

Lensing and cosmological tests of general relativity

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Bernard:

- *Do you think black-holes can couple differently to gravity?*
 - *Do you think dark matter can be a fermionic condensate?*
 - *MOND vs DM (recurent and oscillating topic!)*
-
- Why shall we test general relativity on astrophysical and cosmological scales
 - What should we test?
 - Dark matter and lensing
 - Cosmological tests
 - Conclusions

INTRODUCTION

Goal: remind the hypothesis used in the interpretation of the cosmological data

See JPU, [astro-ph/0605313](#)

The interpretation of the dynamics of the universe and its large scale structure relies on the hypothesis that gravity is well described by General Relativity

Galaxy rotation curves

Introduction of *Dark Matter*

Einsteinian interpretation

Most of the time Newtonian interpretation

Acceleration of the cosmic expansion

Introduction of *Dark Energy*

Einsteinian interpretation

But more important Friedmanian interpretation

The standard cosmological model lies on 3 hypothesis:

H1- Gravity is well described by general relativity

H2- Copernican Principle

On large scales the universe is homogeneous and isotropic

Consequences:

- 1- The dynamics of the universe reduces to the one of the scale factor
- 2- It is dictated by the Friedmann equations

$$3 \left(H^2 + \frac{K}{a^2} \right) = 8\pi G \rho$$
$$\frac{\ddot{a}}{a} = - \frac{4\pi G}{3} (\rho + 3P)$$

H3- Ordinary matter (standard model fields)

Consequences:

- 3- On cosmological scales: pressureless + radiation
- 4- The dynamics of the expansion is dictated by

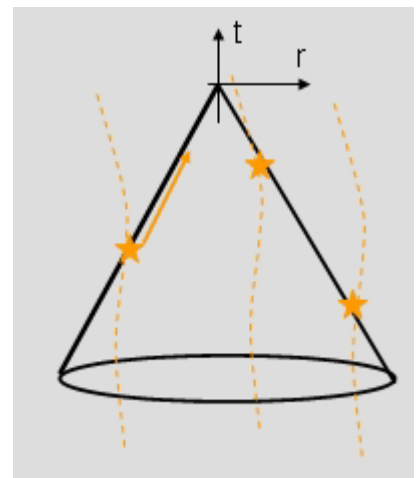
$$\Omega \equiv \frac{8\pi G \rho}{3H^2}$$

$$H^2(z)/H_0^2 = \Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_K^0 (1+z)^2$$

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to $a(t)$.

Consequences:

- $1 + z = \frac{E_{rec}}{E_{em}} \stackrel{H2}{=} \frac{a_0}{a(t)}$
- $a(t) = a_0 \left[1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots \right]$



so that

$$H^2(z)/H_0^2 = 1 + (q_0 + 1)z + \mathcal{O}(z^2)$$

$$q_0 = \Omega_{m0}/2$$

- **Hubble diagram** gives
 - H_0 at small z
 - q_0

Supernovae data (1998+) show

$$q_0 < 0$$



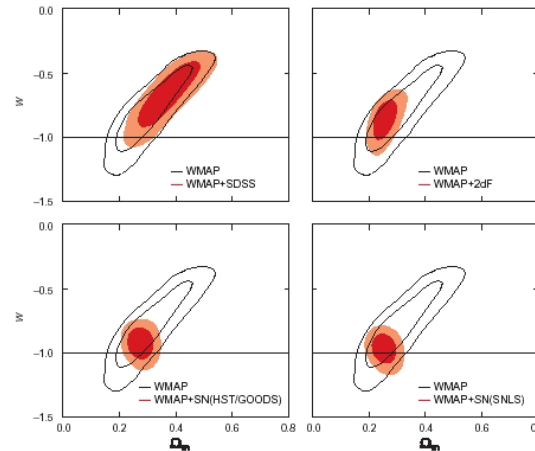
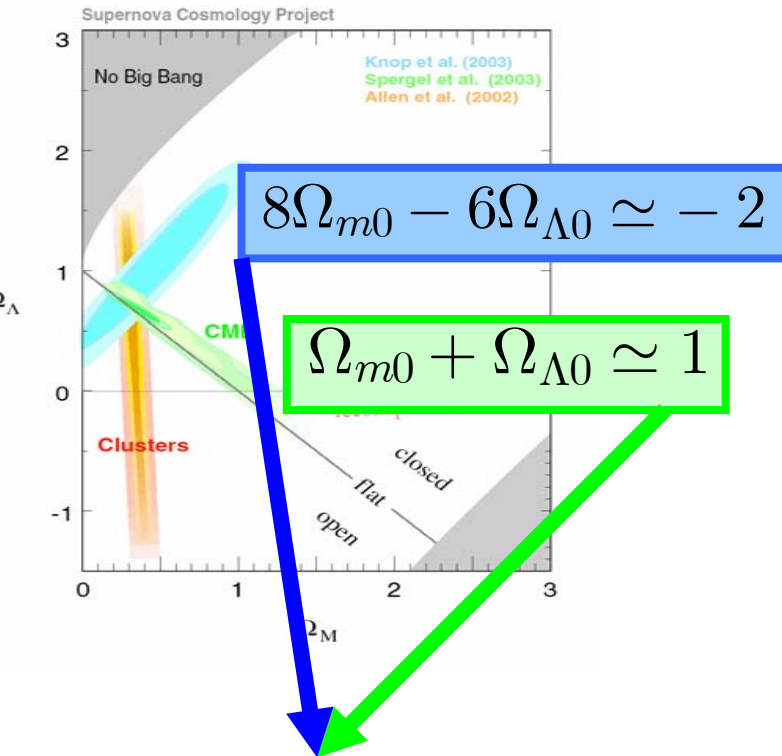
The expansion is now **accelerating**

Λ CDM (REFERENCE) MODEL

The simplest extension consists in introducing a cosmological constant

- constant energy density
- well defined model and completely predictive

$$\rho_{\Lambda} = \frac{\Lambda}{8\pi G} = -P_{\Lambda}$$



Spergel et al., astro-ph/0603449

$$P_{de} = w\rho_{de}$$

Λ CDM consistent with all current data

Observationally, very good
Phenomenologically, very simple
But: cosmological constant problem

$$\Omega_{m0} \sim 0.3, \quad \Omega_{\Lambda0} \sim 0.7$$

The dark sector reflects the fact that the current understanding of the cosmological data drives us to introduce new degrees of freedom.

Dark matter

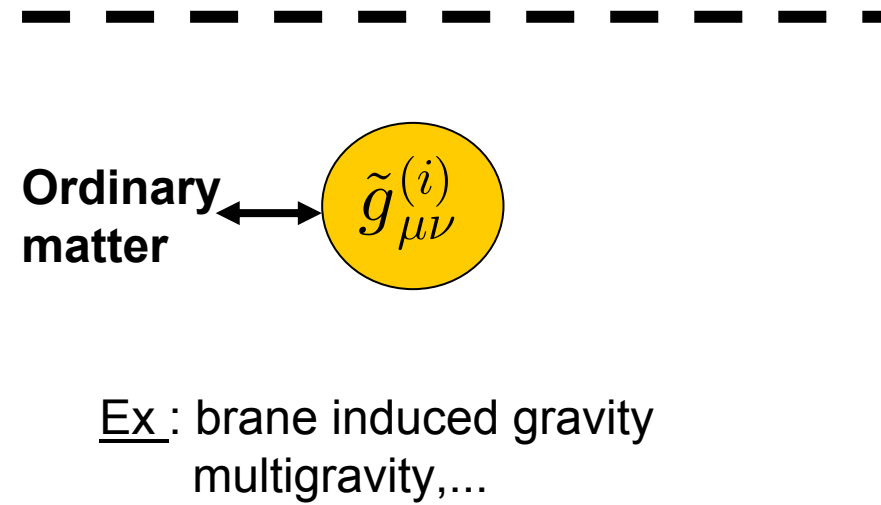
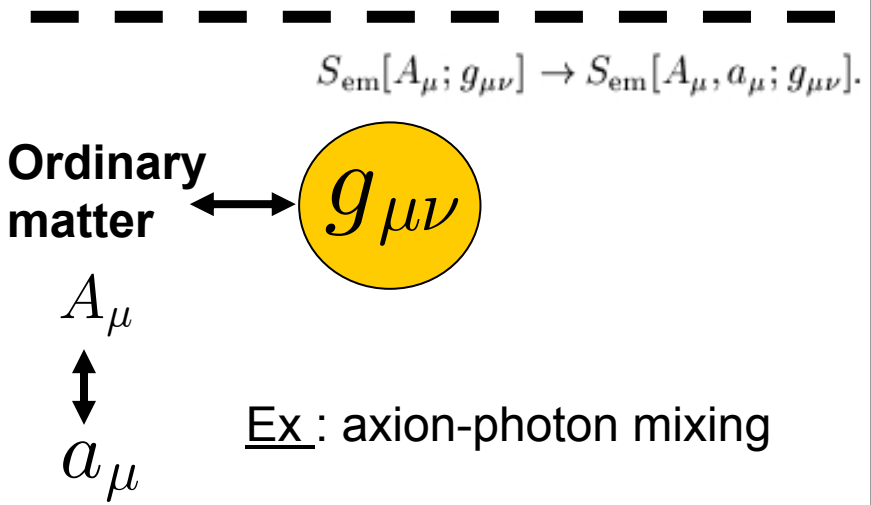
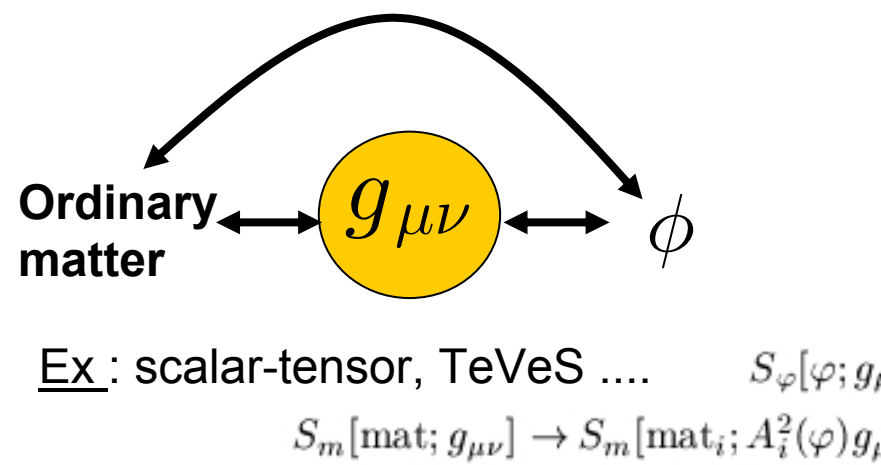
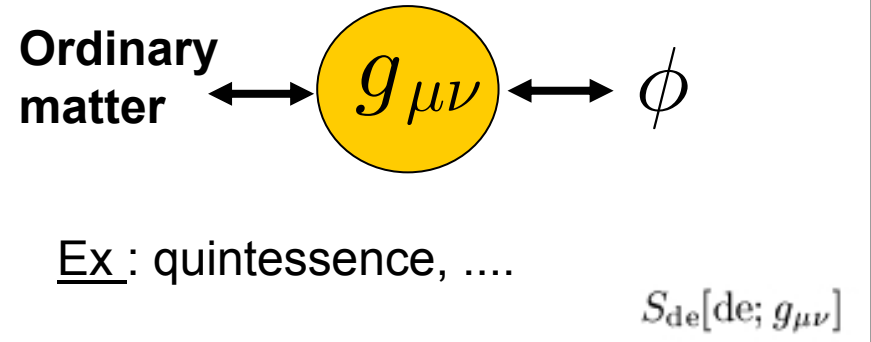
MOND and TeVeS alternative

Dark energy

- 1- The Copernican principle does not hold
- 2- There exists matter such that $\rho + 3P < 0$
- 3- Gravity is not well described by GR on large scales

	Measurement	Scale	Ω_m
1	peculiar velocities: relative rms	20 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20e^{\pm 0.4}$
2	redshift space anisotropy	10 Mpc $\lesssim r \lesssim$ 30 Mpc	0.30 ± 0.08
3	mean relative velocities	10 Mpc $\lesssim r \lesssim$ 30 Mpc	$0.30^{+0.17}_{-0.07}$
4	numerical action solutions	$r \sim$ 1 Mpc	0.15 ± 0.08
5	virgocentric flow	$r \sim$ 20 Mpc	$0.20^{+0.22}_{-0.15}$
6	weak lensing: galaxy-mass	100 kpc $\lesssim r \lesssim$ 1 Mpc	$0.20^{+0.06}_{-0.05}$
7	mass-mass	300 kpc $\lesssim r \lesssim$ 3 Mpc	0.31 ± 0.08

Gravitation = any long range force that cannot be screened



Always need NEW fields

CLASSICAL TESTS OF GR

Goal: remind the tests in the Solar system
understand those that can be generalized

Einstein equivalence principle

universality of free fall

local Lorentz invariance

local position invariance

Metric theories of gravity

spacetime is endowed with a symmetric metric

trajectories of free-falling test bodies are geodesic of that metric

in a freely reference frame, the laws of non-gravitational physics are those written in the language of special relativity

General relativity is a metric theory of gravity

$$S = \frac{1}{16\pi G} \int \sqrt{-g} R dx + \int L_m(\psi, g_{\mu\nu}) \sqrt{-g} dx$$

General relativity is well tested in the Solar system
is our *reference* theory of gravity

Universality of free fall

$$\eta = 2 \frac{|a_1 - a_2|}{|a_1 + a_2|}$$

Local Lorentz invariance

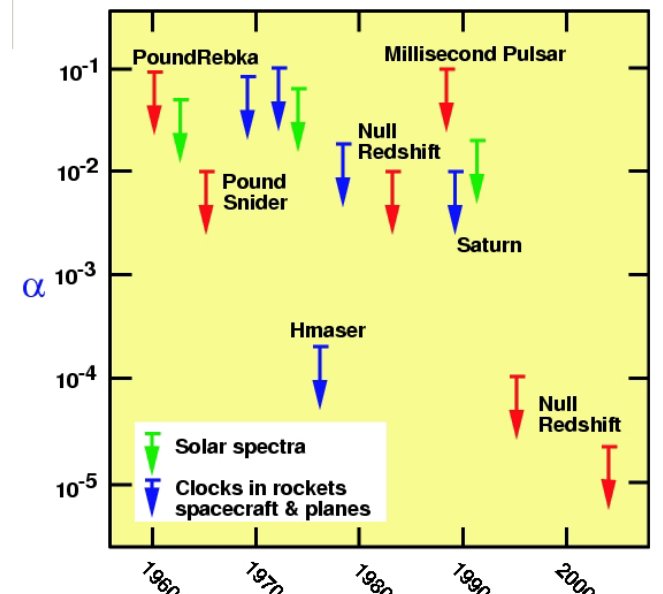
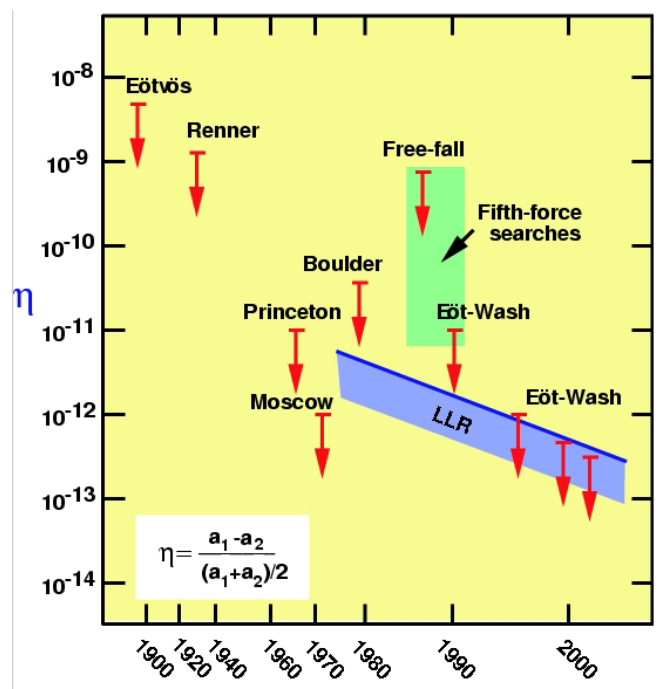
*Michelson-Morley experiments,
isotropy of the speed of light
independence of the speed of light on
velocity of the source*

Local position invariance

gravitational redshift

$$Z = \frac{\delta\nu}{\nu} = (1 + \alpha) \frac{\Delta U_{\text{newt}}}{c^2}$$

constants



Metric theories are usually tested in the PPN formalism

In its simplest form

$$ds^2 = (-1 + 2U + 2(\beta - \gamma)U^2)dt^2 + (1 + 2\gamma U)dr^2 + r^2d\Omega^2$$

$$U = \frac{GM}{rc^2}$$

If gravity is described by GR then $\beta = \gamma = 1$

These parameters can be constrained, independently of a precise theory, from Solar system observations

Light deflection

$$\Delta\theta = 2(1 + \gamma)\frac{GM}{bc^2}$$

Perihelion shift of Mercury

$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2 + 2\gamma - \beta)$$

Shapiro time delay

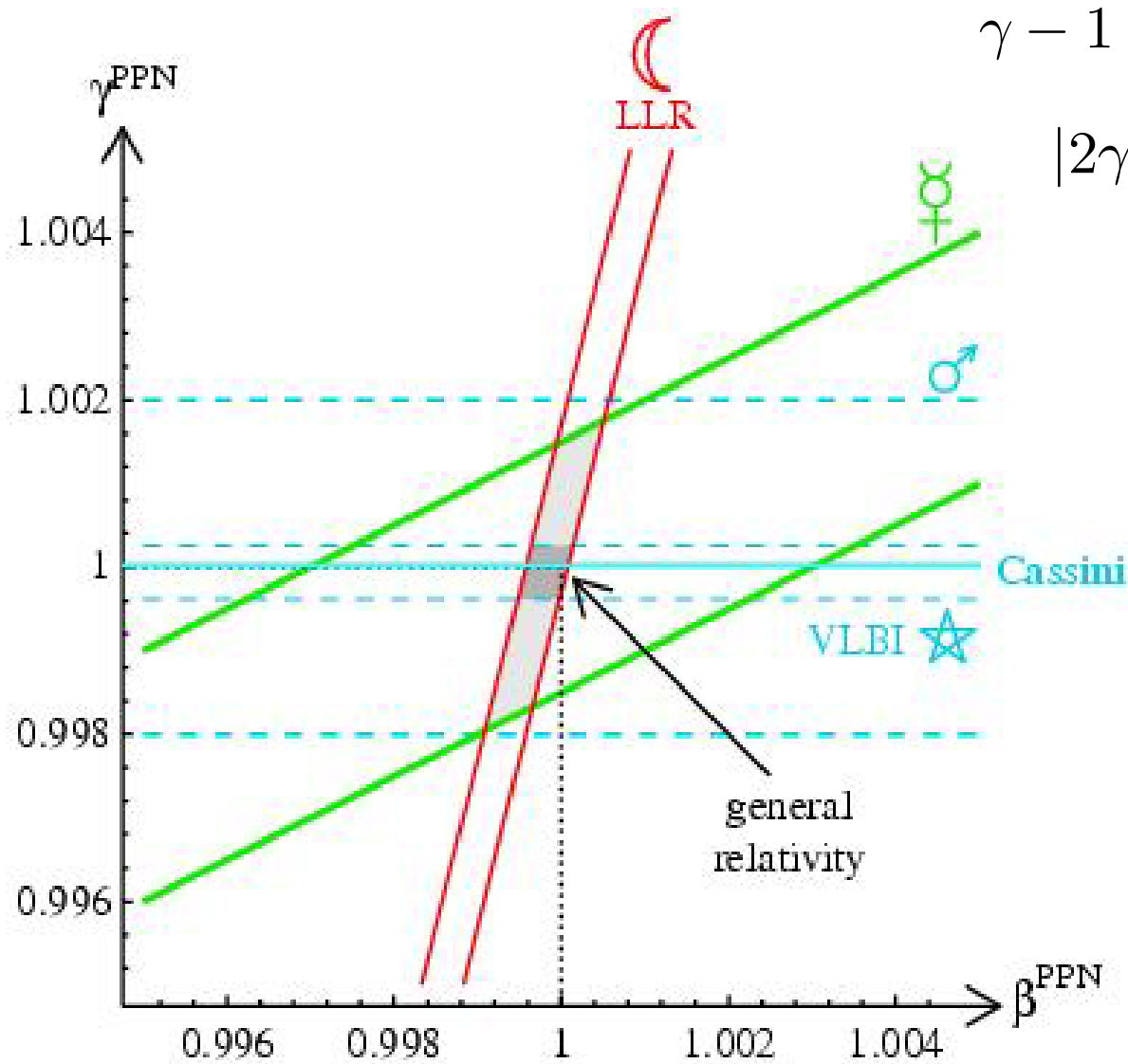
$$\delta t \propto (1 + \gamma)$$

Nordtvedt effect

$$\delta r \sim 13.1(4\beta - \gamma - 3) \cos(\omega_0 - \omega_s)t \quad (\text{m})$$

TESTS OF GR IN THE SOLAR SYSTEM

Courtesy of G. Esposito-Farèse



$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

Among the previous tests, it seems possible to generalize

- **Light deflection**

need to determine independently mass and deflection
cosmology: - we do not measure the deflection but the
distortion of light bundles
- energy of the photons

- **Motion of test-bodies**

growth of structures / velocity fields

- **Constants**

But:

- time evolution (growth of structure):

 - information on the dynamics*

 - evolution effects*

- statistical interpretation and dependence on the initial conditions

- super-Hubble modes

DARK MATTER AND LENSING

For any spherically symmetric metric of the form

$$ds^2 = - B(r)c^2 dt^2 + A(r)dr^2 + r^2 d\Omega^2$$

the deflection angle is

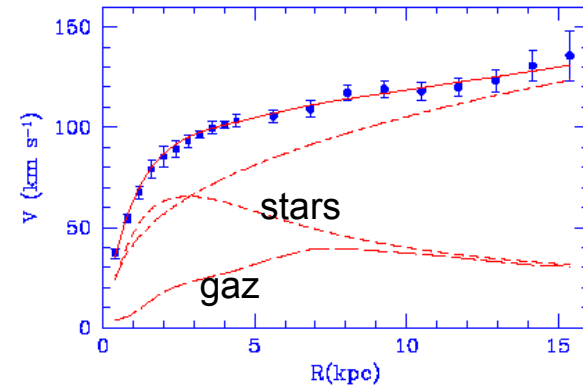
$$\delta\theta = -\pi + 2 \int_b^\infty \frac{dr}{r^2} \sqrt{\frac{A(r)B(r)}{B(r_0)/r_0^2 - B(r)/r^2}}$$

Rotation curves

$$v^2(r) \rightarrow v_\infty^2 \equiv \sqrt{GMa_0}$$

If this dynamics is due to the existence of dark matter, then

$$\delta\theta_{GR} \rightarrow \frac{2\pi\sqrt{GMa_0}}{c^2}$$



MOND alternative

a_0 : limit acceleration

$$a < a_0 : \quad a = \sqrt{a_N a_0} = \sqrt{GMa_0} / r$$

Equivalent to have an effective potential

$$\Phi = -\frac{GM}{r} + \sqrt{GMa_0} \ln r$$

$$r > \sqrt{\frac{GM}{a_0}}, \quad \delta\theta_{MOND} = \frac{2\pi\sqrt{GMa_0}}{c^2}$$

new fields

$$\text{Gravity} + \rho_{\text{dark}}(r) + \rho_b(r)$$

$$\text{Newton} + \rho_{\text{DM}}(r) + \rho_b(r)$$

$$\text{MOND} + 0 + \rho_b(r)$$

$$T_{\mu\nu}^{\text{dark}} \gg T_{\mu\nu}^{\text{b}}$$

$$v^2(r)$$

$$T_{\mu\nu}^{\text{dark}} \ll T_{\mu\nu}^{\text{b}}$$

Dark matter

Gravity

Constraint DM

In the Solar system, we can determine the mass of the Sun and the deflection angle independently

This is why we have a test of GR

Now, one has (at least) 3 notions of mass:

- Baryonic mass, M_b ,
assumed to be proportional to the luminous mass
- Dynamical mass, M_{rot} ,
evaluated from rotation curves
- Deflecting mass, M_{lens} ,
evaluated from lensing

In the standard DM interpretation

$$M_b < M_{DM} \simeq M_{rot} \simeq M_{lens}$$

Let us consider lensing in a large family of gravity theories including General Relativity

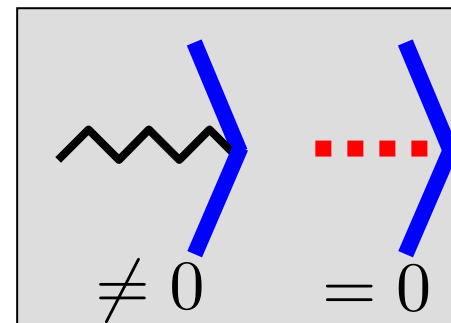
$$S = \frac{c^3}{16\pi G} \int \sqrt{-g} \{ R - 2(\partial_\mu \phi)^2 - V(\phi) \} + S_m \{ \text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \}$$

RAQUAL version $(\partial_\mu \phi)^2 \rightarrow f[(\partial_\mu \phi)^2, \phi]$

Maxwell electromagnetism is conformally invariant in d=4

$$S_{em} = \frac{1}{4} \int \sqrt{-\tilde{g}} \tilde{g}^{ab} \tilde{g}^{cd} F_{ac} F_{bd} d^d x$$

$$= \frac{1}{4} \int \sqrt{-g} g^{ab} g^{cd} F_{ac} F_{bd} A^{d-4}(\phi) d^d x$$

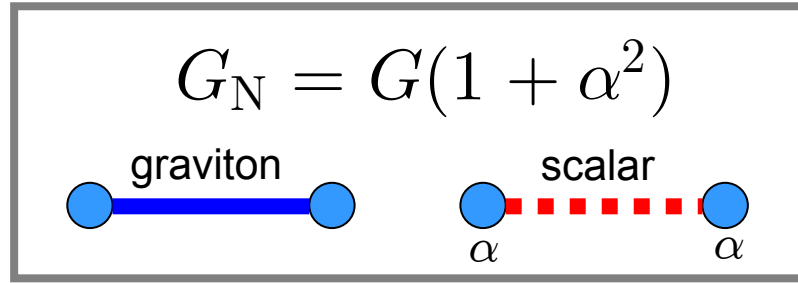


Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$

WHAT IS THE DIFFERENCE?

The difference with GR comes from the fact that massive matter feels the scalar field



$$\alpha = d \ln A / d\phi$$

Motion of massive bodies determines $G_N M$ **not** GM

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_N M}{(1 + \alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

Which means

$$M_{\text{lens}} \leq M_{\text{rot}}$$

A nice trick allows to increase light deflection in scalar-tensor theories

$$\tilde{g}_{\mu\nu} = A^2(\varphi) [g_{\mu\nu} + B(\varphi) \partial_\mu \varphi \partial_\nu \varphi]$$

Bekenstein, gr-qc/921101
 Bekenstein, Sanders,
 gr-qc/931106

Preferred direction
 (radial for spherical system)

The only difference with GR is in the radial component and thus

$$\delta\theta = \delta\theta_{GR} + \int_b^\infty \frac{dr}{r\sqrt{r^2/b^2-1}} B(\partial_r\phi)^2$$

Now, assume that

$$B(\phi)(\partial_r\phi)^2 = 4\sqrt{GMa_0}/c^2$$

then

$$\delta\theta = \delta\theta_{GR}^b + \frac{2\pi\sqrt{GMa_0}}{c^2} \simeq \delta\theta_{GR+DM}$$

The former trick was extended by Bekenstein (TeVSe theory...)

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu} + B(\varphi)V_\mu V_\nu$$

Dynamical unit timelike vector



This is at the basis of the construction of TeVeS theories

When dealing with a specific theory, before determining how well it fits the data, one should investigate if it does not have any pathologies

See Bruneton & Esposito-Farèse, [arXiv:0705.4043](https://arxiv.org/abs/0705.4043)

In conclusion, all we are doing is to test the **compatibility** of the mass distribution measured by different methods.

Early studies:

- Comparison of X-ray and strong lensing

Miralda-Escude & Babul, ApJ **449** (1995) 18

- add weak lensing

Squires et al., ApJ **461** (1996) 572

- Cluster scale (2 Mpc): X-ray vs lensing.

Allen et al. MNRAS **324** (2001) 877

- Use of SZ

Recent data allow to go beyond the spherically symmetric case



Cluster merger at $z=0.296$
Spatial segregation of collisionless
matter/plasma
Lensing reconstruction does not
follow the plasma distribution

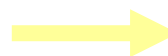
Proof of the existence of DM (...)

Mond in non-spherical geometry (dependence on the version of the theory
and on fitting function)

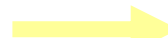
Angus et al., astro-ph/06062

Necessity for 2 eV neutrinos

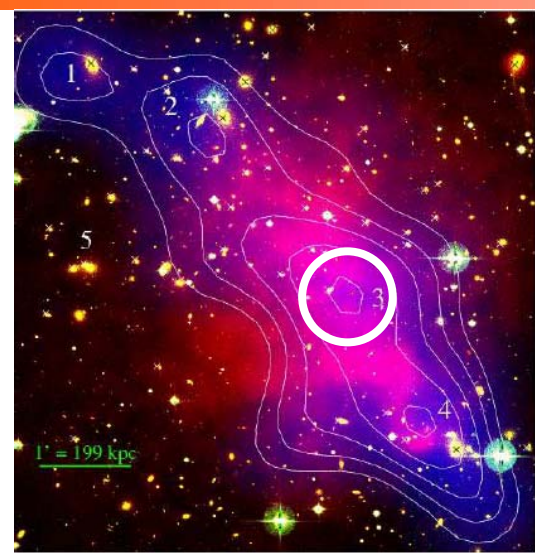
Angus et al., astro-ph/060912



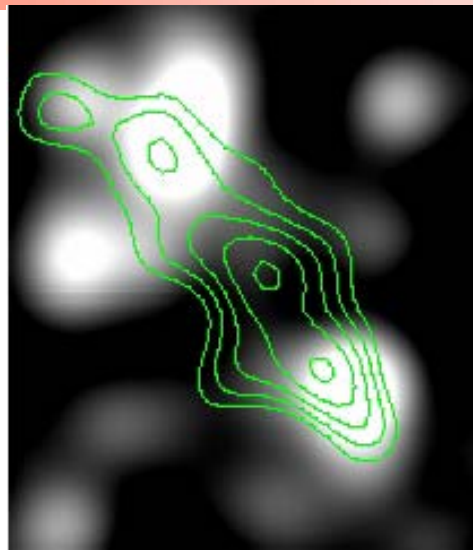
See Robert Sanders talk for more



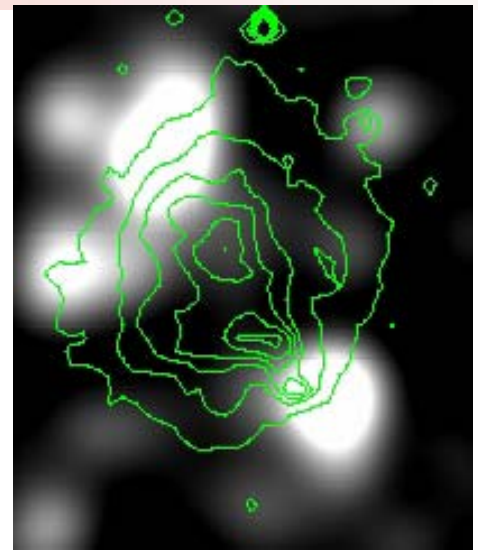
See Douglas Clowe talk for more



X vs lensing



red light vs lensing



red light vs X

Existence of a dark core that coincides with the peak of X-emission

Bernard: MOND regime at 90 kpc....

It is always possible to design coupling to reproduce the deflection angle by DM+GR

We have mostly considered spherically symmetric solutions

The most important issue is how well we can measure the profiles $M_b(r)$, $M_{\text{rot}}(r)$ and $M_{\text{lens}}(r)$

Recent observations drive to go beyond spherical symmetry

Then, conclusions are not straightforward:

- depend on the *version of MOND*
- depend on the *choice of the fitting functions*

See also discussion CL0024+17 this morning

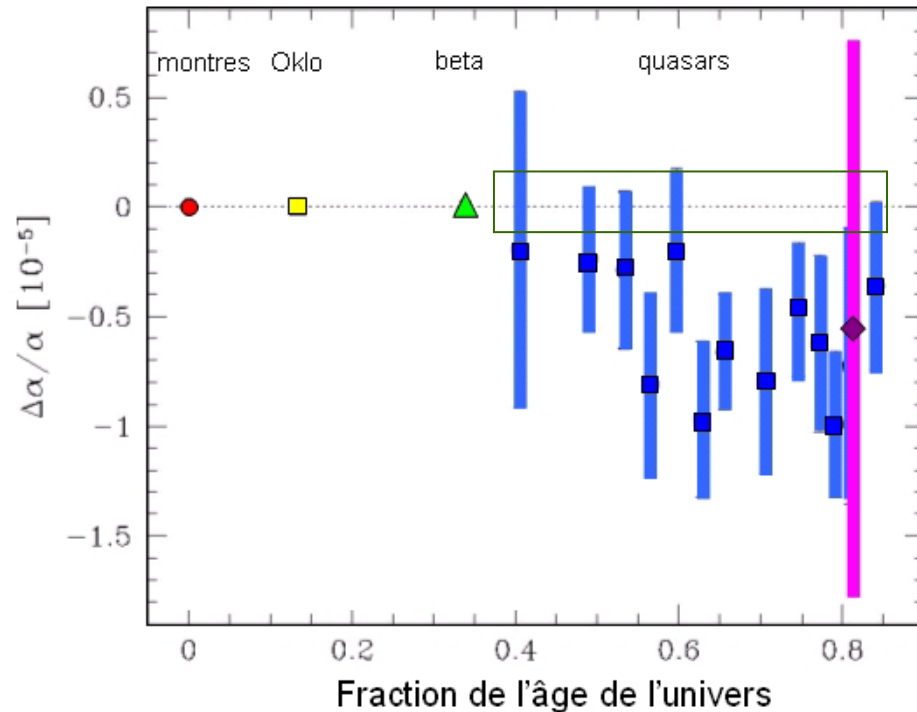
COSMOLOGICAL TESTS

CONSTANTS (LOCAL POSITION INVARIANCE)

Many tests concerning various constants (α , μ , G mainly).

Tests on different time scales:

local	($z=0$)	atomic clocks, Solar System
geophysical	($z=0.1..0.4$)	Oklo, meteorites
astrophysical	($z=0.2-3.5$)	quasars
cosmological	($z=10^3, 10^8$)	CMB, BBN.



General investigation of the link of these constraints and gravity theories

JPU, RMP **75** (2003) 425
astro-ph/0409424

Most observations involve only low- z and sub-Hubble regime
(but CMB and BBN)

$$ds^2 = a^2(\eta) [- (1 + 2\Phi) d\eta^2 + (1 - 2\Psi) \gamma_{ij} dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0 (1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0) (1+z)^2 + \Omega_\Lambda^0$$

Sub-Hubble perturbations

$$\Phi = \Psi$$

$$\Delta\Psi = 4\pi G\rho a^2\delta$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi$$

In the linear regime, the growth of density perturbation is then dictated by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_{\text{mat}}\delta = 0$$

This implies a *rigidity* between the growth rate and the expansion history

Bertschinger, astro-ph/0604485,
JPU, astro-ph/0605313

It can be considered as an equation for $H(a)$

Chiba & Takahashi, astro-ph/070334

$$(H^2)' + 2 \left(\frac{3}{a} + \frac{\delta''}{\delta'} \right) H^2 = 3 \frac{\Omega_0 H_0^2 \delta}{a^5 \delta'}$$

$$\frac{H^2}{H_0^2} = 3\Omega_{m0} \frac{(1+z)^2}{\delta'(z)^2} \int_z \frac{\delta}{1+z} (-\delta') dz$$

Proposal: $D(z)$ from galaxy cluster survey

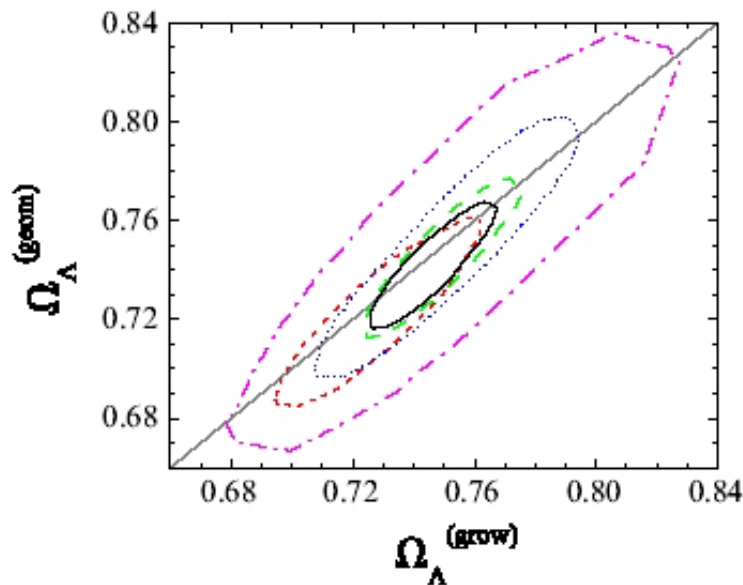
Tang et al, astro-ph/0609028

$H(a)$ from the background (geometry) and growth of perturbation have to agree.

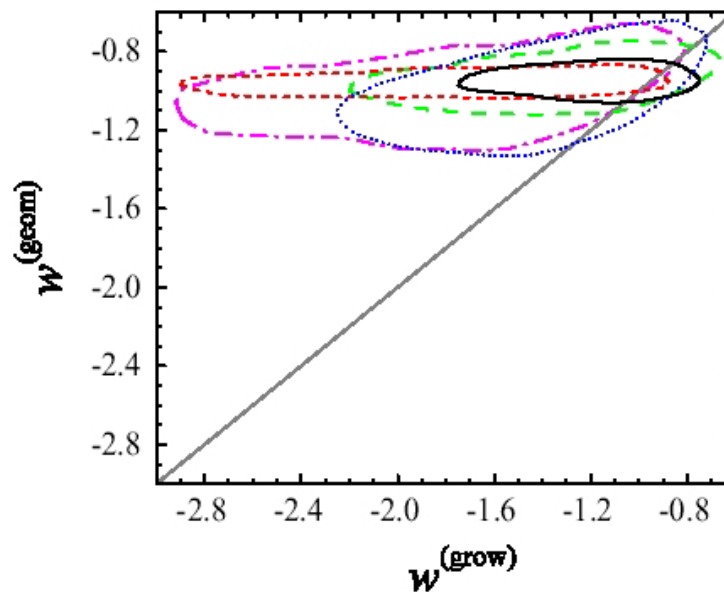
SNLS – WL from 75 deg² CTIO – 2dfGRS – SDSS (luminous red gal)
 CMB (WMAP/ACBAR/BOOMERanG/CBI)

Wang et al., arViv:0705.0165

Flat Λ CDM model



Flat $w = \text{constant}$



Consistency check of any DE model within GR with non clustering DE
 Assume Friedmannian symmetries! (see e.g. [Dunsby and JPU](#))

To go beyond we need a parameterization of the possible deviations

Restricting to low- z and sub-Hubble regime

$$ds^2 = a^2(\eta) [- (1 + 2\Phi) d\eta^2 + (1 - 2\Psi) \gamma_{ij} dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0 (1+z)^3 + (1 - \Omega_m^0 - \Omega_\Lambda^0) (1+z)^2 + \Omega_{de}(z)$$

Sub-Hubble perturbations

$$\Delta(\Phi - \Psi) = \pi_{de}$$

$$-k^2\Phi = 4\pi G_N F(k, H) \rho a^2 \delta + \Delta_{de}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + S_{de}$$

JPU, astro-ph/06053

$$\Lambda\text{CDM} \quad (F, \pi_{de}, \Delta_{de}, S_{de}) = (1, 0, 0, 0)$$

DATA**OBSERVABLE**

Weak lensing

$$\kappa \propto \Delta(\Phi + \Psi)$$

Galaxy map

$$\delta_g = b \delta$$

Velocity field

$$\theta = \beta \delta$$

Integrated Sachs-Wolfe

$$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$$

Various combinations of these variables have been considered

PART OF THE POISSON EQUATION

On sub-Hubble scales, the gravitational potential and density contrast are related by

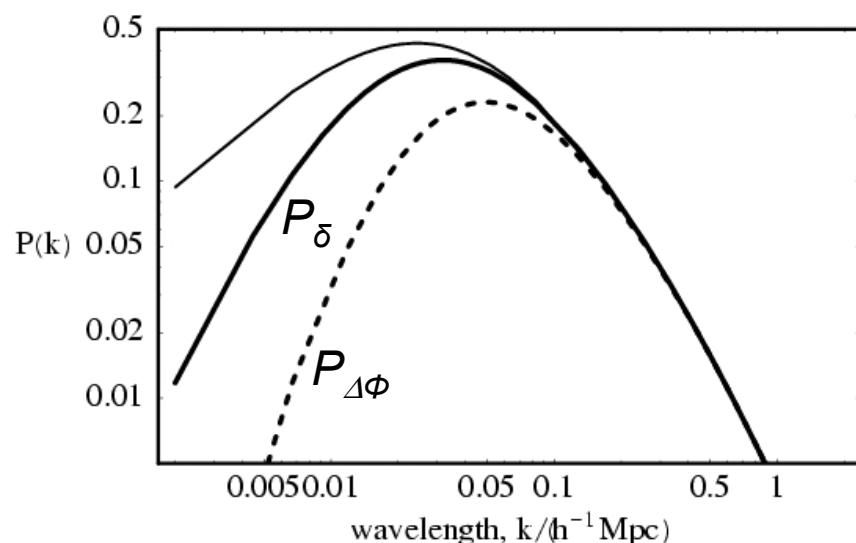
$$\Delta\Phi = 4\pi G\rho a^2\delta$$

Galaxy catalogs (SDSS, 2dF...)

measurement of $\xi(r)$ up to $500h^{-1}\text{Mpc}$

Weak lensing

will be measured up to $100h^{-1}\text{Mpc}$



Toy model: 4D-5D gravity (brane induced)

perturbations freeze on large scales (idem as effect of Λ)

power spectra of Φ and δ are not identical

velocity map

$$\langle \delta_g \dot{\theta} \rangle = b\beta \langle \delta^2 \rangle$$

Galaxy map

$$\langle \delta_g \kappa \rangle \propto b \langle \delta \Delta (\Phi + \Psi) \rangle \stackrel{\Lambda\text{CDM}}{\propto} 8\pi G \rho a^2 b \langle \delta^2 \rangle$$

weak lensing

The ratio of these 2 quantities is independent of the bias

Zhang et al, arXiv:0704.1932

- Assume - no velocity bias ($S_{DE}=0$)
- no clustering of DE ($\Delta_{DE}=0$)

CORRELATIONS

Correlations	Dependence	Limit	Case
$\langle \delta_g \delta_g \rangle - \langle \kappa \kappa \rangle$ JPU-Bernardeau	$(F, \pi_{\text{de}}, \Delta_{\text{de}})$	bias	
$\langle \delta_g \theta \rangle - \langle \delta_g \kappa \rangle$ Zhang et al, arXiv:0704.1932	$(F, \pi_{\text{de}}, \Delta_{\text{de}})$	velocity bias	
$\langle \delta_g \Theta_{SW} \rangle$ Schmidt et al, arXiv:0706.1775		bias	TeVes

A Full study of all the correlations needs to be performed

No test alone can bring a proof of deviation from GR and most studies assume $\Delta_{\text{DE}}=0$

Possible to constrain the cases where $S_{\text{DE}}=\Delta_{\text{DE}}=0$. Quite general.

Null tests for deviation from ΛCDM

At **linear order**, growth factor entangles $H(a)$ and Poisson equation.

$$\delta^{(1)} = D(t)\varepsilon(x)$$

At **second order**

$$\ddot{\delta}^{(2)} + 2H\dot{\delta}^{(2)} = 4\pi G\rho(\delta^{(1)})^2 + a^{-2}\nabla\Phi\cdot\nabla\delta^{(1)} + a^{(-2)}\partial_{ij}u_i^{(1)}u_j^{(1)}$$

$$\langle\delta^3\rangle = \langle(\delta^{(1)})^3\rangle + \langle(\delta^{(1)})^2\delta^{(2)}\rangle$$

$S^3 = \langle\delta^3\rangle/\langle\delta^2\rangle^2$ is independent of $D(t)$. It depends slightly on the cosmological

parameters – dependence on spectral index - Gaussianity

COSMIC SHEAR 3-POINT FUNCTION

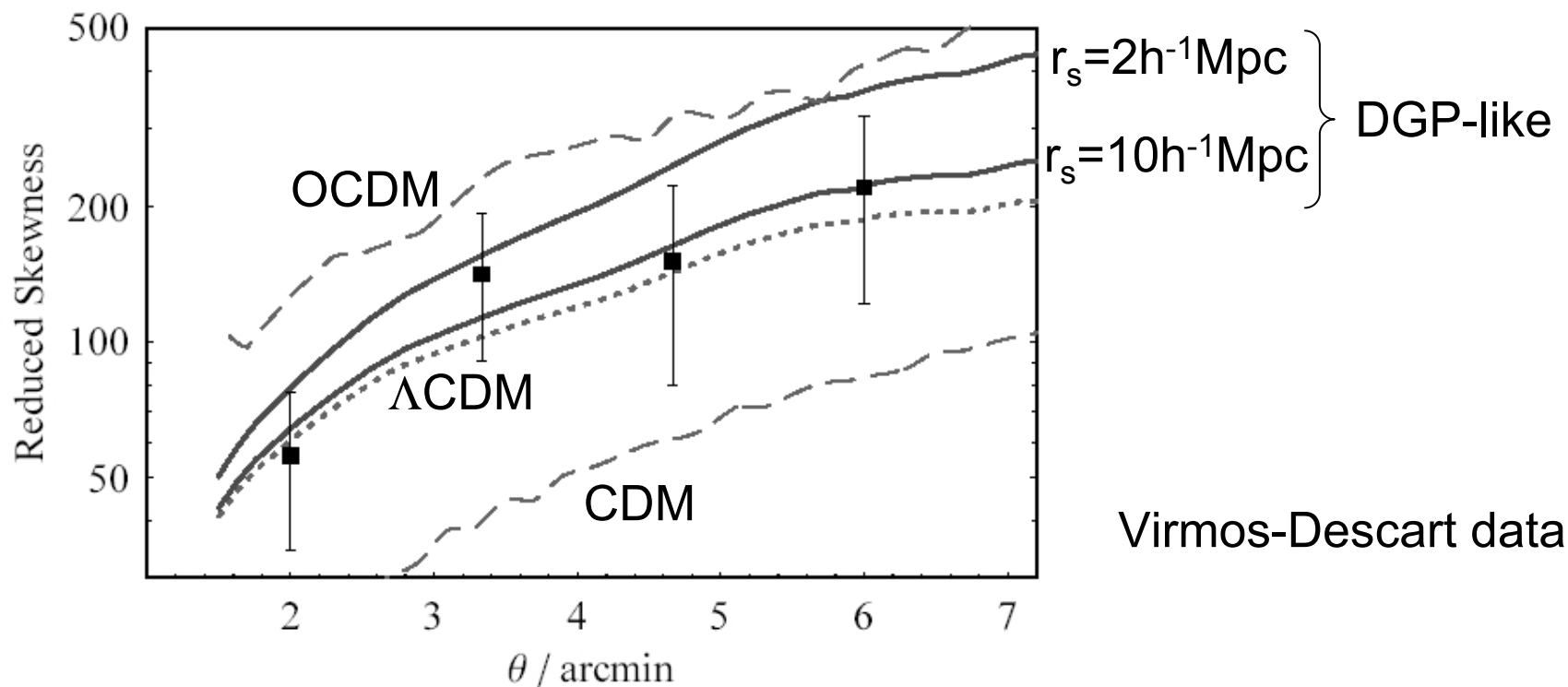
Assume a modified (scale dependent) Poisson equation

Compute the reduced third moment.

Use 3-point correlation of the shear field

Bernardeau et al, A.A.Lett.**389**(2002)2

Pen et al., ApJ**592**(2003)664

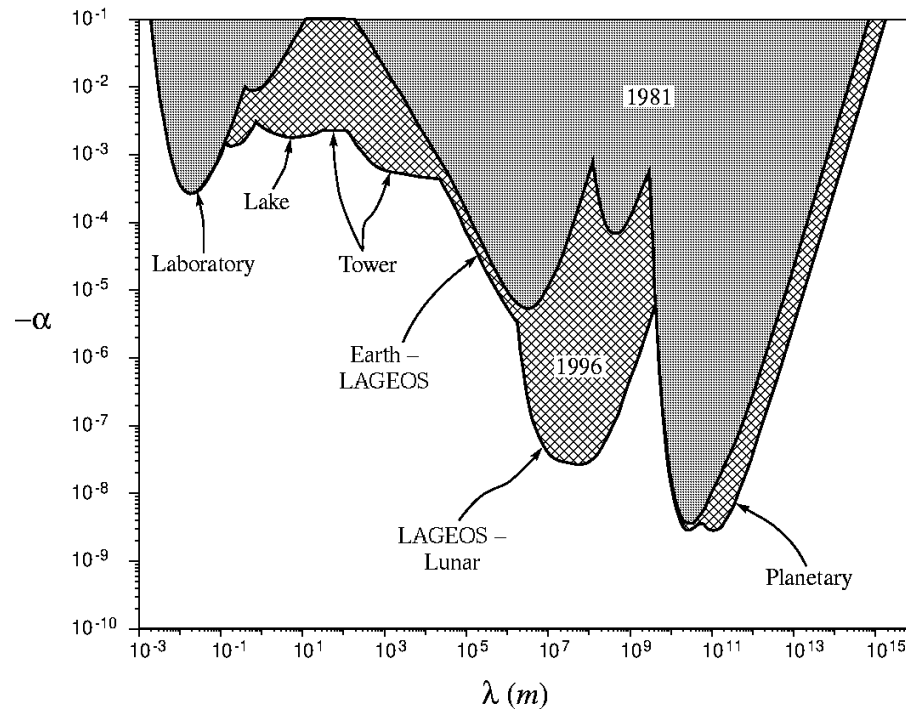


Disfavor $r_s < 2h^{-1}\text{Mpc}$

Various studies have focused on a Yukawa modification of GR

$$U = \frac{Gm}{r} \left(1 + \alpha e^{-r/\lambda} \right)$$

Such a deviation is well constrained in the Solar system

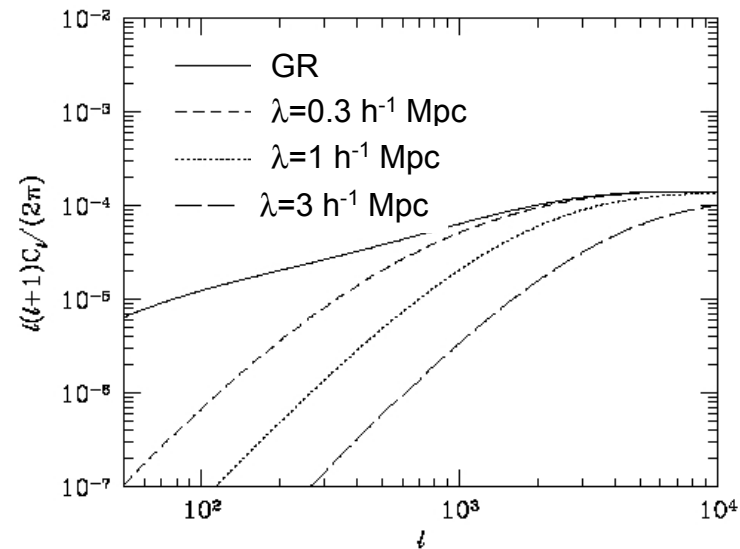


Concerning the growth of structure, it reduces to assuming

$$-k^2\Phi = 4\pi G(1 + f_Y(k\lambda))\rho a^2\delta \quad \Phi = \Psi$$

White & Kochanek, astro-ph/0105227

Weak lensing computed from propagation
of rays through a known density distribution.
No consistent analysis of the growth of structures



Sealfon et al., astro-ph/0404111

Compute power spectrum and bispectrum of LSS

$$\alpha = 0.025 \pm 1.7 \text{ (2dF)} \quad \alpha = -0.35 \pm 0.9 \text{ (SDSS)}$$

on a scale $\lambda \sim 6h^{-1}\text{Mpc}$

Shirata et al., astro-ph/0501366

Linear evolution + Peacock&Dodds for NL

Comparison with SDSS

$$-0.5 < \alpha < 0.6 \quad (\lambda = 5h^{-1}\text{Mpc})$$

$$-0.8 < \alpha < 0.9 \quad (\lambda = 10h^{-1}\text{Mpc})$$

Exclusion plot in (α, λ) less obvious than in Solar system
(dependence on cosmological parameters...)

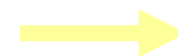
Stabenau & Jain, astro-ph/0604038

N-body simulations on scales 1-100 Mpc

The scale dependence modification of the growth factor in linear regime is enhanced by NL

Peacock&Dodds approach can be extended

Lensing power spectra



See Bhuvnesh Jain

Sereno & Peacock, astro-ph/0605498

Effect is almost degenerate on power spectrum shape with effect of massive neutrinos.

In models involving 2 metrics (scalar-tensor, TeVeS,...), gravitons and standard matter are coupled to different metrics.

In GR:

photons and gravitons are massless and follow geodesics of the same spacetime

$$\delta T_{\gamma g} = T_{\gamma} - T_g = 0$$

In bi-metric:

photons and gravitons follow geodesics of two spacetimes

$$\delta T_{\gamma g} \neq 0$$

Example:

TeVSe model. Observable=SN1987a

$$\delta T_{\gamma g} = - 5.3 \text{ days}$$

DISTANCE DUALITY RELATION

Photons travel on null geodesics
Geodesic deviation equation holds

Etherington, *Phil. Mag.* **15** (1933) 761; Ellis, 1971

$$\text{Reciprocity relation: } r_s = r_o(1+z)$$

If number of photons is conserved

$$D_{\text{lum}}(z) = (1+z)^2 D_A(z)$$

SN Ia data+radio galaxies

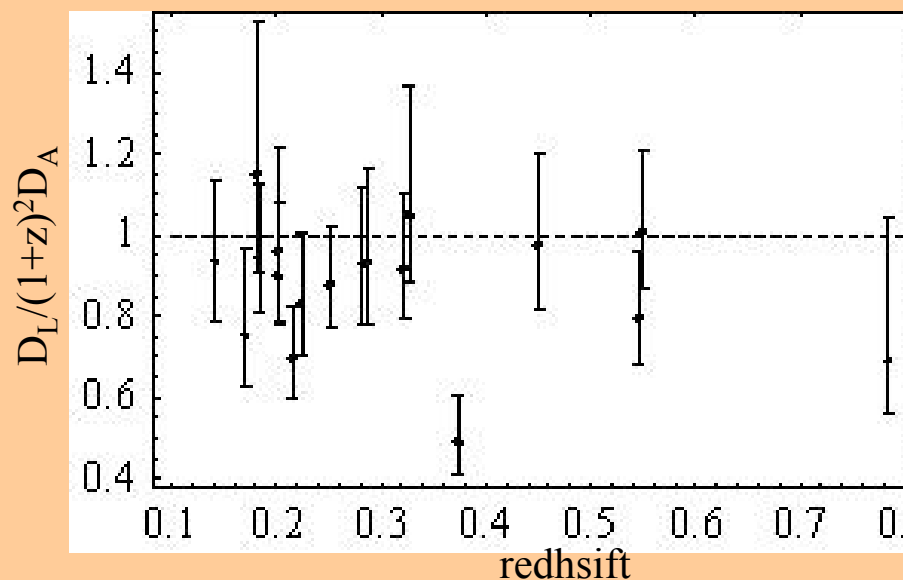
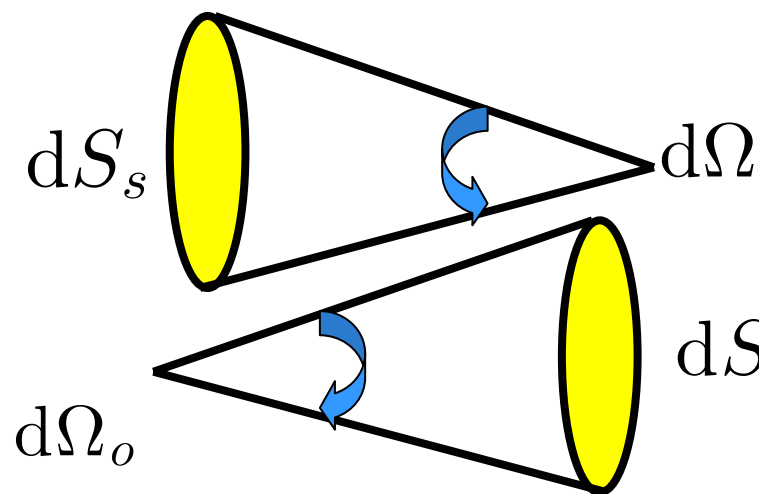
2σ violation

Basset and Kunz, *PRD***69** (2004)101305

X-ray + SZ observation of clusters

no indication of violation

Set constraints on photon-axion mixing



I will not detail the numerous studies in which one given model (TeVeS, DGP, scalar-tensor,...) is compared to combined set of data.

e.g. Amendola et al., arXiv:0704.242
Song, astro-ph/0602598,
Knox et al., astro-ph/0503644,...

General limits:

- **Non-linear regime:** mappings are determined from numerical simulations assuming Newtonian gravity.
- **Effect of massive neutrinos:** can induce scale dependent modification of the power spectrum

Lifting degeneracies:

- background: 1 function $H(a)$
- low z – sub-Hubble: $D(a)$
- one can construct several models reproducing the same subset of data
- needs to include local constraints

CONCLUSIONS

Good motivations to test GR on astrophysical scales

important to understand the parameters we are measuring in Λ CDM.

Are there reasons to extend the Λ CDM framework

- *post- Λ CDM formalism (?)*

- *importance null-tests vs fitting models*

Would allow to design parameterizations adapted to each class of models

Many tests have been proposed but yet no systematic investigation

Dependence on initial conditions and other limitations

Statistical analysis-initial conditions

massive neutrinos

NL regimes

Theoretical limitations

Importance to consider background/perturbation/local tests

Galactic scales / cosmological scales