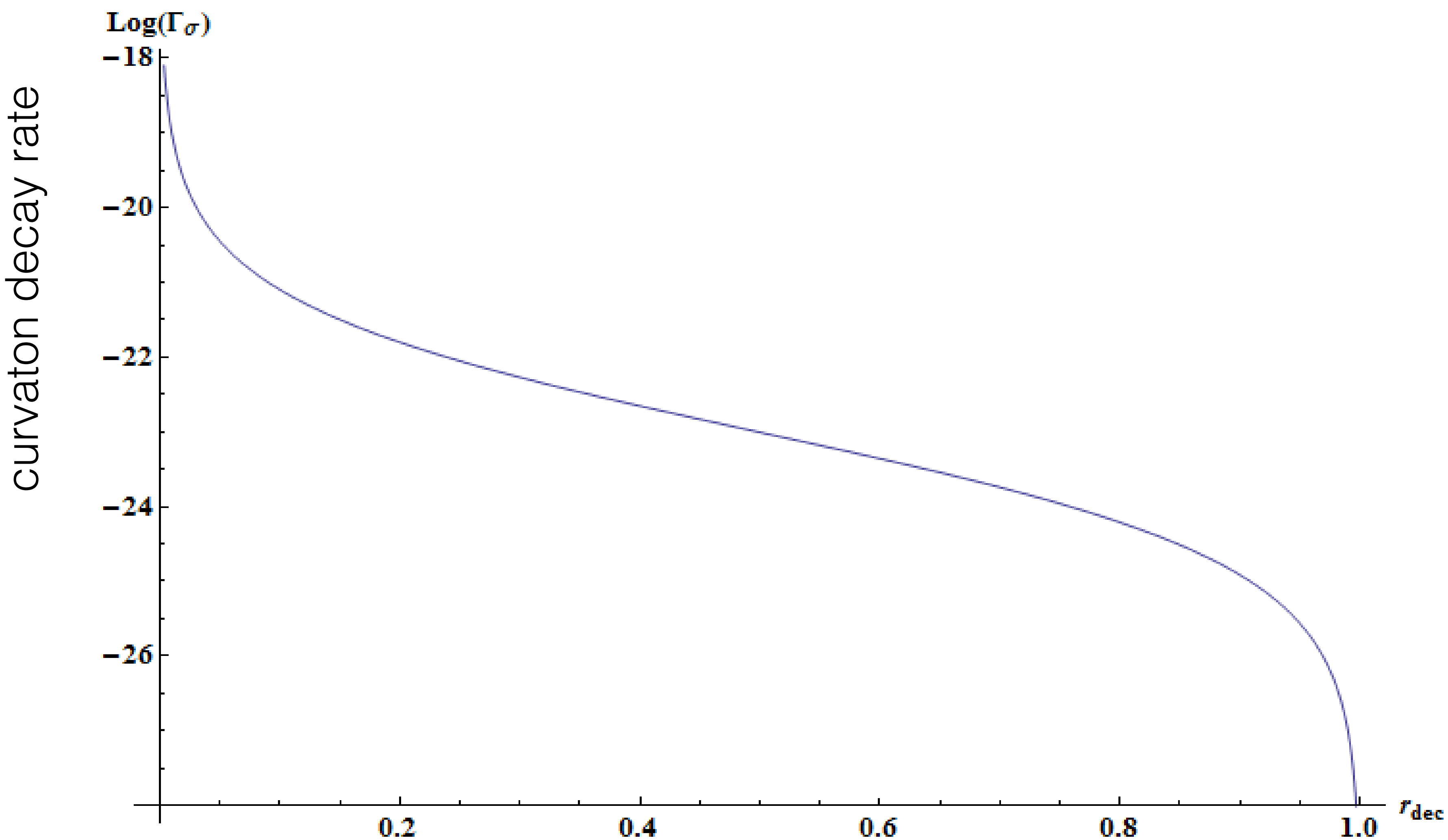
- A priori, we could think of the two extremes as being natural

1. $\frac{\delta\sigma}{\sigma} \sim 1 \Rightarrow r_{\rm dec} \sim 10^{-5}, \quad f_{\rm NL} \sim 10^5$ coming from the COBE normalisation
2. A sufficiently late decaying curvaton, such that $r_{\rm dec} = 1, \quad f_{\rm NL} = -\frac{5}{4}$

- The first case is totally ruled out, so the second case becomes our "prediction". Detecting $f_{NL}=3$ or 7 seems unlikely, although it is compatible with the model

Curvaton decay rate vs r_{dec}

The curve will shift for different choices of masses and initial curvaton vev. But the shape remains the same.
For small curvaton decay rates, r_{dec}->1



Credit: Robert Hardwick

$$r_{\rm dec} \equiv \left.\frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma}\right|_{\rm decay}$$

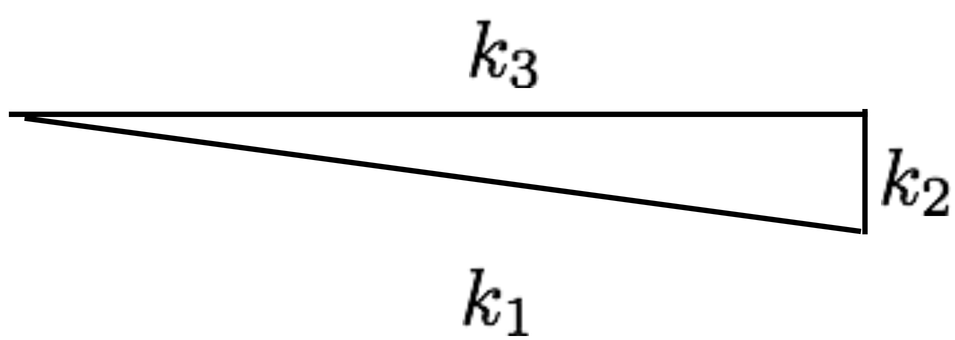
The local model of non-Gaussianity

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x}) \rangle)$$

- The local model which arises from super-horizon evolution of the curvature perturbation
- Zeta is conserved in single-field models on large scales, therefore this model only arises in models with multiple light fields present during inflation
- The Planck constraint (and WMAP9 in brackets) are

$$f_{\rm NL} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9)$$

- Using the power spectrum amplitude, we see that the CMB is at least 99.9% Gaussian for this model.
- This shape has its largest signal in the squeezed limit, when one wavelength is very large
- Because a detection of a squeezed limit bispectrum would rule out all single-field models, the local model has been studied in great depth



$$k_2 \ll k_1 \simeq k_3$$

Komatsu et al; Decadel review 2009

Non-Gaussianity summary

- All single-source models must obey a relation between one trispectrum parameter and f_{NL}

$$\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- If multiple-fields contribute to zeta (eg the curvaton and inflaton), then

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- A large g_{NL} would signal a non-quadratic potential for the curvaton

- f_{NL} will be scale dependent unless the curvaton potential is quadratic and the inflaton fluctuations are negligible

- Previous scale dependence work with Sami Nurmi, Kari Enqvist and Tomo Takahashi. Work in progress with Ewan Tarrant on strong scale dependence case, where the existing formalism breaks down

A general test of single-source models

- For all models in which only one field generates the primordial curvature perturbation (other than the inflaton), there is a consistency relation between one term of the trispectrum and bispectrum

$$\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- In models where multiple fields contribute there is instead the Suyama-Yamaguchi inequality

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- For the mixed inflaton curvaton scenario

$$\tau_{\rm NL} = \frac{P_\zeta}{P_\sigma}\left(\frac{6 f_{\rm NL}}{5}\right)^2 \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- From Planck, $\mathrm{tau}_{\rm NL}$<2800 (95% confidence)

Scale-dependence of f_{NL}

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- Analogously to the power spectrum, f_{NL} is expected to have some scale dependence. This reflects evolution during inflation, e.g. it ends

- It can distinguish between different non-Gaussian scenarios, not just between Gaussian and non-Gaussian models
- The amplitude of f_{NL} can be tuned in most non-Gaussian models, so a precise measurement of f_{NL} wont do this

- In contrast, the scale dependence often can not be tuned independently of:
  1. f_{NL}
  2. spectral index of the power spectrum

- Scale dependence arises from either multiple fields contributing to zeta, or due to self-interactions in the potential (leading to non-linear equations)

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

CB et al 2010

What does the curvaton predict?

- In terms of a linear scale on $-5/4<f_{NL}<10^5$ - 99.99% has already been ruled out
- In terms of a linear scale on $10^{-5}<r_{dec}<1$ - 10% has been ruled out
- These results change a lot if taking a log scale instead of linear

- A priori, we could think of the two extremes as being natural

1. $\frac{\delta\sigma}{\sigma} \sim 1 \Rightarrow r_{\rm dec} \sim 10^{-5}, \;\; f_{\rm NL} \sim 10^5$ coming from the COBE normalisation
2. A sufficiently late decaying curvaton, such that $r_{\rm dec} = 1, \;\; f_{\rm NL} = -\frac{5}{4}$

- The first case is totally ruled out, so the second case becomes our "prediction". Detecting f_{NL}=3 or 7 seems unlikely, although it is compatible with the model

Mixed inflaton-curvaton scenario

All light fields are perturbed during inflation, we will now include the inflaton field perturbations

The power spectra due to the two fields is

$$P_\phi \sim \frac{1}{\epsilon}\left(\frac{H_*}{2\pi}\right)^2, \qquad P_\sigma \sim \Omega_\sigma^2 \frac{1}{\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2,$$

and the total power spectrum is

$$P_\zeta = P_\phi + P_\sigma .$$

The bispectrum is unchanged from the pure curvaton limit

$$B_\zeta = B_\sigma = \frac{1}{\Omega_\sigma} P_\sigma^2$$

but $f_{\rm NL}$ is reduced because the power spectrum is enhanced by the Gaussian inflaton field perturbations

$$f_{\rm NL} \sim \frac{B_\zeta}{P_\zeta^2} = \frac{B_\sigma}{P_\zeta^2} = \frac{1}{\Omega_\sigma}\frac{P_\sigma^2}{P_\zeta^2}.$$

The tensor-to-scalar ratio is also reduced

$$r = 16\epsilon \frac{P_\phi}{P_\zeta}.$$

Higher-order non-Gaussianity

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \left(\zeta_G^2 - \langle \zeta_G^2 \rangle \right) + \frac{9}{25} g_{NL} \zeta_G^3$$

$$\frac{\delta \rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma + \delta\sigma) - V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma} \right)^2$$

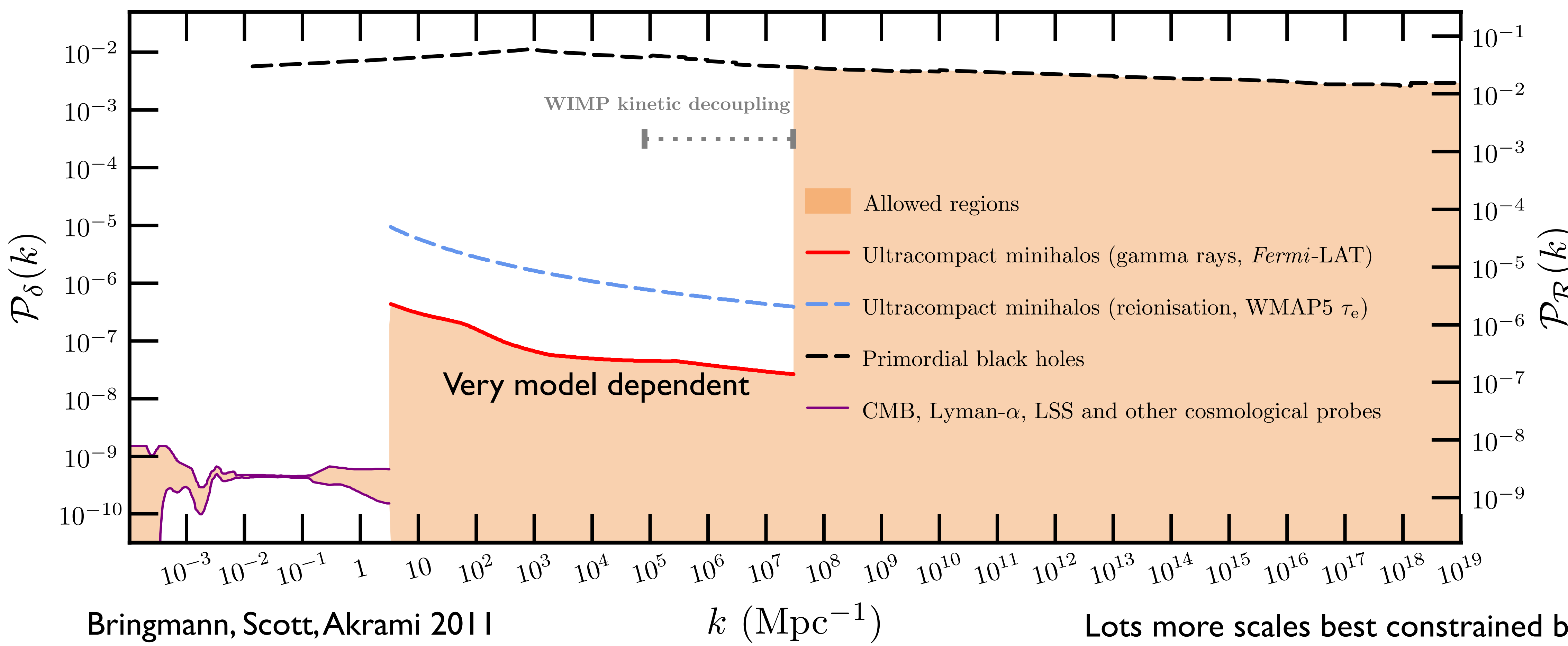
- For a quadratic potential, we may truncate at second order, which implies $g_{NL}=0$. Quadratic potentials are simple to calculate with, so g_{NL} has been unfairly neglected.
- $|g_{NL}| >> f_{NL}^2$ is possible with non-quadratic potentials
- g_{NL} is hard to constrain. The current bound is $|g_{NL}| < 10^6$, Planck has not yet produced a constraint

When f_{NL}=-5/4 makes a big difference

- In order for primordial black holes to form in the very early universe, the amplitude of perturbations needs to be much larger (otherwise the required order unity perturbations will never occur)
- For Gaussian perturbations, one needs zeta~0.1 on the relevant (small) scales in order to form an observable number of primordial black holes
- The curvaton prediction for f_{NL} does not depend on the amplitude of perturbations
- With zeta~0.1, even f_{NL}=-5/4 has a big effect, especially on the tail of the pdf
- This leads to (at least) an order unity change on the allowed amplitude of the power spectrum on small scales

Sam Young & CB 2013

Power spectrum bounds



Bringmann, Scott, Akrami 2011

Lots more scales best constrained by PBHs

The dashed top black line gives the upper bound on the allowed power spectra amplitude from primordial black hole constraints. It is a weak constraint, but the best we have over a large range of scales.

It was calculated assuming exactly Gaussian perturbations, the dashed black line can shift by an order of magnitude for non-Gaussian scenarios