Weak Lensing Goes Flexion

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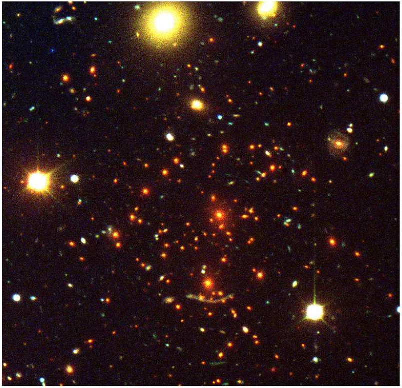
Outline

Observation



Galaxy Cluster Abell 370
(VLT UT1 + FORS1)

ESO PR Photo 47c/98 (26 November 1998)

Arclets, first detected by Bernard Forb et.al (1988)

Theory
Goldberg, D.M.
Bacon, D.J. 2005 APJ, 619, 741,
Bacon,D.J. et al 2006 MNRAS, 365,414

What is flexion ?

- Small scale variation in weak lensing
- Higher order lensing effect
- Gradient of shear

2nd-order expansion of lens equation

- Second-order

$$\beta_i = \theta_i - \psi_{,ij}\theta_j - \psi_{,ijk}\theta_j\theta_k/2$$

- Differential operators

$$\nabla_{\rm c} := \frac{\partial}{\partial\theta_1} + {\rm i}\frac{\partial}{\partial\theta_2}, \;\; \nabla_{\rm c}^* := \frac{\partial}{\partial\theta_1} - {\rm i}\frac{\partial}{\partial\theta_2}$$

  The differential operator $\nabla_{\rm c}$ turns a spin-n field into a spin-(n+1) field, whereas $\nabla_{\rm c}^*$ works opposite. For example, $\nabla_{\rm c}^*\gamma = \nabla_{\rm c}\kappa$ found by Kaiser (1995).

- then lens equation reads

$$\boldsymbol{\beta} = (1-\kappa)\boldsymbol{\theta} - \gamma\boldsymbol{\theta}^* - \frac{1}{4}\nabla_{\rm c}^*\kappa\,\boldsymbol{\theta}^2 - \frac{1}{2}\nabla_{\rm c}\kappa\,\boldsymbol{\theta}\theta^* - \frac{1}{4}\nabla_{\rm c}\gamma\,(\boldsymbol{\theta}^*)^2$$

Mass sheet Degeneracy!

- Under the transformation of $\kappa_\lambda = (1-\lambda) + \lambda\kappa$, the observable shape of the gravitational lens systems doesn't change. The shear is unobservable, but the reduced shear can be measured:

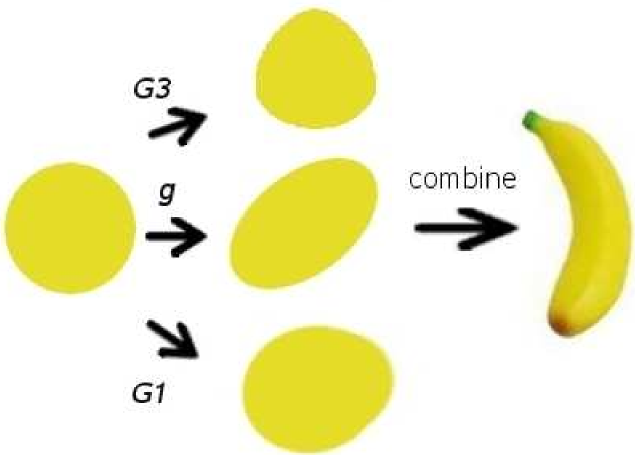
$$g = \frac{\gamma}{1-\kappa}$$

- Then reduced β

$$\hat{\beta} \equiv \frac{\boldsymbol{\beta}}{(1-\kappa)} = \boldsymbol{\theta} - g\boldsymbol{\theta}^* - \Psi_1^*(G_1)\,\boldsymbol{\theta}^2 - 2\Psi_1\,\boldsymbol{\theta}\boldsymbol{\theta}^* - \Psi_3(G_1, G_3)\,(\boldsymbol{\theta}^*)^2$$

  where $G_3 = \nabla_c g,\ G_1 = \nabla_c^* g$ are reduced flexion parameters.
- The lensing equation determined by 3 parameters g, G_1 and G_3.

Weak lensing goes Bananas



Measurements

- How to measure? Which observate characterize flexion

Definition

- Shapelets (Refregier, 2003);
- Brightness Moments

$$Mom[f(\beta)] = \int d^2\beta \, f(\beta) \, I^s(\beta), \quad (Okura\ et\ al, 2006\ HOLICs)$$

We follow this, but in the reduced flexion.

Brightness Moments

- 2nd-order Brightness Moments

$$Q_2 = \frac{1}{S}\int \mathrm{d}^2\theta\ \theta^2 I(\theta),\ Q_0 = \frac{1}{S}\int \mathrm{d}^2\theta\ \theta\theta^* I(\theta),$$

  with S is the total flux, and origin defined by $\int \mathrm{d}^2\theta\theta I(\theta) = 0$.
- then comes to 3rd-order,...

$$T_3 \equiv \frac{1}{S}\int \mathrm{d}^2\theta\ \theta^3 I(\theta);\ T_1 \equiv \frac{1}{S}\int \mathrm{d}^2\theta\ \theta^2\theta^* I(\theta).$$

Moments equation

- By expanding brightness moments of source using lensing equation and determinant, we get relation between source and image moments.

$$Q_2^s = Q_2 - 2gQ_0 + g^2 Q_2^* + A\mathcal{G} - (B\mathcal{G})^2$$

$$\mathcal{T}^s = \tau + C\mathcal{G}$$

A,B are column matrices, C is matric composed of moments, and $\mathcal{G}^T = (G_3^*, G_1^*, G_1, G_3)$, $\mathcal{T}^{s,T} = (T_3^{s*}, T_1^{s*}, T_1^s, T_3^s)$, $\tau = \tau(T_1, T_3, g)$.

Weak approximate

- In the weak lensing case ($g \ll 1$), one can remove some terms, and simplify the brightness moments relation

$$\begin{pmatrix} T_1^s \\ Q_2^s \\ T_3^s \end{pmatrix} = \begin{pmatrix} T_1 \\ Q_2 \\ T_3 \end{pmatrix} + \begin{pmatrix} \frac{12Q_0^2 - 9F_0}{4} & 0 & 0 \\ 0 & -2Q_0 & 0 \\ 0 & 0 & -\frac{3}{4}F_0 \end{pmatrix} \begin{pmatrix} G_1 \\ g \\ G_3 \end{pmatrix}$$

Source assumption

Same as in weak lensing, assuming that the scource ellipticity and orientation are random, $< Q_2^s >= 0, < T_1^s >= 0$ and $< T_3^s >= 0$

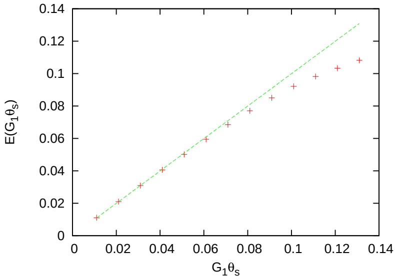
- G_1 and G_3 Estimators

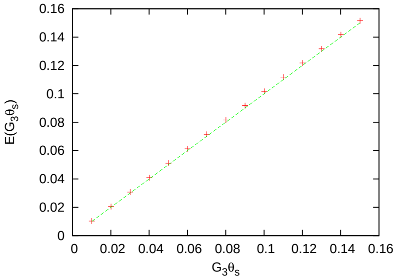
$$G_1 = \frac{4}{9F_0 - 12Q_0^2} T_1$$

$$G_3 = \frac{4}{3F_0} T_3$$

Simulation

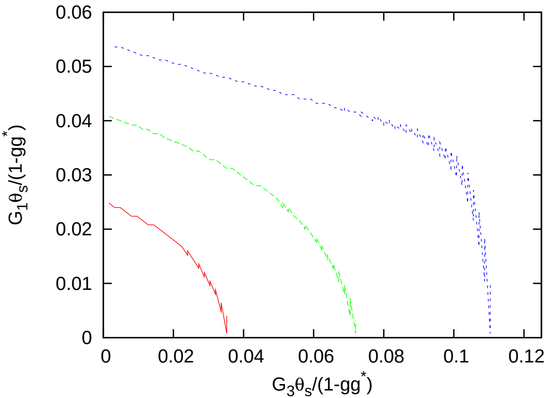
- Mean value of the estimators, over a source population with Gaussian ellipticity distribution, $g = 0.01$.
- The accuracy only depends on the product $G_i \theta_s$ $(i = 1, 3)$.





Error contours

- 5,10,15 percent errors on g for $g = 0.05$



Multiple solutions and Critical curves

- Non-linear lensing equation

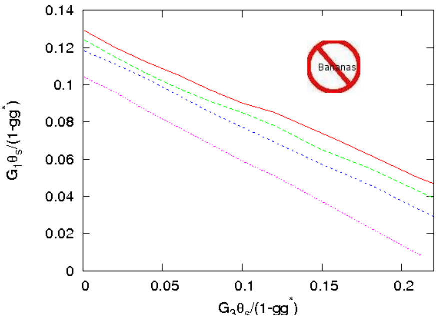
$$\hat{\beta} = \boldsymbol{\theta} - g\boldsymbol{\theta}^* - \Psi_1^*(G_1)\,\boldsymbol{\theta}^2 - 2\Psi_1\,\boldsymbol{\theta}\boldsymbol{\theta}^* - \Psi_3(G_1, G_3)\,(\boldsymbol{\theta}^*)^2$$

More than one solutions of the equation is allowed.

- Critical curves come in !!

Critical curves Limit

- Limits of valid flexion estimator



Summary

- Flexion is the gradient of shear and is simply related to the higher-order brightness moments.
- Only reduced shear and reduced flexion are measurable.
- Estimator bias is small, and depends on the dimensionless quantities $G_1\theta_s, G_3\theta_s$ and g.
- Flexion estimators have a limit by critical curves
- Future application
  Galaxy-galaxy lensing
  Substructure clusters